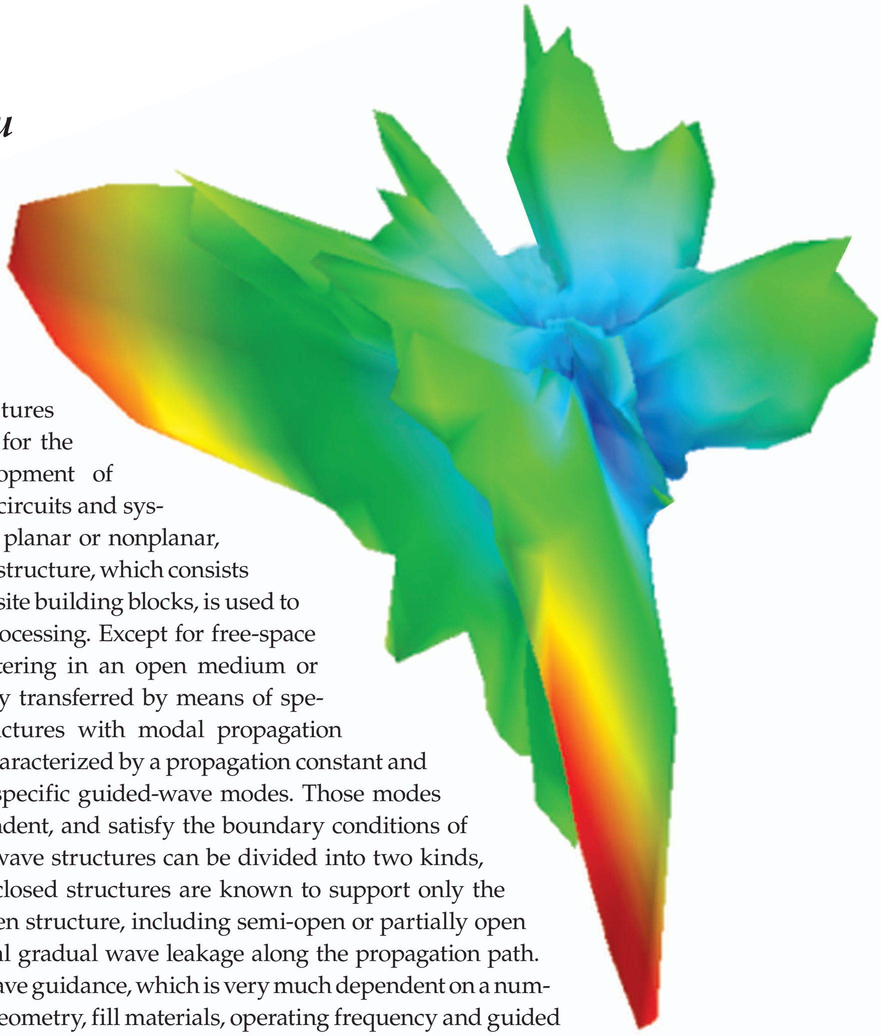
Understanding Leaky-Wave Structures

Feng Xu and Ke Wu



Guided-wave structures are the foundation for the design and development of RF and microwave circuits and systems. Whether it is planar or nonplanar, periodic or straight, a guided-wave structure, which consists of metallic and/or dielectric composite building blocks, is used to support signal propagation and processing. Except for free-space propagation, diffraction ,and scattering in an open medium or space, microwave energy is usually transferred by means of specially designed guided-wave structures with modal propagation behavior that are fundamentally characterized by a propagation constant and transmission loss with respect to specific guided-wave modes. Those modes are structure and frequency dependent, and satisfy the boundary conditions of a guided-wave structure. Guided-wave structures can be divided into two kinds, open and closed. The completely closed structures are known to support only the wave guided therein, while the open structure, including semi-open or partially open structures, are subject to a potential gradual wave leakage along the propagation path. Wave leakage is always related to wave guidance, which is very much dependent on a number of factors including structural geometry, fill materials, operating frequency and guided mode. Such leakage can be used positively to develop a leaky-wave structure called a leaky-wave antenna, which enjoys some distinctive properties such as beam-scanning with frequency. Leaky-wave radiating structures are easily fabricated at millimeter wave frequencies compared with other antennas. Although leaky-wave and guided-wave structures have

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The earliest example of a leaky-wave antenna was a rectangular waveguide with a continuous slit cut along its side [4], [5].

similar characteristics and can be designed by common methods, the simulation and design of leaky-wave structures are usually much more complicated. This is because leaky-wave structures are not closed and their attenuation constant related to leakage needs to be considered. In this article, we focus on the presentation of basic operating principles and special features of positive leaky-wave structures (leaky-wave antennas) ranging from straight to periodic geometries.

A leaky-wave antenna can be regarded as a direct geometrical development of a guided-wave structure that permits energy leakage along its longitudinal direction. A leaky-wave structure is a special class of waveguide and one of the waveguide modes is used to generate leakage along the guided-wave path. For a closed guided-wave structure, microwave energy may leak out when such a closed structure is disturbed. Such a disturbance could be a slit cut on the narrow wall of a rectangular waveguide that disturbs the surface currents of TE_{10} mode (Figure 1), providing leakage of guided energy. For an open guided-wave structure, a special arrangement such as adjusting the structure geometry or selecting appropriate field modes needs to be applied to allow microwave energy radiation because surface waves usually propagate in these open structures.

