# Math 239 Lec 10

Graham Cooper

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How many compositions of n are there where each part is odd? (any number of parts)

Define $N_{odd} = \{1, 3, 5, 7...\}$
Let $S = N_{odd} \cup N_{odd} \cup ... = \bigcup_{k \geq 0} N_{odd}^k$
Define the weight function of a composition to be the sum of its parts

$$\Phi_{N_{odd}}(x) = x + x^3 + x^5 + ... = \frac{x}{1 - x^2}$$

$$\Phi_{N_{odd}^k}(X) = (\Phi_{N_{odd}}(x))^k = (\frac{x}{1 - x^2})^k$$

by product lemma

$$\Phi_S(x) = \sum_{k \geq 0} \Phi_{N_{odd}^k}(x) = \sum_{k \geq 0} (\frac{x}{1 - x^2})^k$$

$$= \frac{1}{1 - \frac{x}{1-x^2}} = \frac{1 - x^2}{1 - x - x^2}$$

Let A(x) $= \sum_{n \geq 0} a + nx^n = \frac{1-x^2}{1-x-x^2}$
$[a_n - a_{n-1} - a_{n-2} = 0] n \geq 3$
$a_0 = 1$ $a_1 = 1$ $a_2 = 1$
$\iff$ $a_n = a_{n-1} + a_{n-1} + a_{n-2}$ for $n \geq 3$ Fibbonacci recurence
1,1,1,2,3,5,8,13,21,34... $a_0, a_1, a_2$
Let $S_n$ be the set of all compositions of n where each part is odd. The recurrence implies that $|S_n| = |S_{n-1}| + |S_{n-2}|$ for $n \geq 3$
Find a bijection between $S_n$ and $S_{n-1} \cup S_{n-2}$

Define f: $S_n \to S_{n-1} \cup S_{n-2}$ where for each $(a_1, ...a_k) \in S_n$
f$(a_0, ...a_k) =$
$(a_1, ...a_k - 1)$ if $a_k = 1$
$(a_1, ..a_{k-1}, a_{k-2})$ if $a_k \geq 3 \leftarrow$ in $S_{n-2}$
The inverse $f^{-1} : S_{n-1} \cup S_{n-2} \to S_n$ where for each $(b_1, ...b_l) \in S_{n-1} \cup S_{n-2}$

$f^{-1}(b_1...b_l)^{-1} =$
$(b_1, ...b_l, 1)$ if $b_1 + ... + b_l =$ n - 1
$(b_1, ..., b_{l-1}, b_{l+2})$ if $b_1 + ...b_l = n - 2$

$\implies$ f is a bijection

Recusively build $S_n$ based on $S_{n-1}$ and $S_{n-2}$
Add 1 part of 1 to any $S_{n-1}$
Add 2 to the last part of any $S_{n-2}$

$S_6 = \{(5,1), (1,1,3,1), (1,3,1,1), (3,1,1,1), (1,1,1,1,1,1), (3,3), (1,5), (1,1,1,3)\}$
from $S_4$

# Binary Strings

A binary string is a sequence of 0's and 1's
Terminology:

- The length of a string is the total number of 0's nad 1's in the string
- There is one string of length 0 and that is the empty (null string) "$\epsilon$"
- The concentration of a and b is ab, a = 001, b = 10, ab = 00110
- b is a substring of S if s = abc for some strings a,c (and possibly $\epsilon$)
- A block is a maximal non-empty substring of all 0's or all 1's