ENTROPY AND GENETIC ALGORITHMS: DEFINITION, AND SOME GRAPHS

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A. Introduction

Although the entropy concept has been used extensively in the study of complex systems (1), it has received little attention in the theory of genetic algorithms (2).

The purpose of this note is to propose a definition and present some graphs of the entropy of the population of the strings, during the genetic search.

B. Definition of Entropy

On the analogy of the physical systems, we divide the genotypically defined phase space of the population of the strings into compartments, where the different points of a compartment represent phenotypically identical populations. These populations may differ genotypically.

The entropy of the population of the strings is a measure of the volume of the compartment containing the phase space point which represents the population (1).

The entropy E_i of a string is defined as:

$$E_i = -p_i \cdot \ln p_i,$$

where p_i is the phenotypic probability of the string.

Consequently the entropy of a population of n strings is defined as:

$$E = \sum_{i=1}^{n} -p_i \cdot \ln p_i$$

C. Objective Function

The objective function to be optimized was the 120-bit, multimodal, deceptive function, formed by summing 40 copies of the 3-bit deceptive function defined by Goldberg (3).

D. Initial Population

The initial population was set to 600.

E. Operators

The following operators were used:

- Crossover.

One-point crossover was used.

- Mutation.

The probability for mutation (p_m) is defined as probability per bit.

- Transposition.

Let us denote with A_i the ith symbol of a string A, of length l. Assume that a substring of length d of the string A is transposed from the site g to the site h. Then the string A' will be generated. Assuming that $g>h$ and that $d<g-h$, we have:

$$\forall j \in N: (1 \le j < h) \Rightarrow A'_j = A_j$$

$$\forall j \in N: (h \le j < h+d) \Rightarrow A'_j = A_x \text{ where } x = j+g-h$$

$$\forall j \in N: (h+d \le j < g+d) \Rightarrow A'_j = A_y \text{ where } y = j-d$$

$$\forall j \in N: (g+d \le j \le l) \Rightarrow A'_j = A_j$$

In a similar way, we can define transposition when $d>g-h$, when $g<h$, and so on.

The g,h, and d are defined randomly.

The probability for transposition (p_t) is defined as probability per string.

F. Selection Schemes

The following selection schemes were used:

- Binary tournament selection (4).

The probability for selecting the fittest string was set to 1.0 .

- Deterministic crowding (5).
- Sharing (6).

Sharing was based on the Hamming distance. σ_{share} was set to $l/2$=60 and α to 1. The niche count was estimated by sampling the 1/10 of the population.

G. Phenotypic Probability

The phenotypic probability p_i of each string was calculated using a recursive algorithm.

H. Number of Generations

The genetic search continued for 250 generations.

I. The Graphs

The graphs of the entropy vs the genetic search for different combinations of selection schemes, probability for mutation, and probability for transposition, are presented.

The graphs of each front page present the maximum and mean fitness and the entropy of the population of the strings during five consecutive runs.

The graphs of each back page present the maximum and mean fitness and the entropy of the population of the strings during one randomly selected run.

J. Discussion

The graphs of the entropy vs the generation number show distinct patterns for each combination of selection scheme, probability for mutation, and probability for transposition. It can be easily seen that the genetic algorithms can create a great amount of order. Considering the optimization of our objective function, the entropy of the population of the strings, at the end of the genetic search, is about 30 orders of magnitude less than the entropy of the initial population. On the other hand, the graphs describe quantitatively the randomness that mutation and transposition reintroduce into the population.

Obviously, the graphs of the entropy vs generation give us more information about the process of the genetic search, than the commonly used graphs of the mean and maximum fitness. Therefore, the mathematical analysis of the entropy during the genetic search could offer a new insight into the nonlinear dynamics of the genetic algorithms and their operators. The analysis will be probably based on the schema theorem.

A particularly intriguing prospect is that such analysis could possibly show that the most efficient genetic search is performed at the boundary between order and randomness, as it happens to other complex adaptive systems (7).

I hope that this note will stimulate a creative exchange of ideas, in these directions.

H. Acknowledgments

I thank Theophanes T. Hatjimihail for the helpful discussions, during the preparation of this note.

I. References

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6. Goldberg DE, Deb K, Horn J. Massive multimodality, deception, and genetic algorithms. In Maenner R and Manderick B. Parallel problem solving from nature 2. Amsterdam: Elsevier Science Publishers B.V. 1992:37-46.
7. Ruthen R. Adapting to complexity. Scientific American 1993;268(1):110-117.

Parameters of the genetic search:
$p_c = 1.0$, $p_m = 0.0000$, $p_t = 0.000$

Selection scheme:
Binary tournament selection

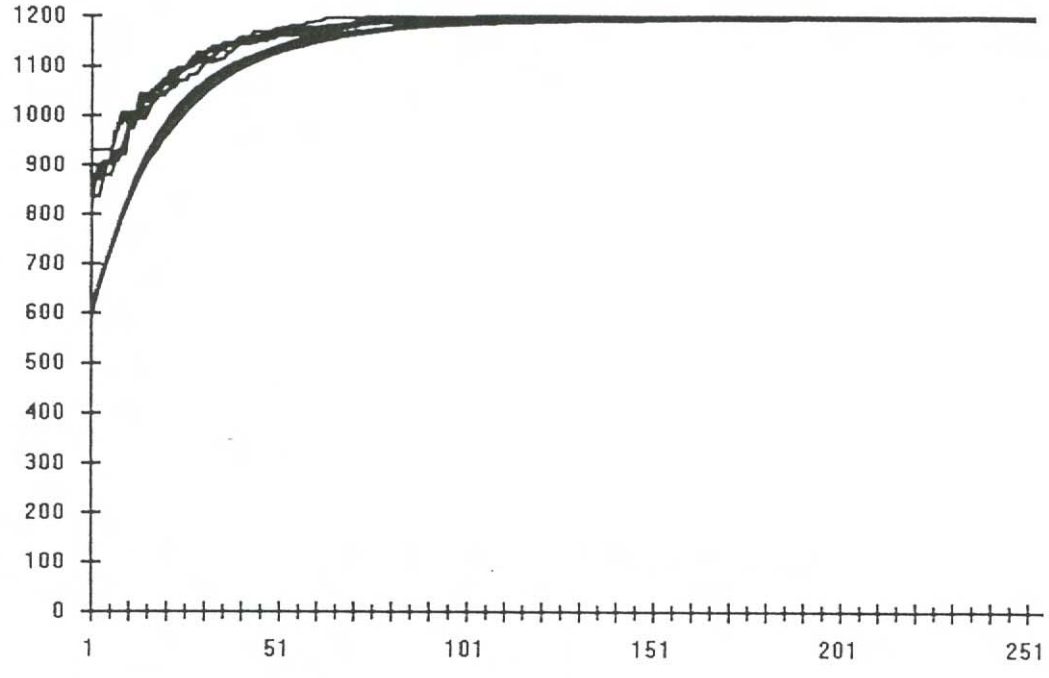
Front page:

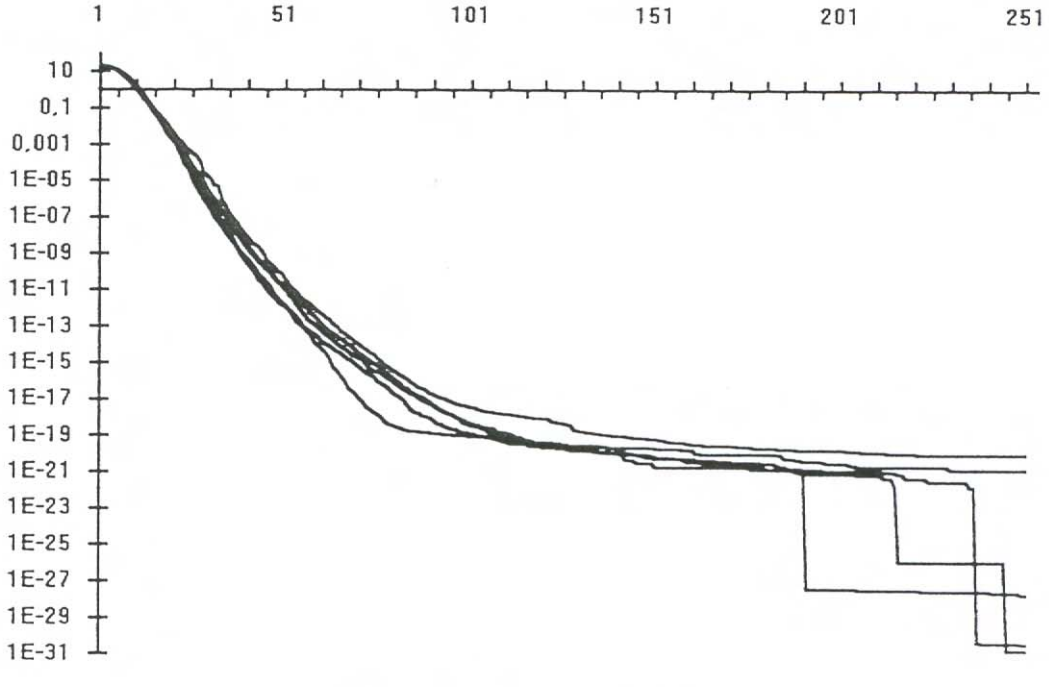
The results of five consecutive runs.
The optimal solution was found during 2/5 runs.

Back page:

The results of one run.
Fittest solution found: 1196, during the 118th generation.

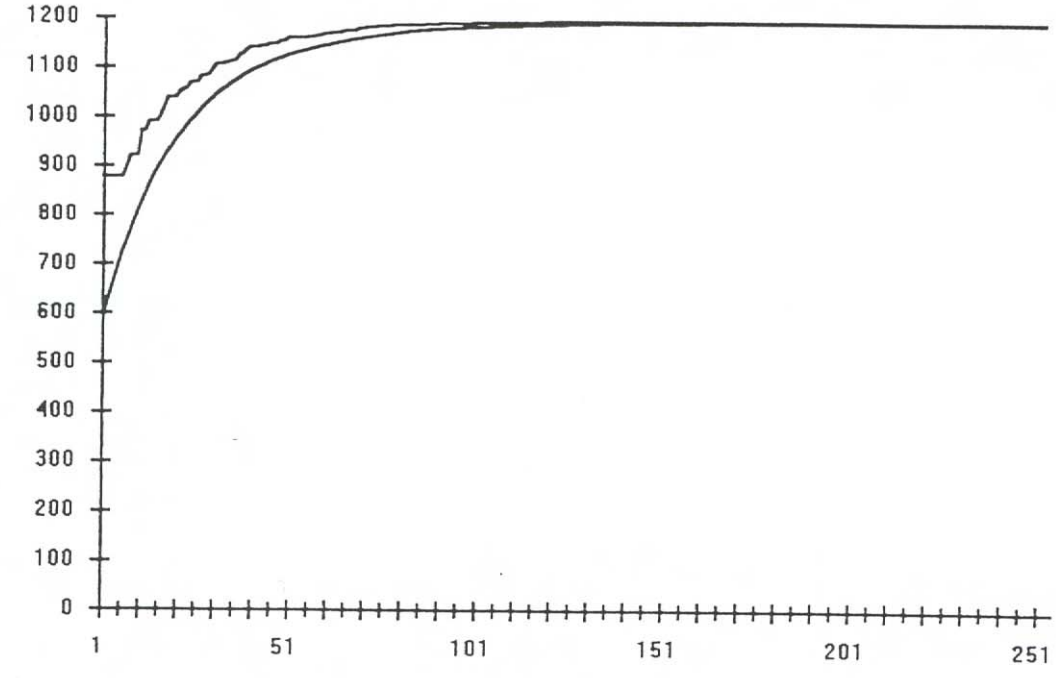
Maximum and Mean Fitness (vs generation)

