Project problems for IE426, Fall 2014

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Restaurant scheduling

After faithfully serving the O.R. profession for 50 years, you decide to retire and open a restaurant. Among the hundreds of details with opening a restaurant, you need to hire and schedule employees. Based on the foot traffic of other restaurants in the area, you expect that you will need the following number of employees each day:

Employees Schedule

Day of week	Employees Needed
Monday	4
Tuesday	5
Wednesday	5
Thursday	10
Friday	12
Saturday	12
Sunday	2

Your employees will work four consecutive days and then have three days off. They will be paid $100 for each day they work. In your rush to get the restaurant started, you haphazardly hire 17 employees. Five will start on Monday, five will start on Thursday and seven will start on Friday. This schedule satisfies the above work requirements, but you have no idea how optimal this is.

Questions:

1. How much money would you save each week from your current schedule if you optimized your workforce?
2. How much additional money would you save or lose each week if you switched your employees to a "five days on, two days off" schedule at $80 per day?

Combination Locks

Figure 1

Many people store their valuables in home safes because they protect against burglaries and fires. They are a good place for storing insurance documents, car titles, cash, and many other valuables.

Figure 1 shows six dials that are on the front of your home safe. In order to open the safe, you must set each of the dials to one number. When the correct numbers are selected on each dial, the safe will open. Unfortunately you have forgotten the combination. All you can remember is that the numbers on all of the dials summed to 419.

Question: What numbers should you select in order to unlock the safe?

Farmer

As a farmer, the decisions you make when planting crops influence whether your year will end with significant profits or in bankruptcy. As the owner of 1000 acres, you need to make the following decisions for your upcoming planting season:

1.) Plant Corn or Soybeans
2.) Buy crop insurance
3.) Use fertilizer

Corn has a profit of $190 per acre but you will lose $190 per acre if your crop fails. Corn fertilizer costs $30,000 and Corn Insurance costs $35,000. Soybeans have a profit of $170 per acre but you will lose $170 per acre if your crop fails. Soybean fertilizer costs $10,000 and soybean insurance costs $20,000.

The table below shows the probability of reaping a full harvest for either crop. For simplicity, assume that you will either successfully reap all 1000 acres or there will be total crop failure.

	Probability of reaping a full harvest (%)
Buy crop insurance and use fertilizer	100%
Use fertilizer only	95%
Buy crop insurance only	90%
Do not buy insurance and do not use fertilizer	85%

Question: What is your expected profit when the best decisions are made?

Fast food

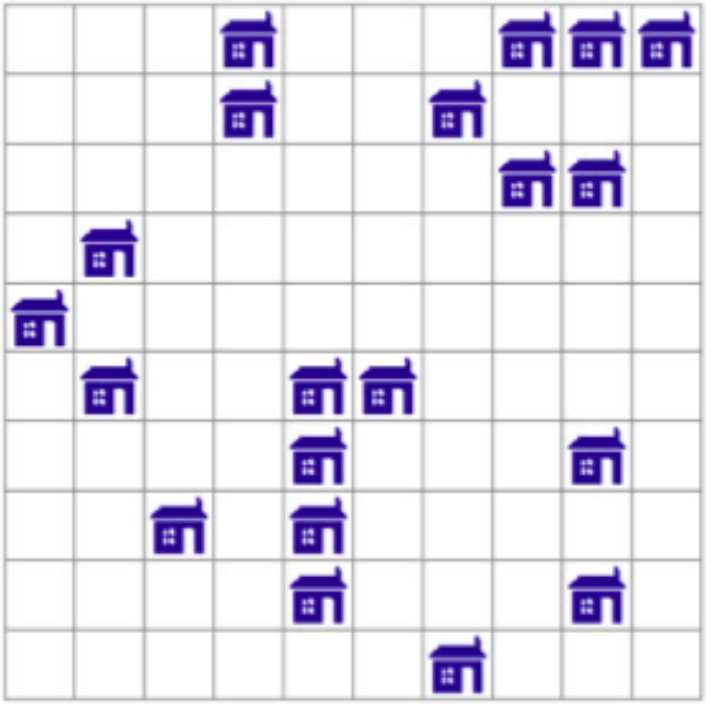


Figure 1

Deciding how many fast food restaurants to build in a town takes careful planning. Building too many will result in wasted capital and building too few will result in lost business.

