# Micro homework 9

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Q1

The origin is locally stable for the system $(x^+ = f(x),\ f(0)=0)$ if, for any $\varepsilon > 0$, there exists a $\delta > 0$ such that $|x(0)| < \delta$ implies $|x(k)| < \varepsilon$ for all $k \geq 0$.

It means that if you start close to the origin you will remain close to it.

Q2 Global stability enforces local stability and global attraction, meaning that as $k \to \infty$ you will end up in $x(k) = 0$.

The difference is then that the local stability can have solutions which does not converge to the origin but remain close to it, while global asymptotic stability will end in the origin.

$x(k+1) = 0{,}3x + 5u \Rightarrow$ pole in $z = 0{,}3$ which is inside the unit circle and hence stable.

The system will end in the origin regardless of initial state, (provided input is zero, $u=0$) hence the tem is globally asymptotically stable.

Q3

① $V(x) \geq \alpha_1(|x|)$
② $V(x) \leq \alpha_2(|x|)$ } catch "strange" cases
③ $V(f(x)) - V(x) \leq -\alpha_3(|x|)$ ← Think as energy should decrease for a stable system.

$V(x) = x^T S x = xSx$ (common according to lecture notes.)

$V(x^+) = (0{,}3x) S (0{,}3x) = x\,0{,}3\,S\,0{,}3\,x = \{0{,}3S0{,}3 = S - Q\} = xSx - xQx =$

$= V(x) - xQx$

③ $V(f(x)) - V(x) = V(x^+) - V(x) = -xQx \leq -\underbrace{\alpha_3}_{\geq 0}(|x|)$

Don't see how to acctualy check conditions 1 & 2

Q4 The terminal state set $\mathbb{X}_f$ plays a big role in the closed loop stability.

Q5 Choosing the terminal set $\mathbb{X}_f$ as a control invariant set means there exists a $u \in \mathbb{U}$ so that $f(x,u) \in \mathbb{X}_f$, which means we can reach the terminal set with our control signals, this seems intuitive.

However an intuitive explanation I can not come up with, I'll have to attend the lecture and find out.