CS240 Tutorial 3

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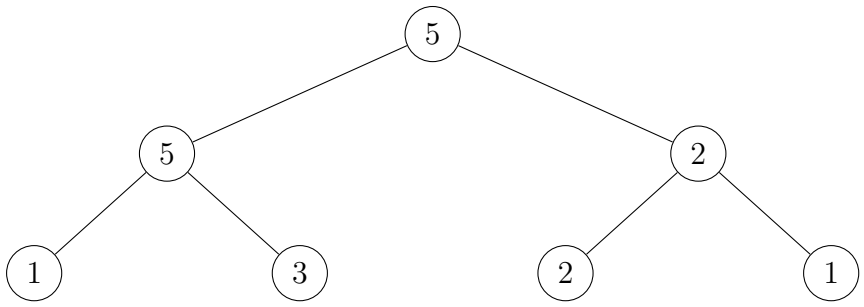
Heaps

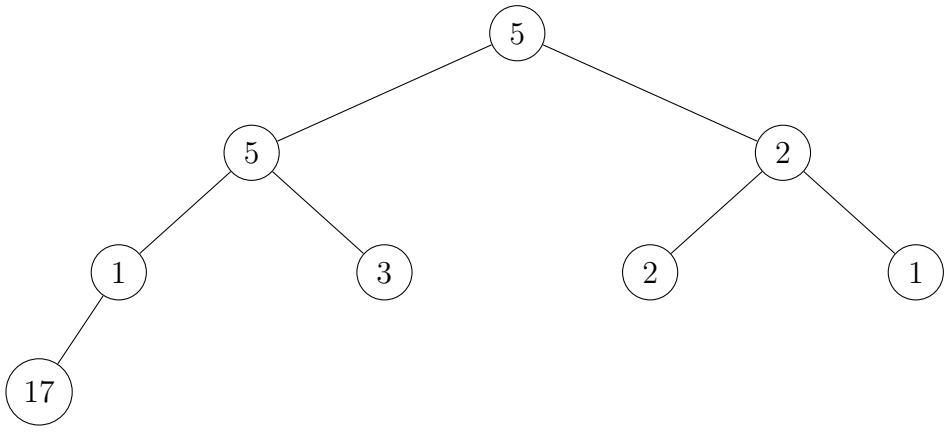
Consider the following heap (max-heap):

