

Taller final de matemáticas

① Determina el dominio y rango de las siguientes funciones (graficar a mano o en software).

• $S = f(t) = 4 - t^2$

(21)

	P ₁	P ₂	P ₃	P ₄	P ₅
x	-2	-1	0	1	2
y	0	3	4	3	0

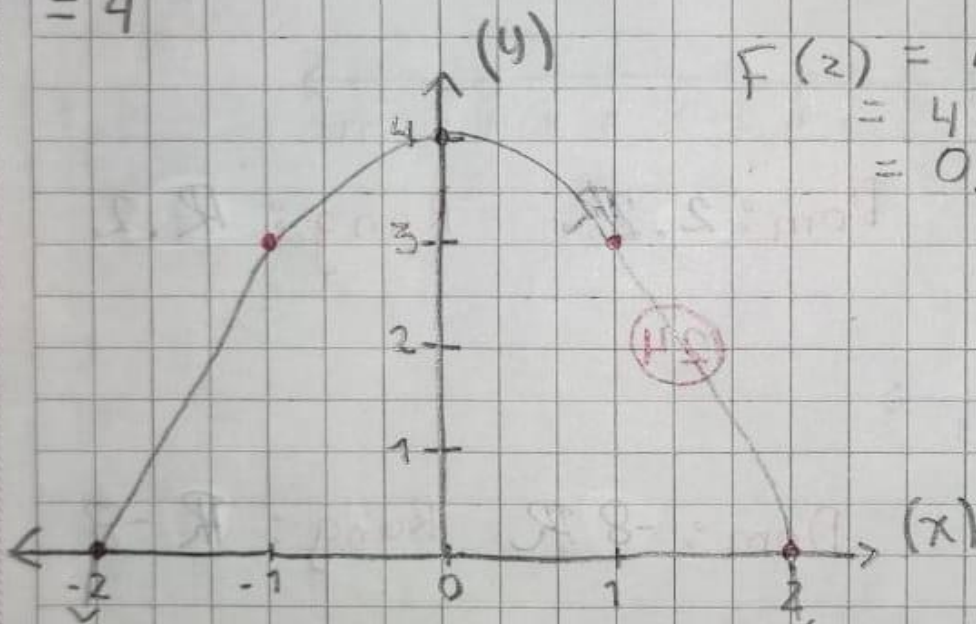
$$\begin{aligned} f(-2) &= 4 - (-2)^2 \\ &= 4 - (4) \\ &= 4 - 4 \\ &= 0 \end{aligned}$$

$$\begin{aligned} f(-1) &= 4 - (-1)^2 \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

$$\begin{aligned} f(0) &= 4 - (0)^2 \\ &= 4 - 0 \\ &= 4 \end{aligned}$$

$$\begin{aligned} f(1) &= 4 - (1)^2 \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

$$\begin{aligned} f(2) &= 4 - (2)^2 \\ &= 4 - (4) \\ &= 0 \end{aligned}$$



DOM / \mathbb{R}

RANGO / $(-\infty, 4]$
Menor Mayor

• $f(x) = 5 - 2x^2$

(22)

x	-2	-1	0	1	2
y	-3	3	5	3	-3

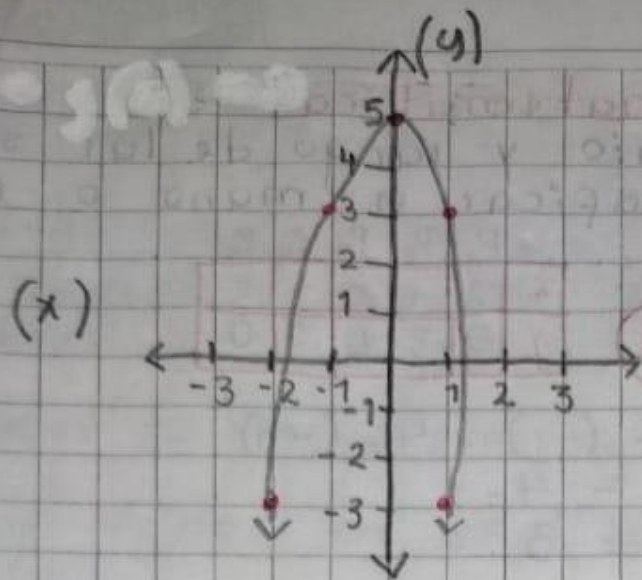
$$\begin{aligned} f(-2) &= 5 - 2(-2)^2 \\ &= 5 - 2(4) \\ &= 5 - 8 \\ &= -3, \end{aligned}$$

