ISE 339 HW#1 Name: Bolun Xu.

CH3, 40. solution:

(a) $E(X) = P_1 EX_1 + P_2 EX_2 + P_3 \cdot EX_3 = 1.9 (d)$

(b) There exist 5 Event: A_1, A_2, A_3, A_4, A_5. $A_1: 3$; $A_2: 1\to 3$; $A_3: 2\to 3$; $A_4: 1\to 2\to 3$; $A_5: 2\to 1\to 3$.
denote X_i as the days before prisoners get freedom in Event i, $i=1,2,3,4,5$.

$E(X) = \sum_{i=1}^{5} P(A_i) \cdot E(X_i)$

$= \frac{1}{3}\times 0 + \frac{1}{6}\times 2 + \frac{1}{6}\times 3 + \frac{1}{6}\times(2+3)\times 2$

$= 2.5 (d)$.

(c) ⓐ $Var(X) = E(X^2) - (EX)^2 = 2^2\times 0.5 + 3^2\times 0.3 - 1.9^2 = 1.09$

ⓑ $Var(X) = E(X^2) - (EX)^2 = \frac{1}{3}\times 0 + \frac{1}{6}\times 2^2 + \frac{1}{6}\times 3^2 + \frac{1}{6}\times 5^2 \times 2 - 2.5^2 = 4.25$

44. solution:

Let N denote the number of customers.
Y denote the sum of money.
X_i denote the money spent by ith customer.

① $E(Y) = E\left(\sum_{i=1}^{N} X_i\right) = E(X_i)\cdot E(N) = 50\cdot\lambda = 500$

② $Var(Y) = E[Var(Y|N)] + Var[E(Y|N)]$

$= E[N\cdot \sigma_x^2] + Var[N\cdot \mu_x]$

$= \lambda(\sigma_x^2 + \mu_x^2)$

$= 10\times\left(\frac{(100)^2}{12} + 50^2\right)$

$= 33{,}333.33$

CH3.

49. Solution:

Let A be Event that A wins the game.

X be the number of games played.

Y be the number of games A has won in the first 2 games.

(a) $P(A)=\sum_{i=0}^{2} P(A|Y=i)\,P(Y=i) = 0 + P(A)\cdot 2p\cdot(1-p)+p^2$

$\therefore\ P(A) = \dfrac{p^2}{1-2p(1-p)}$

(b) $E(X) = \sum_{i=0}^{2} P(Y=i)\cdot E(X|Y=i)$

$= 2(1-p)^2 + (2+E(X))\,2p\cdot(1-p) + 2p^2$

$\therefore\ E(X) = \dfrac{2}{1-2p(1-p)}$

CH4. 28. Solution:

Let state 1 be the event that win ✓, have dinner ✓

2 ... ; ✗ ; ✓

3 ... ; ✓ ; ✗

4 ... ; ✗ ; ✗

Then transition Matrix $P = \begin{pmatrix} 0.56 & 0.04 & 0.24 & 0.16 \\ 0.21 & 0.14 & 0.09 & 0.56 \\ 0.56 & 0.04 & 0.24 & 0.16 \\ 0.21 & 0.14 & 0.09 & 0.56 \end{pmatrix}$

$$\begin{cases} \pi_1 = 0.56\pi_1 + 0.21\pi_2 + 0.56\pi_3 + 0.21\pi_4 \\ \pi_2 = 0.04\pi_1 + 0.14\pi_2 + 0.04\pi_3 + 0.14\pi_4 \\ \pi_3 = 0.24\pi_1 + 0.09\pi_2 + 0.24\pi_3 + 0.09\pi_4 \\ 1 = \pi_1 + \pi_2 + \pi_3 + \pi_4 \end{cases} \Rightarrow \begin{cases} \pi_1 = \frac{7}{15} \\ \pi_2 = \frac{1}{15} \end{cases}$$

The answer is $\pi_1 + \pi_2 = \frac{8}{15} = 0.53$

CH4 . 29. solution:

$$\begin{cases} \pi_0 = 0.7\pi_0 + 0.2\pi_1 + 0.1\pi_2 \\ \pi_1 = 0.2\pi_0 + 0.6\pi_1 + 0.4\pi_2 \\ 1 = \pi_1 + \pi_2 + \pi_3 \end{cases} \Rightarrow \begin{cases} \pi_0 = \frac{6}{17} \\ \pi_1 = \frac{7}{17} \\ \pi_2 = \frac{4}{17} \end{cases}$$

$\therefore$ $\frac{6}{17}$ in classification 0

$\frac{7}{17}$ - - - - 1

$\frac{4}{17}$ - - - - 2

36. solution: Let state 0 be 0, good

1 be 0, bad

2 be 1, good

3 be 1, bad.

$$P = \begin{pmatrix} 0.4p_0 & 0.4q_0 & 0.6p_1 & 0.6q_1 \\ 0.4p_0 & 0.4q_0 & 0.6p_1 & 0.6q_1 \\ 0.2p_0 & 0.2q_0 & 0.8p_1 & 0.8q_1 \\ 0.2p_0 & 0.2q_0 & 0.8p_1 & 0.8q_1 \end{pmatrix}$$

(a) $p_0 p_{00} + p_1 p_{01} = 0.4p_0 + 0.6p_1$

(b) $p_0 p_{00}^4 + p_1 p_{01}^4 = 0.2512p_0 + 0.7488p_1$

(c) $p_0\pi_0 + p_1\pi_1 = \frac{p_0}{4} + \frac{3}{4}p_1$

(d) Not a M.C., as Y_{n+1} doesn't depend on the value of Y_n.

CH4. 52. solution:

$$P = \begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix}$$

$$\begin{cases} 0.6\pi_0 + 0.4\pi_1 = \pi_0 \\ \pi_0 + \pi_1 = 1 \end{cases} \Rightarrow \begin{cases} \pi_0 = \frac{3}{7} \\ \pi_1 = \frac{4}{7} \end{cases}$$

$$E(X) = \pi_0 P_{00} \cdot X_{AA} + (\pi_0 P_{01} + \pi_1 P_{10}) \cdot X_{AB} + (\pi_1 \cdot P_{11} \cdot X_{BB})$$

$$= \frac{3}{7} \times 0.6 \times 6 + \left(\frac{3}{7} \times 0.4 + \frac{4}{7} \times 0.3\right) \times 12 + \frac{4}{7} \times 0.7 \times 8$$

$$= \frac{62}{7} = 8.86$$