品职教育
PZACADEMY.COM

Derivatives
CFA一级知识框架图

扫码查看电子版勘误

讲师：李斯克
www.pzacademy.com

Module 1

Derivative Instrument and Derivative Market Features

Basic Concept

	Basic Concept

Definition	• A derivative is a financial instrument (***contract***) that ***derives(transform, not pass through)*** its performance from the performance of an underlying asset.
	    • Cash (spot) market → Cash (spot) price
	    • Derivative market → Contract price→ ***High leverage***
	• Derivatives are ***created in the form of legal contracts***.
	    • ***The buyer (the long or the holder)***: purchases the derivative, his value changes in a way similar to a long position.
	    • ***The seller (the short)***: has exposure similar to a short position.
	    • ***Stand-alone derivative***: a distinct derivative contract (a derivative on a stock or bond).
	    • ***Embedded derivative***: a derivative within an underlying (a callable, puttable, or convertible bond).

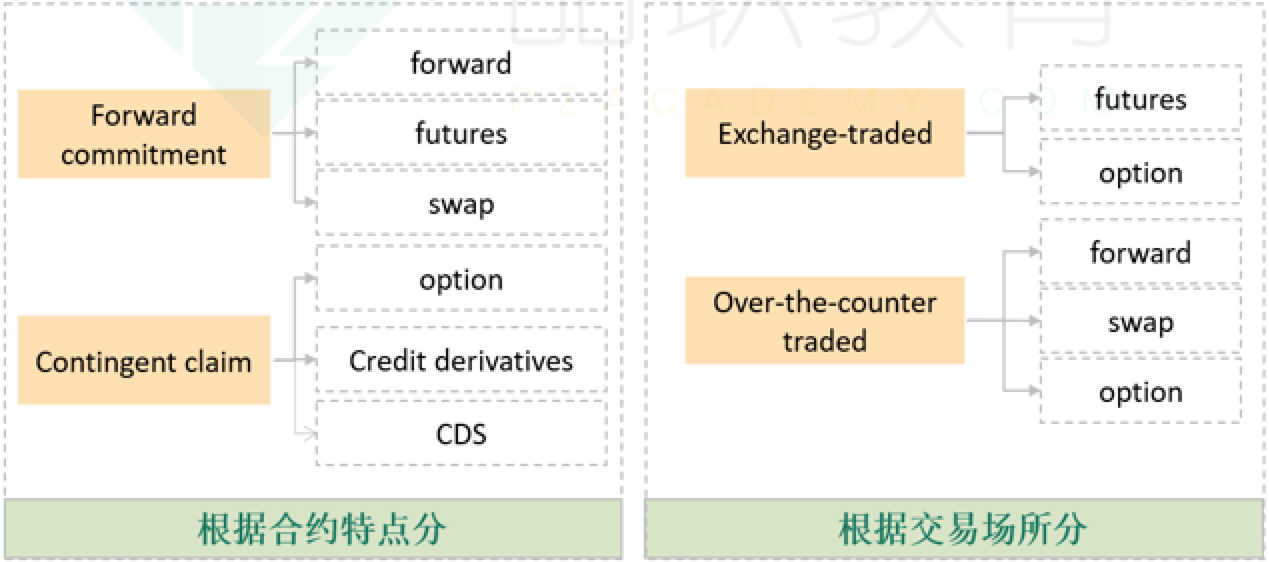
Classification	• ***Forward commitments***:
	    • Provide the ability to ***lock in a price*** at which one might buy or sell the underlying.
	    • ***forward contracts, futures contracts, and swaps.***
	• ***Contingent claims***:
	    • Provides ***the right but not the obligation*** to buy or sell the underlying at a pre-determined price.
	    • The primary contingent claim is called an ***option***.

Derivative Markets

	Derivative Markets
OTC Derivative Markets	• Over-the-Counter (OTC) Derivative Markets: • can be ***formal*** organizations or ***informal*** networks. • can ***be customized*** to match a desired risk exposure and ***flexible*** to hedge a specific underlying. • involve ***derivatives end users*** and ***dealers***, or ***financial intermediaries.***
ETD Derivatives Markets	• Exchange- traded (ETD) Derivatives Markets: • are more ***liquid and transparent.*** • require ***collateral*** on deposit upon inception and during the life of a trade. • ***have transparency***, full information on all transactions is disclosed to exchanges and national regulators. • Exchange memberships are held by ***market makers*** (or dealers) and ***speculators.***
Central Clearing Of OTC Market	• Central counterparties (CCPs): ***eliminate bilateral counterparty credit risk*** and ***provide clearing and settlement services***. • The central clearing mandate ***transfers the systemic risk*** of derivatives transactions from the counterparties, typically financial intermediaries, to the CCPs. • One concern is the ***centralization*** and ***concentration of risks*** in CCPs.

