SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Q1-5 ECDCD
Q6-10 ABEBC
Q11-15 EADAE
Q16-20 AEDCB

Question 1

E

The graph of $f(x) = \frac{e^x}{x-1}$ does **not** have a

A. horizontal asymptote.
B. vertical asymptote.
C. local minimum.
D. vertical axis intercept.
E. point of inflection.

Question 2

C

The asymptote(s) of the graph of $f(x) = \frac{x^2+1}{2x-8}$ has equation(s)

A. $x = 4$

B. $x = 4$ and $y = \frac{x}{2}$

C. $x = 4$ and $y = \frac{x}{2} + 2$

D. $x = 8$ and $y = \frac{x}{2}$

E. $x = 8$ and $y = 2x + 2$

Question 3 D

The implied domain of the function with rule $f(x)=1-\sec\left(x+\frac{\pi}{4}\right)$ is

A. R

B. $[0, 2]$

C. $R\setminus\left\{\frac{(4n-1)\pi}{4}\right\},\ n\in Z$

D. $R\setminus\left\{\frac{(4n+1)\pi}{4}\right\},\ n\in Z$

E. $R\setminus\left\{\frac{(2n-1)\pi}{2}\right\},\ n\in Z$

Question 4 C

The expression $i^{1!}+i^{2!}+i^{3!}+\ldots+i^{100!}$ is equal to

A. 0

B. 96

C. $95+i$

D. $94+2i$

E. $98+2i$

Question 5 D

Let $z=x+yi$, where $x, y\in R$. The rays $\text{Arg}(z-2)=\frac{\pi}{4}$ and $\text{Arg}\left(z-(5+i)\right)=\frac{5\pi}{6}$, where $z\in C$, intersect on the complex plane at a point (a, b).

The value of b is

A. $-\sqrt{3}$

B. $2-\sqrt{3}$

C. 0

D. $\sqrt{3}$

E. $2+\sqrt{3}$

Question 6
A

Let $z, w \in C$, where $\text{Arg}(z) = \frac{\pi}{2}$ and $\text{Arg}(w) = \frac{\pi}{4}$.

The value of $\text{Arg}\left(\frac{z^5}{w^4}\right)$ is

A. $-\frac{\pi}{2}$

B. $\frac{\pi}{2}$

C. π

D. $\frac{5\pi}{2}$

E. $\frac{7\pi}{2}$

Question 7
B

The length of the curve defined by the parametric equations $x = 3\sin(t)$ and $y = 4\cos(t)$ for $0 \le t \le \pi$ is given by

A. $\int_0^{\pi} \sqrt{9\cos^2(t) - 16\sin^2(t)}\, dt$

B. $\int_0^{\pi} \sqrt{9 + 7\sin^2(t)}\, dt$

C. $\int_0^{\pi} \sqrt{1 + 16\sin^2(t)}\, dt$

D. $\int_0^{\pi} \left(3\cos(t) - 4\sin(t)\right) dt$

E. $\int_0^{\pi} \sqrt{3\cos^2(t) + 4\sin^2(t)}\, dt$

Question 8
E

With a suitable substitution, $\int_1^5 (2x-1)\sqrt{2x+1}\, dx$ can be expressed as

A. $\frac{1}{2}\int_1^5 \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$

B. $2\int_3^{11} \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$

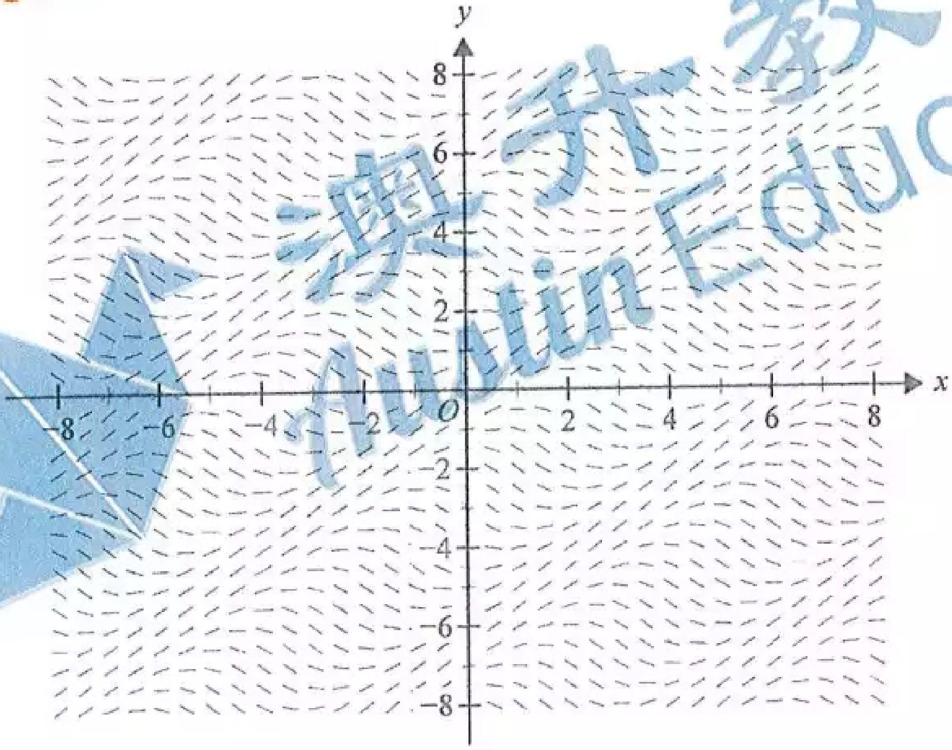
C. $2\int_1^5 \left(u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right) du$

