Neap

Trial Examination 2023

VCE Mathematical Methods Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 23 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2023 VCE Mathematical Methods Units 3&4 Written Examination 2.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

Let $f\colon R \to R, f(x) = 2 + 3\sin(\pi x)$.

The period and range of this function are respectively

A. 2π and $[1, 5]$

B. 2 and $[-1, 5]$

C. π and $[1, 5]$

D. π and $[-1, 5]$

E. 2 and $[1, 5]$

Question 2

Which one of the following is **not** true for the function $f(x) = 2x^3 + 3x^2 - 36x$?

A. $f'(x) > 0$ when $x > 2$.

B. The graph of f has two stationary points.

C. The graph of f does not have any points of inflection.

D. The graph of f has three x-intercepts.

E. $f(x)$ does not have an inverse function.

Question 3

A box contains four green marbles, five blue marbles and one red marble. Two marbles are drawn at random from the box without replacement.

The probability that the two marbles are different colours is

A. $\dfrac{16}{45}$

B. $\dfrac{21}{50}$

C. $\dfrac{8}{15}$

D. $\dfrac{29}{50}$

E. $\dfrac{29}{45}$

Question 4

The graph of the function f has the asymptotes $x = 3$ and $y = -1$.

The rule for f could be

A. $f(x) = \log_e(x - 3)$

B. $f(x) = e^x - 1$

C. $f(x) = -1 + \dfrac{1}{x-3}$

D. $f(x) = 3 - \dfrac{1}{x+1}$

E. $f(x) = \dfrac{-1}{(x-3)^2}$

Question 5

If $f(x) = 4x - 7$, then $f^{-1}(2 + f(2))$ is

A. $\dfrac{2}{5}$

B. 2

C. $\dfrac{5}{2}$

D. 3

E. 5

Question 6

A discrete random variable X has the following probability distribution.

x	0	1	2	3
$\Pr(X = x)$	0.2	0.3	0.4	a

What is the mean of X?

A. 1.4

B. 1.5

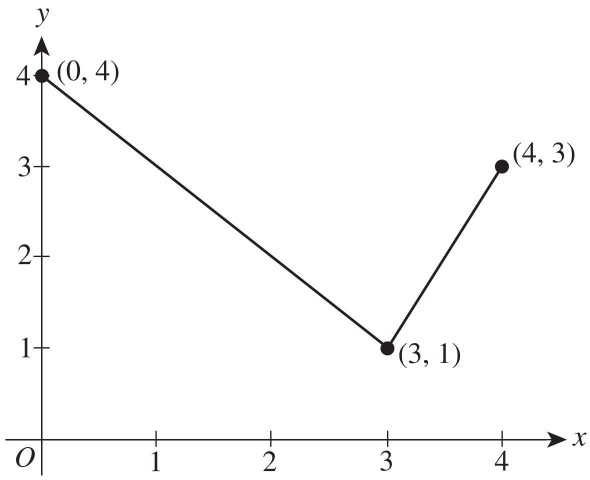
C. 1.6

D. 1.7

E. 1.8

Question 7

The graph of the function f is as follows.



The average value of f in the interval $0 \le x \le 4$ is

A. 2

B. $\frac{19}{8}$

C. $\frac{5}{2}$

D. $\frac{11}{4}$

E. $\frac{14}{5}$

Question 8

The function $f(x) = ax^3 + bx^2 + cx$ has a stationary point at $x = 1$ and a stationary point of inflection at $(–2, 4)$.

What is the value of $f'(0)$?

A. $-\frac{12}{5}$

B. $-\frac{3}{5}$

C. $\frac{2}{5}$

D. $\frac{3}{5}$

E. $\frac{12}{5}$

 MMU34_QB_Ex2_2023

Question 9

Consider the following algorithm.

```
input angle, unit
      if unit = "radians"
                print angle × 180 / π
      else if unit = "degrees"
                print angle × π / 180
      end if
```

The algorithm can be used to

A. convert an angle from radians to degrees only.

B. convert an angle from degrees to radians only.

C. convert an angle between radians and degrees.

D. find the unit of an angle.

E. find an angle.

Question 10

An estimate of one of the x-intercepts for the function $f(x) = \sin(2x) - x$ is $x_0 = 1$.