ETD VS OTC

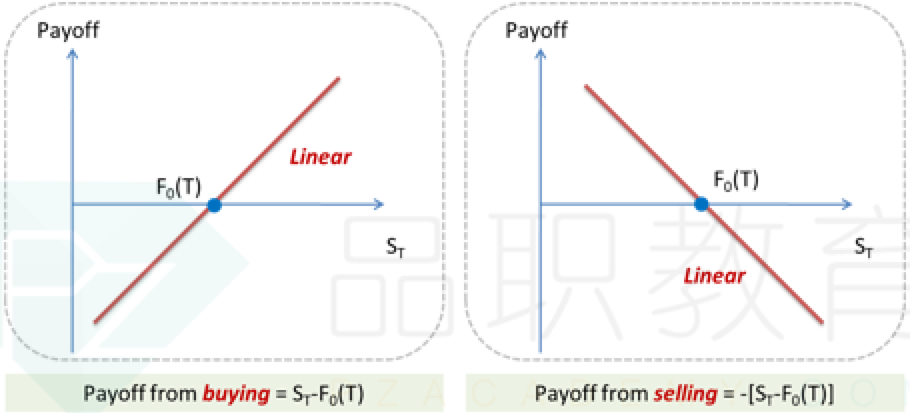
Exchange-traded (ETD)	Over-the-counter (OTC)
Standardized→ less flexibility	Customized→ greater flexibility
More transparent(Regulated)	Less transparent (more counterparty risk)
lower trading and transaction costs	Higher trading and transaction costs
More liquid	less liquid



Module 2

Forward Commitment and Contingent Claim Features and Instruments

Forward

<table>
<tr><th></th><th>Forward Contracts</th></tr>
<tr><td>Payoff</td><td></td></tr>
<tr><td>Purposes of Forward</td><td>• Hedge Risk: locking in the future buying or selling price of underling asset.
    • Hedge risk may not always be good for hedger.</td></tr>
<tr><td>Characteristics of Forward</td><td>• A forward commitment.
• Both parties are exposed to default risk (counterparty risk).
• No payments will be made at t=0.
• Zero-sum game.
• Linear derivative.</td></tr>
<tr><td>Settling</td><td>• Settling at expiration: Physical vs Cash settlement.
• Settling prior to expiration: Offsetting with the original party can avoid credit risk.</td></tr>
</table>

Futures

Forwards	Futures
Private contracts → more privacy	Exchange-traded → more transparent
Less regulation	Regulated
Unique customized contracts → more flexible	Standardized contracts
May have default risk	Guaranteed by clearinghouse
Settlement at maturity	Daily settlement
No margin required	Margin required and adjusted

	Managing Credit Risk
Margin	• ***Initial margin:*** The first deposit which is posted into a futures margin account before any trading takes place. • ***Maintenance margin***: below the maintenance margin will get a ***margin call***. • ***Variation margin***: used to bring the margin balance back up to the ***initial margin level***.
Daily Price Limit	• How much the contract price can change from the ***previous day's settlement price.***
Marking To Market	• At the end of every day there is a ***daily settlement*** process called marking to market. • ***Clearing house moves money*** between participants.

Swaps

	Swaps

Compared with Forward	• ***Similarities***
	  • ***Forward commitment***.
	  • Privately negotiated and subject to ***default***.
	  • ***No payment*** required by either party at initiation ***except*** the principal values exchanged in ***currency swaps***.
	  • Either party can default but only one party can ***default*** at a particular time.
	• ***Differences***
	  • Forwards involve ***one*** future exchange of cash flows.
	  • while swap contracts involves ***more than one*** exchange of future cash flows.

IRS (Interest rate swap)	• IRS in which ***a fixed rate is exchanged for a floating rate*** is the most common swap contract. • The counterparty paying the variable cash flows as the floating-rate payer (or ***fixed-rate receiver***) and the counterparty paying fixed cash flows as the ***fixed-rate payer*** (floating-rate receiver). • IRS can be used to ***change nature of debt*** to get desired interest rate payment. • The ***notional principal is usually not exchanged***, it is used for interest payment calculations. • ***The fixed rate (the swap rate)*** on the swap is determined by a process that forces the value to zero. • $V_0=0$. • As market conditions change, the ***value*** of a swap will ***deviate from zero.***

Options

	Option
Definition	• Option Price Vs. Exercise Price (or Strike Price) • Option Premium
Position	• The option ***buyer has the right*** but not the obligation to transact the trade. • The option ***seller has the obligation*** to fulfill the transaction as chosen by the option buyer.
Payoff and Profit	• For call option buyer: ***Payoff***: C_T=max[0, S_T-X] ; ***Profit***: Π =max[0, S_T-X]-C_0 • For call option seller: ***Payoff***: -C_T=-max[0, S_T-X] ; ***Profit***: Π =C_0-max[0, S_T-X] • For put option buyer: ***Payoff***: P_T=max[0, X-S_T] ; ***Profit***: Π =max[0, X-S_T] – P_0 • For put option seller: ***Payoff***: -P_T=-max[0, X-S_T] ; ***Profit***: Π =P_0-max[0, X-S_T]
Counterparty Credit Risk	• ***Only*** the ***option buyer faces the counterparty credit risk*** of the option seller equal to the option payoff at maturity.

