# Math 239 Lecture 25

Graham Cooper

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Items:

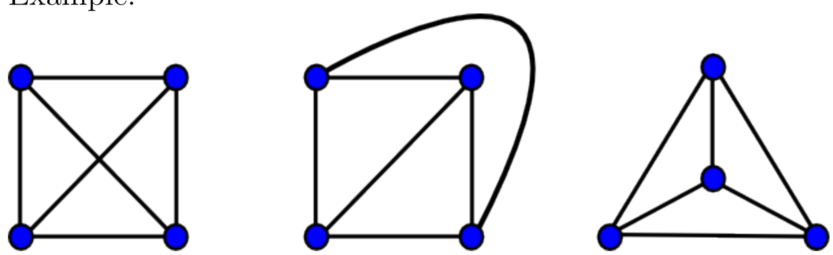
- Planar Graphs
- Euler's Formula

# Planar Graphs

## Definitions

**Definition:** A planar embedding of a graph G is adrawing on a plane such that vertices are at different points and edges do not cross each other. A graph that has a planar embedding is called a planar graph

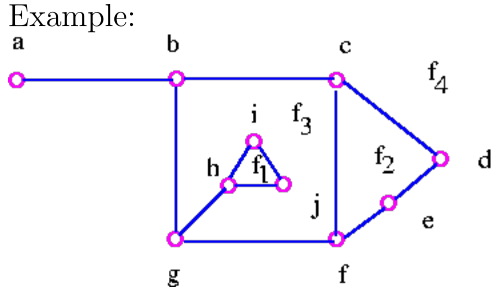
Example:



The first graph is NOT planar, the other two are.

**Definition** A face of a planar embedding is a connected region on the plane. Two faces are adjacent if they share at least one edge

Example:




F4 is the outer face, the unbounded face

**<u>Definition:</u>** For a connected graph, the <u>boundary walk</u> of a face is a cloased walk using the edges around teh boundary.

Example:
f3: {b,c,f,g,b}
f1: {i,j,h,i}

**<u>Definition:</u>** The <u>degree</u> of a face deg(f) is the length of its boundary walk

## Theorems

**<u>Handshaking Lemma for faces:</u>** Let G be a plannar graph with a planar embedding wehre F is the set of all faces. Then

$$\sum_{f \in F} deg(f) = 2|E(G)|$$

**<u>Proof</u>**: Each edge contrubutes 2 to the sume of degrees, 1 for each side of the edge.

A bridge has the same face on both sides. A non-bridge has different faces on each side.

**<u>Jordan curve theorem:</u>** Every simple closed curve on the plan seperates the plane into 2 parts, one inside, one outside

This is not true on the surface of a torus

G is connected and plannar. G has only 1 face if and only if G is a tree. (no cycles). IF G has at least 2 faces, then each face must be seperated from other faces via a cycle on its boundary $\implies$ degree of each face is at least 3 contains a cycle