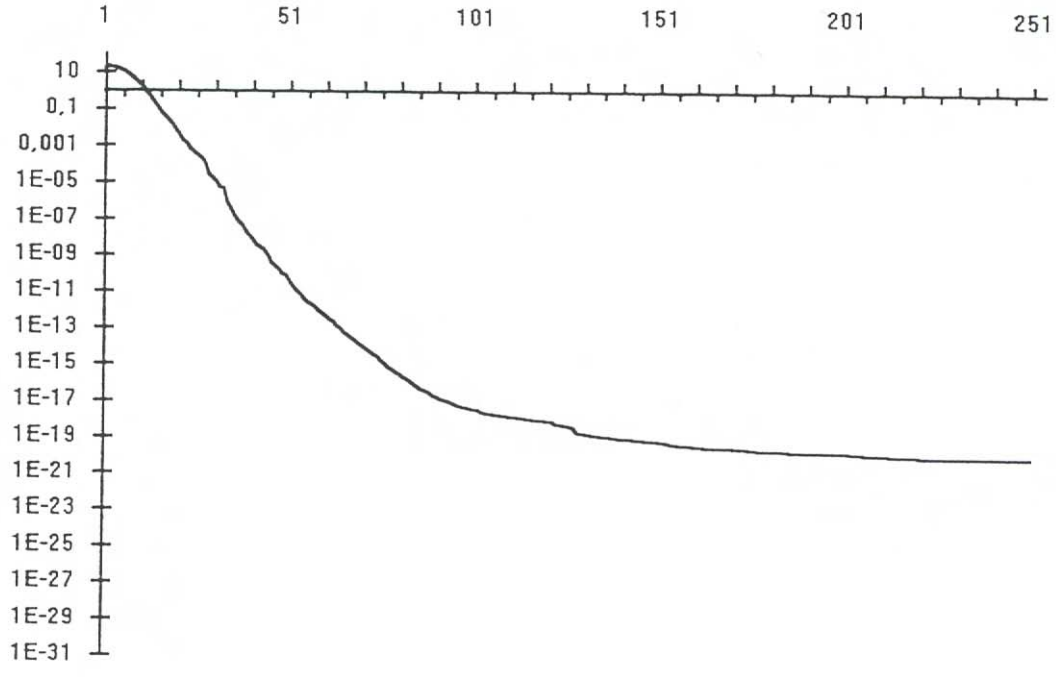




ENTROPY (vs generation)

Maximum and Mean Fitness (vs generation)





ENTROPY (vs generation)

Parameters of the genetic search:
$p_c = 1.0$, $p_m = 0.0000$, $p_t = 0.000$

Selection scheme:
Deterministic Crowding.

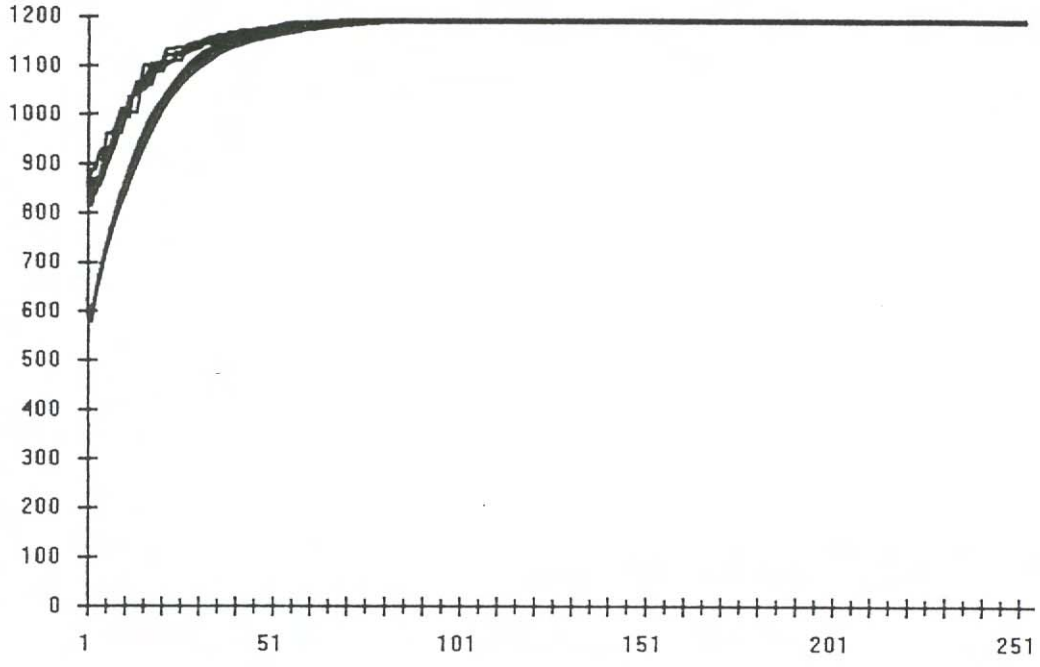
Front page:

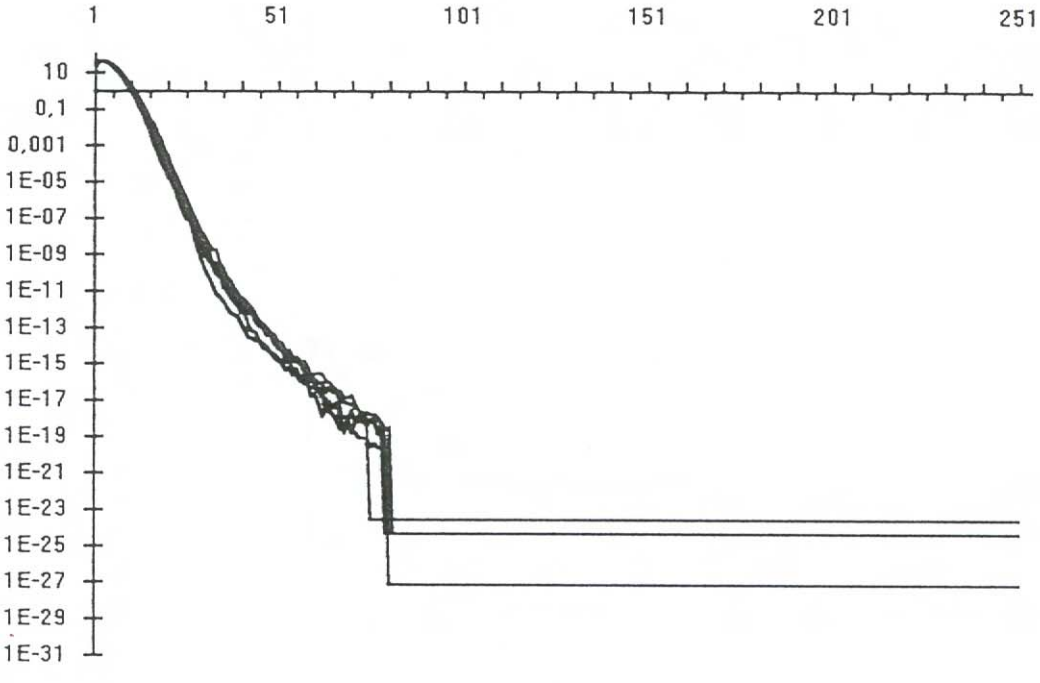
The results of five consecutive runs.
The optimal solution was found during 0/5 runs.

Back page:

The results of one run.
Fittest solution found: 1194, during the 68th generation.

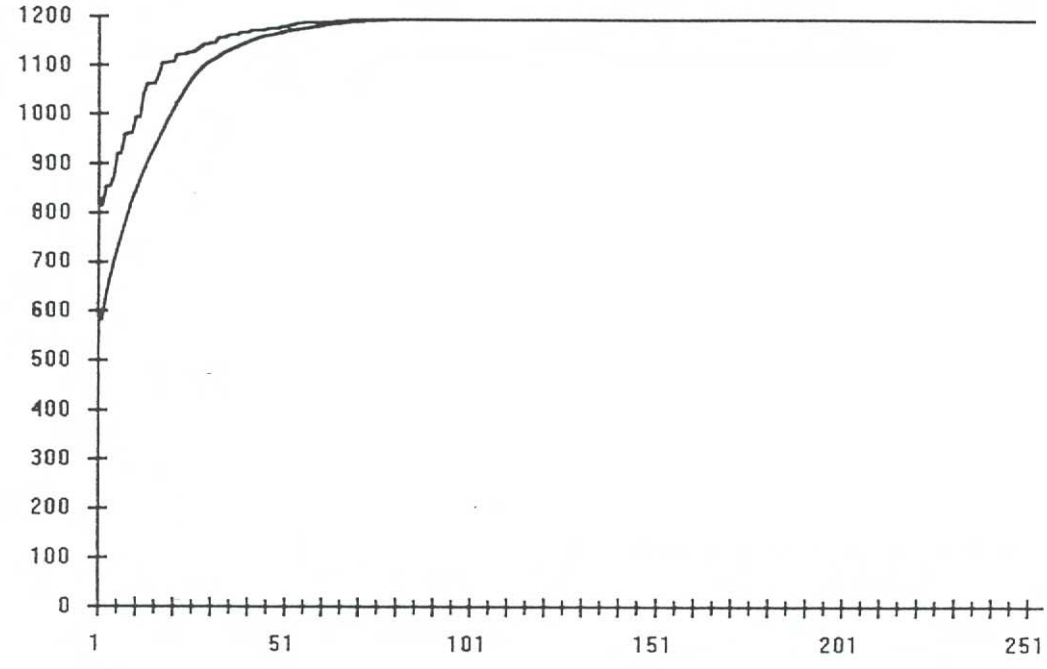
Maximum and Mean Fitness (vs generation)

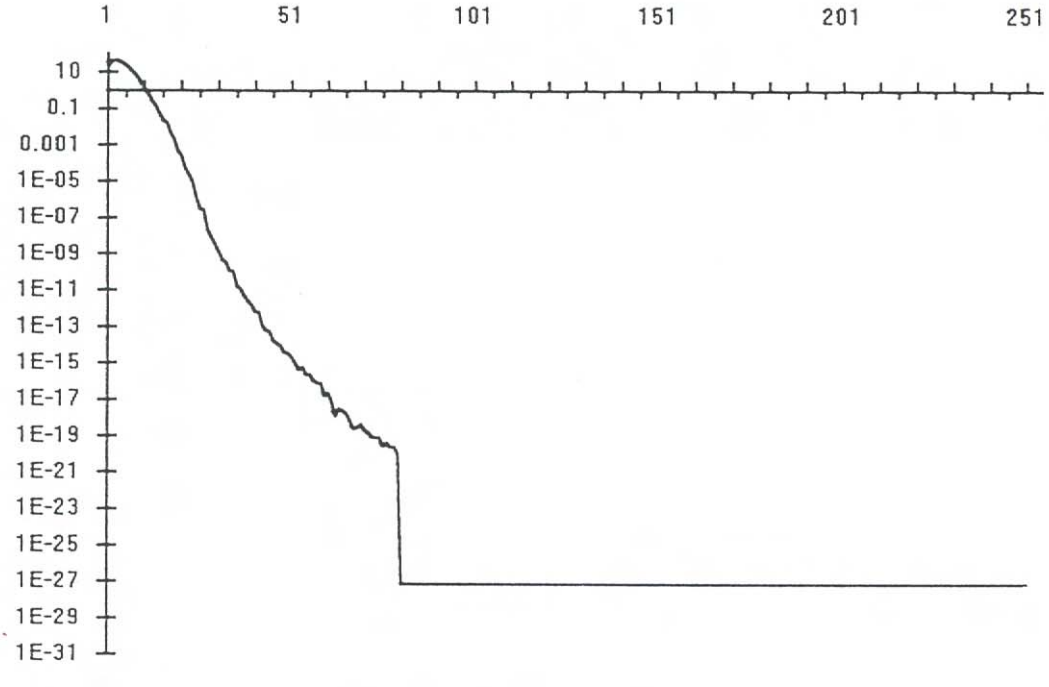




ENTROPY (vs generation)

Maximum and Mean Fitness (vs generation)





ENTROPY (vs generation)

Parameters of the genetic search:
$p_c = 1.0$, $p_m = 0.0000$, $p_t = 0.000$

Selection scheme:
Sharing.

