HW4

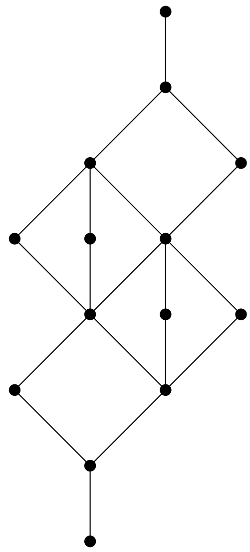
PEOPLE

Date

Problem 1 (Some (counter)-example)**.**

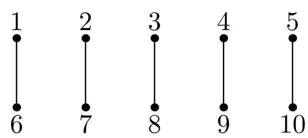
(i) Give an example of a finite graded poset P with the Sperner property, together with a group G acting on P, such that P/G is *not* Sperner.

(ii) Consider the poset P whose Hasse diagram is given by

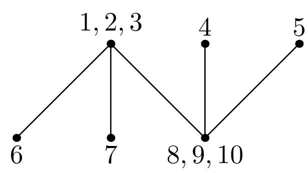


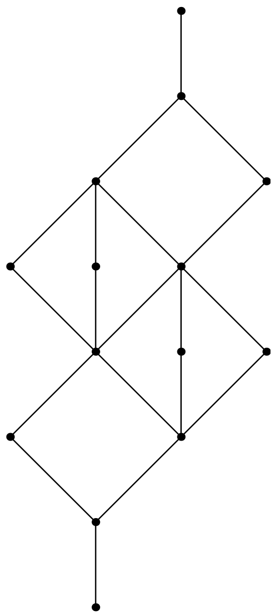
Find a subgroup G of S_7 such that $P \cong B_7/G$ or else prove that such a group does not exist.

Proof. (i) We draw a Hasse diagram for P:



We see that P is Sperner by inspection; its largest antichain is of length four, and each rank has four elements. Let $G = ((1, 2, 3), (8, 9, 10))$ be the group generated by the permutations $(1, 2, 3)$ and $(8, 9, 10)$. By drawing the Hasse diagram of P/G, we see that is clearly not Sperner:



(ii) 

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Problem 2 (Binary Necklace Poset). A $(0,1)$-*necklace* of *length* n and *weight* i is a circular arrangement of i 1's and $n-i$ 0's. For instance, the $(0,1)$-necklaces of length 6 and weight 3 are (writing a circular arrangement linearly) $000111, 001011, 010011$ and 010101. Cyclic shifts of a linear word represent the same necklace.

(i) (easy) Show that N_n is rank-symmetric, rank-unimodal and Sperner.

Proof. We have the same number of length n necklaces of weight i and $n-i$ because there is a bijection from simply flipping all digits. Thus, $p_i = p_{n-i}$, so N_n is rank-symmetric. We can also calculate p_i by first counting the $\binom{n}{i}$ necklaces with order and dividing by n to account for cyclic shifts. Thus,

$$p_i = \frac{1}{n}\binom{n}{i},$$

so the unimodality of N_n arises from the unimodality of the binomial coefficients. ∎