$$\begin{aligned} f(0) &= 5 - 2(0)^2 \\ &= 5 - 2(0) \\ &= 5 - 0 \\ &= 5 \end{aligned}$$

$$\begin{aligned} f(-1) &= 5 - 2(-1)^2 \\ &= 5 - 2(1) \\ &= 5 - 2 \\ &= 3, \end{aligned}$$

$$\begin{aligned} f(1) &= 5 - 2(1)^2 \\ &= 5 - 2(1) \\ &= 5 - 2 \\ &= 3 \end{aligned}$$

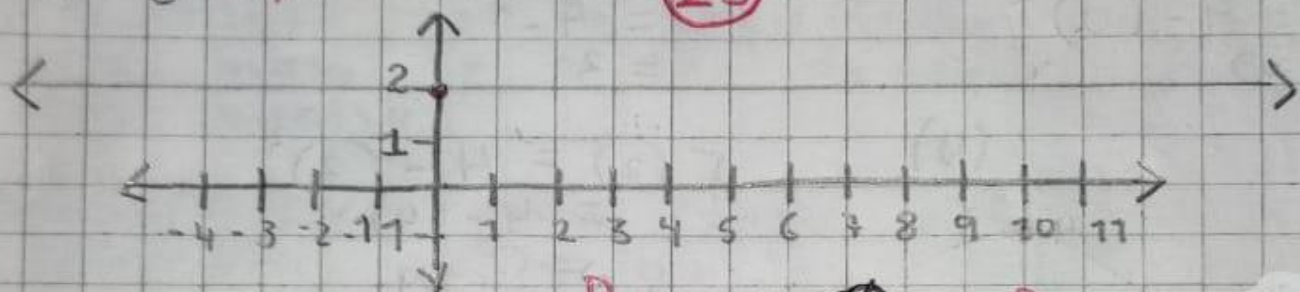
$$\begin{aligned} f(2) &= 5 - 2(2)^2 \\ &= 5 - 2(4) \\ &= 5 - 8 \\ &= -3 \end{aligned}$$



Dom : \mathbb{R}
Rango : $(-\infty, 5)$

• $y = g(x) = 2$

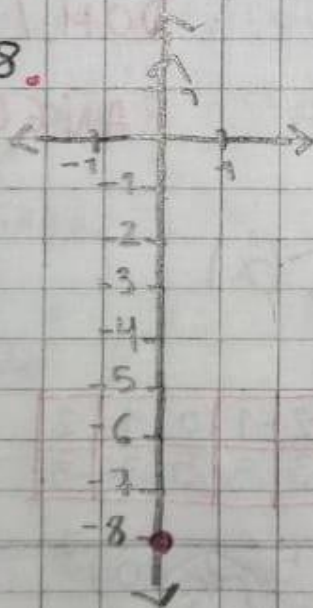
(23)



Dom : \mathbb{R} Rango : $\{2\}$

• $f(5) = -8$

(24)



