Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 1&2

Section Two: Calculator-assumed

SOLUTIONS

Your name _____

Teacher's name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (98 Marks)**

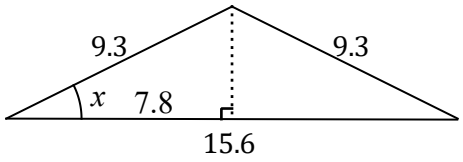
This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

(a) An isosceles triangle has two sides of length 9.3 cm and a base of length 15.6 cm. Sketch this triangle and hence determine the angle between the base and one of the sides. (3 marks)

Solution



$$\cos x = \frac{7.8}{9.3} \quad \text{OR} \quad \cos x = \frac{9.3^2 + 15.6^2 - 9.3^2}{2(9.3)(15.6)}$$

$$x = 33.0°$$

Specific behaviours

✓ sketch, indicates required angle
✓ indicates expression for cosine of angle
✓ correct angle

(b) Clearly show use of Heron's rule to determine the area of a triangle that has sides of length 12 cm, 20.9 cm and 24.1 cm. (3 marks)

Solution

$$s = \frac{12 + 20.9 + 24.1}{2} = 28.5$$

$$\begin{aligned} A &= \sqrt{28.5(28.5 - 12)(28.5 - 20.9)(28.5 - 24.1)} \\ &= \sqrt{28.5 \times 16.5 \times 7.6 \times 4.4} \\ &= 125.4 \text{ cm}^2 \end{aligned}$$

Specific behaviours

✓ calculates semi-perimeter
✓ substitutes correctly
✓ calculates area

See next page

Question 10 **(5 marks)**

(a) The length of a hallway in a building is 21 metres. On a scale drawing of the building, the hallway has a length of 14 cm. Determine the scale used to make the drawing. Give your answer in the form $1:n$, where n is a whole number. (2 marks)

Solution
$14 \text{ cm}: 21 \text{ m}$ $14: 2100$ $1: 150$
Specific behaviours
✓ ratio, using same units ✓ ratio in required form

(b) A plan for an area outside the building is drawn to a scale of $1:250$, and on this plan, a rectangular courtyard is 6.3 cm wide and 8.1 cm long. If this courtyard is paved with square pavers measuring 45×45 cm, determine how many pavers it contains. (3 marks)

Solution
$w = 6.3 \times 250 = 1575 \text{ cm}$ $l = 8.1 \times 250 = 2025 \text{ cm}$ $1575 \div 45 = 35 \text{ pavers wide}$ $2025 \div 45 = 45 \text{ pavers long}$ $35 \times 45 = 1575 \text{ pavers.}$
Specific behaviours
✓ uses scale correctly ✓ indicates appropriate method ✓ correct number of pavers

See next page

Question 11 **(5 marks)**

A single person living away from home and looking for a job qualifies for a government allowance of $\$496.50$ per fortnight, provided their fortnightly income is no more than $\$150$.

In any fortnight that such a person earns more than $\$150$, the allowance will reduce by 50 cents for each dollar of income they have between $\$150$ and $\$250$. If their income is over $\$250$, the payment will further reduce by 60 cents for each dollar of income over $\$250$.

Chi qualifies for the allowance. Each week he works two, 5-hour shifts and one, 4-hour shift at a hotel that pays $\$20.75$ per hour.

(a) Calculate his fortnightly earnings. (2 marks)

Solution
$\text{Hours} = (2 \times 5 + 4) \times 2 = 28$ $\text{Earnings} = 28 \times 20.75 = \$581$
Specific behaviours
✓ indicates correct hours worked each fortnight ✓ calculates earnings

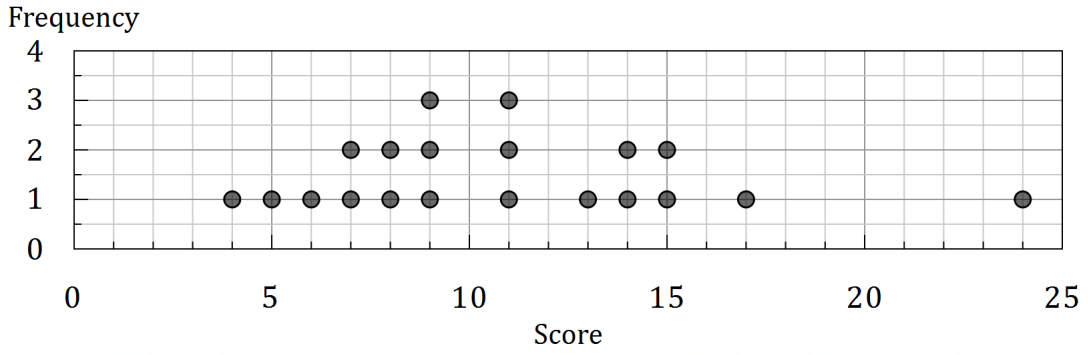
(b) Determine his fortnightly government allowance. (3 marks)

