

**The Mathematical Association of Victoria
SOLUTIONS: Trial Exam 2014
SPECIALIST MATHEMATICS
Written Examination 2**

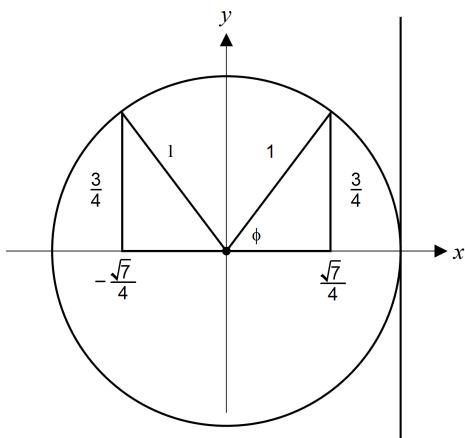
SECTION 1: Multiple Choice

ANSWERS

1. D 2. D 3. A 4. E 5. E 6. D
 7. D 8. B 9. B 10. E 11. B 12. A
 13. C 14. A 15. C 16. B 17. A 18. D
 19. C 20. D 21. E 22. B

Question 1

Answer: D



$$\cot(x) = \frac{\pm\sqrt{7}}{4} = \frac{\sqrt{7}}{3} \text{ or } -\frac{\sqrt{7}}{3}$$

1.2 2.1 3.1 *SM_2_2014

```
solve(csc(x)=4/3,x)|0≤x≤2·π
x=sin⁻¹(3/4) or x=π-sin⁻¹(3/4)
```

$\cot(\sin^{-1}\left(\frac{3}{4}\right))$	$\frac{\sqrt{7}}{3}$
$\cot(\pi-\sin^{-1}\left(\frac{3}{4}\right))$	$-\frac{\sqrt{7}}{3}$

✓ Edit Action Interactive

```
solve(1/sin(x)=4/3,x)|0≤x<π
{x=sin⁻¹(3/4),x=-sin⁻¹(3/4)+π}

1/tan(sin⁻¹(3/4))
sqrt(7)/3

1/tan(-sin⁻¹(3/4)+π)
-sqrt(7)/3
```

Question 2

Answer: D

For f , domain $R \setminus \{-2, 2\}$ and range $R \setminus \{1\}$

The graph of g is a translation of 1 unit to the right, therefore:

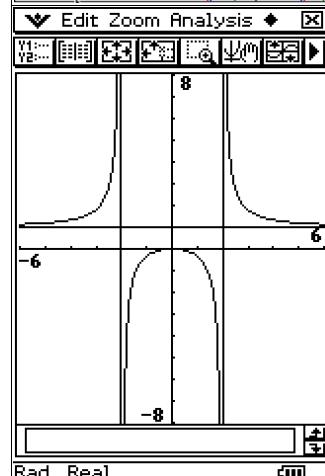
For f , domain $R \setminus \{-1, 3\}$ and range $R \setminus \{1\}$

6.2 6.3 7.1 *SM_2_2014

$y=1$ $x=-2$ $x=2$

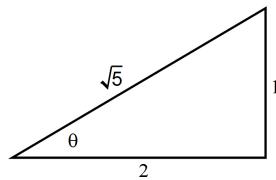
$f_1(x) = \frac{x^2}{x^2-4}$

$f_2(x) = f_1(x-1)$



Question 3**Answer: A**

$$\text{Let } \theta = \tan^{-1}\left(\frac{1}{2}\right)$$



$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\sin(2\theta) = 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{4}{5}$$

Question 4**Answer: E**

$$\begin{aligned} \frac{u}{6-2u} &= \frac{3+4i}{6-(6-8i)} \\ &= \frac{3+4i}{8i} \times \frac{8i}{8i} \\ &= \frac{-32+24i}{-64} = \frac{1}{2} - \frac{3}{8}i \end{aligned}$$

