# MGMTMFE 400: Investments
# Mock Midterm Exam

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## Instructions

The midterm exam is a written, in-person, closed-book exam, but you can bring a one-page (double-sided) "cheat sheet."

You may use a financial calculator that does not have access to the internet.

The length of the exam is 2 hours.

Show your work. Partial credit is given to answers that are numerically incorrect but that show a correct understanding of the solution method.

If the question is not clear, state your assumptions and if they are reasonable, you will be given credit.

A total of 100 points are available for this exam. Allocate your time optimally.

Make sure to write your student ID on every page of the exam.

## Problems

1. The capital budgeting committee of Magna Corporation is contemplating a new investment. Given the project's characteristics, the cost of capital is estimated at 7% per year. The project requires an initial capital expenditure of \$15 million. A year later the project will generate a cash flow equal to \$1 million. Cash flows will then grow at a rate of 2% forever.

   (a) Compute the NPV of this project. (5 points)

   (b) Based on the NPV, should Magna invest? Why? (5 points)

2. Suppose that bond markets reveal the following discount factors:

| **Time** (in years) | **Discount factor** (in \$) |
|---|---|
| 1 | 0.9804 |
| 2 | 0.9518 |
| 3 | 0.9151 |

   (a) Compute the corresponding spot rates (i.e., $r_1$, $r_2$, and $r_3$). (5 points)

   (b) Compute the present value of a three-year annuity with annual payments of \$100. (5 points)

   (c) You buy a three-year zero-coupon bond and sell it after one year. If the return on your investment turns out to be 4%, what is the two-year spot rate one year from now? (10 points)

3. You observe 5- and 10-year zero-coupon bonds with yields of 3% per year. However, you are convinced that the current spread between the two bonds (the 10-year rate minus the 5-year rate) will widen over the next six months.

   (a) Compute the price and modified duration of these two bonds. Assume annual interest compounding. (10 points)

   (b) Despite your strong view of the yield curve, you have no view on the level of interest rates over the next six months, and worry that both rises and falls are possible. You decide to construct a combination of the two bonds that has no duration exposure to general interest rate movements, but benefits from an increase in 10-year yields relative to 5-years. What does this combination look like? [**Hint**: Use the prices and durations computed in (a) to set up a portfolio with zero dollar sensitivity to overall yield changes. Such a portfolio may cost something and, if so, you need to invest the required amount.] (10 points)

(c) Suppose you invest in the portfolio in (b). Which interest rate scenario(s) do you fear? That is, in which scenario(s) will you lose money? When interest rates generally increase/decrease? What risk measure could capture the scenario(s)? This part of the problem does not require any calculations, but try to be explicit when answering. (5 points)

4. You are about to price and evaluate the riskiness of a two-year interest rate swap. The buyer is paying fixed (annual payments) and receiving floating (semi-annual payments). The payments on the floating rate bond are linked to a six-month money market rate. The cash flows of the swap are calibrated such that the initial value of the swap is a zero. Assume that the notional amount on the swap is $100. Suppose your research department has calculated the following discount factors:

| **Time** (in years) | **Discount factor** (in $) |
|---|---|
| 0.5 | 0.9806 |
| 1 | 0.9615 |
| 1.5 | 0.9429 |
| 2 | 0.9246 |

(a) What is the two-year swap rate? Explain how you arrive at the answer. (10 points)

(b) From the buyer's perspective, what is the dollar duration of the swap? To assist you, the research department has computed the modified duration of a six-month zero-coupon bond to be 0.490 and the two-year par bond with annual payments to be 1.886. (10 points)

(c) Interpret how a change in interest rates affect the value of the swap (immediately after the initialization). (5 points)

5. On market closing June 26, 2023, the share price of Coca-Cola Co was \$61.22. At the same time, the trailing annual dividend per share was \$1.78 and analysts' expectations suggested a stable 3% per year growth rate in dividends thereafter. There were 4.32 billion shares outstanding. The analysts finally provided an estimate of Coca Cola's cost of equity of 6.25% per year.

   (a) What is the market capitalization of Coca-Cola? (5 points)

   (b) What is the share price according to the Gordon growth model? (5 points)

   (c) How would you adjust the estimated cost of capital to make the model price equal the market price? Provide calculations. (5 points)

   (d) A colleague suggests that you, as a complement, infer a price estimate from the P/E ratios of Coca-Cola and its peers. You note that the Coca-Cola P/E ratio is 23.6 and that the industry prospective P/E ratios are in the range 19.7–25.0. What price range do you infer for Coca Cola? (5 points)

## Suggested solutions

Note that computations below are exact. If you compute in steps it is easy to get rounding errors.

