

Tutorial 5 CS240

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Topics:

- lower bounds
- counting sort / radix sort
- - applications

Q1)

- Supposed we have n blue jugs and red jugs
- for each blue jug there is a corresponding red jug with the same capacity
- the capacities are unique among jugs of the same colour
- FIND: a set of pairs such that every blue jug is paired with a red jug having the same capacity
- Only allow comparisons between a red and blue jug
- GOAL: Show $\Omega(n \log n)$ comparisons are needed.

B:	5	3	8
R:	3	8	5

$B_1 \rightarrow R_3$

$B_2 \rightarrow R_1$

$B_3 \rightarrow R_2$

Proof: Label blue jugs with $B = \{1, 2, \dots, n\}$ and red jugs $R = \{1, 2, \dots, n\}$

→ A solution has the form:

$A = \{(i, \pi(i)), i \in B, \pi \text{ is a permutation of } R\}$

Decision making: for i, j in B and R respectively, if $i < j$ go to left path

if $i = j$ go to middle path

if $i > j$ go to the right path

→ Suppose that our tree has height h

- at most 3^n leaves

- at least $n!$ leaves

$$3^n \geq n! \geq \left(\frac{n}{e}\right)^n \implies h \geq n \log_3 \left(\frac{n}{e}\right) \in \Omega(n \log n)$$

$\left(\frac{n}{e}\right)^n$ comes from Stirling's approximation.

Q2) - match psuedocode from slides

Use countsort to sort:

$B = (6, 1, 2, 0, 10, 6, 6, 2, 9, 1, 6, 7, 0)$

Range $[0 - 10]$

$\text{inc}(C[B[i]])$

$C = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

$C = (0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0)$

$C = (0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0)$

$C = (0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0)$

... Increment at the corresponding index

Eventually becomes:

$C = (2, 2, 2, 0, 0, 0, 4, 1, 0, 1, 1)$

$I[0] = 0 : I[k] = I[k-1] + I[k-1]$

$I = (0, 2, 4, 6, 6, 6, 10, 11, 11, 11)$

A is the sorted array, same amount of elements as B

$A[I[B[i]]] = B[i]$

Increase($I[B[i]]$)

$A = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

$A = (0, 0, 1, 0, 2, 0, 6, 0, 0, 10)$

.. Repeat procedure $A = (0, 0, 1, 1, 2, 2, 6, 6, 6, 7, 9, 10)$

Q3)

Use Radix Sort to Sort (LSD)

A = (2751, 68, 215, 155, 214, 313, 135, 38, 351, 51)

Sort individual digits using any stable sort.

1st: (2751, 351, 51, 313, 214, 215, 155, 135, 68, 38)

2nd: (313, 214, 215, 215, 135, 38, 2751, 351, 51, 155, 68)

3rd: (051, 068, 135, 155, 214, 215, 313, 351, 2751)

4th: (051, 068, 135, 155, 214, 215, 313, 351, 2751)

Things stay the same in the 4th iteration, but it is not always this way.

1 Q4)

Unsorted array H of n non-negative elements. All of the elements are smaller than n^3 , sort A in $\Theta(n)$

$\Theta(m(n + b))$, b is the base, m is the number of digits. We need to pick a base such $m(n + b) = \Theta(n)$

We are going to pick base n numbers. Therefore we only need 3 digits to represent them $\log_n(n^3)$

$$\Theta(m(n + b)) \implies \Theta(3 \times 2n) \in \Theta(n)$$