

## Report

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### <commit history>

Commits on Apr 20, 2025

support task1, task2

HelloHe110 committed 16 hours ago

2e9315c



Here is the first commit – for supporting task1 and task2. Task1 is about to generate network.graph. And here's the thing I do:

- Read network.pos (provided by TA)
- Precompute angular grid for array pattern where the beam resolution is set to  $[0.5 : 0.5 : 90]$
- For every (GS, SAT) pair, I computed the (horizontal/vertical/total) distance of the pair; after that, I got the elevation angle by  $\text{atan2d}(\text{horizontal}, \text{vertical})$ , and since the elevation angle could only be in range of 0 to 90, I convert the  $180 > \theta_1 > 90$  into  $\theta_2$  by  $\theta_2 = 180 - \theta_1$  just like what I did in lab2! Furthermore, I select the closest tx beam direction and the AoD resolution index for the following computation about the gain table and the final (strongest) gain I picked from gain table.
- (Still applied for every (GS, SAT) pair) I computed the Friis path loss and find out the Power of rx. Here is a thing that needs to be considered: if the link (Prx\_dBm) is too weak  $P_{rx(dBm)} < rx_{thresh(dBm)}$ , we could say that this link (path) doesn't exist in the network graph. On the other hand, if the link is ok, I continue to compute data rate and store the Shannon\_capacity's value in the links (array).
- Finally, after traversing through all the (GS, SAT) pair, I wrote the {GS, SAT, DataRate} into the file *network.graph* for the further usage in task2.

The previous description is about how task1 is implemented (the compute\_datarate.m a.k.a bf.m) will be renamed in the following commit. And there are also some correlated files to support this MATLAB file, such as, lab2\_bf\_referece.m and ewa\_functions.

Now, let's move on to the task2. The following are the process I do:

- I read the *network.graph* file which is located at *BasicExample/src/network.graph* due to the program will be run under the cmake\_or-tools.
- After that, I construct the adjacency list by data-structure  $std::vector < std::vector < std::pair < int, double >>>$  of size |GS|. Therefore, for each station v, we can get a list of (satellite\_id, time) connected to this station. You can note that the time is simply compute by  $1000.0/rate$  (sec) which converts the data rate into a transmission time of seconds for sending a 1000kb unit.
- Before modeling, the code loops over all v from 0 to v-1 and ensures adj[v] is nonempty; if any station lacks valid links, it reports an infeasibility error and exits. This prevents creating an impossible constraint that would force a station with no edges to choose a satellite.

- d. Now, it's time to initialize and setup the solver. A CBC-based MIP solver is

instantiated via `auto solver = MPSolver::CreateSolver("CBC_MIXED_INTEGER_PROGRAMMING");`

where MPSolver is OR-Tools' primary interface for LP/MIP problems. After that, I set up variables. For each ground station  $v$ , and for each outgoing link index  $k$ , the code creates a Boolean decision variable `x[v].push_back(solver->MakeBoolVar("x_v_s"));` indicating whether station  $v$  connects to satellite  $s$ . These variables are stored in a 2D vector  $x$ , and the corresponding satellite IDs in  $sat\_of$  for later lookup.

- e. Also, I set up the max-time variable, a continuous variable  $T$  is declared with bounds  $[0, \infty]$  to represent the maximum collection time across all satellites.

```
const MPVariable* T = solver->MakeNumVar(0.0, MPSolver::infinity(), "T");
```

- f. Moreover, I setup constraints (1) the assignment constraint: for each station  $v$ , a row constraint enforces  $\sum_k x_{v,k} = 1$  ensuring each ground station picks exactly 1 sat. (2) load-balancing constraints: To tie  $T$  to each satellite's load, the code builds, for each satellite  $s$ , a constraint

$$T - \sum_{(v,k): sat\_of[v][k]=s} t_{v,k} \cdot x_{v,k} \geq 0$$

by using a map to accumulate coefficients. This set of constraints guarantees that  $T$  is at least as large as every satellite's total assigned transmission time.

- g. For the objective and solve, the objective function is set to minimize  $T$  via

```
solver->MutableObjective()->SetCoefficient(T, 1);  
solver->MutableObjective()->SetMinimization();
```

and calling `solver->Solve()` runs CBC's branch-and-cut algorithm; the code checks for `MPSOLVER::OPTIMAL` to confirm a valid solution was found.

- h. Finally, I wrote the result into the `network.ortools.out`!

Commits on Apr 22, 2025

support task3 and update task2, file\_name

HelloHe110 committed 2 days ago

017d9dc

Ok, and here's my second commit for supporting task3 – the greedy approach and modify the output order in task2, and also the file name. And the following are what I've done in task3:

- a. Same as task2, I read the `network.graph` file which is located at `BasicExample/src/network.graph` due to the program will be run under the `cmake_or-tools`.