D. $2\int_3^{11} \left(u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right) du$

E. $\frac{1}{2}\int_3^{11} \left(u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right) du$

澳升教育 Austin Education

Question 9 B



The differential equation that has the diagram above as its direction field is

A. $\dfrac{dy}{dx} = \sin(y - x)$

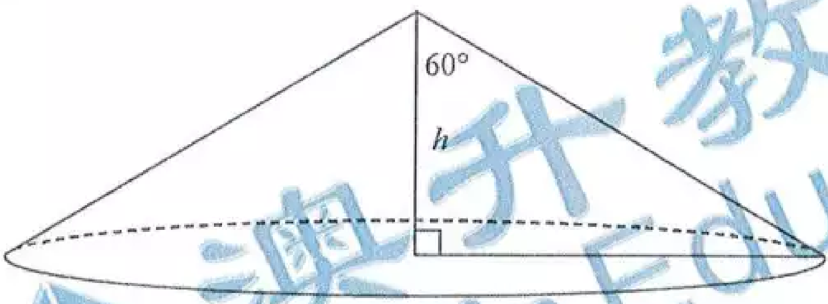
B. $\dfrac{dy}{dx} = \cos(y - x)$

C. $\dfrac{dy}{dx} = \sin(x - y)$

D. $\dfrac{dy}{dx} = \dfrac{1}{\cos(y - x)}$

E. $\dfrac{dy}{dx} = \dfrac{1}{\sin(y - x)}$

Question 10 C



Sand falls from a chute to form a pile in the shape of a right circular cone with semi-vertex angle 60°. Sand is added to the pile at a rate of 1.5 m^3 per minute.

The rate at which the height h metres of the pile is increasing, in metres per minute, when the height of the pile is 0.5 m, correct to two decimal places, is

A. 0.21
B. 0.31
C. 0.64
D. 3.82
E. 3.53

Question 11 E

Let point M have coordinates $(a, 1, -2)$ and let point N have coordinates $(-3, b, -1)$.

If the coordinates of the midpoint of $\overline{MN}$ are $\left(-5, \frac{3}{2}, c\right)$ and a, b and c are real constants, then the values of a, b and c are respectively

A. -13, 2 and $-\frac{1}{2}$

B. -2, $\frac{1}{2}$ and -3

C. -7, -2 and $-\frac{3}{2}$

D. -2, $-\frac{1}{2}$ and -3

E. -7, 2 and $-\frac{3}{2}$

Question 12 A

The vector resolute of $\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ in the direction of $m\underset{\sim}{i} + n\underset{\sim}{j} + p\underset{\sim}{k}$ is $2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$, where m, n and p are real constants.

The values of m, n and p can be found by solving the equations

A. $\dfrac{m(m+n-p)}{m^2+n^2+p^2} = 2$, $\dfrac{n(m+n-p)}{m^2+n^2+p^2} = -3$ and $\dfrac{p(m+n-p)}{m^2+n^2+p^2} = 1$

B. $\dfrac{m(m+n-p)}{m^2+n^2+p^2} = 1$, $\dfrac{n(m+n-p)}{m^2+n^2+p^2} = 1$ and $\dfrac{p(m+n-p)}{m^2+n^2+p^2} = -1$

C. $m+n-p=6$, $m+n-p=-9$ and $m+n-p=-3$

D. $m+n-p=3m$, $m+n-p=3n$ and $m+n-p=-3p$

E. $m+n-p=2\sqrt{3}$, $m+n-p=-3\sqrt{3}$ and $m+n-p=\sqrt{3}$

Question 13 D

Two forces, $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$, both measured in newtons, act on a mass of 3 kg, producing an acceleration of $\sqrt{3}\underset{\sim}{i} + \underset{\sim}{j}$ ms^{-2}.

