Math 239 Lecture 23

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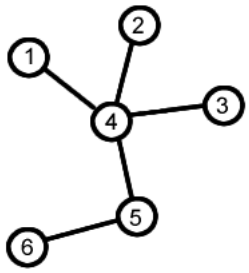
Items:

- Trees
- Spanning Trees

Trees

Evrey tree qwith 22 vertices has 22 leaves



Theorem: Every tree with n vertices has n-1 edges.

Proof: By induction on n.
Base Case: When n = 1, there is only one tree with 1 vertex and that tree has n-1 = 0 edges

Induction Hyptothesis: Assume every tree with n-1 vertices has n-2 edges.

Induction Step: Let T be a tree with n vertices. Let v be a leaf of T and let e be the only edge incident wiht v. Remove e and v from T to get T'. When we remove e from T there are 2 components, one consists of only v and the other is T'. So T' is connected. Also, T' has no cycles since T has no cycles. So T' is a tree with n-1 vertices. By induction hypothese T' has n-2 edges, so T has n-1 edges

Q: How many edges does a forest with n vertices and k components have?

A: n-k, each component has one feqer edfe than vertices. $\implies$ k compnents mean k fewer edges in total

Theorem: There is a unique path between every pair of vertices in a tree

Proof: Suppose BWOC there are two u,v-paths P1, P2. There is an edge e=xy in one path but not the other. WLOG e $\in$ P1, P1 = e,v1,...,vk,x,y,vk+1,...v. Then x,vk,...,u,P2,V,...,vk+1, y is an x,y-walk not using e. So there is an x,y-path P3, without using e. Then P + e is a cycle in the tree, conradiction.
Theorem: A tree is bipartite
Proof: By induction!

Spanning Trees

Definition: T is a spanning tree of a graph G is T is a subgraph of G that is a tree and uses very vertex in G