MGMTMFE 400 Investment

Homework 2

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Group 2:

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In [1]:
```
import numpy as np
import sympy as sp
```

Assignment 2

Problem 1

You will receive $\$1$ million one year from now, but will not need to use it until three years from now. You thus need to invest the $\$1$ million for two years.

(a) To guarantee a rate today for your investment, what is the relevant forward rate?

If we want to guarantee a rate today for the investment, the relevant rate is the **forward rate for the two-year period starting one year from now** and ending three years from now, which is $f_{1,3}$.

(b) The one-year spot rate is 1% and the three-year spot rate is 3%, compute the forward rate in (a).

$$f_{1,3} = \sqrt{\frac{1.03^3}{1.01}} - 1 = 4.0148\%$$

In [2]:
```
# Given spot rates
# s1: One-year spot rate
s1 = 0.01   # 1%
# s3: Three-year spot rate
s3 = 0.03   # 3%

# Calculate forward rate
f2 = (((1 + s3)**3 / (1 + s1))**(1/2)) - 1

print(f"Two-year forward rate starting one year from now: {f2 * 100
```

```
Two-year forward rate starting one year from now: 4.0148%
```

(c) Explain how to use zero-coupon bonds to lock in the forward rate in (a).

We want to use zero-coupon bonds to replicated the return of forward rate, by **selling 1 year bond for money to buy the 3 years bonds that pay back with the same amount that reinvesting with forward rates does**.

At year 0, issue 10,000 one year zero-coupon bonds at par, the payment at maturity should right equal to 1 million, the amount that will be received in time year 1. Therefore the price of this 1 year zero-coupon bond currently at time year 0 is:

$$\frac{1,000,000}{1+1\%} = \$990,099.01$$

Then buy a 3 year zero-coupon bond with this amount of cash.

At year 1, pay the 1 million payment of 1 year zero-coupon bond with the money received.

At year 3, the payment from 3 year zero-coupon bond will be:

$$990,099.01 \times (1+3\%)^3 = \$1,081,907.92$$

then the 2 year return

$$\sqrt{\frac{1,081,907.92}{1,000,000}} - 1 = 4.0148\%$$

gives the same rate of return as the forward rate $f_{1,3}$ does.

In [3]:
```
# future value in year 1
fv = 1000000
# Calculate present year 1 bond prices
one_year_pv = fv / (1 + s1)**1
# Calculate the future value of the 3 years bond
three_year_fv = one_year_pv * (1 + s3) ** 3

# return
f2 = (three_year_fv / fv) **(1/2) - 1

print(f"Price of the one-year bond present value: ${one_year_pv:.2f
print(f"Price of the three-year bond future value: ${three_year_fv:

print(f"f2: {f2*100:.2f}%")
```

```
Price of the one-year bond present value: $990099.01
Price of the three-year bond future value: $1081907.92
f2: 4.01%
```

Problem 2

Consider the following four US Treasury bonds (the par value is $\$100$:

Bond A is a 5-year bond with a 1% annual coupon rate.
Bond B is a 10-year bond with a 1% annual coupon rate.
Bond C is a 5-year bond with a 4% annual coupon rate.
Bond D is a 10-year bond with a 4% annual coupon rate.

Currently trade at 3%, pay semi-annual coupons.

(a) Compute the current prices of bonds

The coupon payment is $\$100 \times (\frac{1\%}{2} = \$0.50$ for Bond A and B; the coupon payment is $\$100 \times (\frac{4\%}{2} = \$2.00$ for Bond C and D.

Then calculate the prices:

$$\mathbf{P}_A = \sum_{n=1}^{10} \left[\frac{0.5}{(1+\frac{3\%}{2})^n} \right] + \frac{100.5}{(1+\frac{3\%}{2})^{10}} = \$90.78$$

$$\mathbf{P}_B = \sum_{n=1}^{20} \left[\frac{0.5}{(1+\frac{3\%}{2})^n} \right] + \frac{100.5}{(1+\frac{3\%}{2})^{20}} = \$82.83$$

$$\mathbf{P}_C = \sum_{n=1}^{10} \left[\frac{2}{(1+\frac{3\%}{2})^n} \right] + \frac{102}{(1+\frac{3\%}{2})^{10}} = \$104.61$$

$$\mathbf{P}_D = \sum_{n=1}^{20} \left[\frac{2}{(1+\frac{3\%}{2})^n} \right] + \frac{102}{(1+\frac{3\%}{2})^{20}} = \$108.58$$

