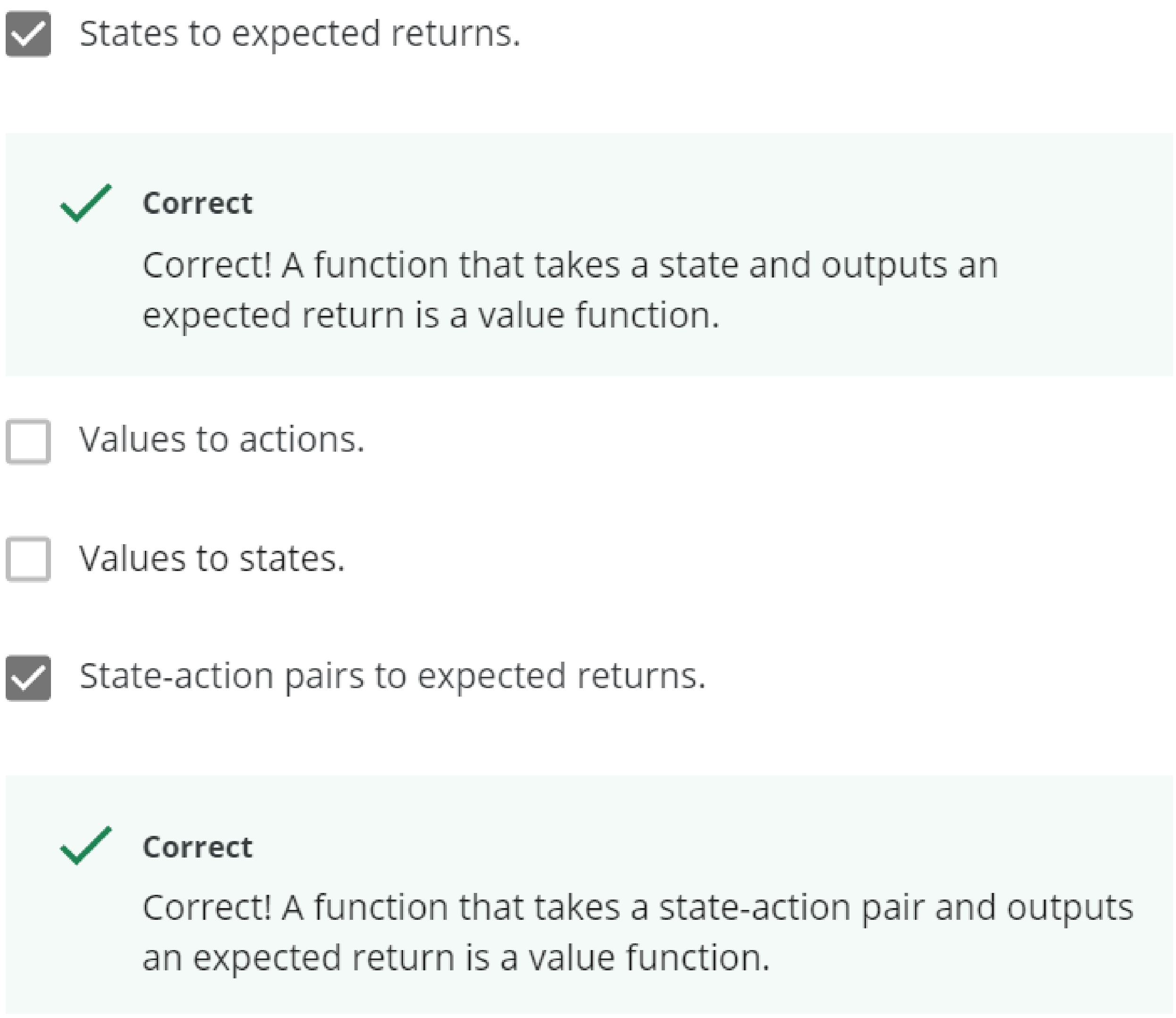
Quiz03-Value-Functions-and-Bellman-Equations

1. A function which maps _ to _ is a value function. [Select all that apply]



☑ States to expected returns.

