



Support Vector Machine (SVM) classifier

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1. How SVM works : theory

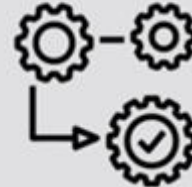
SVM is a supervised classification method that separates data using hyperplanes.



Supervised machine
learning algorithm

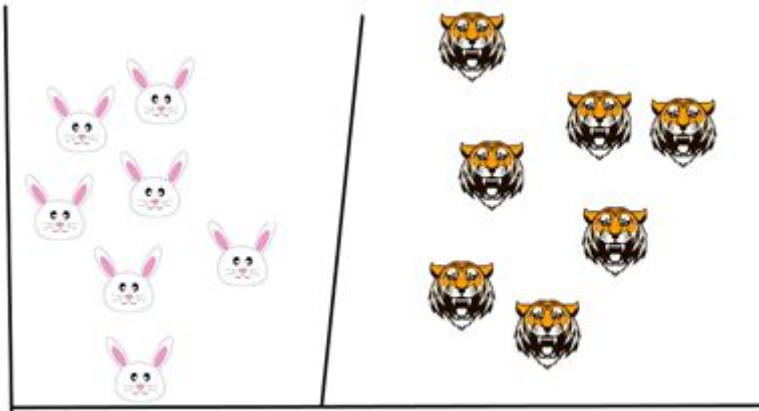


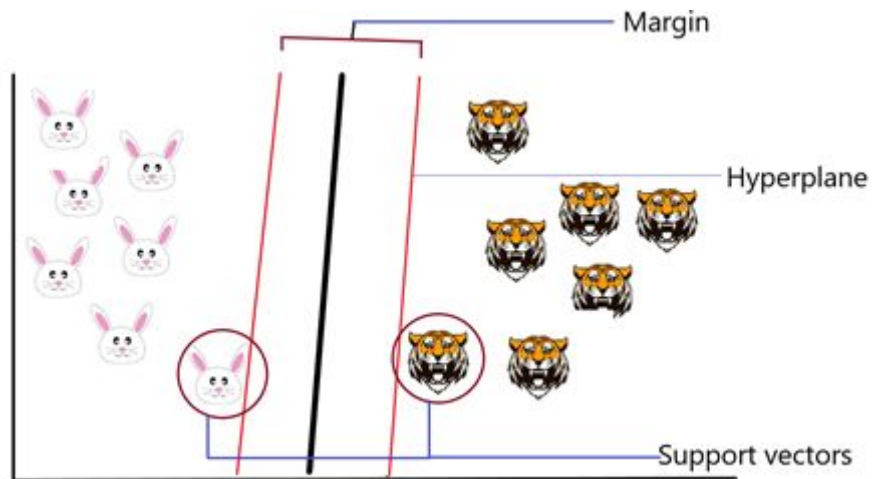
Classification &
Regression algorithm

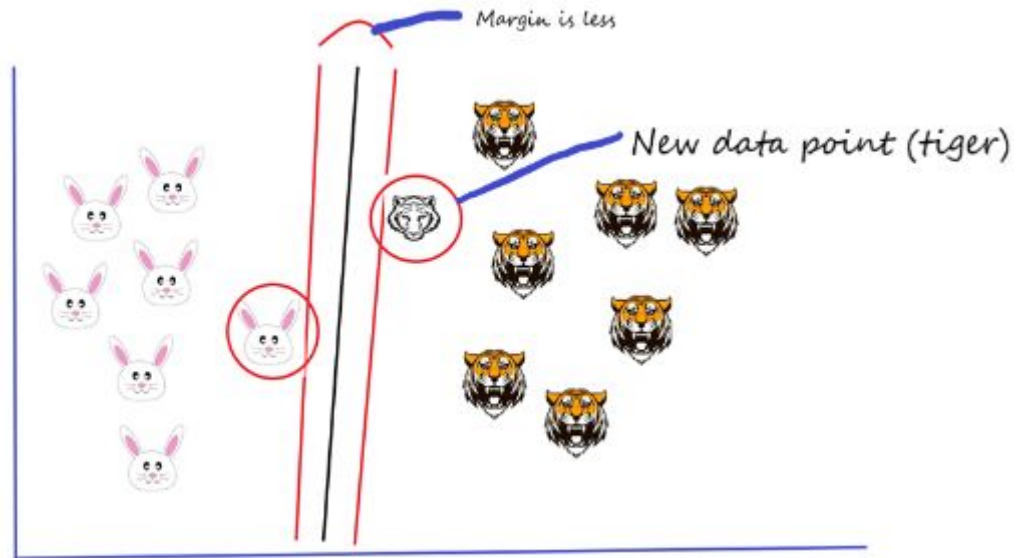


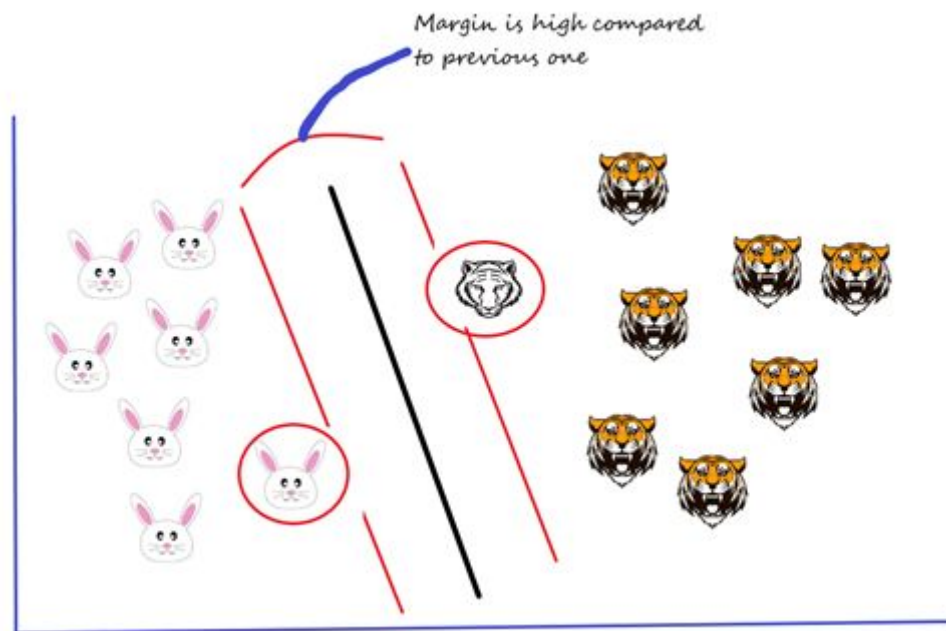
SVM kernel
functions









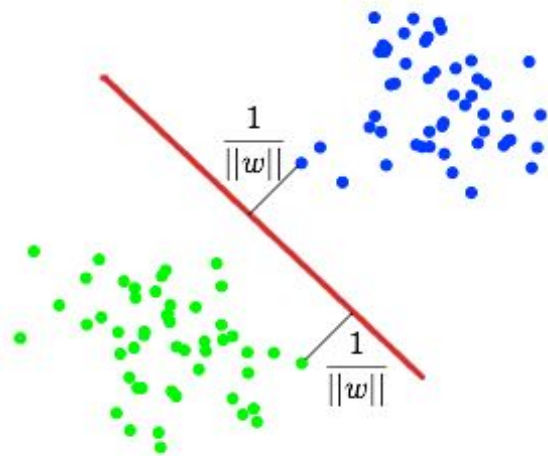


Finding the most suitable hyperplane

$$(x_1, y_1), \dots, (x_m, y_m), \quad x_i \in \mathbb{R}^n, \quad y_i \in \{-1, 1\}.$$

