# ISE420 hw#4 Name: Bolun Xu

9.10

2.

```
#Model

set row = 1..6;

set col = 1..6;

param A{row};

param B{row};

param pu{row};

param pd{row};

param s{row, col};

param beta{row, col};

var theta{row} >= 0;

var Ps{row} ;

var p{row,col};


minimize MinCost : sum {i in row} A[i] * Ps[i]* Ps[i] + sum {i in row} B[i] * Ps[i];

Con1{i in row, j in col}: p[i,j] = beta[i,j] * ( theta[i] - theta[j] );

Con2{i in row}: sum{j in col} p[i,j] = Ps[i];

Con3{i in row}: Ps[i] <= pu[i];

Con4{i in row}: Ps[i] >= pd[i];

con5{i in row, j in col}: -1 * s[i,j] <= p[i,j];

con6{i in row, j in col}:  s[i,j] >= p[i,j];

option solver cplex;

param A := 1 0.1 2 0 3 0 4 0.2 5 0 6 0 ;

param B := 1 1 2 0 3 0 4 0.5 5 0 6 0 ;

param pu := 1 20 2 -4 3 -5 4 60 5 -4 6 -5 ;

param pd := 1 0 2 -4 3 -5 4 10 5 -4 6 -5 ;

param s :
1 2 3 4 5 6   :=
1   0   20  30  0   0   0
2   20  0   0   44  40  0
3   30  0   0   34  0   0
4   0   44  34  0   0   56
5   0   40  0   0   0   34
6   0   0   0   56  34  0
;

param beta :
1 2 3 4 5 6   :=
1   0    4    2.2 0    0    0
2   4    0    0   5.3  2.8  0
3   2.2  0    0   3.5  0    0
4   0    5.3  3.5 0    0    5.5
5   0    2.8  0   0    0    4.3
6   0    0    0   5.5  4.3  0
;
```

```
CPLEX 12.6.1.0: optimal solution; objective 39.40000001
11 separable QP barrier iterations
No basis.
ampl: display theta;
theta [*] :=
1  22.7066
2  21.4796
3  21.3011
4  21.8463
5  20.4612
6  20.7284
;

ampl: display Ps;
Ps [*] :=
1   8
2  -4
3  -5
4  10
5  -4
6  -5
;

ampl: display p;
p [*,*]
:       1          2          3          4          5          6          :=
1    0       4.90799    3.09201    0          0          0
2   -4.90799    0          0         -1.94341    2.85141    0
3   -3.09201    0          0         -1.90799    0          0
4    0       1.94341    1.90799    0          0          6.14859
5    0      -2.85141    0          0          0         -1.14859
6    0          0          0         -6.14859    1.14859    0
;
```

A: Optimal quantities of gens: P1=8;P4=10;

All load is satisfied:

```
Ps [*] :=
1   8
2  -4
3  -5
4  10
5  -4
6  -5
;
```

Total cost =39.4;

3.

```
#Model

set row = 1..6;

set col = 1..6;

param A{row};

param B{row};

param pu{row};

param pd{row};

param s{row, col};

param beta{row, col};

var theta{row} ;

var Ps{row} ;

var p{row,col};


minimize MinCost : sum {i in row} A[i] * Ps[i]* Ps[i] + sum {i in row} B[i] * Ps[i];

Con1{i in row, j in col}: p[i,j] = beta[i,j] * ( theta[i] - theta[j] );

Con2{i in row}: sum{j in col} p[i,j] = Ps[i];

Con3{i in row}: Ps[i] <= pu[i];

Con4{i in row}: Ps[i] >= pd[i];

con5{i in row, j in col}: -1 * s[i,j] <= p[i,j];

con6{i in row, j in col}:  s[i,j] >= p[i,j];

con7{i in row, j in col}:  p[i,j] + p[j,i] = 0;

con8: theta[1]=0;

option solver cplex;
```

Put qij=0 means set θi=0;