✓ **Correct**

Correct! A function that takes a state and outputs an expected return is a value function.

□ Values to actions.

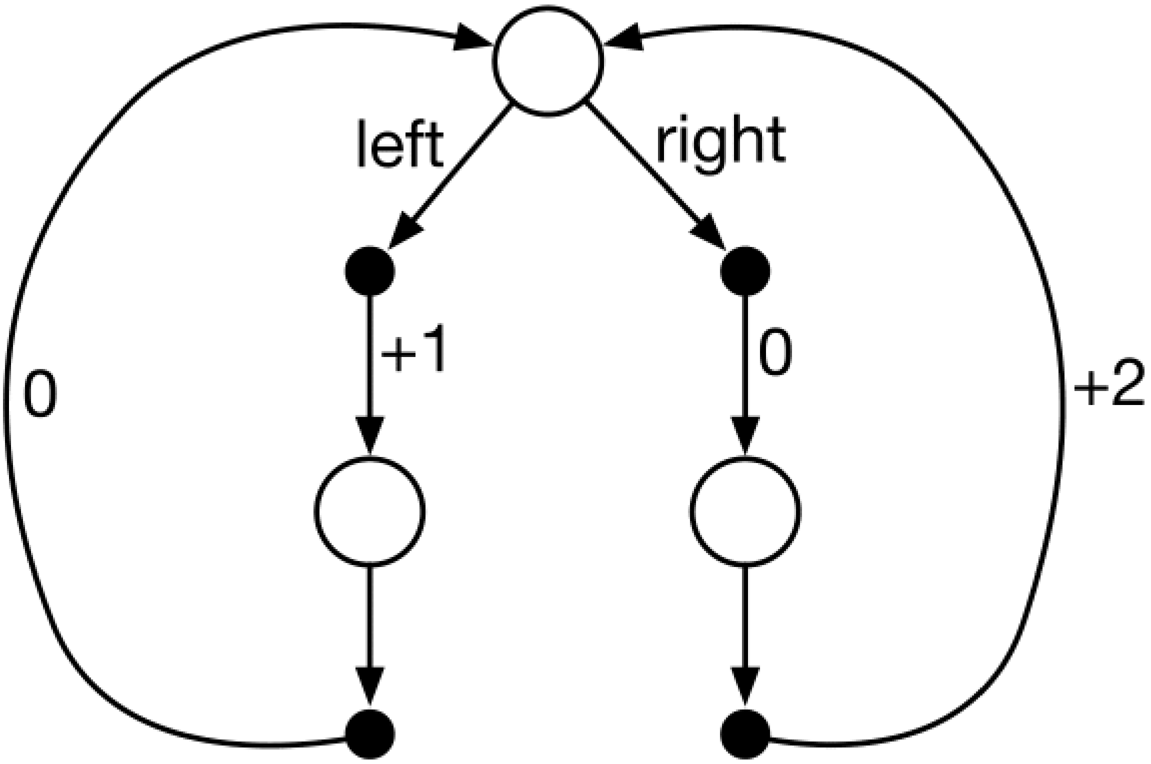
□ Values to states.

☑ State-action pairs to expected returns.

✓ **Correct**

Correct! A function that takes a state-action pair and outputs an expected return is a value function.

2. Consider the continuing Markov decision process shown below. The only decision to be made is in the top state, where two actions are available, left and right. The numbers show the rewards that are received deterministically after each action. There are exactly two deterministic policies, π_{left} and π_{right}. Indicate the optimal policies if $\gamma = 0$? If $\gamma = 0.9$? If $\gamma = 0.5$? [Select all that apply]



