

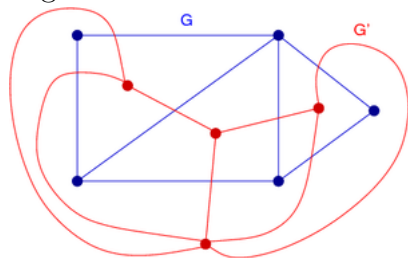
# TITLE

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## Dual Graphs

**Definition:** Let  $G$  be a planar graph with an embedding. The dual  $G^*$  of the embedding has one vertex  $V_f$  corresponding to each face  $f$  of  $G$ , and for each edge in  $G$  whose two sides are  $f_1, f_2$ ,  $G^*$  has a corresponding edge  $V_{f_1}, V_{f_2}$ .



It is possible for the dual graph to have multiple edges and loops.

### Properties of Duality:

1. IF  $G$  is planar then  $G^*$  is also planar ( From inside a face, draw lines to the midpoints of the edges in the boundary)
2.  $(G^*)^* = G$
3. # of faces in  $G = \#$  of vertices in  $G^*$   
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# of edges in  $G = \#$  of edges in  $G^*$
4. degree of a vertex in  $G = \deg$  of the corresponding face in  $G^*$   
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5. The dual of a platonic graph is platonic

**Theorem:** The dual of an Eulerian planar graph is bipartite.

**Proof:** IF a graph is Eulerian, every vertex has even degree. So every face has even degree. By A9 last Question, the dual is bipartite.

# Matchings

**Definition:** A matching of a graph is a set of edges where no two edges share a common vertex (or they form a subgraph of max degree 1)

**General Question:** What is the maximum size of a matching in a graph?

**Definition:** A vertex incident with an edge in a matching is called saturated. It is unsaturated otherwise.

A matching that saturates all vertices is a perfect matching.

If a graph has a perfect matching, then it has even # of vertices.