Dom : \mathbb{R} Rango : $\{-8\}$

• $f(t) = -t^3$

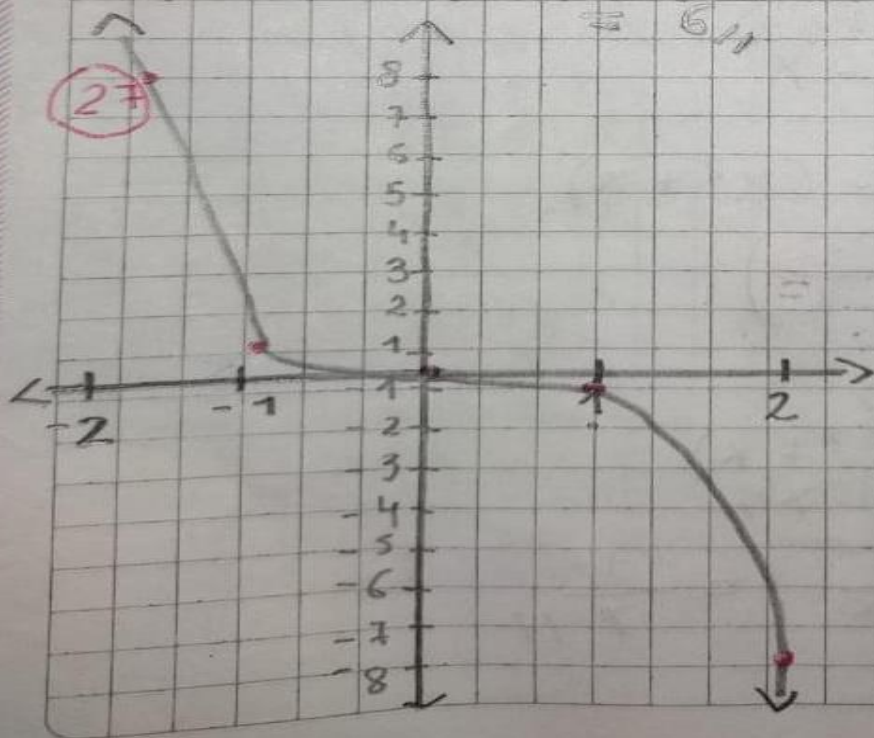
(27)

x	-2	-1	0	1	2
y	-8	-1	0	1	8

$f(-2) = (-2)^3$
 $f(-2) = -8$

$f(-1) = -(-1)^3$
 $f(-1) = -1$

$f(0) = -(0)^3$
 $f(0) = 0$



• $p = h(q) = q(3+q)$

x	-2	-1	0	1	2
y	-2	-2	0	4	10

* $p = h(-2) = -2(3+(-2))$
 $p = h(-2) = -2$

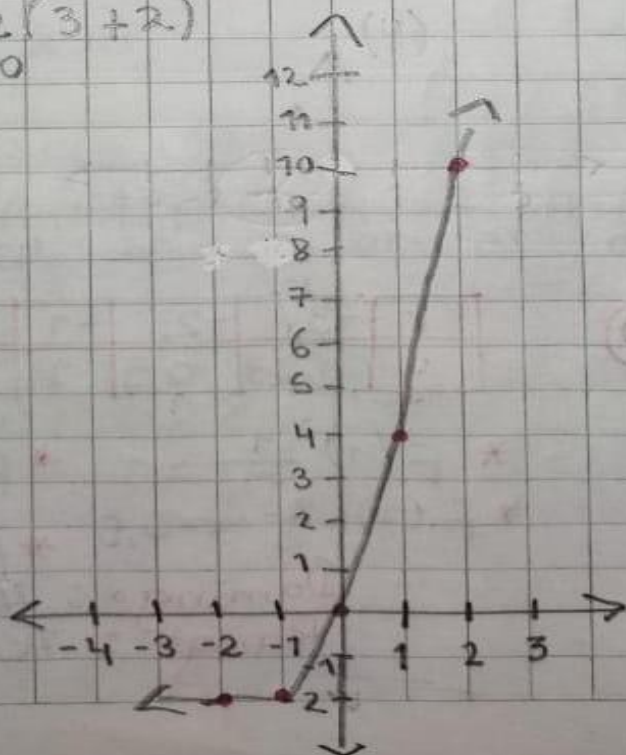
* $p = h(-1) = -1(3+(-1))$
 $p = h(-1) = -1(2)$
 $p = h(-1) = -2$

* $p = h(0) = 0(3+0)$
 $p = h(0) = 0$

* $p = h(1) = 1(3+1)$
 $p = h(1) = 4$

* $p = h(2) = 2(3+2)$
 $p = h(2) = 10$

(28)