☑ For $\gamma = 0.5$, π_{left}

✓ **Correct**

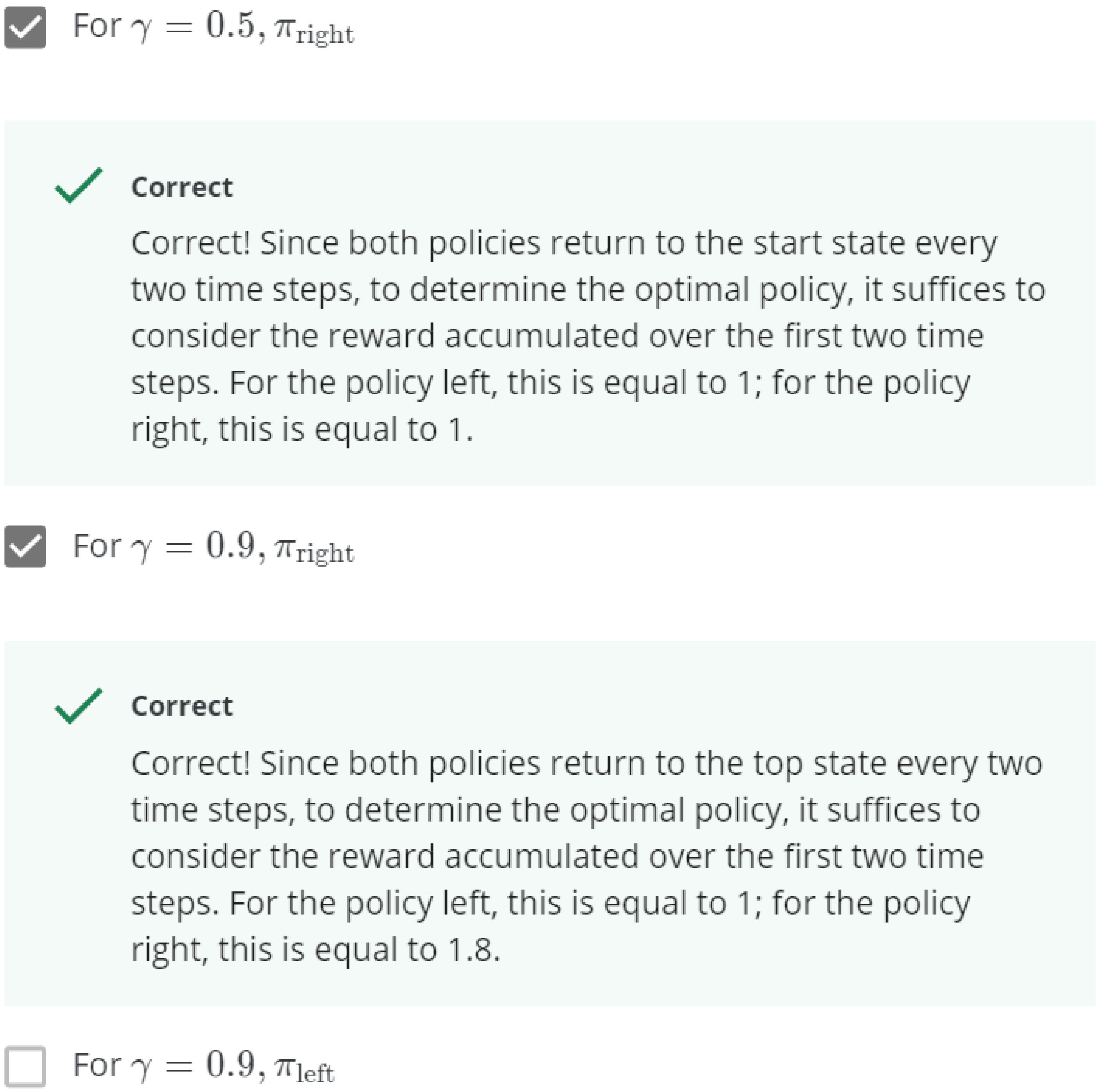
Correct! Since both policies return to the start state every two time steps, to determine the optimal policy, it suffices to consider the reward accumulated over the first two time steps. For the policy left, this is equal to 1; for the policy right, this is equal to 1.

□ For $\gamma = 0$, π_{right}

☑ For $\gamma = 0$, π_{left}

✓ **Correct**

Correct! Since both policies return to the top state every two time steps, to determine the optimal policy, it suffices to consider the reward accumulated over the first two time steps. For the policy left, this is equal to 1; for the policy right, this is equal to 0.



☑ For $\gamma = 0.5$, π_{right}

✓ **Correct**

Correct! Since both policies return to the start state every two time steps, to determine the optimal policy, it suffices to consider the reward accumulated over the first two time steps. For the policy left, this is equal to 1; for the policy right, this is equal to 1.

☑ For $\gamma = 0.9$, π_{right}

✓ **Correct**

Correct! Since both policies return to the top state every two time steps, to determine the optimal policy, it suffices to consider the reward accumulated over the first two time steps. For the policy left, this is equal to 1; for the policy right, this is equal to 1.8.

□ For $\gamma = 0.9$, π_{left}

3. Every finite Markov decision process has __. [Select all that apply]

□ A unique optimal policy

☑ A unique optimal value function

✓ **Correct**

Correct! The Bellman optimality equation is actually a system of equations, one for each state, so if there are N states, then there are N equations in N unknowns. If the dynamics of the environment are known, then in principle one can solve this system of equations for the optimal value function using any one of a variety of methods for solving systems of nonlinear equations. All optimal policies share the same optimal state-value function.

☑ A deterministic optimal policy

✓ **Correct**

Correct! Let's say there is a policy π_1 which does well in some states, while policy π_2 does well in others. We could combine these policies into a third policy π_3, which always chooses actions according to whichever of policy π_1 and π_2 has the highest value in the current state. π_3 will necessarily have a value greater than or equal to both π_1 and π_2 in every state! So we will never have a situation where doing well in one state requires sacrificing value in another. Because of this, there always exists some policy which is best in every state. This is of course only an informal argument, but there is in fact a rigorous proof showing that there must always exist at least one optimal deterministic policy.

4. The _ of the reward for each state-action pair, the dynamics function p, and the policy $\pi$is ___ to characterize the value function v_π. (Remember that the value of a policy π at state s is

$v_\pi(s) = \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)[r + \gamma v_\pi(s')].)$

○ Distribution; necessary

● Mean; sufficient

✓ **Correct**

Correct! If we have the expected reward for each state-action pair, we can compute the expected return under any policy.

5. The Bellman equation for a given a policy \pi$\pi$: [Select all that apply]

☑ Expresses state values $v(s)$ in terms of state values of successor states.

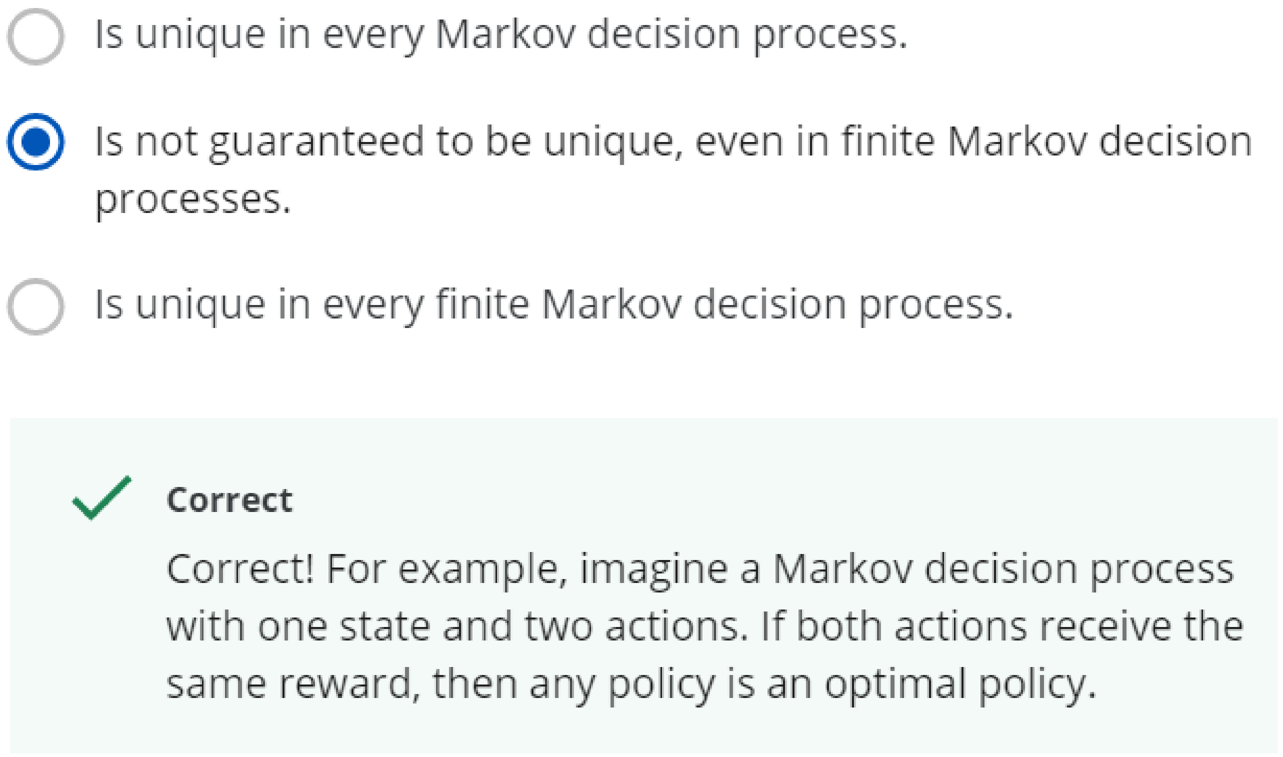
✓ **Correct**

Correct!

□ Holds only when the policy is greedy with respect to the value function.

□ Expresses the improved policy in terms of the existing policy.