1.

   (a) The NPV is given by:

$$\text{NPV} = c_0 + \frac{c_1}{r - g} = –\$15.0\text{m} + \frac{\$1.0\text{m}}{0.07 - 0.02} = \$5.0\text{m}.$$

   (b) As the NPV is positive it is a "go." The investment creates value in expectations.

2.

   (a) The spot rates are given from the discount factors:

$$d_j = \frac{1}{(1 + r_j)^j} \qquad \text{or} \qquad r_j = \left(\frac{1}{d_j}\right)^{(1/j)} - 1.$$

   The spot rates are 2.00%, 2.50%, and 3.00%.

   (b) The present value of the annuity (with payments of \$100 for three years) are:

$$\begin{aligned} \text{PV} &= 100 \times d_1 + 100 \times d_2 + 100 \times d_3 \\ &= 100 \times 0.9804 + 100 \times 0.9518 + 100 \times 0.9151 \\ &= 284.73. \end{aligned}$$

   That is, the present value of the annuity is \$284.73.

   (c) The one-year holding period return solves the following equation:

$$R_1 = \frac{P_{sales}}{P_{purchase}} - 1,$$

   which should be 4% (given in the problem). The purchase price is:

$$P_{purchase} = 100 \times d_3 = 100 \times 0.9151 = 91.51.$$

The sales price in one year is:

$$P_{sales} = \frac{100}{(1 + r_2(1))^2},$$

where $r_2(1)$ is the two-year spot rate in one year. You then find $r_2(1)$ by solving the following equation:

$$\frac{\frac{100}{(1+r_2(1))^2}}{0.9151 \times 100} = 1.04.$$

Solving for $r_2(1)$ gives a future two-year spot rate of 2.51%. Note that the lower future spot rate results in a high sales price and a high return (higher than the spot rate at the time of the purchase).

3. This is a yield spread trade. Note that the yield curve is flat so yields equal spot interest rates.

   (a) The prices are given by:
   $$P_j = \frac{100}{(1 + r_j)^j}.$$
   For the 5-year bond the price is \$86.2609 and for the 10-year bond the price is \$74.4094. The Macaulay durations (the $D_j^{Macaulay}$s) for the two bonds equal their maturities, that is, 5 and 10 years. Their modified durations are given by:
   $$D_j = \frac{D_j^{Macaulay}}{(1 + r_j)}.$$
   They are 4.854 and 9.709 for the 5-year and 10-year bond, respectively. Importantly, the 10-year bond has greater interest rate sensitivity than the 5-year bond.

   (b) To benefit from a widening of the spread, you have to go *long* the 5-year and *short* the 10-year. The intuition is that the spot rate on the 10-year bond increase relative the yield on the 5-year bond, which means that the price on the 10-year bond decreases relative the price of the 5-year bond. As you have no strong view on the level of interest rates, you want to be duration neutral. The trick is to find quantities of the two bonds that leaves you with no sensitivity to equal movements in both yields.

   By investing in $x_5$ number of 5-year bonds and in $x_{10}$ number of 10-year bonds, the portfolio value is:

$$P_P = x_5 P_5 + x_{10} P_{10}$$

The change in value is given by:

$$\Delta P_P = x_5 \Delta P_5 + x_{10} \Delta P_{10}$$

Recall that:

$$\Delta P \approx -D \times P \times \Delta y.$$

The task is then to find quantities $x_5$ and $x_{10}$ such that the portfolio is not sensitive to parallel decreases or increases in yields, where $x_5$ is positive (long position) and $x_{10}$ is negative (short position). That is, solve for $x_5$ and $x_{10}$ in:

$$0 = x_5 D_5 P_5 + x_{10} D_{10} P_{10}.$$

The ratio between $x_5$ and $x_{10}$ is thus:

$$\frac{x_5}{x_{10}} = -\frac{D_{10} P_{10}}{D_5 P_5} = -1.725.$$

This means that for each 10-year bond you short, you have to buy 1.725 of the 5-year bond.

(c) It is not necessary to show any calculations when answering this question (economic reasoning is enough). The trade is set up to make money if the spread widens (i.e., the yield curve becomes steeper). It obviously loses money if there is a inversion of the yield curve. As the trade is duration neutral, you (to a first-order approximation) do not make or lose money with parallel shift in the yield curve. However, there is also negative convexity in the trade so (to a second-order approximation) you lose money with parallel shifts.