Forward Commitments VS Contingent Claims
• ***Long forward*** position / ***long call option*** position / ***sell put position*** benefits from ***a rise in the underlying price.*** • ***Short forward*** position / ***short call option*** position / ***long put option*** benefits from ***a decrease in the underlying price***.

Credit Derivatives

	CDS
Definition	• Credit protection buyer receives a payment from the credit protection seller if ***credit events*** occurs. • The ***contingent payment*** equals the issuer loss given default for the CDS contract notional amount.

Credit Default Swaps (CDS)	• CDS contracts are ***contingent claims*** with some features of ***firm commitments***.
	    • The timing of exercise and payment upon exercise vary depending on the underlying issuer(s).
	    • A CDS contract priced at a par spread has a zero net present value, and the notional amount is not exchanged, serving as a basis for spread and settlement calculations.
	• ***CDS is similar to insurance.***
	    • The periodic premium over the life of the contract agreed to upfront, but the timing and size of the loss under the credit event being unknown.
	    • ***The seller's position*** is similar to that of a long risk position in the issuer's underlying bond.
	    • ***The buyer*** may suffer a loss from the credit event but will ***receive a payment from the CDS contract*** that will ***offset that loss***.
	• ***A credit protection seller's position is similar to that of an underlying cash bond investment.***

Derivatives Underlyings

	Derivatives Underlyings
Equities	• Derivatives on ***individual stocks*** are ***primarily options***. • ***Index derivatives*** in the form of ***options***, ***forwards***, ***futures***, and ***swaps*** are very popular. • Equity swaps: useful in asset allocation strategies. • Options, futures, and swaps are available based upon the ***realized volatility*** of equity index prices. • ***Employee compensation***: Options on individual stocks are used by issuers as compensation.
Fixed- Income and Interest Rates	• ***Options***, ***forwards***, ***futures***, and ***swaps*** on ***bonds*** are widely used. • One of the ***largest derivative underlyings is an interest rate***(not an asset, such as MRR). • ***Interest rate swaps***: used to convert from fixed to floating interest rate exposure.
Currencies	• Options, forwards, futures, and swaps based upon ***sovereign bonds and exchange rates*** are used to ***manage currency risk.***
Commodities	• Commodity derivatives are widely used to ***manage the price risk of an individual commodity or a commodity index*** separate from physical delivery.
Credit	• Credit derivative contracts are based upon ***the default risk*** of a single issuer or a group of issuers in an index. • ***Credit default swaps (CDS):*** like an ***insurance contract*** for the reference obligation which is the bond or a loan.
Other	• Weather, cryptocurrencies, and longevity.

Module 3

Derivative Benefits, Risks, and Issuer and Investor Uses

Derivative Benefits

	Benefits
Risk allocation, transfer, and management	• Using derivative contracts to ***allocate or transfer risk across time and among market participants***. • ***Create exposures*** unavailable in cash markets.
Information Discovery	• Derivative instrument prices serve a ***price discovery function*** beyond the underlying cash or spot market. • ***Futures prices*** are often seen as ***revealing information*** about the direction of cash markets in the future. • ***Option prices*** reflect several characteristics of the underlying, including the implied volatility.
Operational Advantages	• Derivatives have ***lower transaction costs*** than the underlying. • Derivative markets also typically have ***greater liquidity*** than the spot markets. • ***Reduced cash outlay***. • Easier than the underlyings to ***go short***.
Market Efficiency	• Less costly to ***exploit arbitrage opportunities or mispricing***.

Derivative Risks

	Risks
Greater Potential for Speculative Use	• ***Implicit leverage*** for some derivative strategies ***magnifies the realized returns and risks*** and contributes to the severity of derivative-related losses. • These ***risks are mitigated*** through a combination of trading and exposure risk management, daily marking to market, the use of collateral arrangements, transaction and exposure limits, and centralized counterparties.
Lack of Transparency	• Derivatives ***add portfolio complexity***. • This risk increases when a combination of derivatives and/or embedded derivatives is involved, ***such as structured notes***.
Basis Risk	• Potential ***divergence between the expected value*** of a derivative instrument versus an underlying or hedged transaction.
Liquidity Risk	• The ***daily settlement*** of gains and losses in the futures market can ***give rise to liquidity risk***.
Counterparty Credit Risk	• ***Differences in the current price versus the expected future settlement price*** give rise to counterparty credit exposure.
Destabilization and Systemic Risk	• Excessive risk taking and use of leverage in derivative markets may ***contribute to market stress***, as in the 2008 financial crisis.

