Problem Set 1
Convex Optimization

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Ref.	Exercises
[1]	2.2, 2.12 (d)(e)(g), 2.18, 2.24, 3.3, 3.18, 3.19(a)

Matlab Assignment

Problem 1. Let the set $\mathcal{S}$ be described by,

$$\mathcal{S} = \left\{(y_1, y_2) | 2x_1^4 + x_2^4 + y_1 x_1 x_2^3 + y_2 x_1^3 x_2 \geq 0 \;\; \forall (x_1, x_2) \in \mathbb{R}^2\right\}.$$

(a) Use Matlab to draw the set $\mathcal{S}$ and investigate its convexity.

(b) Show your conclusion in part (a) theoretically.

Problem 2. Let Q_1 and Q_2 be arbitrary $n \times n$ symmetric matrices ($n > 2$).

(a) Use Matlab to draw the set $\mathcal{S} = \left\{\left(x^T Q_1 x, x^T Q_2 x\right) \mid \|x\|_2 = 1\right\}$ and investigate its convexity.

(b) Extra point: Can you show your conclusion in part (a) theoretically?

Problem 3. Let A be a real $m \times n$ matrix with a singular value decomposition given by $A = U\Sigma V^T$ (as discussed in class). For a positive integer $k \leq \min\{m, n\}$, we let A_k denote an $m \times n$ matrix which is an "approximation" of the matrix A obtained from its top k singular values and singular vectors, i.e.,

$$A_k = U_k \Sigma_k V_k^T,$$

where U_k has the first k columns of U, V_k has the first k columns of V , and Σ_k is the upper left $k \times k$ block of Σ.

(a) To provide a good approximation for A, consider the cost function $\|A - X\|_2$ where X is restricted to be an $m \times n$ matrix with $\text{rank}(X) \leq k$. It can be shown that A_k is the minimizer of the cost function $\|A - X\|_2$. Download the file HajiFirouz.jpg. Read this file in Matlab by typing:

```
A=imread('HajiFirouz.jpg');
A=im2double(A) ;
A=rgb2gray(A) ;
```

Figure 1: Haji Firouz in Problem 8

The result is a 395×665 matrix A, with each entry representing a single pixel in the picture with a number between 0 and 1.
For different values of k, use Matlab to compute A_k, construct a compressed image with A_k (You can used the command `imwrite`), and report the value of $\|A - A_k\|_2$.

(b) Based on your experiments in part (a), provide a good compressed image for HajiFirouz and explain your interpretations.

References

[1] Boyd, Stephen, Stephen P. Boyd, and Lieven Vandenberghe. Convex optimization. Cambridge university press, 2004.