



Semester Two Examination, 2017

Question/Answer booklet

**MATHEMATICS  
APPLICATIONS  
UNITS 1 AND 2**  
Section Two:  
Calculator-assumed

# SOLUTIONS

Student Number: In figures

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In words

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Your name

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**Time allowed for this section**

Reading time before commencing work: ten minutes  
Working time: one hundred minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer booklet  
Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					100

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section Two: Calculator-assumed**

**65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

**Question 7**

**(5 marks)**

Six variables are listed below.

- A Height of trees
- B Hair colour
- C Country of birth
- D Movie rating using a scale of 1 to 5 stars
- E Daily minimum temperature
- F Driver's licence number

(a) In the spaces below, write the letter corresponding to **one** of the above of variables that is

(i) a categorical variable that can be classified as ordinal. (1 mark)

Solution
D or F
Specific behaviours
✓ suitable variable

(ii) a numerical variable that can be classified as continuous. (1 mark)

Solution
A or E
Specific behaviours
✓ suitable variable

(iii) a categorical variable that can be classified as nominal. (1 mark)

Solution
B or C
Specific behaviours
✓ suitable variable

(b) Give an example of a numerical variable that is not listed above and can be classified as discrete. Explain why it has this classification. (2 marks)

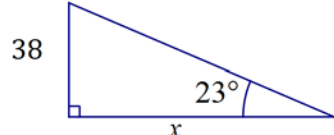
Solution
Number of students in a class, etc, etc.
The value of such variables can be obtained by counting.
Specific behaviours
✓ suitable variable
✓ explanation

**Question 8****(6 marks)**

From the top of a 38 m tall building, the angle of depression to a small dog sitting on a path, level with the base of the building, is  $23^\circ$ .

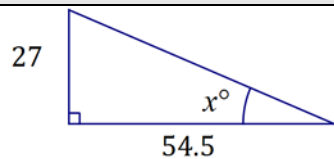
- (a) Calculate the distance the dog is from the base of the building.

**(3 marks)**

Solution	
 $\tan 23 = \frac{38}{x} \Rightarrow x = 89.5 \text{ m}$	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ sketch diagram with measurements</li> <li>✓ uses tangent ratio</li> <li>✓ states distance</li> </ul>	

The dog walks 35 m directly towards the building and then stops.

- (b) Determine the angle of elevation from the dog to a window positioned 11 m below the top of the building. **(3 marks)**

Solution	
 $\tan x = \frac{27}{54.5} \Rightarrow x = 26.4^\circ$	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ sketch with new measurements</li> <li>✓ uses tangent ratio</li> <li>✓ states angle</li> </ul>	

Question 9

(6 marks)

- (a) If concrete can be bought for \$260 per cubic metre, calculate the cost per square metre to make a rectangular concrete slab measuring 6.2 m by 8.5 m and with a uniform thickness of 14 cm. (3 marks)

Solution
$A = 6.2 \times 8.5 = 52.7 \text{ m}^2$ $V = 52.7 \times 0.14 = 7.378 \text{ m}^3$ $C = 7.378 \times 260 = \$1918.28$ $\text{Cost per m}^2 = 1918.28 \div 52.7 = \$36.40$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ calculates slab volume</li> <li>✓ calculates slab cost</li> <li>✓ calculates slab cost per <math>\text{m}^2</math></li> </ul>

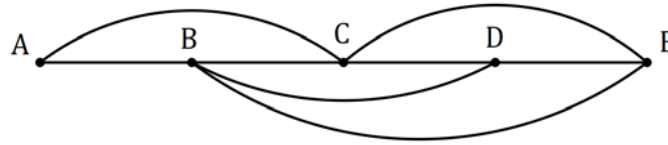
- (b) After pouring another rectangular concrete slab, the builder wanted to check that the corners of the slab were right-angled. The builder measured the diagonal of the slab to be 9.7 m and the sides of the slab to be 7.2 m and 6.5 m.

Use your knowledge of mathematics to determine whether the corners of the slab were right-angled. (3 marks)

Solution
$7.2^2 + 6.5^2 = 51.84 + 42.25 = 94.09$ $9.7^2 = 94.09$ <p>As sum of squares of two shorter sides is equal to square of diagonal, then the triangle satisfies Pythagoras' Theorem, and so the angle in the corner is a right-angle.</p>
Specific behaviours
<ul style="list-style-type: none"> <li>✓ uses Pythagoras' Theorem</li> <li>✓ adds squares of two shorter sides</li> <li>✓ writes conclusion</li> </ul>

**Question 10****(6 marks)**

The network below shows direct bus services between five city landmarks,  $A, B, C, D$  and  $E$ . For example, a direct bus service exists between  $A$  and  $C$  but not between  $A$  and  $D$ .