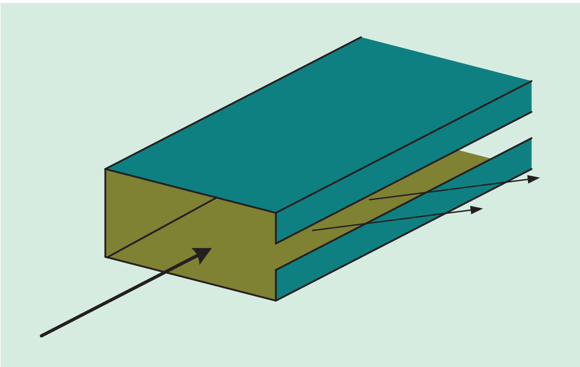

Figure 1. *A rectangular waveguide with a slit cut along the length.*

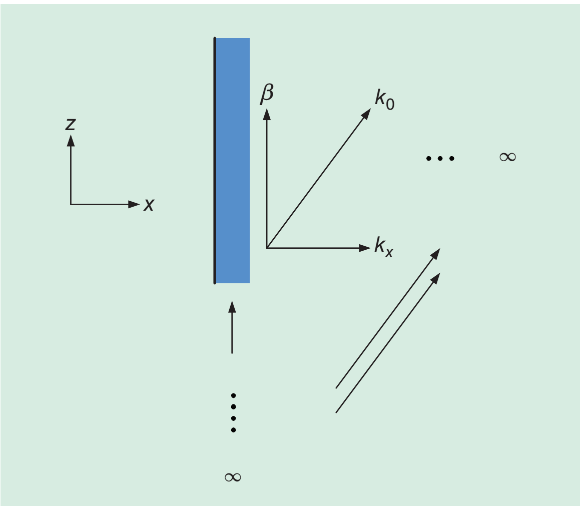

Figure 2. *Radiation of a leaky wave and research of forming this leaky wave.*

The operating principle of leaky-wave antennas has been known for many years [1]–[3]. The earliest example of such an antenna was a rectangular waveguide with a continuous slit cut along its side [4], [5]. As shown in Figure 1, microwave energy leaks out from the slit and radiates into space. This long slit can be replaced by a series of closely spaced holes in order to obtain narrower beams [6]. These are termed one dimensional (1-D) uniform or quasi-uniform leaky-wave antennas. The earliest case of the 1-D periodic leaky-wave antennas was a sandwich wire antenna [7]. By using a simple form of asymmetry, the idea of producing a leaky wave in the open symmetrical guided-wave structures was first proposed in [8]. The earliest form of two-dimensional (2-D) leaky-wave antennas was realized by using a periodic partially reflective screen (PRS) over a ground plane [9]. Different versions of leaky-wave antennas have been proposed and extensively studied with various technologies and structures [10]–[36]. Examples include various topologies of dielectric waveguides, groove guides, rectangular waveguides, microstrip line, coplanar waveguide (CPW), substrate integrated waveguide (SIW), and substrate integrated image guide (SIIG). Recently, metamaterial structures have been introduced in the research of leaky-wave antennas. These alternative points of view provide useful information while a simple explanation in terms of leaky waves has provided the most physically fundamental way to explain the operation principle of leaky-wave antennas.

Conditions for Forming Leaky Waves

Since a leaky wave is derived from its guided wave counterpart, it is important to examine the basic characteristics of the guided wave and on which condition this evolution will result in controllable leakage. Such controllable leakage can be exploited for the design and development of the leaky-wave antenna. As shown in Figure 2, suppose a guided wave ($e^{-j\beta z}$) travels in the $+z$ direction with phase constant β. If this guided wave produces a leaky wave in the x direction (k_x), there is a relationship between k_x and β

$$k_x^2 = k_0^2 - \beta^2. \tag{1}$$

From (1), the leaky wave will start to appear if and only if k_x becomes a real number. As a result, $\beta < k_0$ is the condition for generating a real leaky wave. This is an interesting and fundamental observation because the value of propagation constant is mode dependent. This suggests that the same structure can be used to support guided-wave and/or leaky-wave propagation, depending on the operating modes, which may also be multiple in numbers. It is necessary to look into the relationship between these propagation constants in order to avoid potential confusion in connection with the leaky-wave concept. When a leaky wave is formed, the electromagnetic wave will dampen or become attenuated in the $+z$ direction if the guided wave propagates in this direction. In addition to the phase constant β, an attenuation constant α should also be introduced. The leaky wave travels in the $+z$ direction according to $e^{-\gamma z}$, where

$$\gamma = \alpha + j\beta. \tag{2}$$

In the x direction, a similar equation also exists and so (1) should be revised as

