Math 239 Lecture 8

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May 22th, 2015

Product Lemma

Recall:

Sets A,B with weight α, β
Set A $\times$ B, with weight w(a,b) = $\alpha(a) + \beta(b)$
Then $\Phi_{A \times B}(x) \cdot \Phi_B(x)$

Proof of the Product Lemma

$$\Phi_A(x) \cdot \Phi_B(x) = (\sum_{a \in A} x^{\alpha(a)})(\sum_{b \in B} x^{\beta(b)})$$

$$= \sum_{a \in A} \sum_{b \in B} x^{\alpha(a)} x^{\beta(b)}$$

$$\sum_{(a,b) \in A \times B} x^{\alpha(a)+\beta(b)}$$

$$= \sum_{(a,b) \in A \times B} x^{w(a,b)}$$

$$= \Phi_{A \times B}(x)$$

Example: Let $N_0 = \{0, 1, 2, 3, ...\}$ w(a) = a. Then:

$$\Phi_{N_0}(x) = 1 + x + x^2 + x^3 + ... = \frac{1}{1-x}$$