Week 5: Estimators and Estimation Methods, Nonlinear Regression, Quantile Regression

Advanced Econometrics 4EK608

Vysoká škola ekonomická v Praze

Outline

1. Estimators and estimation methods
   - Properties of estimators - repetition from BSc courses
   - Method of moments
   - Maximum likelihood estimation
2. Nonlinear regression models
3. Quantile regression

Estimators and estimation methods

Notation:

- θ - population parameter
- $(x_1, x_2, \ldots, x_n)$ - random sample of n observation of x
- $\hat{\theta} = \hat{\theta}(x_1, x_2, \ldots, x_n)$ is an estimator of θ

Basic notions:

- All estimators posses sampling distribution
  mean: $\mathbf{E}(\hat{\theta})$
  variance: $\mathbf{E}[(\hat{\theta} - \mathbf{E}(\hat{\theta}))^2]$
  etc.
- Estimators $\times$ estimate
- Many estimators exist for a parameter (population mean):

$$\hat{\theta}_1 = \overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\hat{\theta}_2 = \tilde{x} = \frac{1}{2}(x_{max} + x_{min})$$

Estimators and estimation methods

Small sample properties of estimators & definitions:

- **Unbiasedness**: the mean of sampling distribution equals the parameter being estimated
- **Efficiency**: an estimator is efficient if it is unbiased and no other unbiased estimator has a smaller variance. This is usually difficult to prove, that is why we simplify the concept:
  - Relative efficiency
  - Linear unbiased estimators instead of unbiased estimators (linear estimator is linear function of sample observations)

Estimators and estimation methods

Small sample properties of estimators & definitions:

Best Linear Unbiased Estimator (BLUE) is linear, unbiased and no other linear unbiased estimator has a smaller variance. It is not necessarily the best estimator.

- Non-linear estimators can be better
- Biased estimators can have smaller Mean Square Error: sum of variance and the squared bias

Large sample properties of estimators & definitions:

- Sampling distribution of an estimator changes with the size of sample.
- Asymptotic distribution for any estimator is that distribution to which the sampling distribution tends as the sample becomes larger. Its 1^{st} and 2^{nd} moments are asymptotic mean and asymptotic variance.
- When the sampling distribution collapses onto a single value when the sample becomes larger, we call this value probability limit. We say estimator converges in probability to that value

Large sample properties of estimators & definitions:

- Asymptotic unbiasedness

- **Consistency**
- Unbiased estimators are not necessarily consistent.
- If $\hat{\theta}$ is an unbiased estimator of θ and $\text{plim}(var(\hat{\theta})) = 0$ i.e. [$var(\hat{\theta}) \to 0$ as $n \to \infty$], then $\text{plim}(\hat{\theta}) = \theta$.
- Consistent estimators: unibased & their variance shrinks to zero as sample size grows (entire population is used).
  - Minimal requirement for estimator used in statistics or econometrics.
  - If some estimator is not consistent, then it does not provide estimates of population θ values, even with unlimited data.

Estimators and estimation methods

Large sample properties of estimators & definitions:

- Asymptotic efficiency: An estimator is asymptotically efficient if it is asymptotically unbiased and no other asymptotically unbiased estimator has smaller asymptotic variance.
- Asymptotic efficiency is usually difficult to prove, that is why we simplify the concept:
  - Relative asymptotic efficiency
  - Linear asymptotically unbiased estimators instead of asymptotically unbiased estimators

Method of moments

- With the method of moments, we simply estimate population moments by corresponding sample moments.
- Under very general conditions, sample moments are consistent estimators of the corresponding population moments, but NOT necessarily unbiased estimators.

Application example 1

Sample covariance is a consistent estimator of population covariance.

Application example 2

OLS estimators we have used for parameters in the CLRM can be derived by the method of moments.