$$k_o^2 = (\beta - j\alpha)^2 + (k_x - j\alpha_x)^2, \tag{3}$$

where α_x is the leakage constant of the leaky wave in the $+x$ direction. To make (3) valid and rational, α and α_x should have opposite signs, which can render the imaginary part of the right side of (3) equal to zero. As a result, the leakage constant α_x should be less than zero since $\alpha > 0$. Another rule for the formation of a leaky wave is that $\alpha_x < 0$. When $\alpha_x < 0$, the leaky-wave energy will increase in the $+x$ direction ($e^{-\alpha_x x} e^{-jk_x x}$). This observation may be counterintuitive because it appears that the leaky-wave energy would increase to infinity in the transverse direction ($+x$ direction). This observation is logical because, as shown in Figure 2, the propagation direction of the leaky wave is in the k_0 direction and the infinite leaky-wave energy in the $+x$-direction is partly related to the leaky wave in the $-z$-direction. The leaky-wave energy in the $-z$-direction should be infinity. If not, the electromagnetic wave is not able to travel to the $z = 0$ direction ($e^{-\alpha z} e^{-j\beta z}$) since $\alpha > 0$. The leaky-wave structure itself is always finite in size and the leaky wave is defined only in a sector of space in proximity to the leaky-wave structure, which never reaches infinity in the transverse direction. The understanding and mechanism of this leaky-wave generation allow establishment of design principles of leaky-wave antennas in a straightforward manner. When the imaginary part in (3) is set equal to zero, (3) can be rewritten as

$$k_o^2 = \beta^2 - \alpha^2 + k_x^2 - \alpha_x^2. \tag{4}$$

Since the conditions $\beta \gg \alpha$ and $k_x \gg \alpha_x$ are fulfilled in the design of leaky-wave antennas, (4) can be simplified as (1). The condition $\beta < k_0$ can be accurately used as a judging rule in the design of a leaky-wave antenna and has been widely used because of its simplicity. It is straightforward to describe the design mechanisms of these two basic types of leaky-wave antennas, namely the uniform leaky-wave antenna and the periodic leaky-wave antenna.

There are two different kinds of leaky-wave structures that cover the complete spectrum of leaky-wave design techniques.

Uniform Leaky-Wave Structure

There are two different kinds of leaky-wave structures that cover the complete spectrum of leaky-wave design techniques. One is related to uniform guided-wave structures while the other consists of a large array of periodic guided-wave structures. These two kinds of leaky-wave structures must feature semi-open or unbounded geometry for leakage. They both are derived from specific guided wave counterparts and follow the same rules of formation of leaky wave. Figure 1 shows a classic uniform leaky-wave structure. Usually, a fast wave ($\beta < k_0$) travels in closed guided-wave structures such as rectangular metallic waveguide. A physical cut that disturbs the guided wave along the longitudinal propagation direction will produce a leaky wave because the condition of forming a leaky wave has been fulfilled in this case. For open guided-wave structures, the dominant modes in these waveguides may be bounded even though the geometry is open because the slow wave ($\beta > k_0$) usually travels along the structures. In this case, special techniques are necessary to allow the generation of wanted leakage. Such methods include the introduction of asymmetry, the use of other geometrical modifications, or the selection of appropriate traveling-wave modes. For example, the quasi-TEM mode is a typical slow wave along a microstrip line (a very large portion of guided-wave energy is confined to the dielectric substrate region) and cannot be used to generate a leaky wave scenario. Ermert first studied the radiation of fields from the higher-order modes on microstrip line [10]. Menzel then proposed and presented a traveling-wave antenna on microstrip line fed in its first-higher mode (TE_{10} mode), and this first practical microstrip line leaky-wave antenna operating in proximity of the cutoff state of that higher-order mode [11]. Oliner, who contributed significantly to the understanding and development of leaky-wave antennas, and Lee investigated and explained the nature of leakage from the higher-order modes on microstrip line [19]. Since the higher mode is related to the traveling of a fast wave, a leaky-wave structure can be formed as shown in Figure 3. The use of this fast wave again confirms the basic operation and generation of a leaky-wave structure. Only the fast wave allows for traveling waves and radiation into air from the

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bounded or guided regions. Note that slow wave and fast wave propagation correspond to low-impedance and high-impedance, respectively, according to the theory of high-frequency circuits. High-impedance is always wanted to create a matching condition for radiation between the leaky-wave structure and free-space (generally high impedance, too).

When the frequency increases, conductor loss and fabrication tolerances ultimately limit the potential applications of closed guided-wave structures, which may become difficult for their conversion into leaky-wave structures with complicated mechanical cuts or geometrical deformations. In the millimeter-wave region, the open guided-wave structures have been widely used as they exhibit lower transmission loss and simpler mechanical topology. Open guided-wave structures such as groove guides or nonradiative dielectric (NRD) guides are good candidates for the design and development of leaky-wave antennas because it is quite easy to fabricate them as uniform leaky-wave antennas. They generally have very flexible radiation characteristics. Since the dominant modes along these structures belong to the category of dispersive slow waves, special schemes such as introducing asymmetry or shortening specific dimensions in the cross section should be applied to the design of the guided-wave structure so that desired fast waves can be produced. As shown in Figure 4, the dominant mode has a symmetrical field distribution and belongs to the category of slow (bounded) waves even though the slow speed is not so pronounced if there is no dielectric material or slab involved along the guided-wave channel. The effective dielectric constant is close to the value of one if air is used. This is a very important point, and so the term "slow wave" should be carefully used. By placing a narrow horizontal metal strip in the longitudinal direction on one of the vertical groove guide walls just outside of the central region, parallel to the propagation direction, asymmetry is introduced and an excited TEM mode propagates at an angle to the open ends and radiates power with horizontal polarization. The attenuation constant is controlled by the location and width of the metal strip. It is not easy to fabricate this type of leaky-wave antenna. As a result, it has been replaced by a simpler solution in which the L-shaped groove leaky-wave antenna was developed. This is based on the fact that all the higher modes of the groove guide are related to fast waves. The first higher asymmetry mode is usually selected.

