



Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section Two:

Calculator-assumed

SOLUTIONS

WA student number:

In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work:

ten minutes

Working time:
minutes

one hundred

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (98 Marks)

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(8 marks)

A random sample of 46 drivers was taken at a test centre. Each pair of letters shown below represents one driver. The first letter shows the driving test outcome (Pass, P or Fail, F) for the person and the second letter shows if they were taking the test for the first time (Yes, Y or N, No).

PN PY PN PY FY PY PY PY FN PN FY PY FN PY PN FN PY FY PN PY FN PY PY
PN FN PY PN FN FY PN PY PY PN PN FY PN PY PY PY PY PY FN PN PN FN FN

- (a) Two categorical variables have been recorded for each driver. Name one of the variables and explain why it is categorical. (2 marks)

Solution
One variable is driving test outcome and it is categorical because the outcomes are categories - pass or fail.
Specific behaviours
<ul style="list-style-type: none"> ✓ names a variable ✓ explains categorical

- (b) Summarise the data by completing the two-way frequency table below. (2 marks)

	Yes	No
Pass	19	13
Fail	5	9

Solution
See table
Specific behaviours
<ul style="list-style-type: none"> ✓ one correct entry ✓ all correct entries

- (c) Convert the two-way frequency table to show column percentages (correct to the nearest percentage). (2 marks)

%	Yes	No
Pass	79	59
Fail	21	41

Solution
See table
Specific behaviours
<ul style="list-style-type: none"> ✓ one correct percentage ✓ all correct percentages -1 rounding

- (d) Discuss whether this sample data suggests the presence of an association between passing the driving test and taking the test for the first time. (2 marks)

Solution
Yes, an association is present. The percentages in the Pass row are quite different, indicating that a higher percentage of drivers pass the test on their first attempt (79%) than if they are repeating (59%).
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates association present ✓ justifies by comparing different row percentages

