Math 239 Lecture 27

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Kuratowski's Theorem

Definition: An Edge SUbdivision of a graph is obtained by replacing each edge with a path of length at least 1 (or introduce vertices of deg(2) to the edges).
Theorem: A graph is planar if and only if any edge subdivision of the graph is planar.

Kuratowski's Theorem: A graph is planar if and only if it does not have any edge subdivision of k_r or $k_{3,3}$ as a subgraph

To prove that a graph is non-planar, find an edge subdivision of k_5 or $k_{3,3}$ in the graph. Note: Other than the 5/6 main vertices in k_5 / $k_{3,3}$ no edges are repeated and any vertex is used in at most one path.
Usually there is a $k_{3,3}$ subdivision.

Colouring

Definition: A k-colouring of a graph is an assignment of a colour to each vertex using at most k colors so that adjacent vertices receive different colours. A graph that has a k-colouring is k-colourable.

If a graph is k-colourablem then it is also (k+1) colourable.

General Question: What is the minimum number of colours needed to colour a graph?

Theorem: K_n is n-colourable, bit not (n-1)-colourable
Theorem: A graph is 2-colouring if and only if it is bipartite