HW4

People

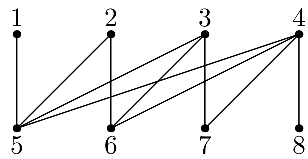
Date

Problem 1 (Some (counter)-example)**.**

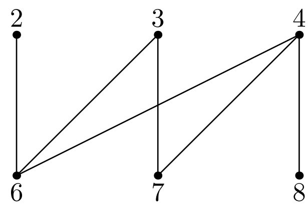
(i) Give an example of a finite graded poset P with the Sperner property, together with a group G acting on P, such that P/G is *not* Sperner.

(ii) Consider the poset P whose Hasse diagram is given by

Proof. (i) We draw a Hasse diagram for P:



We see that P is Sperner by inspection; its largest antichain is of length four, and each rank has four elements. Let $G = (1,2)(5,6)$ be the group generated by the permutations $(1,2)$ and $(5,6)$, effectively collapsing $1,2$ and $5,6$ together, for a Hasse diagram of P/G:



(ii) Notice that P is rank-symmetric, rank-unimodal, and Sperner. Also, P is the Hasse diagram identical to the poset of nonisomorphic simple graphs with 7 vertices. We define our subgroup G to be the permutations $\hat{\pi}$ induced by automorphisms $\pi \in S_7$, where $\hat{\pi}\{i,j\} = \{\pi \cdot i, \pi \cdot j\}$. $\square$

Problem 2. THings and stuff.

Proof. N_n is rank-symmetric because we have the same number of necklaces of weight i and $n-i$, due to the bijection of simply flipping all digits. There are $\square$