Method of moments (MM)

Population moments, stochastic variable X

- $\mathbf{E}(X^r)$: r^{th} population moment about zero
- $\mathbf{E}(X)$: the population mean is the first moment about zero
- $\mathbf{E}[(X - \mathbf{E}(X))^2]$: the population variance is the second moment about the mean

Sample moments, sample observations $(x_1, x_2, \ldots, x_n)$

- $\frac{\sum_{i=1}^{n} x_i^r}{n}$: r^{th} sample moment about zero
- $\frac{\sum_{i=1}^{n} x_i}{n} = \overline{x}$: sample mean is the first moment about zero
- $\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n-1}$: sample variance is the second sample moment about the mean

Estimators and estimation methods

- In a LRM: $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \cdots + \hat{\beta}_k x_k$, the $k+1$ parameters are **OLS**-estimated by minimizing:

$$\sum_{i=1}^{n} \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_{i1} - \cdots - \hat{\beta}_k x_{ik} \right)^2 \tag{1}$$

- In MM, population moment assumptions $E(u) = 0$ and $E(x_j \cdot u) = 0$ are used for sample-based estimation (identical to 1^{st} order conditions for (1) - OLS is a type of MM estimator):

$$\sum_{i=1}^{n} \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_{i1} - \cdots - \hat{\beta}_k x_{ik} \right) = 0$$

$$\sum_{i=1}^{n} x_{i1} \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_{i1} - \cdots - \hat{\beta}_k x_{ik} \right) = 0$$

$$\cdots$$

$$\sum_{i=1}^{n} x_{ik} \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_{i1} - \cdots - \hat{\beta}_k x_{ik} \right) = 0$$

Estimators and estimation methods

- Let $h(\boldsymbol{w}) = h(y, \boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\theta})$ define a regression model, so that $Eh[(y_i, \boldsymbol{x}_i, \boldsymbol{z}_i, \boldsymbol{\theta})] = 0\,, \quad \forall i.$ where $\boldsymbol{z}$ is a set of r instruments (IVs) - see Week 8. For simplicity, we can start by assuming $\boldsymbol{x} \equiv \boldsymbol{z}$.
- **Method of moments estimator $\hat{\boldsymbol{\theta}}_{MM}$** minimizes:

$$\min_{\hat{\boldsymbol{\theta}}} : \left[\frac{1}{n}\sum_{i=1}^{n} h(\boldsymbol{w}_i, \hat{\boldsymbol{\theta}})\right]' \left[\frac{1}{n}\sum_{i=1}^{n} h(\boldsymbol{w}_i, \hat{\boldsymbol{\theta}})\right]$$

- If $\boldsymbol{x} \neq \boldsymbol{z}$ and # IVs > # regressors (overidentification), **Generalized method of moments** is used (GMM)

$$\min_{\hat{\boldsymbol{\theta}}} : \left[\frac{1}{n}\sum_{i=1}^{n} h(\boldsymbol{w}_i, \hat{\boldsymbol{\theta}})\right]' \boldsymbol{W}_n \left[\frac{1}{n}\sum_{i=1}^{n} h(\boldsymbol{w}_i, \hat{\boldsymbol{\theta}})\right]$$

  where $\boldsymbol{W}_n$ is a conveniently chosen $(r \times r)$ matrix. (any positive definite matrix that may depend on data but not on θ, e.g. I_r. Optimum $\boldsymbol{W}_n$: see e.g. Greene, chapter. 13.4.2)

MM - consistency conditions

- **Convergence of the moments:** Sample moments converge in probability to their population counterparts.
- **Identification:** Parameters are identified in terms of the moment equations.
  - **Order condition:** # IVs $\geq$ # model variables.
  - **Rank condition:** Moment equations are not redundant
  - Identification will be discussed in detail during Week 8
- **Limiting Normal distribution for the sample moments:** Population moments obey central limit theorem (CLT) or some similar variant.

MM - summary

- MM is robust to differences in "specification" of the data generating process (DGP). $\rightarrow$ i.e. sample mean or sample variance estimate their population counterparts (assuming they exist) regardless of DGP.
- MM is free from distributional assumptions.
- "Cost" of this approach: if we know the specific distribution of a DGP, MM does not make use of such information $\rightarrow$ inefficient estimates.
- Alternative approach: method of maximum likelihood utilizes distributional information and is more efficient (provided this information is valid).