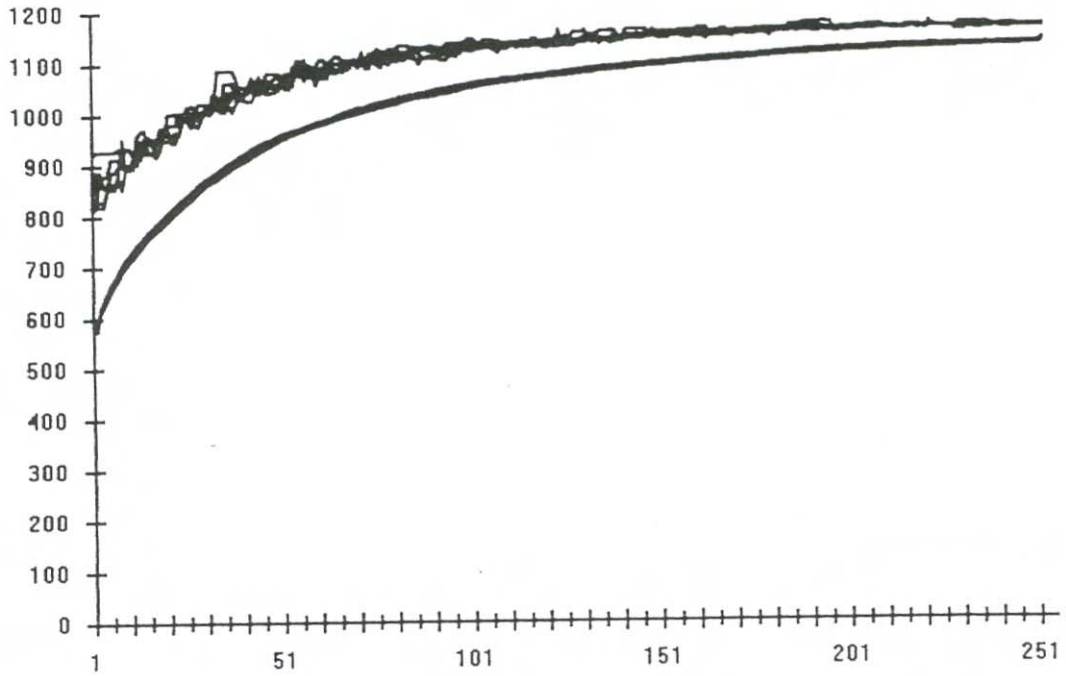
Front page:

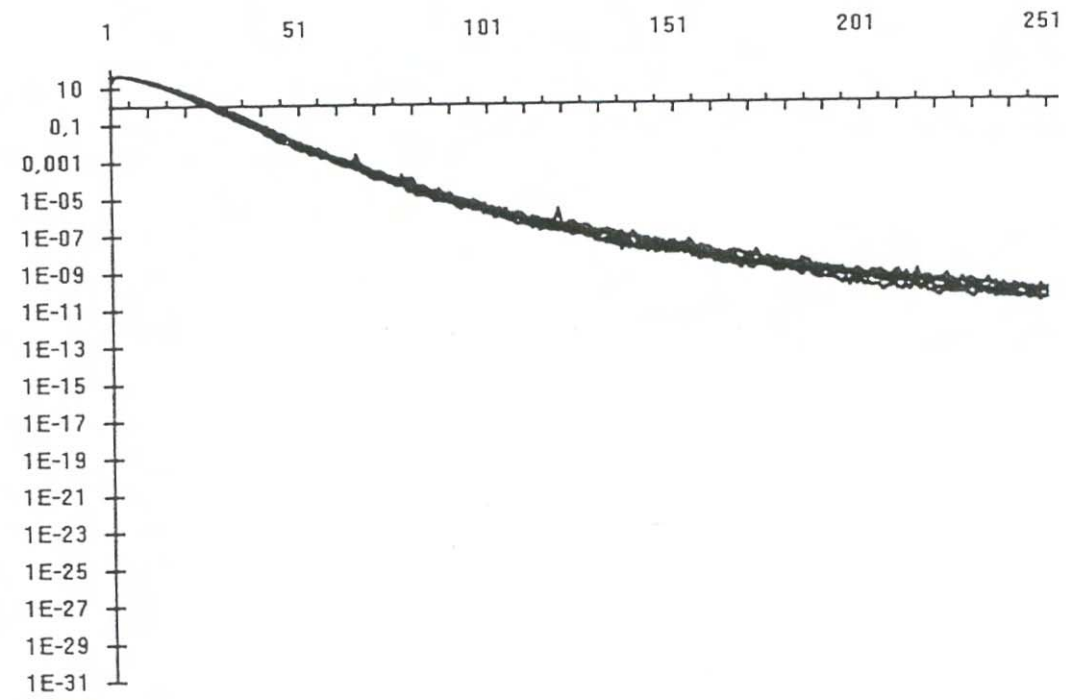
The results of five consecutive runs.
The optimal solution was found during 0/5 runs.

Back page:

The results of one run.
Fittest solution found: 1168, during the 245th generation.

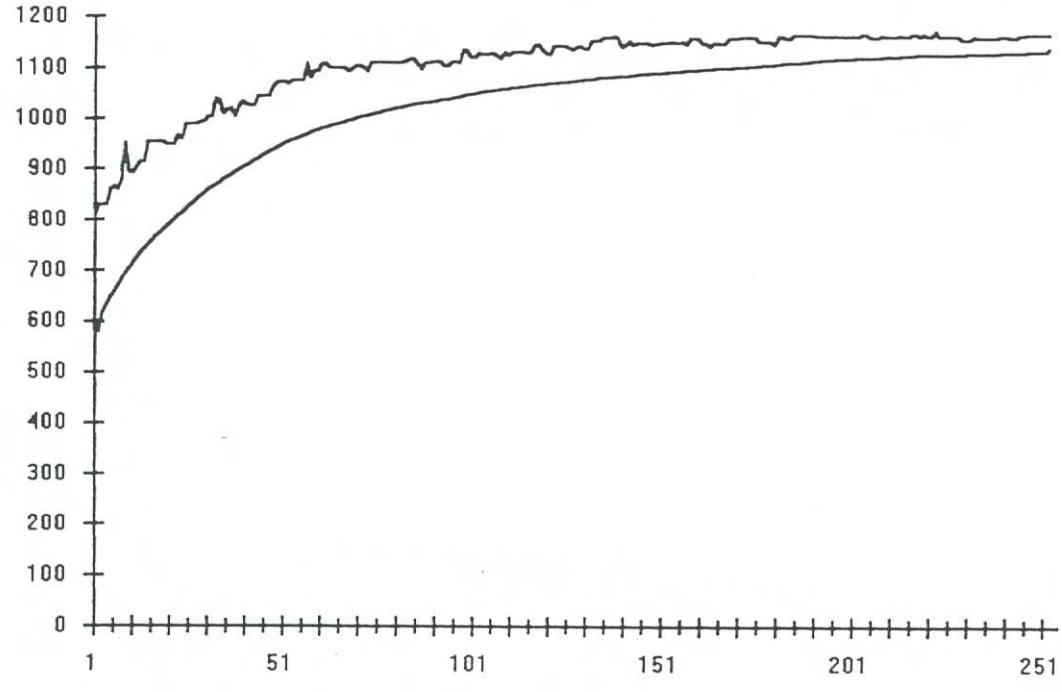
Maximum and Mean Fitness (vs generation)

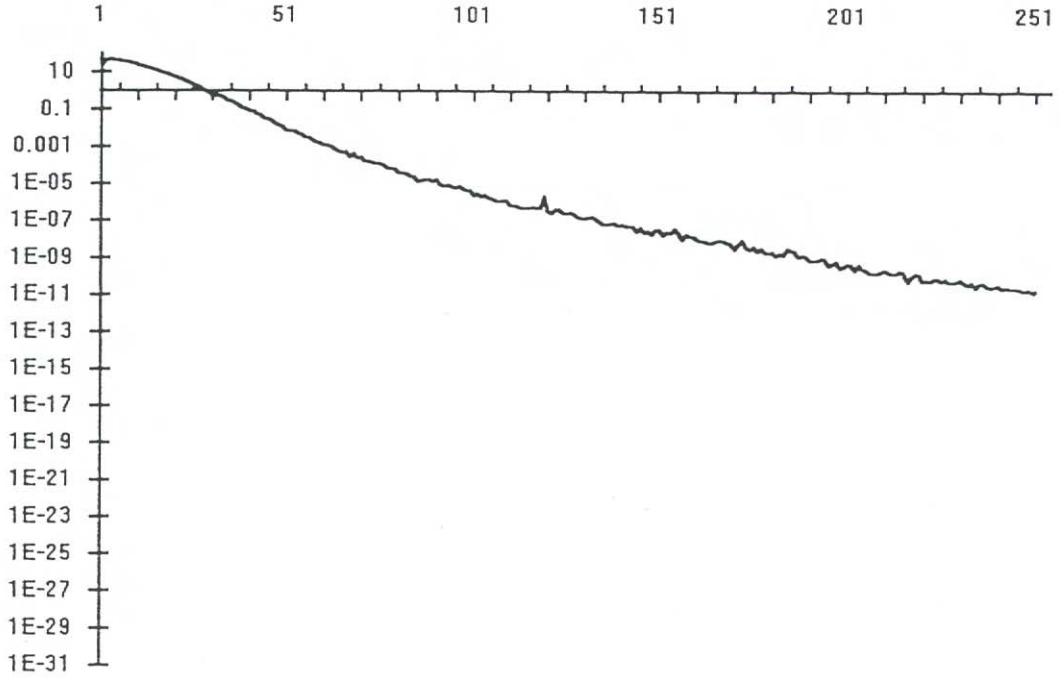




ENTROPY (vs generation)

Maximum and Mean Fitness (vs generation)





ENTROPY (vs generation)

Parameters of the genetic search:
$p_c = 1.0,\ p_m = 0.0001,\ p_t = 0.000$

Selection scheme:
Binary tournament selection

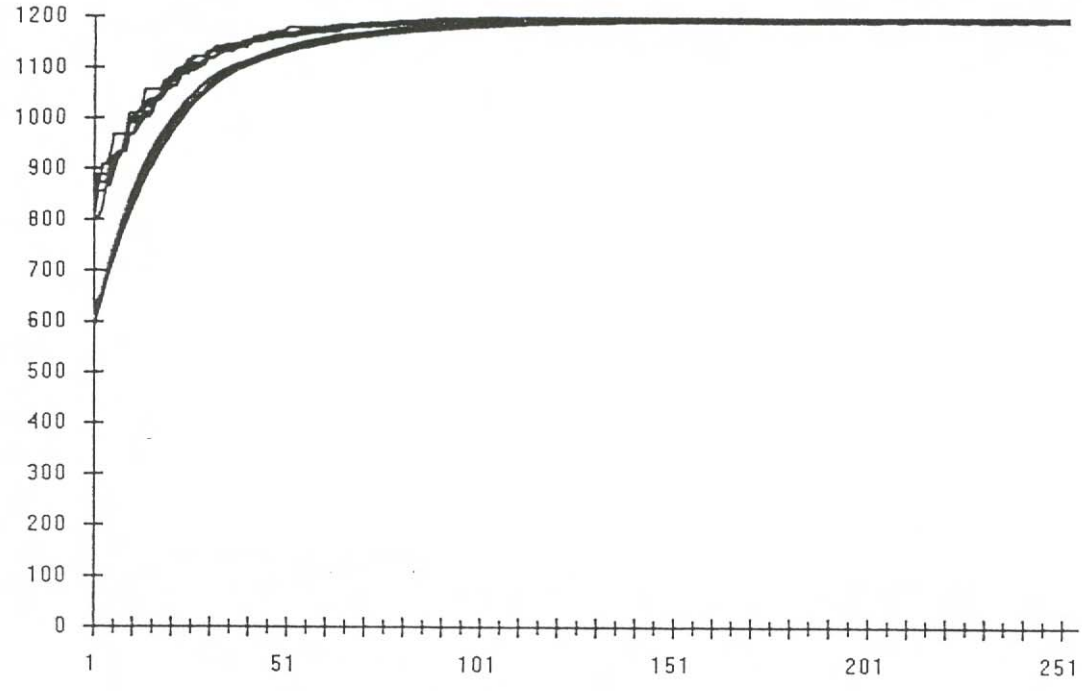
Front page:

