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E-1121

B. E. 1st Semester (Main & Re) Examination, Dec. – 2019

MATHEMATICS - I

Branch : (CE, CSE, ECE, EE, ME)

Time : Three Hours] *[Maximum Marks : 60*

Note : Attempt *all* questions from **Section** – A, *four* questions from **Section** – B and *three* questions from **Section** – C.

SECTION – A

Note : Fill the blanks/choose the most appropriate alternative. 1 × 10 = 10

1. If two eigen values of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are equal to 1 each, then third eigen value of the matrix A is equal to :

(a) 2 (b) 3

(c) 4 (d) 5

2. If $u = \log\left(\dfrac{x^2 + y^2}{x + y}\right)$, then value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is equal to ....

3. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then grad $(\log r)$ is equal to ....

4. If $f(x) = \begin{cases} x\cos\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then

(a) $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$

(b) $f(x)$ is continuous and differentiable at $x = 0$

(c) $f(x)$ is not continuous at $x = 0$

(d) None of these

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5. $\lim_{x\to\infty} \dfrac{3x^3-4x^2+6x-1}{2x^3+x^2+5x+7}$ is equal to :

(a) 0
(b) 3/2
(c) 4
(d) −1/7

6. If the vectors (1, −2, λ), (2, −1, 5) and (3, −5, 7λ) are linearly dependent then value of λ is equal to :

(a) 14/5
(b) 15/5
(c) 16/5
(d) 5/14

7. If $\vec{F}=(3x^2-3yz)\hat{i}+(3y^2-3xz)\hat{j}+(3z^2-3xy)\hat{k}$ then curl $\vec{F}$ is equal to ....

8. If $y=\log\left(\dfrac{3-2x}{5+4x}\right)$, then n^{th} derivative of y is equal to ....

9. The value of $\left[-\dfrac{1}{2}\right.$ is equal to ....

10. Give an example of a monotonic increasing sequence which divergent.

SECTION – B

$5 \times 4 = 20$

1. Reduce the matrix A to its normal form and hence find its rank, where,

$$A=\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

2. Expand $f(x)=4x^2+7x+5$ in powers of $(x-3)$ by Taylor's theorem.

3. Prove that $\beta(m, n)=\dfrac{\lfloor m \lfloor n}{\lfloor (m+n)}$

4. Find the directional derivative of $\phi=(x^2+y^2+z^2)^{-1/2}$ at the point P (3, 1, 2) in the direction of the vector $yz\hat{i}+zx\hat{j}+xy\hat{k}$.

5. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.

6. Test the series $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \ldots\ldots\ldots\ldots\infty$ for convergent or divergent.

SECTION – C

$12 \times 3 = 36$

1. Find the characteristics equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find A^{-1}. Also find the matrix represented by : $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

2. If $y = [x + \sqrt{1 + x^2}]^m$, find y_n at $x = 0$.

3. A rectangular box, which is open at the top, has a capacity of 32 m³. Determine, using the Lagrange's method of multipliers, the dimension of the box such that the least material is required for the construction of the box.

4. Verify the Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$.

5. Test the series :

$$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \ldots\ldots\ldots\ldots + \frac{x^n}{n^2+1} + \ldots\ldots\ldots\ldots$$