

CS240 - Tutorial 4

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QuickSelect Example

Q1

Find the 3rd smallest element in A

Pivot = A[0]

A =

8	17	10	1	6	20	2	9	7	13
<u>8</u>	<u>17</u>	10	1	6	20	2	9	<u>7</u>	13
8	7	10	1	6	20	2	9	17	13
8	17	<u>10</u>	1	6	20	<u>2</u>	9	7	13
8	7	2	1	6	20	10	9	17	13

Place Pivot

6	7	2	1	8	20	10	9	17	13
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Recurse on the left hand side (6,7,2,1)

pivot = A[0]

6	<u>7</u>	2	<u>1</u>
6	1	2	7

Recurse on left hand side (2, 1)

Since the pivot is at index 3, it corresponds to the 3rd smallest element, we are done.

Q2)

Assume A has distinct elements.

```
Bogo(A){
  shuffle(A) //O(n)
  if A is sorted{
    return A;
  }
  else {
```

```

return Bogo(A);
}
}

```

Best case is $O(n)$
Worst case is $O(\infty)$
May not terminate

$$T_{avg}(n) = 1 \cdot cn + \underbrace{\left(\frac{1}{n!} \cdot d\right)}_{Aissorted} + \underbrace{\left(1 - \frac{1}{n!}\right)}_{Aisnotsorted} T_{avg}(n)$$

$$T_{avg}(n) \left[1 - \left(1 - \frac{1}{n!}\right)\right] = cn + \frac{1}{n!} \cdot d$$

$$T_{avg}(n) = cn \cdot n! + d \in O(n \cdot n!)$$

$$EE[x] = \sum_{x \in X} P_r(x) \cdot RunningTime(X)$$

Q3)

Toss identical balls at random into buckets (or bins), one at a time, uniformly at random. How many tosses can we expect to make such that every bucket contains at least 1 ball.

- Define a toss in which a ball falls into an empty bucket as a hit and a non-empty bucket as a miss
- Partition the tosses into stages
 - The i^{th} stage consists of the tosses after the $(i-1)^{th}$ hit until (And including) the i^{th} hit.

Ex. 4 Buckets

Toss sequence: 2|,2,3|,4|,3,3,2,4,1|

During the i^{th} stage.

- (i-1) non-empty buckets
- (b - i + 1) empty buckets (b is the number of buckets)
- $\Pr(\text{throwing in empty bucket}) = \frac{b-i+1}{b}$

Define n_i = number of throws in stage i

$$n = \sum_{i=1}^b n_i$$

$$\begin{aligned} EE[n] &= EE\left[\sum_{i=1}^b n_i\right] \\ &= \sum_{i=1}^b EE[n_i] \end{aligned}$$

the above by linearity of expectation

$n_i \approx$ geometric dist.

$$EE[n_i] = 1/p$$

$$\begin{aligned} &= b \sum_{i=1}^b \frac{1}{b-i+1} \\ &= b \sum_{i=1}^b \frac{1}{i} \in \Theta(b \ln b) \end{aligned}$$

Expected Time Analysis

Probability Review

Intro to Lower Bounds