Maximum likelihood estimator

Single θ parameter case:

- 1^{st} step: deriving a likelihood function $L = L(\theta, y_1, y_2, ..., y_n)$, where y_i is observation of Y (stochastic), θ is parameter of the distribution.
- 2^{nd} step: finding maximum of L with respect to θ, that maximum is $\tilde{\theta} = \theta_{MLE}$

With more parameters: $\boldsymbol{\theta} = (\theta_1, \dots, \theta_m)'$

$$L = L(\theta_1, \theta_2, ...\theta_m, y_1, y_2, ..., y_n)$$

We find MLEs of the m parameters by partially differentiating the likelihood function L with respect to each θ and then setting all the partial derivatives obtained to zero.

Estimators and estimation methods

Likelihood function:
$f(y_1, y_2, \ldots, y_n|\boldsymbol{\theta}) = \prod_{i=1}^{n} f(y_i|\boldsymbol{\theta}) = L(\boldsymbol{\theta}|\boldsymbol{y})$
where $f(y|\boldsymbol{\theta})$ is the pdf of y, conditioned on set of parameters $\boldsymbol{\theta}$.

Maximum likelihood estimation of CLRM parameters:

CLRM: $y_i = \alpha + \beta x_i + \varepsilon_i$ $\quad\quad$ $\mathbf{E}(y_i) = \alpha + \beta x_i = \boldsymbol{x}_i'\boldsymbol{\beta}$

$var(y_i) = var(\varepsilon_i) = \sigma^2$

Probability density function for **Normal distribution**:

$f(y|\boldsymbol{\theta}) = (2\pi\sigma^2)^{-0.5}\, exp[-(y-\mu)^2/2\sigma^2]$

In the case of CLRM, for each $y_i = \alpha + \beta x_i + \varepsilon_i$:

$f(y_i|\boldsymbol{x}_i, \boldsymbol{\theta}) = (2\pi\sigma^2)^{-0,5}\, exp[-(y_i - \mathbf{E}(y_i))^2/2\sigma^2]$, that is:
$f(y_i|\boldsymbol{x}_i, \boldsymbol{\theta}) = (2\pi\sigma^2)^{-0,5}\, exp[-(y_i - \boldsymbol{x}_i'\boldsymbol{\beta})^2/2\sigma^2]$

Estimators and estimation methods

Log-likehood (LL) function, **Normal distribution** assumed, estimation of CLRM parameters:

$$
\begin{aligned}
logL(\boldsymbol{\theta}|\boldsymbol{y},\boldsymbol{X}) &= \sum_{i=1}^{n} log[f(y_i|\boldsymbol{x}_i,\boldsymbol{\theta})] = \\
&= -\frac{1}{2}\sum_{i=1}^{n}\left\{log(2\pi)+log(\sigma^2)+\frac{1}{\sigma^2}[y_i-\boldsymbol{x}_i'\boldsymbol{\beta}]^2\right\} = \\
&= -\frac{n}{2}log(2\pi)-\frac{n}{2}log(\sigma^2)-\frac{1}{2\sigma^2}\sum_{i=1}^{n}[y_i-\boldsymbol{x}_i'\boldsymbol{\beta}]^2
\end{aligned}
$$

numerical iterative method is used for $\boldsymbol{\theta}=(\alpha,\beta,\sigma^2)$ estimation (by maximizing the log-likelihood function).
if CLRM assumptions hold $\Rightarrow$ MLE estimators $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\sigma}^2$ are identical to OLS-generated estimators $\hat{\alpha}$, $\hat{\beta}$ (and $\hat{\sigma}^2$).