Given that $\underset{\sim}{F}_1 = 2\underset{\sim}{j}$, the acute angle between $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$ is

A. $\arccos\left(\dfrac{1}{\sqrt{3}}\right)$

B. $\pi - \arccos\left(\dfrac{1}{\sqrt{3}}\right)$

C. $\dfrac{\pi}{6}$

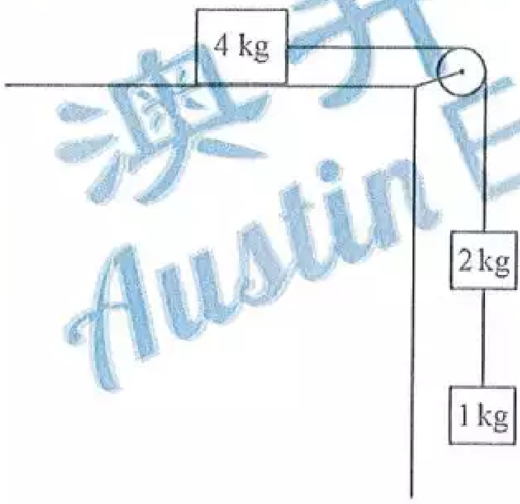
D. $\arccos\left(\dfrac{1}{2\sqrt{7}}\right)$

E. $\pi - \arccos\left(\dfrac{1}{2\sqrt{7}}\right)$

Question 14

A

A 4 kg mass is held at rest on a smooth surface. It is connected by a light inextensible string that passes over a smooth pulley to a 2 kg mass, which in turn is connected by the same type of string to a 1 kg mass. This is shown in the diagram below.



When the 4 kg mass is released, the tension in the string connecting the 1 kg and 2 kg masses is T newtons. The value of T is

A. $\frac{4g}{7}$

B. $\frac{3g}{7}$

C. $\frac{g}{7}$

D. $\frac{6g}{7}$

E. g

Question 15

E

A particle is moving along the x-axis with velocity $\underset{\sim}{v} = u\underset{\sim}{i}$, where u is a real constant.

At time $t = 0$, a force acts on the particle, causing it to accelerate with acceleration $\underset{\sim}{a} = \alpha\underset{\sim}{j}$, where α is a negative real constant.

Which one of the following statements correctly describes the motion of the particle for $t > 0$?

A. The particle slows down, stops momentarily and then begins to move in the opposite direction to its original motion.

B. The particle continues to travel along the x-axis with decreasing speed.

C. The particle travels parallel to the y-axis.

D. The particle moves along a circular arc.

E. The particle moves along a parabola.

Question 16

A

A variable force acts on a particle, causing it to move in a straight line. At time t seconds, where $t \geq 0$, its velocity v metres per second and position x metres from the origin are such that $v = e^x \sin(x)$.

The acceleration of the particle, in ms^{-2}, can be expressed as

A. $e^{2x}\left(\sin^2(x) + \frac{1}{2}\sin(2x)\right)$

B. $e^x \sin(x)\left(\sin(x) + \cos(x)\right)$

C. $e^x\left(\sin(x) + \cos(x)\right)$

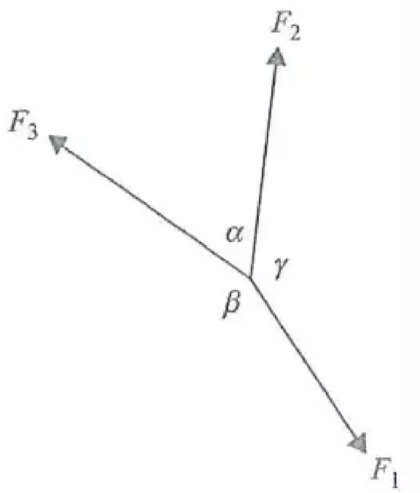
D. $\frac{1}{2}e^{2x}\sin^2(x)$

E. $e^x \cos(x)$

Question 17

E

A particle is held in equilibrium by three coplanar forces of magnitudes F_1, F_2 and F_3. The angles between these forces are α, β and γ, as shown in the diagram below.



If $\beta = 2\alpha$, then $\frac{F_1}{F_2}$ is equal to

A. $\frac{1}{2}\sin(\alpha)$

B. $2\sin(\alpha)$

C. $\frac{1}{2}\text{cosec}(\alpha)$

D. $\frac{1}{2}\cos(\alpha)$

E. $\frac{1}{2}\sec(\alpha)$

Question 18 D

The masses of a random sample of 36 track athletes have a mean of 65 kg. The standard deviation of the masses of all track athletes is known to be 4 kg.

A 98% confidence interval for the mean of the masses of all track athletes, correct to one decimal place, would be closest to

A. (51.0, 79.0)
B. (63.6, 66.4)
C. (63.3, 66.7)
D. (63.4, 66.6)
E. (64.3, 65.7)

Question 19 C

X and Y are independent random variables where each has a mean of 4 and a variance of 9.

