Midterm '15

1. Consider a single factor experiment with three levels of the factor and three replicates. The response data turned out to be $y_{11} = 24$, $y_{12} = 22$, $y_{13} = 20$, $y_{21} = 27$, $y_{22} = 26$, $y_{23} = 28$, $y_{31} = 32$, $y_{32} = 34$, $y_{33} = 36$.

a) What is the value of $y_{..}$?

$66 + 81 + 102 = 147 + 102 = \cancel{250}\ 249$

b) What is the value of $\bar{y}_{..}$?

$\bar{y}_{..} = \frac{249}{9} = 27.67$

c) What are the values of $\bar{y}_{1\cdot}$, $\bar{y}_{2\cdot}$ and $\bar{y}_{3\cdot}$?

$\bar{y}_{1\cdot} = 22 \quad \bar{y}_{2\cdot} = 27 \quad \bar{y}_{3\cdot} = 34$

d) Find the value of SS_E for the corresponding ANOVA model.

$SS_E = 2^2 + 2^2 + 1^2 + 1^2 + 2^2 + 2^2 = 16 + 2 = 18$

e) What is the number of degrees of freedom for SS_T, $SS_{\text{Treatments}}$ and SS_E in the corresponding ANOVA model?

SS_T	SS_{TRT}	SS_E
8	2	6

f) What is the value of $\hat{\sigma}^2$ for the corresponding ANOVA model?

$\hat{\sigma}^2 = MS_E = \frac{18}{6} = 3$

2. A single factor experiment was performed for 4 levels of the factor with 4 replicates. It was found that $\bar{y}_{1\cdot} = 25$, $\bar{y}_{2\cdot} = 21$, $\bar{y}_{3\cdot} = 19$, $\bar{y}_{4\cdot} = 28$ and $MS_E = 9$.

The experimenter suspects that the average of the first three treatments is different from the fourth.

a) Propose a contrast to check the experimenter's suspicion.

$$\Gamma_1 = \mu_1 + \mu_2 + \mu_3 - 3\mu_4$$

b) Propose two more contrasts so that all three of them are orthogonal.

$$\Gamma_2 = \mu_1 + \mu_2 - 2\mu_3$$
$$\Gamma_3 = \mu_1 - \mu_2$$

c) Use Scheffe's method to test the original experimenter's suspicion hypothesis at $\alpha = 0.05$

$$C_1 = 25 + 21 + 19 - 3 \cdot 28 = 65 - 76 = -19$$

$$S_{C_1} = \sqrt{\frac{9}{4}(1+1+1+9)} = \frac{3}{2}\sqrt{12} = \frac{3 \cdot 2\sqrt{3}}{2} = 3\sqrt{3} \qquad |C_1| > 16.8$$

$$S_{\alpha,1} = 3\sqrt{3} \cdot \sqrt{3 \cdot f_{0.05,3,12}} = 9\sqrt{3.49} = 16.8 \qquad \cancel{|C_1| < 16.8}$$

d) Use Scheffe's method perform the corresponding tests on the two contrasts suggested by you, at $\alpha = 0.05$. What is the result?

$$C_2 = 25 + 21 - 2 \cdot 19 = 46 - 38 = 8$$

$$S_{C_2} = \sqrt{\frac{9}{4}(1+1+4)} = \frac{3}{2}\sqrt{6} \qquad |C_2| < 11.9$$

$$S_{\alpha,2} = \frac{3}{2}\sqrt{6}\sqrt{3 \cdot 3.49} = 11.9$$

$$C_3 = 25 - 21 = 4$$

$$S_{C_3} = \sqrt{\frac{9}{4}(1+1)} = \frac{3}{2}\sqrt{2} \qquad |C_3| < 6.9$$

$$S_{\alpha,3} = \frac{3}{2}\sqrt{2} \cdot \sqrt{3 \cdot 3.49} = 6.9$$

$$I = ABDE = BCDF = ACEF$$

3. A 2^{6-2} experiment needs to be performed. The experimenter has decided to use $E = ABD$ and $F = BCD$ as design generators.

a) List all runs the experimenter will have to perform.

0	0	(1)	def	0	1
1	1	ae	adf	1	2
1	0	bef	bd	1	1
2	1	abf	abde	2	2
1	1	cf	cde	1	2
2	2	acef	acd	2	3
2	1	bce	bcdf	2	2
3	2	abc	abcdef	3	3

← these numbers indicate # of letters in runs that are common with block generators from (c). They are needed to answer (d)

b) Write down the alias chains involving the main effects.

$A = BDE = ABCDF = CEF$

$F = ABDEF = BCD = ACE$

$B = ADE = CDF = ABCEF$

$C = ABCDE = BDF = AEF$

$D = ABE = BCF = ACDEF$

$E = ABD = BCDEF = ACF$

c) The experimenter has learned that it would be possible to perform only four runs in a single day. Therefore she would need to block the design accordingly. Propose a set of block generators. Explain why you've chosen this set.

$ABC = CDE = ADF = BEF$

$ACD = BCE = ABF = DEF$

$BD = AE = CF = ABCDEF$ only 3 2-way inter. are confounded

We can use any member of the alias chain as generator for block construction

d) Write down the runs that would go in each block.

block 1 (e e)	block 2 (e o)	block 3 (o e)	block 4 (o o)
(1)	abf	bef	ae
acef	bce	adf	cf
abde	def	cde	bd
bcdf	acd	abc	abcdef

4. In a factorial design with four factors, the following runs were performed: $d = 20$, $ac = 22$, $b = 26$, $abcd = 30$.

a) What is the full defining relation for this design?

(1)	d
a	ac
b	b
ab	abcd

$D = AB$

$C = A$

$I = ABD = AC = BCD$

b) Find the estimates for the main factor effects assuming all interactions are negligible. Are any of the main factors aliased with each other?

$$A = \frac{30+22}{2} - \frac{20+26}{2} = 26 - 23 = 3$$

$$B = \frac{26+30}{2} - \frac{20+22}{2} = 28 - 21 = 7$$

$$C = A = 3$$

$$D = \frac{30+20}{2} - \frac{22+26}{2} = 25 - 24 = 1$$

c) Suppose additional four runs have been performed: $ad = 24$, $c = 28$, $ab = 20$, $bcd = 26$. What is the full defining relation for this fraction? Is it a fold-over? What kind of fold-over?

It is foldover on A

$I = -ABD = -AC = BCD$

d) Combine the two sets of runs to find the estimates of the main effects. What is the full defining relation of the combined design?

$I = BCD$

$$A = \frac{30+22+24+20}{4} - \frac{20+26+28+26}{4} = 24 - 25 = -1$$

$$B = \frac{26+30+20+26}{4} - \frac{20+22+24+28}{4} = 25.5 - 23.5 = 2$$

$$C = \frac{30+22+28+26}{4} - \frac{20+26+24+20}{4} = 26.5 - 22.5 = 4$$

$$D = \frac{30+20+24+26}{4} \; 4 - \frac{22+26+28+20}{4} = 25 - 24 = 1$$