Math 239 Lecture 31

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Matchings

Definition: An Alternating path P with respect to a matching M is a path where consecutive edges alternate between being in M and not in M. An augmenting path is an alternating path that starts and ends with unsaturated vertices.

If there is an augmenting path, we can "switch" edges between the path and M to get a larger matching.

Theorem If there is an augmenting path with respect to M, then M is not a maximum matching.(By switching along the path, we saturate 2 more vertices, and get a matching that has one more edge.)
Theorem If there is no augmenting path with respect to M, then M is a maximum matching

Vertex Cover

Defintition A vertex cover C of a graph G is a set of vertices such that each edge of G has at least one end in C.

General Question: Given a graph, what is the smallest size of a vertex cover?

Theorem: If M is any matching and C is any cover, then $|M| \leq |C|$.

PRoof: For each edge uv in M, at least one of u or v is in C. For different edges in M, different vertices are in C since matchings use distinct vertices So $|M| \leq |C|$

Corollary: If a mathing M and acover C satisfy $|M| = |C|$ then M is a max matching and C is a min cover