6. An optimal policy:



○ Is unique in every Markov decision process.

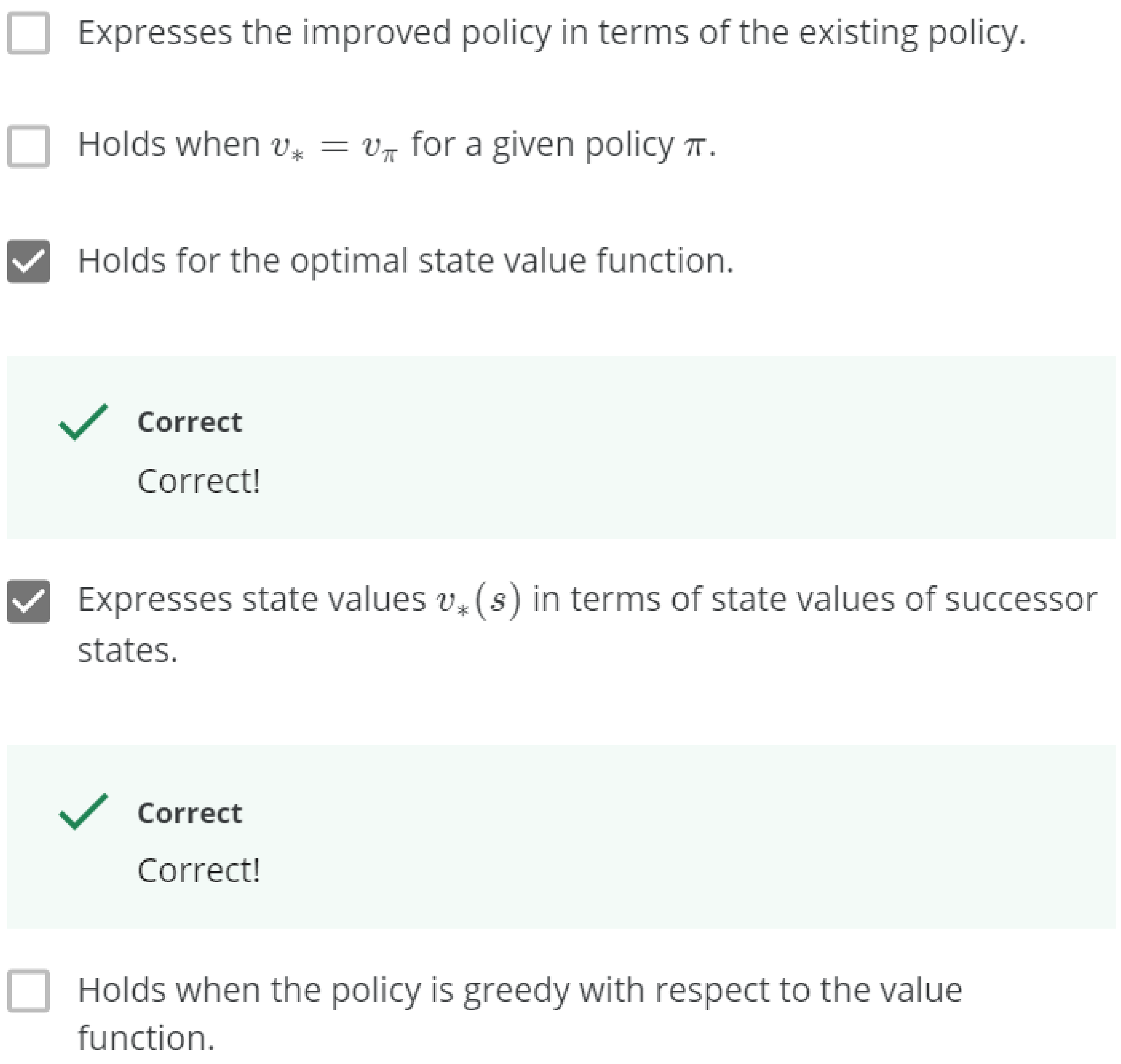
● Is not guaranteed to be unique, even in finite Markov decision processes.