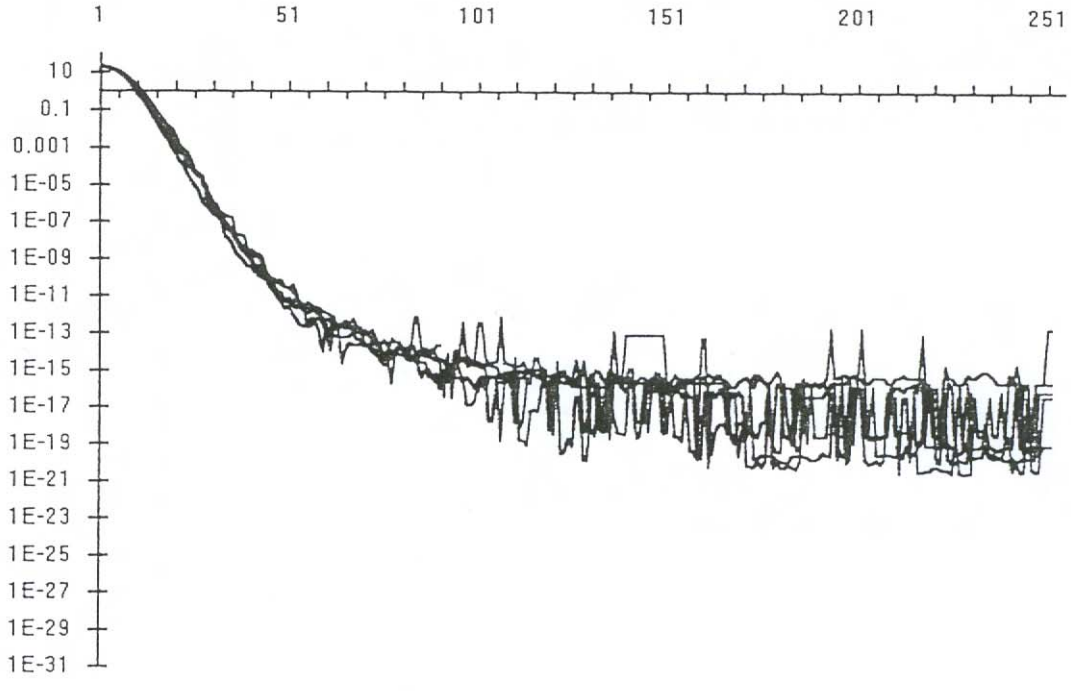
The results of five consecutive runs.
The optimal solution was found during 1/5 runs.

Back page:

The results of one run.
Fittest solution found: 1196, during the 107th generation.

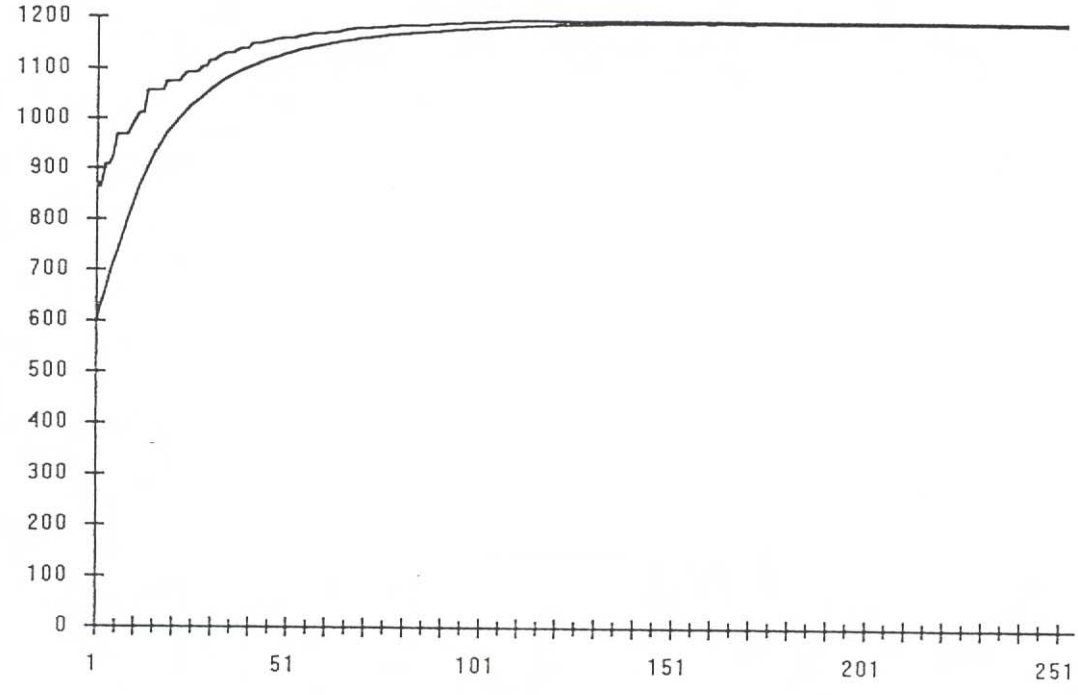
Maximum and Mean Fitness (vs generation)

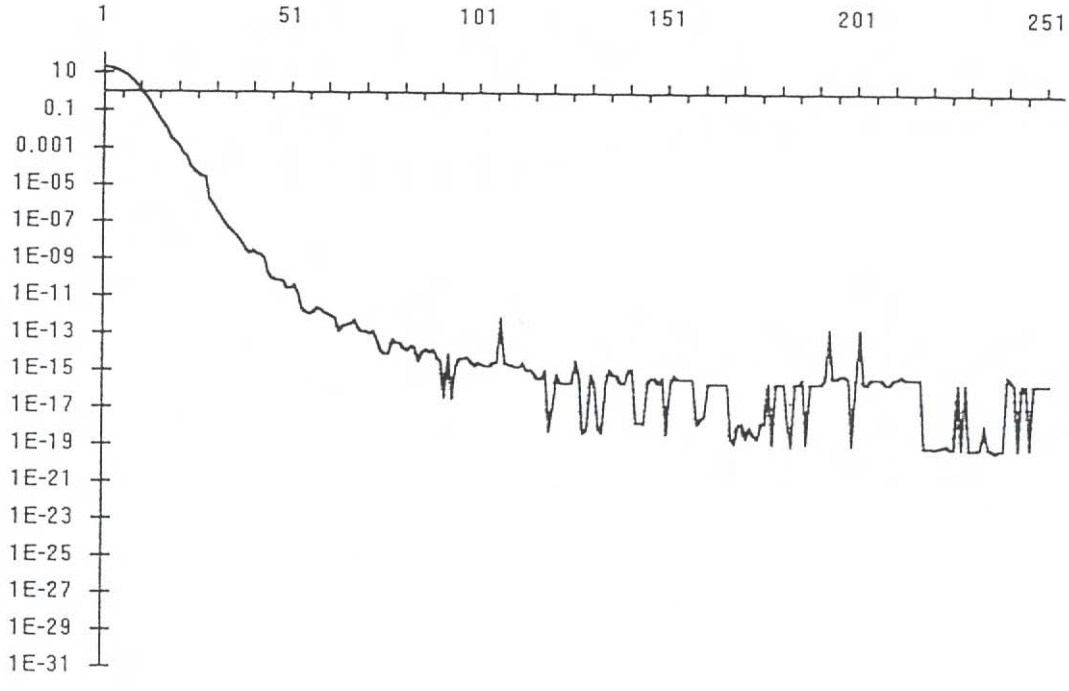




ENTROPY (vs generation)

Maximum and Mean Fitness (vs generation)





ENTROPY (vs generation)

Parameters of the genetic search:
$p_c = 1.0$, $p_m = 0.0001$, $p_t = 0.000$

Selection scheme:
Deterministic Crowding.

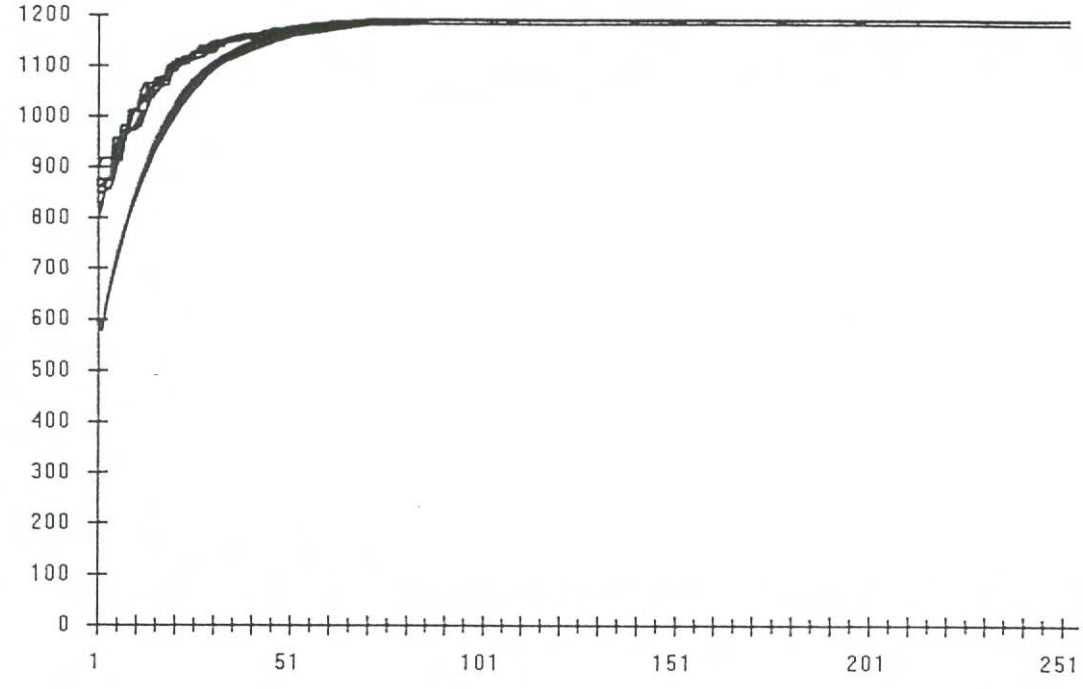
Front page:

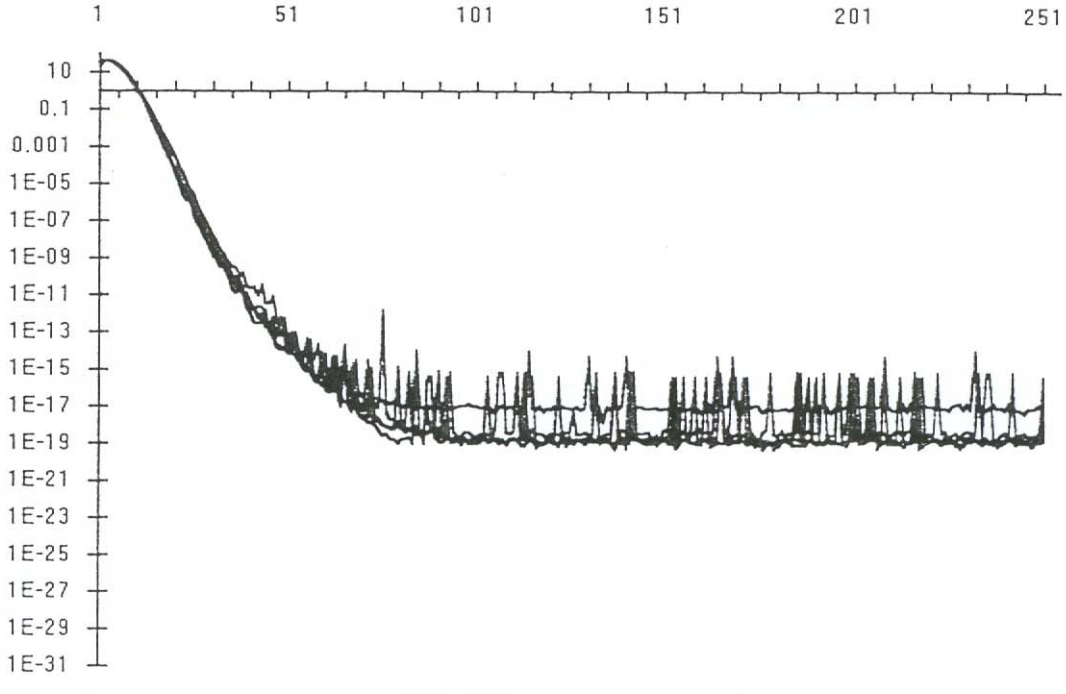
The results of five consecutive runs.
The optimal solution was found during 0/5 runs.

