UCLA Anderson School of Management

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Problem Set #2

You may work on this problem set in groups . Hand in one solution per group. You may discuss the problems only with members of your group. Answers should be typed (or printed legibly) and are due **at the beginning of the week 3 class. No late assignments will be accepted.**

Problem 1. Use Ito's Lemma to prove that

$$\int_0^t W_s^2 dW_s = \frac{1}{3} W_t^3 - \int_0^t W_s ds$$

Problem 2. Check whether the process $X(t) = W_1(t) \times W_2(t)$ is a martingale, where $W_1(t), W_2(t)$ are (independent) brownian motions

Problem 3. Let $g(s)$ be a bounded, deterministic function of time. Show that

$$X_t = \int_0^t g(s)\, dW_s$$

is normally distributed with mean zero and standard deviation $\sqrt{\int_0^t g^2(s)\, ds}$. Hint: a) Use Ito's Lemma to show that $Z_t = e^{-\frac{\eta^2}{2}\int_0^t g^2(s)ds + \eta \int_0^t g(s)dW_s}$ is a martingale. b) Use this observation to conclude that

$$E\left(e^{\eta X_t}\right) = e^{\frac{\eta^2}{2}\int_0^t g^2(s)ds}$$

which is the moment generating function of a normally distributed variable with zero mean and standard deviation equal to $\sqrt{\int_0^t g^2(s)\, ds}$.

Problem 4. Let

$$\beta_k(t) = E\left[W_t^k\right] \text{ for } k = 0, 1, 2..$$

Use Ito's formula to prove that

$$\beta_k(t) = \frac{1}{2} k (k-1) \int_0^t \beta_{k-2}(s)\, ds$$