After one iteration of Newton's method, a more accurate estimate for the x-intercept, correct to three decimal places, is

A. 0.947

B. 0.948

C. 0.949

D. 0.950

E. 0.951

Question 11

X is a normally distributed random variable with a mean of 18 and a variance of 0.16.

If the random variable Z has a normal distribution, then $\Pr(17.2 \le X \le 18.4)$ is equal to

A. $\Pr(-2 \le Z \le 2)$

B. $\Pr(-1 \le Z \le 2)$

C. $\Pr(0 \le Z \le 2)$

D. $\Pr(-1 \le Z \le 1)$

E. $\Pr(-2 \le Z \le 1.5)$

Question 12

Eggs are packed in boxes of 12, and the expected number of broken eggs per box is 0.7.

If the probability of an egg being broken is binomially distributed, what is the closest probability that a box contains at least two broken eggs?

A. 0.004

B. 0.029

C. 0.152

D. 0.203

E. 0.514

Question 13

The function $f: R \to R, f(x) = 2x^3 + px^2 - (p + 1)x + 5$ has no inverse when

A. $p^2 + 6p + 6 > 0$

B. $4p^2 + 24p + 24 < 0$

C. $p^2 + 6p > 0$

D. $p^2 + 24p + 24 < 0$

E. $p^2 + 24p + 24 > 0$

Question 14

A continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} 2\sin\left(\frac{x}{4}\right) & 0 \le x \le k \\ 0 & \text{elsewhere} \end{cases}$$

$\Pr(X > 1)$ is closest to

A. 0.25

B. 0.30

C. 0.45

D. 0.50

E. 0.75

Question 15

The sum of three integers is 100. One of the integers is twice the amount of one of the other integers.

What is the maximum product of the three integers?

A. 32 124

B. 32 448

C. 32 562

D. 32 798

E. 32 912

Question 16

A discrete random variable X has a binomial distribution with a probability of success of $p = 0.24$ for n trials.

If the probability of obtaining more than three successes after n trials is at least 0.8, what is the smallest possible value of n?

A. 17

B. 18

C. 19

D. 21

E. 27

Question 17

The following transformations are applied to the curve $y = \sin(2x)$ in order.

1. reflection in the y-axis
2. dilation by a factor of 2 from the y-axis
3. translation of $\frac{\pi}{2}$ units in the positive direction of the x-axis

The equation of the resulting curve could be

A. $y = -\sin\left(x + \frac{\pi}{2}\right)$

B. $y = \sin\left(x - \frac{\pi}{2}\right)$

C. $y = \cos(x)$

D. $y = -\sin\left(2x + \frac{\pi}{2}\right)$

E. $y = -\sin\left(2x - \frac{\pi}{2}\right)$

Question 18

The tangent to the graph of $y = e^{2x} + 1$ crosses the x-axis at $x = 2$.

