# Homework 3

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## 1. Linear SVM

Choose the linear model of class `sklearn.svm.SVC`. The mathematical formulation can be written as:

$$
\begin{aligned}
&\min_{w,b,\xi} \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}\xi_i \\
\text{s.t.} \quad & y_i\left(w^\top x_i + b\right) \geq 1 - \xi_i \\
& \xi_i \geq 0
\end{aligned}
$$

And the dual problem is:

$$
\begin{aligned}
&\min_{\alpha} \frac{1}{2}\alpha^\top Q\alpha - e^\top \alpha \\
\text{s.t.} \quad & y^\top \alpha = 0 \\
& 0 \leq \alpha \leq C
\end{aligned}
$$

where

$$Q_{ij} = y_i y_j x_i^\top x_j$$

Due to the lackness of hyper-parameters of linear svm model, I just use the default parameters.
With all hyper-parameters be default value (`C=1.0`), the accuracy on test dataset after training convergence is:

```
Accuracy of Linear SVM: 0.846
```

Here are the tasks specifically for **Linear SVM**:

1. **How many support vectors are used to calculate the parameter $w$? I.e., please count the number of training samples with $\alpha_i > 0$.**
   Using the attribute `n_support_` of `sklearn.svm.SVC` to obtain the number of support vectors for each class. The result is sum of number of each classes.

   ```
   Number of support vectors: 3714
   ```

2. **How many positive samples and how many negative samples are among these support vectors?**
   Using the attribute `n_support_` of `sklearn.svm.SVC` to obtain the number of support vectors for each class.

   ```
   Number of negative support vectors: 1845
   Number of positive support vectors: 1869
   ```