Basic MLE assumptions

- **Parameter space:** Gaps and nonconvexities in parameter spaces would generally collide with estimation algorithms (settings such as $\sigma^2 > 0$ are OK).
- **Identifiability:** The parameter vector $\boldsymbol{\theta}$ is identified (estimable), if for two vectors, $\boldsymbol{\theta}^* \neq \boldsymbol{\theta}$ and for some data observations $\boldsymbol{x}$, $L(\boldsymbol{\theta}^*|\boldsymbol{x}) \neq L(\boldsymbol{\theta}|\boldsymbol{x})$.
- **Well-behaved data:** Laws of large numbers (LLN) apply. Some form of CLT can be applied to the gradient (i.e. for the estimation method).
- **Regularity conditions:** "well behaved" derivatives of $f(y_i|\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$ (see Greene, chapter 14.4.1).

MLE properties

- **Consistency:** $\text{plim}(\hat{\boldsymbol{\theta}}) = \boldsymbol{\theta}_0$ ($\boldsymbol{\theta}_0$ is the true parameter)
- **Asymptotic normality** of $\hat{\boldsymbol{\theta}}$
- **Asymptotic efficiency:** $\hat{\boldsymbol{\theta}}$ is asymptotically efficient and achieves the Cramér-Rao lower bound for consistent estimators (see Greene, chapter 14.4.5)
- **Invariance:** MLE of $\boldsymbol{\gamma}_0 = \boldsymbol{c}(\boldsymbol{\theta}_0)$ is $\boldsymbol{c}(\hat{\boldsymbol{\theta}})$ if $\boldsymbol{c}(\boldsymbol{\theta}_0)$ is a continuous and countinuously differentiable function.

MLE - summary

- MLE is only possible if we know the form of the probability distribution function for the population (Normal, Poisson, Negative Binomial, etc.).
- MLEs possess the large sample properties of consistency and asymptotic efficiency. There is no guarantee that they possess any desirable small-sample properties.
- Under CLRM assumptions, MLE estimator are identical to OLS estimators.
- MLE-related tests (Likelihood ratio, Wald, LM) will be discussed separately, with reference to a specific model type (e.g. LDVs in Weeks 11 to 13).

Nonlinear regression: linear vs. nonlinear models

Linear model:

$$y_i = \boldsymbol{x}_i'\boldsymbol{\beta} + \varepsilon_i$$

Nonlinear model:

$$y_i = h(\boldsymbol{x}_i\,,\,\boldsymbol{\beta}) + \varepsilon_i$$

- Linear model is a special case of the nonlinear model.
- Linear models are linear in parameters (encompass regressors such as x_i^2, etc.)
- Many nonlinear model may be transformed into linear models (log-transformation)
- For nonlinear models, nonlinear LS (NLS) are available.
- $\partial h(\boldsymbol{x}_i\,,\,\boldsymbol{\beta})/\partial \boldsymbol{x}$ is no longer equal to $\boldsymbol{\beta}$ (interpretation based on estimated model ...)

Assumptions of the nonlinear regression model

1. **Functional form:** The conditional mean function for y_i, given $\boldsymbol{x}_i$ is:

$$\mathbf{E}[y_i|\boldsymbol{x}_i] = h(\boldsymbol{x}_i\,,\,\boldsymbol{\beta})\;, \quad i = 1, 2, \ldots, n$$

2. **Identifiability of model parameters:** The parameter vector in the model is identified (estimable) if there is no nonzero parameter $\boldsymbol{\beta}^0 \neq \boldsymbol{\beta}$ such that $h(\boldsymbol{x}_i, \boldsymbol{\beta}^0) = h(\boldsymbol{x}_i, \boldsymbol{\beta})$ for all $\boldsymbol{x}_i$.

3. **Zero mean of the disturbance:** For $y_i = h(\boldsymbol{x}_i\,,\,\boldsymbol{\beta}) + \varepsilon_i$, we assume

$$\mathbf{E}[\varepsilon_i|h(\boldsymbol{x}_i\,,\,\boldsymbol{\beta})] = 0\;, \quad i = 1, 2, \ldots, n$$

i.e. disturbance at observation i is uncorrelated with the conditional mean function.

