Math 239 - Lecture 9

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Binary Strings

General Questions

How many binary strings of length n have some propertise?

Approach: let S by the set of all strings with these properties.
Define weight function w(σ) to be the length of σ. Find generating series $\Phi_S(x)$ with respect to w.
Our answer is $[x^n]\Phi_S(x)$ (number of strings in S with weight n)

Example:
S = {01, 001, 010, 01100}
Length: 2, 3, 3, 5
$\Phi_S(x) = x^2 + x^3 + x^3 + x^5$

T = {e, 0, 00, 000, 0000, ...}
$\Phi_T(x) = 1 + x + x^2 + x^3 + x^4 + ... = \frac{1}{1-x}$

Two Operations

Concatenation of sets of strings

If A, B are sets of strings, then AB = {ab $|a \in A, b \in B\}$

Example:
A = {0,11}
B = {1,11}
AB = {01, 011, 111, 1111}
This is similar to the cartesian product

$A^k = AA...A = \{a_1, a_2, ...a_k | a_i \in A\}$
Example:
A = {0,11}
A^5 includes 00000, 01111011

Start (*) operator

$A^* = A^0 \cup A^1 \cup A^2 ... = \bigcup_{n\geq 0} A^n$

Example:
A = {0,1}
$01101 \in \{0,1\}^5$ which is in $\{0,1\}^*$

Any string of length n is in $\{0,1\}^n$ which is in $\{0,1\}^*$ So $\{0,1\}^*$ includes all binary strings

Example:
A = $\{0\}\{00\}^*$ strings of odd length with only 0's
$\{00\}^*$ = {e, 00, 0000, 000000, ...} even length

B = $\{0, 111\}^*$ strings where blocks of 1s have lengths divisible by 3.

C = $\{0\}^*(\{1\}\{0\}^*)^*$
Take any string, break it just before each 1. We have a number of copies of $\{1\}\{0\}^*$ except $\{0\}^*$ at the beginning

This is the set of all strings. These are "decompositions" of strings

Generating Series on Strings

Example:
A={1,11}, B = {00, 000}, $w(\sigma)$ = length of σ

$\Phi_A(x) = x + x^2$
$\Phi_B(x) = x^2 + x^3$

AB = w(ab) = w(a) + w(b)

Assuming concat works like the cartesian product,
$\Phi_{AB}(x) = \Phi_A(x)\Phi_B(x)$

AB = {100,1000,1100,11000}

Example:

B = {0,111}* = $\bigcup_{k\geq 0}\{0,111\}^k$
$\Phi_{\{0,111\}^k}(x) = (\Phi_{\{0,111\}}(x))^k$
$= (x+x^3)^k$

By the sum lemma, $\Phi_B(x) = \sum_{k\geq 0}\Phi_{\{0,111\}^k}(x) = \sum_{k\geq 0}(x+x^3)^k$
$= \frac{1}{1-(x+x^3)}$

Unambiguity of Strings

Example:
A = {1,11}
B = {1,11}

$A \times B = \{(1,1),(1,11),(11,1),(11,11)\}$
AB = {11,111,1111}

$\Phi_{A\times B}(x) = x^2 + 2x^3 + x^4$
$\Phi_{AB}(x) = x^2 + x^3 + x^4$

Definition: AB is ambiguous if there exists distinct pairs. $(a_1,b_1),(a_2,b_2) \in A \times B$ such that $a_1b_1 = a_2b_2$