The map in Figure 1 shows the locations of 20 homes in a small town. Sadly, there are no McEverywhere restaurants where the residents can eat. As the planner for McEverywhere corporation, you have been asked to build restaurants so that no resident has to travel more than 4km to reach a restaurant. You can build as many restaurants as you like and restaurants can be built on any cell (including one that has a home on it).

Use a direct line between cells to calculate travel distance. The distance between two adjacent cells is 1km and the distance between two diagonal cells is 1.41 km.

Question: What is the minimum number of McEverywhere restaurants needed so that no resident has to travel more than 4km to reach one?

Lost ship

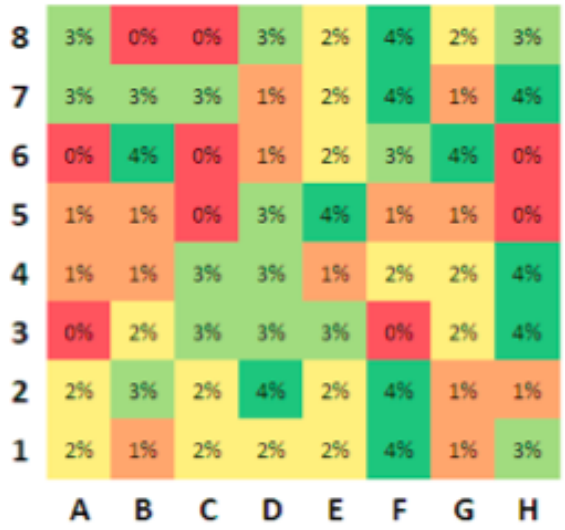


Figure 1

Searching for a lost ship at sea is a time sensitive task that requires skill and urgency. Finding a lost ship quickly means the difference between life and death.

The map in Figure 1 shows a section of ocean divided into 64 cells. Somewhere in this grid, a ship has been lost. Each cell has a number that represents the probability of finding the lost ship when that cell is searched (based on last known position, ocean currents, and debris sightings). For example, if you searched cell A1, you would have a 2% chance of finding the lost ship there.

As the leader of the search and rescue team, your goal is to find the ship with all survivors. Unfortunately it takes you 1 day to search a cell and the lost sailors have only enough food and water to survive for 10 days. This allows you to search a total of 10 cells before the lost sailors perish.

You may start your search in any of the 64 cells. You are only allowed to move to adjacent cells (you cannot move diagonally) and you are not allowed to revisit any cells. Add up the percentages in the 10 cells you have searched to get the probability of finding the lost ship.

Question: What is the greatest probability of finding the lost ship?

Popsicle scheduling

Workforce management is central to efficient operations and good customer service. Proper scheduling of employees can mean the difference between profitability and business failure.

As the manager of a popsicle stand, you are required to hire and set the weekly work schedule for your employees. The required staffing levels for the week are as follows. Total employees required: Monday=5, Tuesday=7, Wednesday=7, Thursday=10, Friday=16, Saturday=18, Sunday=12. Assume the same staffing requirements continue week after week.

Full Time employees work 5 consecutive days and earn $100 per day. Part Time employees work 2 consecutive days and earn $150 per day.

Question: What is the minimal weekly staffing cost you can achieve while meeting the required staffing levels?

Voyage Planning

	Skills			
Name	Fishing	Sailing	Navigation	Salary
Amy	3	5	1	$46,000
Bill	1	2	5	$43,000
Carl	3	4	2	$47,000
Dan	4	3	1	$36,000
Eva	4	2	2	$43,000
Fred	1	3	4	$55,000
Greg	3	1	5	$68,000
Henry	5	4	2	$64,000
Ida	3	3	3	$60,000

Table 1

Successfully navigating the waters during sea voyages is a challenging task. A captain's most important decision is selecting the right crew for the voyage. A mix of different skill sets are required to sail the ship efficiently, navigate to the destination, and fish for food along the way.

Table 1 shows a list of crew members that are available for you to hire for the voyage. Each crew member demands a salary for the voyage and has different skill levels of Fishing, Sailing, and Navigation.