The tangent will cross the y-axis closest to

A. –634

B. –598

C. –505

D. –489

E. –445

Question 19

The domain of the function $f(x) = 5\tan\left(\frac{x}{3}\right) - 2$ is

A. $R \setminus \left\{\frac{\pi}{2}(2k-1)\right\},\ k \in Z$

B. $R \setminus \left\{\frac{3\pi}{2}(2k+3)\right\},\ k \in Z$

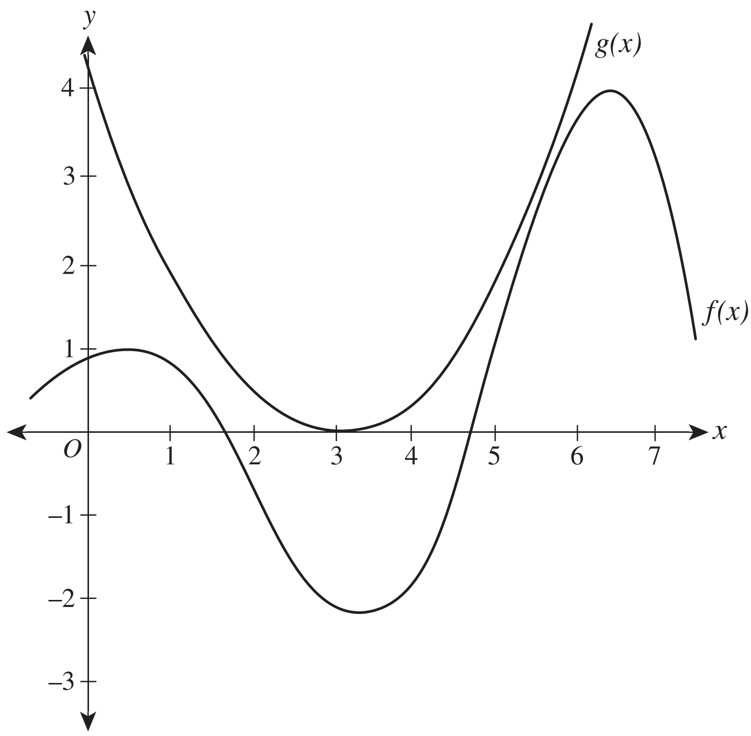
C. $R \setminus \left\{\frac{\pi}{2}(2k+1)\right\},\ k \in Z$

D. $R \setminus \left\{\frac{3\pi}{2}(k-2)\right\},\ k \in Z$

E. $R \setminus \left\{\frac{3\pi}{2}(k+1)\right\},\ k \in Z$

Question 20

The graphs of the functions $f(x)$ and $g(x)$ are as follows.



How many stationary points does $f\left(g(x)\right)$ have for $0 \le x \le 5$?

A. 2

B. 3

C. 4

D. 5

E. 6

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

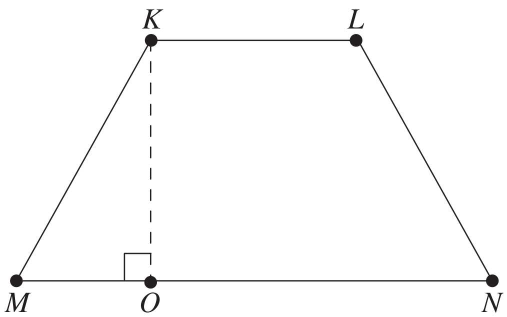
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (9 marks)

The isosceles trapezium *KLNM* shown below has a perimeter of 80 cm. $KL = 2x$ cm and $MN = 6x$ cm.



a. Show that $KM = 40 - 4x$. 1 mark

b. Find an expression for the height *KO* and hence show that the area, *A*, of the trapezium is $A = 8x\sqrt{3x^2 - 80x + 400}$. 3 marks

c. State all possible values of x. 1 mark

d. Find the largest possible area of the trapezium. Give your answer correct to one decimal place. 2 marks

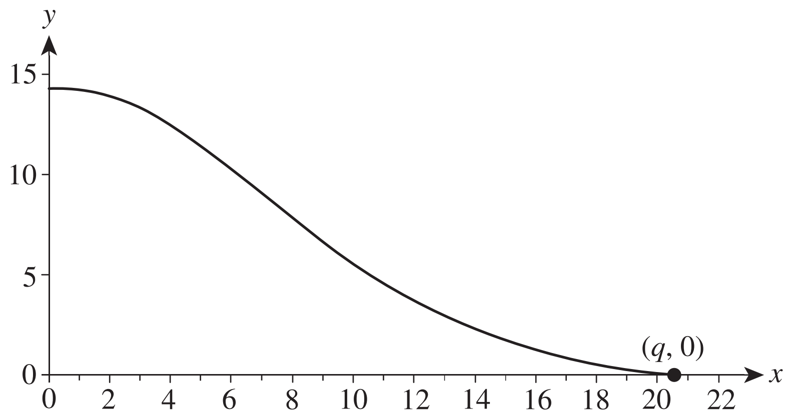
e. Find the average value of the area of the trapezium. Give your answer correct to one decimal place. 2 marks

