

Trial Examination 2021

# VCE Mathematical Methods Units 3&4

Written Examination 1

**Suggested Solutions** 

Neap<sup>®</sup> Education (Neap) Trial Exams are licensed to be photocopied or placed on the school intranet and used only within the confines of the school purchasing them, for the purpose of examining that school's students only. They may not be otherwise reproduced or distributed. The copyright of Neap Trial Exams remains with Neap. No Neap Trial Exam or any part thereof is to be issued or passed on by any person to any party inclusive of other schools, non-practising teachers, coaching colleges, tutors, parents, students, publishing agencies or websites without the express written consent of Neap.

### Question 1 (3 marks)

a. 
$$y = \frac{3}{2}\cos\left(\frac{3x}{2}\right)$$
$$\frac{dy}{dx} = -\frac{3}{2} \times \frac{3}{2}\sin\left(\frac{3x}{2}\right)$$
$$= -\frac{9}{4}\sin\left(\frac{3x}{2}\right)$$
A1

**b.** 
$$f'(x) = -e^{-x} \times \log_e(-x) + e^{-x} \times -\frac{1}{-x}$$
 M1

Note: Product or quotient rule should be used.

$$= -e^{-x} \log_e(-x) + \frac{1}{xe^x}$$

$$f'(-1) = -e^{-(-1)} \log_e(-(-1)) + \frac{1}{(-1)e^{(-1)}}$$

$$= -e$$
A1

# Question 2 (3 marks)

$$\int_{1}^{5} \frac{1}{1 - 2x} dx = -\int_{1}^{5} \frac{1}{2x - 1} dx$$

$$= -\frac{1}{2} \left[ \log_{e}(2x - 1) \right]_{1}^{5}$$

$$= -\frac{1}{2} \left( \log_{e}(9) - \log_{e}(1) \right)$$

$$= -\log_{e}(3)$$

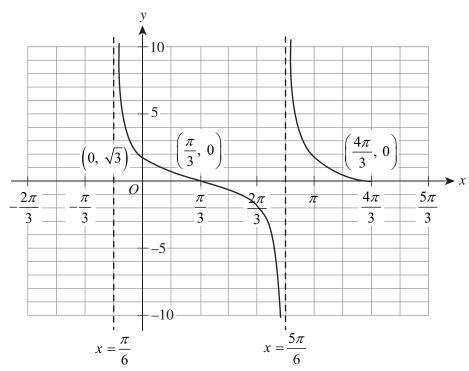
$$= \log_{e} \left( \frac{1}{3} \right)$$

$$\therefore b = \frac{1}{3}$$
A1

Question 3 (7 marks)







correct intercepts A1 correct asymptotes A1 correct shape A1

c. 
$$f(x) = \tan\left(\frac{\pi}{3} - x\right)$$

$$f'(x) = -\sec^2\left(\frac{\pi}{3} - x\right)$$

$$f'(0) = -\sec^2\left(\frac{\pi}{3}\right)$$

$$= -\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right)^2$$

$$= -4$$

$$m_T = -4 \text{ and point } \left(0, \sqrt{3}\right)$$

$$y - \sqrt{3} = -4x$$
$$y = -4x + \sqrt{3}$$
A1

#### Question 4 (3 marks)

$$\log_2(2x+4) - 2\log_2(x+2) - 1 = 0$$

$$\log_2(2x+4) - \log_2(x+2)^2 = 1$$

$$\log_2\left(\frac{2x+4}{(x+2)^2}\right) = \log_2(2)$$

$$\frac{2x+4}{(x+2)^2} = 2$$
M1

$$2x + 4 = 2(x + 2)^{2}$$
$$x + 2 = (x + 2)^{2}$$

$$(x+2)^{2} - (x+2) = 0$$
$$(x+2)(x+2-1) = 0$$
$$(x+2)(x+1) = 0$$

$$x = -2$$
 or  $x = -1$ 

But x > -2 and therefore x = -1 only.

A1

M1

### **Question 5** (3 marks)

$$X \sim Bi(4, p)$$

$$Pr(X = 1) = {}^{4}C_{1} p(1-p)^{3}$$
$$= 4p(1-p)^{3}$$

$$Pr(X = 3) = {}^{4}C_{3} p^{3} (1-p)$$

$$= 4p^{3} (1-p)$$
M1

$$4 \Pr(X = 1) = \Pr(X = 3)$$

$$4 \times 4p(1-p)^3 = 4p^3(1-p)$$

$$4p(1-p)^3 = p^3(1-p)$$

$$4p(1-p)^3 - p^3(1-p) = 0$$

$$p(1-p)(4(1-p)^2-p^2)=0$$

$$p(1-p)(4(1-2p+p^2)-p^2)=0$$

$$p(1-p)(3p^2 - 8p + 4) = 0$$

$$p(1-p)(p-2)(3p-2) = 0$$

$$\therefore p = 0 \text{ or } p = \frac{2}{3} \text{ or } p = 1 \text{ as } 0 \le p \le 1$$

# Question 6 (2 marks)

$$\sigma = \sqrt{\frac{p(1-p)}{n}}$$
Let 
$$\sqrt{\frac{\frac{3}{5} \times \frac{2}{5}}{n}} = \frac{1}{10}.$$

$$\sqrt{\frac{\frac{3}{5} \times \frac{2}{5}}{n}} = \frac{1}{10}$$

$$\sqrt{\frac{6}{25n}} = \frac{1}{10}$$

$$\frac{6}{25n} = \frac{1}{100}$$

$$25n = 600$$

$$n = 24$$

**A**1

M1

#### **Question 7** (5 marks)

**a.** Let 
$$f(x) = 0$$
.

$$x\cos\left(x^2\right) = 0$$

$$x = 0 \text{ or } \cos(x^2) = 0$$

 $x^2 = \frac{\pi}{2}$  gives first positive solution.