Solution
$\text{Reduction } 1 = (250 - 150) \times 0.5 = \$50$ $\text{Reduction } 2 = (581 - 250) \times 0.6$ $= 331 \times 0.6 = \$198.60$ $\text{Allowance} = 496.50 - 50 - 198.60$ $= \$247.90$
Specific behaviours
✓ indicates first reduction ✓ indicates second reduction ✓ correct allowance

See next page

Question 12 **(10 marks)**

A class of students sat a general knowledge quiz that had 25 multiple choice questions, each with one correct answer. The score of each student in the quiz is shown in the dot plot below.



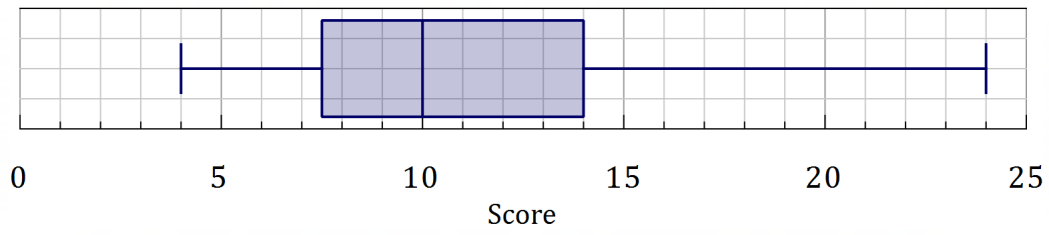
(a) State the number of students in the class. (1 mark)

Solution
20 students.
Specific behaviours
✓ correct number

(b) Name the statistical variable shown in the dot plot and classify this variable as ordinal, nominal, discrete or continuous. (2 marks)

Solution
Score (or **Quiz score**) . Classification is **discrete**.
Specific behaviours
✓ correct variable

(c) Represent the data from the dot plot as a box plot on the axes below. (4 marks)



Solution
Minimum is 4 and maximum is 24. Median is 10. Q_1 is is 7.5 and Q_3 is 14.
Specific behaviours
✓ indicates correct minimum and maximum ✓ indicates correct median ✓ indicates correct quartiles ✓ uses values to draw box plot neatly

See next page

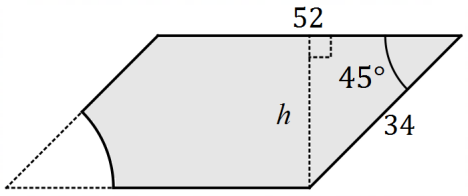
(d) State the interquartile range for this data and use it in a calculation to determine whether the highest score can be classified as an outlier. (3 marks)

Solution
$IQR = 14 - 7.5 = 6.5$ $Q_3 + 1.5 \times IQR = 14 + 1.5 \times 6.5 = 14 + 9.75 = 23.75$ Hence any score above 23.75 is an outlier, and so the highest score in the dataset of 24 is an outlier.
Specific behaviours
✓ correct IQR ✓ calculates $Q_3 + 1.5 \times IQR$ ✓ states 24 is the outlier

See next page

Question 13 **(7 marks)**

The shape of a metal reinforcing plate is a parallelogram with a sector of a circle removed, as shown in the diagram (not to scale).
The acute angle between the sides of 52 cm and 34 cm is $45°$, and the radius of the sector removed is 18 cm.



