

ISE429. Homework 5

1) (weight 0.25) $\{X_t\}$ is a standard one-dimensional Brownian motion. (“Standard” means that variance parameter is 1, $X_0 = 0$, and the drift parameter is 0.) Find the following probabilities:

$$P(X_2 > -2),$$

$$P(X_1 > X_2),$$

$$P(X_2 < X_1 < X_3),$$

$$P(X_t = 0 \text{ for some } t \text{ with } 2 \leq t \leq 3),$$

$$P(X_t < 4 \text{ for all } t \text{ with } 0 < t \leq 9),$$

$$P(X_t < -10 \text{ for all } t > 10).$$

Comment. You do not have to produce actual numbers. It is ok to leave answers in terms of integrals, without numerically evaluating them. But, it has to be an expression that can be numerically evaluated. (If you also produce the numbers, that is a useful thing to practice. But, it will not affect the grade.)

2) (weight 0.25) $\{X_t\}$ and $\{Y_t\}$ are independent standard one-dimensional Brownian motions. Find the following probability:

$$P(X_t = Y_t \text{ for infinitely many } t > 10).$$

3) (weight 0.25) $\{X_t\}$ is a standard one-dimensional Brownian motion. Find the density of $M = \max\{X_t : 0 \leq t \leq 10\}$ and compute its expectation and variance.

Comment.. You can leave answers in terms of integrals, that can be evaluated numerically. (If you also produce the numbers, that is a useful thing to practice. But, it will not affect the grade.)

4) (weight 0.25) $\{X_t\}$ is a standard one-dimensional Brownian motion. Find the density of $T = \min\{t : X_t = 1\}$ and prove that $ET = \infty$.