If the random variable $Z = aX + bY$ has a mean of 8 and a variance of 90, possible values of a and b are

A. $a = 1, b = 1$
B. $a = 4, b = -2$
C. $a = 3, b = -1$
D. $a = 1, b = 3$
E. $a = -2, b = 4$

Question 20 B

The random number function of a calculator is designed to generate random numbers that are uniformly distributed from 0 to 1. When working properly, a calculator generates random numbers from a population where $\mu = 0.5$ and $\sigma = 0.2887$

When checking the random number function of a particular calculator, a sample of 100 random numbers was generated and was found to have a mean of $\bar{x} = 0.4725$

Assuming H_0: $\mu = 0.5$ and H_1: $\mu < 0.5$, and $\sigma = 0.2887$, the p value for a one-sided test is

A. 0.0953
B. 0.1704
C. 0.4621
D. 0.8296
E. 0.9525

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (11 marks)

A curve is defined parametrically by $x = \sec(t) + 1$, $y = \tan(t)$, where $t \in \left[0, \frac{\pi}{2}\right)$.

a. Show that the curve can be represented in cartesian form by the rule $y = \sqrt{x^2 - 2x}$. 2 marks

$x - 1 = \sec(t)$
$y = \tan(t)$ $\Rightarrow$ $y^2 + 1 = (x-1)^2$
$\tan^2(t) + 1 = \sec^2(t)$ $\therefore$ $y^2 + 1 = x^2 - 2x + 1$
$y^2 = x^2 - 2x$
$\because t \in [0, \frac{\pi}{2})$
$\therefore y \geq 0$, must take +ve option $\therefore$ $y = \sqrt{x^2 - 2x}$

b. State the domain and range of the relation given by $y = \sqrt{x^2 - 2x}$. 2 marks

dom: $x \in [2, \infty)$
range: $y \in [0, \infty)$

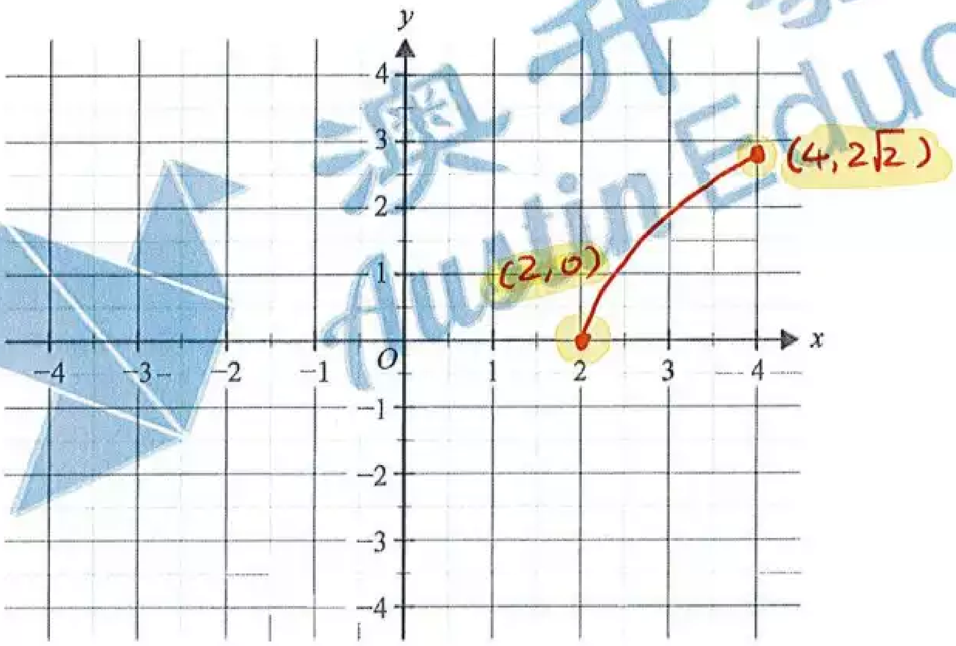
c. **i.** Express $\frac{dy}{dx}$ in terms of $\sin(t)$. 2 marks

$\frac{dy}{dt} = \frac{1}{\cos^2(t)}$
$\frac{dx}{dt} = \frac{\sin(t)}{\cos^2(t)}$
$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1}{\sin(t)}$

ii. State the limiting value of $\frac{dy}{dx}$ as t approaches $\frac{\pi}{2}$. 1 mark

$\left.\frac{dy}{dx}\right|_{t \to \frac{\pi}{2}} = 1$

d. Sketch the curve $y = \sqrt{x^2 - 2x}$ on the axes below for $x \in [2, 4]$, labelling the endpoints with their coordinates. 2 marks



e. The portion of the curve given by $y = \sqrt{x^2 - 2x}$ for $x \in [2, 4]$ is rotated about the y-axis to form a solid of revolution.