(a) Show that the perpendicular distance h between the two long sides is 24 cm. (1 mark)

Solution
$h = 34 \times \sin(45°)$ $= 24.0$ cm
Specific behaviours
✓ shows correct trig expression

(b) Determine the area of the sector removed from the parallelogram. (2 marks)

Solution
$A = \frac{45}{360} \times \pi \times 18^2$ $= 127.23$ cm^2
Specific behaviours
✓ indicates correct fraction of circle ✓ calculates area

(c) Determine the area of the metal plate. (2 marks)

Solution
Parallelogram area: $A = 52 \times 24 = 1248$ Area of plate: $A = 1248 - 127 = 1121$ cm^2
Specific behaviours
✓ area of parallelogram ✓ area of plate

(d) Every edge of the plate is welded to the main structure. Determine the total length of the five edges of the plate. (2 marks)

Solution
Curved edge: $L = \frac{45}{360} \times 2\pi \times 18 = 14.1$ Total length: $L = 14 + 2 \times 52 + 2 \times 34 - 2 \times 18 = 150$ cm
Specific behaviours
✓ length of curved edge ✓ calculates total length

See next page

Question 14 **(8 marks)**

The current share price of an industrial company is $\$5.24$.

(a) A shareholder owns 950 shares in the company, and it has announced a dividend per share of 3.5% of the share price. Determine the dividend the investor will receive. (2 marks)

Solution
$\text{Dividend per share} = 0.035 \times 5.24 = \$0.1834$ $\text{Dividend} = 0.1834 \times 950 = \$174.23$
Specific behaviours
✓ indicates dividend per share ✓ correct dividend

(b) Determine the total cost for the shareholder to buy another 300 shares in the company at their current price, given that their broker charges a fee of $\$35$ plus a commission of 2.5% of the value of the shares bought. (2 marks)

Solution
$\text{Value} = 300 \times 5.24 = \$1572$ $\%\ \text{commission} = 1572 \times 0.025 = \$39.30$ $\text{Total cost} = 1572 + 35 + 39.30 = \$1646.30$
Specific behaviours
✓ indicates value of shares ✓ indicates percentage commission and calculates total

(c) Over the course of a year, the share price of the company increased from $\$4.67$ to its current price. Determine the percentage increase in the share price. (2 marks)

Solution
$\text{Increase} = 5.24 - 4.67 = 0.57$ $\%\ \text{Increase} = 0.57 \div 4.67 \times 100 = 12.2\%$
Specific behaviours
✓ indicates increase in value ✓ correct percentage increase

(d) The company reported that its latest earnings per share was 48.5 cents. Determine the price to earnings ratio for this company. (2 marks)

Solution
$\text{P/E} = 524 \div 48.5 = 10.8$
Specific behaviours
✓ indicates correct calculation ✓ correct value

See next page

Question 15 **(10 marks)**

A commuter recorded the time of 80 trips from home to work and this data is summarised in the following table.

Time (t mins)	Frequency f
$12 < t \leq 15$	6
$15 < t \leq 18$	19
$18 < t \leq 21$	22
$21 < t \leq 24$	16
$24 < t \leq 27$	11
$27 < t \leq 30$	5
$30 < t \leq 33$	0
$33 < t \leq 36$	0
$36 < t \leq 39$	1

(a) A student looked at the above data and stated that the shortest trip of all those recorded took just 12 minutes. Comment on the validity of this statement. (2 marks)

Solution

Statement is not valid. The summary shows that 6 trips took between 12 and 15 minutes and all of them took longer than 12 minutes.

Specific behaviours

✓ states not a valid statement
✓ reasonable explanation

(b) Use the mid-point of each class interval to determine the mean and standard deviation of the trip times. (2 marks)

