ISE 426
Optimization models and applications

instructor:	Katya Scheinberg
	Mohler Lab #479
phone:	(610) 758 4039
office hrs:	after 5pm on Tuesdays (if I am available)
	and by appointment
email:	katyas at lehigh dot edu
web page:	`http://coral.ie.lehigh.edu/ katyas`

Introduction

- What do I expect from this course?
- Homeworks, TAs (Sahar Tahernejad and Secil Sozuer) and lab time.
- Use of cell phones and laptops.
- How to take lecture notes.
- Introductions.
- A few practical issues.

Evaluation

Homework:	25%	(one every two weeks?)
Quiz #1:	10%	(first part of October)
Quiz #2:	10%	(early November)
Case study:	20%	(assigned in November)
Final exam:	25%	
In-class participation:	10%	

Case studies

- study an Optimization problem,
- propose a model, and
- solve it using a tool[1] of their choice.

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[1] An optimization tool...

Lecture plan

- Convexity, relaxations, lower/upper bounds
- Linear Programming (+ duality)
- Integer Programming
- Nonlinear Programming
- Relaxation and Decomposition
- Stochastic Programming
- Robust Optimization
- Multicriteria Optimization

Material

- "Introduction to Mathematical Programming: Applications and Algorithms", Volume 1, by W.L. Winston and M. Venkataramanan;
- Select chapters of "Introduction to Operations Research" by F.S. Hillier and G.J. Lieberman, McGraw-Hill: New York, NY, 1990;
- Select chapters of "Operations Research: Applications and Algorithms" by Wayne L. Winston, PWS-Kent Pub. Co., 1991;
- modeling language: "AMPL: A Modeling Language for Mathematical Programming" by Robert Fourer, David M. Gay, and Brian W. Kernighan.
- Lecture slides.

Modeling languages

They are similar, and each has its own pros/cons. All have limited version available to students.

Mosel: very nice Graphical User Interface (GUI)

AMPL: preferred. No GUI, but I and TAs know it better (can help) also helps the formulation process.

GAMS: Has version with even nicer GUI (Aimms)

What is optimization?

- When to use it?
- What to use it for?
- How to use it?

Optimization models…

- Optimization aims at finding the best configuration of processes, systems, products, etc.
- It relies on a theory developed mostly in the past 50 years
- Applying Optimization in an industrial, financial, logistic context yields a better use of budget/resources ($$$) or a higher revenue ($$$)

…and applications

Source: http://www.informs.com
(see also http://www.ScienceOfBetter.org)

yr	company	result
86	Eletrobras (hydroelectric energy)	43M$ saved
90	Taco Bell (human resources)	7.6M$ saved
92	Harris semicond. prod. planning	50% $\rightarrow$ 95% orders "on time"
95	GM – Car Rental	+50M$
96	HP printers — re-designed prod.	2x production
99	IBM — supply chain	750M$ saved
00	Syngenta — corn production	5M$ saved

A few examples from IBM Research

- Circuit design
- Sorting facility optimization
- Limousine driver/fleet optimization

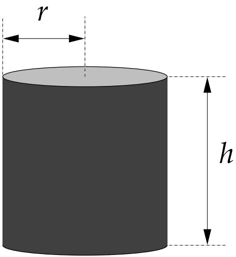
An example

- You work at a company that sells food in glass containers only. Today, your boss has a bright idea! The **tin can**®. It's a cylinder made of tin.
- The can must contain $V = 20$ cu.in. (11 fl.oz., 33 cl)
- Cut and solder tin foil to produce cans
- Tin (foil) is expensive, use as little as possible

Boss: "What is the ideal can? Tall and thin or short and fat?"

You: A cylinder with volume V using as little tin as possible.

