Polarisation Basis Solution

Qcamp 2019, Experimental Session 20/06/2019

1 Introduction

In Mission 2, you construct a quantum communication channel, where the information is encoded in light polarisation. As a result, you are able to construct symmetric secure key by comparing the measurement basis. However, we might have skipped some of the technical aspect. In this handout, we will review some of them and hopefully you will come to appreciate this implementation more.

1.1 Polarisation

Any form of transverse waves have an interesting property known as polarisation, which essentially indicates the direction of oscillation (not propagation!).

Malus' law states that the intensity of a linear polariser output is:

$$I = I_0 \cos^2 \theta \tag{1}$$

when the incoming light is linearly polarised with intensity I_0, and θ is the angle between the polarisation axis of the incoming light and the polariser. A simple picture for this formula is that the polariser allows only the component of the electric field (a vector quantity) that is parallel to the transmission axis, given by $\cos \theta$. Since the intensity is proportional to the electric field, the intensity is reduced by $\cos^2 \theta$.

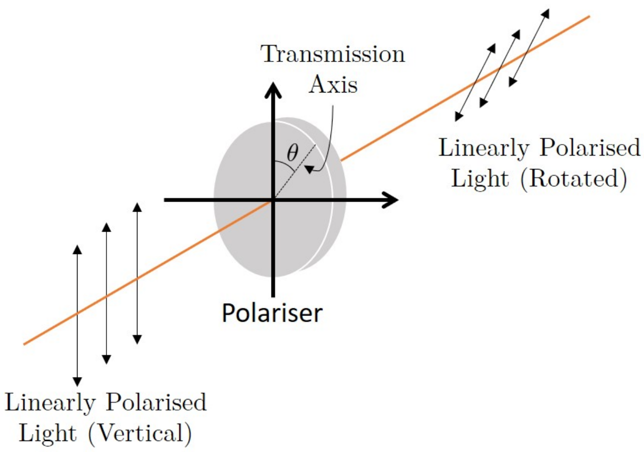

Figure 1: The axis of polarisation of light can be rotated by sending light through a polariser. It is not exactly a perfect rotation as some light is lost, as reflected in Malus' law.

1.2 Basis

To make any form of sensible measurements, we need to first decide on a way to appropriately describe the measurements. Such a process involves choosing a basis. That is, we need to provide a consistent reference to which our measurements are made. A simple example is when you and your friend are facing each other, and the both of you are describing the location of an object nearby. An object to the left of you would be found by your friend to be on his right. No one is more correct than the other in claiming the "leftness" or "rightness" of the object, but confusion would arise if the choice of basis is not made known beforehand.

In the experiment, we define two types of measurement basis: (HV) horizontal–vertical and (DA) diagonal–antidiagonal. In the HV basis, the $|H\rangle$ (horizontally polarised) light will pass through, while the $|V\rangle$ (vertically polarised) light will be blocked completely. In the DA basis however, the $|D\rangle$ (diagonally polarised) light will pass through, while the $|A\rangle$ (anti-diagonally polarised) light will be blocked completely.

From this, we infer that the polarisation axis angle between $|H\rangle$ and $|V\rangle$ is $90°$. If we then define $|D\rangle$ to be halfway in between $|H\rangle$ and $|V\rangle$, i.e. $|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$, then we can also infer that the polarisation axis angle between $|H\rangle$ and $|D\rangle$ is $45°$. Thus, we define the angle of $|H\rangle$, $|D\rangle$, $|V\rangle$, and $|A\rangle$ as $0°$, $45°$, $90°$, and $135°$ respectively.

2 Assignment

Task 1 [2 pts] Verify that if you set the measurement basis to be HV, and you measure $|D\rangle$ polarised light, the light intensity will be half of the original $|D\rangle$ light.

Since the angle between the $|D\rangle$ and the HV basis is $45°$, and $\cos^2 45 = 0.5$, the intensity of the measurement is necessarily half.

Task 2 [2 pts] In Mission 2, you measured the intensity matrix of the 4 incoming light polarisation ($|H\rangle$, $|D\rangle$, $|V\rangle$, and $|A\rangle$) and 2 measurement basis (HV and DA). Assuming that the incoming light intensity is I, construct the matrix and write down the expected values (in units of I).

	H	V	D	A
H	1	0	0.5	0.5
V	0	1	0.5	0.5
D	0.5	0.5	1	0
A	0.5	0.5	0	1

Task 3 [2 pts] Out of those 16 entries in the matrix, there are only a few entries that matter, i.e. important to construct the key. Identify those entries.

Hint: Bob only measures in two basis: (H) horizontal and (D) diagonal. If say, Bob measures in (H) basis and Alice sends $|V\rangle$, then Bob should measure ~ 0. If Alice sends $|H\rangle$ instead, then Bob should measure ~ 1. If Alice sends $|D\rangle$ or $|A\rangle$, it does not matter anyway because that measurement result will be discarded. So, one basically can eliminate those "measurements" that do not matter, as the result will not matter to the final key.

The entries with 1 or 0 are the only ones that matter! :)

Task 4 [2 pts] As a sender, let's say you want to run a sequence of 8 states as follows:

$$|H\rangle, |D\rangle, |A\rangle, |H\rangle, |A\rangle, |V\rangle, |D\rangle, |A\rangle$$

Now, to save time, you only allow a movement of $90°$ maximum. Write the list of angles that you need to set to run the sequence, given that the first two angles are $0°$ and $45°$.

$0°, 45°, 135°, 180°, 135°, 90°, 45°, 135°$

Task 5 [2 pts] Another issue to think about is the synchronisation between Alice and Bob. During the experiment, Alice's and Bob's stepper motor can be rotated in sync. This is due to a synchronisation pulse at the start of the sequence. However, Arduino's crystal oscillator (the device that generates the ticking of clock) is only accurate up to 10^{-4}, i.e. in 10000 seconds, the oscillator might drift around ~ 1 second. This might pose a problem, as the movement of Alice and Bob will go out of sync after some time.

If each bit takes around 1.5 s to transmit (as in the experiment), and between Alice and Bob there can only be at most 50 ms of timing difference, estimate how many bits can Alice send to Bob before doing another synchronisation procedure!

The key to answering this question is to note that we only need to estimate :)

50 ms of timing difference may be accumulated over 0.05×10000 s = 500 s.

500 s of experiment time corresponds to about 333 bits! :)