Assumptions of the nonlinear regression model

4 **Homoskedasticity and nonautocorrelation:**

conditional homoskedasticity:

$$\mathbf{E}[\varepsilon_i^2 | h(\boldsymbol{x}_i\,,\,\boldsymbol{\beta})] = \sigma^2, \quad i = 1, 2, \ldots, n$$

nonautocorrelation:

$$\mathbf{E}[\varepsilon_i \varepsilon_j | h(\boldsymbol{x}_i\,,\,\boldsymbol{\beta}), h(\boldsymbol{x}_j\,,\,\boldsymbol{\beta})] = 0, \quad \text{for all } i \neq j$$

Assumptions of the nonlinear regression model

5. **Data generating process:** DGP for $\boldsymbol{x}_i$ is assumed to be a well-behaved population such that first and second sample moments of the data can be assumed to converge to fixed, finite population counterparts. The crucial assumption is that the process generating $\boldsymbol{x}_i$ is strictly exogenous to that generating ε_i
6. **Underlying probability model** There is a well-defined probability distribution generating ε_i. At this point, we assume only that this process produces a sample of uncorrelated, identically (marginally) distributed random variables ε_i with mean zero and variance σ^2 conditioned on $h(\boldsymbol{x}_i, \boldsymbol{\beta})$. Thus, at this point, our statement of the model is **semi-parametric**.

Nonlinear Regression: NLS

NLS: estimator of the nonlinear regression model

- NLS: min: $S(\boldsymbol{\beta}) = \sum[y_i - h(\boldsymbol{x}_i, \boldsymbol{\beta})]^2$
- Using standard procedure, we can get k first order conditions for the minimization:

$$\frac{\partial S(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}} = \sum_{i=1}^{n} [y_i - h(\boldsymbol{x}_i, \boldsymbol{\beta})] \frac{\partial h(\boldsymbol{x}_i\,, \boldsymbol{\beta})]}{\partial \boldsymbol{\beta}} = \mathbf{0}$$

- The above first order conditions are also moment conditions and this defines the NLS estimator as a GMM estimator.

NLS: estimator of the nonlinear regression model

- NLS being a GMM estimator allows us to deduce that the NLS estimator has good large sample properties: consistency and asymptotic normality (if assumptions are fulfilled).
- Hypothesis testing: The principal testing procedure is the Wald test, which relies on the consistency and asymptotic normality of the estimator. Likelihood ratio and LM tests can also be constructed.

Nonlinear regression: computing NLS estimates

For nonlinear models, a closed-form solution (NLS estimator) usually does not exist.

- Most of the nonlinear maximization problems are solved by an **iterative algorithm**.
- The most commonly used of iterative algorithms are **gradient methods**.
- The template for most gradient methods in common use is the **Newton's method**.
- Look at your software packages which methods are available for computing NLS estimates.

Examples 7.4 & 7.8 (Greene):

Analysis of a Nonlinear Consumption Function

NLS with starting values equal to 0

NLS with starting values equal to the parameters from the OLS estimation (c(3) equal to 1).

Depednent Variable: REALCONS
Method: Least Squares (Marquard - EViews legacy)
Date: 09/19/16 Time 16:31
Sample 1950Q1 2000Q4
Included observations: 204
REALCONS=C(1)+C(2)*REALDPI

	Coeficient	Std.Error	t-Statistic	Prob.
C(1)	-80.35475	14.30585	-5.616915	0.0000
C(2)	0.921686	0.003872	238.0540	0.0000

R-squared	0.996448	Mean dependent var	2999.436
Adjusted R-squared	0.996431	S.D. dependent var	1459.707
S.E. of regression	87.20983	Akaike info criterion	11.78427
Sum squared resid	1536322	Schwarz criterion	11.81680
Log likelihood	-1199.995	Hannan-Quinn criter.	11.79743
F-statistics	56669.72	Durbin-Watson stat	0.092048
Prob(F-statistics)	0.000000		

