Math 239 Lecture 18

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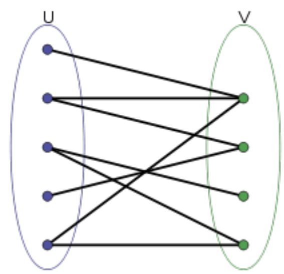
Special Graphs

Bipartite

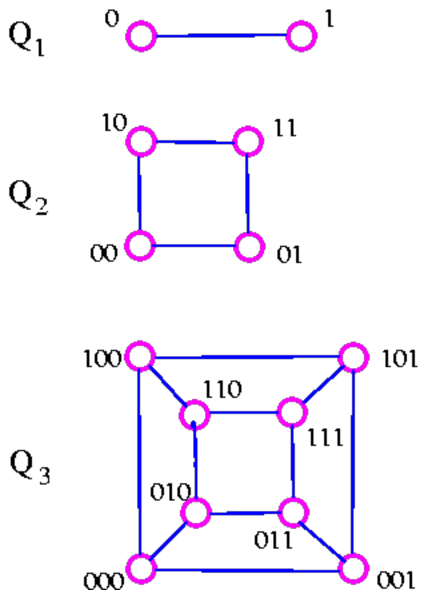


For m,n ∈ N, the complete bipartite graph, $K_{m,n}$ with bipartition (A,B) where $|A|$ = m, $|B|$ = n and it contains all possible edges joining a vertex in A with a vertex in B

How many edes are in $K_{m,n}$? mn, m chocies for a vertex in A, each paired with teh n ertices in B.

N-cube

The n-cube is the graph where the vertices are all binary strings of length n, and two strings are adjacent if and only if they differe in exactly one bit.



Properties of the n-cube:

1. 2^n vertices

2. n-regular. For a string of length n, we can change one of the n-bits to get a neighbour $\implies$ degree n

3. $\frac{n \cdot 2^n}{2} = n \cdot 2^{n-1}$ edges. Total degree is n $\cdot 2^n$ by handshaking lemma, the number of edges is half of it.

4. It is bipartite. Consider the bipartition (A,B) where:
   A consists of strings with even number of 1's and
   B consists of strings with odd number of 1's
   Let s,t be strings where st is an edge. Suppose wlog $s \in A$. We get t be either chaning a 0 to a 1 (increases number of 1s by 1) or changing a 1 to a 0 (decrease numbre of 1's by 1). Since s has even number of 1s t must have an odd number of 1's. So $t \in B$. Therefoe the n-cube is bipartite.

Recursive construction of the n-cube:

1. Take 2 copies of the (n-1)-cube

2. Attach 0 in front of all strings in one copy, attach a 1 in front of all strings in the other copy

3. Join corresponding vertices withedges.

End of midterm material!!!!