ISE429. Homework 2

Unless stated otherwise, the problems are from the Ross textbook (11th edition):

1) (weight 0.20) 5-12 (a,b,c).

2) (weight 0.15) 6-9

3) (weight 0.15) 6-15. *Comment: If Markov chain is in a stationary regime, with stationary distribution $\{\pi_j\}$, then the time-average rate at which transtions $i \to j$ occur is $\pi_i q_{ij}$.*

4) (weight 0.15) 6-22

5) (weight 0.15) 6-23

6) (weight 0.20) Consider the continuous-time birth-death (Markov) process with state space $\mathcal{X} = \{0, 1, 2, 3, \ldots\}$. The transition rate to the "left" ("death" rate) is always equal to 1. The transition rate to the "right" (birth rate) depends on state n and is equal to λ_n.
(a) Suppose $\lambda_n = (1/2) + 10e^{-n}$, $n = 0, 1, 2, \ldots$. Is this Markov chain transient?
(b) Suppose $\lambda_n = 1 - (1/2)e^{-n}$, $n = 0, 1, 2, \ldots$. Is this Markov chain positive recurrent?