Back page:

The results of one run.
Fittest solution found: 1182, during the 69th generation.

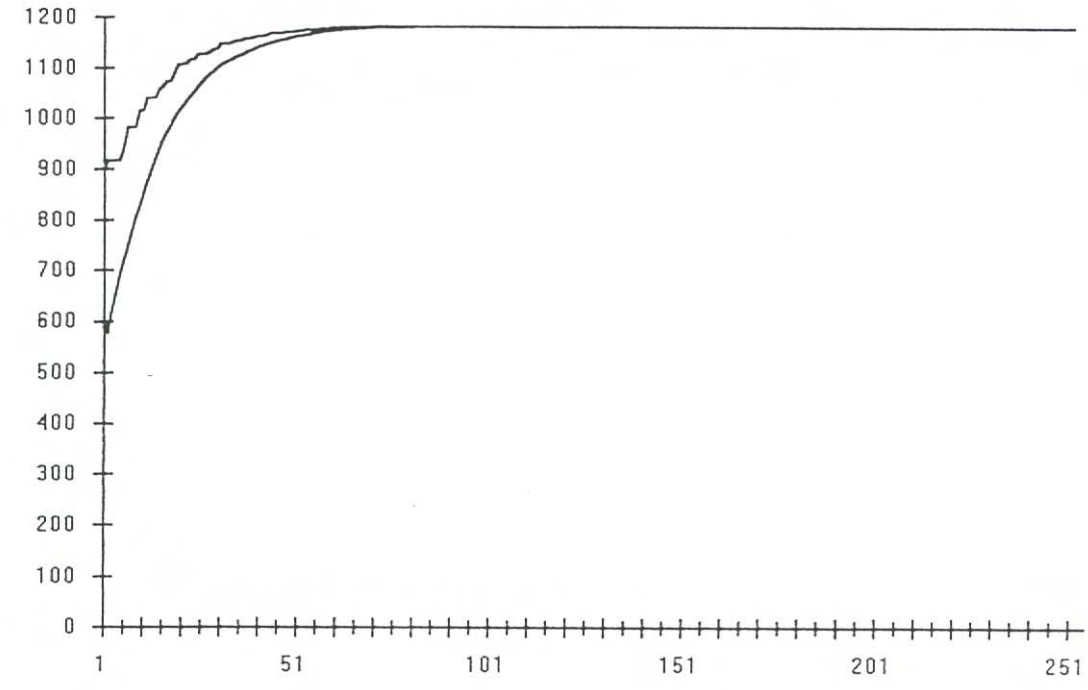
Maximum and Mean Fitness (vs generation).

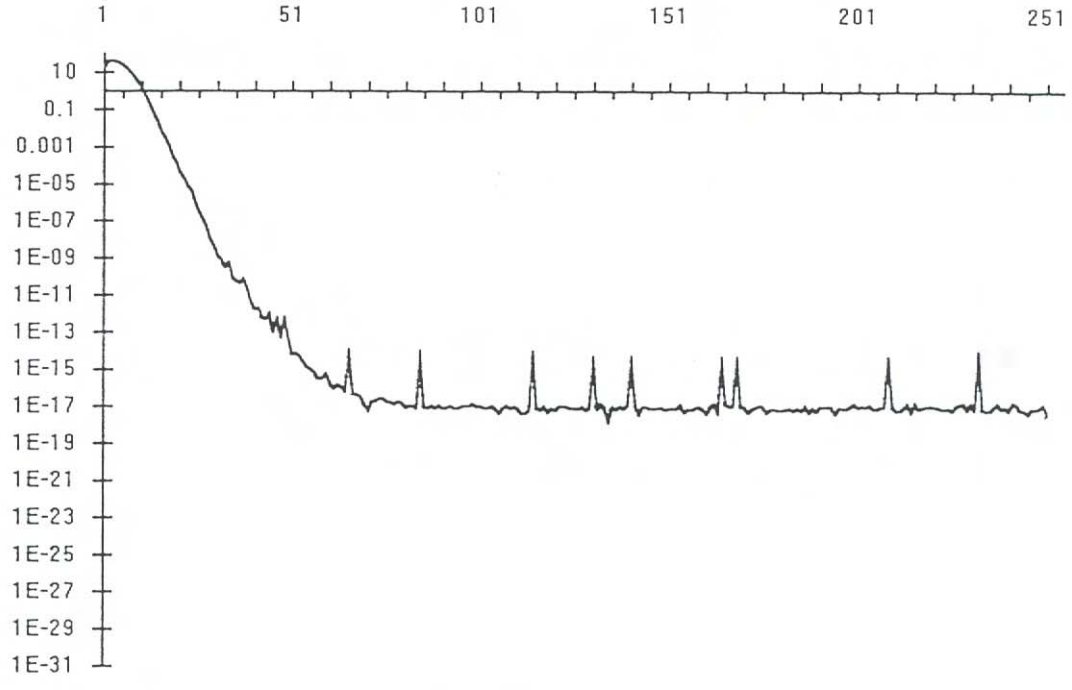




ENTROPY (vs generation)

Maximum and Mean Fitness (vs generation).





ENTROPY (vs generation)

Parameters of the genetic search:
$p_c = 1.0$, $p_m = 0.0001$, $p_t = 0.000$

Selection scheme:
Sharing.

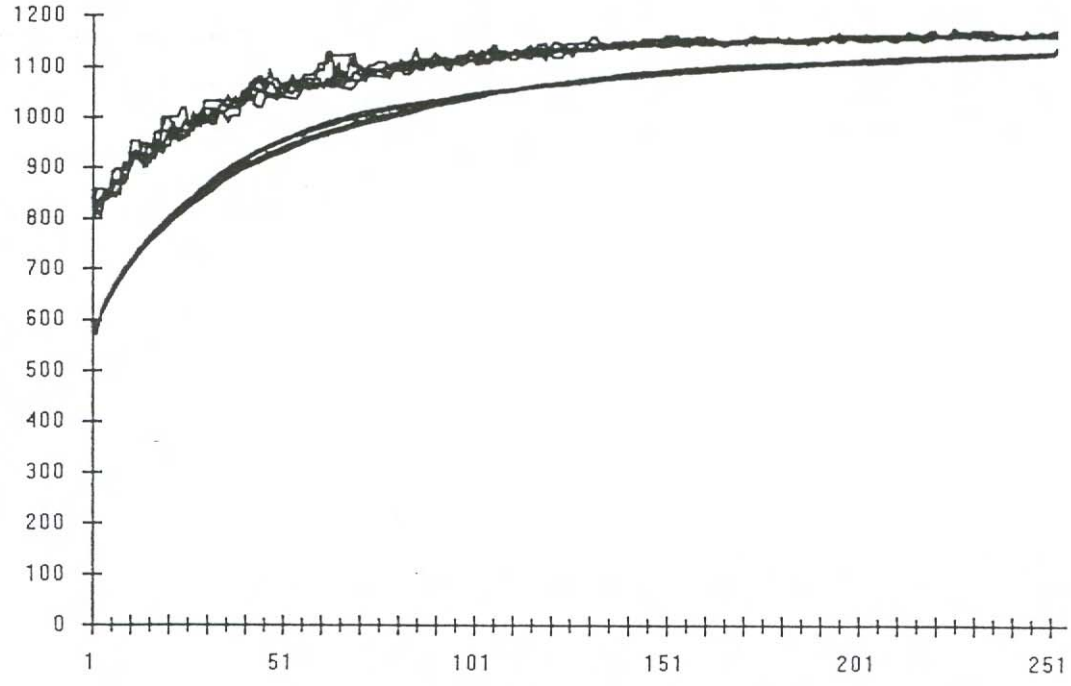
Front page:

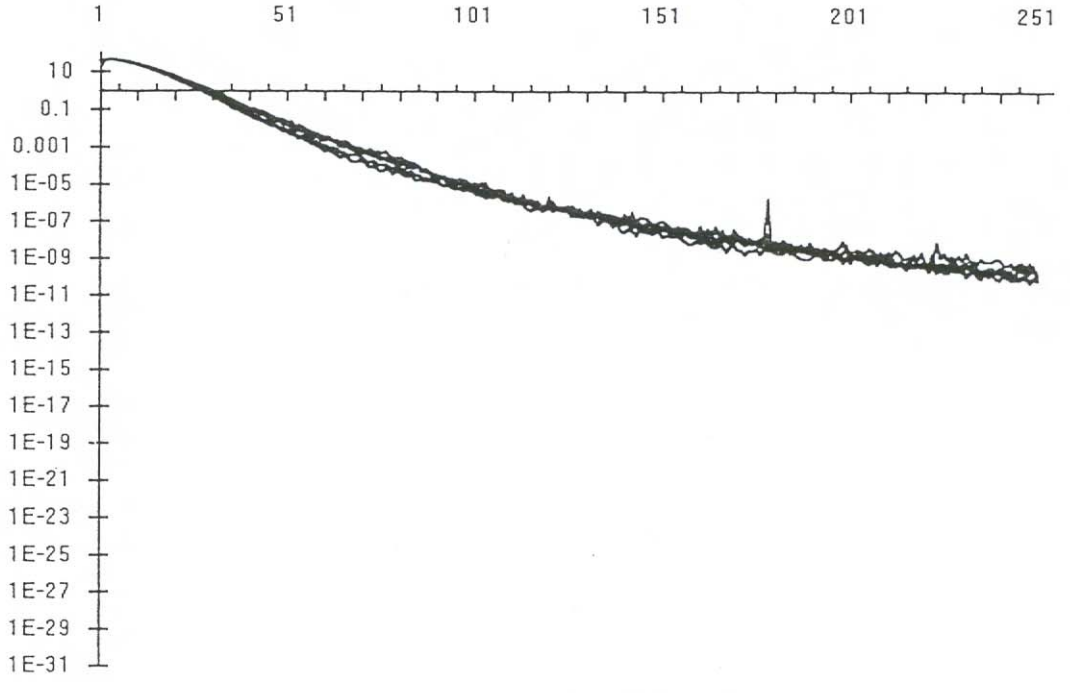
The results of five consecutive runs.
The optimal solution was found during 0/5 runs.

Back page:

The results of one run.
Fittest solution found: 1162, during the 250th generation.

