```
In [ ]: import gym
        import numpy as np
        import matplotlib.pyplot as plt
        import seaborn as sns
        from mpl toolkits.axes grid1 import make axes locatable
        from tqdm import tqdm
        from tabulate import tabulate
        np.set printoptions(formatter={'float': lambda x: "{0:4.2f} ".format(x)})
        env = gym.make('Blackjack-v1', natural=True)
        /usr/local/lib/python3.8/dist-packages/gym/core.py:317: DeprecationWarning: WARN: Initializing wrapper in old step API
        which returns one bool instead of two. It is recommended to set `new_step_api=True` to use new step API. This will be t
        he default behaviour in future.
          deprecation(
        /usr/local/lib/python3.8/dist-packages/gym/wrappers/step api compatibility.py:39: DeprecationWarning: WARN: Initializin
        g environment in old step API which returns one bool instead of two. It is recommended to set `new step api=True` to us
        e new step API. This will be the default behaviour in future.
          deprecation(
In [ ]: # Reference: https://github.com/jng985/BlackjackEnv gym/blob/master/plot utils.py
        # https://github.com/dennybritz/reinforcement-learning
        # Make some small adaptation
        def plot blackjack values(V):
            def get Z(x, y, usable ace):
                return V[x,y,usable ace]
            def get figure(usable ace, ax):
                x_range = np.arange(11, 22)
                y range = np.arange(1, 11)
                X, Y = np.meshgrid(x range, y range)
                Z = np.array([get Z(x,y,usable ace) for x,y in zip(np.ravel(X), np.ravel(Y))]).reshape(X.shape)
                surf = ax.plot surface(X, Y, Z, rstride=1, cstride=1, cmap=plt.cm.coolwarm, vmin=-1.0, vmax=1.0)
                ax.set xlabel('Player\'s Current Sum')
                ax.set ylabel('Dealer\'s Showing Card')
                ax.set zlabel('State Value')
                ax.view init(ax.elev, -120)
```

```
fig = plt.figure(figsize=(30, 12))
    ax = fig.add subplot(121, projection='3d')
    ax.set title('Usable Ace')
    get figure(1, ax)
    ax = fig.add subplot(122, projection='3d')
   ax.set title('No Usable Ace')
    get figure(0, ax)
    plt.show()
def plot policy(matrix, input type='Q'):
    matrix can be Q value or policy matrix
    def get Z(x, y, usable ace):
        if input type == 'Q':
            prob = matrix[x,y,usable ace]
            if prob[0] == prob[1]:
                return 1
            else:
                return np.argmax(prob)
        else:
            return matrix[x,y,usable ace]
    def get figure(usable ace, ax):
        x range = np.arange(1, 11)
        y range = np.arange(21, 10, -1)
        X, Y = np.meshgrid(x range, y range)
        Z = np.array([[get Z(y,x,usable ace) for x in x range] for y in y range])
        surf = ax.imshow(Z, cmap=plt.get cmap('Pastel2', 2), vmin=0, vmax=1, extent=[0.5, 11.5, 10.5, 21.5])
        plt.xticks(x range)
        plt.yticks(y range)
        ax.set xlabel('Dealer\'s Showing Card')
        ax.set ylabel('Player\'s Current Sum')
        ax.grid(color='w', linestyle='-', linewidth=1)
        divider = make_axes_locatable(ax)
        cax = divider.append axes("right", size="5%", pad=0.1)
        cbar = plt.colorbar(surf, cax=cax)
        cbar.ax.set yticklabels(['0 (STICK)','1 (HIT)'])
   fig = plt.figure(figsize=(15, 15))
    ax = fig.add subplot(121)
```

```
ax.set title('Usable Ace')
            get figure(1, ax)
            ax = fig.add subplot(122)
            ax.set title('No Usable Ace')
            get figure(0, ax)
            plt.show()
In [ ]: def get winrate(r):
            x = np.arange(1, len(r)+1)
            win = np.where(r==-1, 0, r)
            win cum = np.cumsum(win)
            return win cum/x
        # Result from 1 Epoch
        def plot winrate(r):
            win rate = get winrate(r)
            plot start = 50
            x = np.arange(1,len(win_rate)+1)
            fig = plt.figure(figsize=(12,8))
            plt.title("Blackjack")
            plt.xlabel("Number of Episode")
            plt.ylabel("Win Rate")
            plt.grid(color = 'gray', linestyle = '--', linewidth = 0.5)
            plt.axhline(0.420, color = 'green', linestyle = '-', linewidth = 0.5)
            plt.plot(x[plot start:], win rate[plot start:])
            plt.show()
        # Average through mutiple Epoch
        def plot arrays(win rate, label):
            win rate = win rate[:,50:]
            mean = np.mean(win rate, axis=0)
            std = np.std(win rate, axis=0)
            plt.plot(range(len(mean)), mean, label=label)
            plt.fill between(range(len(mean)), np.maximum(mean-std, 0.3), np.minimum(mean+std, 0.5), alpha=0.3)
In [ ]: def create optimal solution():
            # Create the Optimal Action, Calculated by Math
            USABLE ACE = np.zeros((32,11), dtype=int)
            NON_USABLE_ACE = np.zeros((32,11), dtype=int)
            USABLE ACE[18,1] = 1
```

