Midterm '14

1. Consider a single factor experiment with three levels of the factor and two replicates. The response data turned out to be $y_{11} = 20$, $y_{12} = 22$, $y_{21} = 24$, $y_{22} = 26$, $y_{31} = 28$, $y_{32} = 30$.

a) What is the value of $y_{..}$?

b) What is the value of $\bar{y}_{..}$?

c) What are the values of $\bar{y}_{1.}$, $\bar{y}_{2.}$ and $\bar{y}_{3.}$?

d) Find the value of $SS_{\text{Treatments}}$ for the corresponding ANOVA model.

e) What is the number of degrees of freedom for SS_E in the corresponding ANOVA model?

f) What is the value of MS_E for the corresponding ANOVA model?

2. A single factor experiment was performed for 4 levels of the factor with 8 replicates. It was found that $\bar{y}_{1\cdot} = 15$, $\bar{y}_{2\cdot} = 11$, $\bar{y}_{3\cdot} = 13$, $\bar{y}_{4\cdot} = 18$ and $MS_E = 4$.

First, Consider the following two contrasts: $\Gamma_1 = \mu_1 - \mu_2$ and $\Gamma_2 = \mu_3 - \mu_4$.

a) Are these contrasts orthogonal and why?

b) If they are orthogonal, construct a third contrast orthogonal to both of them.

c) Use Scheffe's method to test the hypothesis $\Gamma_1 = 0$ at $\alpha = 0.05$

d) Suppose a Dunnett's two-sided test for all pairwise mean comparisons with a control is performed, with the level 1 acting as the control. Which means would be pronounced significantly different from the control at $\alpha = 0.05$?

3. A single factor experiment was performed at four levels of the factor so that each replicate constituted a single block. The total number of runs was equal to 20. The following results of the ANOVA analysis are known: $SS_T = 1000$, $MS_{\text{Treatments}} = 200$, $\hat{\sigma}^2 = 10$.

a) What was the value of SS_E?

Since $MS_E = \hat{\sigma}^2 = 10$ and the number of degrees of freedom for SS_E is equal to $(a-1)(b-1) = 3 \cdot 4 = 12$, we obtain $SS_E = 10 \cdot 12 = 120$.

b) What was the value of $SS_{\text{Treatments}}$?

$$SS_{\text{Treatments}} = MS_{\text{Treatments}} \cdot (a-1) = 200 \cdot 3 = 600$$

c) What was the value of MS_{Blocks}?

$$SS_{\text{Blocks}} = SS_T - SS_{\text{Treatments}} - SS_E = 1000 - 600 - 120 = 280$$

and

$$MS_{\text{Blocks}} = \frac{SS_{\text{Blocks}}}{b-1} = \frac{280}{4} = 70$$

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d) What was the value of MS_E?

$$MS_E = \hat{\sigma}^2 = 10$$

e) What was the value of the test statistic for the F-test on the significance of the main factor?

$$f_0 = \frac{MS_{\text{Treatments}}}{MS_E} = \frac{200}{10} = 20$$

4. A 2^{5-2} experiment was performed, with $D = AB$ and $E = AC$ acting as design generators.

a) List all the runs that were performed.

Using A, B and C as "independent" factors we can begin with full set of runs for these three and then use the generators to find the signs of D and E. The resulting runs are de, a, be, abd, cd, ace, bc, $abcde$.

b) Write down the alias relationships for all effects up to second-order.

$$A = BD = CE$$
$$B = AD$$
$$C = AE$$
$$D = AB$$
$$E = AC$$
$$BC = DE$$
$$BE = CD$$

c) Suppose the effects A, C and E were found to be significant (using Lenth's method, for example). What are the possible ambiguities here arising due to aliases? What kind of fold over sequential experiments would be able to resolve them?

Since $A = CE$, $C = AE$ and $E = AC$, it is not clear which of these three are really significant. It could be all three or just any two of them plus their interaction. Either a full fold over or a fold over on any of these three factors would be able to resolve this ambiguity.

d) List the runs for the full fold over and the fold over on factor C

Full fold over: abc, $bcde$, acd, ce, abe, bd, ade, (1).

Fold over on C: cde, ac, bce, $abcd$, d, ae, b, $abde$.

e) For the two fold-overs from (d), write down alias relationships for effects up to second order, for the combined design.

The quickest way to answer this question is by noting that a full fold over applied to a resolution III design de-aliases all main effects from two-factor interactions. Thus the only effects (counting those up to second order only) that will remain aliased will be $BD = CE$, $BC = DE$ and $BE = CD$.

For a fold over on one of the factors, we can note that the resulting design will have this factor and its two-way interactions de-aliased from the rest of such effects (up to second order). Thus (using a fold over on C, for example) we have the remaining aliased effects: $A = BD$ and $B = AD$.