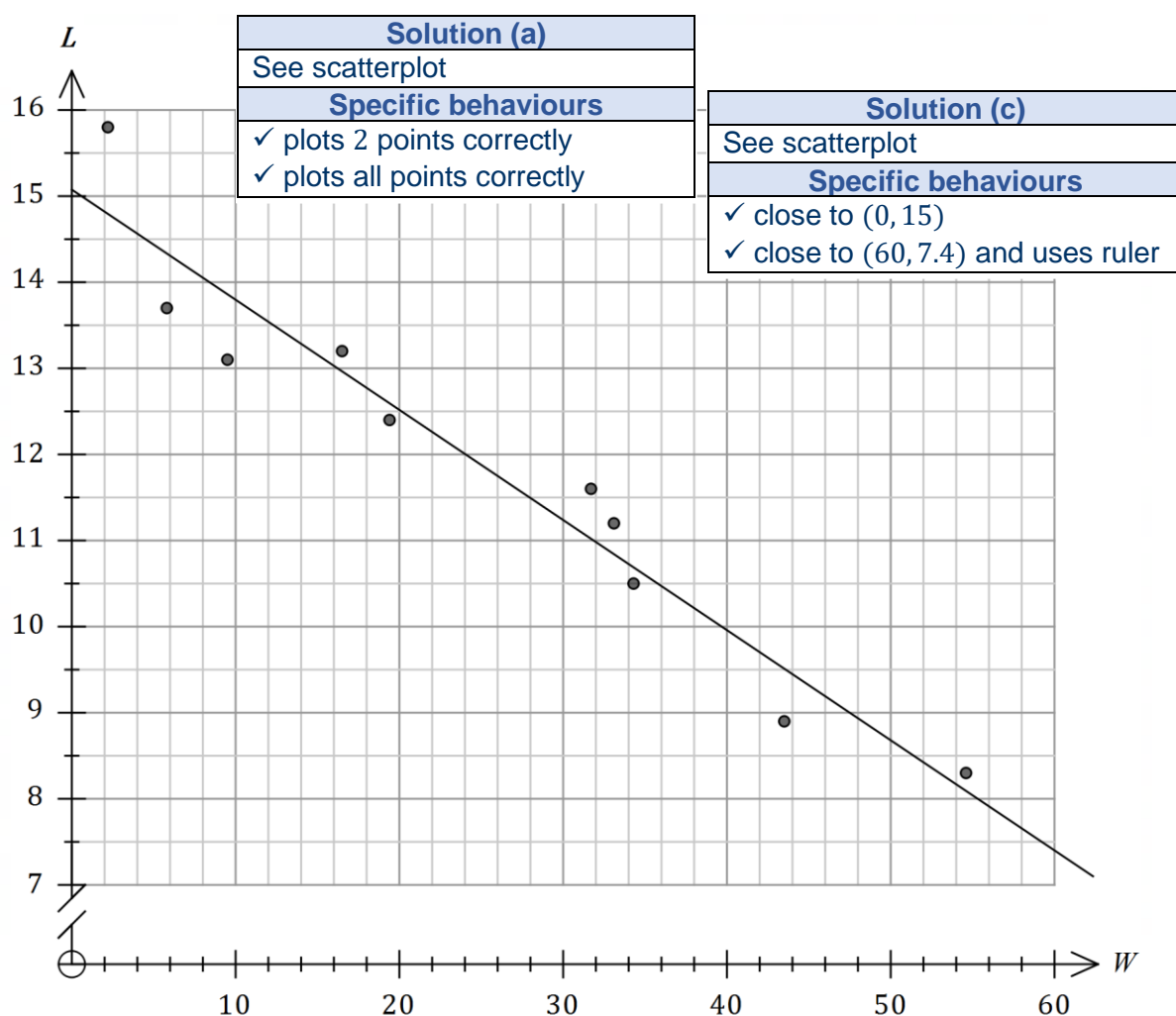
Question 10

(10 marks)

The table below shows the average lifespan L years and the average adult weight W kg of male dogs for a variety of breeds.

Breed	Weight W (kg)	Lifespan L (years)
English Setter	33.1	11.2
Jack Russell	5.8	13.7
Saint Bernard	43.5	8.9
Chihuahua	2.2	15.8
Beagle	9.5	13.1
Bullmastiff	54.6	8.3
Golden Retriever	31.7	11.6
Border Collie	16.5	13.2
German Shepherd	34.3	10.5
Spaniel	19.4	12.4

- (a) Complete the scatterplot below by plotting the missing breeds bolded in the table above (English Setter, Saint Bernard, Golden Retriever and German Shepherd). (2 marks)



(b) Determine

- (i) the correlation coefficient between W and L . (1 mark)

Solution
$r = -0.967$
Specific behaviours
✓ coefficient to at least 2 dp

- (ii) the equation of the least-squares line that can be used to predict L from W . (2 marks)

Solution
$L = -0.128W + 15.08$
Specific behaviours
✓ coefficients
✓ uses correct variables

- (c) Add the least-squares line to the scatterplot. (2 marks)

(d) A breed of dog has an average male weight of 52.5 kg.

- (i) Predict the average lifespan of males of this breed. (1 mark)

Solution
$L(52.5) = 8.4$ years
Specific behaviours
✓ lifespan

- (ii) Briefly discuss two factors that support the validity of your prediction. (2 marks)

Solution
Correlation: The strength of the relationship between the two variables is very strong, with $r = -0.967$.
Interpolation: The prediction involves interpolation, since the weight of 52.5 kg lies between 2.2 and 54.6 kg.
Specific behaviours
✓ indicates strong correlation
✓ indicates interpolation

Question 11

(7 marks)

The recursive rule $A_{n+1} = 1.0036A_n - 320$, $A_0 = 7\,500$ can be used to model the repayment of a loan, where A_n is the amount owing in dollars after n weekly repayments of \$320.

(a) Determine

(i) the initial amount of the loan.

(1 mark)

Solution
$A_0 = \$7\,500$
Specific behaviours
✓ correct amount

(ii) the amount owing after 8 repayments to the nearest cent.

(1 mark)

Solution
$A_8 = \$5\,126.25$
Specific behaviours
✓ correct amount

(iii) the minimum number of repayments required to reduce the amount owing to no more than \$1 500.