In order for your journey to be successful, you must have a cumulative skill of 15 or more in each of the three skill categories from all of your chosen crew members. You may choose as many crew members as you like.

Question: What is the minimum achievable cost for the voyage?

Workforce scheduling

The QED Company must draw up a production program for the next nine weeks. Jobs last weeks and once started must be carried out without interruption. During each week a certain number of skilled workers are required to work full time on the job. Thus is a job i lasts p_i weeks, $l_{i,u}$ workers are required in week u for $u = 1, \ldots, p_i$.The total number of workers available in week t is L_t. Typical job data is shown below

Job	Length	Week 1	Week2	Week 3	Week 4
1	3	2	3	1	-
2	2	4	5	-	-
3	4	2	4	1	5
4	4	3	4	2	2
5	3	9	2	3	-

- Formulate the problem of finding a feasible schedule as an IP.
- Formulate the problem to minimize the maximum number of workers used each week.
- Solve each problem with AMPL/CPLEX.

Nurse staff scheduling

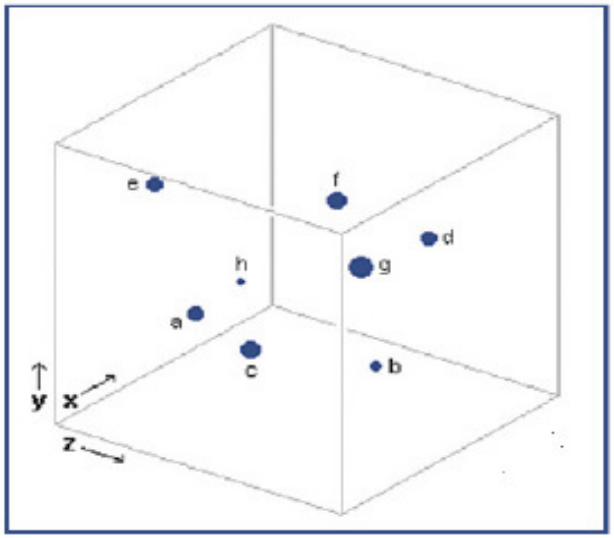
To provide adequate medical service to its constituents at a reasonable cost, hospital administrators constantly seek ways to hold staff level as low as possible while maintaining sufficient staffing to provide satisfactory levels of health care. An urban hospital has three departments: the emergency room (Department 1), the neonatal intensive care nursery (Department 2) and the orthopedics (Department 3). The hospital has three work shifts, each with different level of necessary staffing for nurses. The hospital would like to identify the minimum number of nurses required to meet the following three constraints: (1) the hospital must allocate at least 13, 32 and 22 nurses to the three departments (over all shifts); (2) the hospital must assign at least 26, 24 and 19 nurses to the three shifts (over all departments); and (3) the minimum and maximum number of nurses allocated to each department in a specific shift must satisfy the following limits:

Shift \ Depart	1	2	3
1	(6,8)	(11,12)	(7,12)
2	(4,6)	(11,12)	(7,12)
3	(2,4)	(10,12)	(5,7)

- Identify the approach to solve the problem as a maximum flow problem to identify the minimum number of nurses required to satisfy the constraints.
- Solve each problem with AMPL/CPLEX.

April 2008 - Traveling Spaceman Problem

Below is a three-dimensional map of the universe containing nine galaxies that you, as the traveling spaceman, wish to visit. Each galaxy's position in the universe is indicated by the accompanying table.



COORDINATES			
Galaxy	**x**	**y**	**z**
a	26	38	30
b	75	6	55
c	3	46	66
d	73	59	75
e	37	72	7
f	42	83	67
g	21	77	91
h	80	18	4

Questions:

1. Starting (and ending) at galaxy 'a,' in what order should you visit each galaxy to minimize the traveled distance? You must visit each galaxy and you cannot visit any galaxy more than once.
2. What is the total distance traveled

Relief Mission

	1	2	3	4	5	6	7	8	9	10
A					■					
B					■				■	■
C	■					■		■	■	■
D		■					■			■
E		■							■	
F								■		
G		■								
H		■				■				
I										
J								■		■

