

Course Logistics

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Digital Logic Design

Truth Tables and Laws of Boolean Algebra

Course Notes Module 02

DeMorgan's Law:

$$\overline{X + Y} = \overline{X} \cdot \overline{Y}$$

$$\overline{XY} = \overline{X} + \overline{Y}$$

First we will look at Logic Gates : Come Back to This

Distributive :

$$X + YZ = (X + Y)(X + Z) \text{ why?}$$

$$= XX + XZ + XY + YZ$$

Factor out X :

$$= X (X + Z + Y) + YZ$$

If X is false

Distributive :

$$X + YZ = (X + Y)(X + Z) \text{ why?}$$

$$= XX + XZ + XY + YZ$$

Factor out X :

$$= X (X + Z + Y) + YZ$$

Just **X** being True is enough

To make the first part of the expression true.

Therefore bracketed term drops

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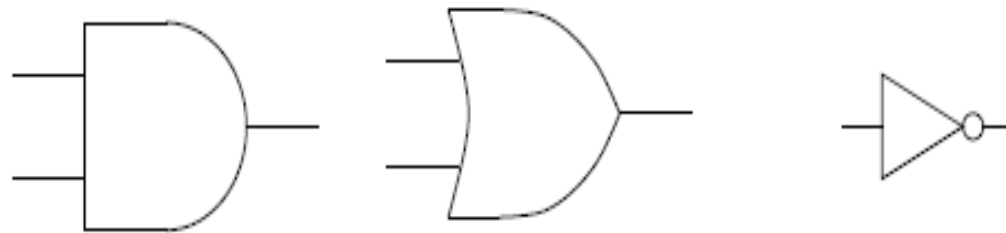
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Binary Numbers: 0010 ? Binary Number system: Review

Using Gates in Logic Design

- Here are symbols for AND, OR, NOT gates



- NOT often drawn as “bubble” on input or output
- AND, OR can be generalized to many inputs (useful)

Review Each of these Gates and Their Truth Tables: On Board

X	Y	Z	F	G
0	0	0	0	1
0	0	1	1	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	0

Formula Simplification Using Laws

- We can use algebraic manipulation (based on laws) to simplify formulas
- An example using the previous truth table

$$\begin{aligned}
 F &= \bar{X}\bar{Y}Z + X\bar{Y}Z + XY\bar{Z} + XYZ \\
 &= \bar{Y}Z(\bar{X} + X) + XY(\bar{Z} + Z) \\
 &= \bar{Y}Z + XY
 \end{aligned}$$

- Difficult even for humans, tricky to automate
- Seems inherently hard to get “simplest” formula
- Is simplest formula the best for implementation?

X	Y	Z	F	G
0	0	0	0	1
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1	0	0	0	1
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What was the formula for G?

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Deriving Truth Table from Circuit

- Label intermediate gate outputs
- Fill in truth table in appropriate order