USABLE ACE[18,9:11] = 1

```
for i in range(17, -1, -1):
                USABLE_ACE[i,:] = 1
            for j in range(16, 12, -1):
                NON USABLE ACE[j,1] = 1
                NON USABLE ACE[j,7:11] = 1
            NON USABLE ACE[12,1:4] = 1
            NON USABLE ACE[12,7:11] = 1
            for k in range(11, -1, -1):
                NON USABLE ACE[k,:] = 1
            return np.dstack((NON USABLE ACE, USABLE ACE))
        OPTIMAL SOLUTION = create optimal solution()
        # Visualization Policy if needed
        # plot policy(OPTIMAL_SOLUTION, 'policy')
In [ ]: def select action(Q, state, method, epsilon, T, N, N e, R plus):
            Action: 0(Stick), 1(Hit)
            if method == 'optimal':
                return select action optimal(state)
            elif method == 'greedy':
                return select_action_epsilon_greedy(Q, state, epsilon)
            elif method == 'softmax':
                return select action softmax(Q, state, T)
            elif method == 'optimistically':
                return select_action_optimistically(Q, state, N, N_e, R_plus)
            else:
                print('Invalid Action Method')
        def select action optimal(state):
            return OPTIMAL SOLUTION[state]
        def select_action_epsilon_greedy(Q, state, epsilon):
            r = np.random.uniform(low=0, high=1)
            if r < epsilon:</pre>
                return np.random.randint(low=0, high=2)
            else:
```

```
return np.argmax(Q[state])
        def select action_softmax(Q, state, T):
            max value = np.max(Q[state]/T)
            sum = np.sum(np.exp(Q[state]/T - max value))
            prob = (np.exp(Q[state]/T - max value)) / sum
            return np.argmax(prob)
        def select action optimistically(Q, state, N, N_e, R_plus):
            optimistic = np.where(N[state] < N_e, R_plus, Q[state])</pre>
            return np.argmax(optimistic)
        def convert state(state):
            Input: state(int, int, bool)
            the Goal of the function is to change the last component to be int
            otherwise cannot properly use numpy array index
            return state[:2] + tuple([1 if state[-1] == True else 0])
        def check black jack(state):
            if state[0] == 21:
                return True
            else:
                return False
        def softmax normalize(Q):
            sum = np.sum(np.exp(Q), 3)
            sum = np.dstack((sum, sum)).reshape(32,11,2,2)
            prob = (np.exp(Q)) / sum
            return np.max(prob,3)
In [ ]: def Q learning(env, num episodes, action method='optimistically', eval method='mc', gamma=0.9, epsilon = 0.1, T=10, N e
            Conducts active O-learning to learn optimal O-values. O-values are updated during each step for a fixed number of
            episodes.
            :param env: The environment with which the agent interacts
            :param num episodes: The number of training episodes during which experience can be collected to learn the Q-values
            :param action method: Action selection strategy for exploration - one of {"optimistic", "softmax", "epsilon greedy"
            :param eval method: evaluation method to update Q value - one of {"mc", "td"} "Monte Carlo" or "Temporal Difference
            :param gamma: Discount factor, in (0, 1]
            :param epsilon: The probability for selecting a random action in [0, 1])
            :param T: The temperature for softmax action selection
```

```
:param N_e: Number of times a state-action pair is visited before expected utility is used instead of optimistic est
    :param R plus: The best possible reward obtainable in any state
   Q = np.zeros((32,11,2,2)) # Q value
   N = np.zeros((32,11,2,2)) # Count
   episode rewards = np.zeros(num episodes)
   for i in tqdm(range(num episodes)):
        state = env.reset()
        state = convert state(state)
        # done = check black jack(state)
        # reward = 1.5 * done # Win 1.5 if it is blackjack
        done = False
        episode = []
        # Go through one episode
        while not done:
            action = select action(Q, state, action method, epsilon, T, N, N e, R plus)
            next state, reward, done, info = env.step(action)
           next state = convert state(next state)
            N[state][action] += 1
           if eval method == 'td':
                Q[state][action] = Q[state][action] + (reward + gamma * np.max(Q[next state]) - Q[state][action]) / N[s]
            episode.append((state, action, reward))
            state = next state
        episode rewards[i] = reward
        if eval method == 'mc':
            g = episode[-1][2] * gamma ** len(episode)
            for item in episode:
                state, action, reward = item
                Q[state][action] = Q[state][action] + (g - Q[state][action]) / N[state][action]
   return Q, episode rewards
def SARSA(env, num episodes, action method='optimistically', gamma=0.9, epsilon = 0.1, T=10, N e=10, R plus=1):
   Conducts active Q-learning to learn optimal Q-values. Q-values are updated during each step for a fixed number of
   episodes.
    :param env: The environment with which the agent interacts
    :param num episodes: The number of training episodes during which experience can be collected to learn the Q-values
```

