Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

SOLUTIONS

Section Two: Calculator-assumed

WA student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

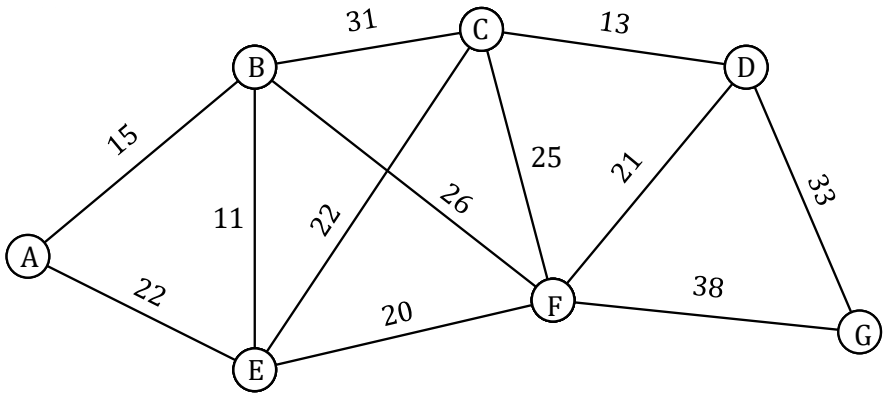
Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(7 marks)**

The vertices A to G in the graph below represent major bus stations in a city and the edge weights represent the travel time between pairs of stations in minutes.



(a) Determine the minimum travel time and corresponding route between the following pairs of stations:

(i) D and E. (2 marks)

Solution
Route is DCE with a least time of 35 minutes.
Specific behaviours
✓ correct route ✓ correct time

(ii) A and G. (3 marks)

Solution
Route is $ABFG$ with a least time of 79 minutes.
Specific behaviours
✓ evidence of checking two or more routes (markings on diagram or multiple routes listed) ✓ correct route ✓ correct time } no evidence required for FULL marks if correct route & correct time stated

(b) It is possible to reduce the travel time between stations E and F. Determine the reduction required so that the current minimum travel time between stations B and G is equal to the minimum travel time between these stations, via station E, after the reduction. (2 marks)

Solution
Route BFG currently minimum - 64 mins. Using $BEFG$: $11 + 20 + 38 = 69$. Hence reduction must be 5 minutes.
Specific behaviours
✓ identifies minimum time without E ✓ correct reduction (FULL marks if 5 mins given)

See next page

Question 10 **(7 marks)**

A grain silo stood empty at the start of a harvest. Over the next month, the weight of barley in the silo, W_n tonnes at the end of the n^{th} day, was modelled by $W_{n+1} = 0.76W_n + 48,\ W_0 = 0$.

(a) Determine, to the nearest tonne, the change in the weight of barley in the silo from the end of day 3 to the end of day 5. (3 marks)

Solution
$W_3 = 112.2, \quad W_5 = 149.3$ $W_5 - W_3 = 37$ t
Specific behaviours
✓ calculates first weight ✓ calculates second weight ✓ calculates difference, to nearest tonne

(b) At the end of which day did the weight of barley in the silo first exceed 185 tonnes? (1 mark)

Solution
Day 10.
Specific behaviours
✓ correct day

(c) Eventually, the weight of barley will reach a steady state. At the end of which day did the weight of barley in the silo first come within one-tenth of a tonne of the steady state? Justify your answer. (3 marks)

Solution
Steady state is 200 tonnes. Need $W_n = 200 - 0.1 = 199.9$ Hence $n = 28$ - the end of day 28.
Specific behaviours
✓ indicates steady state ✓ indicates required weight ✓ correct day

See next page

Question 11 **(8 marks)**

The following table shows the compressive strength, in megapascals, achieved by concrete after one week for different water-cement ratios, as a percentage, used in its mixture.