For the uniform leaky-wave antenna, the beam direction can be written as

$$\sin\theta_m \approx \frac{\beta}{k_0}, \tag{5}$$

where θ_m is the angle of the maximum of the beam, measured from the broadside direction. From (5), it can be concluded that the beam direction will change with frequency because the phase constant β changes with frequency (dispersion in connection with fast waves). Thus, the uniform leaky-wave antenna can be scanned by varying frequency and its scanning range is extended from broadside to end fire (forward area), with the beam nearer to end fire at higher frequencies.

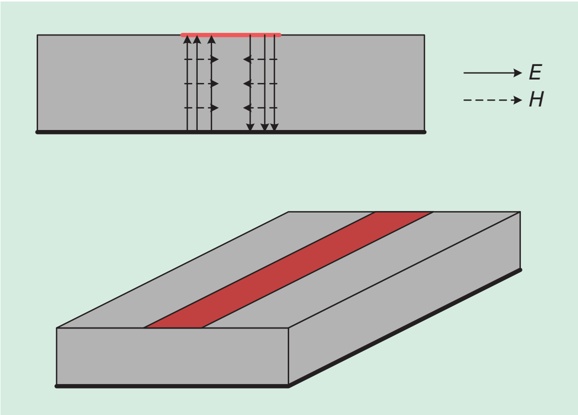

Figure 3. *A leaky-wave antenna of the microstrip version working on the high mode (TE_{10}).*

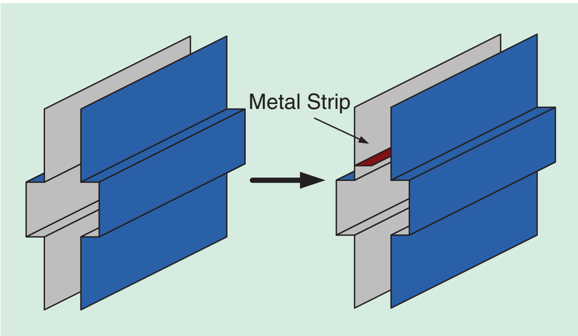

Figure 4. *A leaky-wave antenna of the groove guide version, created by introducing the mechanism of asymmetry.*

Periodic Leaky-Wave Structure

The other type of leaky-wave antenna is based on the periodic leaky-wave structure. Figure 5 shows a typical example of a periodic leaky-wave antenna: a rectangular dielectric rod on which a periodic array of metal strips is placed or printed. The dominant mode in the pure dielectric rod is classified as a slow wave ($\beta > k_0$), meaning that the leaky waves cannot be formed from the dominant mode even if this is an open structure. It is the periodic modulation of the guided-wave structures that allows the generation of leakage. When a periodic array of strips is added, the periodicity introduces an infinite number of space harmonics according to Floquet's theorem, each characterized by phase constants β_n:

$$\beta_n d = \beta_0 d + 2n\pi, \tag{6}$$

where d is the periodic length of the guided-wave structure and β_0, the fundamental space harmonic, is the phase constant of the dominant mode of the original uniform dielectric waveguide, but now perturbed somewhat in value because of the addition of the strips. From (6), even if the dominant mode (the fundamental space harmonic) is slow wave, some other space harmonics may be fast. The phase constant of the first space harmonic ($n = -1$) can be written as

$$\beta_{-1} = \beta_0 - \frac{2\pi}{d}. \tag{7}$$

By carefully designing a leaky-wave structure over a certain frequency range with specific geometrical conditions, the condition of $\beta_{-1} < k_0$ can be fulfilled. Since a single beam is usually needed, the structure is designed so that only the first space harmonic ($n = -1$) is fast.

The beam direction of the periodic leaky-wave antenna can be expressed as

$$\sin\theta_m \approx \frac{\beta_{-1}}{k_0}, \tag{8}$$

where θ_m is the angle of the maximum of the beam, measured from the broadside direction. From (8), when $-k_0 < \beta_{-1} < 0$, the beam of the periodic leaky-wave antenna is directed into the backward area. As frequency increases, the beam swings up from the backward end fire towards the broadside, then through the broadside into the forward quadrant. The range in the forward quadrant is usually limited by the appearance of the $n = -2$ beam from the backward end fire because the antenna is useful only if a single and controllable beam is radiated. An electromagnetic fields distribution [using a three-dimensional (3-D) electromagnetic simulation] of an SIW version of a leaky-wave antenna is shown in Figure 6 [34]. The white strips and squares in Figure 6 represent conductors. Figure 6 illustrates the backward beam direction formed by the first space harmonic β_{-1} which is introduced by the periodic modulation. The fundamental mode is selected as the TE_{20} waveguide mode since this asymmetric mode exhibits better leakage properties than the TE_{10} waveguide mode. It is the periodic modulation of the conductors that gives the periodic leaky-wave antenna a larger scanning range and a more flexible design scheme than the uniform leaky-wave antenna. This

The other type of leaky-wave antenna is based on the periodic leaky-wave structure.

observation is applicable to other periodic leaky-wave antennas. A simulated 3-D pattern of this periodic leaky-wave antenna is shown in Figure 7.