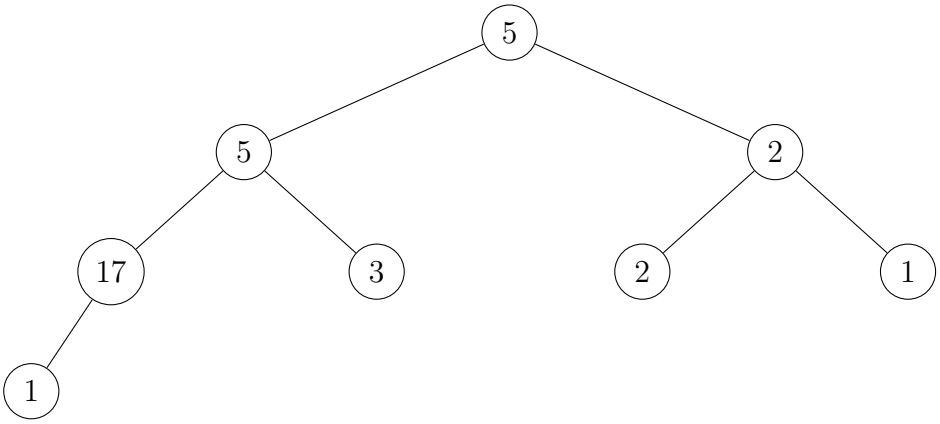


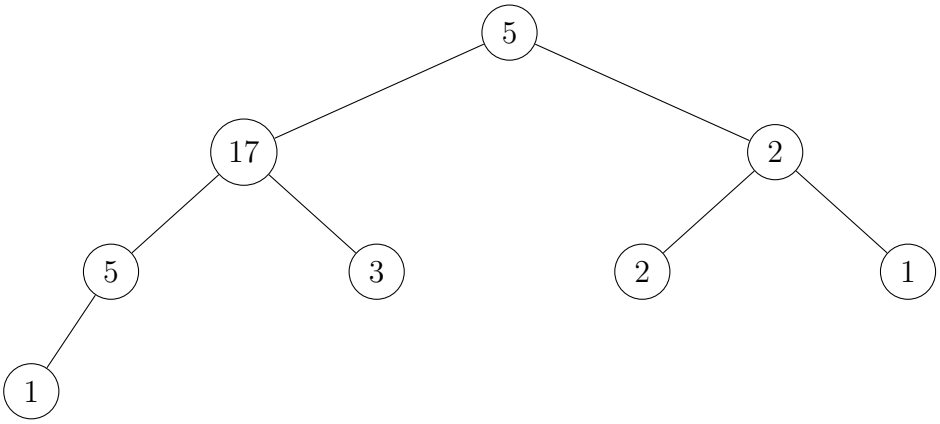
A: [5,5,2,1,3,2,1]

