# Math 239 Lecture 23

## Graham Cooper

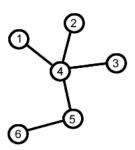
July 3rd, 2015

#### Items:

- Trees
- Spanning Trees

### **Trees**

Evrey tree qwith 22 vertices has 22 leaves



**Theorem:** Every tree with n vertices has n-1 edges.

**Proof:** By induction on n.

Base Case: When n=1, there is only one tree with 1 vertex and that tree has n-1=0 edges

Induction Hyptothesis: Assume every tree with n-1 vertices has n-2 edges.

Induction Step: Let T be a tree with n vertices. Let v be a leaf of T and let e be the only edge incident with v. Remove e and v from T to get T'. When we remove e from T there are 2 components, one consists of only v and the other is T'. So T' is connected. Also, T' has no cycles since T has no cycles. So T' is a tree with n-1 vertices. By induction hypothese T' has n-2 edges, so T has n-1 edges

**Q**: How many edges does a forest with n vertices and k components have?

 $\underline{\mathbf{A}}$ : n-k, each component has one feqer edfe than vertices.  $\implies$  k components mean k fewer edges in total

**Theorem:** There is a unique path between every pair of vertices in a tree

**Proof:** Suppose BWOC there are two u,v-paths P1, P2. There is an edge e=xy in one path but not the other. WLOG  $e \in P1$ , P1 = e,v1,...,vk,x,y,vk+1,...v. Then x,vk,...,u,P2,V,...,vk+1, y is an x,y-walk not using e. So there is an x,y-path P3, without using e. Then P+e is a cycle in the tree, conradiction.

Theorem: A tree is bipartite

**Proof:** By induction!

## **Spanning Trees**

**<u>Definition:</u>** T is a spanning tree of a graph G is T is a subgraph of G that is a tree and uses very vertex in G