A stop-band (or band-gap) region will appear near the broadside for the periodic structure. In this region ($\beta_0 = 2\pi/d$), the value of attenuation constant

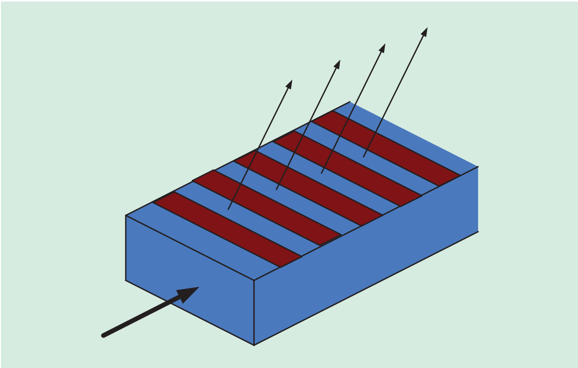

Figure 5. *A rectangular dielectric rod with a periodic array of metal strips.*

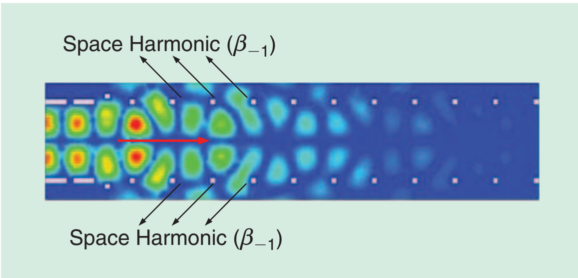

Figure 6. *An SIW version of the periodic leaky-wave antenna and its backward beam direction provoked by the first space harmonic [34].*

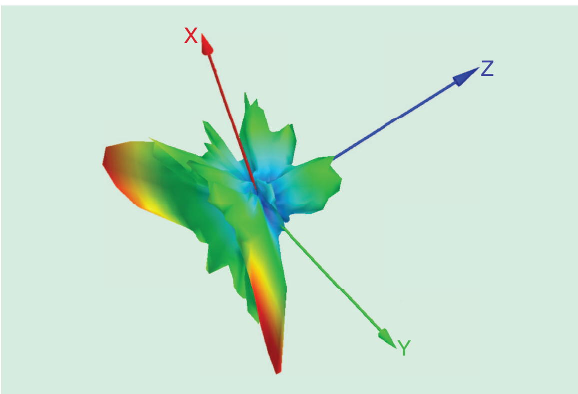

Figure 7. *Three-dimensional pattern of the SIW version of the periodic leaky-wave antenna.*

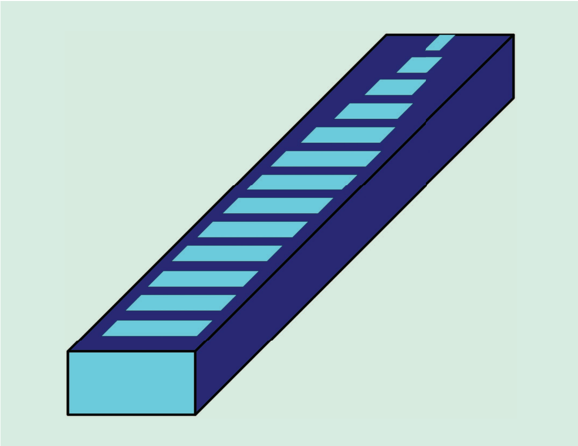

Figure 8. *A waveguide leaky-wave antenna with a group of tapering series cuts.*

α becomes large. The power is reflected back to the source rather than being radiated.

Complex Propagation Constant $\gamma\,(\gamma = \alpha + j\beta)$ and Basic Design Procedures

Uniform leaky-wave antennas and periodic leaky-wave antennas have different physical mechanisms. If one uses the first space harmonic β_{-1} of the periodic leaky-wave antenna to replace the fundamental phase constant β_0 of the original guided-wave structure, both types of leaky-wave antennas share the same design procedure. Propagation characteristics of the leaky mode in the longitudinal direction are governed by phase constant β and attenuation constant (or leakage constant) α per unit length. Length L of the leaky-wave structure forms the effective aperture of a line-source antenna unless the leakage rate is so large that the power totally leaks away before reaching the end of line where a match load is usually placed. From (5) and (8), the beam directivity is determined by the phase constant β. Since the phase constant β changes with frequency, the beam direction also changes with frequency. This is why a leaky-wave antenna can be scanned by varying the frequency, a well-known property of the leaky-wave antenna. A large α implies that a large leakage rate produces a short effective aperture, which will generally result in a large beamwidth. Conversely, a narrow beam is related to and conditioned by a low value of attenuation constant α and a long effective aperture. When the antenna aperture is finite and the attenuation constant α is small, the beamwidth is determined primarily by the aperture length while the value of α primarily affects the efficiency of radiation. According to the characteristics of a line-source antenna, the beam direction is given by (5) or (8) and the beamwidth can be described by

$$\Delta\theta \approx \frac{1}{(L/\lambda_0)\cos\theta_m}, \tag{9}$$

where θ_m is the angle of the maximum of the beam, measured from the broadside direction, and L is the length of the leaky-wave antenna. From (9), the beamwidth is determined primarily by the aperture length L and is also influenced by the aperture field amplitude distribution.

