Math 239 Lecture 1: Introduction

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MTF: 11:30am-12:20pm
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Part 1 Enumeration

We will convert problems into sets

[n] = {1, 2, 3, ... n}

$\mathbb{N} = \{1, 2, 3...\}$ (no zero)

Cartesian Product:

If A, B are sets, then the cartesian product of A and B is:

$A \times B = \{(a, b) | a \in A, b \in B\}$

Example: A = {1, 2} B = {2, 4, 6}

$A \times B = \{(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6)\}$
- Order inside the pairs matter
- Order of the pairs in the set does not matter

If A and B are finite sets, then $|A \times B| = |A| \cdot |B|$

$(A \times B) \times C \neq A \times (B \times C)$
$((a, b), c) \neq (a, (b, c))$

$|A \times B \times C| = |A| \cdot |B| \cdot |C|$

$A^n = \{(a_1, a_2, ...a_n)|a_i \in A\}|A^n| = |A|^n$

Example: The results of throwing 2 different 6-sided dice can be enumerated by $[6] \times [6] or [6]^2$
$= \{(a, b)|a, b \in [6]\}$

Disjoint Unions:

Let $S = S_1 \cup S_2$ and $S_1 \cap S_2 = \phi$ Then $|S| = |S_1| + |S_2|$

$S = S_1 \cup ... \cup S_k$ and $S_i \cap S_j = \phi$ where $i \neg j$

Example: Let E be elements of $[6] \times [6]$ whose sum is even.

Partition E into E_1 and E_2 where

$E_1 = \{(a, b) \in [6] \times [6]$ | a, b are even $\}$
$E_2 = \{(a, b) \in [6] \times [6]$ | a, b are odd $\}$

$E_1 = 2, 4, 6 \times 2, 4, 6$ $|E_1| = 3 \cdot 3 = 9$
$E_2 = 1, 3, 5 \times 1, 3, 5$ $|E_2| = 3 \cdot 3 = 9$

So $|E| = |E_1| + |E_2| = 18$ $(E_1 \cap E_2) = \phi$

Review Basic Counting

Permutations

How many ways can we arrange elements of [n] in a line?

There are n choices for the first spot, n-1 for the second... down to 1 choice for the last spot so there are n! different ways.

Combinations

How many subsets of [n] have size k?

First pick k elements in order. There are n choices, then n-1 choices then ... then n - k + 1 choices.

$= \frac{n!}{(n-k)!}$

Each subset of size k is counted k! times in order. So the number of subsets is:

$\frac{n!}{k! \cdot (n-k)!} = \binom{n}{k}$ "N choose k"

Binomial Theorem

$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} \cdot x^k$

We will touch on this later in the course