SSY281 Model Predictive Control

Assignment 6

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Question 1

The H and V representations are shown below in figure 1. The reason for their different appearance is that V representation is bounded while H does not have to be. The H representation is defined as the intersection of half-spaces and half-planes while the V representation is described by finite number of vertices, and hence bounded. In the bounded case the Polyhedron is often refer ed to as Polytope.

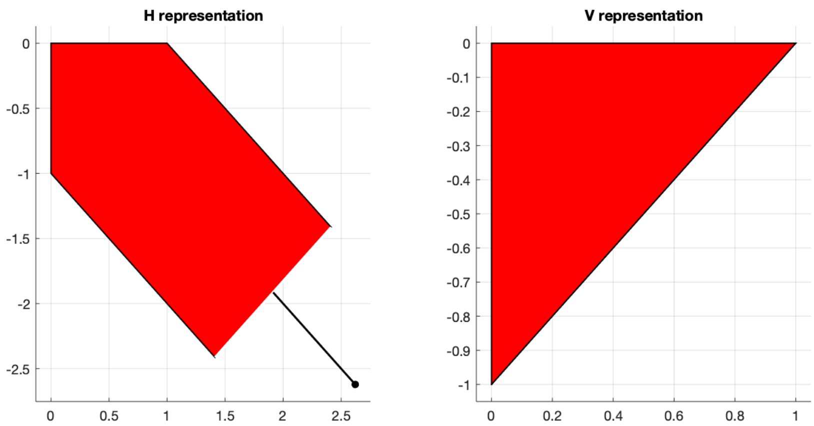

Figure 1: V and H representation

Question 2

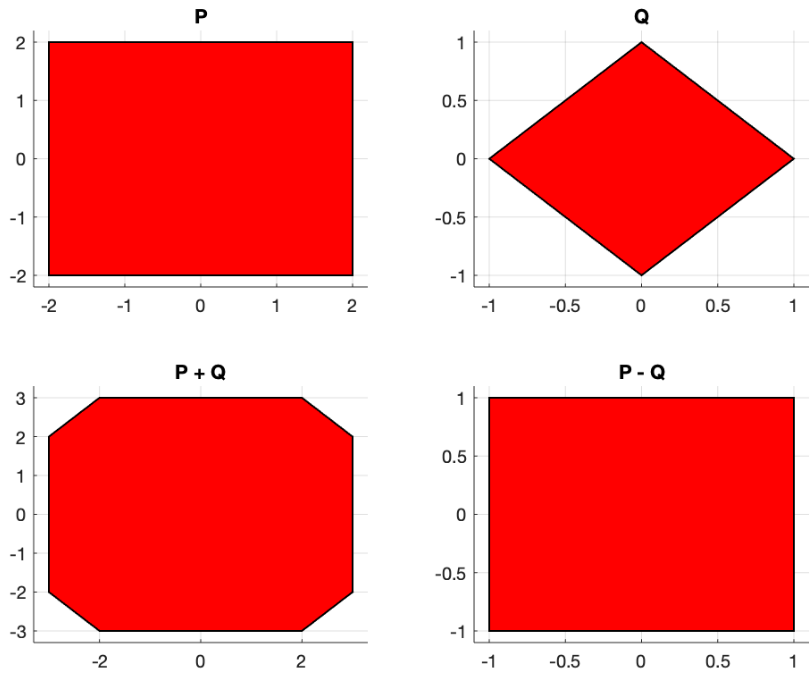

Figure 2: Minkowiski sum and Pontryagin differance

Question 3

The set **S** is positively invariant for the autonomous system if
$x^+ = Ax$ if $x(0) \in \mathbf{S} => x(k) \in \mathbf{S}, \forall k \in N_+$ as stated in the lecture notes. This means that if we let the system propagate one step and remain inside **S** we know that we will not leave **S** for any **k**, meaning that **S** in fact is positively invariant for the system. This can be seen in figure 3. This also seems intuitive since the poles of the system are stable and we don't supply any input so we should approach the origin as time passes.