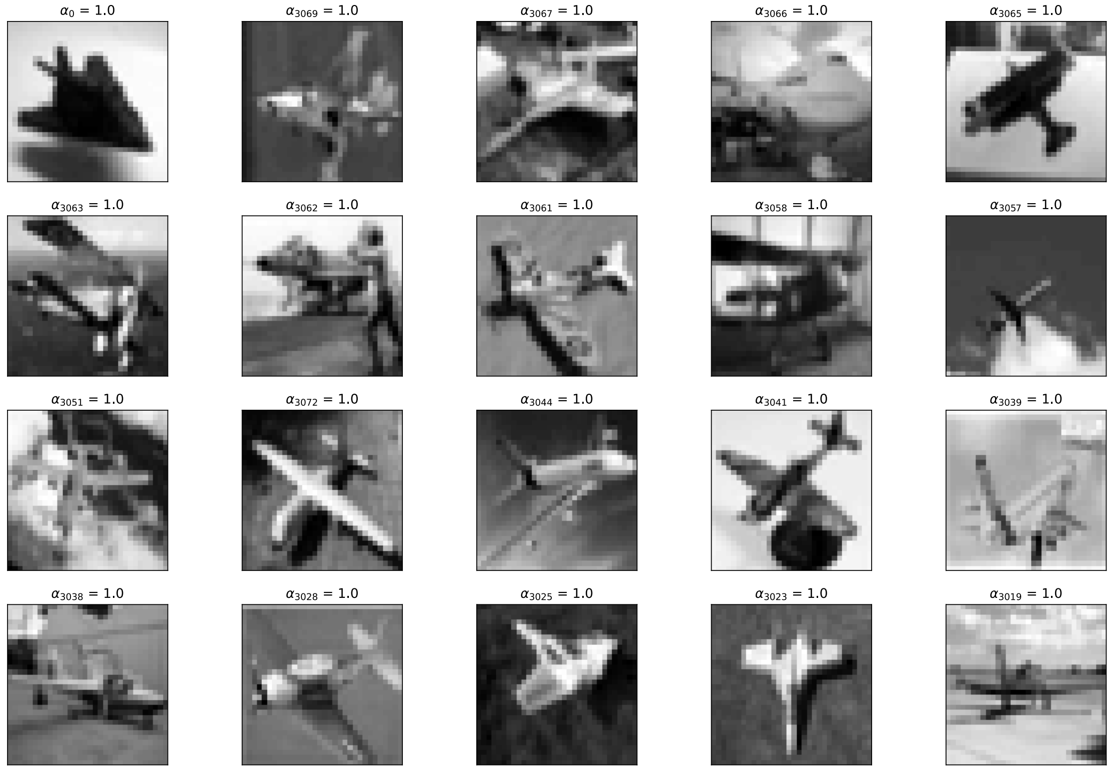
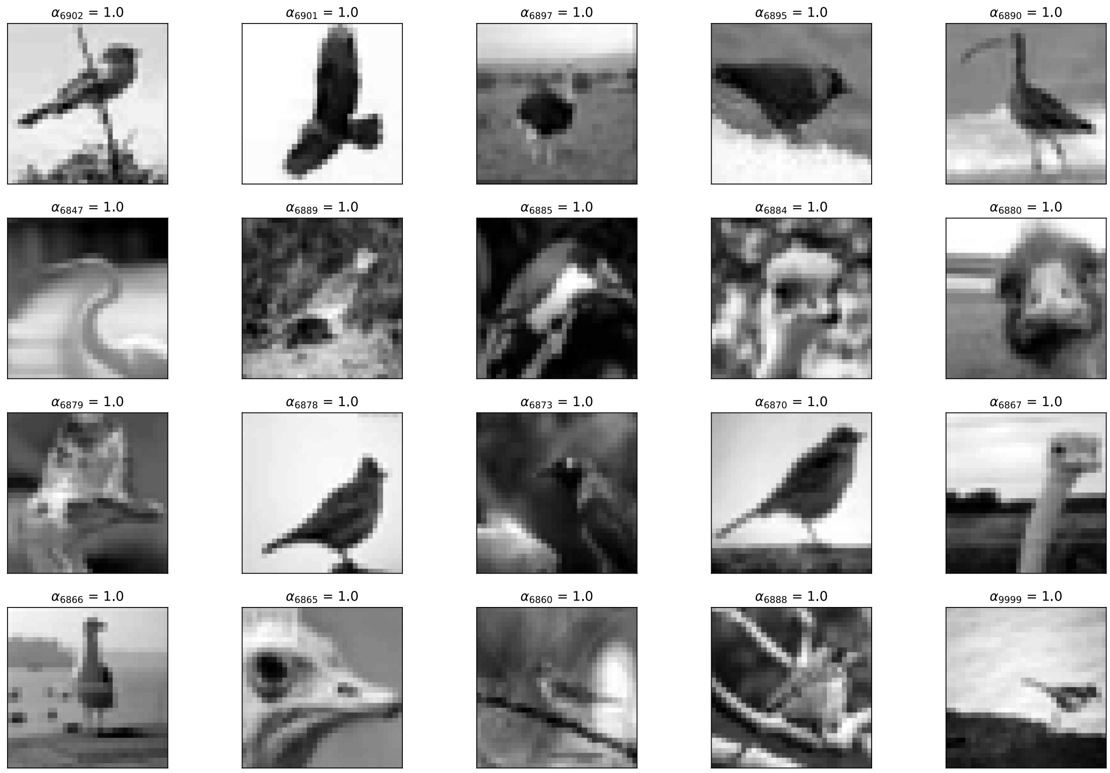
3. **For both positive and negative samples, you need to visualize the top 20 images with the largest values of $\alpha_i$ and attach the value of $\alpha_i$ beside each image.**
   The attribute `dual_coef_` of `sklearn.svm.SVC` gives dual coefficients of the support vector in the decision function, multiplied by their targets. Which is

$$\text{dual\_coef\_} = \alpha_i \cdot y_i$$

Therefore we can obtain each $\alpha$ in dual problem.

We can recover the index of each support vector by comparing the `support_vectors_` with `H_train`.