Data does not change.

```
CPLEX 12.6.1.0: optimal solution; objective 39.40000003
13 separable QP barrier iterations
No basis.
ampl: display theta;
theta [*] :=
1   0
2  -1.227
3  -1.40546
4  -0.860317
5  -2.24536
6  -1.97824
;

ampl: display p;
p [*,*]
:        1          2          3          4          5          6          :=
1    0          4.90799    3.09201    0          0          0
2   -4.90799    0          0         -1.94341    2.85141    0
3   -3.09201    0          0         -1.90799    0          0
4    0          1.94341    1.90799    0          0          6.14859
5    0         -2.85141    0          0          0         -1.14859
6    0          0          0         -6.14859    1.14859    0
;

ampl: display Ps;
Ps [*] :=
1   8
2  -4
3  -5
4  10
5  -4
6  -5
;
```
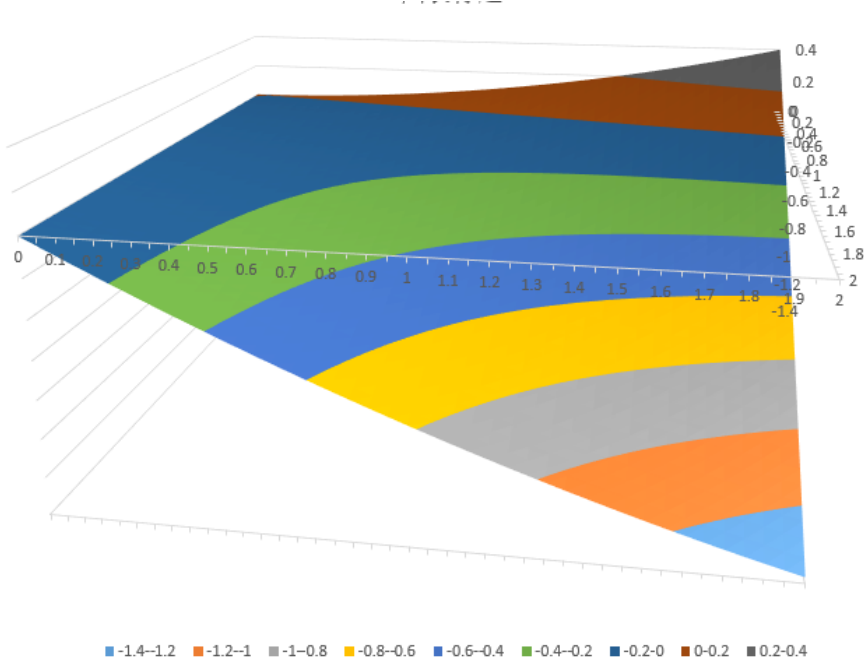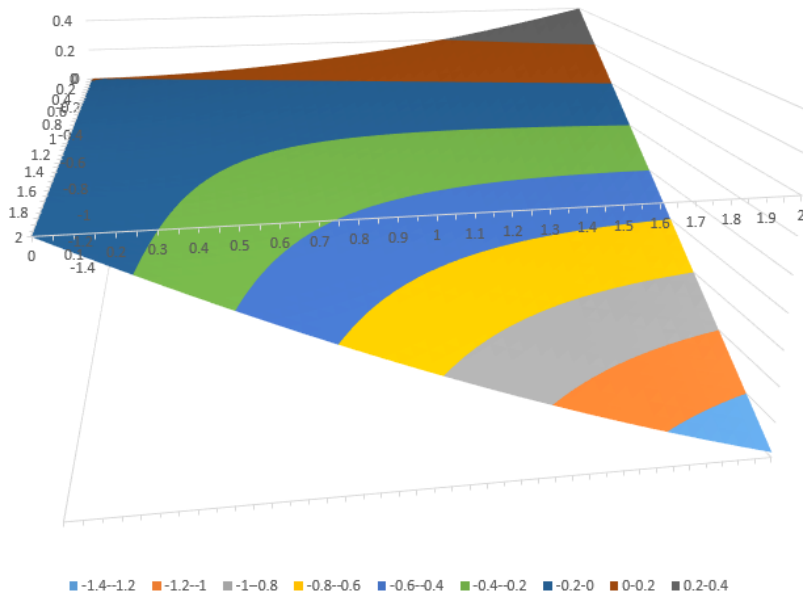
We only add contraints:

pij+pji=0; & qij == 0.

It does not change the optimal solution.

4.1 $p_{ij} = g_{ij}|v_i|^2 - |v_i||v_j|\left(g_{ij}\cos(\theta_i - \theta_j) - b_{ij}\sin(\theta_i - \theta_j)\right)$



■ -1.4--1.2   ■ -1.2--1   ■ -1--0.8   ■ -0.8--0.6   ■ -0.6--0.4   ■ -0.4--0.2   ■ -0.2-0   ■ 0-0.2   ■ 0.2-0.4

4.2 $q_{ij} = b_{ij}|v_i|^2 - |v_i||v_j|\left(g_{ij}\sin(\theta_i - \theta_j) + b_{ij}\cos(\theta_i - \theta_j)\right)$



■ -1.4--1.2   ■ -1.2--1   ■ -1--0.8   ■ -0.8--0.6   ■ -0.6--0.4   ■ -0.4--0.2   ■ -0.2-0   ■ 0-0.2   ■ 0.2-0.4

They are not convex functions of the voltage magnitudes.