Math 239 Lecture 17

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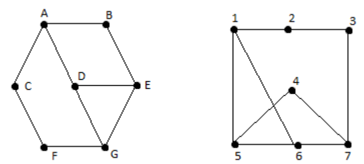
Isomorphism

G_1 is isomorphic to G_2 if $\exists$ a bijection $f : V(G_1) \to V(G_2)$ such that $uv \in E(G_1) \iff f(u)f(v) \in E(G_2)$.



8 Edges in first image, 9 edges in second.
See images on seperate page.
G_1 and G_2 are not isomorphic, there exists 3 mutually adjacent vertices in G_1 but no such vertices exist in G_2.

G_2 and G_3 are isomorpic

v	A	B	C	D	E	F
f(v)	V	III	IV	II	VI	I

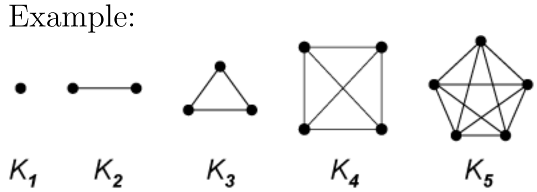
Summary: Isomorphism is a bijection between vertices so that adjacency strcture of the edges is preserved.

To prove 2 graphs are isomorphic, give an isomorphism.
To prove 2 graphs are not ismorphic, find a structure in one graph that does not exist in the other.

Spectial Graphs

Complete Graph

A complete graph is one where every pair of vertices is an edge
A complete graph on n vertices is denoted k_n
Example:



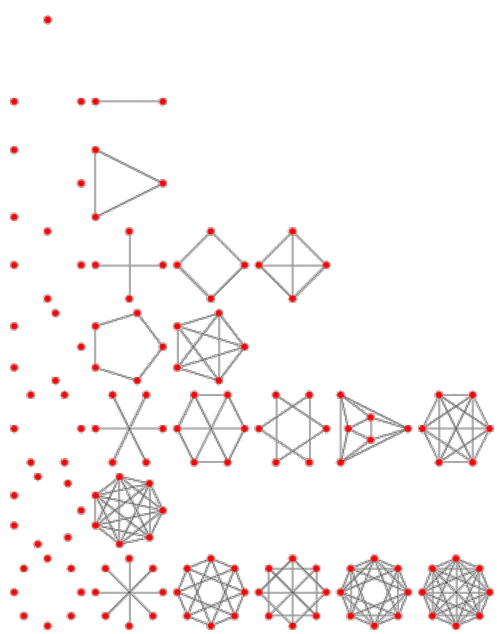
How many edges are in K_n

$$\frac{n(n-1)}{2} = \binom{n}{2}$$

There are $\binom{n}{2}$ pairs, each forming an edge.

K-regular

Example:

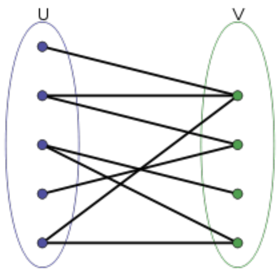


How many eges are there in a k-regular graph with n vertices? (Recall: Handshake Lemma $\sum deg(V) = 2|E(G)|$)
The total degre is nk, so the number of edges is $\frac{nk}{2}$

Bipartite

A graph G is bipartite if there exists a partition (A,B) of V(G) such that each edge in E(G) joins one vertex in A with one Vertex in B.

Example:



If it is bipartite then we get an edge joining 2 vertices of the same part, this is not possible so it is not bipartite. (A triangle)

Any graph containing a griangle is not bipartate, any cycle with an odd number of vertices is not bipartite