

Homework 1

Your Name Here

2023-10-09

Question 1: Expectations

1.a) Define both and explain the difference between (1) the expectation of a random variable and (2) the sample average.

...add answer here...

Question 2: LLN & CLT

2.a) Plot the density of a Beta(5,2) distribution over its domain [0,1]. Make a publication-quality plot by changing any unwanted default plotting behavior and by adding relevant titles and labels. The first parameter of the Beta distribution is often labeled α and the second β . The `dbeta()` and related functions in R label these parameters as `shape1` and `shape2`.

```
# ...add answer here...
```

2.b) State the Law of Large Numbers as simply as you can.

...add answer here...

2.c) Set the seed to the value 1234 (`set.seed(1234)`). Then take 1,000 random draws from the Beta(5,2) distribution using `rbeta()`. Calculate a running sample average. Specifically: calculate $\bar{X}_1 = x_1$, then calculate $\bar{X}_2 = (1/2) \sum_{i=1}^2 X_i$, then calculate $\bar{X}_3 = (1/3) \sum_{i=1}^3 X_i$. Continue until you have calculated $\bar{X}_{1000} = (1/1000) \sum_{i=1}^{1000} X_i$. Create a scatterplot with the values 1–1,000 on the horizontal axis and the 1,000 cumulative average values of \bar{X}_i for $i = 1, \dots, 1000$ you calculated on the vertical axis. Compare your value for \bar{X}_{1000} to the $\mathbb{E}[X] = \alpha/(\alpha + \beta) = 5/7 = 0.7143$.

```
# ...add answer here...
```

2.d) State the Central Limit Theorem as simply as you can.

...add answer here...

2.e) Make two plots. For the first plot, take D=10 draws from the Beta(5,2) distribution and calculate the sample average. Repeat the process of taking D=10 draws and finding the sample average R=10,000 times. Plot a histogram of the 10,000 sample averages. For the second plot, repeat the process with D=100 draws. These two histograms are called “sampling distributions.”

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# ...add answer here...
```

Question 3: Linear Algebra

Suppose X and Y are defined as follows.

$$\mathbf{X} = \begin{bmatrix} 1 & 1 \\ 1 & 4 \\ 1 & 5 \\ 1 & 8 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} 6 \\ 5 \\ 3 \\ 2 \end{bmatrix}$$

3.a) What is the rank of X? Provide a brief (approx 1 sentence) explanation. Check your work via R code using `Matrix::rankMatrix()`.

...add answer here...

```
# ...check answer here...
```

3.b) Calculate $\mathbf{X}'\mathbf{X}$. Use the `bmatrix` environment in Latex to typeset your answer. Check your work via R code.

...add answer here...

```
# ...check answer here...
```

3.c) What is the rank of $X'X$? Provide a brief explanation. Check your work via R code.

...add answer here...

```
# ...check answer here...
```

3.d) Find $(X'X)^{-1}$ “by hand” (as you would with paper and pencil) using the approach outlined on slide 47 of the Class 1 slides. Check your work via R code.

...add answer here...

```
# ...check answer here...
```

3.e) What is the rank of $(X'X)^{-1}$? Provide a brief explanation. Check your work via R code.

...add answer here...

```
# ...check answer here...
```

3.f) Calculate $X'y$. Check your work via R code.

...add answer here...

```
# ...check answer here...
```

3.g) Use your results from 3d and 3f to calculate “by hand” $(X'X)^{-1}X'y$. Check your work via R code.

...add answer here...

```
# ...check answer here...
```