ISE429, Optional sampling (stopping) theorems

Theorem 1 [Optional sampling (stopping) theorem 1]. *Suppose,* $\{M_n,\ n = 0, 1, \ldots\}$ *is a martingale w.r.t. filtration* $\{\mathcal{F}_n\}$*. Suppose,* T *is a stopping time. Suppose,* $T \le K$ *for some constant* K*. Then* $EM_T = EM_0$*.*

Theorem 2 [Optional sampling (stopping) theorem 2]. *Suppose,* $\{M_n,\ n = 0, 1, \ldots\}$ *is a martingale w.r.t. filtration* $\{\mathcal{F}_n\}$*. Suppose,* T *is a stopping time,* $P\{T < \infty\} = 1$*. Suppose,* $E|M_T| < \infty$ *and* $\lim_{n\to\infty} E[|M_n|I\{T > n\}] = 0$*. Then* $EM_T = EM_0$*.*

Theorem 3 [Optional sampling (stopping) theorem 3]. *Suppose,* $\{M_n,\ n = 0, 1, \ldots\}$ *is a uniformly integrable martingale w.r.t. filtration* $\{\mathcal{F}_n\}$*. Suppose,* T *is a stopping time,* $P\{T < \infty\} = 1$*. Suppose,* $E|M_T| < \infty$*. (This condition can be dropped; but we only proved this theorem with it.) Then* $EM_T = EM_0$*.*