(1 mark)

Solution
$A_{19} = 1748.90, \quad A_{20} = 1435.19$
Hence require 20 repayments.
Specific behaviours
✓ correct number of repayments

(b) After 8 repayments, changes to the financial circumstances of the borrower meant that the weekly repayment was reduced by \$140. Determine the change in the minimum number of repayments required to reduce the amount owing to no more than \$1 500. (4 marks)

Solution
$A_{n+1} = 1.0036A_n - 180, A_0 = 5126.25$
$A_{21} = 1608.84, \quad A_{22} = 1434.63$
Now require $8 + 22 = 30$ repayments.
Number of repayments has increased by 10.
Specific behaviours
✓ shows modified recursive rule
✓ new number of payments required (22)
✓ total number of payments required (30)
✓ states the change in number of repayments

Question 12

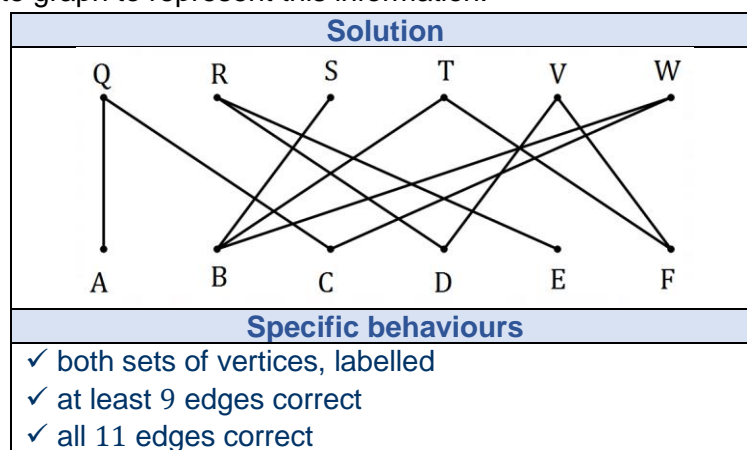
(5 marks)

Six students were asked to create a short presentation to explain the meaning of some chemistry terms. The following table shows which terms each student offered to present.

Student	Terms
Qi	Acid, Cation
Ruby	Dipole, Enzyme
Sid	Base
Tess	Base, Fusion
Vic	Dipole, Fusion
Will	Base, Cation

- (a) Draw a bipartite graph to represent this information.

(3 marks)



- (b) Determine how many more edges must be added to the bipartite graph in (a) so that it would be a complete bipartite graph.

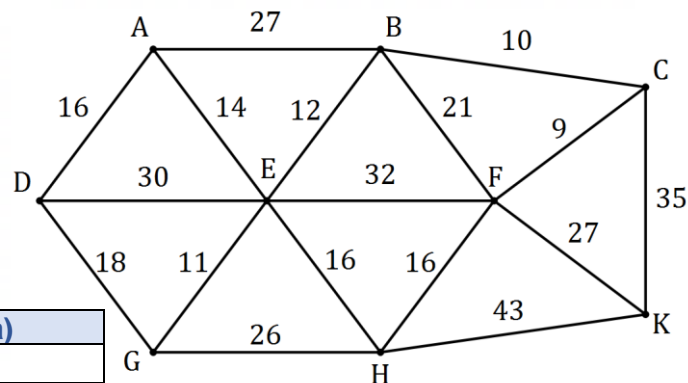
(2 marks)

Solution
Complete bipartite will have $6 \times 6 = 36$ edges. Already have 11 edges, so need another 25 edges.
Specific behaviours
✓✓ correct number to add [25] (1 mark if gives total edges required [36])

Question 13

(7 marks)

In the graph below, the vertices represent towns and the weights on each edge represent the distance, in kilometres, between pairs of towns. A parcel delivery service is based at town D.



Solution (a)
See table
Specific behaviours
✓ 0.5 each, round down

- (a) Complete the table below to show the shortest distance d km from town D to each of the other towns. (4 marks)

Town	A	B	C	E	F	G	H	K
d , km	16	41	51	29	60	18	44	86

- (b) State the route that gives the minimum distance between towns D and K. (1 mark)