Write down, but do not evaluate, a definite integral in terms of t that gives the volume of the solid formed. 2 marks

Now: $y^2 + 1 = (x-1)^2 \quad x \geq 2$

$\therefore x - 1 = \sqrt{y^2+1}, \quad x = \sqrt{y^2+1} + 1$

$y = 0, \ t = 0$

$y = 2\sqrt{2}, \ t = \tan^{-1}(2\sqrt{2})$

$= \sqrt{\tan^2(t)+1} + 1$

$= \sec(t) + 1$ as $t \in [0, \frac{\pi}{2})$

$\therefore V = \pi \int_{y=0}^{y=2\sqrt{2}} x^2 \, dy$

$dy = \sec^2(t)\, dt$

$= \pi \int_0^{\tan^{-1}(2\sqrt{2})} (\sec(t) + 1)^2 \cdot \sec^2(t)\, dt$

Question 2 (10 marks)

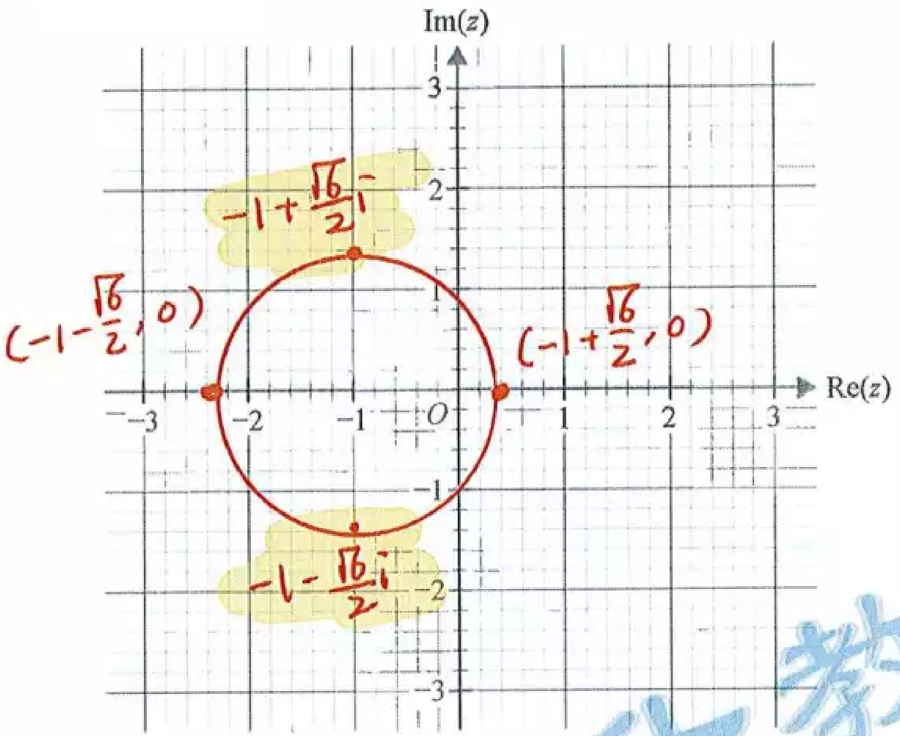
a. **i.** Show that the solutions of $2z^2 + 4z + 5 = 0$, where $z \in C$, are $z = -1 \pm \frac{\sqrt{6}}{2}i$. 1 mark

$$z = \frac{-4 \pm \sqrt{16-4\times2\times5}}{2\times2}$$

$$= \frac{-4 \pm 2\sqrt{6}i}{4}$$

$$= -1 \pm \frac{\sqrt{6}}{2}i$$

ii. Plot the solutions of $2z^2 + 4z + 5 = 0$ on the Argand diagram below. 1 mark



Let $|z+m|=n$, where $m, n \in R$, represent the circle of minimum radius that passes through the solutions of $2z^2+4z+5=0$.

b. **i.** Find m and n. 2 marks

By observation:

$m=-1+0i$

$n=\frac{\sqrt{6}}{2}$

$\therefore m=-1,\ n=\frac{\sqrt{6}}{2}$

ii. Find the cartesian equation of the circle $|z+m|=n$. 1 mark

$(x+1)^2+y^2=\frac{3}{2}$

iii. Sketch the circle on the Argand diagram in **part a.ii.** Intercepts with the coordinate axes do not need to be calculated or labelled. 1 mark

c. Find all values of d, where $d \in R$, for which the solutions of $2z^2+4z+d=0$ satisfy the relation $|z+m| \le n$. 2 marks

$2z^2+4z+d=0$

$|z+1| \le \frac{\sqrt{6}}{2}$

csolve$\left(2z^2+4z+d=0 \text{ and } |z+1| \le \frac{\sqrt{6}}{2}, z\right)$

$z=\ldots, d=5;\ z=\ldots, d=-1$

$-1 \le d \le 5$

d. All complex solutions of $az^2+bz+c=0$ have non-zero real and imaginary parts.

Let $|z+p|=q$ represent the circle of minimum radius in the complex plane that passes through these solutions, where $a, b, c, p, q \in R$.

