

# MATH 239 Final Practice Problems + Abbreviated Solutions

**Note:** These are only abbreviated solutions. You are expected to show all work on the exam.

1.  $2^7 \binom{13}{6}$
2. On the RHS, for  $|A \cap B| = i$ , count the number of ways to choose  $A \cap B, A \setminus B, B \setminus A$  in order.
3. (a)  $A = \{4, 7, 10, \dots\}$ ,  $S = \sum_{k \geq 0} A^k$ ,  $\Phi_S(x) = \frac{1}{1 - \frac{x^4}{1-x^3}}$   
(b)  $a_n - a_{n-3} - a_{n-4} = 0$ ,  $a_0 = 1, a_1 = 0, a_2 = 0, a_3 = 0, a_4 = 1$
4.  $\frac{x}{1-3x^2-x^3+x^4}$
5.  $\{0\}^* (\{1\}\{0\}\{0\}^* \cup \{11\}\{1\}^*\{0, 00, 000\})^* \{1\}^*$
6.  $a_n = 2 \cdot 3^n$
7. Induction.
8. (a)  $a = b$  and  $a, b \geq 2$   
(b)  $a, b$  both even  
(c)  $\min\{a, b\} \leq 2$
9. (a) Defn  
(b) 40
10.  $\frac{5n-10}{3}$  edges,  $\frac{2n-4}{3}$  faces.
11. Nonplanar, planar.
12. 3-colourable, not 3-colourable.
13. Augmenting path  $7, h, 6, g, 4, f$ , new matching  $\{1b, 2a, 3e, 4f, 5d, 6g, 7h, 8i\}$ , cover  $\{1, 2, d, e, f, g, h, i\}$ .
14. False, false, false.