Stochastic Models and Applications

A. Stolyar

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Course Information:

ISE 429
Fall 2015
MW 2:35-4:05
Mohler 375

Course Description

This course is a non-measure theoretic introduction to stochastic processes used in operations research and engineering.

Prerequisites

Elementary probability course; ISE 230 or equivalent; at least two semesters of undergraduate calculus.

Required Text

S. M. Ross, Introduction to Probability Models, 11th Ed., Academic Press, 2014.

Other Relevant Books (optional)

S. M. Ross, Stochastic Processes, 2nd Ed., Wiley, 1996.
G. Lawler. Introduction to Stochastic Processes, 2nd Ed., Chapman & Hall, 2006.
W. Feller. An Introduction to Probability Theory and Its Applications, Vol. 1 and 2.

Assignments

Homeworks.

Two tests: dates TBD.

Final exam.

Final grade: 0.25*Homeworks + 0.25*Tests + 0.5*Final_exam exam + Bonus_points + Class_participation

Bonus_points: you can earn up to 5 points.

Class_participation: up to 5 points, on instructor's discretion, based on, well, class participation.

Worst homework is excluded. Worst test grade is replaced by the final exam grade (if latter is greater than the former).

Policies

Grading scale. (May be calibrated later, if necessary, but the ranges can only be moved down.)

90-100	=	A
85-89	=	A-
80-84	=	B+
75-80	=	B
70-75	=	B-
67-69	=	C+
64-66	=	C
61-63	=	C-
58-60	=	D+
55-57	=	D
52-54	=	D-
<54	=	F

The assignments you do in this course must be your own work. Discussion of subjects, materials, etc., with your classmates or others is fine. Copying somebody else's work is not allowed.

Equitable Community
Lehigh University endorses The Principles of Our Equitable Community (http://www4.lehigh.edu/diversity/principles). We expect each member of this class to acknowledge and practice these Principles. Respect for each other and for differing viewpoints is a vital component of the learning environment inside and outside the classroom

Accommodations for Students with Disabilities: If you have a disability for which you are or may be requesting accommodations, please contact both me and the Office of Academic Support Services, University Center 212 (610-758-4152) as early as possible in the semester. You must have documentation from the Academic Support Services office before accommodations can be granted.

Logistics

All home assignments must be submitted electronically on coursesite, as a scanned PDF file. Late submissions are not accepted.

Course outline (may be adjusted as necessary)

1. Review of basic probability concepts
    a. Sample (probability) space; Probability; Independence; Conditional probability; Formula of complete probability
    b. Random variables; Distribution; Expectation; Independence; Conditional distribution and expectation
    c. Inequalities: Markov, Chebyshev, Jensen
    d. Borel-Cantelli lemma
    e. Law of large numbers (LLN), Strong law of large numbers (SLLN), Central limit theorem (CLT)
    f. Stochastic (random) process; Stationarity; Stationary increments; Independent increments
2. Markov chains (discrete time)
    a. Markov property; Transition probabilities (matrix and graph representation)
    b. Chapman-Kolmogorov equations; n-step transition probabilities
    c. Communication classes; Irreducibility; Recurrence Vs Transience
    d. Positive Vs Null recurrence; Long-term proportions; Stationary distribution
    e. Limiting probabilities; Aperiodicity; Ergodicity
    f. Lyapunov-Foster positive recurrence criterion
    g. Probabilities and average times to reach absorbing states
3. Markov chains (continuous time)
    a. Exponential distribution; Memoryless property
    b. Poisson process; Definition; Interarrival times; Superposition and thinning of Poisson processes
    c. Continuous time Markov chain; Markov property; Transition probabilities (matrix representation); Chapman-Kolmogorov equations
    d. Transition rates (matrix and graph representation); Do transition rates define transition probabilities uniquely? (possible "explosion")
    e. Kolmogorov backward and forward equations
    f. Communication classes; Irreducibility; Recurrence Vs Transience; Positive Vs Null recurrence; Long-term proportions; Stationary distribution and limiting probabilities
4. Reversible Markov chains (MC). Markov chain Monte Carlo (MCMC)
    a. Reversibility w.r.t. a (stationary) distribution; detailed balance conditions; examples (birth-death, simple random walk)
    b. MC truncation preserves reversibility
    c. Time-reversed MC; examples (M/M/1, tandem system)
    d. MCMC: MC with a given stationary distribution; Metropolis algorithm, Gibbs sampler
5. Renewal theory
    a. Regenerative process (renewal points, renewal cycle, renewal time); Examples
    b. Long term proportions
    c. Limiting probabilities; Renewal process; Key renewal theorem; Age, excess time, and current interval of the renewal process
    d. Elementary renewal theorem proof

    e. Renewal reward process
6. Martingales
    a. Conditional expectation of a random variable w.r.t. another random variable (random vector, random element); “Amount of information” (sigma-algebra) associated with a random element; Filtration
    b. Conditional expectation w.r.t. a sigma-algebra; Basic properties
    c. Martingale definition; Examples
    d. Stopping time w.r.t. a filtration; Optional Sampling Theorem; Application examples
    e. Uniform integrability; Another form of the Optional Sampling Theorem
    f. Martingale inequalities: Reflection principle, Doob inequality
7. Brownian motion
    a. Definition and (one) construction. Properties of trajectories
    b. Brownian motion (BRM) is both a martingale and a Markov process
    c. BRM is a Strong Markov process. Application: Reflection principle
    d. Probability one asymptotic properties
    e. Scaling properties
    f. Multi-dimensional BRM. Connection to partial differential equations
    g. Recurrence and transience of a d-dimensional BRM
    h. BRM with drift
8. Elements of stochastic calculus
    a. Integration of one random process with respect to (w.r.t.) another – issues
    b. Ito integral w.r.t. Brownian Motion. Definition and properties
    c. Ito formula
    d. Application: Black-Scholes option pricing formula