Question 5**Answer: E**

$$\text{Let } z^4 = 1 + i\sqrt{3} = 2\text{cis}\left(\frac{\pi}{3}\right)$$

$$z = \sqrt[4]{2}\text{cis}\left(\frac{\pi}{12} + \frac{n\pi}{2}\right), n = -2, -1, 0, 1$$

$$z = \sqrt[4]{2}\text{cis}(\theta), \theta = -\frac{11\pi}{12}, -\frac{5\pi}{12}, \frac{\pi}{12}, \frac{7\pi}{12}$$

Alternatively, solve $z^4 = 1 + i\sqrt{3}$ on CAS – being aware that $e^{i\theta} = \text{cis}(\theta)$.

Question 6 **Answer: D**

Cartesian equations:

$$|z-1| = |z+i| \Rightarrow y = -x$$

$$|z+i| = |z-4+i| \Rightarrow x = 2$$

Intersection is at $(2, -2)$ i.e. $2 - 2i$
Question 7 **Answer: D**If p has integer coefficients, then the factors are at least

$$(z+5)(z-\sqrt{3})(z+\sqrt{3})(z-1+2i)(z-1-2i)$$

Therefore, minimum degree is 5.

Question 8 **Answer: B**

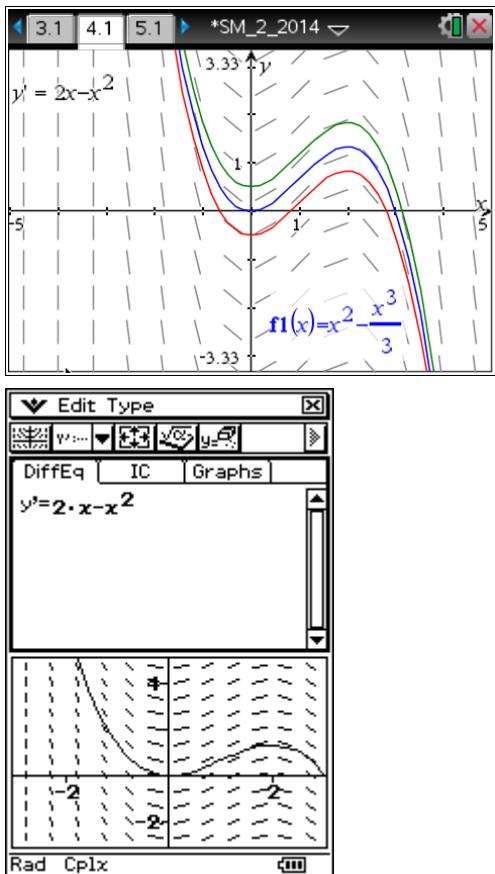
The family of functions could be of the form

$$y = ax^3 + bx^2 + cx + d, a < 0.$$

The DE giving rise could therefore be

$$\frac{dy}{dx} = -x^2 + 2x \text{ (option B), because}$$

$$y = \int (-x^2 + 2x) dx = -\frac{1}{3}x^3 + x^2 + C$$

**Question 9****Answer: B**

$$\text{Let } u = \log_e(\sec(x)) = -\log_e(\cos(x))$$

$$\frac{du}{dx} = -\frac{1}{\cos(x)} \times -\sin(x) = \tan(x)$$

$$x = -\frac{\pi}{6} \Rightarrow u = -\log_e\left(\cos\left(-\frac{\pi}{6}\right)\right) = -\log_e\left(\frac{\sqrt{3}}{2}\right)$$

$$x = \frac{\pi}{3} \Rightarrow u = -\log_e\left(\frac{1}{2}\right) = \log_e(2)$$

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (\tan(x) \log_e(\sec(x))) dx = \int_{-\log_e\left(\frac{\sqrt{3}}{2}\right)}^{\log_e(2)} u du$$

