IE426 – Optimization models and applications

Fall 2012 – Final exam, December 12, 2012

December 10, 2012

First name	
Last name	
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You have three hours. This exam accounts for 25% of the final grade. There are 100 points available. Please write clear and concise statements. Use a **readable** handwriting: it will be hard to grade your answers if they are not readable, let alone give them full points. Use the back of each sheet if you need more space, for example to draw graphs. For each model, clearly specify the meaning of each variable and of each constraint.

1 Duality (20 pts.)

Consider the following minimization problem.

$$\begin{array}{rllll} \min & x_1 & +2x_2 & +5x_3 & \\ \text{s.t.} & x_1 & & +x_3 & = 2 \\ & & x_2 & +x_3 & = 1 \\ & \multicolumn{3}{l}{x_1, x_2 \leq 0; x_3 \geq 0} & \end{array}$$

1. Write its dual (6 pts.);
2. Solve the dual with the graphical method (4 pts.);
3. Find the optimal solution of the primal using that of the dual (8 pts.).

(cont’d)

2 Integer Programming (20 pts.)

My daughter Kyra is trying to arrange a group of her friends into two soccer teams to play each other. Each player has skill level, s_i, $i = 1, \ldots, n$.

- Formulate a integer linear optimization problem to separate the friends into two teams so that the teams are as evenly matched as possible (that is to minimize the different between the total skill levels for each team).

- Assume that there are some kids who insist on being on the same team with their friends. The pairs of such friends are given by the set $E = \{(i, j)\}$. Formulate the same optimization problem as above, but making sure that all pairs of friends are kept together.

3 Integer Programming (10 pts.)

Consider a graph $G = (V, E)$, in Figure 1, and a cost c_i for each node $i \in V$. Suppose you want to find the subset S of V of minimum total cost such that every edge in E is *covered* by at least one node in S, i.e., for every edge $\{i, j\} \in E$, either $i \in S$ or $j \in S$ or both. As an example, for the graph in Figure 1, the dark nodes constitute a feasible solution for this problem.

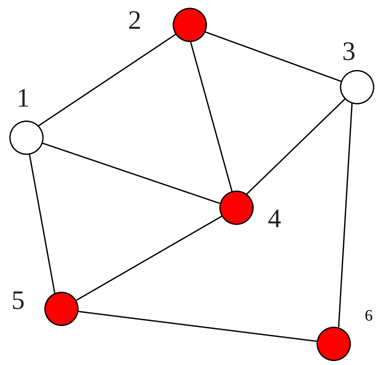

Figure 1:

Formulate this problem as an integer linear programming problem.

4 Stochastic Integer Programming (30 pts.)

Your company is considering to open a few grocery stores, each possibly with a pharmacy, in Pennsylvania. The stores and pharmacies are intended to serve an area with a set C of n cities, and there is a set L of m potential locations.

You are given the distance d_{ij} between any $i \in C$ and any $j \in L$. The cost for building a grocery store at location $j \in L$ is c_j, and the additional cost for adding a pharmacy is f_j.

A pharmacy cannot be built at j unless a grocery store is also built at j. The total budget is B, and the company aims at minimizing the sum of two functions: the first is the sum, among all cities i, of the distance between i and the closest grocery, which can be thought of as the grocery *assigned* to i; the second is analogous to the first and is related to pharmacies.

- Formulate this as an integer linear programming problem.

- Now assume that you build all the groceries first. While you were building them the construction cost may have gone up 20% or down 20%, or it may have remained the same. The three scenarios are equally likely. The change in the construction costs affects only the cost of construction of the pharmacies (but not the groceries). Formulate the new problem as a stochastic integer programming problem. Identify first and second stage variables.

(cont’d)

5 Support Vector Machines (20 pts.)

The convex quadratic formulation of support vector machines

$$\begin{aligned}
\min_{\xi,\mathrm{w},\beta} \quad & \frac{1}{2}w^\top w + c\sum_{i=1}^{n}\xi_i \\
\text{s.t.} \quad & y_i(w^\top x_i + \beta) \geq 1 - \xi_i, \quad i = 1, \ldots, n \\
& \xi \geq 0, \qquad i = 1, \ldots, n.
\end{aligned} \tag{1}$$

can be rewritten as an unconstrained nonsmooth convex optimization problem

$$\min_{\xi,\mathrm{w},\beta} \frac{1}{2}w^\top w + c\sum_{i=1}^{n}\max\{1 - y_i(w^\top x_i + \beta), 0\}$$

Consider the following modification of the problem

$$\min_{\xi,\mathrm{w},\beta} \frac{1}{2}w^\top w + c\sum_{i=1}^{n}(\max\{1 - y_i(w^\top x_i + \beta), 0\})^2$$

Rewrite this as a convex quadratic problem (in the spirit of (1)).