ISE410 – Final 2014

1. In a factorial design with four factors, the following runs were performed: $d = 10$, $ac = 12$, $b = 16$, $abcd = 20$.

a) What is the full defining relation for this design?

b) Find the estimates for the main factor effects assuming all interactions are negligible. Are any of the main factors aliased with each other?

c) Suppose additional four runs have been performed: $ad = 14$, $c = 18$, $ab = 10$, $bcd = 16$. What are the estimates of main factor effects from these four runs (assuming interactions are negligible)? Are any of main factors aliased in this 4-run design?

d) Combine the two sets of runs to find the estimates of the main effects. Are any of them still aliased with each other in the combined design?

e) What is the full defining relation for the combined design?

2. Consider a single factor experiment with four levels of the factor and three replicates. The response data turned out to be $y_{1\cdot} = 27$, $y_{2\cdot} = 36$, $y_{3\cdot} = 36$, $y_{4\cdot} = 45$.

a) What is the value of $y_{\cdot\cdot}$?

b) What is the value of $\bar{y}_{\cdot\cdot}$?

c) Find the value of $SS_{\text{Treatments}}$ for the corresponding ANOVA model.

e) What is the number of degrees of freedom for SS_E in the corresponding ANOVA model?

f) What is the value of the test statistic for the F-test for the significance of the factor if it is known that $\sum_{i=1}^{4}\sum_{j=1}^{3}(y_{ij} - \bar{y}_{i\cdot})^2 = 24$?

3. Using a response surface design, the fitted second order model of the form

$$\hat{y} = 30 + 12x_1 + 6x_2 - 2x_1^2 - 5x_2^2 - 4x_1x_2$$

for the response y was obtained.

a) Find the stationary point in the original coordinates.

b) Find the value $\hat{y}_s$ of the predicted response at the stationary point.

c) Write the model in the canonical form. What is the type of the stationary point?

d) Find the transformation from the original variables to the canonical ones.

4. A factorial experiment with three factors is conducted. Factor A and B are crossed and C is nested under B. B is fixed, A and C are random. The number of levels is 2, 3 and 4 for A, B and C, respectively. There are 2 replicates. Assume the restricted model.

a) Write down the full ANOVA model.

b) Write down the expression for $SS_{C(B)}$.

c) Write down the expressions for $E(MS_{C(B)})$ and $E(MS_{AB})$.

d) Propose an F-test for the significance of factor C.

e) Propose an F-test for the significance of interaction AB.