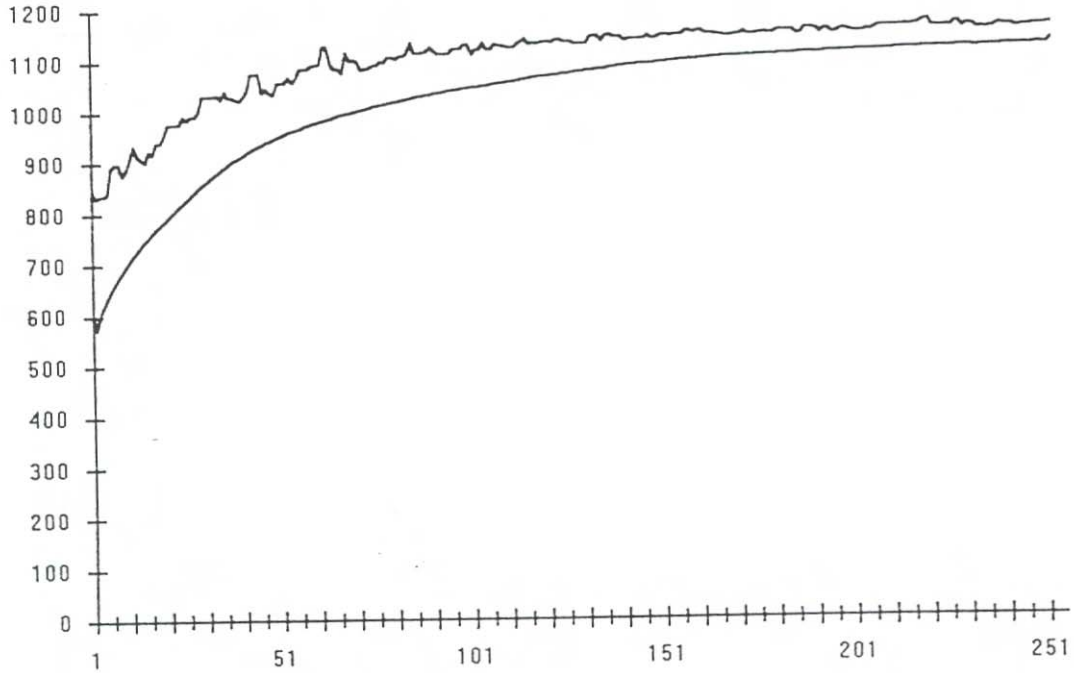
Maximum and Mean Fitness (vs generation)

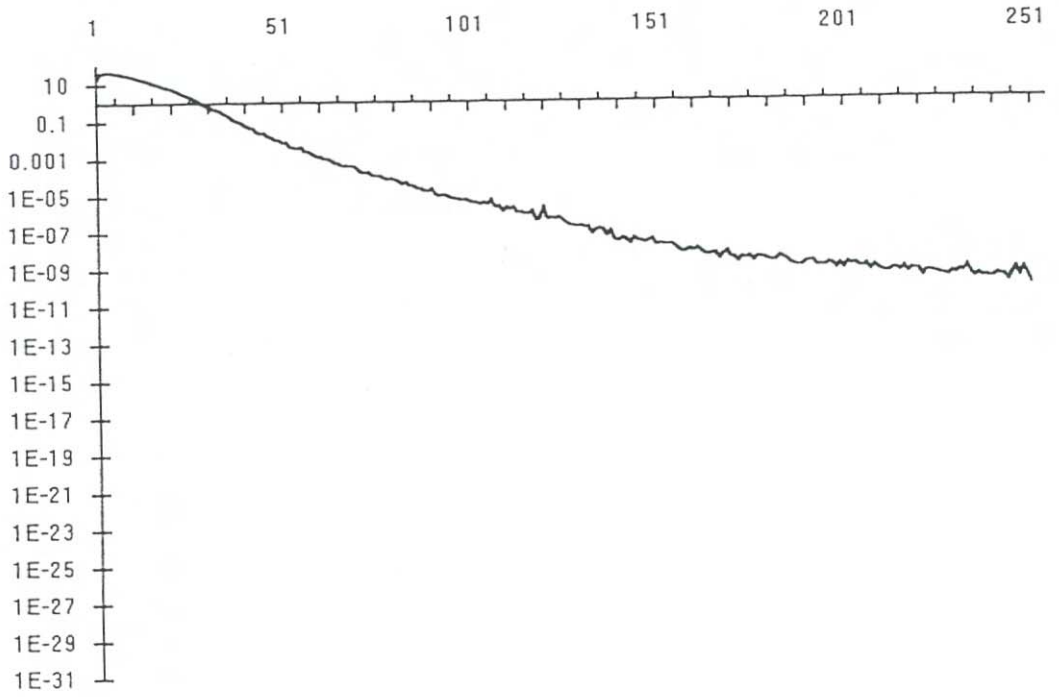




ENTROPY (vs generation)

Maximum and Mean Fitness (vs generation)





ENTROPY (vs generation)

Parameters of the genetic search:
$p_c = 1.0$, $p_m = 0.0000$, $p_t = 0.001$

Selection scheme:
Binary tournament selection

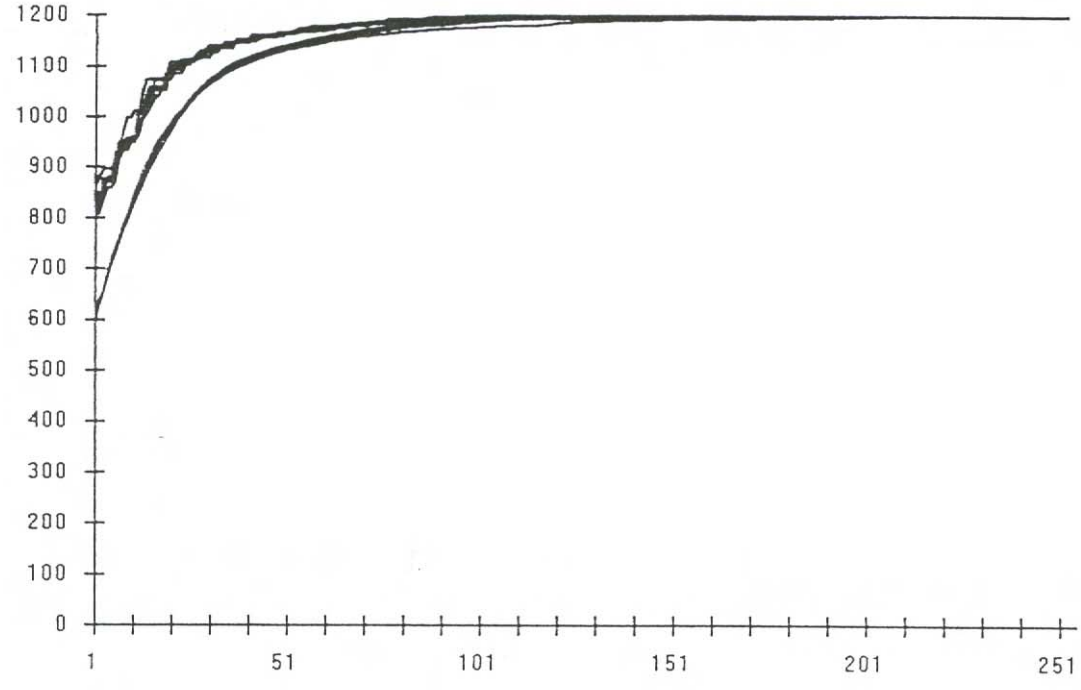
Front page:

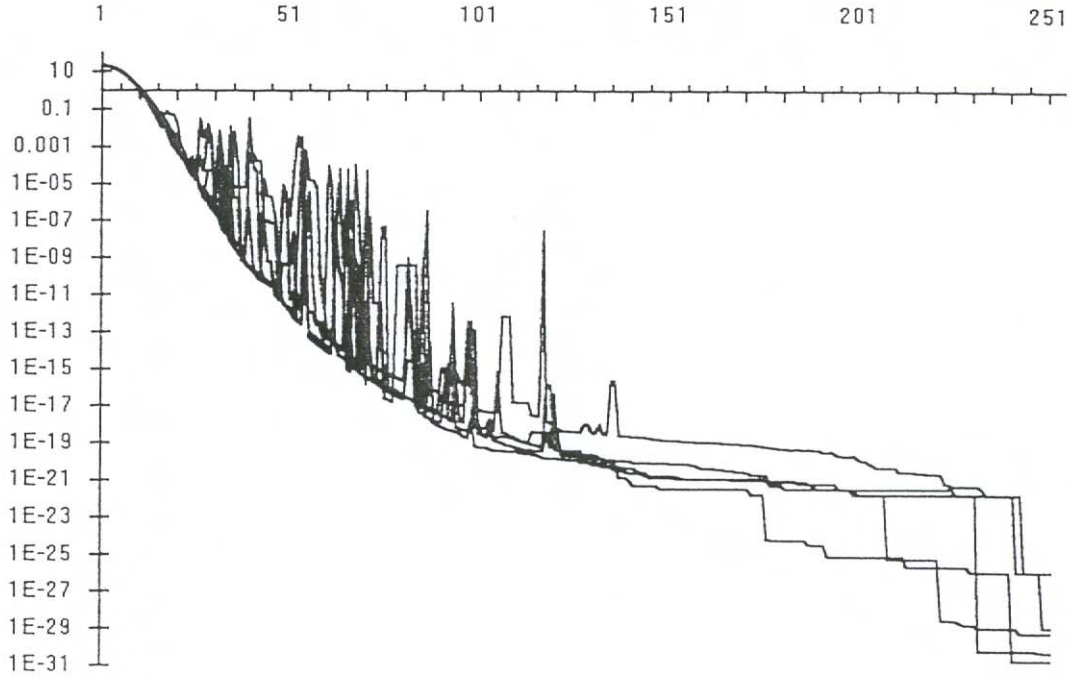
The results of five consecutive runs.
The optimal solution was found during 5/5 runs.

Back page:

The results of one run.
Fittest solution found: 1200, during the 100th generation.