Question 10**Answer: E**

Euler's method: $y_{n+1} = y_n + h f(x_n)$, where

$$f(x) = \log_e\left(\frac{1}{\sqrt{x}}\right) = -\frac{1}{2} \log_e(x),$$

$$h = \frac{1}{10}, x_0 = 1 \text{ and } y_0 = 2.$$

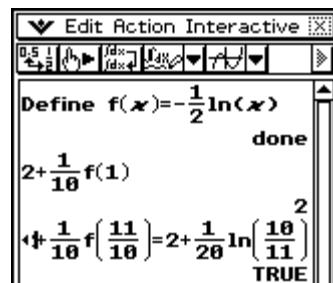
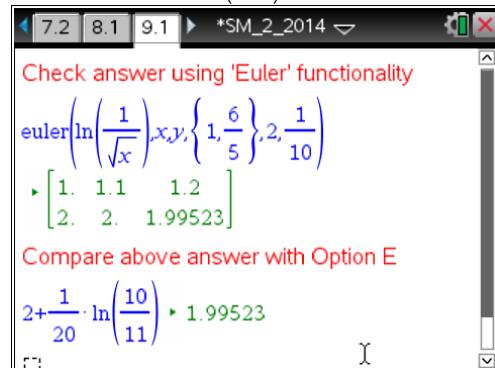
$$y_1 = y_0 + h f(x_0)$$

$$y_1 = 2 + \frac{1}{10} \times -\frac{1}{2} \times \log_e(1) = 2, \text{ when } x_1 = 1 + \frac{1}{10} = \frac{11}{10}$$

$$y_2 = y_1 + h f(x_1)$$

$$y_2 = 2 + \frac{1}{10} \times -\frac{1}{2} \log_e\left(\frac{11}{10}\right)$$

$$y_2 = 2 + \frac{1}{20} \log_e\left(\frac{10}{11}\right)$$

**Question 11****Answer: B**

$$\begin{aligned} V &= \pi \int_0^2 (y^2 - 1) dx \\ &= \pi \int_0^2 ((x^2 + 1)^2 - 1) dx \\ &= \pi \int_0^2 (x^4 + 2x^2) dx \end{aligned}$$

Question 12**Answer: A**

From the graph of f we can deduce

$$g'(x) = 0 \text{ for } x = q$$

$g''(x) = 0$ (points of inflection) somewhere between p and q , and again between q and r .

$$g'(x) < 0 \text{ for } p < x < q$$

$$g'(x) > 0 \text{ for } q < x \leq r$$

Therefore Option A could be the graph of g , as it is the only graph shown that meets these conditions.

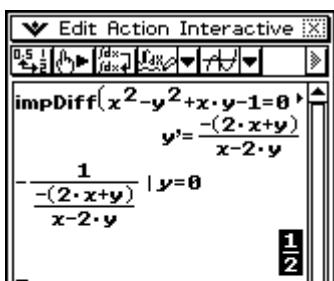
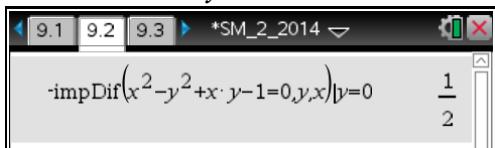
Also note that $g(p) = \int_p^p f(t) dt = 0$ is also satisfied by Option A.

Question 13**Answer: C**

$$\frac{dy}{dx} = \frac{2x+y}{2y-x}$$

$$\text{Gradient of normal} = \frac{x-2y}{2x+y}$$

$$\text{At } y=0, \frac{x-2y}{2x+y} = \frac{1}{2}$$

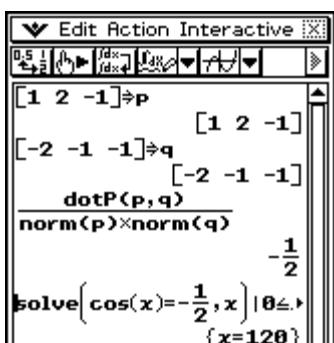
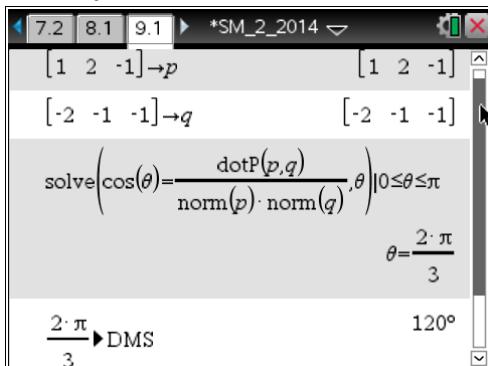
**Question 14****Answer: A**