In [4]:
```
import numpy_financial as npf #imported the library to calculate IR

par = 100
def bond_pricing(maturity,coupon,y):
    return -(npf.pv(y/2,maturity*2,coupon/2,par))

bondA = bond_pricing(5,1,0.03)
bondB = bond_pricing(10,1,0.03)
bondC = bond_pricing(5,4,0.03)
bondD = bond_pricing(10,4,0.03)

print(f'The prices of bonds with 3% yield are :\n Bond A = ${bondA:
```

```
The prices of bonds with 3% yield are :
 Bond A = $90.78
 Bond B = $82.83
 Bond C = $104.61
 Bond D = $108.58
```

(b) Suppose the yield increase from 3% to 3.5%. Compute the new prices and the change in bond's price

When the yields increase, the **prices are expected to decrease**.

In [5]:
```
bondA_dec = bond_pricing(5,1,0.035)
bondB_dec = bond_pricing(10,1,0.035)
bondC_dec = bond_pricing(5,4,0.035)
bondD_dec = bond_pricing(10,4,0.035)

print(f'The prices of bonds with 3.5% yield are :\n Bond A = ${bond
```

```
The prices of bonds with 3.5% yield are :
 Bond A = $88.62
 Bond B = $79.06
 Bond C = $102.28
 Bond D = $104.19
```

In [6]:
```
print(f'The changes in prices of the bonds because of yield change
```

```
The changes in prices of the bonds because of yield change are :
 Bond A = $-2.15
 Bond B = $-3.77
 Bond C = $-2.34
 Bond D = $-4.40
```

(c) Suppose the yield increase from 3% to 2.5%. Compute the new prices and the change in bond's price

When the yields decrease, the **prices are expected to increase**.

In [7]:
```
bondA_inc = bond_pricing(5,1,0.025)
bondB_inc = bond_pricing(10,1,0.025)
bondC_inc = bond_pricing(5,4,0.025)
bondD_inc = bond_pricing(10,4,0.025)

print(f'The prices of bonds with 2.5% yield are :\n Bond A = ${bond
```

```
The prices of bonds with 2.5% yield are :
 Bond A = $92.99
 Bond B = $86.80
 Bond C = $107.01
 Bond D = $113.20
```

In [8]:
```
print(f'The changes in prices of the bonds because of yield change
```

```
The changes in prices of the bonds because of yield change are :
 Bond A = $2.21
 Bond B = $3.97
 Bond C = $2.40
 Bond D = $4.62
```

(d) Compute durations of each of the bonds and use the durations to re-calculate (b) and (c)

$$D = \frac{1}{P} \frac{1}{(1+y)} \sum_{j=1}^{n} j \times \frac{c_j}{(1+y)^j}$$

In [9]:

```
def duration(price, t, n, y, coupon, par):
    j = t*n
    summation = 0
    for i in range(1,j):
        summation += i/2*coupon/(1+y/n)**i

    summation += j/2*(par+coupon)/(1+y/n)**j

    return ((1/price)/(1+y/n)*summation)
```

In [10]:

```
durA = duration(bondA, 5, 2, 0.03, 0.5, 100)
durB = duration(bondB, 10, 2, 0.03, 0.5, 100)
durC = duration(bondC, 5, 2, 0.03, 2, 100)
durD = duration(bondD, 10, 2, 0.03, 2, 100)

print(f'The durations of bonds are :\n Bond A = {durA:.4f}\n Bond B
```

```
The durations of bonds are :
 Bond A = 4.8104
 Bond B = 9.3420
 Bond C = 4.5246
 Bond D = 8.2953
```

Duration also denoted as

$$D = -\frac{dP}{dy}\frac{1}{P}$$

then we can calucate the change in price with 1% change on yield:

$$-\frac{dP}{dy} = D \times P$$

There's a negative sign because prices and yields are negatively related: as yields go down, prices increase and vice vera.