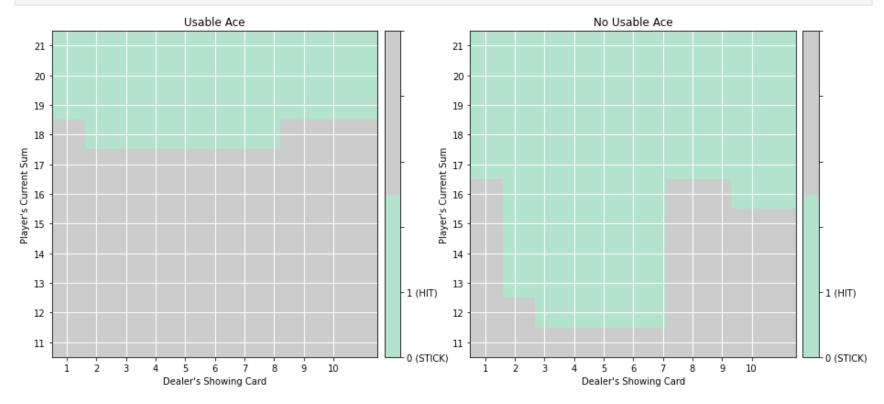
```
:param action_method: Action selection strategy for exploration - one of {"optimistic", "softmax", "epsilon greedy"
:param gamma: Discount factor, in (0, 1]
:param epsilon: The probability for selecting a random action in [0, 1])
:param T: The temperature for softmax action selection
:param N e: Number of times a state-action pair is visited before expected utility is used instead of optimistic est
:param R plus: The best possible reward obtainable in any state
Q = np.zeros((32,11,2,2)) # Q value
N = np.zeros((32,11,2,2)) # Count
episode rewards = np.zeros(num episodes)
for i in tqdm(range(num episodes)):
    state = env.reset()
    state = convert_state(state)
    action = select action(Q, state, action method, epsilon, T, N, N e, R plus)
    # done = check_black_jack(state)
    # reward = 1.5 * done # Win 1.5 if it is blackjack
    done = False
   j = 0
   while not done:
        next state, reward, done, info = env.step(action)
        next state = convert state(next state)
        next action = select action(Q, next state, action method, epsilon, T, N, N e, R plus)
        N[state][action] += 1
        Q[state][action] = Q[state][action] + (reward + gamma * Q[next state][next action] - Q[state][action]) / N[
        state = next state
        action = next action
        j += 1
    episode rewards[i] = reward
return Q, episode rewards
```

```
In []: EPOCHS = 10
    NUM_EPISODES = 400000
    Q_sum = np.zeros((32,11,2,2))

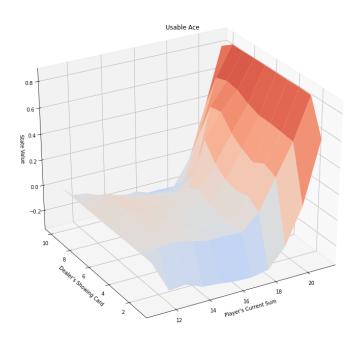
for i in range(EPOCHS):
```