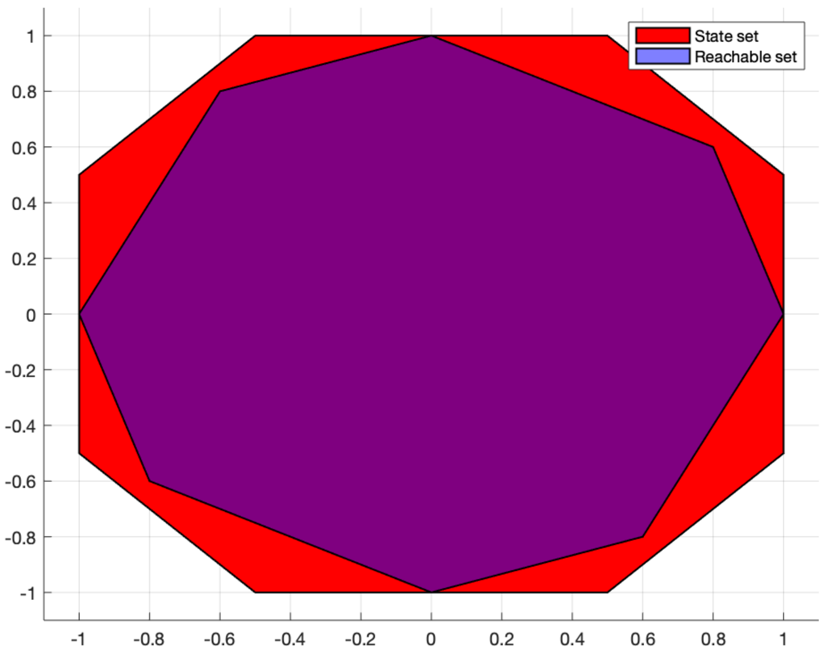

Figure 3: The set S and the 1 step reachable set from S.

Question 4

The one step forward reachable step (Reach) was calculated as shown below and was verified by comparing to **MPT** result as shown in figure 4.

$$Reach(S) = A \odot S \oplus B \odot U \tag{1}$$

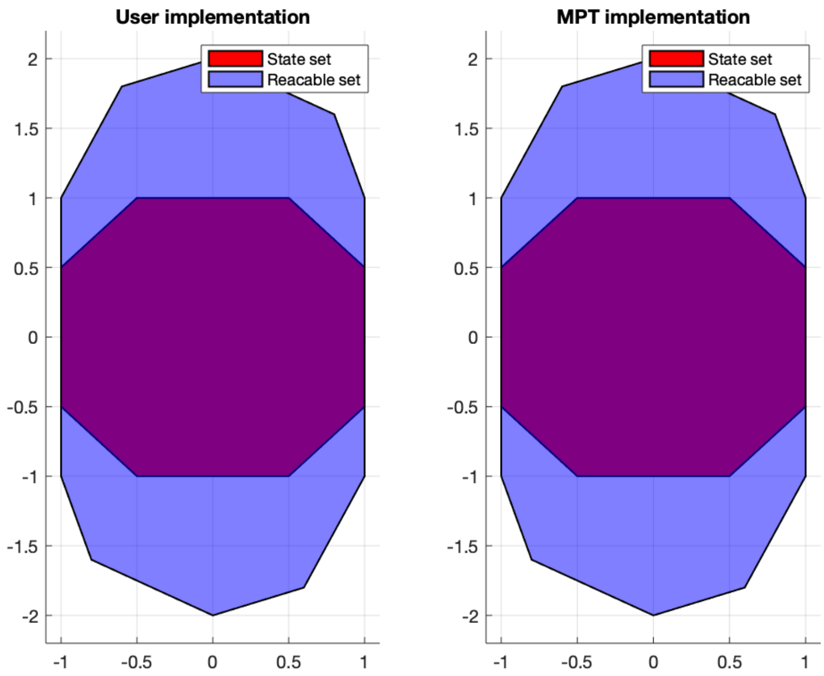

Figure 4: The set S and the 1 step forward reachable set from S.

Question 5

The one step backward reachable step (Pre) was verified by comparing to **MPT** result as shown in figure 5.