- (a) Represent this information in a  $5 \times 5$  matrix  $M$ , with row and column headings in alphabetical order. (2 marks)

Solution	
$M =$	$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$
Specific behaviours	
✓ at least 3 correct rows ✓ all rows correct	

- (b) Calculate  $M^2$ , state the value of entry  $M^2_{3,2}$  and explain what this value means. (3 marks)

Solution	
$M^2 =$	$\begin{bmatrix} 2 & 1 & 1 & 2 & 2 \\ 1 & 4 & 3 & 2 & 2 \\ 1 & 3 & 4 & 2 & 2 \\ 2 & 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 2 & 3 \end{bmatrix}$
$M^2_{3,2} = 3$	
There are 3 ways to travel from $C$ to $B$ using 2 bus services.	
Specific behaviours	
✓ correct matrix ✓ correct element ✓ explanation	

- (c) In how many ways is it possible to use three bus services to start a journey at  $D$  and finish at  $A$ ? (1 mark)

Solution
4 ways (using $M^3$ )
Specific behaviours
✓ correct number of ways

Question 11

(9 marks)

A young person won \$15 000. Not needing the money immediately, they bought 600 TTD shares, 900 AAG shares and placed the remaining \$4 800 in a deposit account.

- (a) During the first year, dividends were paid of 55 cents per share for AAG and of 4.5% on the value of the TTD shares, which were priced at \$11.80 at the time. Calculate the total dividend paid. (3 marks)

Solution
$AAG: 900 \times 0.55 = \$495$ $TTD: 600 \times 11.80 = \$7080, 7080 \times 0.045 = 318.60$ $Total: 495 + 318.60 = \$813.60$
Specific behaviours
✓ dividend per share ✓ dividend as % ✓ total dividend

- (b) The deposit account paid 3.5% per annum interest, compounded annually. Calculate the interest that would accumulate in the account

- (i) during the first year.

(1 mark)

Solution
$I = 4800 \times 0.035 = \$168$
Specific behaviours
✓ uses simple interest

- (ii) during the second year.

(2 marks)

Solution
$P = 4800 + 168 = 4968$ $I = 4968 \times 0.035 = \$173.88$
Specific behaviours
✓ new principle ✓ states interest

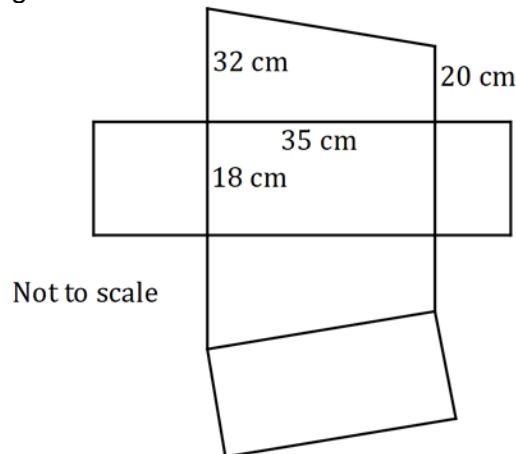
- (c) After two years the young person valued the total of their shares, the dividends received and the money in the deposit account at \$16 340.50. Calculate, to one decimal place, the percentage increase in the value of their winnings. (3 marks)

Solution
$Gain = 16340.50 - 15000 = 1340.50$ $\%Inc = 1340.50 \div 15000 \times 100 = 8.9366\%$ $\%Increase = 8.9\% (1dp)$
Specific behaviours
✓ indicates increase ✓ calculates percentage ✓ rounds correctly

## Question 12

(8 marks)

The net shown below, made from four rectangles and two trapeziums, is folded to form a scale model of a shed with a sloping roof.



- (a) Calculate the volume of the completed model.

(3 marks)

Solution
$A_{TRAP} = \frac{32 + 20}{2} \times 35$ $= 910$
$V = 910 \times 18 = 16\,380 \text{ cm}^3$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ uses trapezium as cross-section</li> <li>✓ calculates area of trapezium</li> <li>✓ calculates volume</li> </ul>

- (b) Determine the total surface area of the completed model.

