

$$\begin{cases} U(x_1, x_2) = (\alpha x_1^\beta + (1 - \alpha)x_2^\beta)^{1/\beta} \longrightarrow \max_{x_1, x_2} \\ I = x_1 P_1 + x_2 P_2 \\ 1 > \alpha > 0 \end{cases}$$

$$U'_{x_1} = \frac{(\alpha \beta x_1^{\beta-1} - \frac{(1-\alpha)\beta P_1 (\frac{I-P_1 x_1}{P_2})^{\beta-1}}{P_2})(\alpha x_1^\beta + (1 - \alpha)(\frac{I-P_1 x_1}{P_2})^\beta)^{\beta^{-1}-1}}{\beta} = 0$$

$$\begin{bmatrix} \alpha \beta x_1^{\beta-1} - P_2^{-1}(1 - \alpha)\beta P_1 (\frac{I-P_1 x_1}{P_2})^{\beta-1} = 0 \\ \alpha x_1^\beta + (1 - \alpha)(\frac{I-P_1 x_1}{P_2})^\beta = 0 \end{bmatrix}$$

$$\alpha \beta x_1^{\beta-1} P_2 = (1 - \alpha)\beta P_1 x_2^{\beta-1}$$

$$\frac{x_1}{x_2} = (\frac{(1 - \alpha)P_1}{\alpha P_2})^{\frac{1}{\beta-1}}$$