Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

MATHEMATICS

3C/3D

Calculator-free

WACE Examination 2013

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

Section One: Calculator Free **(50 marks)**

Question 1 **(5 marks)**

Solve the equation $\dfrac{x+2}{2x-1}=\dfrac{2x+1}{x+4}$.

Solution
$(x+2)(x+4)=(2x+1)(2x-1)$ $\qquad x \neq -4, 0.5$ $x^2+6x+8=4x^2-1$ $0=3x^2-6x-9$ $0=x^2-2x-3$ $0=(x-3)(x+1)$ $x=3$ or -1
Alternative method 1: $\dfrac{x+2}{2x-1}-\dfrac{2x+1}{x+4}=0$ $\dfrac{(x+2)(x+4)-(2x+1)(2x-1)}{(2x-1)(x+4)}=0$ $(x+2)(x+4)-(2x+1)(2x-1)=0$
Alternative method 2: $\dfrac{(x+2)(x+4)(2x-1)}{(2x-1)}=\dfrac{(2x+1)(x+4)(2x-1)}{(x+4)}$ $(x+2)(x+4)=(2x+1)(2x-1)$
Specific behaviours
✓ cross-multiplies correctly, or otherwise obtains equivalent expression ✓ expands correctly ✓ simplifies correctly ✓ factorises correctly ✓ states correct solutions

Question 2 **(7 marks)**

An airline owns three small aircraft: P, Q and R. One day, a total of 80 passengers travelled on the three aircraft. The total number of passengers who travelled on aircraft P and Q was four times the number who travelled on aircraft R.

Each passenger who travelled on aircraft P paid \$200. Those who travelled on aircraft Q paid \$300 each, and those who travelled on aircraft R paid \$100 each. The 80 passengers paid \$19 400 in total.

Let $p =$ number of passengers who flew on aircraft P,
$q =$ number of passengers who flew on aircraft Q, and
$r =$ number of passengers who flew on aircraft R.

(a) Write three equations relating p, q and r that will allow a solution for all three variables. (3 marks)

Solution	
$p+q+r=80$	
$200p+300q+100r=19400$ or $2p+3q+r=194$	
$r=\frac{1}{4}(p+q)$ or $p+q=4r$	
Specific behaviours	
✓	states first equation correctly
✓	states second equation correctly
✓	states third equation correctly

(b) How many passengers flew on each aircraft? (4 marks)

Solution	
$p+q+r=80 \quad \ldots eq1$	
$p+q-4r=0 \quad \ldots eq2$	
$2p+3q+r=194 \quad \ldots eq3$	
$5r=80 \quad \ldots eq1-eq2$	
$r=16 \quad subs\ \ eq2\ \&\ eq3$	
$p+q=64 \quad \ldots eq4$	
$2p+3q=178 \quad \ldots eq5$	
$q=50 \quad \ldots eq5-2eq4$	
$p=14, q=50, r=16$	
Hence 14 passengers flew on aircraft P, 50 passengers flew on aircraft Q and 16 passengers on aircraft R.	
Specific behaviours	
✓	sets up two equations with one variable eliminated
✓	sets up one equation with two variables eliminated
✓	correctly solves for one variable
✓	correctly solves for the other two variables

Question 3 **(4 marks)**

Let $f(x)=\dfrac{1}{x^2}+\dfrac{e^{2x}}{2}$.

Determine the second derivative $f''(x)$.

Solution
$f(x)=x^{-2}+\dfrac{e^{2x}}{2}$ $f'(x)=-2x^{-3}+e^{2x}=\dfrac{-2}{x^3}+e^{2x}$ $f''(x)=6x^{-4}+2e^{2x}=\dfrac{6}{x^4}+2e^{2x}$
Specific behaviours
✓ differentiates $\dfrac{1}{x^2}$ correctly   ✓ differentiates $\dfrac{e^{2x}}{2}$ correctly   ✓ differentiates $\dfrac{-2}{x^3}$ correctly   ✓ differentiates e^{2x} correctly

Question 4 **(10 marks)**

Let $f(x)=(x-1)(x^2-16)$.

(a) Show that $f'(x)=(3x-8)(x+2)$. (3 marks)

Solution
$f'(x)=1(x^2-16)+(x-1)(2x)$ $=x^2-16+2x^2-2x$ $=3x^2-2x-16$ $(3x-8)(x+2)=3x^2-8x+6x-16$ $=3x^2-2x-16$ Hence $f'(x)=(3x-8)(x+2)$
Specific behaviours
✓ differentiates $f(x)$ correctly   ✓ simplifies correctly   ✓ demonstrates equivalence of expressions

(b) Determine the equation of the tangent to the graph of $f(x)$ at the point where $x = 3$.