Examples 7.4 & 7.8 (Greene):
Analysis of a Nonlinear Consumption Function

Depednent Variable: REALCONS
Method: Least Squares (Marquard - EViews legacy)
Sample 1950Q1 2000Q4 Included observations: 204
Convergence achieved after 200 iterations
REALCONS=C(1)+C(2)*REALDPI^C(3)

	Coeficient	Std.Error	t-Statistic	Prob.
C(1)	458.7991	22.50140	20.38980	0.0000
C(2)	0.100852	0.010910	9.243667	0.0000
C(3)	1.244827	0.012055	103.2632	0.0000

R-squared	0.998834	Mean dependent var	2999.436
Adjusted R-squared	0.998822	S.D. dependent var	1459.707
S.E. of regression	50.09460	Akaike info criterion	10.68030
Sum squared resid	504403.2	Schwarz criterion	10.72910
Log likelihood	-1086.391	Hannan-Quinn criter.	10.70004
F-statistics	86081.29	Durbin-Watson stat	0.295995
Prob(F-statistics)	0.000000		

Nonlinear regression: computing NLS estimates

Examples 7.4 & 7.8 (Greene):
Analysis of a Nonlinear Consumption Function

Depednent Variable: REALCONS
Method: Least Squares (Marquard - EViews legacy)
Sample 1950Q1 2000Q4 Included observations: 204
Convergence achieved after 80 iterations
REALCONS=C(1)+C(2)*REALDPI^C(3)

	Coeficient	Std.Error	t-Statistic	Prob.
C(1)	458.7989	22.50149	20.38971	0.0000
C(2)	0.100852	0.010911	9.243447	0.0000
C(3)	1.244827	0.012055	103.2632	0.0000

R-squared	0.998834	Mean dependent var	2999.436
Adjusted R-squared	0.998822	S.D. dependent var	1459.707
S.E. of regression	50.09460	Akaike info criterion	10.68030
Sum squared resid	504403.2	Schwarz criterion	10.72910
Log likelihood	-1086.391	Hannan-Quinn criter.	10.70004
F-statistics	86081.28	Durbin-Watson stat	0.295995
Prob(F-statistics)	0.000000		

Quantile regression (QREG)

- Quantile regression estimates the relationship between regressors and a specified quantile of dependent variable.
- The (linear) quantile model can be defined as $Q[y|\boldsymbol{x}, q] = \boldsymbol{x}'\boldsymbol{\beta}_q$, such that $\text{Prob}[y \leq \boldsymbol{x}'\boldsymbol{\beta}_q|\boldsymbol{x}] = q$, $0 < q < 1$ where q denotes the q-th quantile.
- One important special case of quantile regression is the least absolute deviations (LAD) estimator, which corresponds to fitting the conditional median of the response variable ($q = \frac{1}{2}$).
- QREG (LAD) estimator can be motivated as a robust alternative to OLS (with respect to outliers).

Quantile regression (QREG)

For LRMs, the q-th quantile regression estimator $\boldsymbol{\beta}_q$ minimizes:

$$\min_{\hat{\boldsymbol{\beta}}_q}: \quad Q_n(\hat{\boldsymbol{\beta}}_q) = \sum_{i:\, e_i \geq 0}^{n} q|y_i - \boldsymbol{x}_i\hat{\boldsymbol{\beta}}_q| + \sum_{i:\, e_i < 0}^{n} (1-q)|y_i - \boldsymbol{x}_i\hat{\boldsymbol{\beta}}_q|,$$

where $e_i = (y_i - \boldsymbol{x}_i\hat{\boldsymbol{\beta}}_q)$.

- We use the notation $\hat{\boldsymbol{\beta}}_q$ to make clear that different choices of q lead to different $\hat{\boldsymbol{\beta}}$.
- Slope of the loss function Q_n is asymmetrical (around $e_i = 0$).
- The loss function is not differentiable (at $e_i = 0$) $\rightarrow$ gradient methods are not applicable (linear programming can be used).

