CS240 Midterm Review

Graham Cooper

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Topics:

- First principals

First Principals

1)

$$10n^2 + 11n + 12 \in O(n^2)$$

When n $\geq$ 1

$$\begin{aligned} 10n^2 + 11n + 12 &\leq 10n^2 + 11n^2 + 12n^2 \\ &= 33n^2 \end{aligned}$$

$n_0 = 1, c = 33$

2)

$$n + (logn)^2 \in O(nlogn)$$

when $n \geq 2$

$$\begin{aligned} n + (logn)^2 &\leq n + n \\ &\leq 2nlogn \end{aligned}$$

$c = 2, n_0 = 2$

3)

$$10n^2 - 11n - 12 \in \Omega(n^2)$$

When $n^2 \geq 11n = n \geq 11$

$$\begin{aligned} 10n^2 - 11n - 12 &\geq 10n^2 - n^2 - 12 \\ &= 9^2 - 12 \end{aligned}$$

When $n^2 \geq 12 = n^2 \geq \sqrt{12}$

$$\geq 9n^2 - n^2$$

$$= 8n^2$$

c $= 8$, $n_0 = 11$

4)

$$5n + 15logn - 10\sqrt{n} \in \Omega(logn)$$

when $n \geq 2$

$$5n + 15logn - 10\sqrt{n} \geq 5n - 10\sqrt{n}$$

when $10\sqrt{n} \leq n = 10 \leq \sqrt{n} = 1oo \leq n$

$$\geq 5n - n$$

$$= 4n$$

5)

$$3n^2 10n + 15logn + 2 \in \Theta(n^2)$$

$$3n^2 - 10n + 15logn + 2 \leq 3n^2 + 15logn + 2$$

if n ≥ 1

$$\leq 3n^2 + 15n^2 + 2$$

$$\leq 3n^2 + 15n^2 + 2n^2$$

$$= 20n^2$$

$c_2 = 20$

$$3n^2 - 10n + 15logn + 2 \geq 3n^2 - 10n$$

when $10n \leq n^2 = 10 \leq n$

$$\geq 3n^2 - n^2$$

$$= 2n^2$$

$$2n^2 \leq 3n^2 - 10n + 15logn + 2 \leq 20n^2$$

when $n \geq 10$ $c_1 = 2, c_2 = 20, n_0 = 10$

6)

$n \in o(n^2)$

$$n \leq cn^2$$
$$1 \leq cn$$
$$\frac{1}{2} \leq n$$

$n_0 = \frac{1}{c}$
$c = 0.1$

7)

$$cos(n) \in o(n)$$
$$cos(n) \leq cn$$
$$cos(n) \leq 1 \leq cn$$
$$\frac{1}{c} \leq n$$

$n_0 = \frac{1}{c}$

8)

$n^n \in w(n^20)$

$$n^n \geq cn^20$$

$$\frac{n^n}{n^2 0} \geq c$$

$$n^{n-20} \geq c$$

the below occurs when n $\geq$ 21

$$n^{n-20} \geq n \geq c$$

The inequality holds when $n \geq max(c, 21)$

Loop Analysis

1)

```
foo(n,m,k)
1. for (i = 1 to n)
2. -- for(j = 1 to i)
3. -- -- for(l = 1 to m)
4. -- -- -- print("hello")
5. for(i = 1 to k)
6. -- for(j = 1 to 600)
7. -- -- print("WORld")
```

line 4: $\Theta(1)$
line 7: $\Theta(1)$
lines 6-7 $\sum_{i=1}^{600} 1 = 600 \in \Theta(1)$
lines 5-7 $\sum_{n=1}^{k} \Theta(1) = \Theta(k)$
lines 3-4 $\sum_{l=1}^{m} 1 = m$
lines 2-4 $\sum_{j=1}^{l=1} m = i \cdot m$
lines 1-4 $\sum_{i=1}^{n} i \cdot m$
$= m \sum_{i=1}^{n} i = \frac{m(n(n+1))}{2}$

Total $= \frac{m(n(n+1))}{2} + 600k$
$\in \Theta(mn^2 + k)$

2)

```
foo2(n,m)
1. while (n > m)
2. -- n = n/2
3. -- for (i = 1 to m)
4. -- -- print("x")
```

After t iterations
m' = m
n' = $\frac{n}{2^t}$
$\frac{n}{2^2} \leq m$

$t \geq log(\frac{n}{m}) = log(n) - log(m)$

Aside:
$\frac{n}{2^{t-1}} \geq m \geq \frac{n}{2^t}$
$\frac{2n}{m} > 2^t \geq \frac{n}{m}$
$log(\frac{2n}{m}) > t \geq log(\frac{n}{m})$
$= 1 + log(\frac{n}{m})$

$$\sum_{j=1}^{log(\frac{n}{m})} \sum_{i=1}^{m} 1$$

$$= \sum_{j=1}^{log(\frac{n}{m})} m$$

$$= mlog(\frac{n}{m})$$

3)

```
foo3(n)
1. while(n > 1)
2. -- foo3(n-1)
3. -- n--
4. for(i = 1 to n)
5. -- print("y")
```

After t iterations

$$n' = n - t$$
$$n - 2 \le 1 < n - t + 1$$
$$t = n$$

$$T(n) = \sum_{i=1}^{n} (T(i) + 1) + \sum_{i=1}^{n} 1$$
$$= T(1) + T(2) + ... + T(n-1) + n - 1 + n$$
$$T(n-1) = T(1) + T(2) + ... + T(n-2) + n - 2 + n - 1$$
$$T(n) - T(n-1) = T(n-1) + (n - 1 + n) - (n - 2 + n - 1)$$

$$T(n) = 2T(n-1) + \Theta(1)$$
$$T(n) = 2(2(T(n-2) + \Theta(1)) + \Theta(1)$$
$$2^3 T(n-3) + 6\Theta(1)$$
$$2^k T(n-k) + \Theta(1)$$
$$= 2^k T(1) + \sum_{i=0}^{k-1} 2^i$$

k = n-1 in this case

$$= 2^{n-1} \cdot 1 + \sum_{i=0}^{n-2} 2^i$$
$$= \Theta(2^n)$$

4)

```
foo4(n)
1. if (n = 1) return
2. for (i = 1 to 5)
3. -- foo4(n/3)
4. print("HI")
```

$$T(n) = 5T(\frac{n}{3}) + 1$$
$$T(\frac{n}{3}) = 5(5T(\frac{n}{9}) + 1) + 1$$
$$T(n) = 5(5(5T(\frac{n}{27}) + 1) + 1) + 1$$
$$5^{log_3 n} + 1 + 5 + 25 + 5^{log_3 n}$$
$$T(n) = S^{log_3 n} + 1 + \sum_{i=1}^{log_2 n} 5^i$$
$$\in \Theta(n^{log_3 5})$$

True false

1)

$$f(n) \in \Theta(g(n)) \implies f(n) \in o(g(n)) OR f(n) \in w(g(n))$$

counter example:
f(n) = max(1,nsin(n))
g(n) = max(1, ncos(n))

2)

The average runtime of $f(n) \in \Theta(n)$,
- is the worst case runtime O(n)
- is the worst case runtime $\Omega(n)$
- is the best case O(n)

1 is false, 2 is true, 3 is true