The radiation pattern can be obtained by taking the Fourier transform of the aperture field distribution. If the aperture length is infinite and the aperture field distribution is exponentially decaying, the radiation pattern $(R\theta)$ is given by

$$R(\theta) = \frac{K\cos^2\theta}{(\alpha/k_0)^2 + (\beta/k_0 - \sin\theta)^2}. \tag{10}$$

From (10), a leaky-wave antenna with an infinite length should ideally be free of any side lobes; a finite leaky-wave antenna will possess side lobes. Since power is radiated continuously along the physical extent, the aperture field of a leaky-wave antenna with strictly uniform geometry has an exponential decay. As such, the sidelobe behavior is generally poor. To improve electrical performances of the leaky-wave antennas in connection with the problem of side lobes, the value of α should be designed so that it changes slowly along the geometrical length while the value of β remains constant. As a result, a desired amplitude distribution of the aperture can be obtained to yield the desired sidelobe performance. Figure 8 shows a leaky-wave antenna with a group of tapering series cuts in a rectangular waveguide used to improve its leaky-wave performance with respect to side lobes.

By extracting the complex propagation constants, which include phase constant β and leakage constant α, it is straightforward to design a leaky-wave antenna. From the specifications of a desired radiation pattern, the aperture field amplitude distribution should be decided first. The attenuation constant α is calculated as a function of position along the antenna length in accordance with the aperture field amplitude distribution based on the following equation:

$$2\alpha(z) = \frac{|A(z)|^2}{\dfrac{P(0)}{P(0)-P(L)}\displaystyle\int_0^L |A(\zeta)|^2 d\zeta - \int_0^z |A(\zeta)|^2 d\zeta}, \tag{11}$$

where $A(z)$ is the desired aperture amplitude distribution and $P(z)$ is the power distribution along the antenna length

$$P(z) = P(0)\exp\left[-2\int_0^z \alpha(\zeta)\,d\zeta\right]. \tag{12}$$

The phase constant β must be kept constant along the length so that the radiation from all parts of the aperture point in the same direction. This adds complexity to the design of leaky-wave antennas. From (11), it is common for $P(L)/P(0)$ to be set equal to 0.1, representing 90% of the power radiated. Better performance is obtained when $P(L)/P(0)$ tends to 0; in this case, a very large $\alpha(L)$ is expected from (11).

Comparison Between Uniform and Periodic Leaky-Wave Structures

Compared with uniform leaky-wave antennas, periodic leaky-wave antennas generally have improved backward directivity, larger scanning range, and more flexible design schemes. As a result, periodic leaky-wave antennas have attracted more attention and have been used in millimeter-wave bands due to their electrical and mechanical advantages such as low loss, flexible radiation characteristics, and mechanical simplicity. Various versions of leaky-wave antennas have been proposed and demonstrated and include periodic arrays based on microstrip lines, microstrip periodic patch antennas, the periodic slot array opened in a parallel plate waveguide, the periodic slot array based on CPW, and the laterally shielded rectangular dielectric waveguide leaky-wave antenna. [9], [14]–[16], [22]–[27], [29]–[36].

These two different kinds of leaky-wave antennas present quite different and interesting characteristics, which are summarized in Table 1. As shown in Figure 9(a), uniform leaky-wave antennas provide radiation into the forward quadrant and can yield scanning from broadside to forward end-fire directions. The scanning range for periodic leaky-wave antennas is quite large and reaches from the backward end-fire through broadside directions into part of the forward quadrant, as shown in Figure 9(b).

The dominant mode along a uniform straight leaky-wave antenna is fast wave propagation. The dominant mode of a periodic leaky-wave antenna is in fact a slow wave, which does not radiate whether the structure is open or unbounded. The introduction of a periodic pattern produces an infinite set of space harmonics. This pattern may be cuts or a patch array. Since one desires an antenna that radiates out only with a single beam, the structure should be designed in such a way that only the first space harmonic ($n = -1$) is fast. There is a narrow region around the broadside where the phenomena of stop banding will occur. This should be solved appropriately for the design of periodic leaky-wave antennas. A double periodic structure technique has been proposed to solve this problem [24]. Not all periodic leaky-wave antennas exploit the leakage principle of the first space harmonic β_{-1}. The rectangular waveguide leaky-wave antenna with a group of series cuts shown in Figure 8 makes use of the fundamental space harmonic β_0 to produce leaky waves. In this case, the dominant mode belongs to fast waves. This type of leaky-wave antenna reduces the attenuation constant α in the case of uniform leaky-wave antennas to permit the generation of a narrow radiated beam. The CPW version of the periodic leaky-wave antenna shown in Figure 10 corresponds to the same situation. The dominant mode in the leaky-wave antenna is the fast wave, $\beta_0 < k_0$. This kind of leaky-wave antennas is called the closely spaced periodic leaky-wave antenna and is regarded as a quasi-uniform leaky-wave antenna.

An individual leaky-wave antenna is clearly a line-source antenna; the design produces the desired behavior (usually a narrow beam) in the scan plane, but the

TABLE 1. Differences between uniform and periodic leaky-wave antennas.