Find p and q in terms of a, b and c. 2 marks

Using result from previous part

$z=\frac{-b \pm \sqrt{4ac-b^2}\cdot i}{2a}$

$p=\frac{1}{2}(z_1+z_2)=\frac{-b}{2a}$

$q=\frac{\sqrt{4ac-b^2}}{2a}$

Question 3 (9 marks)

a. The growth and decay of a quantity P with respect to time t is modelled by the differential equation

$$\frac{dP}{dt} = kP$$

where $t \geq 0$.

i. Given that $P(a) = r$ and $P(b) = s$, where P is a function of t, show that $k = \frac{1}{a-b}\log_e\left(\frac{r}{s}\right)$. 2 marks

$t \geq 0$

$p > 0$

$k \cdot \int dt = \int \frac{1}{p}\, dp$

$kt = \ln(p) + C$

$t = a,\ p = r$

$ka = \ln(r) + C$ ①

$t = b,\ p = s$

$kb = \ln(s) + C$ ②

① − ② $\Rightarrow$

$k(a-b) = \ln(r) - \ln(s)$

$k(a-b) = \ln\left(\frac{r}{s}\right)$

$\therefore k = \frac{1}{a-b}\ln\left(\frac{r}{s}\right)$

ii. Specify the condition(s) for which $k > 0$. 2 marks

when $k > 0$

$\frac{1}{a-b} > 0$

$\therefore a > b$

also $\ln\left(\frac{r}{s}\right) > 0$

$\frac{r}{s} > 1$

$r > s$

$\left\{ \therefore a > b \ \& \ r > s \right\}$ and $\underbrace{a, b, r, s \in \mathbb{R}}$

when $k < 0$

$\frac{1}{a-b} < 0$,

$a - b < 0$

$a < b$

$\ln\left(\frac{r}{s}\right) < 0$

$0 < \frac{r}{s} < 1$

$0 < r < s$

b. The growth of another quantity Q with respect to time t is modelled by the differential equation

$$\frac{dQ}{dt} = e^{t-Q}$$

$$\frac{dQ}{dt} = \frac{e^t}{e^Q}$$

where $t \geq 0$ and $Q = 1$ when $t = 0$.

i. Express this differential equation in the form $\int f(Q)dQ = \int h(t)dt$. 1 mark

$$\int e^Q \cdot dQ = \int e^t \cdot dt$$

ii. Hence, show that $Q = \log_e(e^t + e - 1)$. 2 marks

$e^Q = e^t + C$

$Q = 1,\ t = 0$

$\therefore e = e^0 + C$

$C = e - 1$

$\therefore e^Q = e^t + e - 1$

$\Rightarrow Q = \log_e(e^t + e - 1)$

iii. Show that the graph of Q as a function of t does not have a point of inflection. 2 marks

$$\frac{dQ}{dt} = \frac{e^t}{e^t + e - 1}$$

$$\frac{d^2Q}{dt^2} = \frac{(e-1)e^t}{(e^t + e - 1)^2}$$

for $t \geq 0$,

$(e^t + e - 1)^2 > 0$

$(e-1)e^t \geq e - 1$

as $e^t \geq 1$

$\therefore \frac{d^2Q}{dt^2} > 0$

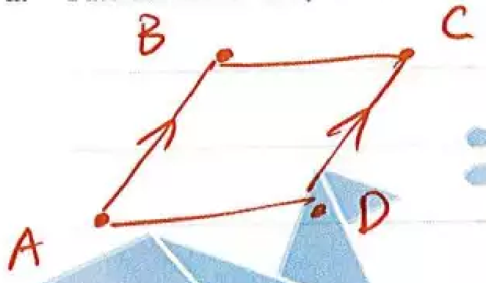
and thus

$Q(t)$ has no point of inflection

Question 4 (11 marks)

The base of a pyramid is the parallelogram $ABCD$ with vertices at points $A(2, -1, 3)$, $B(4, -2, 1)$, $C(a, b, c)$ and $D(4, 3, -1)$. The apex (top) of the pyramid is located at $P(4, -4, 9)$.

a. Find the values of a, b and c. 2 marks



$$\overrightarrow{AB} = \overrightarrow{DC}$$

$$(4-2)\underset{\sim}{i} + (-2-(-1))\underset{\sim}{j} + (1-3)\underset{\sim}{k}$$

$$\underline{\underline{a = 6}}, \quad \underline{\underline{b = 2}}, \quad \underline{\underline{c = -3}}$$

b. Find the cosine of the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$. 2 marks

$$\overrightarrow{AB} = 2\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$$

$$\overrightarrow{AD} = 2\underset{\sim}{i} + 4\underset{\sim}{j} - 4\underset{\sim}{k}$$

$$\cos(\angle BAD) = \frac{\overrightarrow{AB} \cdot \overrightarrow{AD}}{|\overrightarrow{AB}||\overrightarrow{AD}|}$$

$$= \frac{8}{3 \times 6} = \frac{4}{9}$$

c. Find the area of the base of the pyramid. 2 marks

$$\text{Area} = |\overrightarrow{AB} \times \overrightarrow{AD}| = |12\underset{\sim}{i} + 4\underset{\sim}{j} + 10\underset{\sim}{k}|$$

$$= 2\sqrt{65}$$

or

$$\text{Area} = \frac{1}{2} \times |\overrightarrow{AB}||\overrightarrow{AD}| \times \sin(\theta) \times 2$$

$$= 3 \times 6 \times \sin\left(\cos^{-1}\left(\frac{4}{9}\right)\right)$$

$$= 2\sqrt{65}$$

d. Show that $6\underset{\sim}{i}+2\underset{\sim}{j}+5\underset{\sim}{k}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AD}$, and hence find a unit vector that is perpendicular to the base of the pyramid. **3 marks**

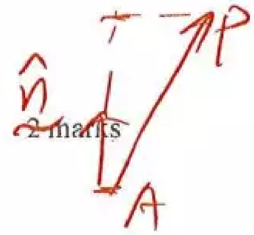
Let $\underset{\sim}{n} = 6\underset{\sim}{i}+2\underset{\sim}{j}+5\underset{\sim}{k}$, $\overrightarrow{AB} = 2\underset{\sim}{i}-\underset{\sim}{j}-2\underset{\sim}{k}$

$\overrightarrow{AD} = 2\underset{\sim}{i}+4\underset{\sim}{j}-4\underset{\sim}{k}$

$\underset{\sim}{n}\cdot\overrightarrow{AB} := 2\times 6+(-1)\times 2+(-2)\times 5 = 12-2-10=0$

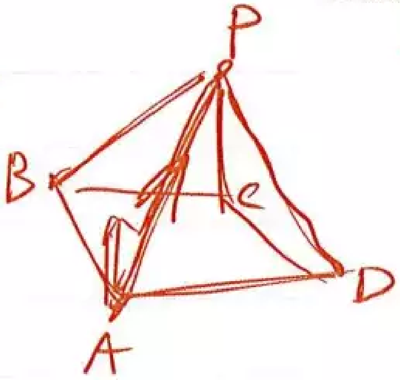
$\underset{\sim}{n}\cdot\overrightarrow{AD} = 6\times 2+2\times 4+5\times -4 = 12+8-20=0$

$\therefore \hat{\underset{\sim}{n}} = \frac{6\sqrt{65}}{65}\underset{\sim}{i}+\frac{2\sqrt{65}}{65}\underset{\sim}{j}+\frac{\sqrt{65}}{13}\underset{\sim}{k}$



e. Find the volume of the pyramid. **2 marks**

Volume $= \frac{1}{3}\times$ Area $\times$ height



height $= \overrightarrow{AP}\cdot\hat{\underset{\sim}{n}}$

$= (2\underset{\sim}{i}-3\underset{\sim}{j}+6\underset{\sim}{k})\cdot\hat{\underset{\sim}{n}}$

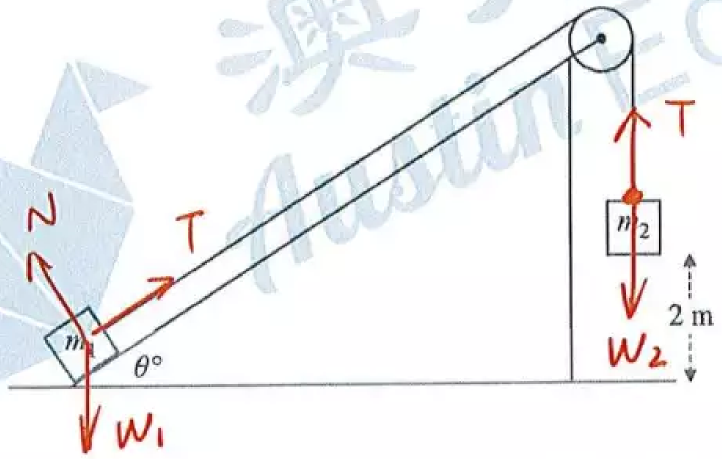
$= \frac{36\sqrt{65}}{65}$

Volume $= \frac{1}{3}\times 2\sqrt{65}\times\frac{36\sqrt{65}}{65} = 24$ units^3

Question 5 (10 marks)

A mass of m_1 kilograms is initially held at rest near the bottom of a smooth plane inclined at θ degrees to the horizontal. It is connected to a mass of m_2 kilograms by a light inextensible string parallel to the plane, which passes over a smooth pulley at the end of the plane. The mass m_2 is 2 m above the horizontal floor.

The situation is shown in the diagram below.



a. After the mass m_1 is released, the following forces, measured in newtons, act on the system:

- weight forces W_1 and W_2
- the normal reaction force N
- the tension in the string T

On the diagram above, show and clearly label the forces acting on each of the masses. 1 mark

b. If the system remains in equilibrium after the mass m_1 is released, show that $\sin(\theta) = \frac{m_2}{m_1}$. 1 mark

$W_1 \sin(\theta°) = T$

$T = W_2$

$N = W_1 \cos(\theta°)$

$\Rightarrow m_1 g \sin(\theta) = T$

$T = m_2 g$

$\Rightarrow m_1 g \sin(\theta) = m_2 g$

$\therefore m_1 \sin(\theta) = m_2$

$\therefore \frac{m_2}{m_1} = \sin(\theta)$

c. After the mass m_1 is released, the mass m_2 falls to the floor.

