ISE 419 HW #2 Answers

1.

$$\max_{1 \le \ell \le k} \left\{ \frac{\sum_{j=1}^{\ell} w_j}{\sum_{j=1}^{\ell} p_j} \right\} = \frac{\sum_{j=1}^{\ell^*} w_j}{\sum_{j=1}^{\ell^*} p_j}$$, where ℓ^* is the job that determines the factor ρ of the chain.

With precedence constraints $\overbrace{1 \to 2}^{\text{chain 1}}$, $\overbrace{3 \to 4 \to 5}^{\text{chain 2}}$ and $\overbrace{6 \to 7}^{\text{chain 3}}$, we have:

i) ρ factor of chain 1 is determined by job 2: $\frac{0+18}{3+6} = 2$
ρ factor of chain 2 is determined by job 3: $12/6 = 2$
ρ factor of chain 3 is determined by job 6: $17/8 = 2.125$
Selecting chain 3: job 6

ii) ρ factor of the remaining part of chain 1 is determined by job 2 : 2
" " 2 " 3 : 2
" " 3 " 7 : 1.78
Here we have two options. Either to select chain 1; jobs 1 & 2 or chain 2, job 3

iii a) Selecting chain 1, jobs 1 & 2:
Continuing with the same analysis as in (ii) we obtain the following sequences (there will be another split decision to make): $\boxed{\begin{matrix} 6-1-2-3-4-5-7 \\ 6-1-2-3-7-4-5 \end{matrix}}$ (1)

iii b) Selecting chain 2, job 3:
Continuing with the analysis as in (ii), after an additional split due the same ρ factor we end up with: $\boxed{\begin{matrix} 6-3-1-2-4-5-7 \\ 6-3-1-2-7-4-5 \end{matrix}}$ (2)

Thus optimal sequences are (1) & (2)

2.

$$C_{max} = (4 + 8 + 12 + 7 + 6 + 9 + 9) = 55$$

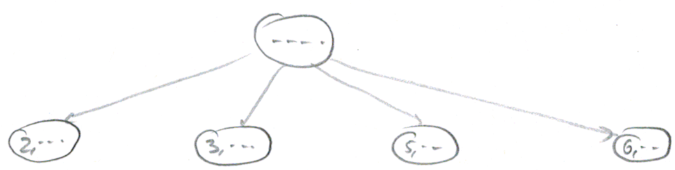
Jobs	1	2	3	4	5	6	7	Selected Job
C(55)	165	77	3025	82.5	77.4	88	77	2
C(47)	141		2209	70.5	76.8	75.2	65.8	4
C(40)	120		1600		76.3	64	56	6
C(31)	93		961		75.5		43.4	7
C(22)	66		484		74.6			1
C(18)			324		74.2			5
C(12)			144					3

Selected Sequence: 3 - 5 - 1 - 7 - 6 - 4 - 2

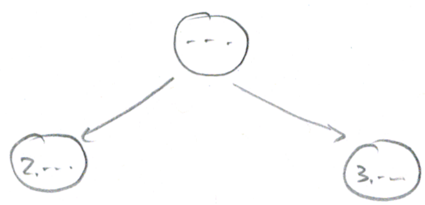
$$\sum_{i=1}^{7} h_i = 539.1$$

3.

With precedence constraints $2 \to 1 \to 4$ & $6 \to 7$, we can start the process with 2, 3, 5 or 6. The B-B tree looks like this:



Because release dates of 5 and 6 are 25, we can not complete jobs 1, 2 or 3. Thus we reject 5 & 6. Now BB looks like this:



2 [0, 8] $L = -36$

1 [8, 14] $L = 6 = L_{max}$

4 [14, 24] $L = 0$

3 [24, 36] $L = -8$

6 [36, 53] $L = -32$

7 [53, 69] $L = 1$

5 [69, 79] $L = -11$

3 [0, 12] $L = -32$

2 [12, 20] $L = -22$

1 [20, 26] $L = 18 > L_{max}$ of chain starting with job 2

Lower bound $= 6 = L_{max}$

The schedule is non preemptive

Opt solution: 2 - 1 - 4 - 3 - 6 - 7 - 5

4. Single machine process - assuming $k=2$

Find I_j for all jobs starting with $t=0$. $I_j = \frac{w_j}{p_j} e^{-\frac{\max(d_j - p_j - t, 0)}{k \cdot \bar{p}(t)}}$

$\bar{p}(t) = \frac{\sum_j p_j}{|J|}$, $j \in J$

$J = \{$ set of all unfinished jobs

@ $t=0$, $\bar{p}(t) = \frac{79}{7} = 11.29$

j	I_j
1	0.153
2	0.1336
3	0.0404
4	0.216
5	0.0029
6	0.116
7	0.0125

→ Job 4 has the largest I, thus we chose it to be the first job.