Solution
D-G-E-B-C-K
Specific behaviours
✓ correct route

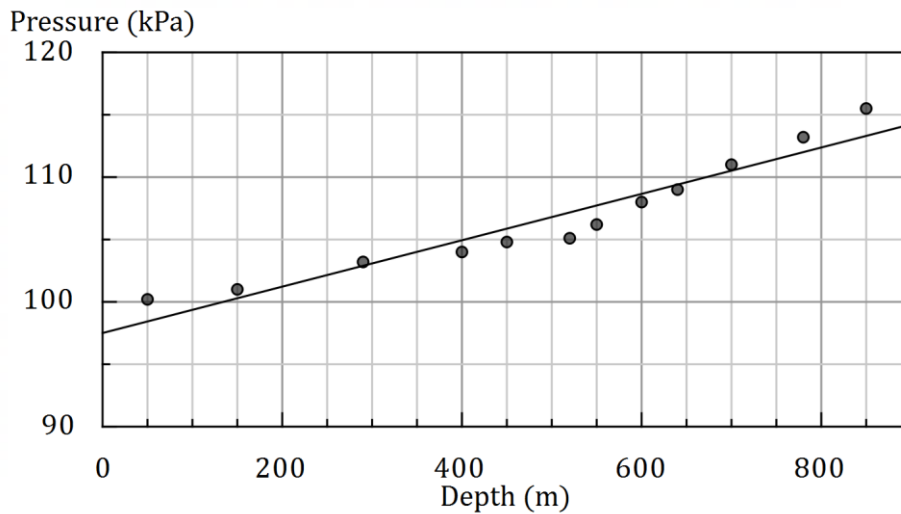
- (c) One day the delivery service has two parcels to deliver, one at A and the other at K. Determine the shortest path from D to K that passes through A and state the length of this path. (2 marks)

Solution
D-A-E-B-C-K
Distance is 87 km.
Specific behaviours
✓ correct route
✓ correct distance

Question 14

(7 marks)

The graph below shows pressure and depth readings collected from a variety of mines in a country, together with the least-squares line for the linear association between the variables.



- (a) The correlation coefficient r for the linear association is one of the values shown in the list below. Circle this value and justify your choice. (3 marks)

{ -0.96, -0.83, -0.41, -0.09, 0.41, 0.83, 0.96 }

Solution
<p>Circles 0.96.</p> <p>r must be close to 1 to reflect the strong positive association.</p>
Specific behaviours
<p>✓ circles correct value</p> <p>✓ indicates positive direction</p> <p>✓ indicates strong association</p>

- (b) Determine the coefficient of determination for the linear association and interpret its value. (2 marks)

Solution
<p>$r^2 = (0.96)^2 = 0.9216$ [accept 0.92]</p> <p>92% of the variation in the pressures at the mines can be explained by the variation in their depths.</p>
Specific behaviours
<p>✓ correct value (decimal or percentage)</p> <p>✓ correct interpretation</p>

- (c) State, with reasons, whether the nature of the relationship between the variables is linear or non-linear. (2 marks)

Solution
<p>Non-linear - 'points lie close to a curve' or 'a pattern would be evident in a residual plot for linear model' (+ve residuals, then -ve, then +ve again).</p>
Specific behaviours
<p>✓ states non-linear</p> <p>✓ reason to support non-linear</p>

Question 15

(7 marks)

The scores of a sample of students who sat two tests are shown in the table below.

Student	1	2	3	4	5	6	7	8
Test A	33	22	30	24	17	20	32	31
Test B	14	18	14	21	24	19	15	10
Residual	1.49	-1.49	-0.42	p	1.33	-1.76	1.85	-3.78

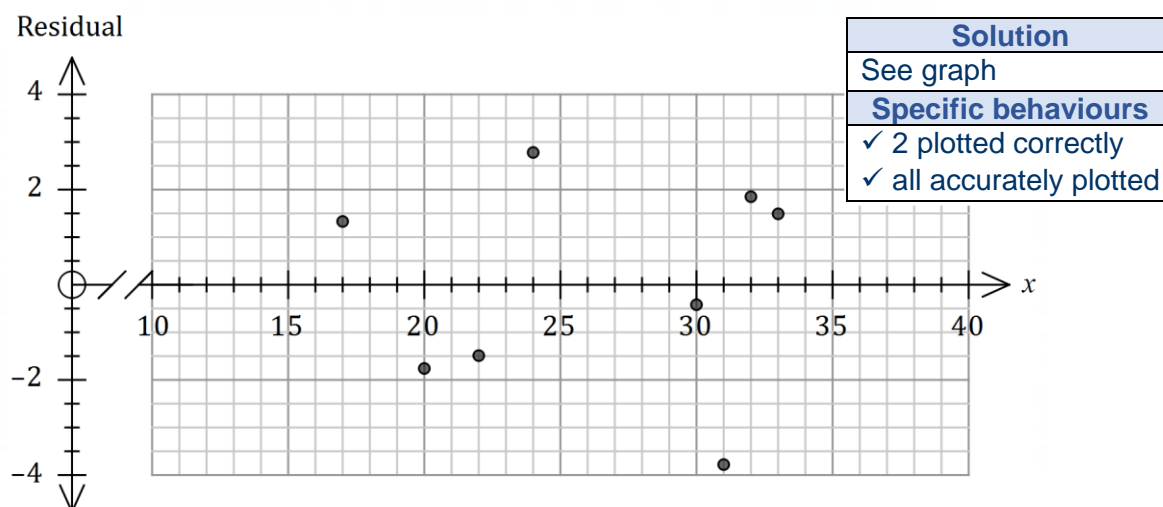
Two students missed Test B and their teacher planned to predict their marks for this test using their scores from Test A and the linear relationship modelled by the least-squares line between the response (y) and explanatory (x) variables.

