ISE426 Optimization Models and Applications, Fall 2015
8 October, Quiz #1 - COMMENTS,

ANSWER 1. **Convexity and relaxations (10 pts.)**

Part a. **(4 pts.)**

Most of you solved this part correctly. Some of you failed to mention which constraint leads to the non-convexity. $|x + y| = 1$ creates non-convexity.

Only drawing the feasible set is not sufficient. If you want to show non-convexity of the feasible set, then you should prove that the line segment (ie, convex combination) between any two points in the set DOES NOT belong to the set.

-2 points are deducted if you only draw the feasible set without any of the arguments above.

Part b. **(6 pts.)**

3 points for the associated upperbound, lowerbound and optimal value of the first problem in part (a). (1 point each)

3 points for the associated upperbound, lowerbound and optimal value of the second problem in part (a). (1 point each)

Note that you should have mentioned your reasoning. Only saying 1 is upperbound for the problem is NOT sufficient. You should have stated why. Sample statement : "For minimization-type problem, any feasible solution gives an upperbound. The point $(65, 97)$ is in the feasible set since it complies with the constraints. Hence its corresponding objective value 926 is an upperbound for this problem".

Another important note: Some of you said something like : "(0,1) is upperbound". This statement is WRONG. The upperbound (and lowerbound) is a value. And this value gives you the upperbound for the objective function value of your original problem.

ANSWER 2. **(Max flow and duality (10 pts.)**

Most of you solved this question correctly.

ANSWER 3. **Linear/Integer programming model, relaxations, duality, upper and lower bounds (20 pts.)**

Part a. **(8 pts.)**

**Part a1. (2 pts.)** Formulate an optimization problem to choose the schools that she can apply to within 10 days and whose total rating is maximized.

This is an integer model and you should have:

$x_i \in \{0, 1\} \quad \forall i \in \{1, 2, ..., 8\}$.

Some of you said $x_i \in (0, 1)$. That is not the correct notation. Also $x_i \in \mathbb{Z} \quad \forall i \in \{1, 2, ..., 8\}$ is not correct since $x_i$ can only be either 0 or 1.

**Part a2. (1 pt.)** Explain why the resulting problem is not a convex problems.

We have non-convexity due to integrality constraint:

$x_i \in \{0, 1\} \quad \forall i \in \{1, 2, ..., 8\}$.

**Part a3. (2 pts.)** Create a linear programming relaxation and solve it (by applying greedy method used in homework and in class for knapsack problems).

Note that you should apply the greedy method to the LP Relaxation Problem, not the original problem. The greedy method solution will give the optimal solution for the LP problem. The optimal soln is $(0.8, 0, 0, 0, 1, 1, 1, 1)$ with objective value 15.2.

**Part a4. (1 pt.)** What does the solution of the relaxation give you?

This is a maximization-type problem. The optimal value of the relaxed problem is upperbound. 15.2 is the upperbound for the problem in part a.1.

**Part a5. (1 pt.)** Can you use this solution to generate a feasible solution of the original problem?

Since $x_1 = 0.8$ is fractional element, you should take this issue into consideration for creating a feasible solution. For instance $(0, 0, 0, 0, 1, 1, 1, 1)$ is a feasible solution. Also $(0, 0, 1, 0, 1, 1, 1, 1)$ is another feasible solution.

**Part a6. (1 pt.)** Can you generate an optimal solution to the original problem? Justify your answers - prove that the solution you obtain is optimal by using the optimal value obtained from the relaxation.

$(0, 0, 1, 0, 1, 1, 1, 1)$ is a feasible solution for the original problem with the objective function value is 15 (ie, 15 is lowerbound). The upperbound is 15.2. The objective function value of the original problem MUST BE integer. This is because, the objective function coefficients are integer and all variables are integer in the original problem. Hence, we cannot obtain a better solution that 15. Then, 15 is both lower and upperbound. Hence it is the optimal value.

Most of you forgot to mention that the objective function value of the original problem MUST BE integer. I put a sign "Intg" and -0.5: in order to point out this aspect.

Part b. **(2 pts.)**

Some of you got the statement wrong. Kyra can only apply to one of the first 3 highschool means that $x_1 + x_2 + x_3 \leq 1$. Hence, Kyra can apply to at most 1 school out of School 1,2 and 3.

The corresponding lowerbound and upperbound arguments will prove that 15 is still the optimal.

Part c. **(7 pts.)**

**Part c1. (3 pt.)** Consider the LP relaxation of the problem defined in part (a). Write down the dual of the linear programming relaxation. (Hint: do not forget the upper bounds on the primal variables).

You should note that there are 8 constraints for: $x_i \in \{0, 1\} \quad \forall i \in \{1, 2, ..., 8\}$.

Hence there should be 8 dual variables associated with the above constaints. Additiaonally, 1 dual variable is coming from the 10-day restriction constraint. There are 9 dual variables in total.

**Part c2. (4 pt.)** Compute the dual optimal solution from complementary slackness conditions and show that this solution is feasible (hence the primal solution you computed by hand is optimal).

Write down the ALL CS conditions. Then find the 9 dual variables values from those equations.

Part d. **(3 pts.)**

You should have derived the value from the CS condition and dual feasibility. This part is closely associated with the previous part.