

Algebraic Combinatorics HW 2

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Problem 1 (**Random Walks on \mathbb{Z}**). Consider a random walk on \mathbb{Z} where we start at 0 and move from i to $i + 1$ or $i - 1$ with equal probability.

- (i) Prove that we eventually return to 0 with probability 1.
- (ii) Compute a_i explicitly and conclude that

$$\sum_{n=0}^{\infty} \frac{a_n}{2^n} = 1$$

What is the sequence a_n called in Math literature?

- (iii) Prove that each number n is visited at least once with probability 1.
- (iv) Let H_n denote the expected # steps needed to reach n for the first time. What is wrong with the following argument? We claim that $H_n = cn$ for some constant c . This is true for $n = 0$. So let $n > 0$. On the average, we need H_1 steps to reach 1, and then H_{n-1} steps to reach n starting from 1. Hence

$$H_n = 1 + H_{n-1} = c + c(n-1) = cn; H_1 = c$$

Proof. (i)

□

Problem 2 (**Some examples of Hitting times**). (i) Find the hitting time between any two vertices of K_n .

- (ii) Find the hitting time between the endpoints of P_n (a path on n vertices).
- (iii) Find the hitting time between an endpoint of P_n and a vertex at distance k from it.
- (iv) Find the hitting time between two vertices of C_n (cycle of n vertices) at distance k .