# Math 239 Lecture 13

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June 3rd, 2015

## Restrictions on Substrings

Block Decomposition: $\{1\}^*(\{0\})\{0\}^*\{1\}\{1\}^*)^*\{0\}^*$

Example: Let S be the set of all strings where every block has length at least 2.

Start with block decomp, remove places were we find blocks of length 1.

$\{0\}\{0\}^* = \{0, 00, 000, 0000, ...\} \to \{00\}\{0\}^*$
$\{1\}\{1\}^* \to \{11\}\{1\}^*$
$\{1\}^* \to \{\epsilon, \{11\}\{1\}^*\}$ ... $\{1\}^* = \{\epsilon, 1, 11, 111...\}$
$\{0\}^* \to \{\epsilon, \{00\}\{0\}^*\}$

So S $= \{\epsilon, \{11\}\{1\}^*\}(\{00\}\{0\}^*\{11\}\{1\}^*)^*\{\epsilon, \{00\}\{0\}^*\}$

$$\Phi_S(x) = (1 + \frac{x^2}{1-x})(\frac{1}{1 - (\frac{x^2}{1-x}\frac{x^2}{1-x})})(1 + \frac{x^2}{1-x}) = \frac{1 - x + x^2}{1 - x - x^2}$$

Example: Let S be the set of all strings where any even block of 0's cannot be followed by an odd block of 1's

Startint with a block decomposition, the only place with 0's followed by 1's is $\{0\}\{0\}^*\{1\}\{1\}^*$

Break into 2 cases
$\to$ even 0's $\{00\}\{00\}^*$
$\to$ odd 0's $\{0\}\{00\}^*\{1\}\{1\}^*$

$S_0S = \{1\}^*(\{00\}\{00\}^*\{11\}\{11\}^* \cup \{0\}\{00\}^*\{1\}\{1\}^*)^*\{0\}^*$

$\Phi_S(x) = \frac{1}{1-x}\frac{1}{1-(\frac{x^2}{1-x^2}\frac{x^2}{1-x^2}+\frac{x}{1-x^2}\frac{x}{1-x})}\frac{1}{1-x} = \frac{1+2x+x^2}{1-3x^2-x^3}$

# String Recursion

Example: Let S be the set of all strings.
S = {0,1}S $\cup\{\epsilon\}$

$\Phi_S(x) = (x + x)\Phi_S(x) + 1 \implies \Phi_S(x) = frac11 - 2x$
$[x^n]\Phi_S(x) = 2^n$

Example: Let S be all the strings with no 000 (Three consecutive zeros)

| $\epsilon$, 0, 00 1 | $\in$ S | |
|---|---|---|

S = {1, 01, 001}S
This does not apply to strings with no 1's
$\Phi_S(x) = (x + x^2 + x^3)\Phi_S(x) + 1 + x + x^2$
$\Phi_S(x) = \frac{1+x+x^2}{1-x-x^2-x^3}$

Example: Let S be the set of all strings with no 1010 as a substring.

Let T be the strings with exactly 1 copy of 1010 at the right end.

1. $\{\epsilon\} \cup S\{0, 1\} = S \cup T$
   $(\subseteq)\epsilon \in S$ For any $\sigma \in$ S, $\sigma\{0, 1\}$ either has no 1010 (in which case it is in S), or it has exactly one copy of 1010 at the very right end (so it is in T).
   $(\subseteq)(otherway)$ A string in S is either $\epsilon$ of by cutting the last bit it is still in S. For a string in T, cutting the last bit destroys the only copy of 1010 in the string so the remaining string is in S.