- b. After getting the network graph, it's time to select the "best" link per station. best\_rate will hold the highest data rate seen so far for station v. And best\_sat will store which satellite gave that highest rate. I looped over all the link to update the value of those 2 vectors. Finally, each station v has chosen its single satellite best\_sat[v] with maximum link rate.

```
19 + // For each ground station: track best satellite (max rate)
20 + std::vector<double> best_rate(V, 0.0);
21 + std::vector<int> best_sat(V, -1);
22 +
23 + for (int i = 0; i < L; ++i) {
24 +     int v, s;
25 +     double rate;
26 +     infile >> v >> s >> rate;
27 +     if (rate > best_rate[v]) {
28 +         best_rate[v] = rate;
29 +         best_sat[v] = s;
30 +     }
31 + }
```

- c. Now, I can compute satellite loads and maximum time. The sat\_time (satellite-load counter) is initialized to 0. And in per-station, I check that station v did find any valid satellite (best\_sat[v] >= 0). After that, I compute its transmission time  $t = 1000/\text{rate}$  and accumulate t into sat\_time[best\_sat[v]]. We can finally get the global maximum by iterating through sat\_time[s] and track the largest into T, which is the worst-case (bottleneck) satellite collection time under the greedy assignment.

```
34 + // Compute per-satellite collection times and global max
35 + std::vector<double> sat_time(S, 0.0);
36 + double T = 0.0;
37 + for (int v = 0; v < V; ++v) {
38 +     if (best_sat[v] < 0) {
39 +         std::cerr << "No valid link for station " << v << std::endl;
40 +         return 1;
41 +     }
42 +     double t = 1000.0 / best_rate[v]; // time for one data unit
43 +     sat_time[best_sat[v]] += t;
44 + }
45 + for (int s = 0; s < S; ++s) {
46 +     if (sat_time[s] > T) T = sat_time[s];
47 + }
```

- d. Finally, we can write the result into network.greedy.out.

Furthermore, I rename compute\_datarate.m to bf.m and add a sorting to make the output of task2 be in order! Moreover, I add the studentID.txt as required.

After all, here is the third commit -> the report is added!

### <Comparison>

Station	Greedy → Satellite	OR-Tools → Satellite
0	1	0
1	2	0
2	2	1
3	3	1
4	3	2
5	3	2
6	3	3
7	4	4
8	5	5
9	6	6
10	4	5
11	5	3
12	6	6
13	8	7
14	8	8
15	8	8
16	8	7
17	8	9
18	8	9
19	5	4

- **Maximum collection time  $T$** 
  - **Greedy:** 0.155097 second
  - **OR-Tools:** 0.0537272 second
- **Per-Satellite Loads:**
  - Greedy creates idle satellites (satellite with index = 0, 7, 9 spends 0s) while others pile up, e.g. satellites with index 8 has 0.155097 seconds.
  - OR-Tools spreads the work by letting all active satellites finish within ~0.0517 to 0.0537 seconds.
- **Execution Time:**
  - **Greedy:** the asymptotic cost is about  $O(L)$  over links, and the measured runtime is about  $< 1ms$ . (Greedy reads each of the  $L$  links once and picks the beat rate per station, so it runs in microseconds on this problem size)
  - **OR-Tools:** the asymptotic cost is about ILP via CBC, and the measured runtime is approximately 0.5 to 3 sec (OR-Tools builds a mixed-integer program and solves it with CBC; we might typically see runtimes on the order of seconds for a  $20 \times 10$  instance).

- **Advantages & Disadvantages:**

Criterion	Greedy	OR-Tools ILP
Solution Quality	Suboptimal worst-case time	Optimal min-max load
Speed	Near instantaneous	Seconds (can be tuned)
Implementation	Simple loops, no external libs	Requires OR-Tools setup & modeling
Scalability	Scales to very large $V, S$	Limited by ILP solver on large data
Determinism	Fully deterministic	Deterministic but solver-tunable

Therefore, we can implement Greedy when we need a rapid, “good-enough” assignment in real-time or on very large networks; on the other hand, we can apply OR-Tools when worst-case performance matters and can afford an offline solve!

The overall comparison toward greedy and or-tools method is that the or-tools solver achieves a much lower maximum collection time ( $\approx 0.0537$  s) than the Greedy heuristic ( $\approx 0.1551$  s), at the cost of a heavier solve routine. Greedy is trivial to implement and near-instantaneous but can leave some satellites idle while others become bottlenecks. OR-Tools balances the load perfectly, minimizing the worst-case satellite time, but requires a full ILP solve which—even on modest hardware—can take seconds rather than microseconds.