(3 marks)

Solution
$f'(3) = (3(3)-8)(3+2) = 5$ Tangent line has equation $y = 5x + c$ $f(3) = (3-1)(3^2-16) = -14$ $-14 = 3(5) + c$ $c = -29$ Tangent line has equation $y = 5x - 29$
Specific behaviours
✓ correctly evaluates $f'(3)$ ✓ correctly evaluates $f(3)$ ✓ correctly determines the equation of the tangent

(c) What is the maximum value of the function over the domain $-4 \le x \le 4$? (4 marks)

Solution
Stationary points where $(3x-8)(x+2) = 0$ $x = \frac{8}{3}$ or -2 The local maximum must occur at $x = -2$. Second derivative test: $f''(x) = 6x - 2$ $f''\left(\frac{8}{3}\right) = 14$ $f''(-2) = -14$ Sign test: $f'(-3) > 0$ $f'(0) < 0$ $f'(3) > 0$ Alternatively, this can be inferred from the shape of the cubic. The maximum over the domain must occur at $x = -2$ or $x = 4$. $f(-2) = 36$ $f(4) = 0$ The maximum value over the domain is 36.
Specific behaviours
✓ finds x-values of stationary points ✓ test stationary points to determine maximum ✓ correctly evaluates $f(-2) = 36$ and $f(4) = 0$ ✓ states correct maximum value

Question 5 **(6 marks)**

A cubic function $f(x) = ax^3 + bx^2 + cx + d$ has these features:

- $f'(x) \geq 0$ only for $-2 \leq x \leq 6$
- $f''(x) \geq 0$ only for $x \leq 2$
- There are exactly two points at which the graph of $f(x)$ meets the x-axis
- $d < 0$.

(a) (i) State the x-coordinate of the point of inflection. (1 mark)

(ii) Is the graph of $f(x)$ horizontal at the point of inflection? Explain your answer. (1 mark)

Solution	
(i)	$x = 2$
(ii)	No, as the stationary points of the cubic must be at $x = -2$ and $x = 6$, so $f'(2) \neq 0$.
Specific behaviours	
✓	states correct x-coordinate
✓	states that the point of inflection is not horizontal with correct explanation

(b) Is a positive or negative? Explain your answer. (2 marks)

Solution	
As $x \to \pm\infty$, $f'(x)$ is negative. This is characteristic of a cubic with $a < 0$	
Specific behaviours	
✓	states that a is negative $(a < 0)$
✓	explains correctly from given information

(c) Determine the coordinates of the local maximum. (2 marks)

Solution	
Local maximum is at (6,0)	
Specific behaviours	
✓	states correct x-coordinate
✓	states correct y-coordinate

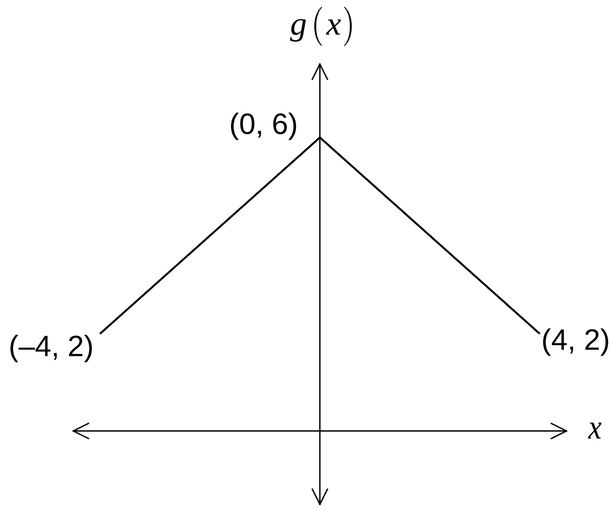
Question 6 **(7 marks)**

A function is defined as $f(x) = x(10-x)$, over the domain $0 \le x \le 10$.

(a) Determine the range of $f(x)$. (2 marks)

Solution	
$0 \le f(x) \le 25$	
Specific behaviours	
✓	correctly determines that $f(x) \ge 0$
✓	correctly determines that $f(x) \le 25$

The graph of a second function $g(x)$ is shown below for the domain $-4 \le x \le 4$.
The coordinates of the endpoints and vertex of the graph are labelled.



