Math 239 Tutorial 7

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1) Prove or Disprove

A) The union of the edges of two distinct u,w-walks contains a cycle.

False
Graph:
V(G) u,w
E(G) {u,w}
Walk1: u,w
Walk2: u,w,u,w

No cycle since there cannot be a cycle in a graph wiht only two vertices

B) The union of the dges of two distinct u,w-paths contains a cycle

True
Path1: u = x1,x2,...xr = w
Path2: u = y1,y2...ys = w
Let i be the smallest index in {1,...r} so that xi = yi but $x_{i+1} \neq y_{i+1}$ (i exists because Path1 is distint from P2)
Let j be the first index in {i+1,i+2,...s} So that yi = xk for some k $\geq$ i + 1. (j exists because s is a candidate)
By choice of j, every vertex in $y_{i+1}, y_{i+2}...y_{j-1}$ is not in Path1
Now C := xi = $y_i, y_{i+1}..., y_{j-1}, y_j = x_k, x_{k-1}...x_i$

2) G graph, every vertex has degree at least 3 $\implies$ G contains a ccle of even length

Let x1,x2...xk be the longest path in G. All the neighbours of x1 are in this path, otherwise we can augment it. Since deg(v1) $\geq$ 3, there exists two neighbours vi, vj, of v1 with $2 < i < j \leq k$

Since either i,j or j-i is even, then either on of the cycles:
C1 = x1,...xi,x1
C2 = x1,...xj,x1
C3 = xi,...xj,x1,xi
has even length

4)Let x,y $\subseteq$ V(G) where x $\cap$ Y = $\emptyset$. Prove that if the cuts induced by X and Y has an even number of edges

E(X,!X) cut induced by X
E(Y,!Y) cut induced by Y
E(X∪Y. !(X∪Y)) cut induced by x ∪ Y
|E(X∪Y. !(X∪Y))| = |E(X,!X)| + |E(Y,!Y)| - 2|E(X,Y)|
The LHS is even since the 2|...| is even.