Example



If we knew radius r and height h,

- the volume would be $\pi r^2 h$
- qty of tin would be $2\pi r^2 + 2\pi r h$

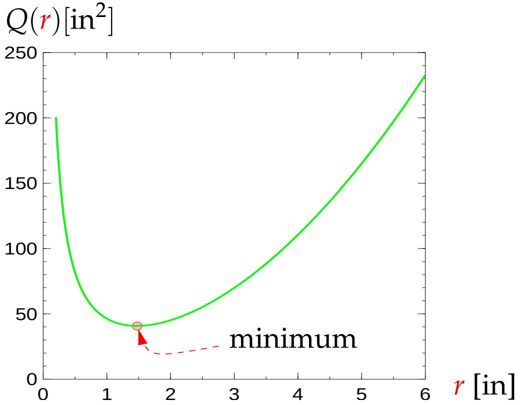
$\pi r^2 h$ must be $V = 20\text{in}^3 \Rightarrow h = \frac{V}{\pi r^2}$

Rewrite the quantity of tin as $Q(r) = 2\pi r^2 + 2\pi r \frac{V}{\pi r^2}$, or

$$Q(r) = 2\pi r^2 + \frac{2V}{r}$$

$\Rightarrow$ Find the minimum of $Q(r)$!

Minimize the quantity of tin



$r = \boxed{\boxed{\textbf{1.471 in}}}$ $\qquad h = \frac{V}{\pi(1.471)^2} = 2.942$ in

Aims of this course

- model Optimization problems
- so that they can be solved
- learn a modeling language
- apply modeling languages to real-world problems

Your first Optimization model

Variables	r: radius of the can's base
	h: height of the can
Objective	$2\pi rh + 2\pi r^2$ (minimize)
Constraints	$\pi r^2 h = V$

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	$h > 0$
	$r > 0$

Optimization Models, in general, have:

Variables: Height and radius, number of trucks, ... The *unknown* (and desired) part of the problem (one thing your boss cares about).

Constraints: Physical, explicit ($V = 20\text{in}^3$), imposed by law, budget limits... They define **all** and **only** values of the variables that give possible solutions.

Objective function: what the boss really cares about. Quantity of tin, total cost of trucks, total estimated revenue, ... a function of the variables

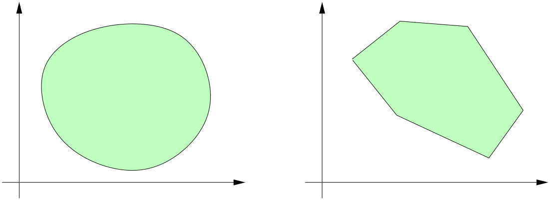
The general optimization problem

Consider a vector $x \in \mathbb{R}^n$ of variables.
An optimization problem can be expressed as:

$$\mathbf{P}: \qquad \begin{array}{rl} \text{minimize} & f_0(x) \\ \text{such that} & f_1(x) \leq b_1 \\ & f_2(x) \leq b_2 \\ & \vdots \\ & f_m(x) \leq b_m \end{array}$$

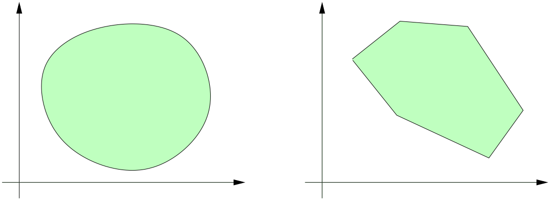
Convexity

Convex sets



Def.: A set $S \subseteq \mathbb{R}^n$ is convex if any two points x' and x'' of S are joined by a segment **entirely** contained in S:

Convex sets



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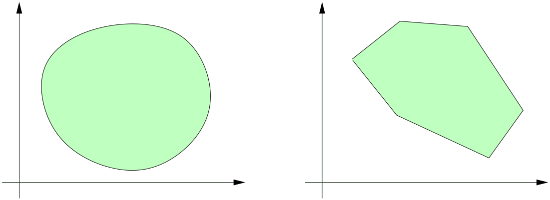

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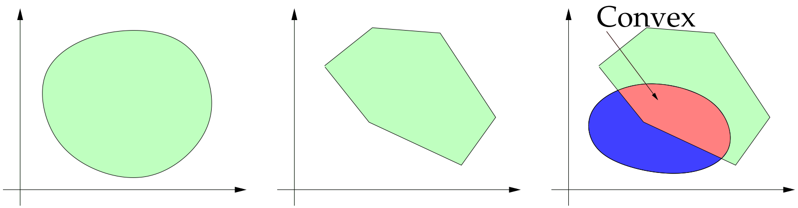
The intersection of two convex sets is convex.