(5 marks)

Solution
Three rectangles: $A_1 = 18 \times (32 + 35 + 20) = 1566$
Triangle: $32 - 20 = 12$ , $\triangle 12$ by $35$
Sloping edge: $x = \sqrt{12^2 + 35^2} = 37$
Roof area: $A_2 = 37 \times 18 = 666$
TSA: $A = 1566 + 666 + 2 \times 910 = 4\,052 \text{ cm}^2$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ calculates areas of three rectangles</li> <li>✓ dimensions of triangle</li> <li>✓ uses Pythagoras' theorem for length of roof</li> <li>✓ area of roof</li> <li>✓ sums six areas for TSA</li> </ul>



**Question 13**

**(7 marks)**

The number of nights that visitors booked into a hotel for on the first day in May this year are summarised in the table below.

Number of nights	1	2	3	4	5	6	7	8	9
Frequency	21	25	12	10	4	8	3	0	1

(a) For this data, determine

(i) the total number of nights booked by visitors.

(1 mark)

<b>Solution</b>
Total nights: 245
Mean: 2.92 nights
SD: 1.86 nights
<b>Specific behaviours</b>
✓ states number of nights
✓ states mean
✓ states SD

(ii) the mean number of nights booked.

(1 mark)

(iii) the standard deviation of the number of nights booked.

(1 mark)

(b) Briefly comment on the distribution of this data.

(1 mark)

<b>Solution</b>
The data is positively skewed.
<b>Specific behaviours</b>
✓ identifies positive skew

(c) Identify, with justification, whether the data contains an outlier.

(3 marks)

<b>Solution</b>
$IQR = 4 - 1.5 = 2.5$ $Q_3 + 1.5(2.5) = 4 + 3.75 = 7.75$ <p>Hence the booking for 9 nights is an outlier, as it exceeds 7.75.</p>
<b>Specific behaviours</b>
✓ indicates IQR ✓ uses $Q_3 + 1.5IQR$ criteria ✓ identifies outlier

## Question 14

(10 marks)

The spreadsheet below shows the wages sheet for some of the staff at a restaurant that is closed on Mondays and Tuesdays. Employees are paid time-and-a-quarter on Saturdays and time-and-a-half on Sundays.

Employee	Base Pay Rate (\$ per hour)	Rate	Hours worked					Weekly Pay (\$)
			Wed	Thu	Fri	Sat	Sun	
			1	1	1	1.25	1.5	
Ahn	17.50		7	8	8	0	0	<b>A</b>
Billy			0	0	0	8	8	334.40
Chris	22.90		6	6	6	6	0	<b>B</b>
Dhu	17.50		0	6	<b>C</b>	0	4	245.00
Eosin	22.90		0	6	6	6	6	652.65

- (a) Calculate the total wages paid to employees on Wednesday.

(2 marks)

Solution	
$7 \times 17.50 + 6 \times 22.90 = 122.50 + 137.40 = \$259.90$	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ uses correct rates and hours</li> <li>✓ correct total</li> </ul>	

- (b) Determine the values of A, B and C in the table.

(4 marks)

Solution	
$A = 17.50(7 + 8 + 8) = 17.50(23) = \$402.50$	
$B = 22.90(6 + 6 + 6 + 6 \times 1.25) = 22.90(25.5) = \$583.95$	
$17.50(6 + C + 4 \times 1.5) = 245.00 \Rightarrow C = 2$	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ value of A</li> <li>✓ value of B</li> <li>✓ calculates total hours or writes equation for Dhu</li> <li>✓ value of C</li> </ul>	

- (c) The restaurant is liable for payroll tax at a rate of 5.5% of all staff payments. Calculate the payroll tax the restaurant must pay this week for the five employees.

(2 marks)

Solution	
Total = 2218.50. $2218.50 \times 0.055 = \$122.02$	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ correct total</li> <li>✓ multiplies total by 0.055, rounding to dollars and cents</li> </ul>	

- (d) Determine Billy's base rate of pay.

(2 marks)

Solution	
$8 \times 1.25 + 8 \times 1.5 = 22$ hours $334.40 \div 22 = \$15.20$ per hour	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ total weighted hours</li> <li>✓ states hourly rate</li> </ul>	

see next page

**Question 15**

**(7 marks)**

A scientific study showed that the lengths of housefly wings are normally distributed with a mean and standard deviation of 4.55 mm and 0.39 mm respectively.