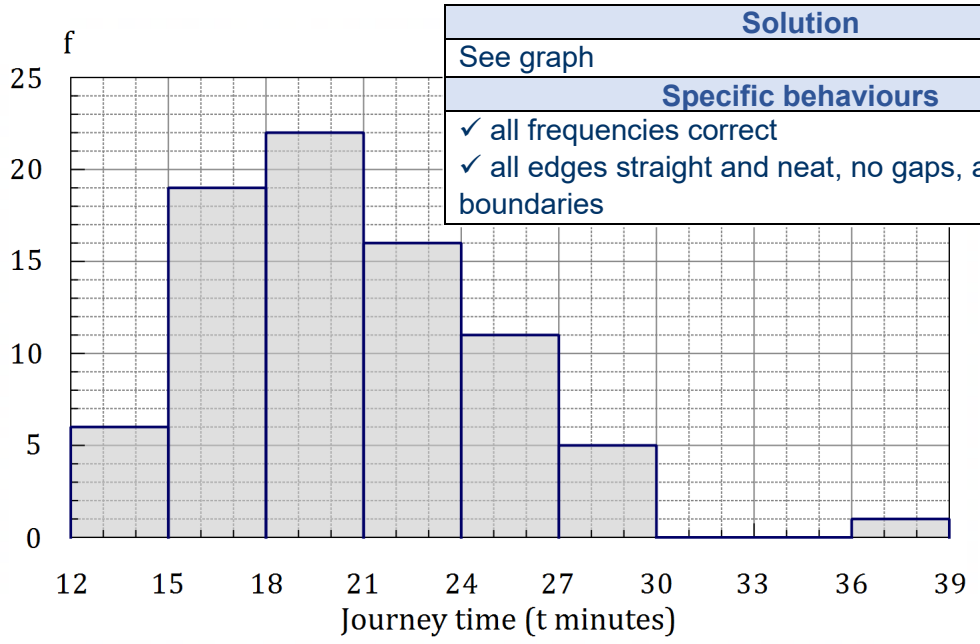
Solution

$\overline{t} = 20.55$ min
$\text{sd} = 4.43$ min

Specific behaviours

✓ mean
✓ standard deviation

(c) Construct a histogram of the data on the axes below. (2 marks)



Solution

See graph

Specific behaviours

✓ all frequencies correct
✓ all edges straight and neat, no gaps, and on boundaries

(d) Describe the distribution of trip times in terms of shape and modality. (2 marks)

Solution
The shape of the distribution exhibits positive skew. The distribution is unimodal, or modal class interval is 18-21.
Specific behaviours
✓ positive skew ✓ one comment on modality

(e) If the outlier was removed from the dataset, describe how your answers to part (b) would change. (2 marks)

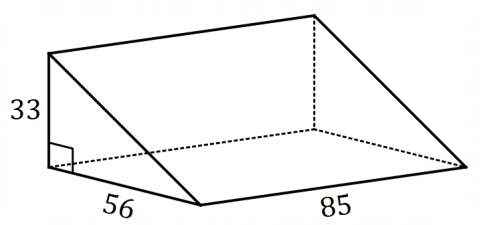
Solution
The mean would decrease. (to 20.34 min) The standard deviation would also decrease. (to 4.02 min)
Specific behaviours
✓ describes change to mean ✓ describes change to standard deviation

See next page

Question 16 **(8 marks)**

A plastic wedge is shown, not to scale.

It takes the form of a prism, with a length of 85 mm and a right triangular cross section measuring 33 mm by 56 mm.



(a) Calculate the volume of the plastic wedge. (2 marks)

Solution
$A = 0.5 \times 56 \times 33 = 924$ $V = 924 \times 85 = 78\,540 \text{ mm}^3$
Specific behaviours
✓ area of triangle ✓ volume of wedge

(b) Calculate the total surface area of the plastic wedge. (4 marks)

Solution
Let x be length of hypotenuse of triangle. $x = \sqrt{33^2 + 56^2} = 65$ Area of rectangles: $A = 85 \times 65 + 85 \times 56 + 85 \times 33 = 13\,090$ Area of triangles: $A = 2 \times 924 = 1848$ TSA: $A = 13\,090 + 1848 = 14\,938 \text{ mm}^2$
Specific behaviours
✓ hypotenuse of triangle ✓ area of rectangles ✓ area of triangles ✓ correct TSA

(c) A scale model of the plastic wedge shown has a length of 17 mm. Determine the total surface area of the scale model. (2 marks)

Solution
Scale factor: $f = 85 \div 17 = 5 \text{ times smaller (or 0.2)}$ TSA of model: $A = 14\,938 \div 5^2 = 597.52 \text{ mm}^2$
Specific behaviours
✓ indicates scale factor ✓ correct TSA

See next page

Question 17 **(7 marks)**

The table below shows the number of tickets sold for seats in the stalls (S), circle (C) and upper circle (U) of a theatre for a show on Tuesday night and Wednesday night.