$$\mathbf{p} \cdot \mathbf{q} = \|\mathbf{p}\| \|\mathbf{q}\| \cos(\theta)$$

$$-3 = \sqrt{6} \times \sqrt{6} \cos(\theta)$$

$$\cos(\theta) = -\frac{1}{2}$$

$$\theta = 120^\circ$$

**Question 15**

Consider Option C

$$\text{LHS} = \left(\vec{AC} - \vec{AB} \right) \cdot \vec{AB}$$

$$= \vec{BC} \cdot \vec{AB}$$

$$= \left| \vec{BC} \right| \left| \vec{AB} \right| \cos(\theta)$$

$$\text{RHS} = \left| \vec{BC} \right|^2 \neq \text{LHS}$$

Question 16**Answer: B**

$$x = \cos(2t) = 1 - 2\sin^2(t) \quad \dots \text{equation 1}$$

$$y = -2\sin(t) \Leftrightarrow \sin(t) = -\frac{y}{2} \quad \dots \text{equation 2}$$

Substituting equation 2 in equation 1

$$x = 1 - 2 \times \frac{y^2}{4}$$

$$y^2 + 2x - 2 = 0$$

Question 17**Answer: A**

$$\frac{dy}{dx} = -12 \int x dx = -6x^2 + c$$

$$\frac{dy}{dx} = -1 \text{ at } x = -1, \text{ therefore } c = 5$$

$$y = \int (-6x^2 + 5) dx = -2x^3 + 5x + c_1$$

$$y = 4 \text{ at } x = -1, \text{ therefore } c_1 = 7$$

$$y = -2x^3 + 5x + 7$$

Question 18**Answer: D**When the ball hits the ground, $y = 0$, therefore

$$20t - 5t^2 = 5t(4-t) = 0$$

 $t = 0$ or 4 . The ball hits the ground at $t = 4$.The horizontal distances travelled is
 $35 \times 4 = 140$ **Question 19****Answer: C**

The displacement from the origin can be found by calculating the signed area between the graph and the horizontal axis.

$$(4 \times 3) + (2 \times 1) + \frac{1}{2}((1 \times 3) + (2 \times 2) - (6 \times 2)) = \frac{23}{2}$$

The coordinates are $\left(\frac{23}{2}, 0 \right)$

Question 20**Answer: D**

$$\frac{d}{dx} \left(\frac{1}{2} v^2 \right) = \frac{9}{x^2}$$

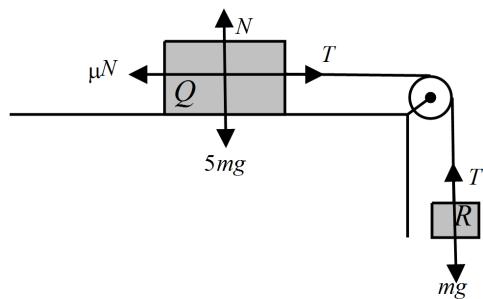
$$\int d \left(\frac{1}{2} v^2 \right) = \int \left(\frac{9}{x^2} \right) dx$$

$$\frac{1}{2} v^2 = -\frac{9}{x} + c$$

When $x = 9$, $v = -2$, therefore $c = 3$

$$\frac{1}{2} v^2 = \frac{3x - 9}{x}$$

$$v = -\sqrt{\frac{6x - 18}{x}} \quad (\text{N.B. } v = -2 \text{ when } x = 9)$$