Then each $\alpha_i$ and original index can be obtained via `dual_coef_`. Sort the index and plot top 20 pictures.

| **class** | **Top 20 images with the largest values of $\alpha_i$** |
|---|---|
| **negative** |   |
| **positive** |   |

## 2. RBF kernel SVM

Choose the rbf model of class `sklearn.svm.SVC`. The mathematical formulation can be written as:

$$
\begin{aligned}
&\min_{w,b,\xi} \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}\xi_i \\
\text{s.t.} \quad & y_i\left(w^\top \phi\left(x_i\right)+b\right) \geq 1-\xi_i \\
& \xi_i \geq 0
\end{aligned}
$$

And the dual problem is:

$$
\begin{aligned}
\min_{\alpha} \ & \frac{1}{2}\alpha^\top Q\alpha - e^\top \alpha \\
\text{s.t.} \quad & y^\top \alpha = 0 \\
& 0 \leq \alpha \leq C
\end{aligned}
$$

where

$$
\begin{aligned}
Q_{ij} &= y_i y_j K\left(x_i, x_j\right) \\
K_{ij} &= \exp\left(-\gamma\|x_i - x_j\|^2\right)
\end{aligned}
$$

First of all, train and test with all default hyper-parameters.

```
Accuracy of RBF kernel SVM with default hyper-parameters: 0.8795
```

Then tune the hyper-parameter $\gamma$ via `GridSearchCV`.
*(code written in `tune.py`)*

```
from sklearn.svm import SVC
from sklearn.model_selection import GridSearchCV

print(f"Default gamma of SVC: {1 / (H_train.shape[1] * H_train.var())}")
param_grid = {'gamma': [0.002, 0.003, 0.004, 0.005, 0.006, 0.007]}
svm = SVC(kernel = 'rbf')
grid_search = GridSearchCV(svm, param_grid, scoring='accuracy', cv=5)
grid_search.fit(H_train, Y_train)
print('Best parameters:', grid_search.best_params_)
print('Best score:', grid_search.best_score_)
```

The output is:

```
Default gamma of SVC: 0.003086419753086423
Best parameters: {'gamma': 0.005}
Best score: 0.8785000000000001
```

We can find that setting `gamma = 0.005` is better than the default `gamma` (which is `"scalar"` option `1 / (H_train.shape[1] * H_train.var())`)

Therefore, train and test the rbf kernal SVM with `gamma = 0.005` and the accuracy is:

```
Accuracy of RBF kernel SVM with gamma = 0.005: 0.8815
```

## 3. Polynomial kernel SVM

Choose the poly model of class `sklearn.svm.SVC`. The mathematical formulation can be written as:

$$
\begin{aligned}
\min_{w, b, \xi} \quad & \frac{1}{2}\|w\|^2 + C \sum_{i=1}^{n} \xi_i \\
\text{s.t.} \quad & y_i \left(w^\top \phi\left(x_i\right) + b\right) \geq 1 - \xi_i \\
& \xi_i \geq 0
\end{aligned}
$$

And the dual problem is:

$$
\begin{aligned}
\min_{\alpha} \quad & \frac{1}{2}\alpha^\top Q \alpha - e^\top \alpha \\
\text{s.t.} \quad & y^\top \alpha = 0 \\
& 0 \leq \alpha \leq C
\end{aligned}
$$

where

$$
\begin{aligned}
Q_{ij} &= y_i y_j K\left(x_i, x_j\right) \\
K_{ij} &= \left(\gamma\langle x_i, x_j\rangle + r\right)^d
\end{aligned}
$$

First of all, train and test with all default hyper-parameters.

```
Accuracy of Polynomial kernel SVM with default hyper-parameters: 0.868
```

Then tune the hyper-parameter $r$ ( `cofe0` ) and $d$ ( `degree` ) via `GridSearchCV` .
*(code written in `tune.py` )*

```
from sklearn.svm import SVC
from sklearn.model_selection import GridSearchCV

param_grid = {'coef0': [0.125, 0.25, 0.375], 'degree': [3, 4, 5, 6]}
svm = SVC(kernel='poly')
grid_search = GridSearchCV(svm, param_grid, scoring='accuracy', cv=5)
grid_search.fit(H_train, Y_train)
print('Best parameters:', grid_search.best_params_)
print('Best score:', grid_search.best_score_)
```

The output is:

```
Best parameters: {'coef0': 0.25, 'degree': 3}
Best score: 0.8803000000000001
```

We can find that setting `coef0 = 0.25` and `degree = 3` is better than the default hyper-parameters.

Therefore, train and test the rbf kernal SVM with `coef0 = 0.25` and `degree = 3` and the accuracy is:

```
Accuracy of Polynomial kernel SVM with coef0 = 0.25, degree = 3: 0.88
```