Figure 1

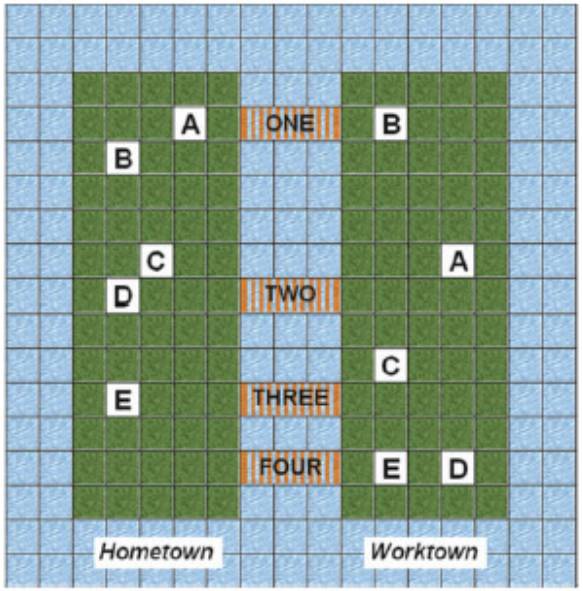
Coordinating relief efforts after catastrophes such as civil unrest and natural disasters can be a logistically complex challenge. Delivering relief to people in need is the immediate focus of any disaster management plan.

The map in Figure 1 shows the locations of 20 villagers, each represented by a 'hut' icon. The villagers are in need of relief supplies contained in the crates attached to parachutes. There are two identical relief packages available. The only delivery option is by air drop. Each package can be dropped on any cell.

After the crates are dropped, each villager will walk to the nearest drop location to pick up relief supplies. Use a direct line between cells to calculate travel distance. For example, the distance between A1 and A2 is 1km and the distance between A1 to B2 is 1.41 km. Assume that each crate contains an unlimited amount of relief supplies.

Question: Which two drop locations will minimize the total distance that all villagers must travel?

Bridges to Somewhere - August 2009



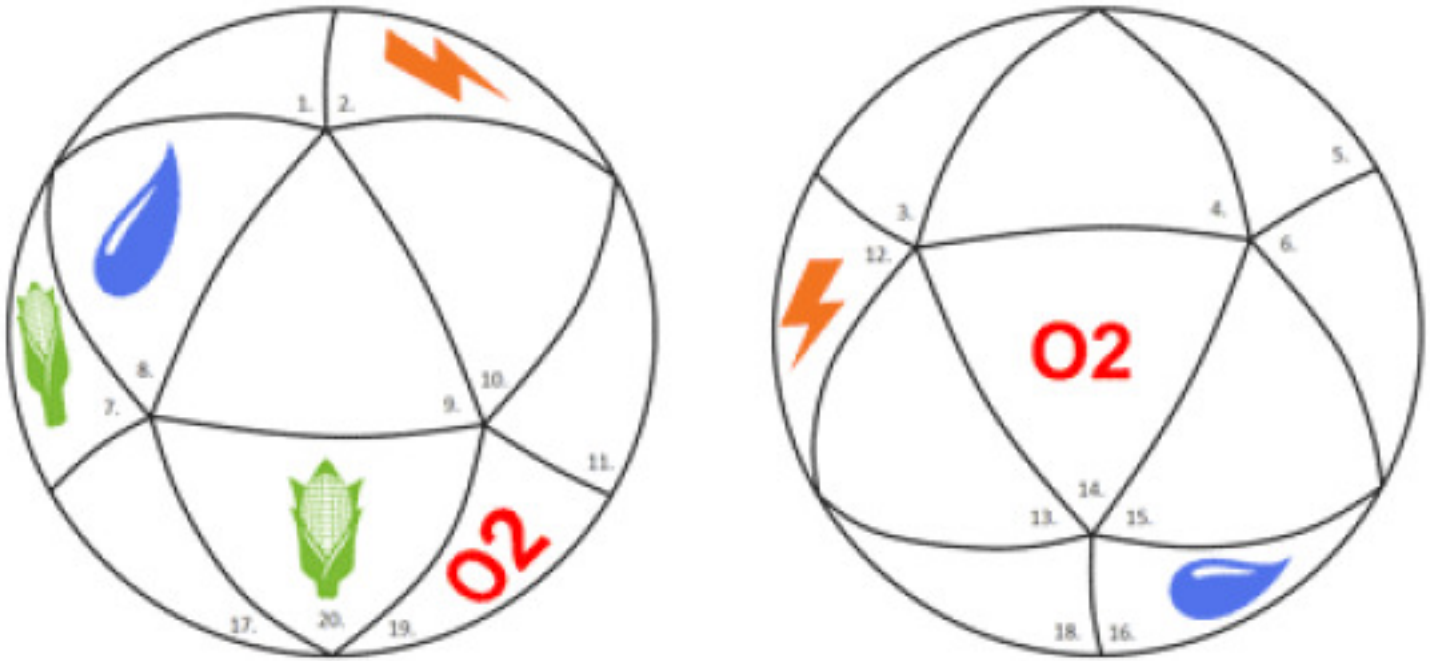
Question: which two bridges should be build in order to minimize the total commuting distance of all residents.