Water-cement ratio R, %	41	44	46	50	53	58	60
Strength S, MPa	28.8	27.7	26.8	22.9	23.3	20.3	18.9

(a) Determine the equation of the least-squares line for the data, with ratio R as the explanatory variable. (2 marks)

Solution
$S = -0.522R + 50.36$
Specific behaviours
✓ correct gradient (at least 2 dp)
✓ correct intercept (at least 2 dp)

(b) In the context of the question, interpret the slope of the least-squares line in part (a). (2 marks)

Solution
For each 1% increase in the water-cement ratio, the strength of the concrete decreases by 0.522 MPa.
Specific behaviours
✓ relates increase in ratio to decrease in strength
✓ quantifies interpretation

(c) State the coefficient of determination and use it to assess the strength of the linear association. (2 marks)

Solution
$r^2 = 0.970$. Since 97% of the variation in strength can be explained by the variation in water-cement ratio, the linear association can be assessed as strong.
Specific behaviours
✓ coefficient as decimal or percentage
✓ interprets r^2 in context

(d) Predict the value of the strength S when the water-cement ratio is 45% and discuss the validity of this prediction. (2 marks)

Solution
$S = -0.522(45) + 50.36 = 26.9$ MPa This prediction is valid since it does not involve extrapolation and the association is strong.
Specific behaviours
✓ calculates strength
✓ states prediction is valid with at least one reason

See next page

Question 12 **(7 marks)**

The balance A_n of an account after n years, in dollars, is modelled by the recurrence relation $A_{n+1} = 1.25A_n,\ A_0 = 75$.

(a) Determine the balance of the account, to the nearest cent, after

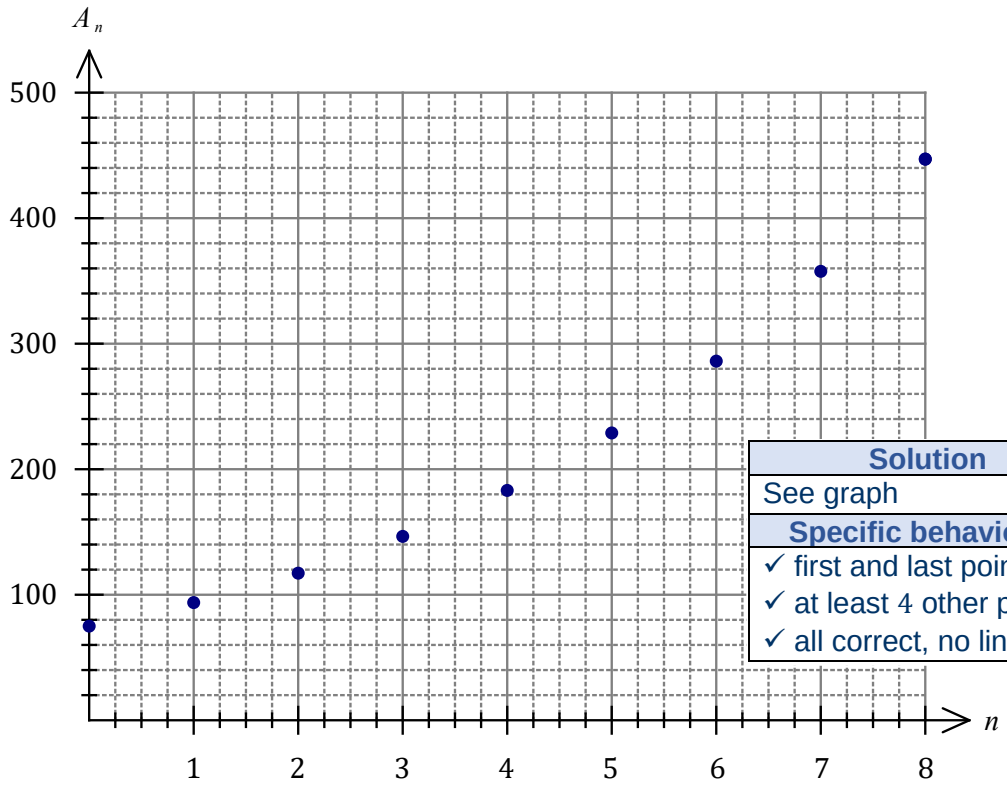
(i) 3 years. (1 mark)

Solution
$A_3 = \$146.48$
Specific behaviours
✓ correct balance

(ii) 6 years. (1 mark)