In [11]:
```
print(f'The changes in price of the bonds because of yield increase
print(f'Then new price of the bonds are:\n Bond A = ${bondA+(-durA*
```

```
The changes in price of the bonds because of yield increase are :
 Bond A = $-2.18
 Bond B = $-3.87
 Bond C = $-2.37
 Bond D = $-4.50

Then new price of the bonds are:
 Bond A = $88.59
 Bond B = $78.96
 Bond C = $102.24
 Bond D = $104.08
```

In [12]:
```
print(f'The change in price of the bond because of yield decrease i
print(f'Then new price of the bonds are:\n Bond A = ${bondA+(-durA*
```

```
The change in price of the bond because of yield decrease is :
 Bond A = 2.18
 Bond B = 3.87
 Bond C = 2.37
 Bond D = 4.50

Then new price of the bonds are:
 Bond A = $92.96
 Bond B = $86.70
 Bond C = $106.98
 Bond D = $113.09
```

The duration and price changes are slightly different than before as we have not accounted for convexity. Convexity will make the changes not in a linear relationship.

(e) Conclusion about reaction of bond prices to changes in yields for bonds with different coupons and maturities.

Generally, **the bonds with higher maturities and lesser coupons have higher durations** (they are more sensitive to interest rate changes)

Looking into details:
Keeping other things the same, the bond with **longer duration will be more sensitive** to the changes in yields, the prices will react with a larger changes.
Keeping other things the same, the bond with **larger coupons will be more sensitive** to the changes in yields, causing the prices to react in a larger changes.
The impact of longer duration is larger than larger coupons, since the difference between the changes of the two pairs is larger in bonds that vary on duration.

Problem 3

Sold a custom-made 31-year bond which pays a single coupon of $\$1$ million 30 years from now and $\$2$ million at maturity. Goal is to hedge the liability. Assume that the term structure is currently flat at 5%.

(a) Construct the hedging portfolio consisting of the following bonds paying annual coupons:

Bond A: coupon rate = 0%, maturity at 31 years
Bond B: coupon rate = 4%, maturity at 30 years
Bond C: coupon rate = 6%, maturity at 30 years

Par values are all $\$100$

We need a porfolio that contains bonds A, B, C such that the payments of their cash flows combined equals to the payments of the liability.

The present value of liability (in million):

$$PV_{Liability} = \frac{2}{(1+5\%)^{31}} + \frac{1}{(1+5\%)^{30}} = \$672,096,398$$

And the present values of bonds (in million):

$$PV_A = \frac{0}{5\%} \times \left[1 - \frac{1}{1.05^3 1}\right] + \frac{100}{1.05^3 1} = \$22.036$$

$$PV_B = \frac{4}{5\%} \times \left[1 - \frac{1}{1.05^3 0}\right] + \frac{100}{1.05^3 0} = \$84.63$$

$$PV_C = \frac{6}{5\%} \times \left[1 - \frac{1}{1.05^3 0}\right] + \frac{100}{1.05^3 0} = \$115.372$$

We here noticed that, bond A is a zero-coupon bond with maturity of 31 years, so band A will be respond to hedge out the $\$2$ million payments in year 31, then the coupons and par payments of bond B and C will be used to hedge the $\$1$ million payments.

Set $\mathbf{X}_A$ = amount of bond A, $\mathbf{X}_B$ = amount of bond B, $\mathbf{X}_C$ = amount of bond C

$$4\mathbf{X}_B + 6\mathbf{X}_C = 0$$

$$104\mathbf{X}_B + 106\mathbf{X}_C = 1,000,000$$

$$100\mathbf{X}_A = 2,000,000$$

We get

$$\mathbf{X}_A = 20,000$$
$$\mathbf{X}_B = 30,000$$
$$\mathbf{X}_C = -20,000$$

In [16]:

```
# python calculation for the system of equaitons of X_B and X_C
sp.init_printing()
# Define the variables
x_B, x_C= sp.symbols('x y')

# Set up the equation for the present value of the portfolio
f = sp.Eq(4*x_B+6*x_C,0)
g = sp.Eq(104*x_B+106*x_C,1000000)

# Solve for x, y, and z such that PV_custom = PV_portfolio
solution = sp.solve([f,g], (x_B, x_C))
print(solution)
```

```
{x: 30000, y: -20000}
```

Therefore, the hedging portfolio will be **purchasing 20,000 units of Bond A and 30,000 units of Bond B, and selling 20,000 units of Bond C**.

(b) Assume having 10- and 15-year zero-coupon bonds available, with par value $\$100$. Construct the hedging porfolio of the two bonds that has the same market value and the same interest-rate sensitivity.

