Problem Set 1

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1 Problem 1

1.1 (a)

We need a portfolio that solves

$$\Delta = \frac{V^u - V^d}{S^u - S^d} = \frac{10 - 0}{65 - 40} = 40$$

$$D = \frac{1}{1.1}\frac{65 \times 0 - 40 \times 10}{65 - 40} = -14.55$$

So, holding 0.4 units of the stock and borrowing \$14.55.

1.2 (b)

$$C^{55}(50) = \Delta S_o + D = 0.4 \times \$50 - \$14.55 = \$5.45$$

1.3 (c)

$$q_u = \frac{S \times (1 + R) - d}{u - d} = \frac{50 \times 1.1 - 40}{65 - 40} = 0.6$$

$$1 - q_u = 0.4$$

1.4 (d)

$$C_0^{55}(50) = \frac{0.6 \times \$(65 - 50 \times 1.1) + 0.4 \times \$0}{1.1} = \$5.45$$

2 Problem 2

If the stock price exceeds \$70 at any point in the tree, a \$15 dividend is paid immediately before the next movement. Here, $S_2^{UU} = \$84.5 > \$70$, which should be replaced by $S_2^{UU} = \$84.5 - \$15 = \$69.5$.

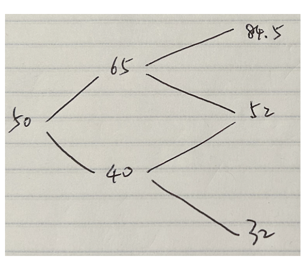

Figure 1: stock payoff diagram

Then we calculate option payoff.

At t=2:
$C_{UU} = max(0, \$69.5 - 55) = \$14.5$
$C_{UD} = max(0, \$52 - 55) = 0$ (Option is not exercised)
$C_{DD} = max(0, \$32 - 55) = 0$ (Option is not exercised)

At t=1:
For the upward node:
$C_U = max(\$65 - 55$, risk-neutral probabilities price)
$C_U = max(\$10$, risk-neutral probabilities price) $= \$10$
For the downward node:
$C_D = max(\$40 - 55, 0) = 0$ (Option is not exercised)

At t=0:
C_0 = avg (\$10 and \$0) = \$5

$$C^{55}(65) = \frac{0.6 \times \$14.5 + 0.4 \times \$0}{1.1} = \$7.91 < \$10$$

Since the immediate exercise value (\$10) is greater than the risk-neutral probabilities price (\$7.91), you should exercise the option early in this hypothetical scenario.
Thus, you should only exercise the option early if the price goes up in the year 1 to get the dividend.