Use of Derivatives among Issuers and Investors

	Use of derivatives among issuers and investors

Issuer use of derivatives	• ***Hedge accounting***
	  ◦ Allows an issuer to offset a hedging instrument (usually a derivative) against a hedged transaction or balance sheet item to ***reduce financial statement volatility***.
	  ◦ Allows corporate issuers to ***recognize derivative gains and losses*** at the same time as the associated underlying hedged transaction.
	• Types: cash flow hedge, fair value hedge, net investment hedge.
	  ◦ ***Cash flow hedge***: ***Absorbs variable cash flow*** of floating-rate asset or liability such as foreign exchange, interest rates, or commodities (forecasted transaction).
	  ◦ ***Fair value hedge***: ***Offsets fluctuation in fair value*** of an asset or liability.
	  ◦ ***Net investment hedge***: Designated as ***offsetting the FX risk*** of the equity of a foreign operation.

Investor Use of Derivatives	• Investors use derivatives to ***replicate a cash market strategy***, ***hedge a fund's value*** against adverse movements in underlyings, or ***modify or add exposures using derivatives***. • ***Investors to be less focused than issuers on hedge accounting treatment***.

Module 4

Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

Arbitrage and Replication

	Arbitrage and Replication
Law Of One Price	• Two securities or portfolios that have ***identical cash flows*** in the future, regardless of future events, should have the ***same price.***
Arbitrage	• Arbitrage is the condition that two ***equivalent assets or derivatives or combinations of assets and derivatives sell for different prices, leading to an opportunity to buy at the low price and sell at the high price***, thereby earning a risk- free profit without committing any capital. • Use arbitrage to price derivatives.

Replication	• ***Long forward commitment replication***:
	  • t=0: borrow at the risk-free rate, r, and buy the underlying asset (S_0)
	  • t=T: repay loan ($S_0(1 + r)^T$)and sell at spot (S_T)
	  • Under the no-arbitrage condition of $F_0(T) = S_0(1 + r)^T$, ***the return is equal to $S_T – F_0$ (T) for both the long forward and the replication strategy.***
	• ***Short forward commitment replication***
	  • t=0: short sale of the underlying asset (S_0) and invest of proceeds at the risk-free rate, r.
	  • t=T: the short sale is covered at S_T
	  • Under the no-arbitrage condition of $F_0(T) = S_0(1 + r)^T$, ***the return is equal to $F_0(T) – S_T$ for both the short forward and the replication strategy.***
	• ***Risk-free trade replication***: Long Asset, Short Forward.

Costs & Benefits Associated With Owning The Underlying

	Costs and benefits
Cost of carry	• ***The net of the costs and benefits*** related to owning an underlying asset for a specific period.
Opportunity cost (risk-free interest rate, r)	• A ***positive risk-free rate*** causes a greater forward price than the underlying spot price. • This opportunity cost ***applies to any asset.***
Other costs of ownership (C, c)	• Such as storage cost, , transportation costs, insurance, spoilage costs. • These costs should be compensated, so a ***forward price is therefore greater than the underlying spot price***, all else equal.
Benefits of ownership (I, i)	• Owning the underlying asset enjoys cash flow or other benefits: stock dividends or bond coupons. • These benefits reduce the forward price, so ***a forward price is therefore lower than the underlying spot price***, all else equal.
Relationships	• Opportunity and Other Cost > Benefit: $F_0(T) > S_0$ • Opportunity and Other Cost < Benefit: $F_0(T) < S_0$ • Opportunity and Other Cost = Benefit: $F_0(T) = S_0$

Different Underlying has Different Costs and Benefits

Different underlying has different costs and benefits	
Individual equity securities **receive the benefit of regular stock dividends**	$F_0(T)=[S_0-PV_0(I)](1+r)^T$
Stock Index Futures **with Dividend Yield**	$F_0(T)=S_0e^{(r-i)T}$

Foreign Exchange Forward	• Spot rate
	  • $S_{0,f/d}$: the number of units of a price currency (f) in the numerator per single unit of a base currency (d) in the denominator.
	• FX (foreign exchange) forward contract
	  • ***A long FX forward*** position involves the ***purchase of the base currency*** and the ***sale of the price currency*** on a future date at a forward price ($F_{0,f/d}$) agreed on at inception.
	$F_{0,f/d}(T)=S_{0,f/d}e^{(r_f-r_d)T}$

Commodities	• ***Costs***: the storage, insurance, transportation, and potential spoilage (in the case of soft commodities) of these physical assets. • A non-cash benefit of holding a physical commodity versus a derivative is known as a ***convenience yield***.

