Math 239 Lecture 16

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Graph Theory

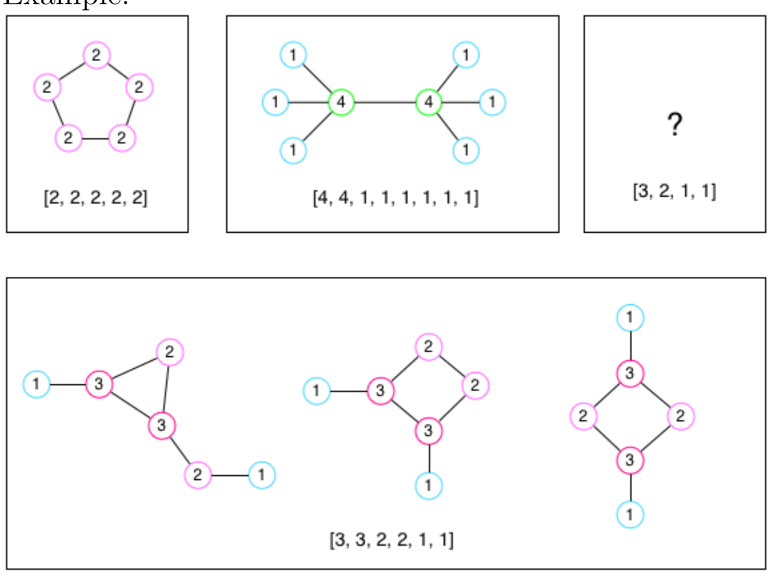
Note:

1. A shorthand for {u,v} is uv.
2. The edges are unordered, uv = vu, if order matters then it is a directed graph, draw lines with arrows
3. We mostly consider "simple" graphs, ie no multiple edges and no loops. Loops are a nod pointing at itself, multiple edges are more than one edge between two nodes
4. All Graphs we consider are finite
5. Usually we don't consider empty graphs

Degree

Definition: The degree of a vertex v is the number of edges incident with v, denoted deg(v)

Example:



Sum of the vertices of the bottom right graph

$$\sum_{v \in V(G)} deg(v) = 12$$

The sum will always be even, because it is the number of edges x 2

Handshaking lemma: For any graph G, $\sum_{v \in V(G)} deg(v) = 2|E(G)|$

Proof: Each edge uv contributes 2 to the sum, 1 for deg(u) and 1 for deg(v)

Corollary: For each graph, the nmber of odd-degree vertices is even
Proof: Let O, E be the vertices of odd and even degrees respectively. Then:

$$\underbrace{\sum_{v \in V(G)} deg(v)}_{A} = \underbrace{\sum_{v \in O} deg(v)}_{B} + \underbrace{\sum_{v \in E} deg(v)}_{C}$$

A is even by the handshaking lemma
C is even since it is a sum of even numbers.
$\therefore$ B is even, Since B is a sum of odd numebrs,there must be an even numbers of them so $|O|$ is even.

Isomorphism

Definition: Two graphs G_1, G_2 are isomorphic. If there exists a bijection $f : V(G_1) \to V(G_2)$ such that $uv \in E(G_1)$ if and only if f(u)f(v) $\leftarrow E(G_2)$ (adjacency is preserved). Such a mapping f is called an isomorphism.

Example: