Assignment 2

In this assignment you will continue to solve fixed-income problems and an equity valuation problem.

1. You will receive \$1 million one year from now, but will not need to use it until three years from now. You thus need to invest the \$1 million for two years.

   (a) Suppose that you want to guarantee a rate today for your investment. What is the relevant forward rate?

   (b) Suppose that the one-year spot rate is 1% and the three-year spot rate is 3%. Compute the forward rate in (a).

   (c) Suppose that you are unable to contact a bank that offers forward rates. You can contact, however, a dealer who trades zero-coupon bonds. Explain how you can use zero-coupon bonds to lock in the forward rate in (a).

2. Consider the following four US Treasury bonds (the par value is \$100):

   Bond A is a 5-year bond with a 1% annual coupon rate.

   Bond B is a 10-year bond with a 1% annual coupon rate.

   Bond C is a 5-year bond with a 4% annual coupon rate.

   Bond D is a 10-year bond with a 4% annual coupon rate.

   Assume all of the bonds are currently trading at a 3% yield and, as their maturity is over one year; they pay semi-annual coupons. Recall that if a bond has a 4% annual coupon rate, it pays 2% of par every six months.

   (a) Compute the current prices of the bonds.

   (b) Suppose the yield increases from 3% to 3.5%. Compute the new prices and compute the change in the bond's price.

   (c) Suppose the yield decreases from 3% to 2.5%. Compute the new prices and compute the change in the bond's price.

   (d) Compute durations of each of the bonds and answer (b) and (c) using duration.

   (e) What can you conclude about reaction of bond prices to changes in yields for bonds with different coupons and maturities?

3. You just sold to a client a custom-made 31-year bond which pays a single coupon of \$1 million 30 years from now and \$2 million at maturity. You would like to hedge this liability. Assume that the term structure is currently flat at 5%.

   (a) You first consider replicating the liability synthetically. Construct the hedging portfolio consisting of the following bonds paying annual coupons:

Bond	Coupon rate (in %)	Maturity
A	0	31 years
B	4	30 years
C	6	30 years

   The par values of the three bonds are \$100.

   (b) You next consider duration-based hedging. Assume that you have 10- and 15-year zero-coupon bonds available, with par values \$100. Construct the hedging portfolio of the two bonds that has the same market value and the same interest-rate sensitivity (measured by duration as a first-order approximation) as your liability.

   (c) Suppose the interest rate fell to 4.8%. Would your hedging portfolio computed in part (b) remain the same? If not, explain why not and comment on the composition of the new hedging portfolio relative to the old one. (No computations are necessary.)

   (d) What are the advantages of the synthetic replication in (a) relative to the approximate hedging in (b)? Why might one use the approximate hedging instead? (No computations are necessary.)

4. For this question, assume that the current yield curve is flat at 2% and that all bonds are issued with a face value of \$100.

   (a) What is the price, modified duration, and convexity of a one-year 2% coupon bond with a single coupon payment at the end of the year?

   (b) What is the price, modified duration, and convexity of a two-year 2% coupon bond with annual coupon payments at the end of years one and two?

   (c) Consider a two-year annual-pay floating rate note (FRN). The cash flows take the following structure:

- At $t = 0$, the first coupon payment is locked in at $y_1 \times F$, where F is the face value and y_1 is the one-year spot rate (currently 2%).
- At $t = 1$, the issuer makes the first coupon payment and the second coupon payment is locked in at the one year spot rate as reported at the end of the first year.
- At $t = 2$, the issuer pays the second coupon payment along with the note's face value.

What is the price of the FRN at issuance?

(d) Define the modified duration and convexity as $D = -\frac{1}{P}\frac{dP}{dy}$ and $C = \frac{1}{P}\frac{d^2P}{dy^2}$, respectively. These definitions are equivalent to that of the lecture slides when applied to fixed rate bonds, but it also extends the concept of duration and convexity to other types of securities. What is the modified duration of the FRN just after issuance? What is its convexity?

Hint: Consider how the price of the FRN changes with a small change to yields.

(e) Provide intuition for your results in parts (c) and (d) as they relate to your answers from parts (a) and (b).

5. A firm has an expected dividend per share next year of \$2. It has an expected earnings per share (EPS) next year of \$4, which is expected to grow at a rate of 4% per year forever. Its cost of equity is 8% per year. Its return on equity (ROE) is 8% per year.

   (a) What is the firm's share price?

   (b) What is the sustainable growth rate with this level of payout?

   (c) What is the firm's expected dividend yield?

   (d) What is the sum of the dividend yield and the growth rate? Explain why this sum is what it is.

   (e) Suppose that the firm switches to paying out all its earnings. What will then be its sustainable growth rate?

   (f) What will then be its share price?

   (g) Can you explain the relationship between the answers to (a) and (f)?
   Hint: What is the NPV of any investment the firm makes in order to grow?