- (a) If a housefly wing is selected at random from those in the study, determine the probability that

- (i) the wing is longer than 5 mm. (1 mark)

Solution
$P(X > 5) = 0.124$
Specific behaviours
✓ correct probability

- (ii) the wing length is within 0.3 mm of the mean. (2 marks)

Solution
$P(4.25 < X < 4.85) = 0.558$
Specific behaviours
✓ indicates lower and upper bounds
✓ correct probability

- (b) If 99.7% of the wing lengths were between  $4.55 - L$  mm and  $4.55 + L$  mm, determine the value of  $L$ . (2 marks)

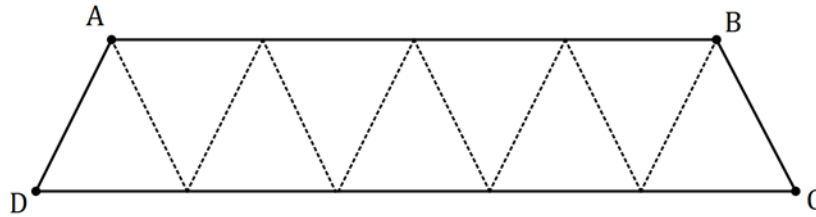
Solution
$99.7\% \Rightarrow \pm 3sd$ $L = 3 \times 0.39 = 1.17 \text{ mm}$
Specific behaviours
✓ indicates within 3 sd of mean
✓ states $L$

- (c) If 300 wings were selected at random, how many would be expected to have lengths less than 4 mm? (2 marks)

Solution
$P(X < 4) = 0.079$ $0.079 \times 300 \approx 24 \text{ wings}$
Specific behaviours
✓ calculates proportion
✓ states correct whole number

**Question 16****(9 marks)**

Nine equilateral triangles are joined together to make a trapezium  $ABCD$  with perimeter of length 132 cm, as shown below.



- (a) Explain why the sides of each triangle must be 12 cm long.

**(2 marks)**

Solution
There are 11 equal length sides making the perimeter. So $132 \div 11 = 12$ cm.
Specific behaviours
✓ indicates 11 equal length sides ✓ uses division

- (b) Use Heron's rule to calculate the area of **one** of the triangles.

**(3 marks)**

Solution
$s = \frac{12 + 12 + 12}{2} = 18$ $A = \sqrt{18(18 - 12)(18 - 12)(18 - 12)}$ $A = 62.35 \text{ cm}^2$
Specific behaviours
✓ calculates semi-perimeter ✓ substitutes into Heron's ✓ calculates area

- (c) Determine the area of the trapezium.

**(1 mark)**

Solution
$TA = 9 \times 62.35 = 561.2 \text{ cm}^2$
Specific behaviours
✓ states area

- (d) Determine the length of diagonal  $AC$  in the trapezium.

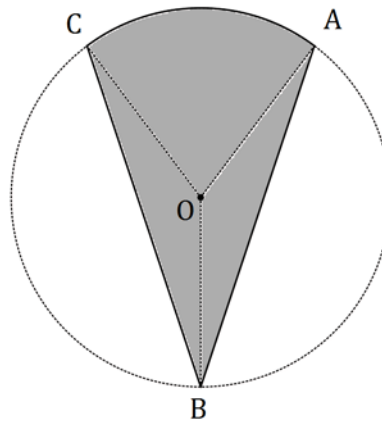
**(3 marks)**

Solution
$AB = 48, BC = 12, \angle ABC = 120^\circ$ $AC^2 = 48^2 + 12^2 - 2(48)(12) \cos 120^\circ$ $AC = 54.99 \text{ cm}$
Specific behaviours
✓ indicates lengths $AB$ , $AC$ and angle $ABC$ ✓ uses cosine rule ✓ states length

Question 17

(8 marks)

The diagram shows points  $A, B$  and  $C$  on the circumference of a circle with centre  $O$  and radius 18 cm. Angle  $BOC$  and angle  $BOA$  are both  $140^\circ$ .



- (a) Calculate the area of triangle  $BOA$ .

(2 marks)

Solution
$A = \frac{1}{2}(18)(18) \sin 140^\circ$ $= 104 \text{ cm}^2$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ uses area formula</li> <li>✓ states area</li> </ul>

- (b) Determine the size of angle  $AOC$ .

(1 mark)

Solution
$360 - 140 - 140 = 80^\circ$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ states angle</li> </ul>

- (c) Calculate the area of the shaded region  $ABC$ .

(3 marks)

Solution
$A_s = \frac{80}{360} \times \pi \times 18^2$ $= 226 \text{ cm}^2$ $A = 2 \times 104 + 226$ $= 434 \text{ cm}^2$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ substitutes into sector area formula</li> <li>✓ calculates sector area</li> <li>✓ calculates total area</li> </ul>

- (d) Determine the new area of the shaded region if the radius of the circle was increased from 18 cm to 72 cm.