Below I illustrate the more subtle convexity effect. The answer includes calculations (but they are not necessary to get maximum points). If you short one of the 10-year bond and buy 1.725 of the 5-year bond, you have to invest:

$$\begin{aligned} P_P &= x_5 P_5 + x_{10} P_{10} \\ &= 1.725 \times \$86.2609 - 1 \times \$74.4094 \\ &= \$74.4094. \end{aligned}$$

Note that while this portfolio has zero duration by construction, it has a negative convexity:

$$\begin{aligned} C_P &= \frac{x_5 P_5}{P_P} C_5 + \frac{x_{10} P_{10}}{P_P} C_{10} \\ &= \frac{1.725 \times 86.2609}{74.4094} 28.278 + \frac{-1 \times 74.4094}{74.4094} 103.686 \\ &= -47.130, \end{aligned}$$

where the convexity of the two bonds are given by:

$$C_5 = \frac{5 \times 6}{1.03^2} = 28.278$$

and

$$C_{10} = \frac{10 \times 11}{1.03^2} = 103.686.$$

As the portfolio has negative convexity, you would lose money on parallel shifts (that is, you lose on both general yield decreases and general yield increases).

4.

a) The initial price of the swap is zero. A swap can be seen as a package of a floating rate bond and a fixed rate bond. From the buyer's perspective, we have:

$$P_{swap} = P_{float} - P_{fixed}.$$

Note that at the time of reset the floater sells at par ($P_{float} = \$100$). This means that the price of the fixed rate bond also needs to be \$100 (a par bond) for the swap to be a zero-cost investment. Hence, the coupon rate on the fixed rate bond should be equal to the yield. It is straightforward to show that the yield (and the swap rate) then equals:

$$y_{swap} = \frac{1 - d_n}{\sum_{j=1}^{n} d_j},$$

where $d_j$ is the discount factor of cash flow $j$ (here in period $j$ and also year $j$). Plugging in numbers for the discount factors on cash flows in one and two years (the fixed rate bond has annual payments, so $j = 1, 2$ and $n = 2$), we have:

$$y_{swap} = \frac{1 - 0.9246}{0.9615 + 0.9246} = 0.04 = 4\%.$$

That is, the two-year swap rate is 4%.

b) To get the dollar duration of the swap, you need to realize that the dollar duration of the swap equals the dollar duration of the floating rate bond minus the dollar duration of the fixed rate bond, that is,

$$\text{Dollar duration of swap} = D_{float} \times P_{float} - D_{fixed} \times P_{fixed}.$$

The prices of the two bonds are both equal to \$100. Note that the modified duration of the floating rate bond equals the Macaulay duration of the bond divided by one plus the yield/2. The durations are given in the problem: 0.490 and 1.886 (you do not need to compute them, but can confirm them using standard modified duration calculations). Taken together, we have the dollar duration of the swap being equal to:

$$\text{Dollar duration of swap} = 0.490 \times 100 - 1.886 \times 100 = -139.6.$$

Using the usual approximation:

$$\Delta P_{swap} \approx -[D_{float} \times P_{float} - D_{fixed} \times P_{fixed}]\Delta y.$$

c) The above expression can be interpreted as follows: An increase in the yield with one percentage point results in an increase of the swap value with \$1.396. Note that this makes sense as the buyer pays fixed and receives floating. An increase in the yield makes the swap more valuable (going from a zero value to a positive value) for the buyer who receives higher (semi-annual) payments and pays fixed.

5.

(a) The market capitalization is the number of shares times the price per share:

MCap = #shares × price per share = 4.32 billion × \$61.22 = \$264.47 billion.

(b) Based on the current dividend of \$1.78, the cost of equity of 6.25%, and the growth rate of 3.0%, the Gordon growth model gives a price per share according to:

$$P_0 = \frac{D_0(1+g)}{r\text{–}g} = \frac{\$1.78(1+0.03)}{0.0625\text{–}0.03} = \$56.41.$$

(c) The Gordon growth model gives a lower price (\$56.41) than the market price (\$61.22). Given the current dividend (observed), a lower $r$ or a higher $g$ can adjust the model price so that it equals the actual market price. The dividend-price ratio equals 2.91% (= 1.78/61.22). The implied required rate of return is:

$$r = g + (1+g)\frac{D_0}{P_0} = 3\% + 1.03 \times 2.91\% = 5.99\%.$$

(d) The earnings per share of Coca-Cola are \$61.22/23.6 = \$2.59. With a low industry P/E ratio of 19.7, the peer comparison would give a low price of 19.7×\$2.59 = \$51.10. With a high industry P/E ratio of 25.0, the peer comparison would give a high price of 25.0×\$2.59 = \$64.85. That is, the Coca-Cola price per share of \$61.22 is in the range suggested by a so-called multiple valuation (it can also be seen from Coca-Cola's P/E ratio, which is inside the industry P/E ratio range).