TITLE

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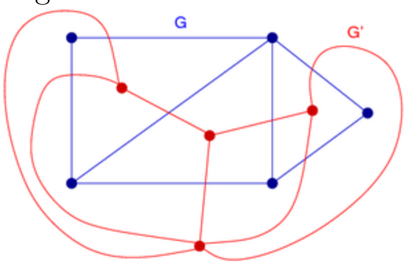
Dual Graphs

Definition: Let G be a planar graph with an embedding. The dual G* of the embedding has one vertex V_f corresponding to each face f of G, and foreach edge in G whose two sides are f_1, f_2, G* has a corresponding edge V_{f_1}, V_{f_2}.



It is possible for the dual graph to have multiple edges and loops.

Properties of Duality:

1. IF G is planar then G* is also planar (From inside a face, draw lines to the midpoints of the edges in the boundary)

2. (G*)* = G

3. # of faces in G = # of vertices in G*
   # of vertices in G = # of faces in G*
   # of edges in G = # of edges in G*

4. degree of a vertex in G = deg of the corresponding face in G*
   degree of a face in G = degree of the corresponding vertex in G*

5. The dual of a platonic graph is platonic

Theorem: The dual of an Eulerian planar graph is bipartite.

Proof: IF a graph is Eulerian, every vertex has even degree. So every face has even degree. By A9 last Question, the dual is bipartite.

Matchings

Definition: A matching of a graph is a set of edges where no two edges share a common vertex (or they for ma subgraph of max dgeree 1)

General Question: What is the maximum size of a matching in a graph?

Definition: A vertex incident with an edge in a matching is called saturated. It is unsaturated otherwise.

A matching that saturates all vertices is a perfect matching.

If a graph has a perfect matching, then it has even # of vertices.