	Uniform Leaky-Wave Antennas	Periodic Leaky-Wave Antennas
Scanning range	From broadside to forward end-fire directions	From backward end-fire through broadside into a part of the forward quadrant
Dominant mode	Fast mode	Slow mode
Leakage mode	Dominant mode (β_0)	Space harmonic (β_{-1})
Stop band	No	Yes

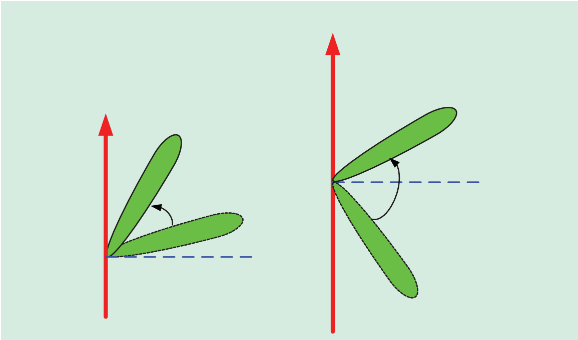

Figure 9. *Scanning range of (a) uniform and (b) periodic leaky-wave antennas.*

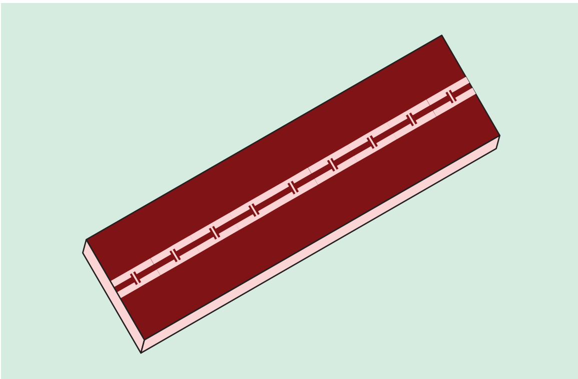

Figure 10. *A CPW version of periodic leaky-wave antenna.*

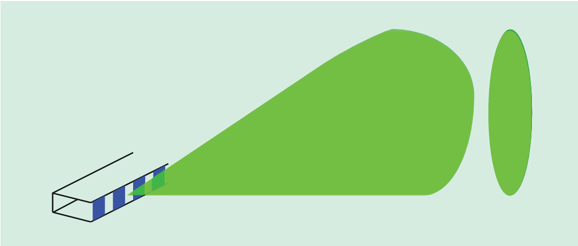

Figure 11. *Fan beam of a leaky-wave antenna.*

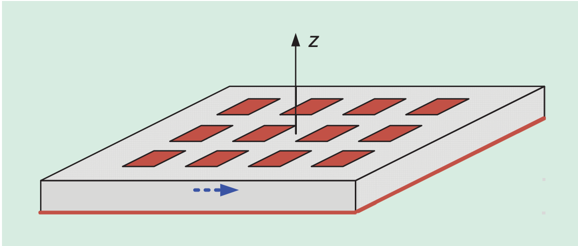

Figure 12. *A 2-D leaky-wave antenna with a partially reflecting screen (PRS) which is composed of a 2-D periodic array of rectangular metal patches.*

radiation pattern in the cross-plane is just a fan beam or pencil beam whose detailed beam shape depends on the cross-sectional dimensions of the leaky-wave antenna as shown in Figure 11. Techniques available for narrowing the beam in the cross-plane to generate a pencil beam include the use of a horn, placing the line-source antenna in an array, or the introduction of a two-dimensional (2-D) periodic leaky-wave antenna [30], [31].

Two-Dimensional Leaky-Wave Antenna

Two-dimensional leaky-wave antennas are constructed on planar structures in order to obtain a highly directive beam which may be a pencil beam at broadside or a conical beam at any desired scan angle [9], [15], [16], [30], [31]. As shown in Figure 12, planar 2-D leaky-wave antennas usually consist of a metal ground plane covered with a dielectric layer on which a PRS is printed. There are many different PRSs that include a metal screen consisting of a periodic array of slots or metal patches (Figure 12), an array of parallel wires or strips, and a stack of dielectric layers. One of the advantages of 2-D leaky-wave antennas is that the exciting source can be very simple, such as a horizontal electric dipole in the middle of the layer or a magnetic dipole on the ground plane. A radially expanding cylindrical wave is excited by an electric or magnetic dipole and in turn produces a narrow radiating beam in the form of an omnidirectional conical beam. The beam angle is approximately given by (5), but the definition of angle θ_0 is different. Now, θ_0 is an angle about z-axis. Accurate analysis of this kind of antenna is also needed to extract the propagation constants of the antenna. Recently, much attention has been directed to the development of 2-D leaky-wave antennas. Some research activities are related to the use of periodic metamaterials to create the 2-D leaky-wave antennas [37], [38]. Some researchers have different opinions on this type of antennas and they often refer to them as Fabry-Pérot antennas [39], [40].

Numerical Analysis

Although commercial software can be used to analyze the periodic structures of leaky wave antennas, the complex propagation constants cannot be obtained directly because there is no accurate single periodic structure model. For the most accurate analysis, two simulations for the same periodic structure but with different lengths should be performed. In this scheme, a numerical calibration technique as discussed in [41], [42] is used to extract the phase and attenuation constants. As a result, accurate complex propagation constants can be obtained from these parameters generated in two simulations. Some commercial software packages provide a single periodic structure model. One such example is the eigenvalue solution model of Ansoft HFSS often used for resonant cavities. By using this solution method and combining with the equivalent resonant cavity model [43], one can accurately extract the propagation constant in some cases. When the attenuation constant is large, such as in the case of leaky-wave antennas, the solution is not accurate, especially when the phase between two periodic planes is close to 180°.