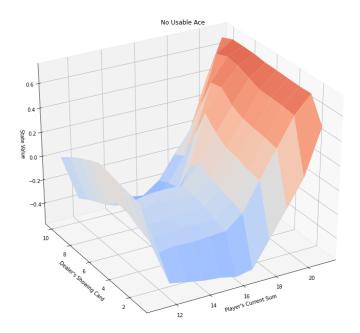
```
Q, r = Q learning(env, NUM EPISODES, action method='greedy', eval method='mc')
    Q sum += Q
100%
                 400000/400000 [01:31<00:00, 4380.10it/s]
100%
                 400000/400000 [01:16<00:00, 5242.32it/s]
100%
                 400000/400000 [01:15<00:00, 5316.92it/s]
100%
                 400000/400000 [01:16<00:00, 5248.89it/s]
100%
                 400000/400000 [01:13<00:00, 5405.62it/s]
100%
                 400000/400000 [01:14<00:00, 5353.28it/s]
100%
                 400000/400000 [01:13<00:00, 5420.65it/s]
100%
                 400000/400000 [01:14<00:00, 5361.24it/s]
100%
                 400000/400000 [01:13<00:00, 5426.76it/s]
100%
                 400000/400000 [01:13<00:00, 5452.50it/s]
```

In []: plot_policy(Q_sum)



In []: plot_blackjack_values(np.max(Q_sum/10, 3))





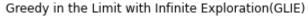
```
In [ ]:
        EPOCHS = 5
        NUM_EPISODES = 100000
        wr optimal mc = np.zeros((EPOCHS, NUM EPISODES))
        wr greedy mc = np.zeros((EPOCHS, NUM EPISODES))
        wr softmax mc = np.zeros((EPOCHS, NUM EPISODES))
        wr optimistically mc = np.zeros((EPOCHS, NUM EPISODES))
        for i in range(EPOCHS):
            Q, r = Q learning(env, NUM EPISODES, action method='optimal', eval method='mc')
            wr optimal mc[i] = get winrate(r)
            Q, r = Q learning(env, NUM EPISODES, action method='greedy', eval method='mc')
            wr greedy mc[i] = get winrate(r)
            Q, r = Q learning(env, NUM EPISODES, action method='softmax', eval method='mc')
            wr softmax mc[i] = get winrate(r)
            Q, r = Q_learning(env, NUM_EPISODES, action_method='optimistically', eval_method='mc')
            wr optimistically mc[i] = get winrate(r)
```