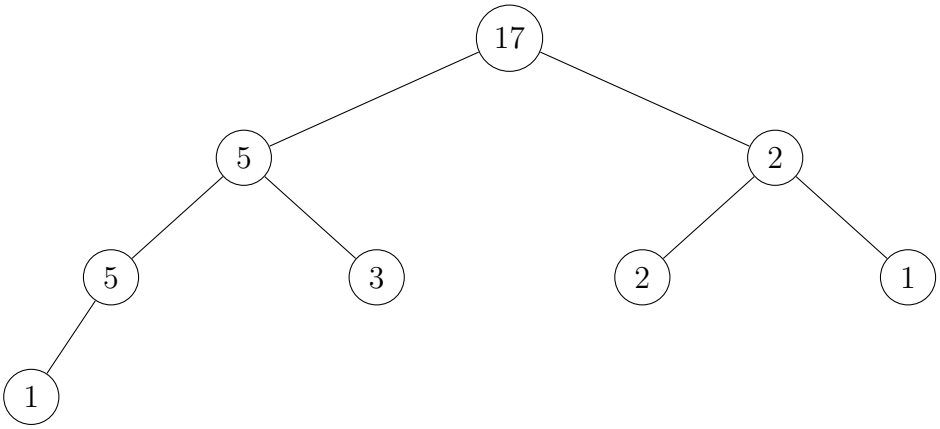
Insert

Suppose we insert 17 and 8 into the heap

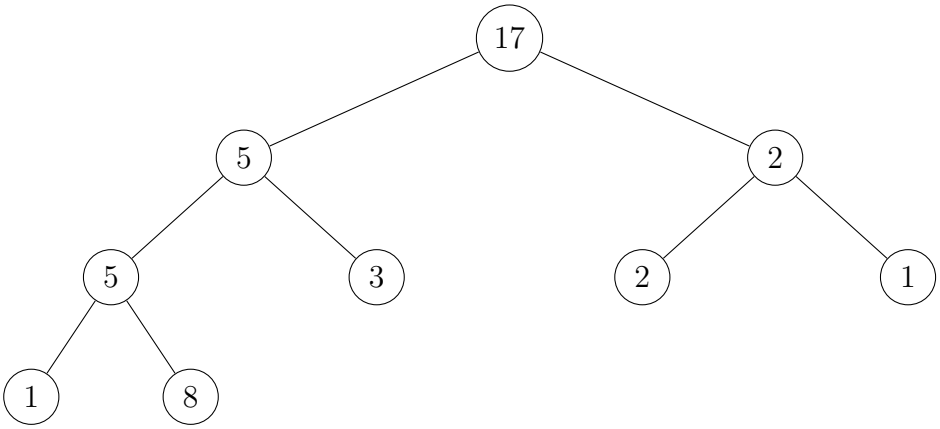


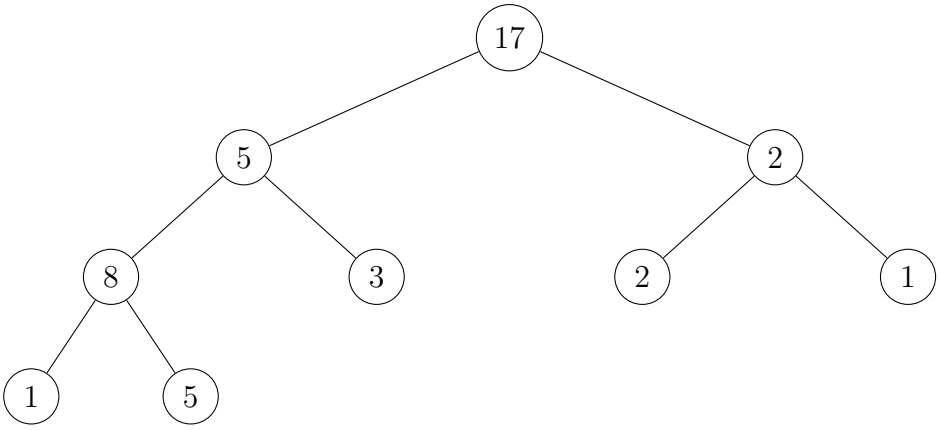


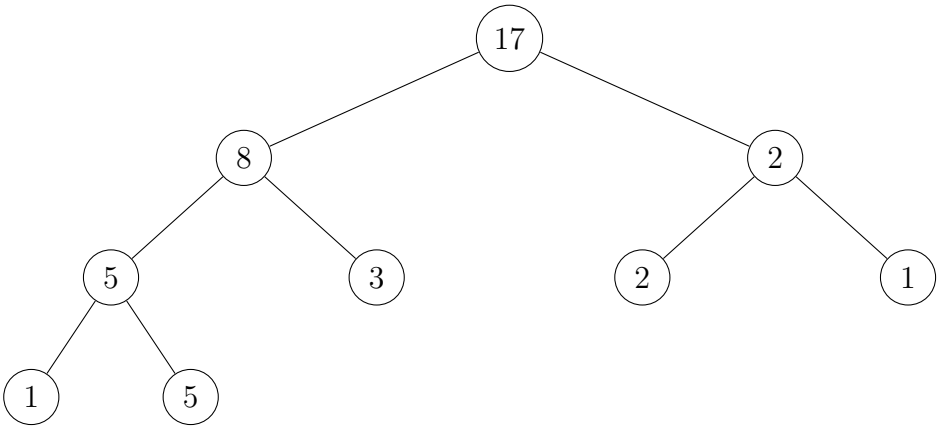




For 8:



