

Choice:

1. Shift symbol from input to stack
2. reduce top of stack by grammar rule

Theorem (Donald Knuth, 1965)

- create tex, became latex - currently writing the art of computer programming

Theorem:

The set $\{wa \mid \exists x, S' \implies *wax\}$

w is the stack

a is the next character

is a regular language.

If it is a regular language, then it can be described by a DFA.

- use the DFA to make shift/reduce decisions

LR Parsing

- left-to-right scan
- rightmost derivations

Example: $S' \rightarrow \vdash E \dashv$

$E \rightarrow E + T$

$E \rightarrow T$

$T \rightarrow \text{id}$

See pretty picture 1 in notes

LR(0) machine (simplest)

Definition: An item is a production with a dot (\cdot) somewhere on the right hand side.

(indicates partially completed rule)

- begin with a state with the starting state production rule and the dot at the beginning of input
- Label an arc with the symbol that follows the dot ; advance the dot in the next state
- If the dot precedes a non-terminal A, add all productions with A on the LHS to the state, dot in the leftmost position.
- Repeat until we get all of the transitions we can

Using the machine

- Start in the start state with empty stack
- Shifting
 - shift char from input to stack
 - follow transition for that char to next state
 - if no transition, error, or reduce
- Reducing
 - "Reduce states" have only one item and the dot is rightmost
 - called a complete item
 - reduce by the rule in the state
 - reduce: pop RHS off the stack, backtrack size(RHS) states in the DFA, push LHS, follow shift transition for the LHS
- Backtracking the DFA
 - must remember the DFA states
 - Push the DFA states onto the stack as well.

Stack	Read	unread	Action
1	ϵ	$\vdash id+id+id \dashv$	S2 (shift and go to 2)
1 \vdash 2	\vdash	$id+id+id \dashv$	S6 (Shift and go to 6)
1 \vdash 2 id 6	$\vdash id$	$+id+id \dashv$	RT \rightarrow id (Pop 1 symbol and 1 state) Now in state 2, Push T g R, E \rightarrow T pop 1 sym, pop 1 state, push E goto 3
1 \vdash 2 T 5	$\vdash id$	$+id+id \dashv$	
1 \vdash 2 E 3	$\vdash id$	$+id+id \dashv$	S7
1 \vdash 2 E 3 + 7	$\vdash id +$	$id+id \dashv$	S6
1 \vdash 2 E 3 + 7 + 6	$\vdash id + id$	$+id \dashv$	R, T \rightarrow id, goto 8
1 \vdash 2 E 3 + 7 T 8	$\vdash id+id$	$+id \dashv$	R, E \rightarrow E + T (Pop 3 sym, 3 states, goto 2)
1 \vdash 2 E 3	$\vdash id + id$	$+id \dashv$	S7
1 \vdash 2 E 3 + 7	$\vdash id + id +$	$id \dashv$	S6
1 \vdash 2 E 3 + 7 id 6	$\vdash id+id+id$	\dashv	R, T \rightarrow id, goto 8
1 \vdash 2 E 3 + 7 T 8	$\vdash id+id+id$	\dashv	R, E \rightarrow E + T goto3
1 \vdash 2 E 3	$\vdash id+id+id$	\dashv	S4
1 \vdash 2 E 3 \dashv 4	$\vdash id+id+id \dashv$	ϵ	Accept

What can go wrong?

What if the state looks like this:

$A \rightarrow \alpha \cdot c \beta$

$B \rightarrow \gamma \cdot$

Shift c or reduce $B \rightarrow \gamma$

This is a shift-reduce conflict!

$A \rightarrow \alpha \cdot$

$B \rightarrow \beta \cdot$

reduce-reduce conflict

Whenever a complete item $A \rightarrow \alpha \cdot$ is not alone in a state there is a conflict and the grammar is not LR(0)

Example: Right-associative

$S' \rightarrow \vdash E \dashv$

$E \rightarrow T + E$

$E \rightarrow T$

$T \rightarrow id$

See pretty picture 2 in notes

Example:

$\epsilon 1 \text{ shift } - > \vdash 2 \text{ shift } - > \vdash id \ 6 \text{ reduce } - > \vdash T \ 5$

Should we reduce $E \rightarrow T$?

Depends: If input is $\vdash id \dashv$ then YES, otherwise, no!

Add a lookahead to fix the conflict

For each $A \rightarrow \alpha \cdot$, attach $\text{Follow}(A)$

$\text{Follow}(E) = \{\dashv\}$

$\text{Follow}(A) = \{+, \dashv\}$

Interpretation:

A reduce action

$A \rightarrow \alpha \cdot X$ ($X = \text{follow}(A)$)

only applies if the next char is in X.

So $E \rightarrow T \cdot$ applies when the next char is \dashv

$E \rightarrow T \cdot + E$ applies when next char is $+$

Conflict resolved!

Result is called an SLR(1) parser. = Simple LR with 1 char lookahead

SLR(1) resolves many, but not all conflicts

LR(1) - more powerful than SLR(1)

- Produces many more states

Building a Parse Tree

Top-Down

$\dashv S$		$S \rightarrow A y B$
$\dashv B y A$		keep S make the new nodes its children

Bottom-up

$\dashv ab$		Reduce $A \rightarrow a b$
$\dashv A$		Use A as parent, make a b as children