Micro homework 11

Isak Åslund (isakas)

Group 14

Question 1. *Consider the following Hankel matrix and find a representation of (A, B, C) when the system as one input and one output.*

$$H = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

Question 2. *In a RH controller, what is the effect of a short control horizon, i.e. small M, on the complexity of the method? How does this effect the existence of a feasible solution?*

Question 3. *Compare Predictive Functional Control (PFC) with RH controller in terms of complexity and optimality.*

(Q1) $H(k) = CA^{k-1}B \qquad A \in \mathbb{R}^{2\times 2} \quad B \in \mathbb{R}^{2\times 1} \quad C \in \mathbb{R}^{1\times 2}$

$H(1) = 0 = CB = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = c_1 b_1 + c_2 b_2 \qquad c_1 = c_2 = 1 \quad b_1 = 1 \quad b_2 = -1$ (choosen by me)

$H(2) = 0.5 = CAB = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} a_1 b_1 \\ a_2 b_2 \end{bmatrix} = a_1 b_1 c_1 + a_2 b_2 c_2 = a_1 - a_2$

$H(2) = 0.5 = CAB = a_1 - a_2 = 0.5$

$H(3) = 1 = CA^2B = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} a_1^2 & 0 \\ 0 & a_2^2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = a_1^2 - a_2^2 = 1$

Matlab gives $\Rightarrow A = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} = \begin{bmatrix} 5/4 & 0 \\ 0 & 3/4 \end{bmatrix}$

$A = \begin{bmatrix} 5/4 & 0 \\ 0 & 3/4 \end{bmatrix} = \begin{bmatrix} 1{,}25 & 0 \\ 0 & 0{,}75 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 \end{bmatrix}$

(Q2) Lower complexity due to smaller amount of opt. var. but decreases the chance of there existing a feasable solution.

(Q3) Computational complexity is reduced, when looking in the book there isn't much so I can't answer the optimality question. If I'd have to guess I would say they are the same since both solve the same opt. prob but choose different ways to express the control sequence