Question 2 (11 marks)

The cross-section of a water slide is modelled by the differentiable function f, where

$$f(x)=\begin{cases} g(x)=-\dfrac{1}{16}x^2-\dfrac{1}{4}x+\dfrac{221}{16} & 0\le x\le 8 \\ h(x)=\dfrac{1}{20}x^2-\dfrac{41}{20}x+k & 8\le x\le q \end{cases}$$

The graph of $y = f(x)$ is as follows, where x is the horizontal distance from the start of the slide and y is the height of the slide. All lengths are in metres.



a. Explain why $k=\dfrac{1681}{80}$. You are not required to show calculations. 2 marks

b. Show that $f(x)$ is differentiable at $x = 8$. 3 marks

c. Find the length and height of the cross-section. 2 marks

d. The area under $y = f(x)$ for $0 \le x \le 8$ will be approximated using the trapezium rule with interval widths of 1.

i. Explain whether the approximation will be more or less than the actual area. You are not required to approximate the area. 1 mark

ii. Find the difference between the actual area and the approximate area. Give your answer correct to three decimal places. 3 marks

Question 3 (14 marks)

The time it takes to travel between two locations in a particular city varies depending on traffic conditions.

The continuous random variable T, which models the travel time t, in minutes, from location A to location B, has the following probability density function f.

$$f(t)=\begin{cases}\dfrac{1}{40}(t-8) & 8\le t\le 12\\ \dfrac{1}{40}(20-t) & 12\le t\le 20\\ 0 & \text{elsewhere}\end{cases}$$

a. Show that the probability that it takes less than 15 minutes to travel from A to B is $\dfrac{11}{16}$. 3 marks

b. Find $\Pr(T\le 12 \mid T\le 15)$. 2 marks

c. Find the average travel time from A to B, in minutes, correct to one decimal place. 2 marks

Ten random trips are taken from A to B and the travel times are recorded. The duration of each trip is independent of any other trip.

d. Find the probability that at least six trips were completed in less than 12 minutes.
Give your answer correct to four decimal places. 2 marks

Traffic lights that are located between A and B are optimised such that travel time U is normally distributed with a mean of 12 minutes and standard deviation of σ.

e. If $\Pr(U < 13) = 0.65$, find σ, correct to four decimal places. 2 marks

f. The number of times that a trip from A to B is interrupted by a red light is normally distributed with a mean of 3.4 and a standard deviation of 0.8. For a sample of 30 trips, $\hat{P}$ is the random variable that represents the proportion of trips that are interrupted by less than three red lights.

Find $\Pr(0.5 \leq \hat{P} \leq 0.8)$, correct to four decimal places. Do not use a normal approximation. 3 marks

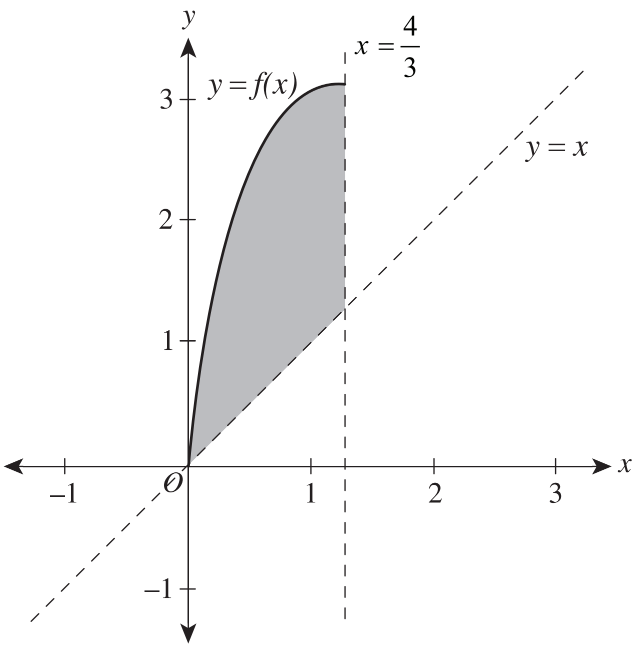
Question 4 (16 marks)

Let $f:[0,k] \to R, f(x) = \sqrt{x}(4-x)$ such that f has an inverse function.

a. Show that $0 < k \leq \frac{4}{3}$. 3 marks

b. Let $k = \frac{4}{3}$.

