ISE426 – Optimization models and applications

Fall 2013 – Quiz #1, October 8, 2013

First name	
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You have 75 minutes. There are two problems. This quiz accounts for 10% of the final grade. There are 40 points available. Please write clear and concise statements. Use a **readable** handwriting: it will be hard to grade your answers if they are not readable — let alone give them full points. Use the back of each sheet if you need more space, for example to draw graphs. Do not use calculators.

1 Convexity and relaxations (8 pts.)

The following problem is not convex, explain why (4 pts.):

$$\begin{array}{ll} \max & y \\ & |x| = 1 \\ & x^2 + y^2 \leq 2 \end{array}$$

Find upper and lower bounds on the optimal value and explain how you know that these are indeed upper and lower bounds. Find the optimal solution graphically and check that it is between the upper and lower bounds. (4 pts.).

2 Linear Programming Model (16 pts.)

Consider the following LP problem, which is a slight modification of the problem from your homework $\#2$:

$$\begin{array}{rlllll}
\min & 2x_1 & +5x_2 & +2x_3 & -2x_4 & \\
\text{s.t.} & x_1 & +x_2 & +2x_3 & +x_4 & \geq 1 \\
 & x_1 & +x_2 & -x_3 & -2x_4 & \geq 2 \\
 & x_2, x_3 & \geq 0 & x_1, x_4 & \leq 0. &
\end{array}$$

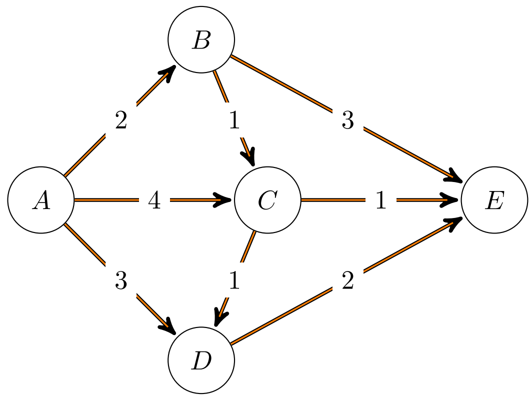
- Write the dual. (5 pts)

- Solve the dual through the graphical method. (3 pts)

- After finding the optimal value of the dual variables, use complementary slackness to find the optimal value of the primal variables. (4 pts)

- Find the range of the right hand side coefficient of the second constraint (which is "2" in the original problem) for which the dual solution that you found remains optimal. Find the answer by checking for which values of this coefficient the primal complementary solution remain feasible. (4 pts)

3 Flow problem (16 pts.)



1. Formulate the shortest path problem for going from A to E as a linear programming problem, using the formulations studied in this course. Note that now the arcs are directed, hence you can only travel one way on each arc. (6 pts)

2. Write the dual of the above shortest path problem. (5 pts)

3. Find the shortest path for the problem, by simple observation. This gives you the primal optimal solution for your LP. Write it down. Using it write down complementarity condition of the optimal dual solution. Observe that the dual optimal solution is not defined uniquely. But these complementarity conditions uniquely define the optimal value of the objective function of the dual. Demonstrate this. (2 pts)

4. Consider the change in the length of the (D, E) edge. It is 2 in the original problem. How low can it get before the shortest path changes. Demonstrate this by showing that if the length of (D, E) gets any lower than this value, then the complementary dual solution will no longer be feasible (you can show this even if you do not compute the unique dual solution). (3 pts)

4 Linear Programming Model (20 pts.)

Sophia goes out to lunch 5 days a week. She is trying to decide on what she will be eating during a given week. Her options are: lo mein, hamburger, fried chicken, soup, pizza and a Subway sandwich. She can eat each of these as many times as she chooses. She eventually needs to have 5 meals. Her budget is $20 and she is trying to maximize her satisfaction. In the table below you see the cost per each type of meal and their satisfaction coefficients.

Food	Lo Mein	Hamb.	Fr. Chic.	Soup	Pizza	Sub.
satisfaction	6	5	4	4	3	3
cost	7	5	5	4	4	3

(a) Formulate an integer optimization problem to choose the meals to maximize Sophia satisfaction (note that each meal can be chosen more than once as long as the total is 5 meals). Create a linear programming relaxation. If you do this correctly, $(0, 2.5, 0, 0, 0, 2.5)$ would be an optimal solution to the relaxation. (6pts)

(b) Write down the dual of the linear programming relaxation defined in part (a). Using the optimal solution of the relaxation given in part (a), compute the dual optimal solution from complementary slackness conditions and show that this solution is feasible (hence the primal solution that is given is optimal). (8 pts)

(c) Consider the satisfaction coefficient of lo mein. How much higher does it have to be to change to optimal solution of the relaxation? Derive your answer from the feasibility of the complementary dual solution. (2pts)

(d) Consider the satisfaction coefficient of the soup. Show that it cannot get any lower without changing the optimal solution. (2 pts)

(e) $(0, 2.5, 0, 0, 0, 2.5)$ is not a feasible solution to the original problem since it is not integer. Does it give you a bound on the optimal solution of the original problem? Find an optimal solution of the original problem and prove that it is optimal using that bound. (2pts) extra sheet