The analysis and design of various leaky-wave antennas mainly consists of the accurate extraction of two critical parameters, namely phase constant β and leakage constant α. A number of numerical methods such as the transverse resonance method, the integral equation method, and the spectral-domain method have been used to extract the complex propagation constant. The finite-difference, frequency-domain (FDFD) method is used in this article due to its high degree of simplicity and versatility in analyzing the characteristics of periodic leaky-wave antennas. This method has been successfully applied to the modeling and extraction of the propagation characteristics of various types of substrate integrated structures [44]–[46] including popular SIW and is therefore directly applicable to the analysis of periodic leaky-wave antennas.

When the FDFD method is used to investigate the characteristics of guided-wave structures, a nonsymmetrical standard eigenvalue problem is presented if the equivalent resonant cavity model is used [45]. As an alternative, it is possible to use a standard FDFD method [44] by eliminating the longitudinal field components. In order to obtain more accurate results, a nonsymmetrical generalized eigenvalue problem can be solved by using a shift-and-invert (SI) Arnoldi technique [46]. For uniform leaky-wave antennas, the simulation domain is a 2-D structure, and for periodic leaky-wave antennas, only a single periodic length structure in the longitudinal direction needs to be set up in the simulation domain. If the SIW periodic leaky-wave antenna in Figure 6 is used as an example, the entire simulation domain includes the substrate subdomain and the air subdomain, as shown in Figure 13. In a periodic guided-wave structure, when the propagation direction is in the z-direction, Floquet's theorem for periodic structures shows that the electromagnetic field components can be expressed as

$$\bar{E}(x,y,z,t) = \bar{e}(x,y,z)e^{-\gamma z}e^{j\omega t} \tag{13a}$$

$$\bar{H}(x,y,z,t) = \bar{h}(x,y,z)e^{-\gamma z}e^{j\omega t}, \tag{13b}$$

where γ is the propagation constant $(\gamma = \alpha + j\beta)$, $e(x,y,z)$, and $h(x,y,z)$ are periodic functions with respect to z. If the periodic length of a periodic guided-wave structure is d, then the periodic boundary conditions in the longitudinal direction are

$$\bar{e}(x,y,z+d) = \bar{e}(x,y,z) \tag{14a}$$

$$\bar{h}(x,y,z+d) = \bar{h}(x,y,z). \tag{14b}$$

After the boundary conditions are applied, a large-scale generalized eigenvalue equation is set up,

$$\mathbf{A}\mathbf{x} = \gamma\mathbf{B}\mathbf{x}. \tag{15}$$

The SI Arnoldi technique can be used to obtain the deterministic solution of a large-scale generalized eigenvalue problem formulated in (15) [47]–[49].

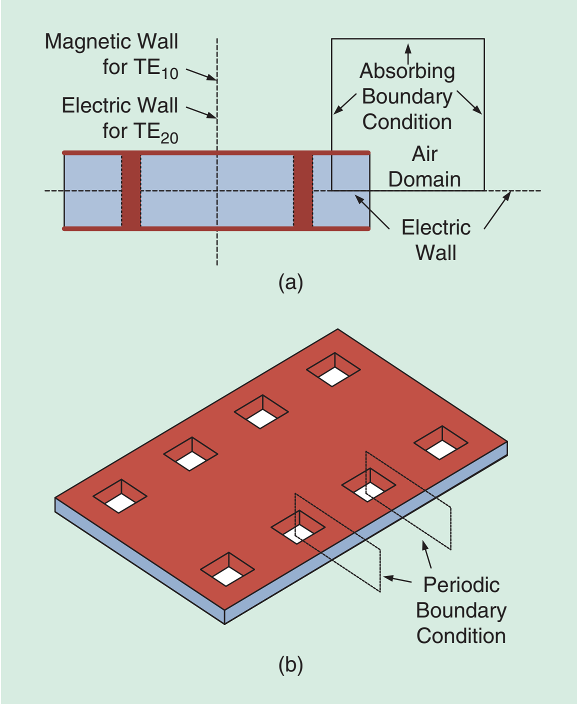

Figure 13. *SIW simulation domain in (a) transversal view and (b) 3-D view.*

Conclusion

A leaky-wave structure is derived from a guided-wave structure that allows energy to leak away along its longitudinal direction. By investigating the relationship between the wave number k_0 in free space and the propagation constant β in the longitudinal direction, the key condition forming a leaky-wave structure ($\beta < k_0$) can be easily obtained. With the help of the complex propagation constant $\gamma(\gamma = \alpha + j\beta)$ extracted by using numerical methods such as FDFD, the aperture field distribution can be calculated, making the design of a leaky-wave antenna easier. There are two different kinds of leaky-wave structures that have been discussed in this article. One is related to uniform guided-wave structures while the other consists of an array of periodic guided-wave structures. They share common features but also exhibit many interesting differences in connection with electrical and mechanical aspects. Significant research in leaky-wave antennas has shown that leaky-wave radiating structures are very promising candidates for millimeter-wave band applications because they exhibit low losses, very flexible radiation characteristics, and excellent mechanical simplicity. Some currently used leaky-wave structures such as SIW antenna structures are well suited for a wide range of applications over millimeter-wave and terahertz frequency bands.

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