○ Is unique in every finite Markov decision process.

✓ **Correct**

Correct! For example, imagine a Markov decision process with one state and two actions. If both actions receive the same reward, then any policy is an optimal policy.

7. The Bellman optimality equation for v_* : [Select all that apply]



□ Expresses the improved policy in terms of the existing policy.

□ Holds when $v_* = v_\pi$ for a given policy π.

☑ Holds for the optimal state value function.

✓ **Correct**

Correct!

☑ Expresses state values $v_*(s)$ in terms of state values of successor states.

✓ **Correct**

Correct!

□ Holds when the policy is greedy with respect to the value function.

8. Give an equation for v_π in terms of $q_\pi$and π.

○ $v_\pi(s) = \max_a \pi(a|s)q_\pi(s,a)$

○ $v_\pi(s) = \sum_a \gamma\pi(a|s)q_\pi(s,a)$

● $v_\pi(s) = \sum_a \pi(a|s)q_\pi(s,a)$

○ $v_\pi(s) = \max_a \gamma\pi(a|s)q_\pi(s,a)$

✓ **Correct**

Correct!

9. Give an equation for q_π in terms of v_π and the four-argument p.

○ $q_\pi(s,a) = \sum_{s',r} p(s',r|s,a)\gamma[r + v_\pi(s')]$

○ $q_\pi(s,a) = \sum_{s',r} p(s',r|s,a)[r + v_\pi(s')]$

○ $q_\pi(s,a) = \max_{s',r} p(s',r|s,a)[r + v_\pi(s')]$

○ $q_\pi(s,a) = \max_{s',r} p(s',r|s,a)[r + \gamma v_\pi(s')]$

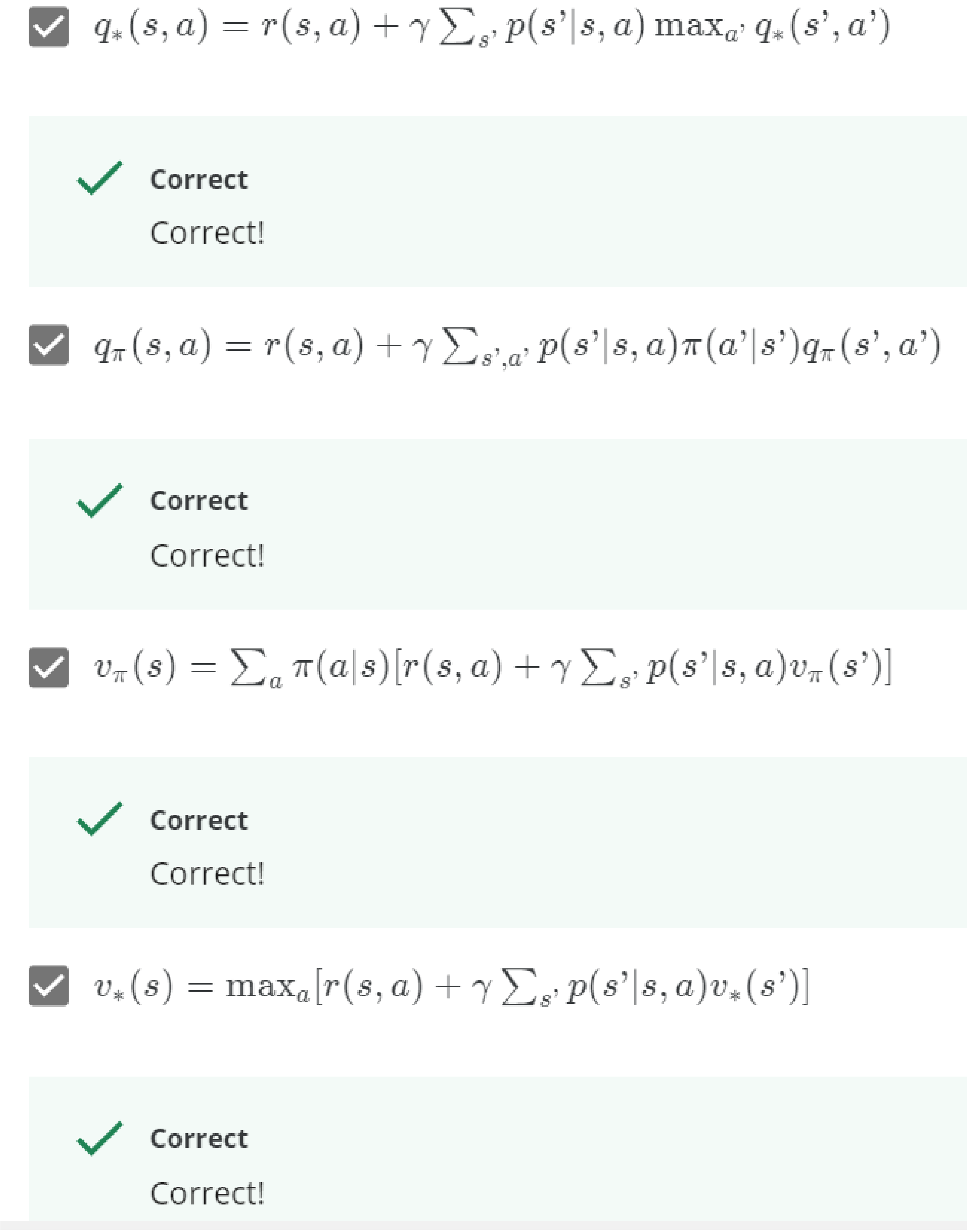
○ $q_\pi(s,a) = \max_{s',r} p(s',r|s,a)\gamma[r + v_\pi(s')]$

● $q_\pi(s,a) = \sum_{s',r} p(s',r|s,a)[r + \gamma v_\pi(s')]$

✓ **Correct**

Correct!

10. Let $r(s, a)$ be the expected reward for taking action aa in state s, as defined in equation 3.5 of the textbook. Which of the following are valid ways to re-express the Bellman equations, using this expected reward function? [Select all that apply]



☑ $q_*(s,a) = r(s,a) + \gamma \sum_{s'} p(s'|s,a) \max_{a'} q_*(s',a')$

✓ **Correct**

Correct!

