Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section One: Calculator-free

SOLUTIONS

WA student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

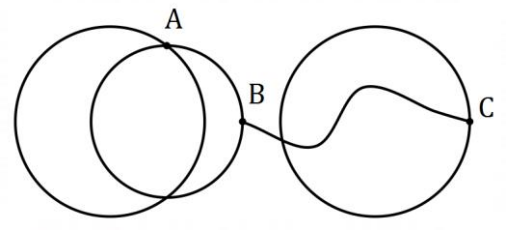
Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

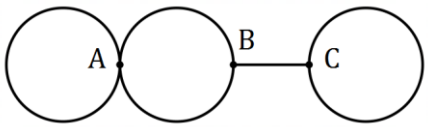
Working time: 50 minutes.

Question 1 **(6 marks)**

Consider the following graph G.



(a) Draw G in the plane, to clearly show that it is planar. (2 marks)

Solution

Specific behaviours
✓ 5 edges, 3 labelled vertices ✓ correctly drawn in the plane

(b) State the degree of each vertex and the degree sum for G. (2 marks)

Solution
$d_A = 4, d_B = 3, d_C = 3$ $d_A + d_B + d_C = 10$
Specific behaviours
✓ correct degrees ✓ correct degree sum

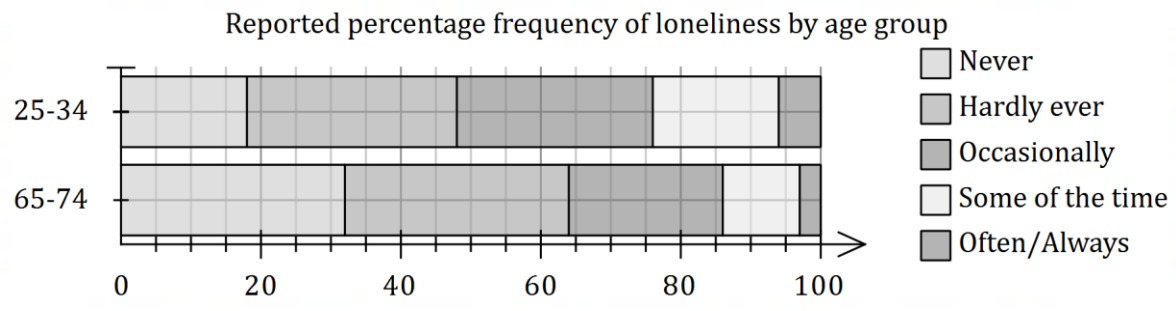
(c) Explain how to recognise a bridge in a connected graph and state, with justification, whether G contains a bridge. (2 marks)

Solution
When a bridge is removed from a connected graph, the graph becomes disconnected. Yes, edge BC is a bridge in G.
Specific behaviours
✓ correct explanation ✓ identifies edge that is bridge

See next page

Question 2 **(7 marks)**

A recent survey asked people their age and the question "How often do you feel lonely?". The responses for the age groups 25-34 years and 65-74 years are shown below. The categories in the key are shown from left to right in the stacked percentage frequency graph.



(a) State the largest category for the 25-34 age group. (1 mark)

Solution
'Hardly ever'.
Specific behaviours
✓ correct category

(b) State the percentage of those

(i) in the 65-74 age group who responded with 'Often/Always'. (1 mark)

Solution
3%
Specific behaviours
✓ correct percentage (accept 0.5 either way)

(ii) in the 25-34 age group who responded with 'Occasionally' or 'Some of the time'. (1 mark)

Solution
$100 - 48 - 6 = 46\%$
Specific behaviours
✓ correct percentage (accept 0.5 either way)

(c) Use the graph to explain why the data suggests an association exists between feeling lonely and age. (2 marks)

Solution
The difference in percentages across the age groups suggests an association. For example: Only 18% of 25-34 age group never feel lonely compared to 32% of the 65-74 age group.
Specific behaviours
✓ indicates different percentages suggest association ✓ example using correct percentages

(d) Briefly discuss non-causal explanations for the observed association. (2 marks)

Solution
The association may be due to coincidence (possibly only a small, biased survey, etc) or due to confounding by another variable (possible that one age group lived in city and the other in rural area, living arrangements, whether working or retired etc).
Specific behaviours
✓✓ gives 2 reasons

See next page

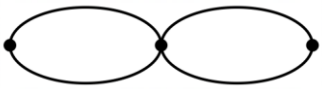
Question 3 **(6 marks)**

(a) Connected planar graph G_1 has 3 vertices and 4 edges.