The graphs of $y = f(x)$, $y = x$ and $x = \frac{4}{3}$ are as follows.

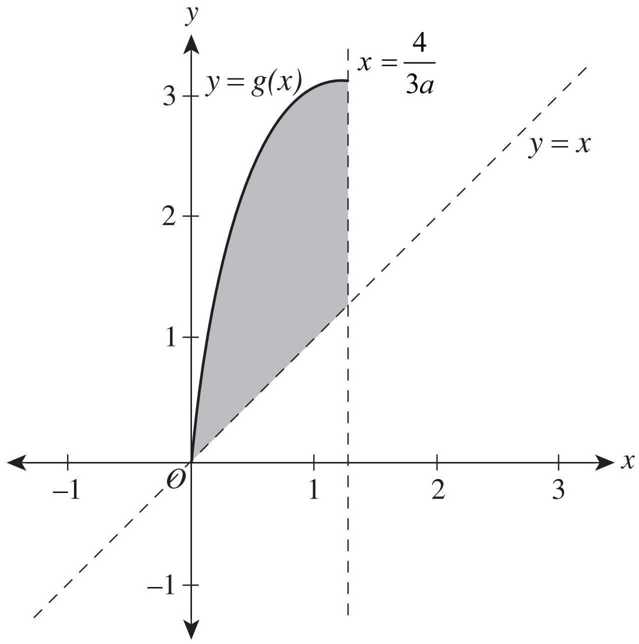


Find the area of the shaded region by writing a definite integral. Give your answer correct to two decimal places. 2 marks

c. Find the area between f and f^{-1} for $0 \leq x \leq \frac{4}{3}$. Give your answer correct to two decimal places. 3 marks

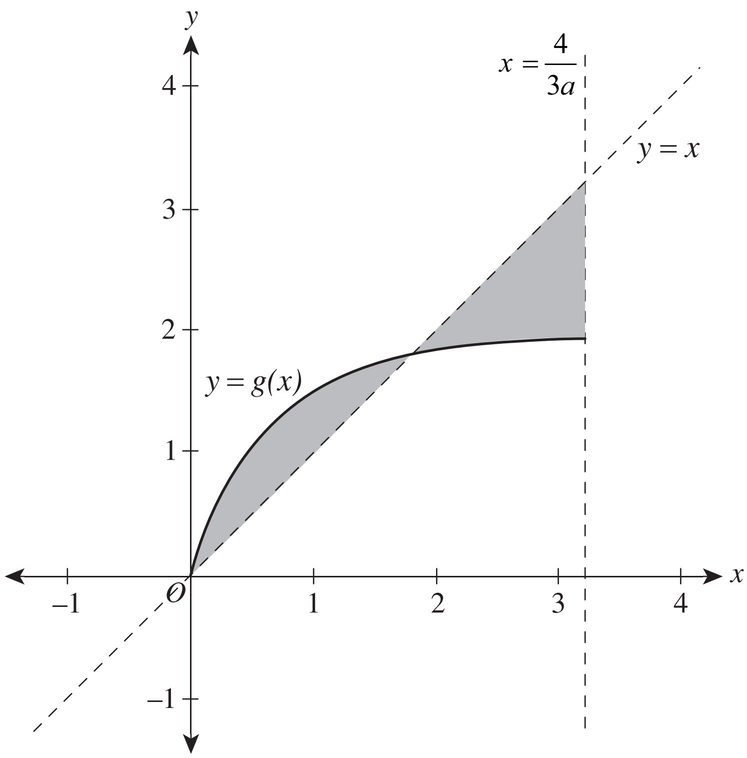
Let $g:\left[0, \dfrac{4}{3a}\right] \to R,\ g(x) = a\sqrt{x}(4 - ax)$.

d. The graphs of $y = g(x)$, $y = x$ and $x = \dfrac{4}{3a}$ are as follows. The region between g and $y = x$ for $0 \le x \le \dfrac{4}{3a}$ is positioned entirely above the line $y = x$.