Solution
$A_6 = \$286.10$
Specific behaviours
✓ correct balance

(b) Plot A_n on the axes below for $n = 0$ to $n = 8$. (3 marks)



Solution
See graph
Specific behaviours
✓ first and last points
✓ at least 4 other points
✓ all correct, no line

(c) Describe the features of the graph in part (b) that illustrate the exponential growth of the balance. (2 marks)

Solution
As n increases, so the balance increases (hence growth) at an increasing rate (hence exponential).
Specific behaviours
✓ describes growth
✓ describes exponential

See next page

Question 13 **(8 marks)**

Participants at a conference were categorised by district they worked in and main area of interest. The table below shows the number of participants in these categories.

		Main area of interest		
		Legal	Accounting	Banking
District	Metropolitan	36	33	59
	Regional	43	53	56

(a) Determine what percentage of participants

(i) had accounting as their main area of interest. (1 mark)

Solution
$86 \div 280 = 30.7\%$
Specific behaviours
✓ correct percentage

(ii) worked in the regional district. (1 mark)

Solution
$152 \div 280 = 54.3\%$
Specific behaviours
✓ correct percentage

(b) Use the above table to complete the following table of row percentages, rounding entries to the nearest whole number. (3 marks)

(%)	Legal	Accounting	Banking
Metropolitan	**28**	**26**	**46**
Regional	**28**	**35**	**37**

Solution
See table
Specific behaviours
✓ at least two correct entries ✓ both rows add to 100 ✓ all correct entries (-1 for not rounding to whole number)

(c) Explain whether the percentaged table above suggest the presence of an association between district worked in and main area of interest for the participants. (3 marks)

Solution
Yes, an association is evident between the variables as the pairs of percentages in the columns for both accounting and banking are different. For example, 46% of those who work metro have banking as main interest compared to only 37% who work regional. But no association for district worked and Legal interest
Specific behaviours
✓ states association between district worked with interests of Accounting or Banking ✓ explanation with a specific example using different percentages from a column in the table ✓ states no association for Legal interest with district worked

See next page

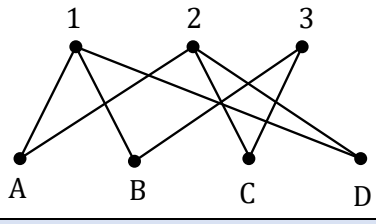
Question 14 **(7 marks)**

A student found a box containing three keys and four padlocks. Some keys will open more than one padlock. A tick in the following table indicates that a key will open that padlock.

		Key		
		1	2	3
Padlock	A	✓	✓	
	B	✓		✓
	C		✓	✓
	D	✓	✓	

(a) Represent this information clearly as a bipartite graph, G. (3 marks)

Solution



Specific behaviours

✓ two distinct sets of vertices
✓ all vertices labelled
✓ correct set of edges

(b) The presence of exactly two odd vertices in G indicates that it is semi-Eulerian. State the definition of a semi-Eulerian graph. (2 marks)

Solution

A semi-Eulerian graph contains an open trail that includes every edge once only.

Specific behaviours

✓ states has an **open trail** or **trail that starts and finishes at different vertices**
✓ states trail **includes every edge once only**

(c) If another edge was added to G, from one odd vertex to the other, state, with reasons, whether G is still:

(i) bipartite. (1 mark)

Solution

No - cannot have an edge joining two vertices in one set.

Specific behaviours

✓ states no, with reason

(ii) semi-Eulerian. (1 mark)

Solution

No - graph will become Eulerian.

Specific behaviours

✓ states no, with reason

See next page

Question 15 **(7 marks)**

An unmanned submarine has to return directly to its host ship, currently at anchor and 138 km away from the submarine. With failing batteries, the submarine can travel 25 km in the first hour, 22.5 km in the second hour and so on, always 2.5 km less than in the previous hour until it no longer moves.