Examples: Convex sets



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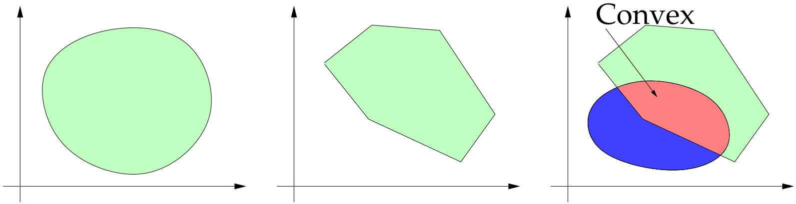
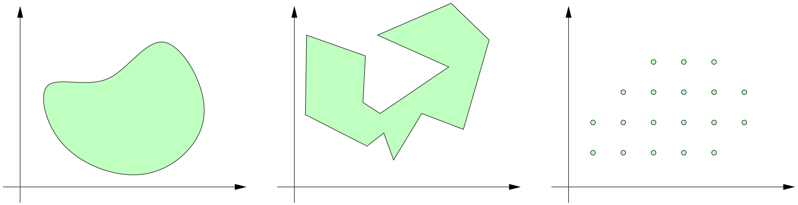


- $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^3$, etc. are convex
- $[a, b]$ is convex
- $\{4\}$ is convex

Examples: Nonconvex sets



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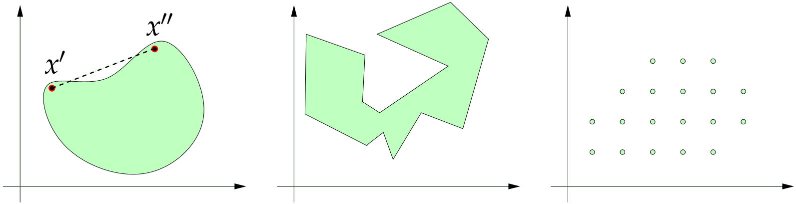

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- $\{x \in \mathbb{R} : x \leq 2 \vee x \geq 3\}$ is nonconvex
- $\mathbb{Z}$ is nonconvex

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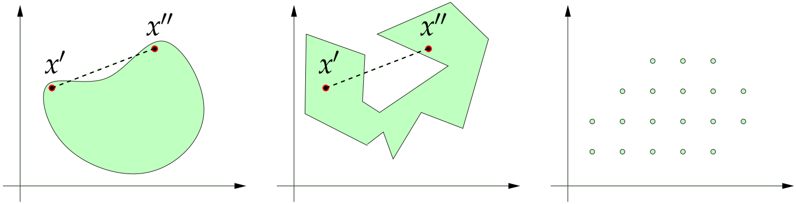

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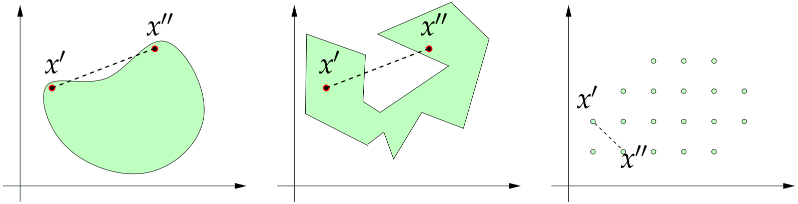

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Convex functions

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- Multiplying a convex function by a positive scalar gives a convex function
- **linear** functions $\sum_{i=1}^{k} a_i x_i$ are convex, irrespective of the sign of a_i's.