The five residents of Hometown live in houses represented by the letters "A" through "E" as shown on the left side of Figure 1. The offices where they will be working are represented by their matching letters on the island of Worktown.

Because a river lies between Hometown and Worktown, the residents are unable to get to work. They have in their budget enough funds to build two bridges that could connect Hometown to Worktown. The locations where these bridges could be built are indicated by the brown 1x3 hashed tiles. The two bridges can only be built in these approved areas.

Once the bridges are built, the residents would then be able to commute to work. A commuter will always take the shortest path from home to work and can only travel in up, down, left or right directions (no diagonals). Each tile represents a 1-km-by-1-km distance. As an example, if bridge four were built, resident "E" would have to travel 10 km to reach his workplace.

Planet colonization



Colonizing a new planet where conditions are unpredictable and harsh is never easy. The primary concern when selecting a landing site for the first colony is the proximity to natural resources. Access to these valuable resources determines whether or not a new colony thrives or dies.

Figure 1 shows the front and rear views of a newly discovered planet. The planet has been divided into 20 areas, each of which is a potential landing site to establish a new colony. Some areas contain valuable resources (represented by colored icons) that are needed in order for the new colony to survive. Food is represented by the green ear of corn, Oxygen by the red O2, water by the blue drop, and energy by the orange lightning bolt.

When choosing a landing site, it is best to minimize the distance between that site and the four needed resources. Distance is calculated by the number of units it takes to get to the resource. For example, if area 9 were selected as the landing site, the total travel distance required to reach all four resources would be 6 units (2 for Energy, 1 for Food, 1 for Water, and 2 for Oxygen).

Question: Which of the 20 areas is the best landing site to minimize the total distance you would have to travel to all four resources?

“Google” problem

The task is to assign integers to the letters $W, D, O, T, G, L, E, C, M$, so the following subtraction is true:

$$\begin{array}{cccccc} W & W & W & D & O & T \\ - \; G & O & O & G & L & E \\ \hline D & O & T & C & O & M. \end{array}$$

To different letters you must assign different numbers.

Model this as an integer program, and write an AMPL code to solve it.

Pipe problem

A company has a series of pipelines laid under the factory floor. The floor consists of a series of heavy square slabs. It is desired to inspect each pipeline and this can be done if the company lifts **one** of the slabs directly above each pipeline. The layout of the slabs is as shown:

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36

with pipelines lying under the five slab groups {2,3,4},{7,16,17,18}, {10,11,12},{13,14},{25,34,33,32}.
When a slab is lifted it suffers damage, as do all other slabs that touch it (unless they touch diagonally). The company wishes to minimize the total number of slabs damaged when all pipelines have been inspected.

Formulate and solve this problem using AMPL.

Sudoku

Write a MIP in AMPL to solve sudoku puzzles. Go to the webpage

`http://www.websudoku.com/`

and run your model on one problem of each difficulty level: easy, medium, hard, evil.

You need to turn in one model file, and 4 data files.