For zero-coupon bonds, their Mac. Duration is the years of maturity. Therefore we have:

$$D_{10}^{Macaulay} = 10$$

$$D_{15}^{Macaulay} = 15$$

Then the Modified Durations are:

$$D_{10}^{Modified} = \frac{10}{1+5\%} = 9.5238$$

$$D_{15}^{Modified} = \frac{15}{1+5\%} = 14.2857$$

Also calculate the PV for zero-coupon bonds and liability:

$$PV_{10} = \frac{100}{1.05^{10}} = \$61.39$$

$$PV_{15} = \frac{100}{1.05^{15}} = \$48.10$$

$$PV_{Liability} = \frac{2,000,000}{(1+5\%)^{31}} + \frac{1,000,000}{(1+5\%)^{30}} = \$672,096.40$$

Use PV of Liability to calculate the Duration for the liability:

$$D_{Liability}^{Macaulay} = \frac{30}{672,096.40} \times \frac{1,000,000}{1.05^{30}} + \frac{31}{672,096.40} \times \frac{2,000,000}{1.05^{31}} = 30.6557$$

$$D_{Liability}^{Modified} = \frac{30.6557}{1.05} = 29.1959$$

In [17]:
```
# mac D for bonds
macD_10 = 10
macD_15 = 15
# modified D for bonds
MD_10 = macD_10/1.05
MD_15 = macD_15/1.05

# PV for bonds and liability
PV_10 = 100/1.05**10
PV_15 = 100/1.05**15
PV_L = 2000000/1.05**31 + 1000000/1.05**30

# mac D and modified D for liability
macD_L = 30/PV_L*(1000000/1.05**30) + 31/PV_L*(2000000/1.05**31)
MD_L = macD_L/1.05

print(f'10-year zero-coupon bond has the PV of ${PV_10:.2f} and the
print(f'15-year zero-coupon bond has the PV of ${PV_15:.2f} and the
print(f'The liability of 31-year coupon bond has the PV of ${PV_L:.
```

```
10-year zero-coupon bond has the PV of $61.39 and the modified dur
ation of 9.5238
15-year zero-coupon bond has the PV of $48.10 and the modified dur
ation of 14.2857
The liability of 31-year coupon bond has the PV of $672096.40 and
the modified duration of 29.1959
```

We know that

$$P_{Liability} = P_{10}\mathbf{X}_{10} + P_{15}\mathbf{X}_{15}$$

$$\Delta P_L = \Delta P_{10}\mathbf{X}_{10} + \Delta P_{15}\mathbf{X}_{15}$$

Plug in $\Delta P_i = -D_i^{Modified} \times P_i \times \Delta y$:

$$-D_L^{Modified} P_L \Delta y = -D_{10}^{Modified} P_{10} \Delta y \mathbf{X}_{10} - D_{15}^{Modified} P_{15} \Delta y \mathbf{X}_{15}$$

$$-D_L^{Modified} P_L = -D_{10}^{Modified} P_{10} \mathbf{X}_{10} - D_{15}^{Modified} P_{15} \mathbf{X}_{15}$$

Therefore, solve the equation system:

$$672,096.40 = 61.39\mathbf{X}_{10} + 48.10\mathbf{X}_{15}$$

$$-(29.1959)(672,096.40) = -(9.5238)(61.39)\mathbf{X}_{10} - (14.2857)(48.10)\mathbf{X}_{15}$$

we have

$$\mathbf{X}_{10} = -34279$$

$$\mathbf{X}_{15} = 57722$$

In [19]:
```
# python calculation for the system of equaitons of X_10 and X_15
sp.init_printing()
# Define the variables
x_10, x_15= sp.symbols('x y')

# Set up the equation for the present value of the portfolio
f = sp.Eq(PV_10*x_10+PV_15*x_15,PV_L)
g = sp.Eq(-MD_10*PV_10*x_10-MD_15*PV_15*x_15,-MD_L*PV_L)

# Solve for x, y, and z such that PV_custom = PV_portfolio
solution = sp.solve([f,g], (x_10, x_15))
print(solution)
```

```
{x: -34278.9958232110, y: 57722.0517709164}
```

Therefore, the hedging porfolio consists of **selling 34279 units of 10-year zero-coupon bonds** and **purchasing 57722 units of 15-year zero-coupon bonds**.

(c) Suppose the interest rate fell to 4.8%, would your hedging portfolio in (b) remain the same? Explain why or why not and comment on the composition of the new hedging portfolio relative to the old one.

My hedging porfolio will change.
The decrease on interest rate will cause small changes on prices and durations. Then the results from the equation system will change too.
As prices and durations increase due to discount facter increase, we need to both **long more and short more on the two bonds** to reach the new liability with higher PV and larger duration. In this case, the hedging portfolio changes.