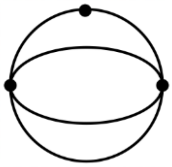
(i) Use Euler's formula to determine the number of faces in G_1. (2 marks)

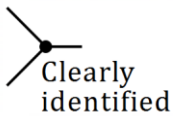
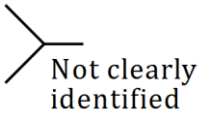
Solution
$3 + f - 4 = 2$ $f = 3$
Specific behaviours
✓ correct use of Euler's formula ✓ correct number of faces

(ii) Sketch a possible graph G_1. (2 marks)

Solution
 (Example only - others exist)
Specific behaviours
✓ correct number of edges and vertices (V=3, E=4) ✓ connected, with correct number of faces (F=3)

(b) Graph G_2 has 3 vertices and is Eulerian. The length of the Euler cycle is 5. Sketch a possible graph G_2. (2 marks)

Solution
 (Example only - others exist)
Specific behaviours
✓ connected with 5 edges and 3 vertices ✓ all vertices with even degree (Eulerian)

Markers note
Penalise 1 mark overall if vertices not clearly identified.
 Clearly identified  Not clearly identified

See next page

Question 4 **(6 marks)**

A recursive rule for a sequence is $T_{n+1} = T_n - 3.5,\ T_3 = 13.5$.

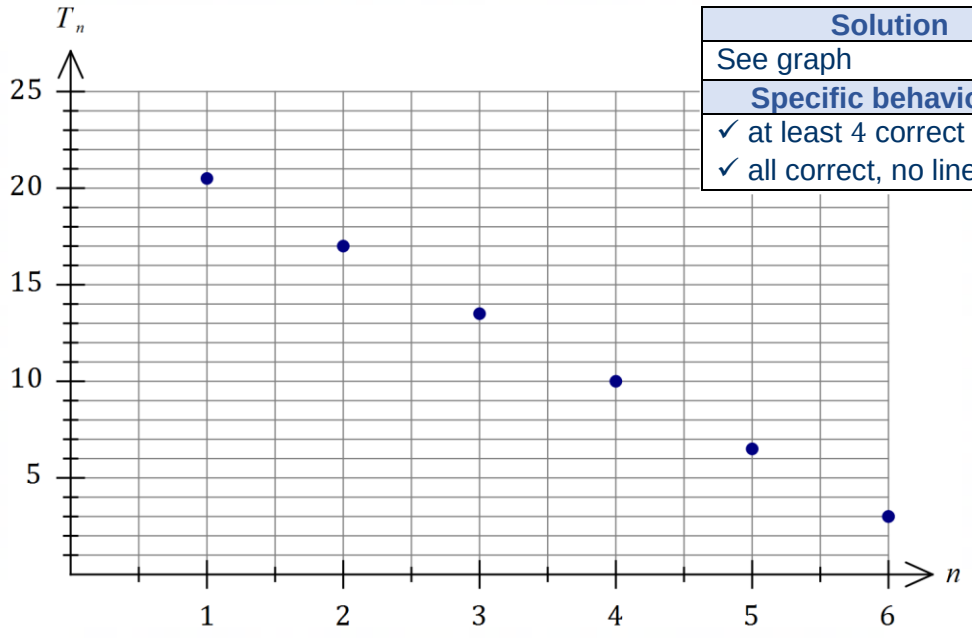
(a) Briefly explain which feature of the recursive rule indicates that the sequence is arithmetic. (1 mark)

Solution
There is a constant difference of -3.5 between consecutive terms.
Specific behaviours
✓ explains the constant difference

(b) Determine T_5 and T_1. (2 marks)

Solution
$T_5 = 13.5 - 3.5 - 3.5 = 6.5$ $T_1 = 13.5 + 3.5 + 3.5 = 20.5$
Specific behaviours
✓ value of T_5 ✓ value of T_1

(c) Graph the first six terms of the sequence on the axes below. (2 marks)



Solution
See graph
Specific behaviours
✓ at least 4 correct points ✓ all correct, no line