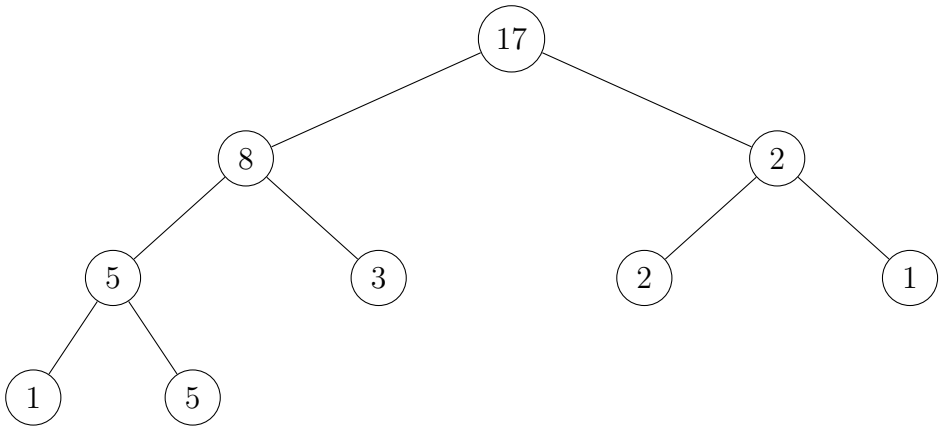




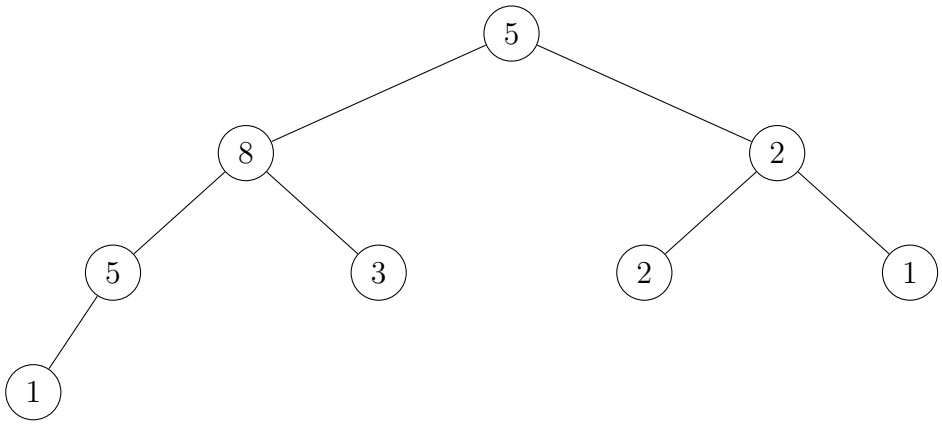
Do not swap with 17.

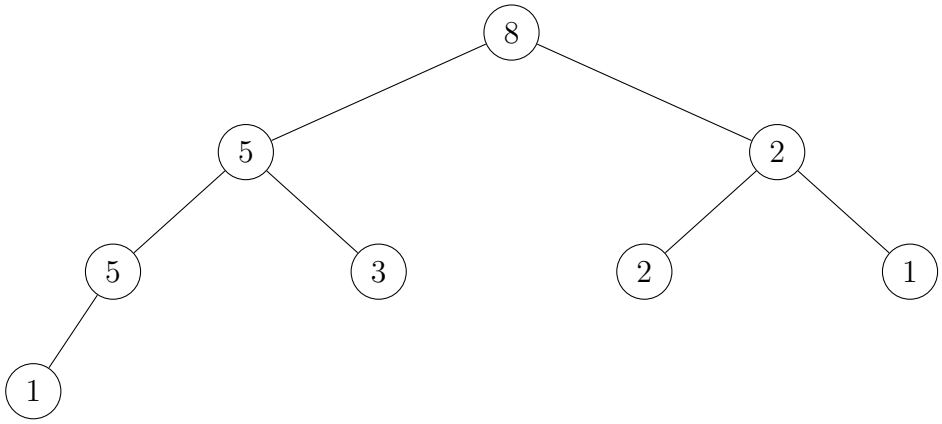
Delete-max

Suppose now we want to delete-max:



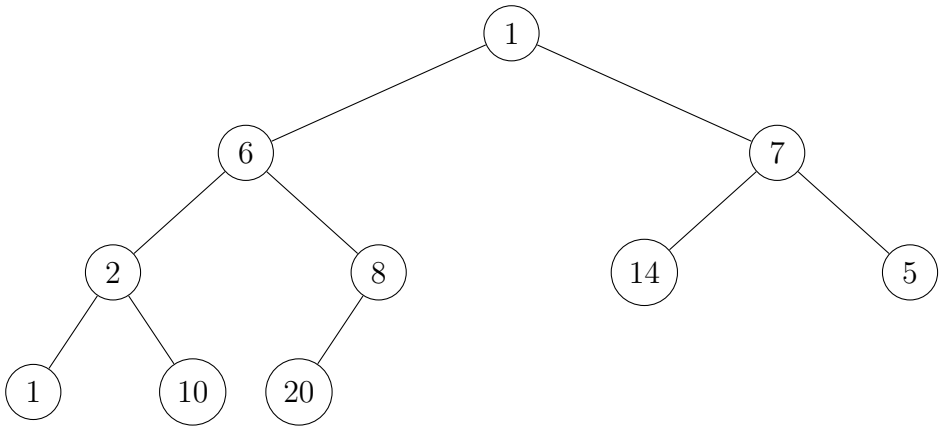
1) Swap root with the right most leaf, (on bottom level)
2) remove the largest element
3) Bubble down the new root