Quantile regression - LAD

- LAD estimator is the QREG for $q = \frac{1}{2}$ (median) and the loss function simplifies to:

$$\min_{\hat{\boldsymbol{\beta}}_q}: \quad Q_n(\hat{\boldsymbol{\beta}}_q) = \sum_{i=1}^{n} |y_i - \boldsymbol{x}_i \hat{\boldsymbol{\beta}}_q|$$

- LAD estimator predates OLS (itself older than 200 years). Until recently, QREG and LAD have seen little use in econometrics, as OLS is vastly easier to compute.
- Different software packages use a variety of optimization algorithms for QREG/LAD estimation.
- Linear programming can be used for finding QREG estimates (Koenkerr and Bassett (around 1980).

Quantile regression

QREG coefficient interpretation example:

(1) $\text{wage}_i = \beta_0 + u_i$

(2) $\text{wage}_i = \beta_0 + \beta_1 \text{female}_i + u_i$

(3) $\text{wage}_i = \beta_0 + \beta_1 \text{female}_i + \beta_2 \, \text{exper}_i + u_i$

The above equations are estimated by OLS / LAD / QREG:

Coefficient	OLS	LAD ($q = \frac{1}{2}$)	QREG ($q = \frac{3}{4}$)
(1) β_0	$\hat{\beta}_0 = \overline{y}$ sample mean	$\hat{\beta}_0 = \tilde{y}$ sample median	$\hat{\beta}_0 = Q_3$ sample 3^{rd} quartile
(2) $\beta_0, \beta_0 + \beta_1$	conditional sample mean wage: male / female	cond. sample median wage: male / female	conditional sample Q_3 wage: male / female
(3) β_2	change in expected mean wage for $\Delta\text{exper} = 1$	change in exp. median wage for $\Delta\text{exper} = 1$	change in expected Q_3 wage for $\Delta\text{exper} = 1$

Quantile regression example

Example 7.9 (Greene):

Cobb-Douglass Production Function

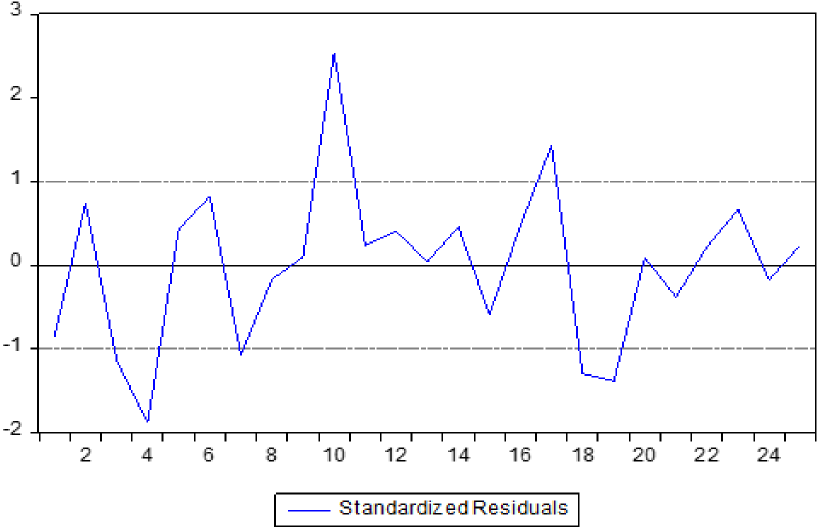
OLS → Standardized residuals indicate two outliers → LAD

Depednent Variable: LNYN
Method: Least Squares
Sample 1 25
Included observations: 25