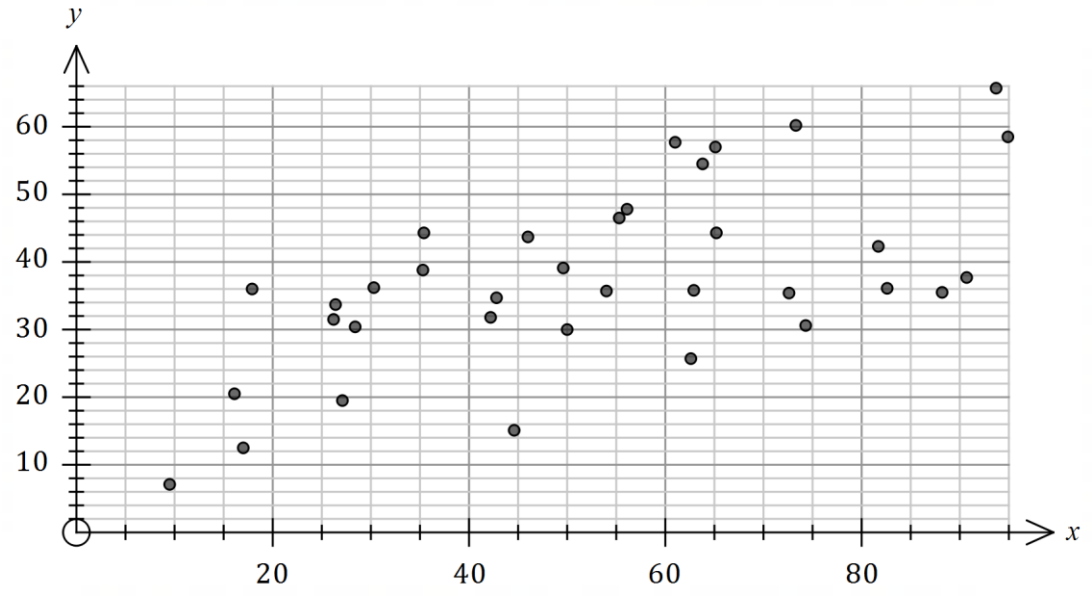
(d) What feature of the graph indicates that the sequence is arithmetic? (1 mark)

Solution
The points are linear / lie in a straight line.
Specific behaviours
✓ indicates linear nature

See next page

Question 5 **(7 marks)**

The scatterplot below shows the number of PC's per 100 people on the x-axis and the GDP/employee, in thousands of dollars, on the y-axis for a selection of countries in 2017.



(a) Describe the strength and direction of the association between the variables. (2 marks)

Solution
The association is of moderate strength and in a positive direction.
Specific behaviours
✓ strength ✓ direction

(b) The equation of the least-squares line for the data is $y = 19.1 + 0.35x$. Interpret the intercept and the slope of this line. (3 marks)

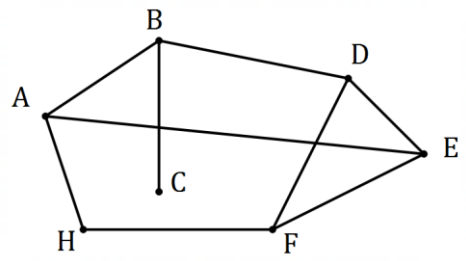
Solution
The intercept means that if a country had no PC's then the GDP/employee is expected to be 19.1 thousand dollars ($19100). The slope means that for every extra PC per 100 people, the GDP/employee is expected to rise by 0.35 thousand dollars ($350).
Specific behaviours
✓ interprets intercept ✓ interprets slope ✓ includes units (thousands of dollars) in interpretations

(c) A newspaper article used the graph to claim that increasing the number of PC's per person in a country caused the GDP/employee to rise. Comment on this claim. (2 marks)

Solution
The claim is unlikely to be true - an observed association does not necessarily mean there is a causal relationship between variables.
Specific behaviours
✓ disputes validity of claim (must say not valid or claim can't be made) ✓ comments on causation

See next page

Question 6 **(6 marks)**

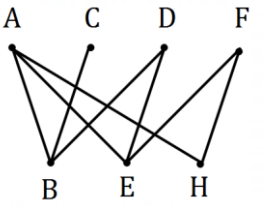


Graph G is shown. G_1 and G_2 are subgraphs of G, so that each subgraph has 7 vertices but one less edge than G.

