5-12. solution:

(a) $P(X_1<X_2<X_3) = P(X_1=\min(X_1,X_2,X_3))\cdot P(X_2<X_3 \mid X_1=\min(X_1,X_2,X_3))$

$= \frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}\cdot P(X_2<X_3)$

$= \frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_2+\lambda_3)(\lambda_2+\lambda_3)}$

(b) $P(X_1<X_2 \mid \max(X_1,X_2,X_3)=X_3) =$

$= \frac{P(X_1<X_2<X_3)}{P(X_1<X_2<X_3)+P(X_2<X_1<X_3)}$

$= \frac{\frac{\lambda_1\lambda_2}{\lambda_2+\lambda_3}}{\frac{\lambda_1\lambda_2}{\lambda_2+\lambda_3}+\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_3}}$

$= \frac{1}{1+\frac{\lambda_2+\lambda_3}{\lambda_1+\lambda_3}}$

(c) $E[\max X_i \mid X_1<X_2<X_3]$.

$= E[(X_3-X_2)+(X_2-X_1)+X_1 \mid X_1<X_2<X_3]$.

$= E[\min X_i \mid X_1<X_2<X_3] + E[X_2 \mid X_2<X_3] + E[X_3-X_2 \mid X_2<X_3]$

$= E[\min X_i] + E[\min(X_2,X_3) \mid X_2<X_3] + E[X_3]$

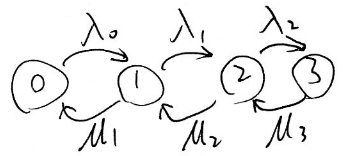
$= \frac{1}{\lambda_1+\lambda_2+\lambda_3} + \frac{1}{\lambda_2+\lambda_3} + \frac{1}{\lambda_3}$.

6-9. solution: if the system is non-empty, it is a poisson process with mean μ.

$$P_{ij} = \frac{e^{-\mu t}(\mu t)^{i-j}}{(i-j)!}, \quad 0 \le j \le i.$$

$$P_{i0}(t) = 1 - \sum_{j=1}^{i} \frac{e^{-\mu t}(\mu t)^{i-j}}{(i-j)!} = \sum_{k=i}^{\infty} \frac{e^{-\mu t}(\mu t)^{k}}{k!}$$

6-15. (a).



$\lambda_0 = \lambda_1 = \lambda_2 = 3$

$\mu_2 = \mu_3 = 4, \ \mu_1 = 2.$

$$\left.\begin{aligned} &P_0\lambda_0 = P_1\mu_1 \Rightarrow P_1 = \tfrac{3}{2}P_0, \\ &P_1\lambda_1 = P_2\mu_2 \Rightarrow P_2 = \tfrac{3}{4}P_1, \\ &P_2\lambda_2 = P_3\mu_3 \Rightarrow P_3 = \tfrac{3}{4}P_2, \\ &P_0 + P_1 + P_2 + P_3 = 1 \end{aligned}\right\} \Rightarrow P_0 = \frac{32}{143}$$

potential customers enter the system is $P_0 + P_1 + P_2 = \frac{116}{143}$

(b) $\mu_1 = 4$, others are the same as above.

$$\left.\begin{aligned} &P_1 = \tfrac{3}{4}P_0, \ P_2 = \tfrac{3}{4}P_1, \ P_3 = \tfrac{3}{4}P_2. \\ &P_0 + P_1 + P_2 + P_3 = 1 \end{aligned}\right\} \Rightarrow P_0 + P_1 + P_2 = \frac{148}{175}$$

2. $\lambda_n = \frac{\lambda}{n+1}$, $n \geq 0$.

$\mu_n = \mu$, $n \geq 1$

$P_1 = \frac{\lambda}{\mu} P_0$ $P_2 = \frac{1}{2} \cdot \left(\frac{\lambda}{\mu}\right)^2 P_0, \cdots$

$P_n = \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n P_0$

$1 = \sum P_i = \left[1 + \sum \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n\right] P_0 \Rightarrow P_0 = e^{-\frac{\lambda}{\mu}} \Rightarrow P_n = \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n e^{-\frac{\lambda}{\mu}} \sim \text{Poisson}\left(\frac{\lambda}{\mu}\right)$

23. (a). $\lambda_0 = \frac{3}{10}$, $\lambda_1 = \frac{2}{10}$, $\lambda_2 = \frac{1}{10}$,

$\mu_1 = \frac{1}{8}$, $\mu_2 = \frac{2}{8}$, $\mu_3 = \frac{2}{8}$.

$$\left.\begin{array}{l} P_1 = \frac{12}{5} P_0 , \ P_2 = \frac{4}{5} P_1 = \frac{48}{25} P_0, \ P_3 = \frac{192}{250} P_0 \\ P_0 + P_1 + P_2 + P_3 = 1 \end{array}\right\} \Rightarrow P_0 = \frac{250}{1522}$$

Average number not in use $= P_1 + 2P_2 + 3P_3 = \frac{1068}{761}$

(b). proportion of time both repairmen are busy $= P_2 + P_3 = \frac{336}{761}$

6).(a) $\begin{cases} P_n = \lambda_0 \cdots \lambda_n P_0 \\ \sum P_n = 1 \end{cases} \Rightarrow$ M.C. is irreducible.

~~for certain~~ if $\forall P_i \nrightarrow 0$, then M.C. is not transient.

~~if $P_i \nrightarrow 0$, then M.C. is not transient.~~

$$P_0 \left(1 + \sum_{i=0}^{\infty} \lambda_0 \lambda_1 \cdots \lambda_i\right) = 1$$

$$1 + \sum_{i=0}^{\infty} \lambda_0 \lambda_1 \cdots \lambda_i = 1 + \sum_{i=0}^{3} \lambda_0 \cdots \lambda_i + \sum_{i=4}^{\infty} \lambda_0 \cdots \lambda_3 \cdot \lambda_4 \cdots \lambda_i$$

$$= 1 + C_1 + C_2 \cdot \sum_{i=4}^{\infty} \lambda_4 \cdots \lambda_i \leq 1 + C_1 + C_2 \cdot \frac{\frac{4}{5}\left(1 - \left(\frac{4}{5}\right)^{\infty}\right)}{1 - \frac{4}{5}} \leq 1 + C_3, \quad C_3, C_1, C_2 > 0$$

$$\therefore P_0 \geq \frac{1}{C_3 + 1}, \quad P_i \geq \frac{\prod_{i=0}^{n} \lambda_i}{C_3 + 1}, \quad C_3 > 0.$$

$\therefore$ M.C. is recurrent, not transient.