(b) Determine:

(i) $f(g(2))$. (2 marks)

Solution	
$f(g(2)) = f(4) = 4(10-4) = 24$	
Specific behaviours	
✓	correctly evaluates $g(2)$ from graph
✓	correctly evaluates $f(g(2))$

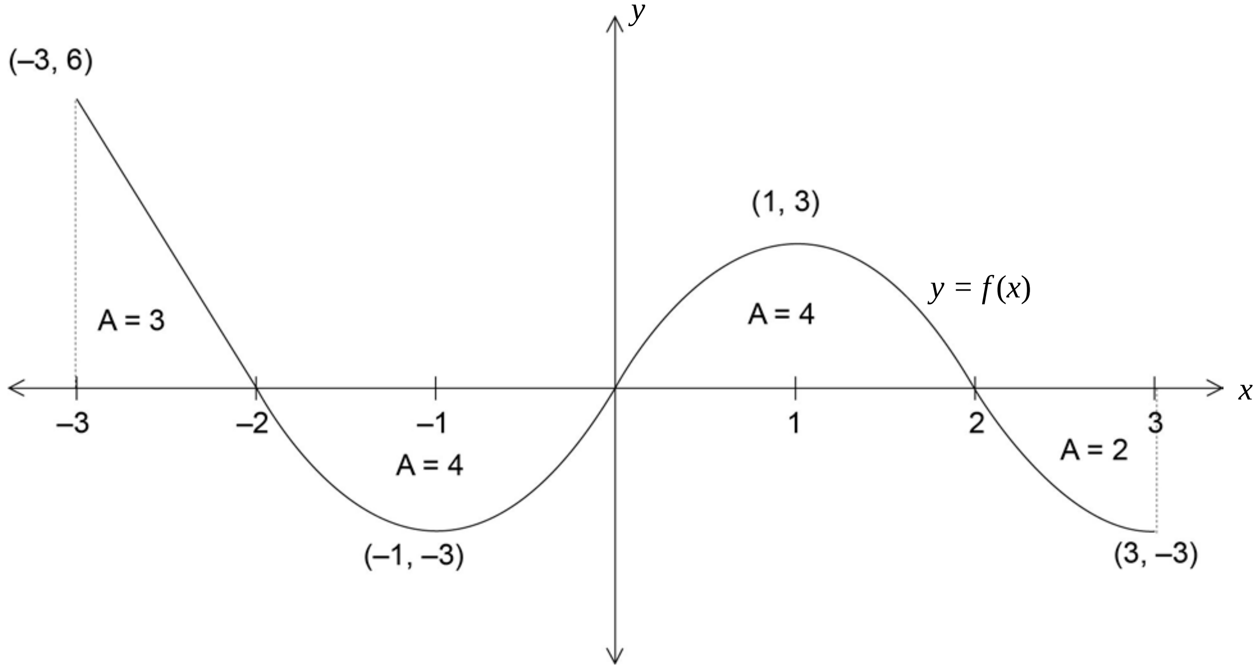
(ii) the domain and range of $f\left(g\left(x\right)\right)$. (3 marks)

Solution
Domain: $-4 \le x \le 4$ $f(2) = 16$ Range: $16 \le f\left(g\left(x\right)\right) \le 25$
Specific behaviours
✓ correctly states domain ✓ correctly determines that $f\left(g\left(x\right)\right) \le 25$ ✓ correctly determines that $f\left(g\left(x\right)\right) \ge 16$

Question 7 **(11 marks)**

The graph of the function $f(x)$ is shown below for $-3 \le x \le 3$.

The areas enclosed between the graph, the x-axis and the lines $x=-3$ and $x=3$ are marked in the appropriate regions.



Determine:

(a) the value of $\int_{-2}^{3} f(x)\,dx$. (2 marks)

Solution
$\int_{-2}^{3} f(x)\,dx = -4+4-2=-2$
Specific behaviours
✓ uses additive property for integrals ✓ correctly uses signed areas

(b) the area enclosed between the graph of $f(x)$ and the x-axis, from $x=-2$ to $x=3$. (2 marks)

Solution
Area = 4 + 4 + 2 = 10
Specific behaviours
✓ uses additive property of areas ✓ correctly selects unsigned areas

(c) the value of $\int_0^3 f(-x)\,dx$. (2 marks)

Solution
$\int_0^3 f(-x)\,dx = \int_{-3}^0 f(x)\,dx = 3-4 = -1$
Specific behaviours
✓ correctly determines the effect of the transformation on the integral ✓ correctly evaluates the integral

(d) the value of $\int_0^2 \left(x - f(x)\right)dx$. (3 marks)

Solution
$\int_0^2 \left(x - f(x)\right)dx = \int_0^2 x\,dx - \int_0^2 f(x)\,dx$ $= \left[\frac{x^2}{2}\right]_0^2 - 4$ $= 2-4$ $= -2$
Specific behaviours
✓ separates integral into correct components ✓ integrates first component correctly ✓ integrates second component correctly

(e) the value of $\int_{-1}^1 f'(x)\,dx$. (2 marks)

Solution
$\int_{-1}^1 f'(x)\,dx = \left[f(x)\right]_{-1}^1 = 3-(-3) = 6$
Specific behaviours
✓ applies Fundamental Theorem of Calculus ✓ evaluates integral correctly

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