(d) What are the advantages of the synthetic replication in (a) relative to the approximate hedging in (b)? Why might one use the approximate hedging instead?

The synthetic replication is set based on the actual term structure of the hedged bonds. So **when there is a shift or change of the term structure, synthetic replication indicates the exact change and adjusts according to it**. In comparison, the hedging approximation uses the price and duration, which don't directly reflect the actual change of the terms (price and duration might remain the same for some changes on term structure).
However, **when there are a lot different bonds that we need to consist in a portfolio, the approximation using duration provides an easier way to compute the change in price or ratio of each bond in the portfolio**.

Problem 4

Assume that the current yield curve is flat at 2% and that all bonds are issued with a face value of $\$100$.

In [20]:
```
def bond_data(par, coupon, r, t, n):
    '''
    par = par value
    coupon = coupon rate
    r = yield
    t = year of maturity
    n = numbers of payments per year
    '''
    coupons = coupon*par/n
    price = 0
    for i in range(1,t*n+1):
        price += coupons/(1+r/n)**i

    price += par/(1+r/n)**(t*n)

    summation = 0
    for i in range(1,t*n+1):
        summation += i*coupons/(1+r/n)**i

    summation += (t*n)*par/(1+r/n)**(t*n)
    duration = 1/par/(1+r)*summation

    summation2 = 0
    for i in range(1,t*n+1):
        summation2 += i*(i+1)*coupons/(1+r/n)**i

    summation2 += (t*n)*(t*n+1)*par/(1+r/n)**(t*n)
    convexity = 1/par/((1+r)**2)*summation2

    return (price, duration, convexity)
```

(a) What is the price, modified duration, and convexity of a 1-year 2% coupon bond with a single coupon payment at the end of the year?

Price

$$P = \frac{102}{1.02} = \$100$$

Modified Duration

$$D^{Modified} = \frac{1}{1.02} = 0.9804$$

or

$$D^{Modified} = \frac{1}{100} \times \frac{1}{1.02} \times (1 \times \frac{102}{1.02}) = 0.9804$$

Convexity

$$C = \frac{1}{100} \times \frac{1}{1.02^2} \times (1 \times 2 \times \frac{102}{1.02}) = 1.9223$$

In [21]:
```
price, duration, convexity = bond_data(100,0.02,0.02,1,1)

print(f'The price is ${price:.2f}.')
print(f'The duation is {duration:.4f}.')
print(f'The convexity is {convexity:.4f}.')
```

```
The price is $100.00.
The duation is 0.9804.
The convexity is 1.9223.
```

(b) What is the price, modified duration, and convexity of a 2-year 2% coupon bond with annual coupon payments at the end of the years 1 and 2?

Price

$$P = \frac{2}{1.02} + \frac{102}{1.02} = \$100$$

Modified Duration

$$D^{Modified} = \frac{1}{100} \frac{1}{1.02} \left(1 \times \frac{2}{1.02} + 2 \times \frac{102}{1.02^2} \right) = 1.9416$$

Convexity

$$C = \frac{1}{100} \frac{1}{1.02^2} \left(1 \times 2 \times \frac{2}{1.02} + 2 \times 3 \times \frac{102}{1.02^2} \right) = 5.6916$$

In [22]:
```
price, duration, convexity = bond_data(100,0.02,0.02,2,1)

print(f'The price is ${price:.2f}.')
print(f'The duation is ${duration:.4f}.')
print(f'The convexity is ${convexity:.4f}.')
```

```
The price is $100.00.
The duation is $1.9416.
The convexity is $5.6916.
```

(c) Consider a 2-year annual-pay floating rate note (FRN) with following structure

- At $t = 0$, the first coupon payment is locked at $y_1 \times F$, F is the face value and y_1 is the one-year spot rate (currently at 2%).
- At $t = 1$, the issuer makes the first coupon payment and the second coupon payment is locked in at the one year spot rate as reported at the end of the first year.
- At $t = 2$. the issuer pays the second coupon payment along with the note's face value

What is the price of the FRN at issuance?

The price of FRN will be $\$100$.
For each period, the bond will be reset to par because each coupon payment is set based on the newly reported previous year's one year spot rate. This is like we have a new 1-year bond each period trade at par, the discounted part is cancelled out by the interest rate. Therefore, the price remains at $\$100$.

