Math 239 Lecture 20

Graham Cooper

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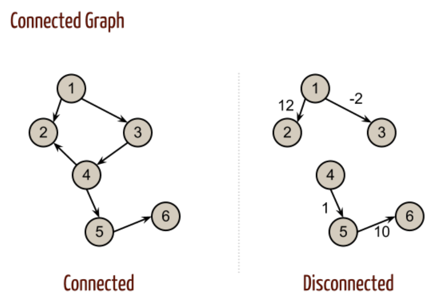
Topics:

- Components and Cuts
- Euler Tours

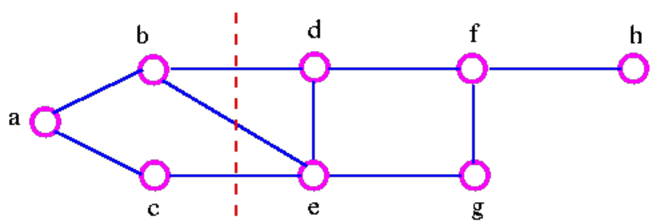
Disconnected Graphs

Definition: A component of graph G is a maximal connected non-empty subgraph of G.



Definition: Maximal means that a graph cannot be enlarged to get another connected subgraph

Definition: Let X be a subset of V(G). The cut induced by X is the set of edges with one end in X and one end is V(G)/X



X = {a,b,c}
The cut induced by X is {bd,be,ce}

Theorem: A graph G is disconnected if an d only if there exists a non-empty proper subset x of V(G) where the cut induced by X is empty.

Proof: $\implies$

IF G is disconnected, then it has at least two components. Let H be one component, hten V(H) is non-empty (by definition) and a proper subset (there is another component) of V(G). IF there is an edge in the cut induced by V(H) then H can be enlarged to get a larger connected subgraph which is not possible since H is maximal. So the cut induced by V(H) is empty.

$\impliedby$

Let X be a non-empty proper subset of V(G) with an empty cut. so there exists u $\in$ X and V $\in$ X that are vertices of G. Suppose there is a u,v-path $v_0, v_1 .... v_k$ where $v_0 = u_1, v_k = v$. We see that v_0 is in X and that V_k is not in X. So there exists i such that $v_0, v_1 ... v_i \in X$ but $v_{i+1} \notin$ X. Then v_i, v_{i+1} is an edge that the cut induced by X, which is not possible. So, no u,v path exists and G is disconnected.

Disconnected Example:

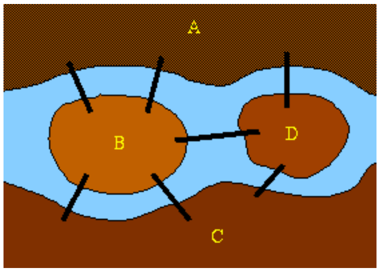
Let G_n be the graph where vertices are binary strings of length n, and two strings are adjacent if and only if they differe by exactly 2 bits

Claim:

G_n is disconnected for all n. Let x be the number of 0's. X is a non-empty proper subset of V(G_n). Suppose st is any edge where $s \in X$. Since we change 2 bits of S to get to T the number of 0's has the same partiy in s and t so $t \in x$ and the cut induced by x is empty. So G_n is disconnected

Euler Tours

Deftition: A Euler Tour is a closed walk which uses every edge of the graph exactly once.


A
B
D
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