(2 marks)

Solution
$sf = 72 \div 18 = 4$ $\text{New area} = 434 \times 4^2 = 6944 \text{ cm}^2$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ indicates scale factor</li> <li>✓ states new area</li> </ul>

## Question 18

(9 marks)

Some currency exchange rates advertised by an Australian bank for one Australian dollar are shown in the table below.

Country (currency)	Buy rate	Sell rate
Denmark (kroner)	5.6650	4.7070
Switzerland (franc)	0.8550	0.6910
South Korea (won)	891.7598	779.4543
Thailand (baht)	27.8070	24.2670

- (a) The midrate is the average of the buy and sell rates. Calculate the midrate for Danish kroner.

(1 mark)

Solution
$(5.665 + 4.707) \div 2 = 5.186$
Specific behaviours
✓ states midrate

- (b) If a Thai tourist in Australia wanted to exchange 5 000 baht for Australian currency, explain why the bank would use the buy rate, and calculate how many Australian dollars they would receive.

(2 marks)

Solution
The bank is <i>buying</i> foreign currency, so will use buy rate.
$5000 \div 27.8070 = \$179.81$
Specific behaviours
✓ explains rates quoted from banks point of view
✓ calculates amount

- (c) The same product, including shipping, can be bought online from a Swiss website for 123 francs or a Danish website for 776 kroner. Compare these prices in dollars for an Australian purchaser and calculate the amount saved by buying from the cheaper site.

(3 marks)

Solution
$123 \div 0.6910 = \$178.00$
$776 \div 4.707 = \$164.86$
Saving = $178.00 - 164.86 = \$13.14$
Specific behaviours
✓ uses sell rates for conversions
✓ converts both to AUD
✓ states saving

- (d) An Australian traveller exchanged \$2 000 into South Korean won, spent 1 250 000 won whilst away and then converted the remainder back into Australian dollars on their return. How much did the traveller receive back?

(3 marks)

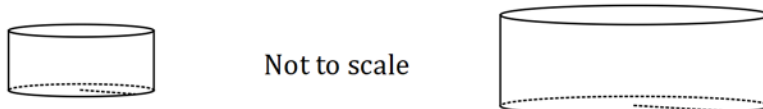
Solution
$2000 \times 779.4543 = 1558908.6$ won
$1558908.6 - 1250000 = 308908.6$ won
$308908.6 \div 891.7598 = \$346.40$
Specific behaviours
✓ uses correct rates both ways
✓ converts to won and subtracts spending
✓ converts back to AUD

**Question 19**

**(8 marks)**

Two similar cylindrical containers, both open at the top and full of water, are made from thin steel. The smaller container has an internal radius of 30 cm and height of 25 cm, and the larger a radius of 48 cm.

Neither cylinder, shown below, has a lid.



- (a) Using the fact that the cylinders are similar, determine the height of the large cylinder.

**(2 marks)**

<b>Solution</b>
Length scale factor: $48 \div 30 = 1.6$
$h = 25 \times 1.6 = 40 \text{ cm}$
<b>Specific behaviours</b>
✓ indicates scale factor (or ratio of dimensions)
✓ states height

- (b) Calculate the internal surface area (the area in contact with water) of the small cylinder.

**(2 marks)**

<b>Solution</b>
$A = \pi \times 30^2 + 2 \times \pi \times 30 \times 25$ $A = 2827 + 4712 = 7539 \approx 7540 \text{ cm}^2$
<b>Specific behaviours</b>
✓ indicates sum of base and wall
✓ states area, reasonable rounding

- (c) 150 mL of paint was required to coat the inside of the small cylinder. Determine how much paint was required to coat (to the same thickness) the inside of the larger cylinder.

**(2 marks)**

<b>Solution</b>
$150 \times 1.6^2 = 384 \text{ mL}$
<b>Specific behaviours</b>
✓ uses area scale factor
✓ states amount

The small cylinder is emptied and water from the large cylinder is then used to refill the small cylinder.

- (d) How many times can the water from the large cylinder be used to completely refill the small cylinder? Justify your answer.

**(2 marks)**

<b>Solution</b>
$V_L = 1.6^3 \times V_S$ $1.6^3 = 4.096 \Rightarrow \text{can refill 4 times}$
<b>Specific behaviours</b>
✓ uses volume scale factor
✓ states correct number

Additional working space

Question number: \_\_\_\_\_



Additional working space

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