1 Log/Exponent Identities

The more common identities you will likely use:

- $(b^a)^c = b^{ac}$
- $b^a b^c = b^{a+c}$
- $\log_b(ac) = \log_b a + \log_b c$
- $\log_b(a^c) = c \log_b a$
- $b^{\log_c a} = a^{\log_c b}$
- $\log_b a = \frac{\log_c a}{\log_c b}$

Because of the last identity the base of the log is often immaterial within big-Oh notation. If $b = 2$, then some authors write $\log_2 a$ more compactly as $\lg a$ or indicate early on in the text that base 2 is assumed and write $\log a$ instead.

2 Common Summations

- Arithmetic series:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

- Useful approximation:

$$\sum_{i=1}^{n} i^k \approx \frac{n^{k+1}}{k+1}$$

- Geometric series (where $a \neq 1$):

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}$$

- Infinite series (where $0 < a < 1$):

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$$

- Harmonic sequence:

$$H_n = \sum_{i=1}^{n} \frac{1}{i} = \ln n + O(1) \in \Theta(\log n)$$

- $$\sum_{i=1}^{n} i\, r^i = \frac{nr^{n+1}}{r-1} - \frac{r^{n+1}-r}{(r-1)^2}$$
- Sum of inverse squares:
$$\sum_{i=1}^{\infty} i^{-2} = \frac{\pi^2}{6}$$
- Stirling Approximation:
$$n! \sim \sqrt{2\,\pi\,n}\left(\frac{n}{e}\right)^n \in \Theta\left(n^{n+1/2}e^{-n}\right)$$
- Log of $n!$
$$\log n! \in \Theta(n \log n)$$