Dominio: \mathbb{R}
Rango: \mathbb{R}

$$s = F(r) = \sqrt{r-5}$$

r	41	30	21	14	9
y	6	5	4	3	2

$$\begin{aligned} f(41) &= \sqrt{41-5} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

$$\begin{aligned} f(21) &= \sqrt{21-5} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

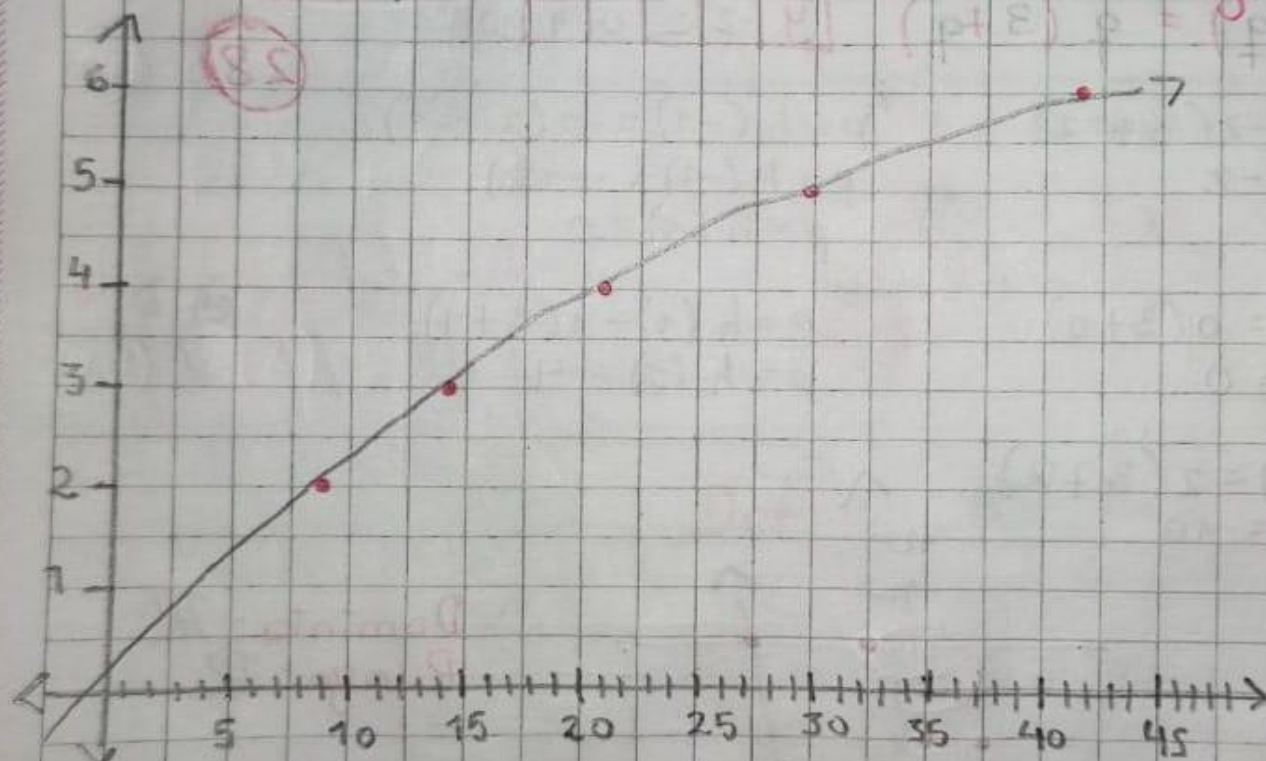
(29)

$$\begin{aligned} f(9) &= \sqrt{9-5} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} f(30) &= \sqrt{30-5} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} f(14) &= \sqrt{14-5} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