$t = 0 + P_4 = 10$

@ $t=10$, $\bar{p}(t) = 11.5$

j	I_j
1	0.167
2	0.262
3	0.064
5	0.0048
6	0.019
7	0.020

→ next job is 2

$t = 10 + P_2 = 13$

@ $t=13$, $\bar{p}(t) = 12.2$

j	I_j
1	0.167
3	0.094
5	0.0079
6	0.030
7	0.031

next job is 1

$t = 13 + P_1 = 24$

And we continue the procedure outlined here until we schedule all the jobs. The final sequence is: 4-2-1-3-7-5

5. Assume $w_i = 1$ for all jobs

Backward process:

n	S^c	$t = \sum_i P_i,\ i \in S^c$	$P_i = (t - d_i) w_i$ Job 1	2	3	4	5	i^* (based on best P_i
5	1,2,3,4,5	16	14	13	10	6	4	5
4	1,2,3,4	11	9	8	5	1		4
3	1,2,3	9	7	6	3			3
2	1,2	7	5	4				2
1	1	4	2					1

Backward process total tardiness penalty, $\sum P_i = 4 + 1 + 3 + 4 + 2 = 14$

Current sequence = {1, 2, 3, 4, 5}, $\mu = 5$

Forward phase (swap to reduce tardy penalty)

k	j	$j-k$	s'	$C_i - d_i$, $C_i = t_i + P_i$, $i \in s'$ Job 1	2	3	4	5	$P(s')$
4	5	1	5,2,3,4,1	0	5	4	2	14	25 > 14
3	4	1	4,2,3,1,5	0	2	1	9	4	16 > 14
3	5	2	1,5,3,4,2	2	0	5	11	13	31 > 14
2	3	1	3,2,1,4,5	0	2	7	1	4	14 matches prev tardiness, thus we have a new sequence s^c = {3, 2, 1, 4, 5}

k	j	$j-k$	s'	$C_i - d_i$, $C_i = t_i + P_i$, $i \in s'$					$P(s')$
4	5	1	5,2,1,4,3	0	5	10	4	10	29 > 14
3	4	1	4,2,1,3,5	0	2	7	5	4	18 > 14
3	5	2	3,5,1,4,2	0	0	9	4	13	25 > 14
2	3	1	1,2,3,4,5						
2	4	2	3,4,1,2,5	0	0	6	8	4	18 > 14
2	5	3	3,2,5,4,1	0	2	0	2	14	27 > 14
1	2	1	2,3,1,4,5	0	0	7	1	4	12 New sequence = {2, 3, 1, 4, 5}

k	j	$j-k$	s'	$C_i - t_i$					$P(s')$
4	5	1	5,3,1,4,2	0	1	9	3	13	26 > 12
3	4	1	4,3,1,2,5	0	0	6	8	4	18 > 12
3	5	2	2,5,1,4,3	0	0	10	4	10	24 > 12
2	3	1	1,3,2,4,5	2	0	6	1	4	13 > 12
2	4	2	2,4,1,3,5	0	0	7	5	4	16 > 12
2	5	3	2,3,5,4,1	0	0	0	2	14	16 > 12
1	2	1	3,2,1,4,5	0	2	7	1	4	14 > 12
1	3	2	2,1,3,4,5	0	5	3	1	4	13 > 12
1	4	3	2,3,4,1,5	0	0	0	9	4	13 > 12
1	4	4	2,3,1,5,4	0	0	7	2	6	15 > 12

As all sequences s' are of larger tardiness than the best found sequence, there is nothing to improve, thus the final sequence is $s^* = \{2, 3, 1, 4, 5\}$ with tardiness of 12