Math 239 - Lec 7

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Power Series

F(x) is a power series if $[x^0]g(x) = 0$
If $[x^0]g(x) \neq 0$, f(g(x)) may or may not be a power series.

$$f(x) = 1 + x$$
$$g(x) = 1 + x$$
$$f(g(x)) = f(1 + x) = 1 + 1 + x = 2 + x$$

Let $A(x) = \sum_{n \geq 0} a_n x^n$ where $A(x) = \frac{1+2x}{1-5x+6x^2}$
Multiply both sides by $1 - 5x + 6^2$.

$$(1 - 5x + 6x^2)A(x) = 1 + 2x$$
$$LHS = (1 - 5x + 6x^2)(a_0 + a_1 x + a_2 x^2 + ...)$$
$$= a_= + a_1 x + a_2 x^2 + a_3 x^3 - 5a_0 x - 5a_1 x^2 - 5a_2 x^3 - ...$$
$$= a_0 + (a_1 - 5a_0)x + \sum_{n \geq 2}(a_n - 5a_{n-1} + 6a_{n-2})x^n$$

This equals to 1+2x by comparing coeff:

$$a_0 = 1$$
$$a_1 - sa_0 = 2 \implies a_1 = 2 + 5 = 7$$
$$a_n - 5a_{n-1} + 6a_{n-2} = 0$$

(the last is for $n \geq 2$)

$$a_n = 5a_{n-1} - 6a_{n-2}$$
$$a_2 = 5a_1 - 6a_0 = 35 - 6 = 29$$
$$a_3 = 5a_2 - 6a_1 = 103$$
$$a_4 = 5a_3 - 6a_2 = 341$$

$$A(x) = 1 + 7x + 29x^2 + 103x^3 + 341x^4 ...$$

In general if $A(x) - \frac{P(x)}{Q(x)}$ where $Q(x) = 1 + q_1 x + q_2 x^2 + ... + q_k x^k$ then $a_n + q_1 a_{n-1} + q_2 a_{n-2} + ... + q_k a_{n-k} = 0$ for $n \geq max(deg(P(X)) + 1, k)$

Sum and Product Lemmas

Generating Series: Set S, weight function w.

$$\Phi_s(x) = \sum_{\sigma \in S} x^{w(\sigma)} = \sum_{n \geq 0} a_n x^n$$

a_n = number of things in S of weight n.

Sum Lemma: Let $S = A \cup B$ where $A \cap B = \emptyset$ (disjoint union)
Let w be a weight function on S. Then:
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