Tuesday			Wednesday		
S	C	U	S	C	U
315	294	198	323	283	206

(a) Represent this information in the 2×3 matrix $\boldsymbol{A}$, so that $\boldsymbol{A}_{1,1}$ is the number of stalls tickets sold for Tuesday and $\boldsymbol{A}_{2,3}$ is the number of upper circle tickets sold for Wednesday. (2 marks)

Solution

$$\boldsymbol{A} = \begin{bmatrix} 315 & 294 & 198 \\ 323 & 283 & 206 \end{bmatrix}$$

Specific behaviours

✓ $\boldsymbol{A}_{1,1}$ and $\boldsymbol{A}_{2,3}$ correct
✓ all coefficients correct

Ticket prices to sit in the stalls, circle and upper circle were $\$92, \$135$ and $\$75$ respectively.

(b) Represent the ticket price information in matrix $\boldsymbol{B}$ so that matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ can be multiplied together to create a meaningful result. (1 mark)

Solution

$$\boldsymbol{B} = \begin{bmatrix} 92 \\ 135 \\ 75 \end{bmatrix}$$

Specific behaviours

✓ correct matrix $\boldsymbol{B}$

(c) State the order in which matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ must be multiplied together, and determine the result of this multiplication, matrix $\boldsymbol{M}$. (2 marks)

Solution

Order is $\boldsymbol{A} \times \boldsymbol{B}$.

$$\boldsymbol{M} = \begin{bmatrix} 83\,520 \\ 83\,371 \end{bmatrix}$$

Specific behaviours

✓ clearly states correct order
✓ correct matrix $\boldsymbol{M}$

(d) State the night on which the value of ticket sales was highest and calculate by how much the sales on this night exceeded the other night. (2 marks)

Solution

$$83\,520 - 83\,371 = 149$$

Sales were highest on Tuesday night, by $\$149$.

Specific behaviours

✓ correct night
✓ correct amount

See next page

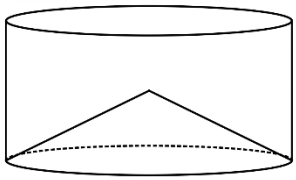
Question 18 **(7 marks)**

A solid cone has a radius of 8 cm and a perpendicular height of 3.9 cm.

(a) Determine the slant height of the cone and hence its total surface area. (3 marks)

Solution
$s = \sqrt{8^2 + 3.9^2} = 8.9 \text{ cm}$ $A = \pi(8)(8.9) + \pi(8)^2$ $= 223.68 + 201.06$ $= 425 \text{ cm}^2$
Specific behaviours
✓ calculates slant height ✓ calculates curved surface area ✓ calculates total surface area

The cone is placed inside a hollow cylinder of radius 8 cm and height 7.5 cm as shown in the diagram (not to scale), and the remaining space inside the cylinder filled with liquid.



(b) Determine the volume of liquid that is required, rounding your answer to the nearest 10 cm^3. (4 marks)

Solution
$V_{CONE} = \frac{1}{3}\pi(8)^2(3.9)$ $= 261.4$ $V_{CYL} = \pi(8)^2(7.5)$ $= 1508$ $V_L = 1508 - 261.4$ $= 1246.6$ $\approx 1250 \text{ cm}^3$
Specific behaviours
✓ evaluates cone volume ✓ evaluates cylinder volume ✓ evaluates volume of liquid ✓ rounds as required, stating units

See next page

Question 19 **(8 marks)**

Mary bought an artwork as an investment at an auction for $\$85\,000$. The auction house charged Mary a handling fee of 0.95% of the sale price.

(a) Determine the handling fee paid by Mary. (1 mark)

Solution
$\frac{0.95}{100} \times 85\,000 = \$807.50$
Specific behaviours
✓ correct amount

One year later, Mary sold the artwork through a gallery for $\$129\,200$. The gallery charged Mary a commission of 32% of the sale price.

(b) Determine the net profit made by Mary, after she had paid the handling fee and sales commission. (3 marks)

Solution
$\frac{32}{100} \times 129\,200 = \$41\,344$ $\text{Profit} = 129\,200 - 85\,000 = \$44\,200$ $\text{Net Profit} = 44\,200 - 807.50 - 41\,344 = \$2048.50$
Specific behaviours
✓ calculates commission ✓ calculates profit ✓ calculates net profit

At the time Mary bought the artwork, she could have placed the money she paid for it in a 12 month term deposit that offered an interest rate of 3.3% p.a. compounded monthly.