The equation is $y = -0.635x + 33.46$ and the correlation coefficient is -0.867 . This equation was used to determine the residuals shown in the table above.

- (a) Determine the value of p in the table above. (2 marks)

Solution
$y = -0.635(24) + 33.46 = 18.22$ $p = 21 - 18.22 = 2.78$
Specific behaviours
✓ correct value of y ✓ correct residual to 2 decimal places

- (b) Complete the residual plot for the data on the axes below by plotting student 4, student 7 and student 8. (2 marks)



- (c) Using the residual plot and other relevant factors, comment on the teacher's plan. (3 marks)

Solution
The teachers plan is sound as (i) no pattern evident in residuals and so use of linear model is appropriate. (ii) the linear relationship is strong, with $r = -0.867$ However, the scores of the missing students in Test A are unknown. If they are not between 17 and 33 then the predictions for Test B will involve extrapolation and should be treated with caution.
Specific behaviours
✓ indicates no pattern in residuals is good ✓ indicates strong correlation is good ✓ indicates possible danger of extrapolation (must be interpolation) and/or small sample space

See next page

Question 16

(8 marks)

A photocopier was purchased for \$5 750. Its value depreciates at a rate of 6.5 cents per copy. Let V_n be the value of the photocopier in dollars after n copies have been made, where $V_n = a + bn$.

- (a) State the value of the constant a and the value of the constant b . (2 marks)

Solution
$a = 5750, \quad b = -0.065$
Specific behaviours
✓ value of a ✓ value of b

- (b) Determine V_{2000} . (1 mark)

Solution
$V_{2000} = 5750 - 0.065 \times 2000$ $= \$5\,620$
Specific behaviours
✓ correct value

- (c) Determine n when $V_n = 4671$. (2 marks)

Solution
$4671 = 5750 - 0.065n$ $n = 16\,600$
Specific behaviours
✓✓ correct value of n (1 mark if correct equation given)

- (d) Calculate the value of the photocopier after 65 000 copies have been made. (1 mark)

Solution
$V_{65000} = 5750 - 0.065 \times 65000$ $= \$1\,525$
Specific behaviours
✓ correct value

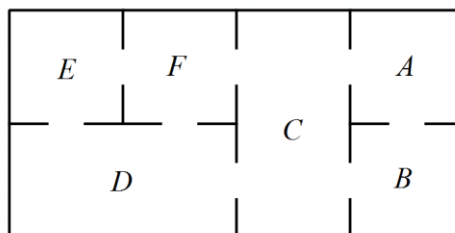
- (e) The photocopier will be replaced as soon as its value falls below \$300. Determine the number of copies the photocopier will make before it is replaced. (2 marks)

Solution
$300 = 5750 - 0.065n$ $n = 83846.15$ Will be replaced after 83 847 copies.
Specific behaviours
✓✓ correct number of copies (1 mark if correct equation given)

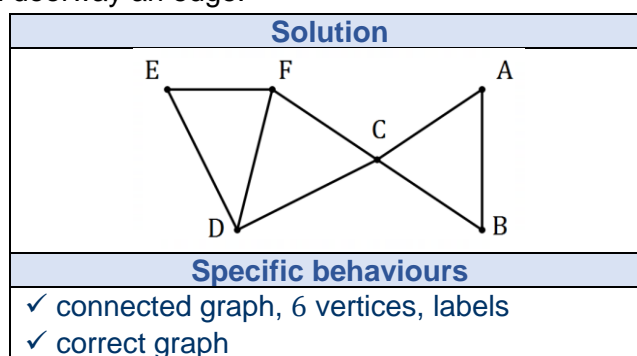
Question 17

(8 marks)

A warehouse has dividing walls that split its interior into six areas, as shown in the plan below. The gaps in the dividing walls are doorways that allow people to move from one area to another.