(a) Determine the total distance travelled by the submarine in the first three hours. (2 marks)

Solution
$D_3 = 20$ Distance $= 25 + 22.5 + 20 = 67.5$ km.
Specific behaviours
✓ indicates distance travelled in third hour ✓ calculates sum of first three terms

(b) Determine a simplified rule for the distance D_n travelled by the submarine in the n^{th} hour. (2 marks)

Solution
$D_n = 25 + (n-1)(-2.5)$ $= 27.5 - 2.5n$
Specific behaviours
✓ substitutes a and d into n^{th} term rule ✓ simplifies and uses D_n

(c) At the start of which hour will the submarine no longer move? (1 mark)

Solution
Start of the 11^{th} hour.
Specific behaviours
✓ states correct hour

(d) State, with reasons, whether the submarine will reach its host ship. (2 marks)

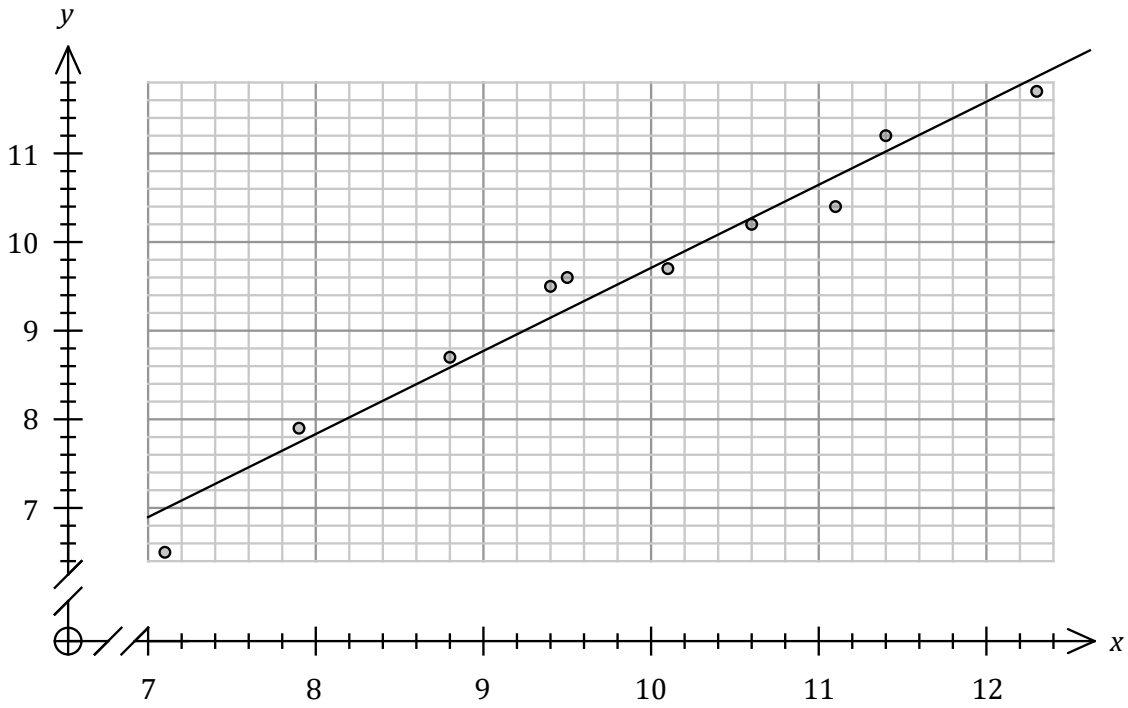
Solution
$25 + 22.5 + 20 + \cdots + 2.5 = 137.5$ km Submarine will not reach its host ship as the total distance it will travel is 137.5 km, stopping 0.5 km away.
Specific behaviours
✓ sums distances travelled ✓ states no, with reasoning

See next page

Question 16 **(9 marks)**

The table and graph below shows the average fuel consumption, in litres per 100 km, achieved by the drivers of different cars before and after they took part in an advanced driving course.

Before x	12.3	9.4	8.8	7.9	10.6	10.1	11.1	9.5	11.4	7.1
After y	11.7	9.5	8.7	7.9	10.2	9.7	10.4	9.6	11.2	6.5



(a) Use the above information to determine

(i) the correlation coefficient r_{xy}. (1 mark)

Solution
$r = 0.983$
Specific behaviours
✓ correct value to at least 2 dp

(ii) the equation of the least-squares line of y on x. (2 marks)

Solution
$y = 0.937x + 0.335$
Specific behaviours
✓ correct slope to at least 2 dp
✓ correct intercept to at least 2 dp

(b) Draw the least-squares line on the graph above. (2 marks)

Solution
See graph
Specific behaviours
✓ draws a ruled straight line
✓ passes close to $(7.1, 7)$ and $(12.2, 11.8)$

See next page

(c) The fuel consumption achieved by the driver of another car was 6.6 litres per 100 km before they took part in the course.