(d) What is the modified duration of the FRN just after issuance? What is its convexity?

$$D = -\frac{1}{P}\frac{dP}{dy}$$
$$C = \frac{1}{P}\frac{d^2P}{dy^2}$$

For FRN, it is like connecting multiple 1-year coupon bonds together, since its coupon will be reset to the market rate of current period. Therefore the price is $\$100$. Therefore, the small change on yields will be the same as 1-year coupon bonds.

Calculate the duration from Mac Duration.

Use the formular of convexity:

$$C = \frac{1}{100}\frac{1}{1.02^2}\left(1 \times 2 \times \frac{2}{1.02} + 2 \times 3 \times \frac{102}{1.02^2}\right) = 5.6916$$

In [23]:
```
D_FRN = 1/1.02
C_FRN = (1/(100*(1.02)**2)) * (2*2/1.02 + 2*3*102/1.02**2)

print(f'For FRN:')
print(f'The duation is {D_FRN:.4f}.')
print(f'The convexity is {C_FRN:.4f}.')
```

```
For FRN:
The duation is 0.9804.
The convexity is 5.6916.
```

(e) Provide intuition for results in parts (c) and (d) as they relate to answers from parts (a) and (b).

The modified duration of an FRN for each period before reset will be the same with modified duration of that period, because the **price will be 100 for every period**, 1 year in this problem, and thus dp/dy is the same as 1 year par bond with the impact of change in yield is only effected on the current period, until the end of this period, when it is reset back to price of 100. Therefore, the **duration will be the same with 1-year bond**.

However, for convexity, it should be considered as all the cashflows until it reaches maturity year, so **FRN have the same convexity as the 2-year bond**.

Problem 5

A firm has an expected dividend per share next year of $\$2$. It has an expected EPS next year of $\$4$, which is expected to grow at a rate of 4% per year forever. Its cost of equity is 8% per year. Its ROE is 8% per year.

(a) What is the firm's share price?

$$P_0 = \frac{\mathbf{E}[\mathbf{D}_1]}{r - g} = \frac{2}{8\% - 4\%} = \$50$$

(b) What is the sustainable growth rate with this level of payout?

$$payout = \frac{2}{4} = \frac{1}{2}$$
$$g = b \times ROE = (1 - \frac{1}{2}) \times 8\% = 4\%$$

(c) What is the firm's expected dividend yield?

$$\mathbf{E}(d) = \frac{\mathbf{E}(D)}{P_0} = \frac{2}{50} = 4\%$$

(d) What is sum of the dividend yield and growth rate? Explain why this sum is what it is.

In Gordon growth model, we have

$$P_0 = \frac{\mathbf{E}(D_1)}{r - g}$$

moving dividend yield and growth rate to one side, we got

$$\frac{\mathbf{E}(D_1)}{P_0} + g = r = 8\%$$

The sum of dividend yield and growth rate is the cost of equity for dividend payer , which is also the rate of return for dividend receiver.

(e) Suppose that the firm switchese to paying out all its earnings. What will then be its sustainable growth rate?

If the firm is going to pay out all earnings, the payout ratio will be 1, so that plowback ratio is 0.

$$g = b \times ROE = 0 \times ROE = 0$$

Then the sustainable growth rate will be equal to zero.

(f) What will then be its share price?

$$P_0 = \frac{(1 - b)\mathbf{E}[\mathbf{X}_1]}{r - g} = \frac{1 \times 4}{8\% - 0\%} = \$50$$

(g) Explain the relationship between (a) and (f)

The Net Present Value, $NPV = PV_{cashflowinflows} - PV_{cashflow_outflows}$ is the diference between presen value of cash inflows and outflows. If the NPV of an investment is zero, the investments earns excatly the rate of return required, which is r_e, te cost of equity. \
In (a), the firm save half of its earnings, thus its value has a growth rate of 4%, and this leads to a price of $\$50$.
In (f), the firm uses all the earnings to pay for didvidends to the shareholders, also has a price of $\$50$.
For both condition the returns to shareholders are same. This is, the cost of equity, r_e and return on equity, ROE are both 8%, the equidation indicates that the NPV is zero, and firm is not making or losing values. The difference is what contributes to such price: (a) is generated by the combination of dividends and the growth of the firm, while (f) is all about the didvidends.
Therefore, **when we have $r_e = ROE$, stating that the return on new investment is equal to the required return on equity, the intrinsic value of the stock will not change, whatever what strategies the firm used on arrangement of its earnings**.