Domínio: \mathbb{R}
Rango: \mathbb{R}



$$F(r) = -\frac{1}{r}$$

(30)

r	-3	-2	-1	1	2	3
y	0,33	0,5	1	-1	-0,5	0,33

$$* f(-2) = -\frac{1}{-2} = 0,5$$

$$* f(-1) = -\frac{1}{-1} = 1$$

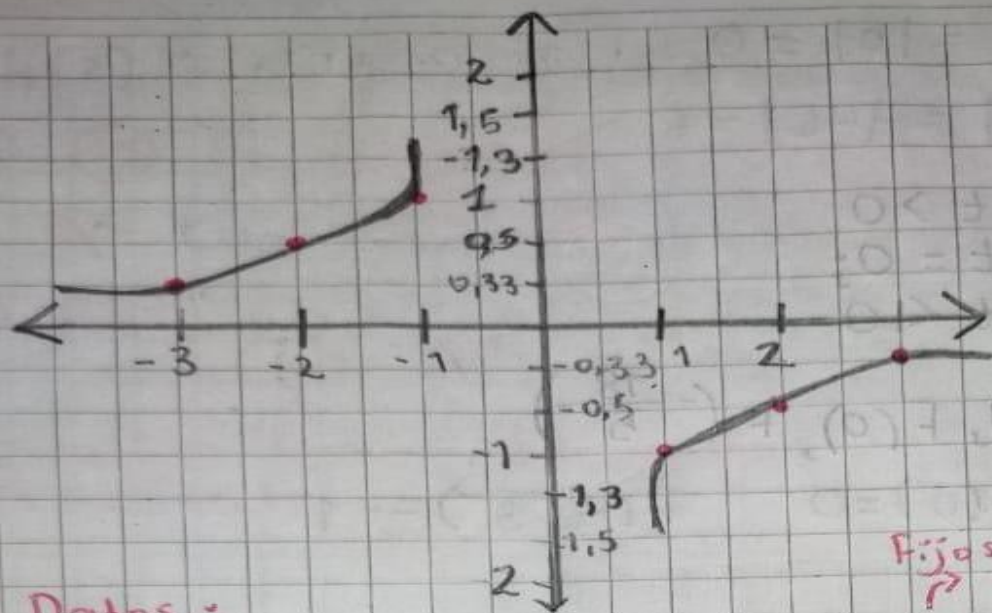
$$* f(1) = \frac{1}{1} = 1$$

$$* f(2) = \frac{1}{2} = 0,5$$

$$* f(3) = \frac{1}{3} = 0,33$$

$$* f(-3) = -\frac{1}{3} = -0,33$$

Domínio: $\mathbb{R} - [0]$
Rango: $\mathbb{R} - [0]$



③ Datos:

Fijas Variables
 $\rightarrow \uparrow$

①⑦ $f(x) = 8$; $f(2)$, $f(t+8)$, $f(-\sqrt{17})$

$$f(x) = 8 \quad f(2) = 8 \quad f(t+8) = 8$$

$$f(-\sqrt{17}) = 8$$

①⑧ $g(x) = |x-3|$; $g(10)$, $g(3)$, $g(-3)$.

$$g(x) = |x-3|$$

$$g(10) = |10-3| = |7| = 7$$

$$g(3) = |3-3| = |0| = 0$$

$$g(-3) = |-3-3| = |-6| = 6$$

$$(19) \quad F(t) = \begin{cases} 1, & \text{si } t > 0 \\ 0, & \text{si } t = 0; \\ -1, & \text{si } t < 0 \end{cases}$$

$$F(10), F(-\sqrt{3}), F(0), F\left(-\frac{18}{5}\right)$$

$$F(10) = 1 \quad F(0) = 0 \quad F(-\sqrt{3}) = -1$$

$$F\left(-\frac{18}{5}\right) = -1$$

$$(20) \quad f(x) = \begin{cases} 4, & \text{si } x \geq 0 \\ 3, & \text{si } x < 0 \end{cases}$$

$$f(3), f(-4), f(0).$$

$$f(3) = 4$$

$$f(-4) = 3$$

$$f(0) = 4$$

$$(21) \quad G(x) = \begin{cases} x, & \text{si } x \geq 3 \\ 2-x^2, & \text{si } x < 3 \end{cases}$$