- (a) Construct a graph to represent the warehouse areas and doorways, with each area being a vertex and each doorway an edge. (2 marks)



- (b) An inspector started in one area and followed a route that went through all doorways exactly once before stopping in another area. State where their route started and stopped and explain how the Eulerian properties of the graph in (a) help to identify these locations. (3 marks)

Solution
Start at <i>F</i> and stop at <i>D</i> (or reverse). The graph is semi-Eulerian, which means that the graph contains an Euler trial but not an Euler circuit - hence start at one odd vertex and stop at the other.
Specific behaviours
<ul style="list-style-type: none"> ✓ correct endpoints ✓ states graph is semi-Eulerian / has Euler trail (or not Eulerian / no Eulerian circuit) ✓ indicates use of odd vertices

- (c) Another inspector wishes to start in an area, follow a route that visits all the other areas exactly once and end up back where they started. Comment on whether this is possible, referring to the Hamiltonian properties of the graph in (a) to justify your response. (3 marks)

Solution
Not possible. The graph is semi-Hamiltonian (not a Hamiltonian cycle). Can not return to start as would have to pass through <i>C</i> twice. Only possible to visit all vertices just once, but unable to return to the start.
Specific behaviours
<ul style="list-style-type: none"> ✓ states not possible ✓ states graph is semi-Hamiltonian / has Hamilton path not cycle ✓ explains meaning of semi-Hamiltonian

Question 18

(9 marks)

The value T_n , in dollars, of a property n years after it was bought can be represented by the rule $T_{n+1} = 1.079T_n$, $T_0 = 450\,000$.

- (a) State the value of the property when it was bought and the annual percentage increase in its value. (2 marks)

Solution
Initial value: \$450 000.
Percentage increase is $0.079 \times 100 = 7.9\%$.
Specific behaviours
✓ initial value
✓ percentage increase

- (b) Determine the value of the property after 2 years. (1 mark)

Solution
$T_2 = \$523\,908.45$
Specific behaviours
✓ correct value

- (c) Determine how many years it will take for the value of the property to at least double. (2 marks)

Solution
$2 \times \$450\,000 = \$900\,000$ $T_9 \approx \$892\,000$, $T_{10} \approx \$963\,000$ Hence it will take 10 years.
Specific behaviours
✓ indicates value of T_9 or T_{10}
✓ correct number of years

- (d) If the annual percentage increase in value of the property changed to 4.4% after 3 years, determine the value (to the nearest \$100) of the property 8 years after it was bought. (4 marks)

Solution
$T_3 = \$565\,297.22$ $T_n = T_n \times 1.044$, $T_0 = 565\,297.22$ $T_5 = \$701\,100$
Specific behaviours
✓ value after 3 years ✓ indicates new rule ✓ calculate value after 5 more years ✓ correct value (to nearest \$100)

Question 19

(7 marks)

The average mid-year commuting times for full-time workers in Melbourne (m minutes) and in Perth (p minutes) between the years 2004 ($t = 4$) and 2011 ($t = 11$) are shown in the table below.

Year, t	4	5	6	7	8	9	10	11
Melbourne, m	29.5	29.9	31.2	32.3	32.8	32.1	33.4	33.5
Perth, p	24.2	25.8	27.2	26.1	27.7	27.1	30.2	29.5

The least-squares line to model the linear relationship between t and p is $p = 0.719t + 21.8$ and $r_{tp} = 0.902$.

