ISE426 Optimization models and applications, Fall 2015
Quiz 2 Solution,

1. Reformulate as a Linear Programming Problem (8pts.)

By introducing new variables y_i for i = 1, ... , k, we have

$$\begin{aligned}
\min \ & c^T x \\
\text{s.t.} \ & \\
& |(a^i)^T x| \leq y_i \qquad \text{for } i = 1, ..., k \\
& \sum_{i=1}^{k} y_i \leq b \\
& y_i \geq 0 \qquad \text{for } i = 1, ..., k
\end{aligned}$$

Since this formulation is not linear, we make it linear in this way

$$\begin{aligned}
\min \ & c^T x \\
\text{s.t.} \ & \\
& (a^i)^T x \leq y_i \qquad \text{for } i = 1, ..., k \\
& -(a^i)^T x \leq y_i \qquad \text{for } i = 1, ..., k \\
& \sum_{i=1}^{k} y_i \leq b \\
& y_i \geq 0 \qquad \text{for } i = 1, ..., k
\end{aligned}$$

2. Mixed Integer/Goal Programming (12 pts.)

2.1.

The nonpreemptive goal programming formulation to minimize the constraint violation is as follows

$$\begin{aligned}
\min \ & y_1^+ + y_2^- + y_3^- \\
\text{s.t.} \ & \\
& -x_1 + 2x_2 \leq -2 + y_1^+ \\
& 2x_1 + x_2 \geq 1 - y_2^- \\
& x_1 - 3x_2 \geq -4 - y_3^- \\
& x_1 \in [-1, 4] \\
& x_2 \in [1, 6] \\
& y_1^+, y_2^-, y_3^- \geq 0
\end{aligned}$$

2.2.

This problem can be formulated by introducing the new binary variables as follows

$$z_i = \begin{cases} 1 & \text{If constraint } i^{th} \text{ is violated.} \\ 0 & \text{Otherwise} \end{cases}$$

$$\min \quad y_1^+ + y_2^- + y_3^- + 5z_1 + 5z_2 + 5z_3 \tag{1}$$
$$\text{s.t.} \tag{2}$$
$$-x_1 + 2x_2 \leq -2 + y_1^+ \tag{3}$$
$$2x_1 + x_2 \geq 1 - y_2^- \tag{4}$$
$$x_1 - 3x_2 \geq -4 - y_3^- \tag{5}$$
$$x_1 \in [-1, 4] \tag{6}$$
$$x_2 \in [1, 6] \tag{7}$$
$$y_1^+ \leq M_1 z_1 \tag{8}$$
$$y_2^- \leq M_2 z_2 \tag{9}$$
$$y_3^- \leq M_3 z_3 \tag{10}$$
$$y_1^+, y_2^-, y_3^- \geq 0 \tag{11}$$
$$z_i \in \{0, 1\} \tag{12}$$

where M_1, M_2 and M_3 are sufficiently large numbers.

2.3.

3 and 8 follow that

$$-x_1 + 2x_2 + 2 \leq M_1 z_1 \tag{13}$$

Based on 13, 6 and 7, by setting x_1 and x_2 to -1 and 6, respectively, we have

$$15 \leq M_1$$

Similarly, 4 and 9 follow that

$$2x_1 + x_2 - 1 \geq -M_2 z_2 \tag{14}$$

According to 14, 6 and 7, by choosing $x_1 = -1$ and $x_2 = 1$, we have

$$2 \leq M_2$$

Similar to the other cases, 5 and 10 follow that

$$x_1 - 3x_2 + 4 \geq -M_3 z_3 \tag{15}$$

Based on 15, 6 and 7, by setting x_1 and x_2 equal to -1 and 6, respectively, we have

$$15 \leq M_3$$

3. Binary optimization modeling (15 pts.)

Let

$$x_i = \begin{cases} 1 & \text{If person i is invited.} \\ 0 & \text{Otherwise} \end{cases}$$

The constraints are

Constraint 1:

$\sum_{i=1}^{12} x_i \leq 8$

Constraint 2:

$x_4 \leq x_6$

$x_6 \leq x_4$

Constraint 3:

$x_7 \leq x_3$

Constraint 4:

$x_2 + x_{10} \leq x_1$

Constraint 5:

$\sum_{i=9}^{12} x_i \leq 1$

Constraint 6:

$x_1 + x_8 + x_9 \leq 2$

Constraint 7:

$$y = \begin{cases} 1 & \text{If she invites no boys.} \\ 0 & \text{If she invites at least two boys.} \end{cases}$$

$\sum_{i=9}^{12} x_i \leq 4(1-y)$
$\sum_{i=9}^{12} x_i \geq 2(1-y)$
$y \in \{0,1\}$

4. Branch and Bound (5+4 pts)

4.1.

Case 1: RHS = 22

The optimal solution of the LP relaxation of node 1 (by using the greedy method) is

$$(1, 1, 0.6, 1, 0, 1)$$

Branching on the variable x_3 generates two nodes as follows

- Node 2 : $x_3 = 0$
  The optimal solution of the LP relaxation of this node is
  $$(1, 1, 0, 1, \frac{3}{4}, 1)$$
- Node 3 : $x_3 = 1$
  The optimal solution of the LP relaxation of this node is
  $$(\frac{5}{7}, 1, 1, 1, 0, 1)$$

Since the solutions of the LP relxation of all three nodes are non-integer, this tree has more than three nodes.

Case 2: RHS = 24

The optimal solution of the LP relaxation of node 1 (by using the greedy method) is

$$(1, 1, 1, 1, 0, 1)$$

Since the optimal solution of the relaxation problem is integer, it is the optimal solution of the main problem and the corresponding tree has only one node.

As a result, case 1 (RHS=22) has larger tree in comparison to case 2 (RHS=24).

4.2.

The tree in the homework has only three nodes; Based on last part, by changing the right hand side to 22, the corresponding tree has more than three nodes so this case has larger tree in comparison to 23. On the other hand, the other case (changing the right hand side coefficient to 24) has only one node, so its tree is smaller.