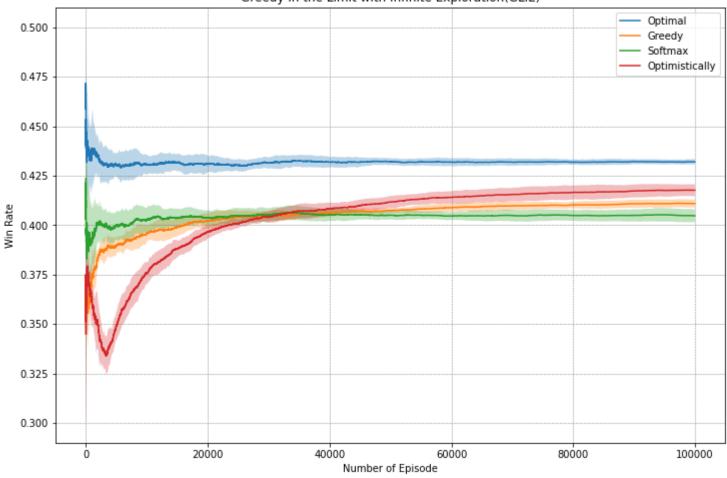
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```
100%
                 100000/100000 [00:18<00:00, 5545.01it/s]
100%
                 100000/100000 [00:18<00:00, 5337.63it/s]
100%
                 100000/100000 [00:21<00:00, 4550.19it/s]
100%
                 100000/100000 [00:18<00:00, 5428.33it/s]
100%
                 100000/100000 [00:16<00:00, 5951.42it/s]
100%
                 100000/100000 [00:18<00:00, 5370.10it/s]
100%
                 100000/100000 [00:21<00:00, 4608.84it/s]
100%
                 100000/100000 [00:18<00:00, 5462.94it/s]
100%
                 100000/100000 [00:16<00:00, 5922.42it/s]
100%
                 100000/100000 [00:18<00:00, 5392.98it/s]
100%
                 100000/100000 [00:22<00:00, 4460.13it/s]
100%
                 100000/100000 [00:18<00:00, 5385.09it/s]
100%
                 100000/100000 [00:16<00:00, 5941.45it/s]
100%
                 100000/100000 [00:18<00:00, 5429.96it/s]
100%
                 100000/100000 [00:21<00:00, 4655.74it/s]
100%
                 100000/100000 [00:18<00:00, 5414.27it/s]
100%
                 100000/100000 [00:16<00:00, 5935.52it/s]
100%
                 100000/100000 [00:18<00:00, 5434.18it/s]
100%
                 100000/100000 [00:22<00:00, 4521.68it/s]
100%
                 100000/100000 [00:18<00:00, 5373.61it/s]
```

```
In []: fig = plt.figure(figsize=(12,8))
    plot_arrays(wr_optimal_mc, 'Optimal')
    plot_arrays(wr_greedy_mc, 'Greedy')
    plot_arrays(wr_softmax_mc, 'Softmax')
    plot_arrays(wr_optimistically_mc, 'Optimistically')

plt.title("Greedy in the Limit with Infinite Exploration(GLIE)")
    plt.xlabel("Number of Episode")
    plt.ylabel("Win Rate")
    plt.grid(color = 'gray', linestyle = '--', linewidth = 0.5)
    plt.legend()
    plt.show()
```





```
In []: wr_optimal_td = np.zeros((EPOCHS, NUM_EPISODES))
    wr_greedy_td = np.zeros((EPOCHS, NUM_EPISODES))
    wr_softmax_td = np.zeros((EPOCHS, NUM_EPISODES))
    wr_optimistically_td = np.zeros((EPOCHS, NUM_EPISODES))