Variable	Coeficient	Std.Error	t-Statistic	Prob.
C	2.293263	0.107183	21.39582	0.0000
LNKN	0.278982	0.080686	3.457639	0.0022
LNLN	0.927312	0.012055	9.431359	0.0000
R-squared	0.959742	Mean dependent var		0.771734
Adjusted R-squared	0.956082	S.D. dependent var		0.899306
S.E. of regression	0.188463	Akaike info criterion		-0.387663
Sum squared resid	0.781403	Schwarz criterion		-0.241398
Log likelihood	7.845786	Hannan-Quinn criter.		-0.347095
F-statistics	262.2396	Durbin-Watson stat		1.937830
Prob(F-statistics)	0.000000			

Quantile regression example

Example 7.9 (Greene): Cobb-Douglass Production Function



Example 7.9 (Greene):
Cobb-Douglass Production Function
(results differ from the textbook)

Depednent Variable: LNYN   Method: Quantile Regression (Median)
Sample 1 25   Included observations: 25
Huber Sandwich Standard Errors & Covariance
Sparsity method: Kemel (Epanechnikov) using residuals
Bandwidth method: Hall-Sheather, bw=0.33227
Estimation successfully identifies unique optimal solution

Variable	Coeficient	Std.Error	t-Statistic	Prob.
C	2.275038	0.179268	12.69071	0.0000
LNKN	0.260365	0.122447	2.126351	0.0449
LNLN	0.927243	0.152593	6.076572	0.0000

Pseudo R-squared	0.794575	Mean dependent var	0.771734
Adjusted R-squared	0.775900	S.D. dependent var	0.899306
S.E. of regression	0.190505	Objective	1.627051
Quantile dependent va...	0.966677	Restr. objective	7.920415
Sparsity	0.594465	Quasi-LR statistic	84.69274
Prob(Quasi-LR stat)	0.000000		

Example 7.10 (Greene):
Income Elasticity of Credit Cards Expenditure
OLS & LAD & Income elasticity at different deciles

Depednent Variable: LOGSPEND
Method: Least Squares
Date: 09/15/16 Time 13:53
Sample (adjusted): 3 13443
Included observations: 10499 after adjustments

Variable	Coeficient	Std.Error	t-Statistic	Prob.
C	-3.055807	0.239699	-12.74852	0.0000
LOGINC	1.083438	0.032118	33.73296	0.0000
AGE	-0.017364	0.001348	-12.88069	0.0000
ADEPCNT	-0.044610	0.010921	-4.084857	0.0000
R-squared	0.100572	Mean dependent var		4.728778
Adjusted R-squared	0.100315	S.D. dependent var		1.404820
S.E. of regression	1.332496	Akaike info criterion		3.412366
Sum squared resid	18634.35	Schwarz criterion		3.415131
Log likelihood	-17909.21	Hannah-Quinn criter.		3.413300
F-statistic	391.1750	Durbin-Watson stat		1.888912
Prob(F-statistic)	0.000000			

Example 7.10 (Greene):
Income Elasticity of Credit Cards Expenditure (LAD)

Depednent Variable: LOGSPEND   Method: Quantile Regression (Median)
Sample (adjusted): 3 13443   Included observations: 10499 after adjustments
Huber Sandwich Standard Errors & Covariance
Sparsity method: Kemel (Epanechnikov) using residuals
Bandwidth method: Hall-Sheather, bw=0.04437
Estimation successfully identifies unique optimal solution

Variable	Coeficient	Std.Error	t-Statistic	Prob.
C	-2.803756	0.233534	-12.00577	0.0000
LOGINC	1.074928	0.030923	34.76139	0.0000
AGE	-0.016988	0.001530	-11.10597	0.0000
ADEPCNT	-0.049955	0.011055	-4.518599	0.0000

Pseudo R-squared	0.058243	Mean dependent var	4.728778
Adjusted R-squared	0.057974	S.D. dependent var	1.404820
S.E. of regression	1.346476	Objective	5096.818
Quantile dependent va...	4.941583	Restr. objective	5412.032
Sparsity	2.659971	Quasi-LR statistic	948.0224
Prob(Quasi-LR stat)	0.000000		

Quantile regression example 2

Example 7.10 (Greene):
Income Elasticity of Credit Cards Expenditure