(i) Predict the fuel consumption this driver will achieve after the course. (1 mark)

Solution
$\hat{y}(6.6) = 6.5 \text{ L/km}$
Specific behaviours
✓ correct value (no penalty for units)

(ii) Explain why the correlation coefficient supports confidence in the above prediction. (1 mark)

Solution
Its closeness to 1 indicates a strong linear relationship.
Specific behaviours
✓ explanation describing strong linear relationship

(iii) Explain why this prediction involves extrapolation and how this affects confidence in the above prediction. (2 marks)

Solution
The 'before' fuel consumption figure of 6.6 lies outside the range of the original data - hence extrapolation. Extrapolation is a dangerous process and lowers the confidence associated with the prediction.
Specific behaviours
✓ explains extrapolation ✓ indicates extrapolation lowers confidence

See next page

Question 17 **(8 marks)**

(a) The value of a painting, initially worth $\$2500$, increases by a 7% of its value each year.

(i) Deduce the n^{th} term rule for the value V_n of the painting after n years. (2 marks)

Solution
$V_n = 2500(1.07)^n$
Specific behaviours
✓ rule in exponential form
✓ correct rule using V_n

(ii) Determine the number of years until the painting is first worth more than $\$6\,000$. (1 mark)

Solution
$V_{12} = 5630.48, \quad V_{13} = 6024.61$ After 13 years.
Specific behaviours
✓ correct number of years

(b) The value of a machine decreases by a fixed percentage of its value each year, so that after 3 years it has a value of $\$2947.80$ and after 4 years it has a value of $\$2505.63$.

(i) Determine the fixed percentage. (3 marks)

Solution
$2947.80 \times r = 2505.63$ $r = 2505.63 \div 2947.80$ $r = 0.85$ Hence decrease is $100\% - 85\% = 15\%$ per year.
Specific behaviours
✓ writes equation for unknown
✓ solves for rate
✓ states percentage decrease

(ii) Determine the initial value of the machine. (2 marks)

Solution
$a(0.85)^3 = 2947.80$ $a = 4800$ Initial value is $\$4\,800$.
Specific behaviours
✓ writes equation for value
✓ states initial value (FULL marks if initial value given without equation)

See next page

Question 18 **(7 marks)**

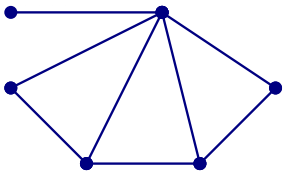
Graph G has 6 vertices with degrees $1, 2, 2, 3, 3$ and 5.

(a) Determine the number of edges that G has. (2 marks)

Solution
$e = \frac{1+2+2+3+3+5}{2} = 8$
Specific behaviours
✓ equates sum of degrees and edges
✓ correct number

(b) Draw G as a simple connected planar graph and state the number of faces it has. (3 marks)

Solution



G has 4 faces.

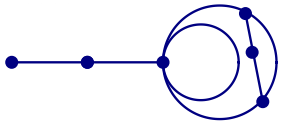
Example shown - mark by checking vertex degrees, no loops, no me's.

Specific behaviours

✓ vertices with correct degrees
✓ connected but no loops or multiple edges
✓ no edges crossing and states number of faces

(c) Draw G as a connected planar graph with two bridges. (2 marks)

Solution



(example)

Specific behaviours

✓ vertices with correct degrees, no edges crossing
✓ connected and exactly two bridges

See next page

Question 19 **(7 marks)**

A person has decided to deposit $\$50$ every month into their savings account. Interest at a rate of 0.17% of the balance will be added to the account just before each deposit is made.

The recurrence relation $A_{n+1} = 1.0017A_n + 50,\ A_0 = 2000$ can be used to model the balance of the savings account, where A_n is the balance in dollars after n deposits.