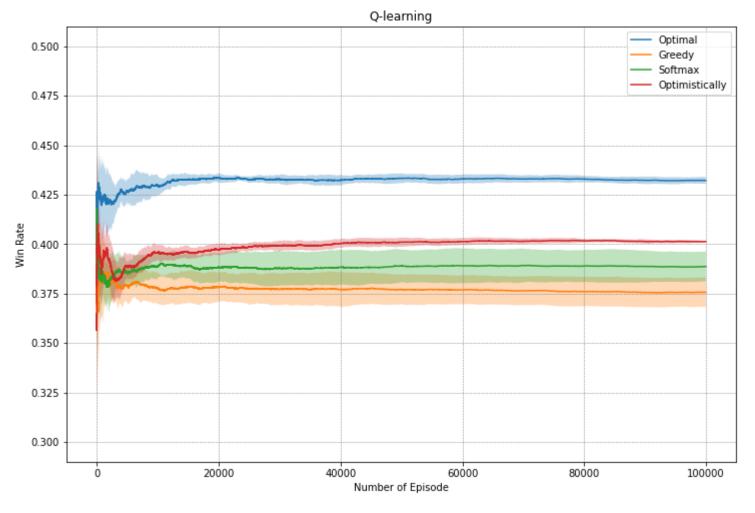
for i in range(EPOCHS):
    Q, r = Q_learning(env, NUM_EPISODES, action_method='optimal', eval_method='td')
    wr_optimal_td[i] = get_winrate(r)

    Q, r = Q_learning(env, NUM_EPISODES, action_method='greedy', eval_method='td')
    wr_greedy_td[i] = get_winrate(r)

    Q, r = Q_learning(env, NUM_EPISODES, action_method='softmax', eval_method='td')
```

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```
wr softmax td[i] = get winrate(r)
            Q, r = Q learning(env, NUM EPISODES, action method='optimistically', eval method='td')
            wr optimistically td[i] = get winrate(r)
        100%
                         100000/100000 [00:18<00:00, 5353.66it/s]
        100%
                         100000/100000 [00:20<00:00, 4859.62it/s]
        100%
                         100000/100000 [00:24<00:00, 4117.86it/s]
        100%
                         100000/100000 [00:20<00:00, 4878.55it/s]
        100%
                         100000/100000 [00:17<00:00, 5582.54it/s]
        100%
                         100000/100000 [00:20<00:00, 4897.70it/s]
        100%
                         100000/100000 [00:23<00:00, 4170.96it/s]
        100%
                         100000/100000 [00:20<00:00, 4870.43it/s]
        100%
                         100000/100000 [00:18<00:00, 5297.47it/s]
        100%
                         100000/100000 [00:20<00:00, 4843.95it/s]
        100%
                         100000/100000 [00:23<00:00, 4174.86it/s]
        100%
                         100000/100000 [00:20<00:00, 4874.11it/s]
        100%
                         100000/100000 [00:17<00:00, 5603.44it/s]
        100%
                         100000/100000 [00:20<00:00, 4876.44it/s]
        100%
                         100000/100000 [00:23<00:00, 4180.48it/s]
        100%
                         100000/100000 [00:20<00:00, 4856.67it/s]
        100%
                         100000/100000 [00:17<00:00, 5585.95it/s]
        100%
                         100000/100000 [00:20<00:00, 4907.12it/s]
        100%
                         100000/100000 [00:25<00:00, 3983.77it/s]
        100%
                         100000/100000 [00:20<00:00, 4830.24it/s]
In [ ]: fig = plt.figure(figsize=(12,8))
        plot arrays(wr optimal td, 'Optimal')
        plot arrays(wr greedy td, 'Greedy')
        plot_arrays(wr_softmax_td, 'Softmax')
        plot arrays(wr optimistically td, 'Optimistically')
        plt.title("Q-learning")
        plt.xlabel("Number of Episode")
        plt.ylabel("Win Rate")
        plt.grid(color = 'gray', linestyle = '--', linewidth = 0.5)
        plt.legend()
        plt.show()
```



```
In [ ]: wr_optimal_sarsa = np.zeros((EPOCHS, NUM_EPISODES))
    wr_greedy_sarsa = np.zeros((EPOCHS, NUM_EPISODES))
    wr_softmax_sarsa = np.zeros((EPOCHS, NUM_EPISODES))
    wr_optimistically_sarsa = np.zeros((EPOCHS, NUM_EPISODES))

for i in range(EPOCHS):
    Q, r = SARSA(env, NUM_EPISODES, action_method='optimal')
    wr_optimal_sarsa[i] = get_winrate(r)

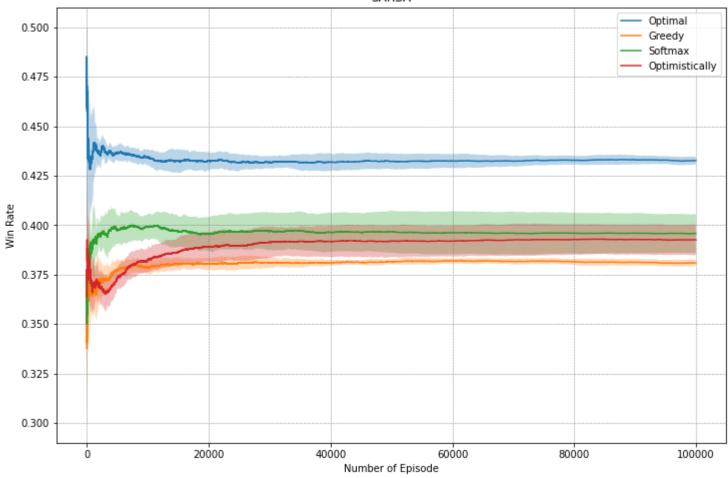
    Q, r = SARSA(env, NUM_EPISODES, action_method='greedy')
    wr_greedy_sarsa[i] = get_winrate(r)

    Q, r = SARSA(env, NUM_EPISODES, action_method='softmax')
```