i. For what values of θ will this occur? Express your answer as an inequality in terms of m_1 and m_2. 1 mark

$W_2 > W_1 \sin(\theta)$, $m_2 g > m_1 g \sin(\theta)$

$0 < \theta < \sin^{-1}\left(\frac{m_2}{m_1}\right)$

ii. Find the magnitude of the acceleration, in ms^{-2}, of the system after the mass m_1 is released and before the mass m_2 hits the floor. Express your answer in terms of m_1, m_2 and θ. 2 marks

$m_2 g - T + T - m_1 g \sin(\theta) = (m_1 + m_2) a$

$a = \dfrac{g(m_2 - m_1 \sin(\theta))}{m_1 + m_2}$

d. After the mass m_1 is released, it moves up the plane.

Find the maximum distance, in metres, that the mass m_1 will move up the plane if $m_1 = 2m_2$ and $\sin(\theta) = \frac{1}{4}$. 5 marks

First stage

Have tension

$m_1 = 2m_2$, $\sin(\theta) = \frac{1}{4}$ $\Rightarrow$ $a = \dfrac{g(m_2 - 2m_2 \times \frac{1}{4})}{3m_2}$

$a = g \times \dfrac{m_2/2}{3m_2} = \dfrac{g}{6}$ m/s^2

Before the $m_2 g$ hits the ground

using $a = +\frac{1}{6} \times 9.8$, $s = 2$, $u = 0$, $V = ?$

$V^2 = 0^2 + 2 \times \frac{1}{6} \times 9.8 \times 2$

$V = \dfrac{7\sqrt{30}}{15}$ m/s

$\therefore$ Total distance $= 2 + \frac{4}{3} = \frac{10}{3}$ m

No tension!

2nd stage: $u = \dfrac{14\sqrt{15}}{15}$, $m_1 a = -\frac{1}{4} m_1 g$, $a = -\frac{1}{4} \times 9.8$

$V = 0$, $S = ?$ $0 = \left(\frac{14}{\sqrt{15}}\right)^2 - 2 \times \frac{1}{4} \times 9.8 \times S$

$S = \frac{4}{3}$

Question 6 (9 marks)

A company produces packets of noodles. It is known from past experience that the mass of a packet of noodles produced by one of the company's machines is normally distributed with a mean of 375 grams and a standard deviation of 15 grams.

To check the operation of the machine after some repairs, the company's quality control employees select two independent random samples of 50 packets and calculate the mean mass of the 50 packets for each random sample.

a. Assume that the machine is working properly. Find the probability that at least one random sample will have a mean mass between 370 grams and 375 grams. Give your answer correct to three decimal places. 2 marks

≥ 1

$\bar{X} \sim N\left(375, \left(\frac{15}{\sqrt{50}}\right)^2\right)$

$\Pr(370 \leq \bar{X} \leq 375)$

$= 0.4907889\ldots$

$Y \sim Bi(2, 0.4907889)$

$\Pr(Y \geq 1)$

$= 1 - \Pr(Y = 0)$

$= 0.7407041\ldots$

≈ 0.741

b. Assume that the machine is working properly. Find the probability that the means of the two random samples differ by less than 2 grams. Give your answer correct to three decimal places. 3 marks

$\Pr(|\bar{X}_1 - \bar{X}_2| < 2)$

$= 2\Pr(0 < \bar{X}_1 - \bar{X}_2 < 2)$

$= 0.49501506\ldots$

$= \boxed{0.495}$

$E(\bar{X}_1 - \bar{X}_2) = 0$

$Var(\bar{X}_1 - \bar{X}_2) = 2Var(\bar{X}_1) = \frac{2 \times 15^2}{50} = 9$

$sd(\bar{X}_1 - \bar{X}_2) = 3$

To test whether the machine is working properly after the repairs and is still producing packets with a mean mass of 375 grams, the two random samples are combined and the mean mass of the 100 packets is found to be 372 grams. Assume that the standard deviation of the mass of the packets produced is still 15 grams. A two-tailed test at the 5% level of significance is to be carried out.

c. Write down suitable hypotheses H_0 and H_1 for this test. 1 mark

$H_0: \mu = 375$

$H_1: \mu \neq 375$

d. Find the p value for the test, correct to three decimal places. 1 mark

p-value $= 0.046$

e. Does the mean mass of the sample of 100 packets suggest that the machine is working properly at the 5% level of significance for a two-tailed test? Justify your answer. 1 mark

No, it's not working properly.
Because p-value < 0.05, and we reject H_0.

f. What is the smallest value of the mean mass of the sample of 100 packets for H_0 to be **not** rejected? Give your answer correct to one decimal place. 1 mark

$\Pr(\bar{X} \le C_1) = 0.025$ $\bar{X} \sim N(375, \frac{15^2}{100})$

$C_1 = \underline{372.1 \text{ grams}}$