- (a) Determine the least-squares line to model the linear relationship between t and m and state the correlation coefficient for this association. (2 marks)

Solution
$m = 0.580t + 27.5$
$r_{tm} = 0.938$
Specific behaviours
✓ correct equation, using correct variables
✓ correct value of r

- (b) Predict the average commuting times in Melbourne and Perth in the year 2022 and state, with justification, which prediction you are most confident in. (3 marks)

Solution
$m(22) = 0.580(22) + 27.5 \approx 40.3$ m (or 40.2 from classpad)
$p(22) = 0.719(22) + 21.8 \approx 37.6$ m
Most confident in prediction for Melbourne as correlation is stronger. OR Not confident in either, as both involve considerable extrapolation.
Specific behaviours
✓ correct Melbourne time
✓ correct Perth time
✓ justifies choice

- (c) Predict the year in which the average commuting time will be the same in both cities and comment on how confident you are of this prediction. (2 marks)

Solution
$0.580t + 27.5 = 0.719t + 21.8$ $t \approx 41$
The times will be the same in year 2041.
Not at all confident in this prediction as it involves considerable extrapolation.
Specific behaviours
✓ correct year
✓ no confidence justified using extrapolation

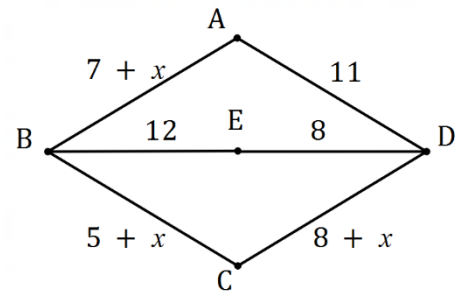
Question 20

(7 marks)

The edges in the graph (not to scale) represent roads and the weight on each edge is the time, in minutes, that it takes to drive along that road. The times to drive along AB , BC and DC vary throughout the day.

The variable x can only take whole number values.

An inspector wishes to drive along each road at least once, starting and finishing at B , in the minimum possible time.



- (a) Briefly explain why the edges on a path between B and D will have to be repeated.

(1 mark)

Solution
B and D are the only odd vertices. This means a semi-eulerian trail exists starting at B and ending at D , requiring a path between B and D to be repeated to get back to B from D .
Specific behaviours
✓ explanation

- (b) List all possible paths between B and D , and state how long each would take, in terms of x where appropriate.

(2 marks)

Solution
BAD - time is $18 + x$ BED - time is 20 BCD - time is $13 + 2x$
Specific behaviours
✓ 3 paths correct or 3 times correct ✓ all listed and correct

- (c) Determine all possible values of x so that BC would be one of the repeated edges.

(2 marks)

Solution
By substitution, BCD takes least time of three paths when: $x = 0, 1, 2, 3$.
Specific behaviours
✓ set of values with no more than 1 error ✓ correct set of values

- (d) For the case when $x = 6$, determine the time required for the inspectors' drive. (2 marks)

Solution
$BAD + BED + BCD = 24 + 20 + 25 = 69$ Repeat BED , so an extra 20 . Total time: $69 + 20 = 89$ minutes.
Specific behaviours
✓✓ correct total time [89 mins] (1 mark for correct sum of weights [69])

Question 21

(8 marks)

A nail is hammered into a piece of wood. The distances moved by the tip of the nail during the first, second and third hits are 20, 14 and 9.8 mm respectively.

- (a) Show that the distances can be modelled by a geometric sequence.

(2 marks)

Solution
$r_1 = 14 \div 20 = 0.7$ $r_2 = 9.8 \div 14 = 0.7$ Hence the distances have a common ratio and can be modelled by a geometric sequence.
Specific behaviours
✓ both ratios correct ✓ states distances have a common ratio

- (b) Write a rule for the distance moved by the tip of the nail D_n during the n^{th} hit of the hammer in the form $D_n = a(r)^{n-1}$.

(1 mark)

Solution
$D_n = 20(0.7)^{n-1}$
Specific behaviours
✓ correct rule

- (c) Determine which hit first moves the tip of the nail less than 2 mm, and state the distance moved during this hit, rounded to one decimal place.

(2 marks)

Solution
$D_7 = 2.35, \quad D_8 = 1.65$ On the 8 th hit, when it moves 1.6 mm (1 dp).
Specific behaviours
✓ correct hit ✓ correct distance to 1 dp

- (d) The piece of wood is 65 mm thick. State, with justification, whether the tip of the nail will pass all the way through the piece of timber, stating any assumptions that you make.

(3 marks)

Solution
Yes. The nail will emerge on the 11 th hit as the sum of the first 11 terms is 65.3 mm. Assumptions: - geometric sequence will continue - nail is driven directly through wood, not at angle. - etc
Specific behaviours
✓ states yes (with justification) ✓ justifies with sum of terms ✓ at least one valid assumption

Supplementary page

Question number: _____

Supplementary page

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