Math 239 Definitions

Enumerations:

Bijection: A bijection between two sets S and T is a mapping $f: S -> T$ that is one to one and onto.

One to one: Every element in T is mapped to a distinct element in S

Onto: Every element in T is mapped to by some S

Composition: defined as an ordered arrangement of k non-negative integers that sum up to n. n has 2^{n-1} compositions

Former Power Series: $A(x) = \sum_i^{\infty} a_i x^i$

$B(x) = \sum_i^{\infty} b_i x^i$

Addition Rule: $(A + B)(x) = \sum_i (a_i b_i) x^i$

Multiplication Rule: $A(x)B(x) = \sum_i (\sum_j^i a_j b_{i-j}) x^i$

The sum of any two formal power series is a formal power series. The inverse of a formal power series exists if the constant term is non-zero. You can substitute fps in for fps if the constant term of the fps you're substituting is 0.

Generating Series: Given a set S where each $\sigma \,\mathcal{E}\, S$ is given a weight $w(\sigma)$, the generating series for S with respect to w is

$$\Phi_s(x) = \sum_{\sigma \mathcal{E} S} x^{w(\sigma)}$$

Sum Lemma: Let S = A U B, then $\Phi_s(x) = \Phi_A(x) + \Phi_b(x)$

Product Lemma: S = A x B for each $(a, b) \,\mathcal{E}\, S$ and

$w(A, B) = w(A) + w(B)$ then $\Phi_s(x) = \Phi_A(x)\Phi_b(x)$

* Lemma: Suppose A* is unambigious, then:

$$\Phi_{A*}(x) = \frac{1}{1 - \Phi_A(x)}$$

Binary Strings: Binary string is a string of symbols $a_1 a_2 \ldots a_n$ *where* $a_i \mathcal{E} \{0,1\}$ and n is the length

Unambiguous Expressions: Let A and B be sets of Binary Strings. The set AB is formed by concatenating A and B. $AB = \{ab: a \,\mathcal{E}\, A, b \,\mathcal{E}\, B\}$.

The concatenation set AB is unambiguous if for every $\sigma \,\mathcal{E}\, AB$, there is exactly one pair (a,b) with $a \,\mathcal{E}\, A, b \,\mathcal{E}\, B. Such\ that\ \sigma = ab.$

A*: The set of binary strings formed by concatenating any number of strings in A.

Blocks: a block is a binary string which is a non-empty substring that contains entirely of 0's and 1's

Block Decomposition: {0,1}* = {1}*({0}{0}*{1}{1}*)*{0}* where the RHS is unambiguous

0 Decomposition: {0,1}* = {1}*({0}{1}*)*

Recurrence Relation: equation that recursively defines a sequence

Homogenous: $a_n + c_1 a_{n-1} + \ldots c_d a_{n-d} = 0$

Non-homogenous: $a_n + c_1 a_{n-1} + \ldots c_d a_{n-d} = f(n)$

Characteristic polynomial: $g(x) = 1 + c_1 x^1 + \ldots + c_k x^k$

Graph Theory:

Vertex: $V(G)$

Edge: $E(G)$

Adjacent: Vertices x and y are adjacent in G if {x,y} $\mathcal{E}\ E(G)$

Degree: The number of neighbours of v in G, denoted by deg(v)

Isomorphism: An isomorphism between 2 graphs G and H is a map $f: v(G) -> V(H)$ such that

1.) f is a bijection and

2.) $xy\ \mathcal{E}\ E(G) iff\ f(x)f(y) \mathcal{E}\ E(H)$

Handshake Lemma: In every graph G, the number of vertices of odd degree is even.

Bipartite: We say a graph G is bipartite of V(G) = A U B where there's no intersection between A and B, and every edge in G has one vertex in A and one vertex in B

Complete Bipartite: $K_{p,q}$has V(G) = A U B where the intersection of A and B is empty and |A| = p |B| = q and $E(K_{p,q}) = \{ab: a\ \mathcal{E}\ A, b\ \mathcal{E}\ B\}$

n-cube: Vertex set $V(Q^n)$is the set of all binary strings of length n.

Walk: sequence of vertices and edges $v_0 e_1 v_1 e_2 v_2 \dots\ e_i = v_{i-1} v_i$. A walk from $v_0\ to\ v_n$ and n is the length of the walk.

Path: A walk in which all vertices are distinct.