(a) Determine

(i) the initial balance of the account. (1 mark)

Solution
$A_0 = \$2000$
Specific behaviours
✓ correct balance

(ii) the balance of the account after 6 deposits. (1 mark)

Solution
$A_6 = \$2321.76$
Specific behaviours
✓ correct balance

(iii) the number of months it would take for the account balance to first exceed double its initial balance. (2 marks)

Solution
$A_n > 4000 \Rightarrow n = 37 \text{ months}$
Specific behaviours
✓ indicates required balance ✓ correct number of months (FULL marks without required balance)

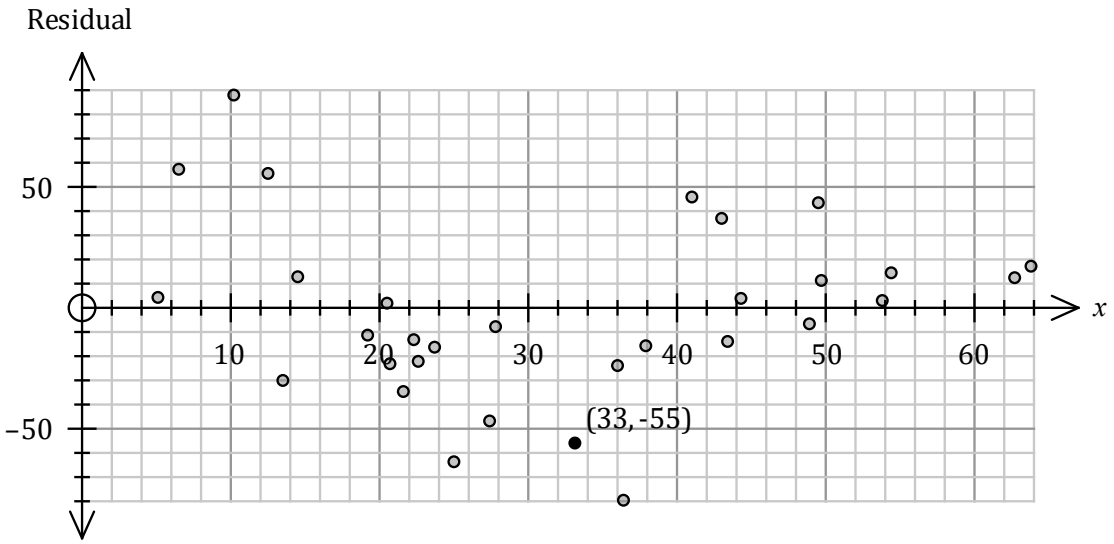
(b) If, after the 12^{th} deposit, the interest rate increased from 0.17% to 0.21% and the monthly deposit decreased from $\$50$ to $\$35$, determine the account balance after a further 12 deposits have been made. (3 marks)

Solution
$A_{12} = 2646.83$ $T_{n+1} = 1.0021T_n + 35, \quad T_0 = 2646.83$ $T_{12} = \$3139.19$
Specific behaviours
✓ calculates new opening balance ✓ states new recurrence relation ✓ correct final balance (2 marks if stated without any other working)

See next page

Question 20 **(8 marks)**

The linear model fitted to a data set had equation $\hat{y} = 14.21x - 67.6$. The correlation coefficient between the variables was $r_{xy} = 0.989$. The residual plot for the linear model is shown below.



(a) The residual for the data point $(33, 346)$ is not shown. Determine the residual for this point and add it to the residual plot. (3 marks)

Solution
$\hat{y} = 14.21(33) - 67.6 = 401.33$ Residual: $346 - 401.33 = -55.33$.
Specific behaviours
✓ calculates $\hat{y}$ ✓ calculates residual ✓ plots residual (must be centre & just below halfway of grid box)

(b) Use the residual plot to assess the appropriateness of fitting a linear model to the data. (2 marks)

Solution
Linear model is not appropriate as a pattern is clearly evident in the residuals.
Specific behaviours
✓ states that linear model is not appropriate ✓ states a pattern evident in the residuals

