IE426 – Optimization models and applications

Fall 2014 – Homework #3

This homework accounts for 5% of the final grade. It is due on Tuesday, November 4, in class. There are 20 points available. For all problems where an AMPL model is required, include the model file, the data file (they can be the same file), and the optimal solution, shown with the command (e.g. `display`) used to print it.

1 Goal programming (5 pts.)

Consider the following Linear Programming problem:

$$
\begin{array}{llllll}
\min & 2x_1 & +x_2 & -11x_3 & +5x_4 & \\
\text{s.t.} & & & -2x_3 & +3x_4 & \leq 0 \\
 & x_1 & & & +x_4 & \geq 1 \\
 & x_1 & & -2x_3 & +2x_4 & = 0 \\
 & & x_2 & & +x_4 & \leq 0 \\
 & x_1 & +x_2 & -x_3 & +x_4 & \geq 2 \\
 & 3x_1 & -x_2 & -x_3 & -x_4 & = 0 \\
 & x_1 & -x_2 & +2x_3 & & \leq 2 \\
 & \multicolumn{5}{l}{x_1, x_2, x_3, x_4 \geq 0}
\end{array}
$$

It is infeasible, but we'd like to *select* a subset S of its constraint that form a feasible set which can be made feasible by using goal programming.

1. Check that the original LP is infeasible by solving it with AMPL.
2. Formulate a nonpreemptive goal programming (as a linear programming problem) problem which gives equal weight to all constraint violations and solve it in AMPL, describe which subset of constraints are feasible for this solution.
3. Write an Mixed Integer Linear Programming model that finds the subset S with maximum cardinality (using the logical constraints we learned in class). Note that, in this case, we do not care by how much a constraint is violated, as long as it is violated. In other words you are trying to satisfy as many of the constraints as possible, and if you violate one - then it does not matter if you violate it by a little but or by a lot.
4. Solve the IP model with AMPL. Based on the result, re-write the new LP that only includes the selected constraints, and solve it with AMPL to check that it really is feasible.
5. Formulate a preemptive goal programming (an LP) that gives first preference to the largest set of feasible constraints which you identified earlier. So you will have two LP problems to solve consecutively: one which should give you constraint violation equal to zero, and the other that minimizes the total violation for the remaining constraints. Solve these problems in AMPL.

2 Logic (10 pts.)

For each of the following propositions write models as a set of MILP constraints so that they hold true: Part 1.

1. $a \vee (b \wedge \neg c)$
2. Write the statement which is the opposite of the above.
3. $\neg(a \wedge b) \Rightarrow (c \vee d)$
4. $(\neg a \wedge (b \vee c)) \Leftrightarrow \neg d$
5. $(a \vee b \vee c) \Rightarrow (\neg a \wedge \neg b)$

Part 2.

1. If $\sum_{i=1}^{n} x_i \leq k$ then $\sum_{i=1}^{n} y_i \geq l$.
2. $\neg(\sum_i x_i \leq 3 \; AND \sum_i y_i \geq 3) \;\Rightarrow\; \sum_i y_i \geq 4$.
3. Exactly one out of
$$\sum_i x_i \geq 5, \sum_i y_i \geq 4$$
should be true.
4. Formulate
$$(x_1 + x_2 + x_3 \leq 2) \; OR \; (x_6 + x_7 + x_8 \geq 1) \; OR \; (x_9 + x_{10} \geq 1)$$

3 Formulations (5 pts.)

1. Formulate the following set as a feasible set of a linear programming problem (you do not need an objective function, but you can include an arbitrary one if it makes you feel better). $S = \{ y \in \mathcal{R}^n \mid \sum_{i=1}^{n} |y_i| \leq 1 \}$.
2. Formulate the following set as a feasible set of a mixed integer programming problem (again, you do not need an objective function, but you can include an arbitrary one). $S = \{ y \in \mathcal{R}^n \mid \sum_{i=1}^{n} |y_i| \geq 1, -M \leq y_i \leq M \, \forall i \}$.
3. Formulate the problem
$$\begin{aligned} \max \quad & c^T y \\ s.t. \quad & Ay \leq b \\ & -M \leq y_i \leq M \, \forall i \\ & y \neq y^* \\ & y \in \mathcal{Z}^n \end{aligned} \tag{1}$$
where $y^* \in \mathcal{Z}^n$ is a fixed vector, as an MIP.