IE426 – Optimization models and applications

Fall 2012 – Homework #5

November 26, 2014

This homework accounts for 5% of the final grade. There are 20 points available. For all problems where an AMPL model is required, include the AMPL model file, the data file, and the optimal solution, shown clearly with the command (`display`) you have used to print it. This homework is due by Thursday, December 4, 5pm. It can be delivered to mail boxes of the professor or TAs in Mohler 421.

1 Support Vector Machines(5pts)

Formulate and solve the classification problem via the linear support vector machines

$$
\begin{array}{lll}
\min_{\xi,\mathrm{w},\beta} & \frac{1}{2}w^\top w + c\sum_{i=1}^{n}\xi_i & \\
\text{s.t.} & y_i(w^\top x_i + \beta) \geq 1 - \xi_i, & i = 1,\ldots,n \\
 & \xi \geq 0, \qquad i = 1,\ldots,n. &
\end{array}
$$

using the following training data (see the text file with the data for simple paste and copy purposes)

$$X =$$

$$
\begin{bmatrix}
-0.0192 & 0.4565 \\
-0.0302 & -0.8531 \\
-0.1170 & -0.9854 \\
0.4454 & 0.3952 \\
-0.7989 & -0.2569 \\
0.0935 & 0.7398 \\
0.2654 & 0.3098 \\
0.6040 & -0.0959 \\
-0.6324 & -0.9139 \\
0.9770 & -0.4862 \\
0.9260 & 0.0075 \\
0.8055 & -0.0103 \\
0.3007 & 0.9564 \\
-0.2771 & 0.1357 \\
0.3782 & 0.6800 \\
-0.5911 & -0.1808 \\
-0.2501 & 0.4231 \\
-0.1130 & 0.8032 \\
0.9353 & -0.2590 \\
-0.1272 & 0.9856 \\
-0.0244 & 0.7780 \\
0.2476 & 0.7701 \\
0.1555 & -0.8341 \\
-0.9507 & -1.0000 \\
-0.6986 & -0.0473 \\
-0.4293 & -0.9466 \\
-0.8917 & 0.2226 \\
0.1545 & 0.4526 \\
-0.8230 & 0.7856 \\
-0.5885 & 0.5231 \\
-0.6578 & -0.7660 \\
-0.2009 & -0.7598 \\
0.0965 & -0.8755 \\
0.3399 & 0.8383 \\
-0.0519 & 0.9419
\end{bmatrix}
$$

$$y =$$

$$\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

Solve 3 instances of the problem: with penalty c=10000, c=100 and c=1. Formulate the appropriate QPs in AMPL, solve it using Cplex. Compare the solutions - report the support vectors, i.e. the points for which $y_i(w^\top x_i+\beta) = 1$ and the points violating classification constraints, i.e. for which $y_i(w^\top x_i+\beta) < 1$

2 Modeling (5pts)

Consider the problem of estimating sparse (with few nonzero entries) solution x which approximately satisfies the system $Ax = b$.

$$\begin{array}{rl} \min_{\mathrm{x}} & \|x\|_1 \\ \text{s.t.} & \|Ax - b\|_\infty \leq \epsilon \end{array}$$

where $\|v\|_\infty = \max_i |v_i|$ is an infinity norm of a vector v. Reformulate this problem as a linear programming problem.

3 Stochastic Integer Programming(5pts)

You have \$20,000 to invest. Stock XYZ sells at \$20 per share today. A European call option to buy 100 shares of stock XYZ at \$15 exactly six months from today sells for \$1000[1]. You can also raise additional funds which can be immediately invested, if desired, by selling these same call options. In addition, a 6-month (riskless zero-coupon) bond with \$100 face value sells for \$90. This means if you buy a bond today at \$90 you will get exactly \$100 in 6 months. You have decided to limit the number of call options that you buy or sell to at most 50.

You consider three scenarios for the price of stock XYZ six months from today: the price will be the same as today (scenario 1), the price will go up to \$40 (scenario 2), or drop to \$12 (scenario 3). Your best estimate is that each of these scenarios is equally likely.

The profit from buying one call option in scenario 1 is $\$100(20-15)-\$1000 = -\$500$ (loss), in scenario 2 the profit is $\$100(40 - 15) - \$1000 = \$1500$, and in scenario 3 the profit (loss) is $-\$1000$, as you will not exercise this option in 6 months.

- Formulate and solve a linear program to determine the portfolio of stocks, bonds, and options that maximizes expected profit, considering the expected value for the profits from each investment expected value.

- (ii) Suppose that the investor wants a profit of at least \$2000 in any of the three scenarios. Write a stochastic linear program that will maximize the investor's expected profit under this additional constraint.

4 Integer Programming(5pts)

You currently own a portfolio of eight stocks. Using the Markowitz model, you computed the optimal mean/variance portfolio. The weights of these two portfolios are shown in the following table:

[1] A European call option is the right to buy a stock at a future determined date at the agreed price

Stock	A	B	C	D	E	F	G	H
Your Portfolio	0.12	0.15	0.13	0.10	0.20	0.10	0.12	0.08
M/V Portfolio	0.02	0.05	0.25	0.06	0.18	0.10	0.22	0.12

You would like to rebalance your portfolio in order to be closer to the M/V portfolio. To avoid excessively high transaction costs, you decide to rebalance only three stocks from your portfolio. Let x_i denote the weight of stock i in your rebalanced portfolio. The objective is to minimize the quantity

$$|x_1 - 0.02| + |x_2 - 0.05| + |x_3 - 0.25| + \ldots + |x_8 - 0.12|$$

which measures how closely the rebalanced portfolio matches the M/V portfolio.

Formulate this problem as a mixed integer linear program.

5 Quadratic Integer Programming - Support Vector Machines (Practice only, do not turn in with your homework)

The following formulation is the standard formulation of support vector machines. The second term accounts for the points that do not satisfy the constraints for separation. This term is called ”hinge loss” and it depends linearly on the ”amount of violation” for each constraint.

$$\min_{\xi,\mathrm{w},\beta} \frac{1}{2}\|w\|^2 + c\sum_{i=1}^{n} \max\{1 - y_i(w^\top x_i + \beta), 0\}$$

Now, imagine instead I want to count and minimize the *number* of points which violate the constraints (the number of ”outliers”), hence I want to optimize

$$\begin{array}{rl} \min_{\xi,\mathrm{w},\beta} & \frac{1}{2}\|w\|^2 + c(\#\text{ of nonzeros}(\xi)) \\ \text{s.t} & y_i(w^\top x_i + \beta) \geq 1 - \xi_i, \quad i = 1, \ldots, n \\ & \xi \geq 0 \end{array}$$

- Formulate this problem as a problem with integer variables, quadratic objective and linear constraints.
- Write down a convex relaxation of this problem.