

Problems in Discreet Math

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December 26, 2024

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Chapter 1

What are proof and formal logic?

Well, before to approach about what is proof, in mathematic point of view, we would like to approach a something that is mixed with language and logic.

The reasons to make it is because the proof is made by words, but its are words supported by objective definitions and these definitions are built by words.

I hope that in this first chapter the reader can be able to understand the mechanism that divide between sentences, propositions and expressions.

1.1 Sentence, Proposition and Expression

In the first place, we would like to define "Sentence". The definition of a proposition is based on the concept of a sentence. However, this is a very difficult step and, at the same time, very important to make it easier to understand the other definitions that will follow.

Well, in other words, we want to say that having good interpretation skills is very important to start studying Mathematics.

We based the definitions about sentence in the following references [Las22], [Sri18], [MAR81] and [Rau10]

Definition 1.1.1 (Sentence). A sentence is: **In first place, put it all definitions about sentence that I could think or find in other references!**

- Expression of a complete thought. 1

The sentences can be the following:

- Affirmative
- Negative

- Imperative
- Exclamation
- Interrogative

Definition 1.1.2 (Open Sentence). A sentence is ...

Definition 1.1.3 (Close Sentence). A sentence is ...

Remark (Expression). The phrase that is not a sentence.

Definition 1.1.4 (Predicate). A predicate is...

Definition 1.1.5 (Proposition). A proposition is a statement (communication) that is either true or false. [Eri17], [Ger16], [Smu14] and [WR10]

Is not considered a proposition the following sentences:

- Interrogative
- Exclamation
- Open sentences
- Imperative sentences
- Without verb

So, what is the key to understand about proposition?? I gave above its definition, but I don't show you yet the mechanism that makes a given sentence to acquire its dichotomous nature.

Where we could to pay our attention to understand if a given sentence has or not dichotomous nature?

Well, in first place, we have to check if a given sentence is or not objective. That is, this sentence doesn't have any subjective feature. For example, if I give the following affirmation:

Example 1. *You are beautiful.*

The example above is a sentence, but is not a proposition. Why?

Because, the subject "You" is not defined.

Ok! So, we have to give an subject to this "You". So, if we affirm the following:

Example 2. *Wonder Woman is beautiful.*

Now, the sentence above became a proposition? No! So why?

Because there is a person who think she is beautiful and there is ones that doesn't agree!

Ok, ok! I got it. So, if everyone agrees the sentences become a proposition? For example, as following sentence:

Example 3 (A sentence that is subjective and everyone agree (this is very difficult)). *Thinking*

Finally, the sentence above is an proposition, right? NOOOOO!!! Is not a proposition, because same any person thinks that the affirmation above is right, we can't see, measure or observe to conclude universally that is true.

Well, an objective sentence is possible to verify, measure or observe? Well, every sentence where it has these feature is objective and is a proposition, but this still doesn't define a proposition.

So, why the sentences where it has features above is a proposition?

Because, the features that is possible to verify, measure or observe its conclusion is the same for any context. For example:

Example 4 (Feature that is possible to verify). *Thinking*

Example 5 (Feature that is possible to measure). *Thinking*

Example 6 (Feature that is possible to observe). *Thinking*

All those examples above is a proposition. But, this is not enough condition to be equivalent to the definition given about proposition. Because, there is an sentence that is objective and affirmative, but is not possible to verify, measure or observe, but is a proposition. Be careful it isn't going to confound with Axioms/Postulates, these have features mentioned above less to be true or false.

Example 7 (A sentence that is proposition, but is not possible to verify, measure or observe.). *Thinking*

Example 8 (Conjecture - An proposition that still not proof if is true or false). *Thinking*

Definition 1.1.6 (Mathematical proof). A mathematical proof of a proposition is a chain of logical deductions leading to the proposition from a base set of axioms.

1.1.1 Binary Connectives

1.1.2 Unary Connectives

Section 1.1 References

- [Eri17] Albert R Meyer Eric Lehman F Thomson Leighton. *Mathematics for Computer Science*. MCS, 2017.
- [Ger16] Judith L. Gersting. *Fundamentos matemáticos para a ciência da computação: Matemática Discreta e Suas Aplicações*. LTC, 2016.
- [Las22] Zolán Gyenis Laszlo Csirmaz. *Mathematical Logic, Exercises and Solutions*. Springer, 2022.
- [MAR81] WITOLD MARCISZEWSKI. *DICTIONARY OF LOGIC AS APPLIED IN THE STUDY OF LANGUAGE, Concepts/Methods/Theories*. Springer, 1981.
- [Rau10] Wolfgang Rautenberg. *A Concise Introduction to Mathematical Logic, Third Edition*. Springer, 2010.
- [Smu14] Raymound M. Smullyan. *A Beginner's Guide to Mathematical Logic*. Dover, 2014.
- [Sri18] Shashi Mohan Srivastava. *A Course on Mathematical Logic, Second Edition*. Springer, 2018.
- [WR10] Whitehead and Russell. *Principia Mathematica, Volume I*. Cambridge University Press, 1910.

1.2 Propositional Logic

Section 2 of the second chapter [exemplochapter1sec2] [Aut24b].

Referências da Seção 1.2

- [Aut24b] capítulo 1 Autor Artigo Secao 2. “Título do Artigo Exemplo”. In: *Revista Exemplo* 1.1 (2024), pp. 1–10.

Chapter 2

Second Chapter

Second Chapter...

2.1 First Section

Section 1 of the first chapter [Aut24d] [Aut24a]

Referências da Seção 2.1

[Aut24a] capítulo 2 Autor Artigo Secao 1. “Título do Artigo Exemplo”. In: *Revista Exemplo* 1.1 (2024), pp. 1–10.

[Aut24d] capítulo 2 Autor Exemplo Secao 1. *Título do Livro Exemplo*. Editora Exemplo, 2024.

2.2 Second Section

Section 2 of the second chapter [Aut24e] [Aut24c]

Referências da Seção 2.1

[Aut24c] capítulo 2 Autor Artigo Secao 2. “Título do Artigo Exemplo”. In: *Revista Exemplo* 1.1 (2024), pp. 1–10.

[Aut24e] capítulo 2 Autor Exemplo Secao 2. *Título do Livro Exemplo*. Editora Exemplo, 2024.