Warewolf

You are visiting a magical forest in which every inhabitant is either a knight or a knave (but not both). Knights always tell the truth and knaves always lie. In addition some of the inhabitants are werewolves and have the annoying habit of sometimes turning into wolves at night and devouring people. A werewolf can be either a knight or a knave. You are interviewing three inhabitants, 1, 2, and 3, and it is known that exactly one of them is a werewolf. They make the following statements:

1. I am a werewolf.
2. I am a werewolf.
3. At most one of us is a knight.

Set up an integer program whose feasible solutions will tell which inhabitant is what. Make sure that your IP formulation is 1) complete, i.e. it incorporates all the available information; 2) minimal, i.e. it does not use more constraints than necessary.

Note that with some logical reasoning we might as well figure out what the only feasible solution is! The point of the IP formulation is to use the smallest amount of logical reasoning to set up the constraints, then leave the rest to the IP solver.

Wine

Five married couples each decided to celebrate with a bottle of wine last week. Each couple had wine on a different night. Five different wines were used. The men are: Carl. Philip, Ray, Roland, and Simon. The women are: Kathy, Margaret, Marie, Olive and Vanessa. The wines are: Chianti, Liebfrauenmilch, Riesling, Soave and Spumante. The nights are: Monday to Friday. We have the following information available:

- Philip is married to Marie. They did not have wine on Wednesday night. Carl had wine on Wednesday night.
- The Soave was not drunk on Friday night, nor was this wine drunk by Simon.
- Simon and his wife had a bottle of wine the night after the couple who had the Spumante, but two nights after Margaret and her husband had wine.
- Kathy did not have wine on Tusday night, but she was the person who had the Chianti.
- Olive and her husband, who is not Ray, enjoyed their wine on Friday.

1. Write an IP whose *feasible* solutions tell us, which 4-tuples of men, women, wines, and nights are possible. Code it in AMPL, and find a feasible solution.

   Make sure that your variables are indexed by names, NOT numbers. Say, if you want $x[i,j]$ to be 1, iff man i is married to woman j, then the domain of i should NOT be $\{1,\ldots,5\}$, but Carl, Philip, Ray, Roland, Simon. Similarly for the women, days, and wines. See the file steel2.dat.

2. Among all possible feasible solutions find the one in which Margaret had wine the *latest* (i.e. on a day closest to Friday).

3. Among all possible feasible solutions find the one in which the Spumante was drunk the *latest* (i.e. on a day closest to Friday).

Multiobjective Showcase Scheduling

Dance-pair	Student	Teacher	Skill
1	Daniel	Mr. Brown	2
2	Camila	Mr. Davis	4
3	Brianna	Ms. Evans	2
4	Eve	Ms. Clark	3
5	Ava	Ms. Anderson	1
6	Camila	Ms. Clark	5
7	Ava	Mr. Davis	3
8	Eve	Ms. Evans	1
9	Camila	Ms. Anderson	4
10	Brianna	Mr. Davis	3

Table 1

A popular dance studio in New York City holds ballroom dancing showcases twice a year to provide its students with an environment for socializing, practice, and improvement. A showcase consists of several heats in which multiple dance-pairs dance at the same time. Because multiple objectives are desired to maximize the quality of the showcase, scheduling the dance-pairs becomes a complex problem that requires OR techniques to solve.

The objectives, when scheduling a showcase, are to minimize the number of heats (in order to minimize the overall duration of the showcase), group similarly skilled dance-pairs in the same heat, and minimize the number of heats that have only one dance-pair.

Table 1 shows the dance-pairs that must be scheduled for the showcase. For example, dance-pair 1 shows student Daniel will dance with teacher Mr. Brown. Their Skill level is an indicator of how well this dance-pair performs together. Each dance-pair must be assigned to one heat. You may schedule as many heats as you like in order to fulfill this requirement but you cannot schedule students or teachers twice to the same heat. For example, you cannot assign dance-pairs 6 and 9 to heat 1 because it would require Camila to dance with both Ms. Clark and Ms. Anderson at the same time.

Scoring:
The quality of the showcase is based on a points system. A dance-pair arrangement requiring 3 total heats is worth 110 points, 4 heats are worth 100 points, 5 heats are worth 90, 6 heats are worth 80, 7 heats are worth 70, etc. For every heat with a standard deviation over 1 there is a 25 point penalty. For every heat with only one dance-pair there is a 10 point penalty.

Question:
What is the optimal way to assign the dance-pairs to heats in order to maximize the quality of the showcase?