ISE429. Homework 1

Unless stated otherwise, the problems are from the Ross textbook (11th edition):

1) (weight 0.15) 4-18. Here "proportion" means "limiting proportion (fraction of time)"

2) (weight 0.20) 4-20 and 4-25

3) (weight 0.15) 4-52

4) (weight 0.15) 4-58

5) (weight 0.15) 4-60

6) (weight 0.20) Discrete time Markov chain $\{X_n,\ n = 0, 1, 2, \ldots\}$ takes values in state space $\mathcal{X} = \{-5, -4, -3, -2, -1, 0, 1, 2, \ldots\}$. It has the following structure:

$$X_n = \max\{X_{n-1} + Y_n, -5\}, \quad n = 1, 2, \ldots,$$

where $Y_n,\ n = 1, 2, \ldots$ are i.i.d. (independent identically distributed) random variables, taking integer values, and such that

$$E|Y_1| < \infty, \qquad EY_1 < 0, \quad P\{Y_1 = 1\} > 0.$$

This Markov chain is irreducible (explain why) and aperiodic (explain why). Is it positive recurrent? *Comment: You must give rigorous proofs.*