$$G(8), G(3), G(-1), G(1)$$

$$G(8) = 8$$

$$G(3) = 3$$

$$G(-1) = 2 - (-1)^2 = 2 - 1 = 1$$

$$G(1) = 2 - (1)^2 = 2 - 1 = 1$$

$$(22) \quad h(r) = \begin{cases} 3r-1, & \text{si } r > 2 \\ r^2-4r+7, & \text{si } r < 2 \end{cases}$$

$$h(3), h(-3), h(2)$$

$$h(3) = 3(3) - 1 = 9 - 1 = 8$$

$$h(-3) = (-3)^2 - 4(-3) + 7$$

$$= 9 + 12 + 7$$

$$= 28$$

③ Datos:
Costo inicial: \$850
Costos adicionales: $3q$

R) a. $C_T(q) = 850 + 3q$

b. $1.600 = 850 + 3q$

$$3q = 1.600 - 850$$

$$3q = 750$$

$$q = \frac{750}{3}$$

$$q = 250.$$

$$C_T = C_F + C_V$$

31

R) Se producen 250 unidades por un costo total de \$1.600.

Punto 4

a. $(f+g)(x) = x^3 + x^2 + x$
 $= 2x^2 + x //$

b. $(f-g)(x) = x^2 - (x^2 + x)$
 $= x^2 - x^2 - x$
 $= -x //$

c. $(f-g)\left(-\frac{1}{2}\right) = x^2 - (x^2 + x)$
 $= -x$
 $= -\left(-\frac{1}{2}\right)$
 $= \frac{1}{2} //$

d. $(fg)(x) = x^2(x^2 + x)$
 $= x^4 + x^3 //$

e. $\frac{f}{g}(x) = \frac{x^2}{x^2 + x} = \frac{1}{x} //$

$$f. \frac{f}{g} \left(-\frac{1}{2} \right) = \frac{x^2}{x^2 + x}$$

$$= \frac{1}{x} = \frac{1}{-\frac{1}{2}} = -\frac{2}{1} = -2 //$$

$$g. (f \circ g)(x) = (x^2 + x)^2$$

$$= (x^2)^2 + 2(x^2)(x) + (x)^2$$

$$= x^4 + 2x^3 + x^2 //$$

$$h. (g \circ f)(x) = (x^2)^2 + (x^2)^1$$

$$= x^4 + x^2 //$$

$$i. (g \circ f)(-3) = (-3)^2 + (-3)^1$$

⑤ ⑮ Ecuación de demanda:

Q = Cantidad

P = Precio

C = Costo

$$\text{Datos: } (q_1, p_1) = (40, 12)$$

$$(q_2, p_2) = (25, 18)$$

Ecuación lineal demanda:

Pendiente:

$$m = \frac{p_2 - p_1}{q_2 - q_1}$$

$$m = \frac{18 - 12}{25 - 40}$$

$$m = \frac{6}{-15}$$

$$m = -0,4$$

Ecuación demanda:

y = Unidades demandadas

$$y - p_1 = m(q - q_1)$$

$$y - 12 = -0,4(q - 40)$$

$$y - 12 = -0,4q + 16$$

$$y = -0,4q + 16 + 12$$

$$y = -0,4q + 28$$

Precio cuando la demanda es 30.

P = y

Q = Cantidad

$$p = -0,4q + 28$$

$$= -0,4(30) + 28$$

$$= -12 + 28$$

$$p = 16 //$$

16) Ecuación de demanda.

Datos:

$$q_1 = 26.000 \text{ (Libros)} \quad q_2 = 10.000 \text{ (Libros)}$$

$$p_1 = 16 \text{ (x und)} \quad p_2 = 24 \text{ (x und)}$$

$$m = \frac{p_2 - p_1}{q_2 - q_1}$$

$$m = \frac{24 - 16}{10.000 - 26.000}$$

$$m = \frac{8}{-16.000}$$

$$m = \frac{1}{2000} //$$

Demanda: $P - P_1 = M (Q - Q_1)$

$$P - 16 = \frac{-1}{2000} (Q - 26000)$$

$$P = \frac{-1}{2000} Q + 13 + 16$$

$$P = \frac{-1}{2000} Q + 29 //$$

Ecuación de demanda.