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```
wr softmax sarsa[i] = get winrate(r)
            Q, r = SARSA(env, NUM EPISODES, action method='optimistically')
            wr optimistically sarsa[i] = get winrate(r)
        100%
                         100000/100000 [00:16<00:00, 5981.10it/s]
        100%
                         100000/100000 [00:20<00:00, 4935.65it/s]
        100%
                         100000/100000 [00:25<00:00, 3848.49it/s]
        100%
                         100000/100000 [00:20<00:00, 4879.57it/s]
        100%
                         100000/100000 [00:16<00:00, 5969.11it/s]
        100%
                         100000/100000 [00:20<00:00, 4937.08it/s]
        100%
                         100000/100000 [00:26<00:00, 3807.67it/s]
        100%
                         100000/100000 [00:21<00:00, 4640.13it/s]
        100%
                         100000/100000 [00:16<00:00, 5939.24it/s]
        100%
                         100000/100000 [00:20<00:00, 4895.60it/s]
        100%
                         100000/100000 [00:26<00:00, 3803.02it/s]
        100%
                         100000/100000 [00:20<00:00, 4853.85it/s]
        100%
                         100000/100000 [00:16<00:00, 5971.66it/s]
        100%
                         100000/100000 [00:20<00:00, 4908.88it/s]
        100%
                         100000/100000 [00:26<00:00, 3814.37it/s]
        100%
                         100000/100000 [00:20<00:00, 4868.70it/s]
        100%
                         100000/100000 [00:17<00:00, 5622.44it/s]
        100%
                         100000/100000 [00:20<00:00, 4921.40it/s]
        100%
                         100000/100000 [00:26<00:00, 3774.71it/s]
        100%
                         100000/100000 [00:20<00:00, 4842.27it/s]
In [ ]: fig = plt.figure(figsize=(12,8))
        plot arrays(wr optimal sarsa, 'Optimal')
        plot arrays(wr greedy sarsa, 'Greedy')
        plot_arrays(wr_softmax_sarsa, 'Softmax')
        plot arrays(wr optimistically sarsa, 'Optimistically')
        plt.title("SARSA")
        plt.xlabel("Number of Episode")
        plt.ylabel("Win Rate")
        plt.grid(color = 'gray', linestyle = '--', linewidth = 0.5)
        plt.legend()
        plt.show()
```



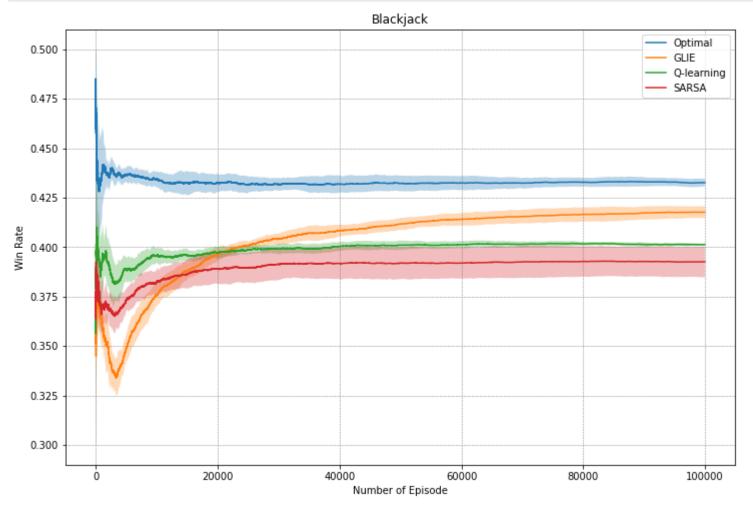