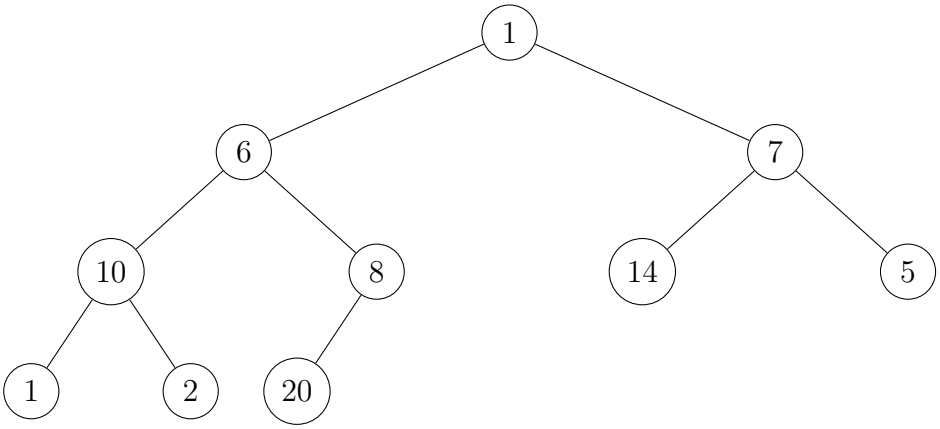


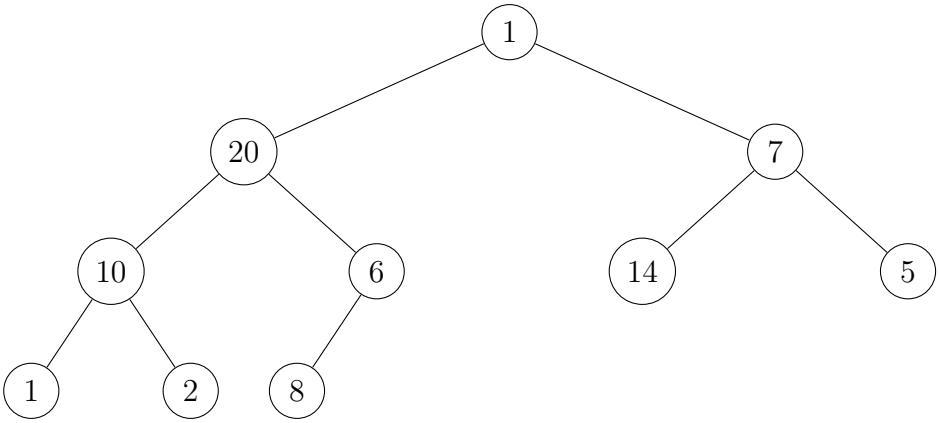


0.1 Heapify:

1. n size(A) 1
2. for i = floor(n/2) down to 1
3. bubble-down(A, i)




