Math 239 - Tutorial 5

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Binary String

$$AB = \{\sigma_1, \sigma_2 | \sigma_1 \in A, \sigma_2 \in B\}$$
$$A \cup B \{\sigma | \sigma \in A or \sigma \in B\}$$
$$A^* \{\epsilon\} \cup A \cup AA \cup AAA...$$

Ambiguous vs unambiguous

ambigiuos expression

$$\{00\}\{1, 01\}$$
$$= \{01, 001, 0001\}$$

2

Prove that $\{0,1\}^*\{000011\}\{0,1\}^*$ is ambigious

000011000011 ϵ

1

Let S = $(\{1\}(\{0\}\{1\}^*\{0\})^*\{1\}\{0\}^*)^*$

a) Find Binary strings at most 4

$\epsilon, 11, 110, 1100, 1001, 1111$

b) Assuming S is unambiguous find $\Phi_S(x)$

$$\Phi_S(x) = \Phi_{A^*}(x) = \frac{1}{1 - \Phi_A(x)}$$

$$\Phi_A(x) = \Phi_{\{1\}(\{0\}\{1\}^*\{0\})^*\{1\}\{0\}^*}(x)$$

$$= \Phi_{\{1\}}(x) \cdot \frac{1}{1 - \Phi_{\{0\}\{1\}^*\{0\}}} \cdot \Phi_{\{1\}}(x) \frac{1}{1 - \Phi_{\{0\}}(x)}$$

$$= x \cdot \frac{1}{1 - \frac{x^2}{1-x}} \cdot x \frac{1}{1-x}$$

$$= \frac{x^2}{1-x} \cdot \frac{1}{\frac{1-x-x^2}{1-x}}$$

$$= \frac{x^2}{1 - x - x^2}$$

$$\Phi_{\{0\}\{1\}^*\{0\}}(x) = \frac{x^2}{1-x}$$

3

Find the unambigious decompositions of the following sets of strings

a) strings that begin and end with the same bit

$$\{0\}\{0,1\}^*\{0\} \cup \{1\}\{0,1\}^*\{1\} \cup \{\epsilon, 1, 0\}$$

b) The set of binary strings where no block is divisible by 3

$$(\{0, 00\}\{000\}^*\{1, 11\}\{111\}^*)$$

$$\{0, 00\}\{000\}^*$$

$$= \{0, 0, 0000, 00000\}$$

$$\{\epsilon\} \cup (\{1, 11\}\{111\}^*)(\{0, 00\}\{000\}^*\{1, 11\}\{111\}^*)^*(\{\epsilon\} \cup \{0, 00\}\{000\}^*) n \{\epsilon\}$$

c) Strings where the length of each block of 1's has the same parity as the length of 0's imediately following it, and the string does not end with 1

$$\{0\}^*(\{11\}\{11\}^*\{00\}\{00\}^* \cup \{1\}\{11\}^*\{0\}\{00\}^*)^*)$$