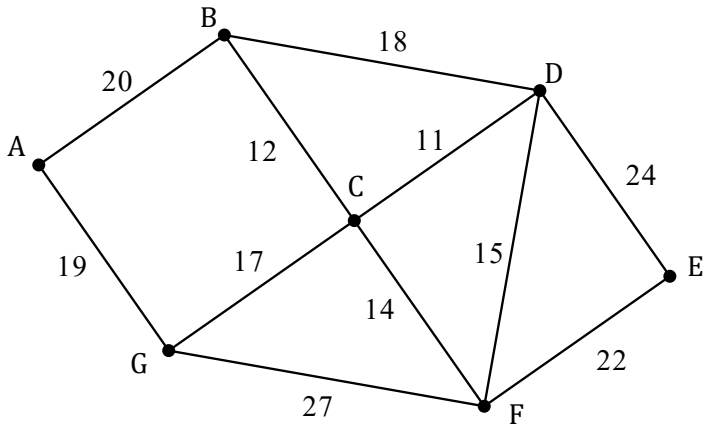
(c) The point shown on the plot above with a residual of 38.6 was derived from the data point $x = a, y = b$. Determine the value of a and the value of b. (3 marks)

Solution
a is the x-coordinate: $a = 43$ b is the y-coordinate: $\hat{y} = 14.21(43) - 67.6 = 543.43$ $b - 543.43 = 38.6 \Rightarrow b = 582$
Specific behaviours
✓ value of a ✓ calculates $\hat{y}$ ✓ value of b

See next page

Question 21 **(8 marks)**

The vertices in graph G_1 below represent towns, the edges represent roads, and each edge weight represents the length of the road between adjacent towns in kilometres.



(a) List, starting with A and in the order visited, the vertices that lie on the Hamiltonian cycle with the minimum total road length and state this minimum length. (3 marks)

Solution
Hamilton cycle is ABDEFCGA. (or in reverse order) Length: $20 + 18 + 24 + 22 + 14 + 17 + 19 = 134$ km.
Specific behaviours
✓ lists vertices in a Hamilton cycle, A … A ✓ identifies shortest Hamilton cycle ✓ calculates the minimum length (must have shortest path to get length mark)

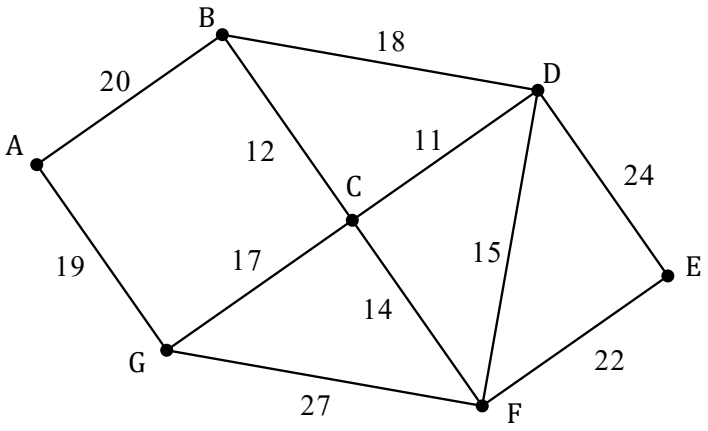
An engineer must drive an inspection vehicle along the entire length of all 11 roads in G_1.

(b) State, with justification, where the inspection should start and where it should finish to minimise the distance that the engineer must drive. (2 marks)

Solution
Choose towns B and G. These towns are the endpoints (the 2 odd vertices) of a semi-Eulerian trail. [Which means they will cover every edge once without repeating.]
Specific behaviours
✓ chooses correct towns ✓ justifies due to semi-Eulerian graph

See next page

(c) For practical reasons, the engineer has to start at town A and must return there at the end of the inspection. Determine, with reasoning, the minimum distance the engineer must drive. A copy of G_1 is provided below. (3 marks)



Solution
Sum of all edge lengths is 199 km. The walk ABDEFDCFGCBCGA repeats edges BC and CG to minimise distance and has length $199 + 12 + 17 = 228$ km.
Specific behaviours
✓ indicates sum of all edge lengths ✓ indicates a sufficient walk with associated distance ✓ correct minimum distance

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____