1
6
7
10
8
14
5
1
2
20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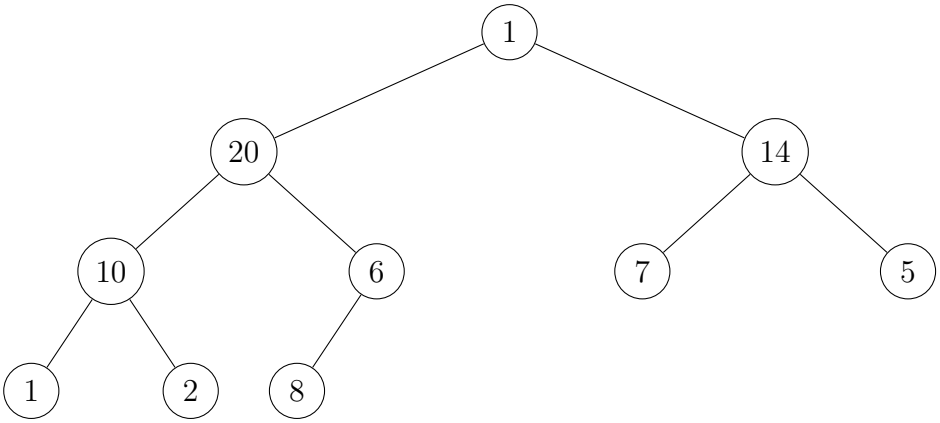
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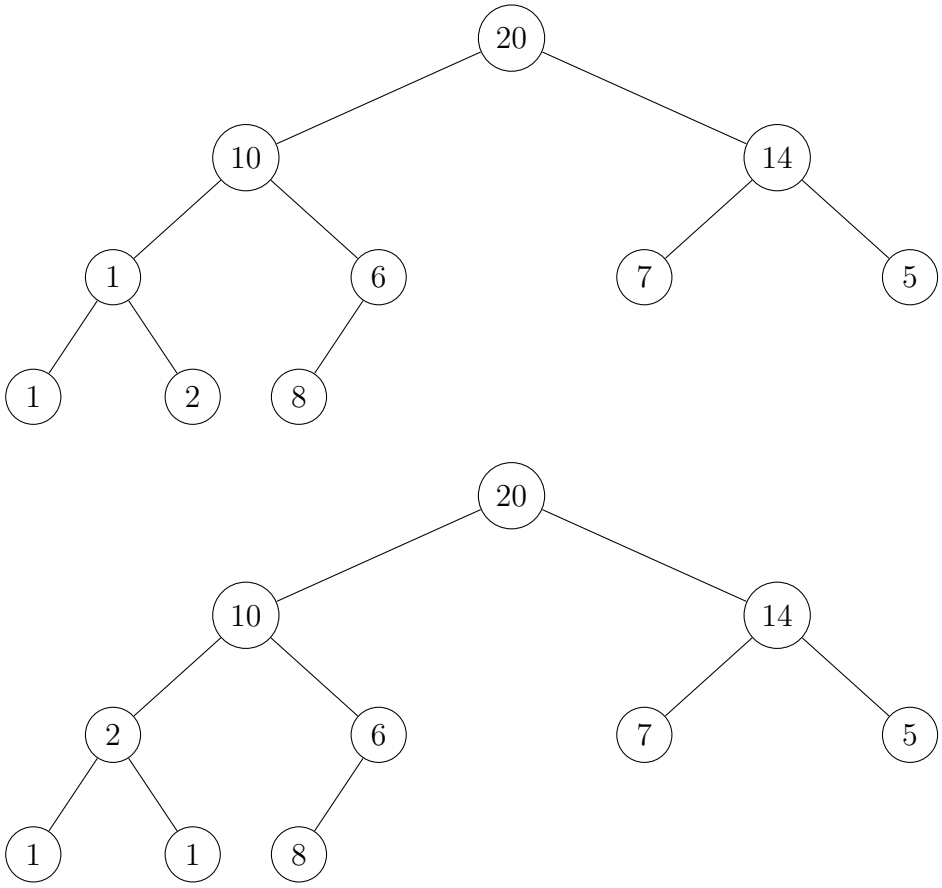

Overall we bubbled down 2, 6, 7, 6 and then 1

Heapsort

Use heapsort to sort the following array:
A: [2520, 1982, 34]800, 34000, 322,159,2845, 9]

1. H $\leftarrow$ Heapify(A)
2. call delete-max(H) n times and store the result in A.

Stack

Q: How can we simulate a stack using a priority queue (heap).

A stack should support push and pop

```
Stack {
- max-heap H
- priority P
}

Push(e) {
    H.heapinsert(p,e);
    p = p + 1;
}
Pop() {
    H.deletemax();
    p = p-1;
}
```

Q: Given K sorted lists, where the combination of all k lists has n elements, combine them into 1 sorted list O(nlogk time. Hint: use priority queues.

First idea (use heapsort)

- use heapify O(n)
- using delete-max n times gives O(nlog(n)) time which is a bit too much

1	7	12	19		
2	9	14	18	21	23
3	8	16			
5	10	17	70	72	

Look the the first collumn to create a min-heap. Then delete-min and add on the next element onto the min-heap.

Note: Always have k items in our heap(height logk) and repeat n times for a total running time of O(nlogk) time