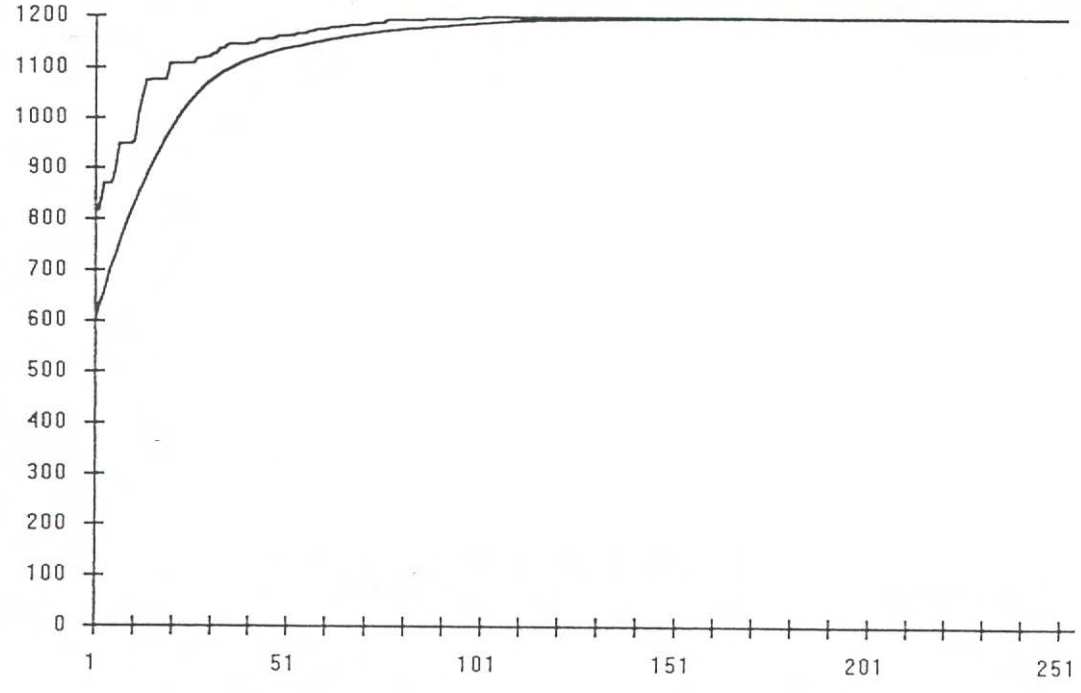
Maximum and Mean Fitness (vs generation)

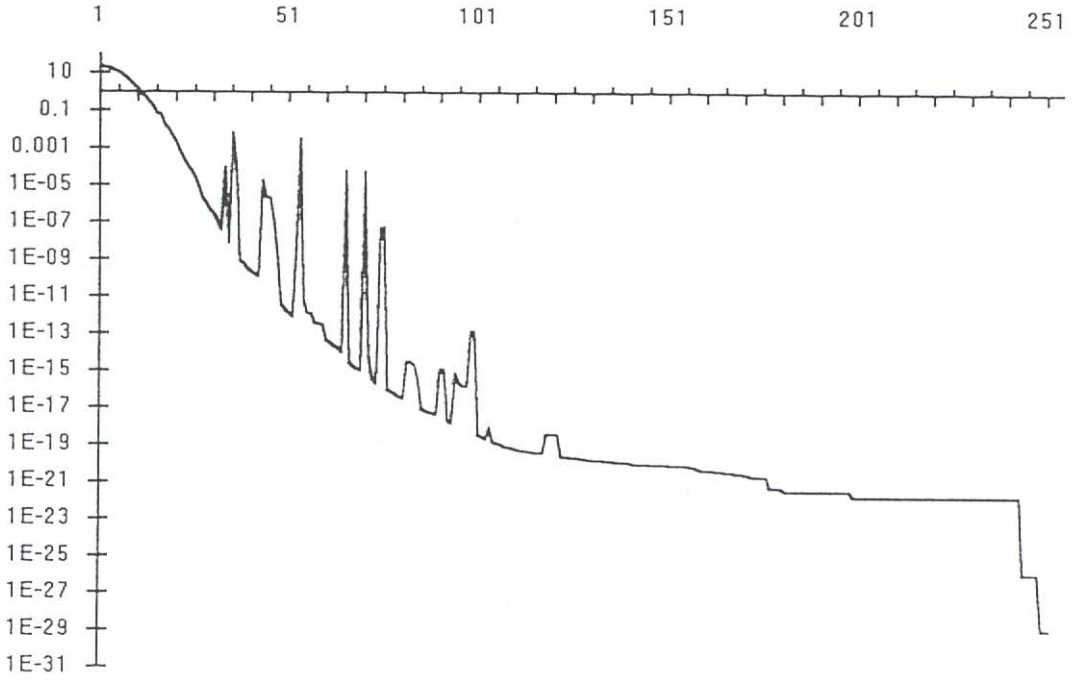




ENTROPY (vs generation)

Maximum and Mean Fitness (vs generation)





ENTROPY (vs generation)

Parameters of the genetic search:
$p_c = 1.0$, $p_m = 0.0000$, $p_t = 0.001$

Selection scheme:
Deterministic Crowding.

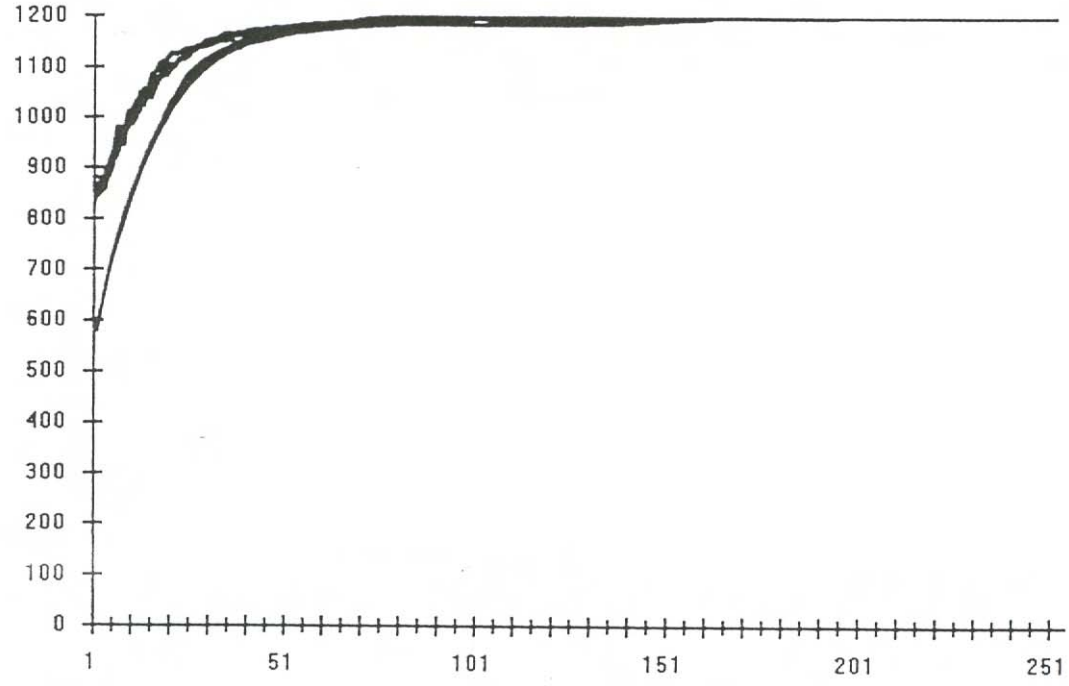
Front page:

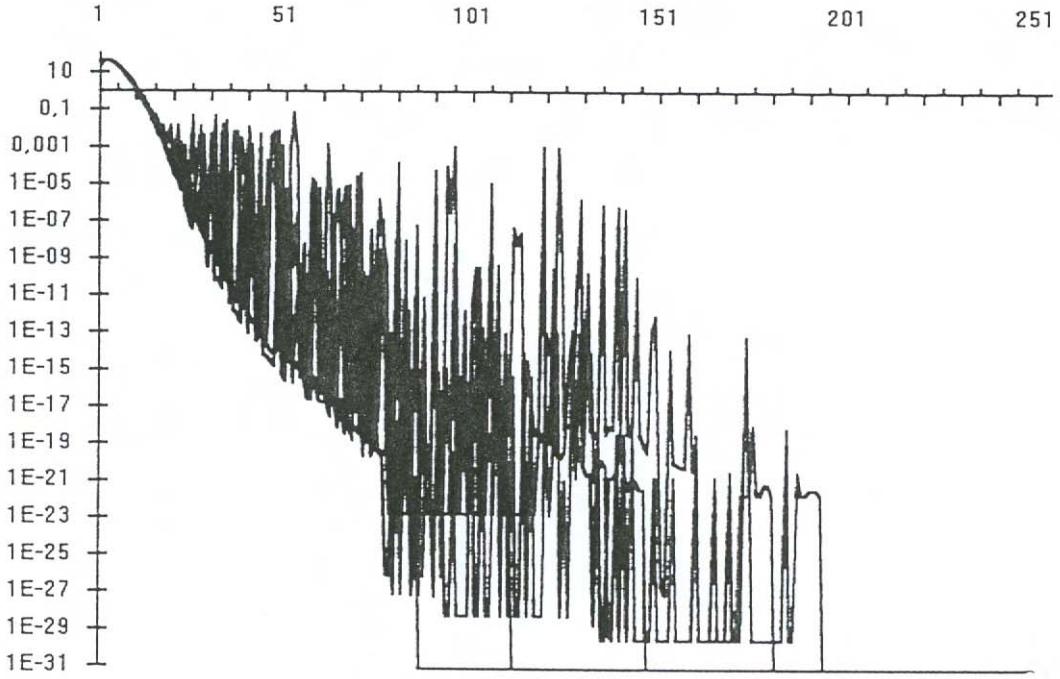
The results of five consecutive runs.
The optimal solution was found during 5/5 runs.

Back page:

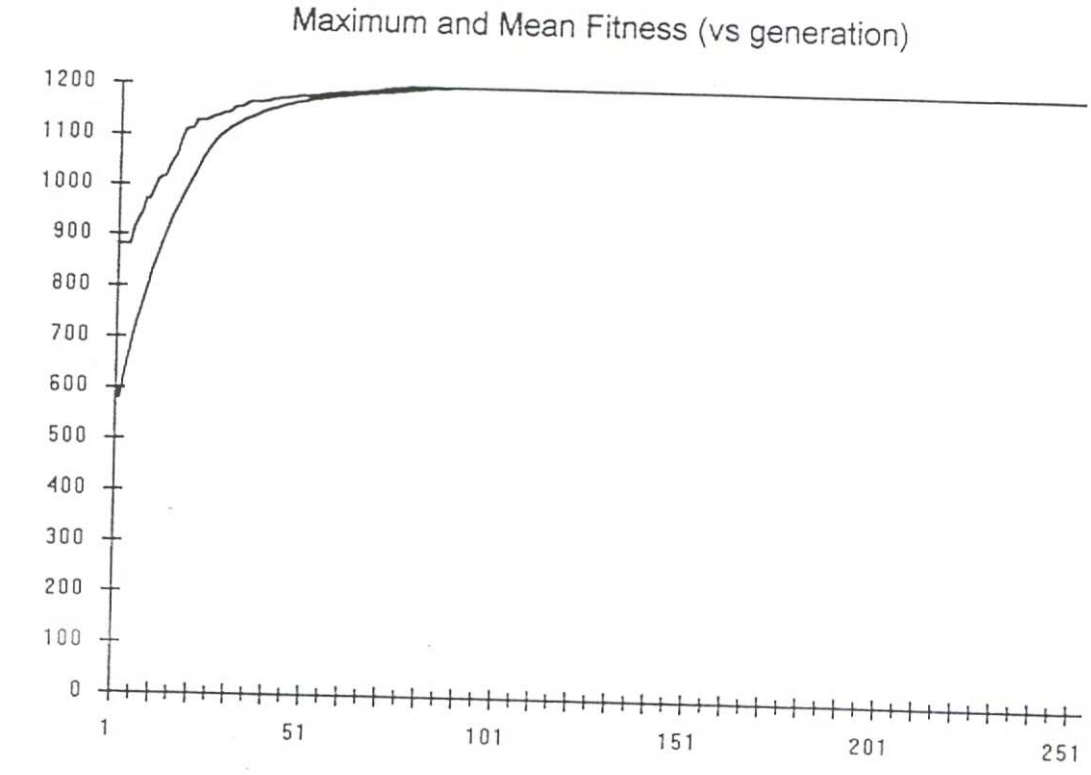
The results of one run.
Fittest solution found: 1200, during the 75th generation.