$$a = \sqrt{\frac{\pi}{2}} \text{ as } a > 0$$
$$= \frac{\sqrt{\pi}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2\pi}}{2}$$

A1

b. 
$$\frac{d(\sin(x^2))}{dx} = 2x \times \cos(x^2)$$

$$\int 2x \cos(x^2) dx = \sin(x^2)$$

$$\int x \cos(x^2) dx = \frac{1}{2} \sin(x^2)$$
M1

$$Area = \int_0^{\frac{\sqrt{2\pi}}{2}} x \cos(x^2) dx$$

$$= \left[\frac{1}{2} \sin(x^2)\right]_0^{\frac{\sqrt{2\pi}}{2}}$$

$$= \left[\frac{1}{2} \sin\left(\frac{\sqrt{2\pi}}{2}\right)^2\right] - \left[\frac{1}{2} \sin\left((0)^2\right)\right]$$

$$= \left[\frac{1}{2} \sin\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{2}$$
A1

### Question 8 (6 marks)

a. 
$$\int_{0}^{p} a\sqrt{p-x} \, dx = 1$$

$$\left[ -\frac{2a}{3}(p-x)^{\frac{3}{2}} \right]_{0}^{p} = 1$$

$$(0) - \left( -\frac{2a}{3}p^{\frac{3}{2}} \right) = 1$$

$$\frac{2a}{3}p^{\frac{3}{2}} = 1$$

$$a = \frac{3}{2p^{\frac{3}{2}}}$$
A1

b. 
$$f(x) = \frac{3}{2p^{\frac{3}{2}}} \sqrt{p-x}$$

$$q = f(0)$$

$$= \frac{3}{2p^{\frac{3}{2}}} \times p^{\frac{1}{2}}$$

$$= \frac{3}{2p}$$
A1

c. Let 
$$g(p) = p + q$$
.  
 $g(p) = p + \frac{3}{2p}$   
 $g'(p) = 1 - \frac{3}{2p^2}$   
Let  $g'(p) = 0$ .

$$1 - \frac{3}{2p^2} = 0$$
 M1

$$p^2 = \frac{3}{2}$$

$$p = \sqrt{\frac{3}{2}} \text{ as } p > 0$$

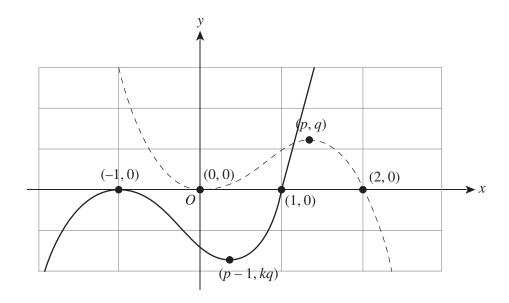
$$p = \sqrt{\frac{3}{2}} \quad \text{as } p > 0$$

$$p+q = \sqrt{\frac{3}{2}} + \frac{3}{2\left(\sqrt{\frac{3}{2}}\right)}$$
$$= \frac{\sqrt{3}}{\sqrt{2}} + \frac{3\sqrt{2}}{2\sqrt{3}}$$
$$= \frac{6+6}{2\sqrt{6}}$$
$$= \sqrt{6}$$

As 
$$(p+q) = \sqrt{m}$$
,  $\sqrt{m} = \sqrt{6}$   
 $\therefore m = 6$ 

### Question 9 (8 marks)

a.



correct x-intercepts A1 correct turning point A1 correct shape and scale A1

**b.** 
$$f(x) = ax^{2}(x-2)$$
 where  $a < 0$   
 $f(x) = a(x^{3} - 2x^{2})$   
 $f'(x) = a(3x^{2} - 4x)$   
Let  $f'(x) = 0$ .  
 $3x^{2} - 4x = 0$   
 $x(3x - 4) = 0$   
 $x = 0$  or  $x = \frac{4}{3}$   
 $\therefore p = \frac{4}{3}$ 

**A**1

c. The average value of g(x) over the interval [-1,1] is equal to -1. Find q in terms of k.

$$\frac{1}{1--1} \int_{-1}^{1} g(x)dx = -1$$

$$\frac{1}{2} \int_{-1}^{1} kf(x+1)dx = -1$$

$$\frac{k}{2} \int_{-1}^{1} f(x+1)dx = -1$$

$$\frac{k}{2} \int_{0}^{1} f(x)dx = -1$$

$$\frac{k}{2} \int_{0}^{2} ax^{2}(x-2)dx = -1$$

$$\frac{kaa}{2} \int_{0}^{2} x^{2}(x-2)dx = -1$$

$$\int_{0}^{2} x^{3} - 2x^{2}dx = \frac{-2}{ka}$$

$$\left[\frac{2^{4}}{4} - \frac{2(2)^{3}}{3}\right] = \frac{-2}{ka}$$

$$\frac{-4}{3} = \frac{-2}{ka}$$

$$f(x) = \frac{3}{2k}(x^3 - 2x^2)$$

$$q = f\left(\frac{4}{3}\right)$$

$$= \frac{3}{2k}\left(\left(\frac{4}{3}\right)^3 - 2\left(\frac{4}{3}\right)^2\right)$$

$$= -\frac{16}{9k}$$

$$q = -\frac{16}{9k}$$

 $a = \frac{3}{2k}$ 

**A**1

M1