IE426 – Optimization models and applications

Fall 2015 – Quiz #2, November 12, 2015

First name	
Last name	
Lehigh email	

You have 75 minutes. This quiz accounts for 10% of the final grade. There are 40 points available plus 4 bonus points. Please write clear and concise statements. Use a **readable** handwriting: it will be hard to grade your answers if they are not readable — let alone give them full points. Do not use calculators. For each model, clearly specify the meaning of each variable and of each constraint.

1 Reformulate as a Linear Programming Problem (8pts.)

Consider the following problem, where the vectors $a^1, \ldots, a^k, c$ and a scalar b are given:

$$\begin{array}{ll} \min & c^T x \\ st. & \sum_{i=1}^{k} |(a^i)^T x| \leq b \end{array}$$

Reformulate this problem using linear constraints as we studied in class introducing appropriate extra variables. (Hint: first consider if the problem is convex or not).

2 Mixed Integer/Goal Programming (12 pts.)

The following LP is infeasible

$$\begin{array}{lrrl} \min & 2x_1 & +x_2 & \\ & -x_1 & +2x_2 & \leq -2 \\ & 2x_1 & +x_2 & \geq 1 \\ & x_1 & -3x_2 & \geq -4 \\ & x_1 & & \in [-1, 4] \\ & & x_2 & \in [1, 6] \end{array}$$

1. Write a nonpreemptive goal programming formulation to minimize the constraint violation. Do not solve! (5pts)

2. In nonpreemptive goal programming we minimize the total sum of the constraint violation. You can think of it as minimizing the cost of violating constraints if each time you violate a constraint by one unit you pay one unit of cost. Now imagine that each time you violate one of the constraints you pay a fixed cost of 5 on top of the per-unit cost of 1. Notice that for the fixed cost we do not care by how much these constraints are violated, as long as they are. Formulate the problem of minimizing the total cost of violating the constraints. (5 pts).

(cont'd)

3. Write the good values for the "big" M constants (2pts).

3 Binary optimization modeling (15 pts.)

Kyra is planning whom to invite for her 13th birthday. The list of possible friends to invite is as follows, together with the expected value of the gift they will bring

1. Danielle - $20
2. Julia - $40
3. Georgina - $10
4. Sophia - $35
5. Leyla - $30
6. Helena - $20
7. Emily -$20
8. Miriam -$25
9. Jacob -$15
10. Benjamin - $45
11. Mario - $40
12. Oliver -$30

1. Kyra wants to maximize the total value of the gift received, but there are constraints on whom she can invite. Formulate an Integer (Binary) Programming model for the problem including the constraints below (DO NOT SOLVE). Each of the constraint has a point value if you write it correctly. The more complicated constraints are worth more points.

1. She can invite at most 8 people (1 pt)
2. If she invited Helena, she has to invite Sophia and vice versa. (1 pt)
3. She can only invite Emily if she invites Georgina, but she can invite Georgina without Emily (Emily is Georgina's older sister). (2 pts)
4. Julia and Benjamin cannot stand each other, so at most one of them can be invited, but not both. (2pts)
5. She has to invite at least 2 boys (the last 4 names). (2pts)
6. If she invites Jacob she cannot invite Miriam and Danielle together. (3pts)
7. Instead of the constraint (or a set of constraints) that there should be at least two boys, write a constraint that either there should be at least two boys or no boys at all. (4 pts)

(cont'd)

4 Branch and Bound (5+4 pts)

Recall the knapsack problem from homework #3, which you solved by Branch and Bound

$$\begin{array}{llllllll} \max & 6x_1 & +5x_2 & +4x_3 & +4x_4 & +3x_5 & +3x_6 & \\ st. & 7x_1 & +5x_2 & +5x_3 & +4x_4 & +4x_5 & +3x_6 \leq 23 \\ & & & & & x_i & \in \{0,1\}\ \forall i. \end{array}$$

Now consider changing the right hand side coefficient from 23 to either 22 or 24. If you now apply B&B to the two new problems, which case will produce a larger B&B tree? Explain (6pts)

Bonus: Will either of them be larger than the tree in the homework (the one given in the posted HW solution)? (4pts)