Module 5

Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

Pricing and Valuation of Forward

	Pricing and Valuation of Forward Contracts
At Initiation Date	• *Valuation*: The value at initiation is *zero*. • *Pricing*: $F_0(T)=\left[S_0-PV_0(I)+PV_0(C)\right](1+r)^T$
During the Life of the Contract	• *Valuation with no additional costs or benefits* $V_t(T)=S_t\text{-}F_0(T)(1+r)^{-(T-t)}$ • *Valuation with additional costs or benefits* $V_t(T)=(S_t-PV_t(I)+PV_t(C)-F_0(T)(1+r)^{-(T-t)})$ • *The MTM value of the FX forward* $V_t(T)=S_{t,f/d}\text{-}F_{0,f/d}(T)e^{-(r_f-r_d)(T-t)}$
At Expiration	• *Valuation* $V_t(T)=S_T\text{-}F_0(T)$

Pricing and Valuation of Interest Rate Forward

	Implied forward rates(IFR)
Discount Factor	• The price equivalent of a zero rate is the present value of a currency unit on a future date, known as a discount factor. • The ***discount factor for period i (DF_i)*** is: $DF_i = \frac{1}{(1+z_i)^i}$

Implied forward rates(IFR)	• Definition
	  • The breakeven reinvestment rate linking a short-dated and a long-dated zero-coupon bond is an ***implied forward rate (IFR).***
	• Quote
	  • $IFR_{A,B-A}$: a forward rate on a bond that ***starts in period A and ends in period B***. Its tenor is B – A periods.
	• Calculation - ***for government benchmark rates***
	$(1+z_A)^A \times (1+IFR_{A,B-A})^{B-A} = (1+z_B)^B$
	• Calculation - ***for short-term market reference rates***
	  • Step 1: Ensure ***the time frames (periodicities)*** of the interest rates ***are the same***.
	$\left(1+\frac{APR_m}{m}\right)^m = \left(1+\frac{APR_n}{n}\right)^n$
	  • Step 2: Calculate the $IFR_{A,B-A}$

	Forward Rate Agreements (FRAs)

Forward Rate Agreements	• *FRA buyer (long FRA* , or *fixed-rate payer):*
	  • Borrower, pay fixed payment and receive float.
	  • Hedging against a *increase* in interest rates.
	• *FRA seller (short FRA* , or *fixed-rate receiver):*
	  • Lender, receive fix and pay float.
	  • Hedging against a *decrease* in interest rates.

FRA is similar to swap	• Fixed versus floating payments on an FRA occur *on a net basis* and the *notional is not exchanged* but is used solely for interest calculations.
FRA Pricing	• The *implied forward rates represent the FRA fixed rate* for a given period where no riskless profit opportunities exist. • This no-arbitrage interest rate ensures that the FRA has a value of zero ($V_0(T) = 0$) to both parties at inception.
FRA Settlement	$(\text{notional principal})\left[\dfrac{\left(\text{MRR}_{B-A}-\text{IFR}_{A,B-A}\right)\left[\dfrac{\text{days}}{360}\right]}{1+\text{MRR}\left[\dfrac{\text{days}}{360}\right]}\right]$

Module 6

Pricing and Valuation of Futures Contracts

Differences between Forward and Futures

	Differences in the *value* of forward and futures
Forward contract	• MTM value changes are captured by the difference between the current spot price, St , and the present value of the forward price, $PV_t\ [F_0(T)]$. • ***MTM is not settled until maturity***, giving ***rise to counterparty credit risk*** over time.
Futures contract	• The daily settlement resets the futures MTM to zero, and variation margin is exchanged to settle the difference, ***reducing counterparty credit risk***. • At maturity and the futures price ***converge to the spot price***, S_T. • The cumulative realized MTM gain or loss on a futures contract is approximately the same as for a comparable forward contract.

