Math 239 Lecture 22

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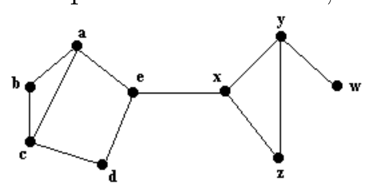
Items:

- Bridges
- Trees

Bridges

Theorem: IF e = uv is a bridge of a connected graph G, then G-e has exactly two components. Moreover, u and v are in different components of G-e.



Bridges: ex, yw

Some edge cuts: {ab, bc, ac}, {ae, ed}

Proof:
(Recall the cut induced by the vertices of a component is empty).
Suppose BWOC that G-e has 3 components. Let H be one component not containing u nor v. The cut induced by V(H) in G-e is empty, but uv is not in the cut induced by V(H) in G, since u,v $\notin$ V(H). So the cut induced by V(H) in G is empty. This contradicts the fact that G is connected.
So G-e has exactly 2 components.

Suppose BWOC u,v are in the same component of G-e. Let J be the component. Sothe cut induced by V(J) in G-e is empty. Since u,v $\in$ V(J), e is not in the cut induced by V(J) in G. So this cut is empty in G, contradiction. So u,v are in differnt component of G-e

Theorem: An edge e = uv in G is a bridge if and only if e is not in any cycle of G.

Contrapositive: e is in a cycle of G if and only if e is not a bridge.

Proof: $\Longrightarrow$: Suppose that e is in a cycle, u,v,v1,v2....u. Then in G-e, there is a u,v-path v,v1,v2....u. so u,v are in the same component of G-e. Using the contrapositive of the previous theorem, e is not a bridge.
$\Longleftarrow$ Suppose e is not a bridge. Let H be the component containing e. Then H-e is still connected. So it contains a u,v-path, which does not includ ee. Then P+e is a cycle containing e.

Trees

Definition: A tree is a connected graph with no cycles.
Definition: A forest is a graph with no cycles.

Lemma: Everye edge in a Tree/Forest is a bridge.

Definition A leaf in a tree is a vertex of degree 1

Theorem: Every tree with at least 2 vertices have at least 2 leaves.
Proof: Let T be a tree and let P = v0,v1 ... vk be the longest path in T. P has lenght at least 1 since there are $\geq$ 2 vertices. Vertex V0 has one neighbour v1. It cannot have any neighbour outside of P (For otherwise P is not the longest path), nor could it have a neighbour on P other than V1 (otherwise it forms a cycle). So deg(v0)=1 and its a leaf. Sone argument applies to Vk, so T has at least 2 leaves.