$$F(\mathbf{x}) = \text{sign}(\langle \mathbf{w}, \mathbf{x} \rangle + b)$$

$$\langle \mathbf{w}, \mathbf{x} \rangle + b = 0$$



$$\begin{cases} \arg \min_{\mathbf{w}, b} ||w||^2, \\ y_i(\langle \mathbf{w}, \mathbf{x}_i \rangle + b) \geq 1, \quad i = 1, \dots, m. \end{cases}$$

$$\begin{cases} \mathbf{L}(\mathbf{w}, \mathbf{b}; \lambda) = \frac{1}{2} \|\mathbf{w}\|^2 - \sum_{i=1}^n \lambda_i (c_i ((\mathbf{w} \cdot \mathbf{x}_i) - b) - 1) \rightarrow \min_{w, b} \max_{\lambda} \\ \lambda_i \geq 0, \quad 1 \leq i \leq n \end{cases}$$



how do I
separate this
data?

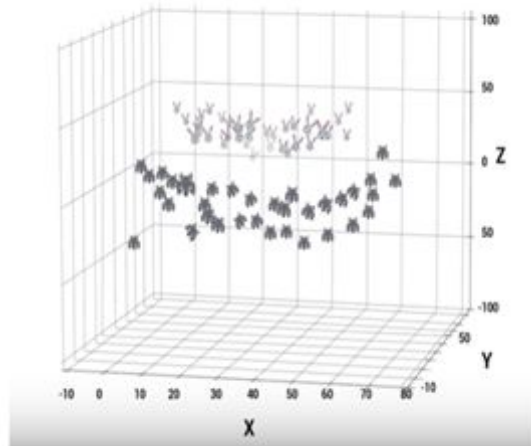


Introduction to non-linear SVMs




how do I
separate this
data?