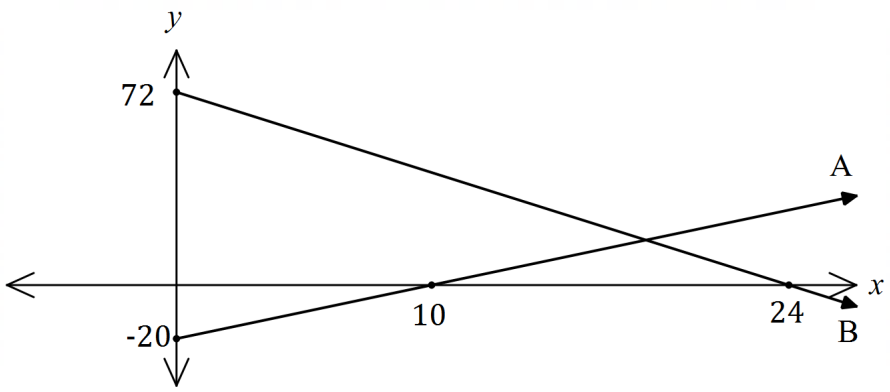
(c) Determine whether the term deposit would have been a better investment for Mary. (4 marks)

Solution
$F = 85\,000\left(1 + \frac{3.3}{12 \times 100}\right)^{12}$ $= \$87\,847.82$ $I = 87\,847.82 - 85\,000 = \$2847.82$ Hence term deposit would have been a better investment.
Specific behaviours
✓ shows correct use of compound interest formula ✓ calculates future value of deposit ✓ calculates interest, to nearest cent ✓ appropriate response to question

See next page

Question 20 **(8 marks)**

The temperatures of component A and component B in a heat exchanger vary in a linear fashion as shown in the graph below, where y is the temperature in $°C$, after x minutes.



(a) Determine a rule for the temperature of component A in terms of x. (3 marks)

Solution
$m = 20 \div 10 = 2$ $c = -20$ $y = 2x - 20$
Specific behaviours
✓ correct gradient ✓ correct intercept ✓ writes rule for y

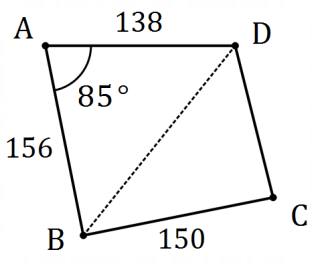
(b) Explaining your method, determine when both components are at the same temperature, and state what this temperature is. (5 marks)

Solution
Require equation for component B: $m = -72 \div 24 = -3$ $c = 72$ $y = -3x + 72$ Now use calculator to graph both lines and find point of intersection at $(18.4, 16.8)$. Hence components reach the same temperature of $16.8\,°C$ after 18.4 minutes.
Specific behaviours
✓ correct gradient for line B ✓ writes rule for B ✓ states time ✓ states temperature ✓ explains or shows method throughout

See next page

Question 21 **(9 marks)**

Field $ABCD$ is a quadrilateral with sides AB, BC and AD having lengths $156, 150$ and 138 m respectively, as shown in the diagram (not to scale). The size of angle BAD is $85°$.



In this question, you must justify your answers by showing use of trigonometry.

(a) Determine the length of the diagonal BD. (2 marks)

Solution
$BD^2 = 128^2 + 156^2 - 2(138)(156)\cos 85°$ $BD = 199$ m
Specific behaviours
✓ correct use of cosine rule ✓ calculates length

(b) Determine the size of angle ABD. (2 marks)

Solution
$\frac{\sin \angle ABD}{138} = \frac{\sin 85°}{199}$ $\angle ABD = 43.7°$
Specific behaviours
✓ correct use of sine rule ✓ calculates angle

It is also known that the size of angle ABC is $76°$.

(c) Calculate the size of angle DBC. (1 mark)

Solution
$\angle DBC = 76 - 43.7 = 32.3°$
Specific behaviours
✓ calculates angle

(d) Determine the area of the field, rounding your answer to the nearest 100 square metres. (4 marks)

Solution
$A_{ABD} = \frac{1}{2}(138)(156)\sin 85° = 10\,723$ $A_{CBD} = \frac{1}{2}(199)(150)\sin 32.3° = 7983$ $\text{Area} = 10\,723 + 7983 = 18\,706$ $\approx 18\,700$ m^2
Specific behaviours
✓ area of one triangle, showing trigonometry ✓ area of second triangle ✓ calculates total area ✓ rounds answer to nearest 100 square metres

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____