Question 21**Answer: E**

$$R = ma \text{ for } Q: T - \frac{1}{10} \times 5mg = 5ma \quad (\text{eqn. 1})$$

$$R = ma \text{ for } R: mg - T = ma \quad (\text{eqn. 2})$$

Add equations 1 and 2

$$\frac{mg}{2} = 6ma$$

$$a = \frac{g}{12}$$

Question 22**Answer: B**

$$-mg - m_kv^2 = ma$$

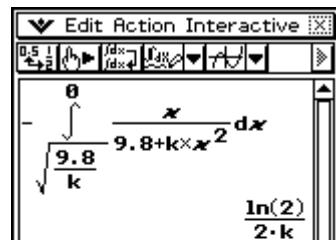
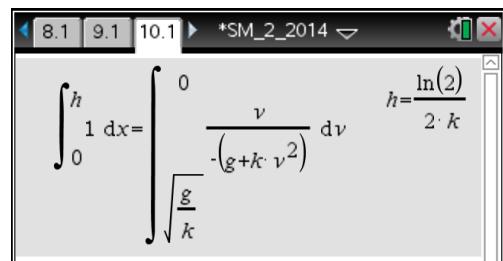
$$-(g + kv^2) = v \frac{dv}{dx}$$

When $x = 0$, $v = \sqrt{\frac{g}{k}}$ and when $x = h$, $v = 0$,where h is the maximum height.

$$-(g + kv^2) = v \frac{dv}{dx}$$

$$\int_0^h dx = - \int_{\sqrt{\frac{g}{k}}}^0 \frac{v}{g + kv^2} dv$$

$$h = \frac{\log_e(2)}{2k}$$

**END OF SECTION 1 SOLUTIONS**

SECTION 2: Extended Response SOLUTIONS**Question 1****1a.i.**

$$w = 4 - 4i$$

$$|w| = \sqrt{4^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}, \text{ as required. } 1M$$

1a.ii.

$$\arg(w) = \tan^{-1}\left(\frac{-4}{4}\right) = \tan^{-1}(-1) = -\frac{\pi}{4} \quad 1M$$

1b.

$$z^5 = 4\sqrt{2} \operatorname{cis}\left(-\frac{\pi}{4}\right) \quad 1M \text{ (de Moivre's theorem)}$$

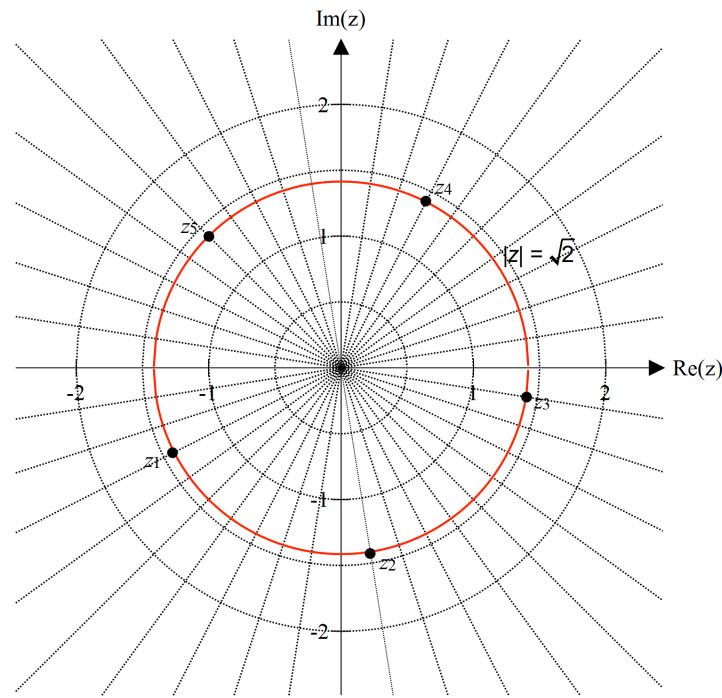
$$z = (4\sqrt{2})^{\frac{1}{5}} \operatorname{cis}\left(-\frac{\pi}{20} + \frac{2k\pi}{5}\right), k = -2, -1, 0, 1, 2 \quad 1M \text{ (5 solutions with } a = \sqrt{2} \text{)}$$