(a) G_1 does not satisfy Euler's formula. State which edge must be removed from G, and show that G_1 does not satisfy Euler's formula. (3 marks)

Solution
Remove edge BC. Substituting the values $V = 7, F = 4, E = 8$ into the expression $V + F - E$ then $7 + 4 - 8 = 3$ but Euler's formula states this value must equal 2. Hence graph does not satisfy Euler's formula.
Specific behaviours
✓ correct edge to remove ✓ correct values of V, F, E ✓ substitutes and shows formula not satisfied

(b) G_2 is bipartite. State which edge must be removed from G, and draw G_2 to clearly show the partite sets. (3 marks)

Solution
Remove edge DF.    *(Bipartite graphs have no odd length cycles)*
Specific behaviours
✓ correct edge to remove ✓ correctly shows partite sets [A,C,D,F and B,E,H] ✓ correctly draws edges

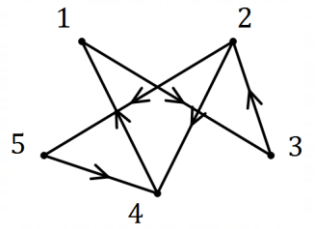
See next page

Question 7 **(7 marks)**

In a set of 5 pages on a website, there are hyperlinks from page 1 to page 3, from page 2 to pages 4 and 5, from page 3 to page 2, from page 4 to page 1, and from page 5 to page 4.

(a) Construct digraph D to show the above information, where pages are represented by vertices and hyperlinks by directed edges. (2 marks)

Solution



Specific behaviours

✓ vertices all have correct degree
✓ edges all directed correctly

(b) Construct an adjacency matrix for D. (2 marks)

Solution

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & \mathbf{1} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} & \mathbf{1} \\ 0 & \mathbf{1} & 0 & 0 & 0 \\ \mathbf{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} & 0 \end{bmatrix} \end{array}$$

Specific behaviours

✓ at least 4 correct rows
✓ all correct

(c) List, starting at page 3 and in the order visited, vertices in D that form a

(i) walk of length 2. (1 mark)

Solution

$\{3, 2, 4\}$ *and* $\{3, 2, 5\}$

Specific behaviours

✓ correct walks . Note: Must have both to get 1 mark)

(ii) cycle of length 4. (1 mark)

Solution

$\{3, 2, 4, 1, 3\}$

Specific behaviours

✓ correct cycle

(iii) trail of length 5. (1 mark)

Solution

$\{3, 2, 5, 4, 1, 3\}$

Specific behaviours

✓ correct trail

See next page

Question 8 **(7 marks)**

Every day, 10% of the water in a tank is drained for crop irrigation and then the tank is topped up with 40 kL of water. The tank has a maximum capacity of 350 kL.

At the start of Day 1, before water is drained for the crops, the tank contains 50 kL.

(a) Determine the amount of water in the tank at the start of Day 2. (2 marks)

Solution
$10\% \times 50 = 5$ $50 - 5 + 40 = 85 \text{ kL}$
Specific behaviours
✓ amount drained ✓ correct amount

(b) Determine a recursive rule for the amount of water, A_n, in the tank at the start of Day n. (2 marks)

Solution
$A_{n+1} = 0.9A_n + 40, \quad A_1 = 50$
Specific behaviours
✓ set up recursive rule with correct multiplier ✓ finish recursive rule with correct addition and first term

(c) Explain why the tank will never empty. (1 mark)

Solution
With this type of recursive rule, the amount in the tank will always increase, tending towards a **steady state** amount. OR Never empty as 90% will always remain in tank and 40 kL added.
Specific behaviours
✓ indicates amount always increasing as amount drained is always less than amount added.

(d) State, with justification, whether the tank will overflow. (2 marks)

Solution
Let x be long-term steady state. $x = 0.9x + 40$ $0.1x = 40$ $x = 400$ Hence tank **will overflow**, as the capacity of the tank (350 kL) is less than the long-term steady state (400 kL).
Specific behaviours
✓ identifies long-term steady state solution is equal to 400 kL ✓ states tank will overflow

End of questions

Supplementary page

Question number: _____