# Math239 Lecture 19

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Topics:

1. Cycles
2. Connectedness

# Cycles

## Hamilton Cycle

**Definition:** A hamilton cycle is a cycle that contains every vertex of the graph

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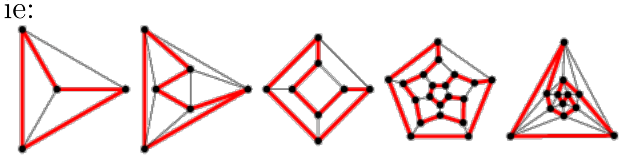
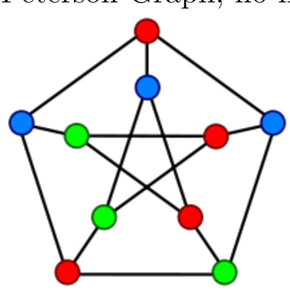


Peterson Graph, no hamilton cycle:



Traveling salesman problem: Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?

### Finding a Ham cycle of the shortest length

Theorem: For $n \geq 2$ the n-cube has a Ham cycle

Find a hamilton cycle for the smaller n-cube then link the two new sections together.

Proof: By induction on n.

For n = 2 it is obviously a ham cycle. as we are just going in a square around the edges. Assume (n-1)-cube has a Ham cycle. The n-cube is built from 2 copies of the (n-1) cube. Take the same Ham-cycle of the (n-1)-cube for both copies. Suppose st is an edge of the Ham cycle for the (n-1)-cube. Then Os, Ot and ls, lt are edges in the n-cube. Remove these two edges and add 0s, ls and ot, lt to get a Ham cycle in the n-cube.

## Connectedness

**Definition:** A graph G is connected if there is a u,v-path for every pair of vertices u,v ∈ V(G)

Theorem: If there exists a vertex u ∈ V(G) such that a u,v-path exist for all v ∈ V(G) then G is connected.

Proof:

Let x,y be any two vertices in G. By assumption, there exists an x,u-path and a u,y-path. By tansitivity, there is an x.y-path so G is connected
Theorem:: The n-cube is connected

Proof: Let $v_0$ be the string of n 0's and let x be any string of length n. Suppose x has k 1's, located at positions $i_1, i_2...i_k$. We produce $v_1, v_2, ...v_k$ by letting $v_j$ be the string with exactly j 1's, at positions $i_1, i_2, ...i_j$. notce that for $j \geq 0$ $v_j$ and $v_{j+1}$ differe in one bit at position $i_{j+1}$ so $v_j v_{j+1}$ is an edge. Hence, $v_0, v_1, v_2...v_k = x$ is a $v_0, x$-path. Therefore the n-cube is connected

## Components and Cuts

<u>**Definition**</u>: A subgraph H of G has vertex set V(H) $\subseteq$ V(G) and E(H) $\subseteq$ E(G) where each edge in E(H) joins two vertices in V(H)

Example: