1.	Please write the formulation for the least square regression, the ridge regression, the kernel regression, and the LASSO regression. least square regression:
	$\ Y-X^ opeta\ ^2$
	ridge regression:
	$\ Y-X^\top\beta\ ^2+\lambda\ \beta\ _2^2$
	kernel regression:
	$\ Y - Kc\ ^2 + \lambda c^\top Kc$
	LASSO regression:
	$\frac{1}{2}\ Y-X^\top\beta\ ^2 + \lambda \beta _1$
2.	Please write analytic solutions to the least square regression, the ridge regression, the kernel regression, and the spline regression. (不用写证明过程) least square regression:
	$\beta = \left(X^\top X\right)^{-1} X^\top Y$
	ridge regression:
	$\beta = \left(X^\top X + \lambda I\right)^{-1} X^\top Y$
	kernel regression:
	$c = \left(K + \lambda I\right)^{-1} Y$

spline regression:

$$\alpha = \left(Z^\top Z + \lambda D\right)^{-1} Z^\top Y$$

3. Please derive the LDA for two classes.

$$orall X_i \in \Omega^+, p\left(X_i|y=+1
ight) \sim \mathrm{N}\left(\mu^+,\mu^-
ight) \ orall X_i \in \Omega^-, p\left(X_i|y=-1
ight) \sim \mathrm{N}\left(\mu^-,\mu^+
ight)$$

$$egin{aligned} \sigma_{ ext{between}}^2 &= \left[(\mu^+ - \mu^-)^ op eta
ight]^2 \ \sigma_{ ext{within}}^2 &= n_{ ext{pos}} \sigma_{ ext{pos}}^2 + n_{ ext{neg}} \sigma_{ ext{neg}}^2 \ n_{ ext{pos}} &= |\Omega^+| \ n_{ ext{neg}} &= |\Omega^-| \ \sigma_{ ext{pos}} &= eta^ op \Sigma^+ eta \ \sigma_{ ext{neg}} &= eta^ op \Sigma^- eta \ S &= rac{\sigma_{ ext{between}}^2}{\sigma_{ ext{within}}^2} \ &= rac{\left[(\mu^+ - \mu^-)^ op eta
ight]^2}{n_{ ext{pos}} \sigma_{ ext{pos}}^2 + n_{ ext{neg}} \sigma_{ ext{neg}}^2 \ &= rac{eta^ op S_B eta}{eta^ op S_W eta} \end{aligned}$$

其中

$$egin{aligned} S_B &= \left(\mu^+ - \mu^-
ight) \left(\mu^+ - \mu^-
ight)^ op \ S_W &= n_{
m pos} \Sigma^+ + n_{
m neg} \Sigma^- \end{aligned}$$

因为 β 模长不改变结果,可令

$$eta^ op S_W eta = 1 \ egin{aligned} eta^ op S_W eta &= 1 \ egin{aligned} & \max_eta eta^ op S_B eta & ext{s.t.} & eta^ op S_W eta &= 1 \ & L &= eta^ op S_B eta - \lambda \left(eta^ op S_W eta - 1
ight) \ & \Rightarrow & rac{\partial L}{\partial eta} &= 2 S_B eta - 2 \lambda S_W eta &= 0 \ & \Rightarrow & S_B eta &= \lambda S_W eta \ & \Rightarrow & S_W^{-1} S_B eta &= \lambda eta \end{aligned}$$

 β 是 $S_W^{-1}S_B$ 的特征向量。

因为 $S_B\beta$ 与 $\mu^+ - \mu^-$ 方向相同,

$$eta \propto S_W^{-1} \left(\mu^+ - \mu^-
ight)$$