17) Ecuación de oferta

Datos:

q = cantidad producida P = precio

$$(3000, 940) \quad (2200, 740)$$

Ecuación de la oferta: ?

$$m = \frac{p_2 - p_1}{q_2 - q_1}$$

$$m = \frac{740 - 940}{2200 - 3000} = \frac{-200}{-800} = \frac{1}{4}$$

$$p_2 - p_1 = m \cdot (q - q_1)$$

$$p = 940 = \frac{1}{4} (q - 3000)$$

$$p = \frac{1}{4} q - 750 + 940$$

$$p = \frac{1}{4} q + 190$$

$$P = \frac{1}{4} q + 190$$

19) Ecuación de costo

$$P_1 = 40$$

$$P_2 = 70$$

$$q_1 = 10$$

$$q_2 = 20$$

$$m = \frac{P_2 - P_1}{q_2 - q_1}$$

$$m = \frac{70 - 40}{20 - 10}$$

$$m = \frac{30}{10}$$

$$m = 3$$

$$y - 40 = 3(x - 10)$$

$$y = 3x - 30 + 40$$

$$y = 3x + 10$$

Substitución:

$$C(q) = 3q + 10$$

$$q = 35$$

$$C(35) = 3(35) + 10$$

$$C(35) = \$115 //$$

Punto 6.

$$\textcircled{31} \begin{cases} 2x - y = 6 \\ 3x + 2y = 5 \end{cases}$$

$$1) (2) \quad 4x - 2y = 12$$

$$2) \quad 3x + 2y = 5$$

$$\hline 7x = 17$$

$$x = \frac{17}{7}$$

$$\textcircled{32} \begin{cases} 8x - 4y = 7 \\ y = 2x - 4 \end{cases}$$

$$\begin{aligned} 8x - 4y &= 7 \\ -2x + y &= -4 \end{aligned}$$

$$1) (4) \quad \begin{aligned} -8x + 4y &= 16 \\ 8x - 4y &= 7 \end{aligned}$$

NO TIENE SOLUCIÓN

$$\textcircled{34} \begin{cases} 3x + 6y = 9 & (-4) \\ 4x + 8y = 12 & (-3) \end{cases}$$

$$1) -12x - 24y = -36$$

$$2) -12x - 24y = -36$$

$$= 0$$

$$\textcircled{37} \begin{cases} 3x - 2y + z = -2 & (1) \\ 2x + y + z = 1 & (2) \\ x + 3y - z = 3 & (3) \end{cases}$$

$$\textcircled{3} x + 3y - z = 3 \quad (-3)$$

$$\textcircled{1} 3x - 2y + z = -2$$

$$-3x - 9y + 3z = -9$$

$$3x - 2y + z = -2$$

$$\boxed{-11y + 4z = -11}$$

$$\textcircled{35} \begin{cases} \frac{1}{4}x - \frac{3}{2}y = -4 & (-3) \\ \frac{3}{4}x + \frac{1}{2}y = 8 \end{cases}$$

$$\frac{3}{4}x + \frac{1}{2}y = 8$$

$$\textcircled{1} -\frac{3}{4}x + \frac{9}{2}y = 12$$

$$\textcircled{2} \frac{3}{4}x + \frac{1}{2}y = 8$$

$$0 \quad 5y = 20$$

$$y = \frac{20}{5}$$

$$y = 4$$

$$\textcircled{3} x + 3y - z = 3 \quad (-2)$$

$$2x + y + z = 1$$

$$-2x - 6y + 2z = -6$$

$$2x + y + z = 1$$

$$-5y + 3z = -5$$

$$\begin{cases} -11y + 4z = -11 & \textcircled{4} \quad (-3) \\ -5y + 3z = -5 & \textcircled{5} \quad (-4) \end{cases}$$

$$\begin{cases} -11y + 4z = -11 & \textcircled{4} \quad (-3) \\ -5y + 3z = -5 & \textcircled{5} \quad (-4) \end{cases}$$

$$\textcircled{4} 33y - 12z = 33$$

$$-5y + 12z = -20$$

$$13y = 13$$

$$y = \frac{13}{13}$$

$$y = 1$$