ISE426 – Optimization models and applications

Fall 2015 – Quiz #1, October 8, 2015

First name	
Last name	
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You have 75 minutes. There are two problems. This quiz accounts for 10% of the final grade. There are 40 points available. Please write clear and concise statements. Use a **readable** handwriting: it will be hard to grade your answers if they are not readable — let alone give them full points. Use the back of each sheet if you need more space, for example to draw graphs. Do not use calculators.

1 Convexity and relaxations (10 pts.)

The following two problems are not convex, explain why (4 pts.):

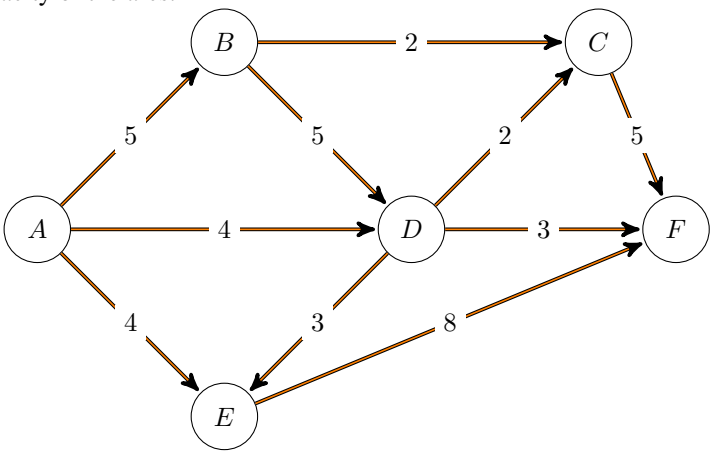
$$(1)\ \min \quad x^2 + y^2 \quad |x+y| = 1 \quad -1 \le x \le 1 \quad -1 \le y \le 1$$

$$(2)\ \max \quad x \quad |x+y| = 1 \quad x^2 + y^2 \le 1$$

For each problem, find upper and lower bounds on the optimal value and explain how you know that these are indeed upper and lower bounds. Can you find an optimal solution just from the lower and upper bounds? If yes, explain, if not, find the optimal solution graphically or by simple observation and check that it is between the upper and lower bounds. (6 pts.).

2 Max flow and duality (10 pts.)

Consider the problem of sending the largest possible amount of flow from source A to sink F using the following network, where the numbers on the arcs represent the capacity of the arcs.



1. Formulate this max flow problem as a linear programming problem, using the formulations studied in this course. (5 pts.).

2. Write the dual of the above MAX FLOW problem (5 pts.).

3 Linear/Integer programming model, relaxations, duality, upper and lower bounds (20 pts.)

Kyra is applying to several high schools. She has 8 high schools to choose from, each high school has a ranking (higher is better) and each of them requires several days to complete the applications. Below in the table are the list of high schools and the their ranking and the time their applications require. She has 10 days in total. She wants to maximize the total ranking of all schools she applies to.

School	#1	#2	#3	#4	#5	#6	#7	#8
rating	4	5	3	2	4	3	3	2
# of days	5	9	4	3	2	1	2	1

(a) Formulate an optimization problem to choose the schools that she can apply to within 10 days and whose total rating is maximized. Explain why the resulting problem is not a convex problems. Create a linear programming relaxation and solve it (by applying greedy method used in homework and in class for knapsack problems). What does the solution of the relaxation give you? Can you use this solution to generate a feasible solution of the original problem? Can you generate an optimal solution to the original problem? Justify your answers - prove that the solution you obtain is optimal by using the optimal value obtained from the relaxation. (8pts)

(b) There is an additional constraint, which says that Kyra can only apply to one of the first three high schools (#1, #2 or # 3). Add this constraint to your formulation. What is the optimal solution of the new problem? Can you prove that it is an optimal solution? (2pts)

(c) Consider the LP relaxation of the problem defined in part (a). Write down the dual of the linear programming relaxation. (Hint: do not forget the upper bounds on the primal variables). Compute the dual optimal solution from complementary slackness conditions and show that this solution is feasible (hence the primal solution you computed by hand is optimal). (7 pts)

(d) Consider the LP relaxation in part (a). How much higher the ranking of the 4th school should be for Kyra to consider applying there. Derive your answer from complementarity conditions and feasibility of the dual solution. (3 pts)

extra sheet