```
In []: fig = plt.figure(figsize=(12,8))

plot_arrays(wr_optimal_sarsa, 'Optimal')
plot_arrays(wr_optimistically_mc,'GLIE')
plot_arrays(wr_optimistically_td,'Q-learning')
plot_arrays(wr_optimistically_sarsa,'SARSA')

plt.title("Blackjack")
plt.xlabel("Number of Episode")
plt.ylabel("Win Rate")
plt.grid(color = 'gray', linestyle = '--', linewidth = 0.5)
```

```
plt.legend()
plt.show()
```



```
table = tabulate(m, headers, tablefmt="fancy_grid")
# Show it.
print(table)
```

	Mean	Std
Optimal	0.432612	0.00196645
GLIE	0.417636	0.00295745
Q_learning	0.401348	0.000475285
SARSA	0.392608	0.00773201

```
In [ ]: fig = plt.figure(figsize=(30, 8))
        ax = fig.add subplot(131)
        plot arrays(wr optimal td, 'Optimal')
        plot arrays(wr greedy td, 'Greedy')
        plot_arrays(wr_softmax_td, 'Softmax')
        plot arrays(wr optimistically td, 'Optimistically')
        plt.title("Q-learning")
        plt.xlabel("Number of Episode")
        plt.ylabel("Win Rate")
        plt.grid(color = 'gray', linestyle = '--', linewidth = 0.5)
        plt.legend()
        ax = fig.add subplot(132)
        plot arrays(wr optimal mc, 'Optimal')
        plot arrays(wr greedy mc, 'Greedy')
        plot_arrays(wr_softmax_mc, 'Softmax')
        plot arrays(wr optimistically mc, 'Optimistically')
        plt.title("Greedy in the Limit with Infinite Exploration(GLIE)")
        plt.xlabel("Number of Episode")
        plt.grid(color = 'gray', linestyle = '--', linewidth = 0.5)
        plt.legend()
        ax = fig.add subplot(133)
        plot arrays(wr optimal sarsa, 'Optimal')
        plot_arrays(wr_greedy_sarsa, 'Greedy')
```

```
plot_arrays(wr_softmax_sarsa, 'Softmax')
plot_arrays(wr_optimistically_sarsa, 'Optimistically')
plt.title("SARSA")
plt.xlabel("Number of Episode")
plt.grid(color = 'gray', linestyle = '--', linewidth = 0.5)
plt.legend()
plt.show()
```

