Math 239 Lecture 32

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Vertex Covers

If M is a matching and C is a cover then $|M| \leq |C|$.

<u>**Corollary:**</u> IF M is a matching and C is a cover where $|M| = |C|$, then M is a maximum matching and C is a minimum cover.

<u>**Proof**</u> Let M' be any matching. Then by previous result $|M'| \leq |C|$. But $|C| = |M|$ so $|M'| \leq |M|$ So M is a max matching.

Let C' be any cover. Then $|C'| \geq |M|$. But $|M| = |C|$, so $|C'| \geq |C|$ So C is a min cover.
One way to prove that M is maximum is by providing a cover C where $|M| = |C|$

Konig's Theorem

<u>**Theorem:**</u> In a bipartite graph, the size of a maximum matching is equal to the size of a minimum cover.

<u>**Proof:**</u> Let M be a maximum matching. Let X_0, X, Y be the sets obtained at the end of the algorithm. There is no edge joining a vertex in X wit ha vertex in B/Y(if such an edge exists, it extends an alternating path starting from X_0 and the vertex in B/Y. Should he been in Y)
Therefore, Y∪(A/X) is a vertex cover

XY-Construction Algorithm

<u>**Strategy**</u>: Find all possible alternating paths starting from an unsaturated vertex in A. Any augmenting path that starts in A must end in B.