	Differences in the *price* of forward and futures
Forward contract	• The forward contract price, $F_0(T)$, ***remains fixed*** until the contract matures.
Futures contract	• Futures contract prices ***fluctuate daily*** based upon market changes. • The difference depends on both interest rate volatility and ***the correlation between interest rates and futures prices.***

Pricing and Valuation of Futures

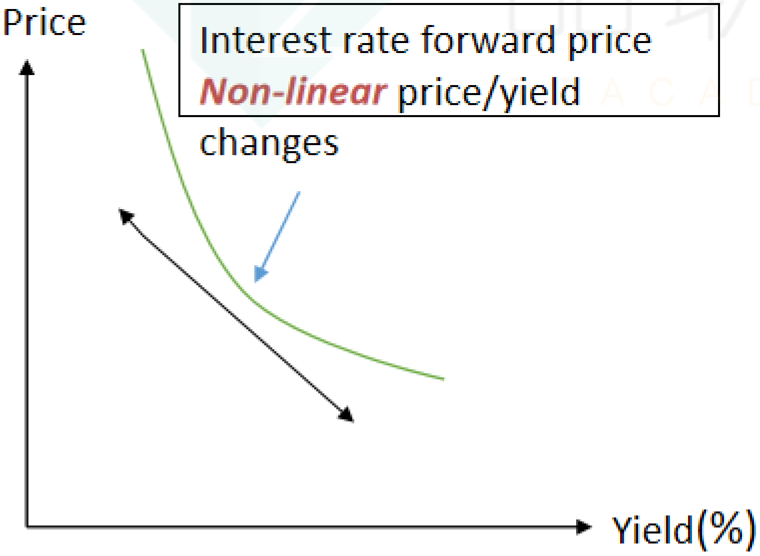
<table>
<tr><th></th><th>Pricing and Valuation of Futures Contracts</th></tr>
<tr><td>Effect of central clearing of OTC derivatives</td><td>• The emergence of derivatives central clearing has introduced futures-like margining requirements for over-the-counter (OTC) derivatives, such as forwards.
• Central clearing reduces the difference in futures and forward prices.</td></tr>
<tr><td>Prices of Futures vs. Forward Contracts</td><td>
<table>
<tr><th>Correlation between the Futures prices and interest rate</th><th>Forward vs. Futures</th></tr>
<tr><td>Positive</td><td>The futures price > forward contract.</td></tr>
<tr><td>Zero</td><td>The futures price = forward contract.</td></tr>
<tr><td>Negative</td><td>The futures price < forward contract.</td></tr>
</table>
</td></tr>
</table>

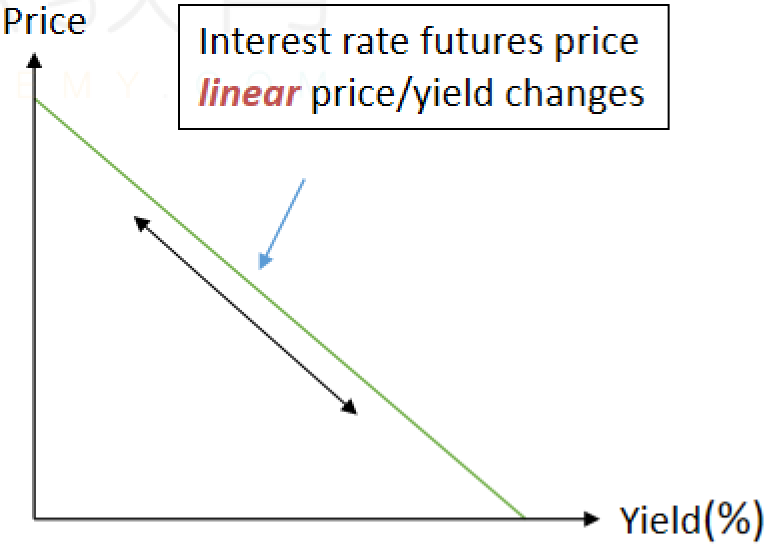
Interest Rate Futures

<table>
<tr><th></th><th>Interest rate futures</th></tr>
<tr><td>Price convention</td><td>$$f_{A,B-A} = 100 - (100 \times MRR_{A,B-A})$$
• $f_{A,B-A}$ represents the futures price for a market reference rate for B−A periods that begins in A periods ($MRR_{A,B-A}$).</td></tr>
<tr><td>Interest rate futures daily settlement</td><td>• Interest rate futures daily settlement:
$$Futures\ Contract\ BPV = Notional\ Principal \times 0.01\% \times Period$$
• One basis point (0.01%) change in MRR leads to contract BPV change $25.
• The interest rate futures contract has a fixed linear payoff profile for a given basis point change, the FRA settlement does not.</td></tr>
<tr><td>Interest rate futures VS FRAs</td><td><table>
<tr><th>Contract type</th><th>Gains from rising MRR</th><th>Gains from falling MRR</th></tr>
<tr><td>Interest rate futures</td><td>Short futures contract</td><td>Long futures contract</td></tr>
<tr><td>Forward rate agreement</td><td>Long FRA: FRA fixed-rate payer (FRA floating-rate receiver)</td><td>Short FRA: FRA floating-rate payer (FRA fixed-rate receiver)</td></tr>
</table></td></tr>
</table>

Interest Rate Futures

	convexity bias
Convexity bias	• There is a price difference between interest rate futures and forward rate agreements (FRAs) due to ***convexity bias.*** • The convexity bias between interest rate futures and interest rate forwards causes the percentage price change to be ***greater*** (in absolute value) when MRR falls than when it rises for a forward contract.





