Tutorial 5 CS240

Graham Cooper

June 3rd, 2015

Topics:

- lower bounds
- counting sort / radix sort
- - applications

Q1)

- Supposed we have n blue jugs and red jugs
- for each blue jug there is a corresponding red jug with the same capacity
- the capacities are unique among jugs of the same colour
- FIND: a set of pairs such that every blue jub is paired with a red jug having the same capacity
- Only allow comparisons between a red and blue jug
- GOAL: Show $\Omega(nlogn)$ comparisons are needed.

B:	5	3	8
R:	3	8	5

$B_1 \rightarrow R_3$
$B_2 \rightarrow R_1$
$B_3 \rightarrow R_2$

Proof: Label blue jugs with B = {1,2, ... n} and red jugs R = {1,2, ... n}
$\rightarrow$ A solution has the form:
A = {(i, $\pi(i)), i \in B, \pi$ is a permutation of R }

Decision making: for i, j in B and R respectively, if $i < j$ go to left path
if $i = k$ go to middle path
if $i > j$ go to the right path

$\rightarrow$ Suppose that our tree has height h

- at most 3^n leaves
- at least n! leaves

$$3^n \geq n! \geq (\frac{n}{e})^n \implies h \geq nlog_3(\frac{n}{e}) \in \Omega(nlogn)$$

$(\frac{n}{e})^n$ comes from Stirling's approximation.

Q2) - match psuedocode from slides

Use countsort to sort:
B = (6,1,2,0,10,6,6,2,9,1,6,7,0)
Range $[0 - 10]$

inc(C[B[i]])

C = (0,0,0,0,0,0,0,0,0,0,0)

C = (0,0,0,0,0,0,1,0,0,0,0)
C = (0,1,0,0,0,0,1,0,0,0,0)
C = (0,1,1,0,0,0,1,0,0,0,0)
... Increment at the corresponding index
Eventually becomes:
C = (2,2,2,0,0,0,4,1,0,1,1)

I[0] = 0 : I[k] = I[k-1] + I[k-1]

I = (0,2,4,6,6,6,10,11,11,11)

A is the sorted array, same amount of elements as B

A[I[B[i]]] = B[i]
Increase(I[B[i]])

A = (0,0,0,0,0,0,0,0,0,0,0)
A = (0,0,1,0,2,0,6,0,0,0,10)
.. Repeat procedure A = (0,0,1,1,2,2,6,6,6,6,7,9,10)

Q3)

Use Radix Sort to Sort (LSD)
A = (2751, 68, 215, 155,214,313,135,38,351,51)
Sort individual digits using any stable sort.

1st: (2751,351,51,313,214,215,155,135,68,38)
2nd: (313,214,215,215,135,38,2751,351,51,155,68)
3rd: (051, 068, 135,155,214,215,313,351,2751)
4th: (051, 068, 135,155,214,215,313,351,2751)
Things stay the same in the 4th iteration, btu it is not always this way.

1 Q4)

Unsorted array H of n non-negative elements. All of the elements are smaller than n^3, sort A in $\Theta(n)$

$\Theta(m(n+b))$, b is the base, m is the number of digits. We need to pick a base such m(n + b) = $\Theta(n)$

We are going to pick base n numbers. Therefore we only need 3 digits to represent them $log_n(n^3)$

$\Theta(m(n+b)) \implies \Theta(3 \times 2n) \in \Theta(n)$