Cycle: is a walk and a path from $v_0 \dots v_{n-1}\ and\ v_0 =\ v_n$

Connected: The graph G is said to be connected if for any two vertices x and y in G, there exists a path in G from X to Y

Component: A component of G is a maximal connected subgraph of G. H is a component of G means:

1.) H is a subgraph of G

2.) H is connected

3.) H is not combined in a larger subgraph H of G that is also connected

Cut induced by X: Let G be a graph and let $x \leq V(G)$. The cut induced by X is the set of edges $\{xy\ \mathcal{E}\ E(G): x\ \mathcal{E}\ X, y\ \mathcal{E}\ V(G) -\ x\}$

Bridge: Let G be a connected graph. Let e be an edge, e is a bridge of G if G-e is disconnected.

Tree: A connected graph with no cycles

Spanning Tree: A spanning tree of G is a subgraph T of G that is 1.) a tree, 2.) Spanning (ie. V(T) = V(G))

BFST: Spanning tree of G obtained by the following algorithm:

Input: A connected non-empty G

Output: A spanning tree T* of G

1.) Let r be an arbitrary vertex of G. Call r the root. Put r into V(T) and set pr(r)=0.

2.) If no vertex in T has a neighbour outside T then stop and set T: = T*, output T*

3.) Else let u be the vertex of T (with a neighbour outside T) that was added to T earliest. u is called the active vertex.

Choose a neighbour V that does not belong to V(T) and put v into V(T), put uv into E(T) and set pr(v) = u.

Exhausted Vertex: A vertex v is exhausted when is no longer used.

Active Vertex: vertex u that was added to a graph T.

Level: Let T be a BFST with root r in a connected graph G. The set of vertices of G that are of distance k from r in T is called the kth level of T.

Root: pr(r) = 0, and the edges are directed away from the root

Parent function pr(): Let T be a tree. Let x, y be vertexes in the tree.. Assume that we have a path xy, then the pr(y) is the preceding vertex, x.

Planar: A graph G is said to be planar if it has a drawing in the graph so that

1.) No 2 vertices coincide

2.) No 2 edges intersect except at their common endpoint

Planar Drawings: Drawings of planar graphs

Face: Let G be a graph, then the face is the region that is surrounded by a cycle.

Outer Face: The region that is outside the entire graph

Boundary: For a face f of a planar drawing G, the set of vertices and edges on the perimeter of f is the boundary of f.

Face Shake Lemma: Let G be a connected planar graph. Let F(G) be the set of faces of G then

$$\sum_{f \varepsilon F(G)} deg(f) = 2|E(G)|$$

Girth: The length of the shortest cycle in a graph G.

Subdivision: Let H be a graph. A subdivision of H is any graph obtained by replacing each edge of H by a path of length ≥ 1, such that all vertices on all paths are distinct

Colouring: A colouring of a graph is a function $f: V(G) -> \{1,2,3\ \dots\ n\}\ such\ that$ $f(v) \neq f(w)$ for every edge $vw \epsilon E(G)$

K-Colouring: if $f: V(G) -> \{1,2,3\ \dots\ k\}$ is a colouring we say G is k-colourable

Planar Dual: Let G be a planar graph drawn in the plane. The planar dual of G denoted G* is a planar graph drawn in the plane as follows:

-G* has a vertex face for each face f of G drawn inside f

-G* has an edge for each edge e of G joining the two faces incident with e, drawn to cross e

Multigraph: a multigraph is a graph which is permitted to have multiple edges, or edges that have the same end vertices.

Contraction of an edge: Let G be a graph and let e=xy be an edge of G. The graph G G/e formed by contracting e has vertex set

V(G)\{x,y}U{z} where z is a new vertex.

$E(G/e) = \{uv \in E(G): \{u,v\} \cap \{x,y\} = 0\}\ uz: u \notin \{x,y,z\}\ and\ ux \in E(G)\ or\ uy \in E(G)$

Platonic Graphs: A graph is platonic if it has a planar drawing in which all the vertices have the same degree d, and all the faces have the same degree d*.

Matching: a matching in graph G is a set of edges so that no vertex in G is incident to more than one edge in m.

A matching in G is called perfect if has a size |V(G)|/2 (every vertex of G is incident to exactly one edge in the matching)

Neighbourhood: For $s \subseteq V(G)$ the neighbourhood N(S) is

$$N(S) = \{Z \in V(G) | x, z \in E(G)\}\ for\ some\ x\ \in S$$

Maximum Matching: A maximum matching in G is a matching in G of largest possible size.

M-saturated: We say a vertex x is saturated by a matching m if x is incident to an edge of m, otherwise we say x is m-exposed.

M-alternating: Let m be a matching in G. An m alternating path in G is a path in which every shared edge is in m.

M-augmenting: An M-augmenting path is an m-alternating path that has length ≥ 1 which begins and ends with an m-exposed vertex and length must be odd.

Cover: Let G be a graph. A cover of G is a set of vertices C of G such that every edge of G is incident to a vertex in C.

Minimum Cover: Vertex cover of minimum size.