Module 7

Pricing and Valuation of Interest Rates and Other Swaps

Swaps VS. Forwards

	Swaps VS. forwards
Similarities	• The ***single cash flow of an FRA*** is similar to a ***single-period swap***. • Both contracts are firm commitments with ***symmetric payoff profiles*** and ***no cash is exchanged at inception***. • Both interest rate forward and swap contracts ***involve counterparty credit exposure***.
Differences	• ***The FRA has a single settlement***, which occurs at the beginning of an interest period, while a standard ***swap has periodic settlements***, which occur at the end of each respective period. • Different FRA fixed rates usually exist for different times to maturity. • An interest rate ***swap has a constant fixed rate over its life***, which includes multiple periods.

Benefits of using swaps over forwards	• Swaps ***allow issuers*** to match the periodic cash flows of a specific balance sheet liability to transform their ***interest rate exposure profile***.
	• Swaps ***enable investors*** to actively adjust their interest rate exposure profile without buying or selling underlying securities.
	• Swaps enable ***both issuers and investors*** to avoid the administrative burden of entering into and managing multiple forward contracts.
	• Interest rate swap can be used to ***change the portfolio duration***.
	    • A pay-fixed swap is similar to a short bond position that reduces duration;
	    • A receive-fixed swap is similar to a long bond position that increases duration.

Pricing and Valuation of Swap

	Pricing and Valuation of Swap Contracts
Swap Pricing	• ***The fixed rate*** on an interest rate swap is priced such that the present value of the floating payments ***is equal to*** the present value of the fixed payments at t = 0.

Swap Valuation	• At inception:
	  • ***the value of a swap is zero*** (ignoring transaction and counterparty credit costs).
	• On any settlement date:
	  • the value of a swap = ***the current settlement value*** + ***the present value of all remaining future swap settlements***.
	  • For the fixed-rate payer on a swap, the ***periodic settlement value*** is:
	  Periodic settlement value = $(MRR - s_N) \times$ Notional amount $\times$ Period.
	• The ***fixed-rate payer realizes an MTM gain*** on the swap if:
	$\sum PV(\text{Floating payments received}) > \sum PV(\text{Fixed payments paid})$
	• The ***fixed-rate payer realizes an MTM loss*** on the swap if:
	$\sum PV(\text{Floating payments received}) < \sum PV(\text{Fixed payments paid})$

IRS VS a bond portfolio	• The ***fixed-rate payer (receiver)*** is equivalent to ***long (short) a floating-rate note (FRN)*** priced at the MRR and ***short (long) a fixed-rate bond*** with a coupon equal to the fixed swap rate.

Module 8

Pricing and Valuation of options

Option Values

<table>
<tr><th></th><th>Option values</th></tr>
<tr><td>Option exercise value</td><td>

- Call option exercise value: ***Max (0, $S_t - X(1 + r)^{-(T-t)}$).***
- Put option exercise value: ***Max (0, $X(1 + r)^{-(T-t)} - S_t$).***

Before maturity
</td></tr>
<tr><td>Option moneyness</td><td>

- Moneyness expresses the ***relationship between the underlying price and the exercise price：***

<table>
<tr><th>Moneyness</th><th>Call Option (c_t)</th><th>Put Option (p_t)</th></tr>
<tr><td>In-the-money (ITM)</td><td>$S_t > X$</td><td>$S_t < X$</td></tr>
<tr><td>At-the-money (ATM)</td><td>$S_t = X$</td><td>$S_t = X$</td></tr>
<tr><td>Out-the-money (OTM)</td><td>$S_t < X$</td><td>$S_t > X$</td></tr>
</table>

- The degree to which an option is in or out of the money affects ***the sensitivity of an option's price to underlying price changes***.
</td></tr>
<tr><td>Option time value</td><td>

- ***Option price = exercise value + time value***
- Call option time value: c_t - Max(0, $S_t - X(1 + r)^{-(T-t)}$).
- Put option time value: p_t - Max(0, $X(1 + r)^{-(T-t)} - S_t$).
- The time value of an option is ***always positive*** but declines to zero at maturity, a process referred to as ***time value decay***
</td></tr>
</table>