Now I can easily
classify the data




$$\varphi: \mathbb{R}^n \rightarrow X. \quad F(\mathbf{x}) = \text{sign}(\langle \mathbf{w}, \varphi(\mathbf{x}) \rangle + b).$$

$$\mathbf{k}(\mathbf{x}, \mathbf{x}') = \langle \varphi(\mathbf{x}), \varphi(\mathbf{x}') \rangle$$

$$\mathbf{k}(\mathbf{x}, \mathbf{x}') = (\langle \mathbf{x}, \mathbf{x}' \rangle + \text{const})^d$$

$$\mathbf{k}(\mathbf{x}, \mathbf{x}') = e^{-\gamma \|\mathbf{x} - \mathbf{x}'\|^2}, \gamma > 0.$$

$$\mathbf{k}(\mathbf{x}, \mathbf{x}') = e^{-\frac{\|\mathbf{x} - \mathbf{x}'\|^2}{2\sigma^2}}.$$

$$\mathbf{k}(\mathbf{x}, \mathbf{x}') = \tanh(\kappa \langle \mathbf{x}, \mathbf{x}' \rangle + c), \kappa > 0, c < 0.$$



Tuning parameters of SVM

$$\begin{cases} \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^n \xi_i \rightarrow \min_{w, b, \xi_i} \\ c_i (\mathbf{w} \cdot \mathbf{x}_i - b) \geq 1 - \xi_i, \quad 1 \leq i \leq n \\ \xi_i \geq 0, \quad 1 \leq i \leq n \end{cases}$$

Cupcakes



versus

Muffins



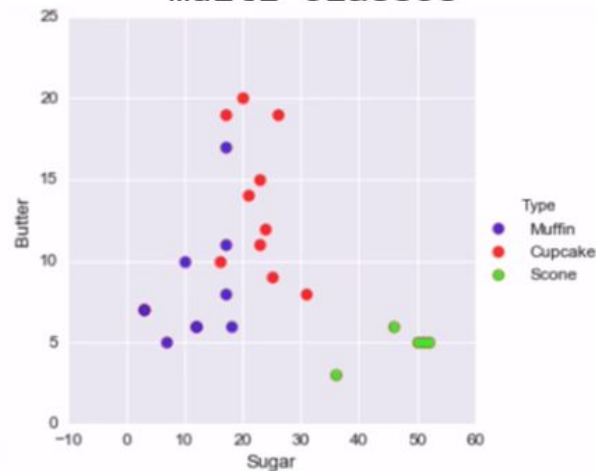
“a cupcake is just a muffin with frosting”

“a muffin is just a cupcake with random bits of stuff in it”

Datset

Type	Flour	Milk	Sugar	Butter	Egg	Baking Powder	Vanilla	Salt
Muffin	55	28	3	7	5	2	0	0
Muffin	47	24	12	6	9	1	0	0
Muffin	47	23	18	6	4	1	0	0
Muffin	50	25	12	6	5	2	1	0
Muffin	55	27	3	7	5	2	1	0
Muffin	54	27	7	5	5	2	0	0
Muffin	47	26	10	10	4	1	0	0
Muffin	50	17	17	8	6	1	0	0
Muffin	50	17	17	11	4	1	0	0
Cupcake	39	0	26	19	14	1	1	0
Cupcake	34	17	20	20	5	2	1	0
Cupcake	39	13	17	19	10	1	1	0
Cupcake	38	15	23	15	8	0	1	0
Cupcake	42	18	25	9	5	1	0	0
Cupcake	36	14	21	14	11	2	1	0
Cupcake	38	15	31	8	6	1	1	0
Cupcake	36	16	24	12	9	1	1	0
Cupcake	34	17	23	11	13	0	1	0

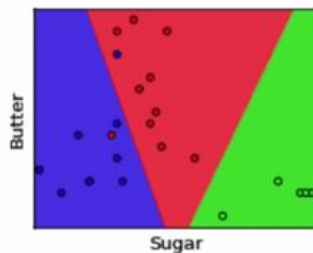
Multi Classes



Higher Dimensions

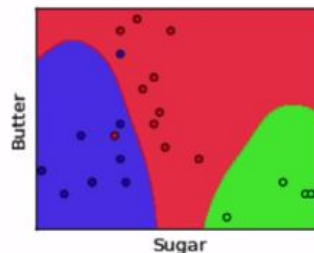
C Parameter

Kernel Trick: Comparison



Kernel: Linear
C: 1

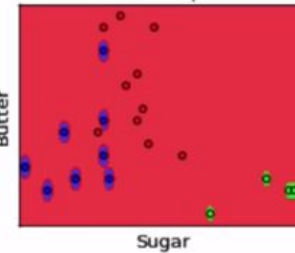
- Muffin
- Cupcake
- Scone



Kernel: RBF
C: 1

Gamma: 2^{-5}

Small Gamma:
Less complexity



Kernel: RBF
C: 1

Gamma: 2^1

Large Gamma:
More complexity

Higher Dimensions

C Parameter

Multiple Classes

Kernel Trick



Thank you.