Find the value of a that gives the maximum area and hence find the maximum area.
Give your answer correct to two decimal places. 3 marks

e. The graphs of $y = g(x)$, $y = x$ and $x = \dfrac{4}{3a}$ are shown below. The region between g and $y = x$ for $0 \le x \le \dfrac{4}{3a}$ is split into **two** parts such that it is not entirely above the line $y = x$.



i. Write an interval for the possible values of a. Give your answer correct to two decimal places. 2 marks

ii. Let $a = \frac{1}{2}$.

Express the area of the shaded region using definite integrals and hence find its value correct to two decimal places. 3 marks

Question 5 (10 marks)

Let $f(x) = 2\sin(2x) + \cos(x)$.

a. Find $f'(x)$. 1 mark

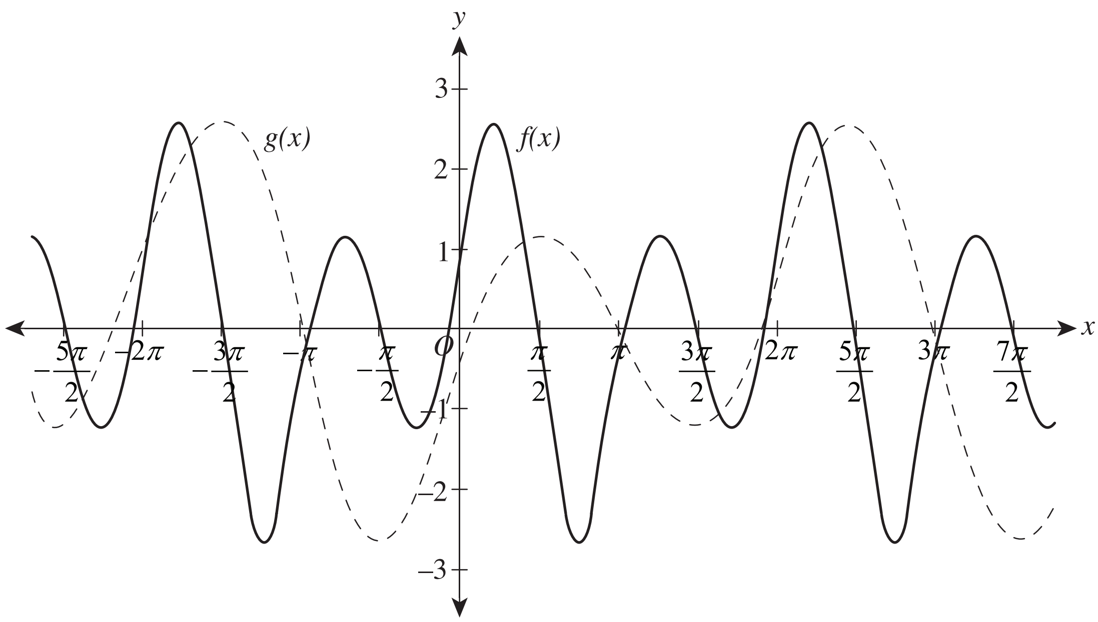
b. An estimate of one of the roots for the equation $f(x) = 0$ is $x_0 = 1$.

Using two iterations of Newton's method, find a more accurate estimate for this root. Give your answer correct to four decimal places. Compare the estimate with the actual root. 3 marks

c. **i.** State the period of f. 1 mark

iI. State the range of f. Give your answer correct to two decimal places. 1 mark

d. The graphs of $y = f(x)$ and $y = g(x)$ are as follows.



i. State the values of a and b if $g(x) = af(bx)$. 2 marks

ii. Show that $f\left(\dfrac{x}{2} - \pi\right) = g(x)$. 2 marks

END OF QUESTION AND ANSWER BOOKLET