Math 239 - Lecture 12

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Ambiguity

AB works like $A \times B$ if it is unambiguous. $A \cup B$ is unambigious if $A \cap B = \emptyset$

Gemerating Series:
A = {1,11} B = {00,000}

$$\begin{aligned} \Phi_{AB}(x) &= \Phi_A(x)\Phi_B(x) \\ &= (x + x^2)(x^2 + x^5) \end{aligned}$$

S = {0,111}*

$$\Phi_S(x) = \sum_{n \geq 0} \Phi_{\{0,111\}^n}(x)$$
$$= \sum_{n \geq 0} (x + x^3)^n = \frac{1}{1 - (x + x^3)}$$

Theorems

Theorem (sum and product lemmas for strings) Let A, B be sets of strings.

1. If $A \cap B = \emptyset$ then $\Phi_{A \cup B}(x) = \Phi_A(x) + \Phi_b(x)$
2. If AB is unambiguous, then $\Phi_{AB}(x) = \Phi_A(x)\Phi_B(x)$
3. If A^* is unambiguous, then $\Phi_{A^*}(x) = \frac{1}{1 - \Phi_A(x)}$

Proofs

1. Sum Lemma
2. There is a bijection between $A \times B$ and AB when AB is unambiguous, (a,b) $\rightarrow$ ab. (The inverse is possible due to the unambiguity of AB). The product lemma applies.
3. Since A^* is unambiguous by sum lemma, $\Phi_{A^*}(x) = \sum_{n \geq 0} \Phi_{A^n}(x) = \sum_{n \geq 0} (\Phi_A(x))^n = \frac{1}{1 - \Phi_A(x)}$ Since the constant term of $\Phi_A(x)$ is 0. If const term is not 0, then $\epsilon \in A$ lnA^*, we acn get $\epsilon = \epsilon\epsilon = \epsilon\epsilon\epsilon = ...$

Basic Decompositions

3 basic unambiguous decomposition rules for the sets of al strings

1. $\{0,1\}^*$ cut any string after every bit, only one way
2. $\{0\}^*(\{1\}\{0\}^*)^*$ cut any string just before each 1. 00|1|10|1000
3. Block decomposition $\{0\}^*(\{1\}\{1\}^*\{0\}\{0\}^*)^*\{1\}^*$ 00|111100|100|111|11100|10|111
   Cut off any string after each block of 0's

Restrictions on Substrings

Example: Let S be the set of all strings with no 3 consecututive 0's. Start with $\{0\}^*(\{1\}\{0^*\})^*$ where can we find 000?

In $\{0\}^*$ remove all instances of 000 in $\{0\}^*$ to get $\{\epsilon, 0, 00\}$ So S $= \{\epsilon, 0, 00\}(\{1\}\{\epsilon, 0, 00\})^*$

This is unambiguous since we are removing elements from an unambigious expression.

$$\begin{aligned}\Phi_S(x) &= (1+x+x^2)\frac{1}{1-(x(1+x+x^2))}\\ &= \frac{1+x+x^2}{1-x-x^2-x^3}\end{aligned}$$

The number of strings in S of length n is $[x^n]\frac{1+x+x^2}{1-x-x^2-x^3}$

In general, start with one of the 3 basic decompositions. Remove parts of it that violate our conditions

If we start with block decomp, $\{0\}^* \to \{\epsilon, 0, 000\}$
$\{0\}\{0\}^* \to \{0\}\{\epsilon, 0\} \to \{0, 00\}$
S $= \{\epsilon, 0, 00\}(\{1\}\{1\}^*\{0, 00\}^*)^*\{1\}^*$
$\Phi_S(x) = (1+X+x^2)\frac{1}{1-X\frac{1}{1-x}(x+x^2)}\frac{1}{1-x}$
$= (1+x+x^2)\frac{1-x}{1-x-(x)(x+x^2)}\frac{1}{1-x} = \frac{1+x+x^2}{1-x-x^2-x^3}$