Arbitrage and Replication

	Arbitrage and replication
No-arbitrage price bounds	• **Price bounds for call option:** ***$Max\ (0, S_t - X(1+r)^{-(T-t)}) < c_t \le S_t$*** $c_{t,\text{Lower bound}} = \text{Max}(0, S_t - X(1+r)^{-(T-t)})$ $c_{t,\text{Upper bound}} = S_t$ • **Price bounds for put option:** ***$Max\ (0, X(1+r)^{-(T-t)} - S_t) < p_t \le X$*** $p_{t,\text{Lower bound}} = \text{Max}(0, X(1+r)^{-(T-t)} - S_t)$ $p_{t,\text{Upper bound}} = X$

Replication	• An option has an ***asymmetric payoff profile***, and the likelihood of option exercise changes over time, transactions ***replicating an option contract must be adjusted over time***.
	• A forward commitment has a ***symmetric payoff profile*** that will settle with certainty at a future date, and therefore the replicating transactions ***do not require adjustment*** over the contract life.
	• **Replicate the long call option,** at inception (t = 0):
	    • ***Borrow a proportion of $X(1+r)^{-T}$***, which is based on the likelihood of exercise at time T, closely associated with the moneyness of an option, at the risk free rate, r.
	    • Use the proceeds to ***purchase the underlying at a price of S_0***.
	• **Replicate the short put option,** at inception (t = 0):
	    • ***Sell the underlying short at a price of S_0.***
	    • ***Lend a proportion of $X(1+r)^{-T}$***, which is based on the likelihood of exercise at time T, closely associated with the moneyness of an option, at the risk-free rate, r.

Factors Affect the Value of an European Option

Sensitivity Factor	Calls	Puts
Underlying price	Positively related	Negatively related
Volatility	Positively related	Positively related
Risk-free rate	Positively related	Negatively related
Time to expiration	Positively related	* May be negatively related for European put
Exercise price	Negatively related	Positively related
Payments on the underlying	Negatively related	Positively related
Carrying cost	Positively related	Negatively related

Module 9

Option Replication Using Put–Call Parity

Put－Call Parity

	Put–call Parity

Put–call Parity	• ***Protective put*** committed funds of $S_0 + p_0$, while ***fiduciary call*** committed funds of $c_0 + X/(1 + r)^T$.
	$$c_0 + X/(1+r)^T = S_0 + p_0$$
	• ***Key points***
	  • ***Calculation***
	  • ***Synthetic***
	  • ***Arbitrary***

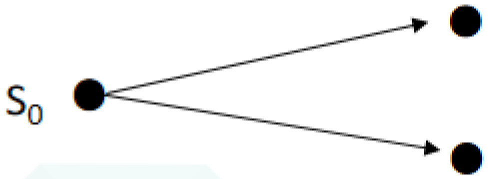
Put–Call–Forward Parity	• ***Protective Put with Forward***
	  • ***Long a forward contract*** and a ***risk- free bond in which the face value is the forward price. ($F_0(T)$, T, X)***; ***Long put. (p_0, T, X).***
	• ***Fiduciary Call***
	  • ***Long risk- free bond. ($F_0(T)$, T,)***; ***Long call. (c_0, T, X).***
	$$F_0(T)/(1+r)^T + p_0 = c_0 + X/(1+r)^T$$

Applications	• Put–call parity demonstrates:
	  • ***Equity holders have a position equivalent to a purchased call option on the value of the firm with unlimited upside;***
	  • ***Debtholders have a sold put option position on firm value with limited upside.***

Module 10

Valuing a Derivative Using a One-Period Binomial Model

Binomial Model

	Binomial Model
Binomial Model	 $S_1^u = R^u S_0$ $C_1^u = Max(0, S_1^u - X) = S_1^u - X$ $S_1^d = R^d S_0$ $C_1^d = Max(0, S_1^d - X) = 0$

No Arbitrage Method	• ***Create a hedge of a call***
	  • ***Sell one call and hold h units of the underlying.*** The value today of a combination of ***h units of the underlying and one short call is:*** $V_0 = hS_0 - c_0$.
	  • Depending on which of the two paths is taken by the underlying, the value of this portfolio at time 1 will be $V_1^u = hS_1^u - C_1^u$; $V_1^d = hS_1^d - C_1^d$.
	$h = \frac{C_1^u - C_1^d}{S_1^u - S_1^d}\,(shares\ per\ option)$
	$V_0 = \frac{V_1}{1+r} \rightarrow hS_0 - c_0 = \frac{V_1}{1+r} \rightarrow c_0 = hS_0 - \frac{V_1}{1+r}$

Risk Neutral Method	$c_0 = \frac{\pi c_1^u + (1-\pi)c_1^d}{(1+r)^T}$ $p_0 = \frac{\pi p_1^u + (1-\pi)p_1^d}{(1+r)^T}$ $\pi = \frac{1+r-R^d}{R^u - R^d}$ • ***The actual probabilities of the up and down moves do not matter***


Thank You!