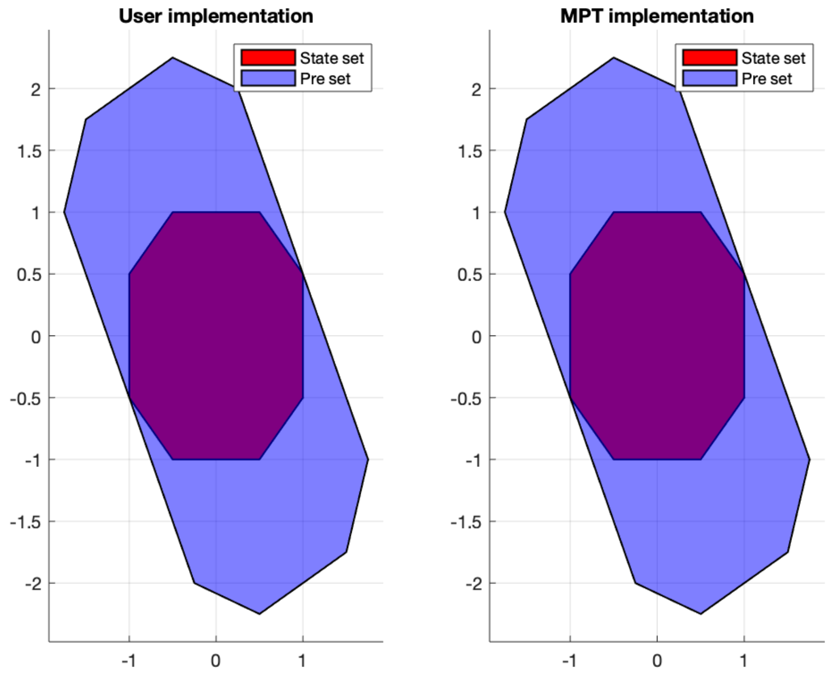

Figure 5: The set S and the 1 step backward reachable set from S.

Question 6

(1)

The shortest N for the RH controller that makes it feasible was found to be $N = 26$. See **ShortestN_14ṁ** for implementation.

(2)

The RH controller was still found to be feasible until convergence to the origin. See **RHCXf_14ṁ** for implementation.

(3)

An important note when calculating the feasible initial states is that we may not violate the state or input constraints at any time and hence have to take the intersection after each backward reachable (Pre) state operation with the constraint state set.

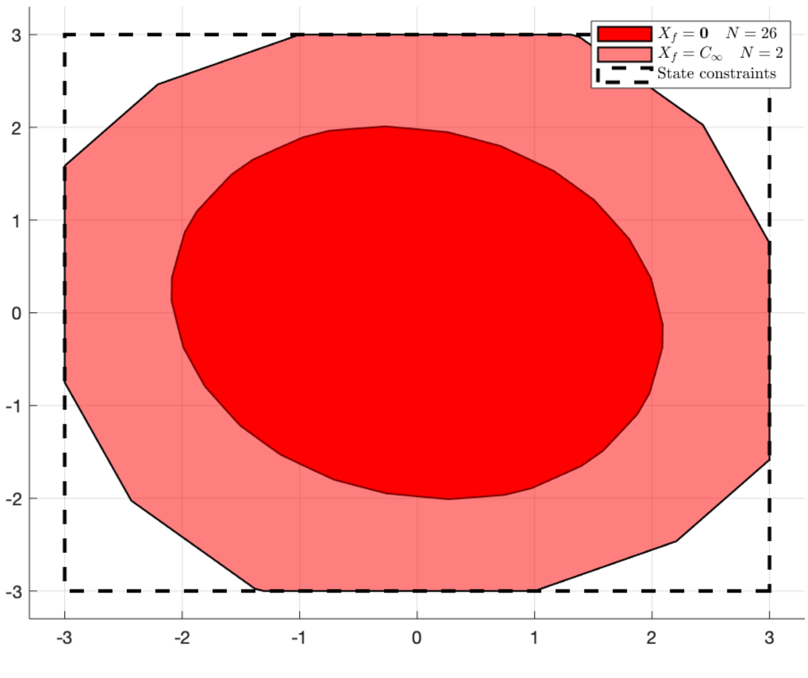

Figure 6: The feasible initial state sets for the two controllers.

When it comes to the size of the optimization problem I am uncertain if we mean the off line quadratic optimization problem we did to find the shortest N and to check if the controller still was feasible. If this is the case the size is dependant on the the number of optimization variables which is dependant on the prediction horizon. The number of constraint is dependant on the prediction horizon and hence a longer prediction horizon will create a bigger optimization problem.

If we however are talking about the online optimization problem, that is when we have an explicit controller where we need to check which state we are in and which affine control law to use. Then the optimization problem size is dependant on our calculated Polyhedron which are in $\mathcal{R}^2$ and have a number of constraints. If we look at them (see below in figure 7) we can see that the number of constraints (52 vs 14 inequality constraints) goes up with an extended horizon and thus creates a bigger optimization problem.

```
>> XPre26
Polyhedron in R^2 with representations:
    H-rep (irredundant) : Inequalities  52 | Equalities   0
    V-rep (irredundant) : Vertices  52 | Rays   0
Functions : none
>> XPre2
Polyhedron in R^2 with representations:
    H-rep (irredundant) : Inequalities  14 | Equalities   0
    V-rep (irredundant) : Vertices  14 | Rays   0
Functions : none
```

Figure 7: Matlab output for the 2 ploted polyhedrons