☑ $q_\pi(s,a) = r(s,a) + \gamma \sum_{s',a'} p(s'|s,a)\pi(a'|s')q_\pi(s',a')$

✓ **Correct**

Correct!

☑ $v_\pi(s) = \sum_a \pi(a|s)[r(s,a) + \gamma \sum_{s'} p(s'|s,a)v_\pi(s')]$

✓ **Correct**

Correct!

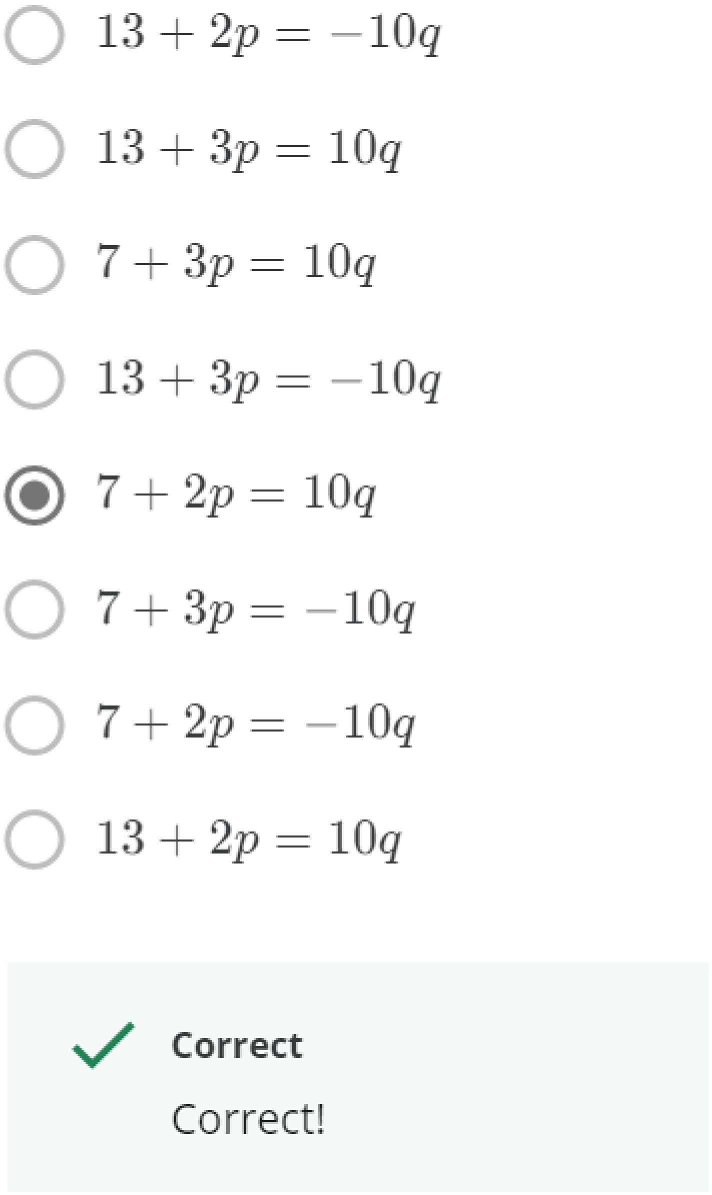
☑ $v_*(s) = \max_a [r(s,a) + \gamma \sum_{s'} p(s'|s,a)v_*(s')]$

✓ **Correct**

Correct!

11. Consider an episodic MDP with one state and two actions (left and right). The left action has stochastic reward 11 with probability p and 3 with probability $1-p$. The right action has stochastic reward 0 with probability q and 10 with probability $1-q$. What relationship between p and q makes the actions equally optimal?

$$\begin{aligned} p+3-3p &= 10-10q \\ -2p+3 &= 10-10q \\ -2p-7 &= -10q \\ 7+2p &= 10q \end{aligned}$$



○ $13+2p=-10q$

○ $13+3p=10q$

○ $7+3p=10q$

○ $13+3p=-10q$

● $7+2p=10q$

○ $7+3p=-10q$

○ $7+2p=-10q$

○ $13+2p=10q$

✓ **Correct**

Correct!