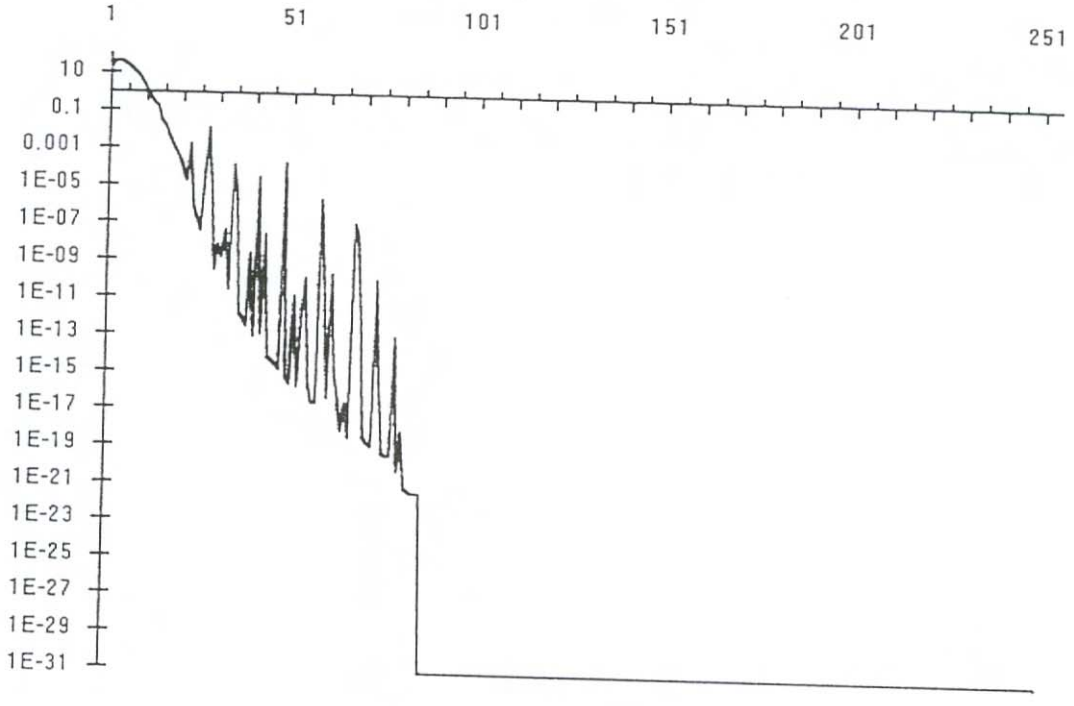
Maximum and Mean Fitness (vs generation)





ENTROPY (vs generation)





ENTROPY (vs generation)

Parameters of the genetic search:
$p_c = 1.0$, $p_m = 0.0000$, $p_t = 0.001$

Selection scheme:
Sharing.

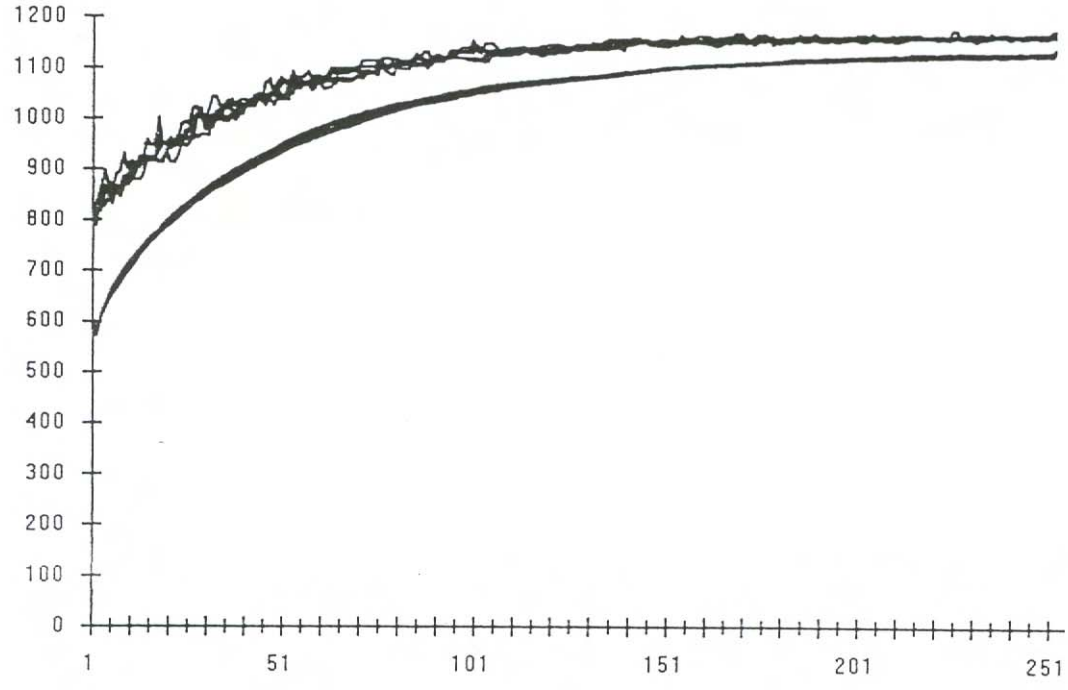
Front page:

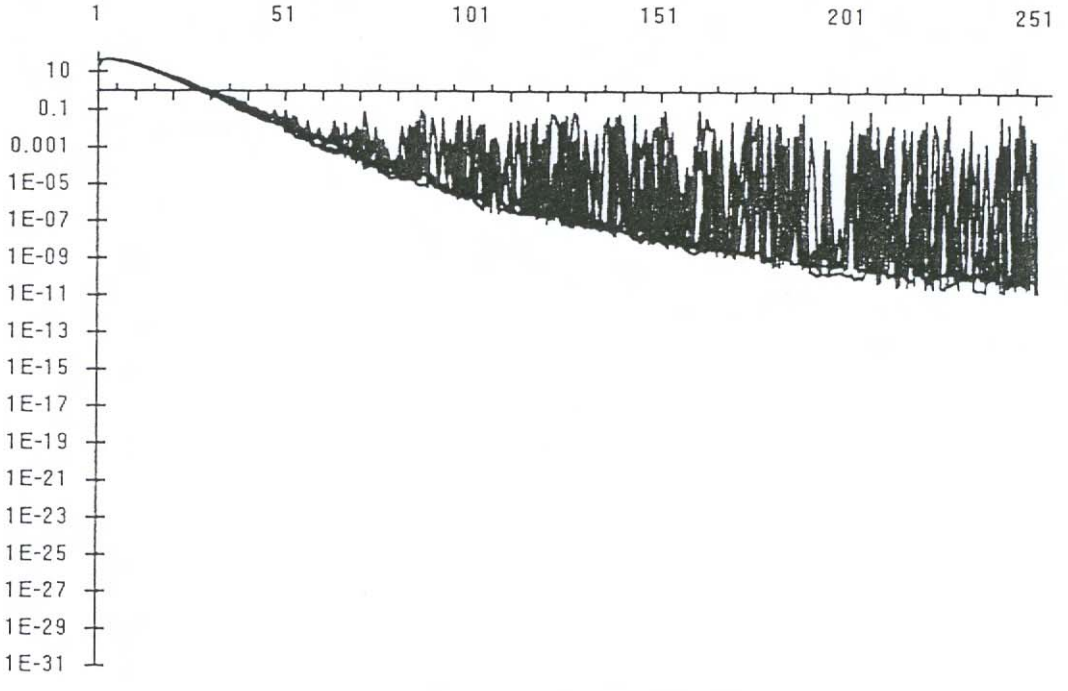
The results of five consecutive runs.
The optimal solution was found during 0/5 runs.

Back page:

The results of one run.
Fittest solution found: 1176, during the 250th generation.

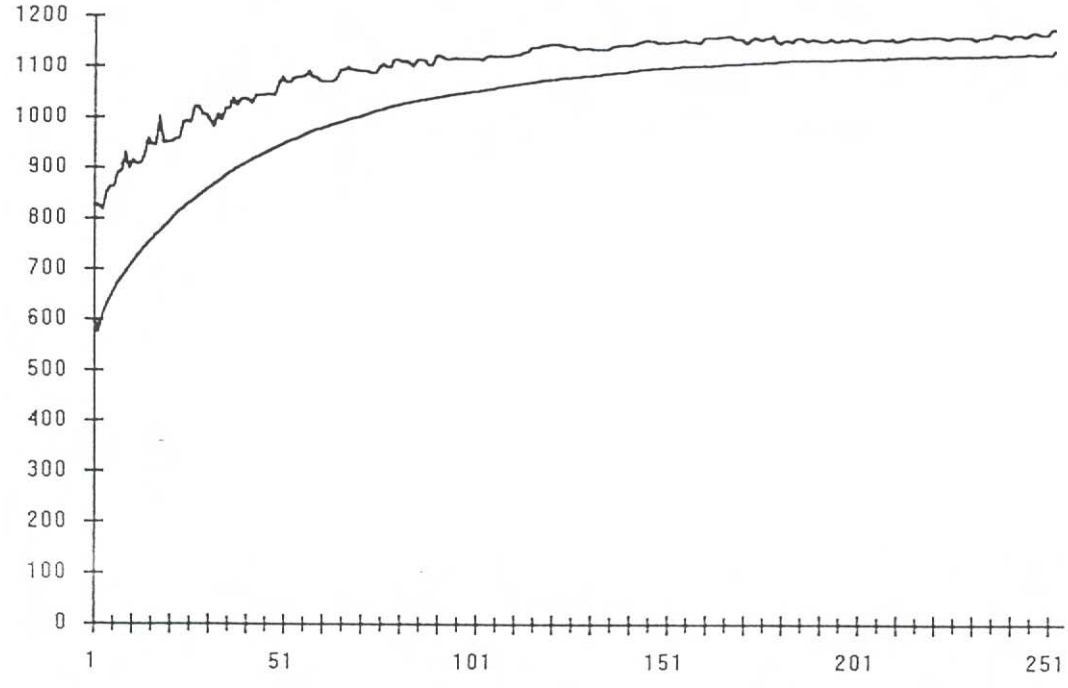
Maximum and Mean Fitness (vs generation)

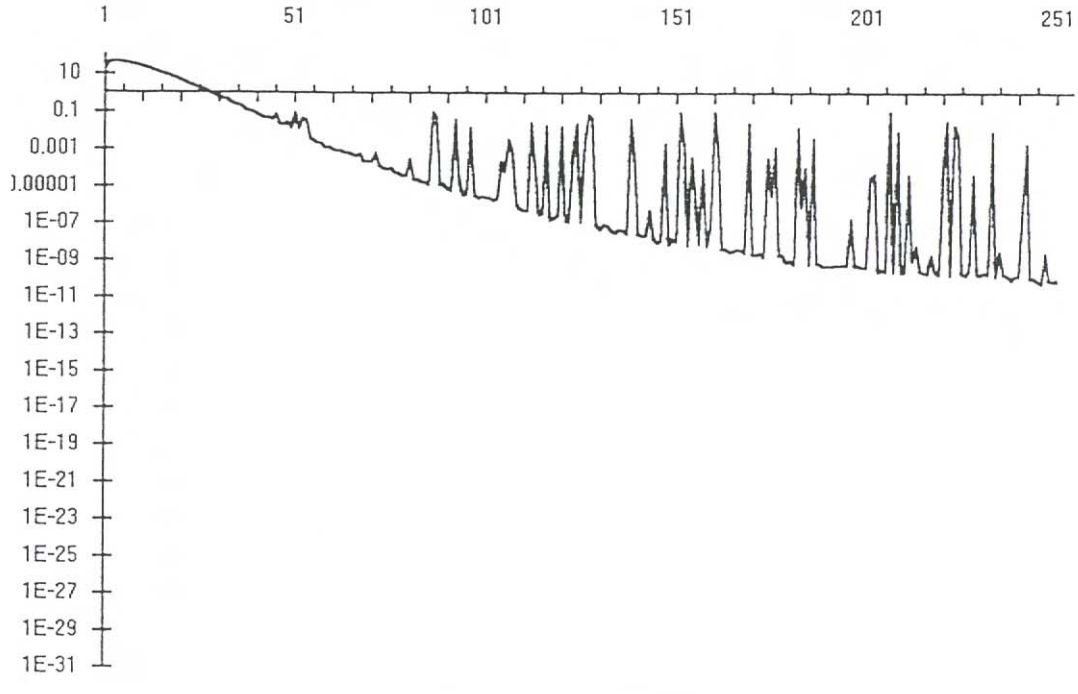




ENTROPY (vs generation)

Maximum and Mean Fitness (vs generation)





ENTROPY (vs generation)