$$z = \sqrt{2} \operatorname{cis}\left(-\frac{17\pi}{20}\right) \text{ or } z = \sqrt{2} \operatorname{cis}\left(-\frac{9\pi}{20}\right) \text{ or } z = \sqrt{2} \operatorname{cis}\left(-\frac{\pi}{20}\right) \text{ or } z = \sqrt{2} \operatorname{cis}\left(\frac{7\pi}{20}\right) \text{ or } z = \sqrt{2} \operatorname{cis}\left(\frac{3\pi}{4}\right)$$

1A (five correct solutions with $\theta \in (-\pi, \pi]$)

The left window shows the input $(4\sqrt{2})^5$ and the output: $\frac{-\pi + 2\pi k}{20} + \frac{2\pi \cdot k}{5} |_{k=\{-2, -1, 0, 1, 2\}}$, which expands to $\left\{ \frac{-17\pi}{20}, \frac{-9\pi}{20}, \frac{-\pi}{20}, \frac{7\pi}{20}, \frac{3\pi}{4} \right\}$.

The right window shows the input $(\sqrt{2})^5$ and the output: $\text{seq}\left(-\frac{\pi}{20} + \frac{2\pi k}{5}, k, -2, 2, 1\right)$, which expands to $\left\{ \frac{-17\pi}{20}, \frac{-9\pi}{20}, \frac{-\pi}{20}, \frac{7\pi}{20}, \frac{3\pi}{4} \right\}$.

**1c.i.**1A (Circle centred at the origin with radius $\sqrt{2}$ - i.e. slightly less than the 1.5 on the scale provided),**1c.ii.**1A (5 points with correct angles on the circle of radius $\sqrt{2}$)**1d.****Method 1** (by hand)

$$u = \sqrt{3} - i$$

$$|u| = 2 \text{ and } \arg(u) = -\frac{\pi}{6}$$

Using de Moivre's theorem,

$$u^k = 2^k \left(\cos\left(-\frac{k\pi}{6}\right) + i \sin\left(-\frac{k\pi}{6}\right) \right) \quad 1M$$

When $u^k \in R^+$,

$$\operatorname{Im}(u^k) = \sin\left(-\frac{k\pi}{6}\right) = 0$$

Therefore, $k = 6n$, where n is a positive integer. 1MBut $\cos\left(-\frac{(6n)\pi}{6}\right)$ is positive only when n is an even integer.Therefore the least value of k is $6 \times 2 = 12$. 1A**Method 2** (CAS-assisted)Solve for k , $\operatorname{Im}(u^k) = 0$ 1M $k = 6n$, where n is a positive integerWhen $n = 1$, $u^k = -64 \notin R^+$ 1M (show that k is a positive multiple of 6)When $n = 2$, $u^k = 4096 \in R^+$ Therefore the least value of k is $6 \times 2 = 12$. 1A

Method 3 ('Guess and check')

'Guess and check' without other relevant working attracts only 1 mark out of 3 marks.

The least value of k is 12. 1A**1e.**

$$\text{Let } P(z) = z^9 + 16(1+i)z^3 + c + id$$

If $u = \sqrt{3} - i$ is a root of $P(z)$, then

$$P(u) = u^9 + 16(1+i)u^3 + c + id = 0 \quad 1M$$

$$\text{Therefore, } (c+128) + i(d+384) = 0 + i0 \quad 1M$$

Equating real and imaginary parts

$$c = -128 \text{ and } d = -384 \quad 1A$$

Question 2**2a.**

$$|\tilde{m}| = \sqrt{49} = 7, \text{ hence } \hat{m} = \frac{1}{7}(-6i + 2j + 3k)$$

$$\text{Therefore, } \cos(\phi) = \frac{2}{7} \quad 1A$$

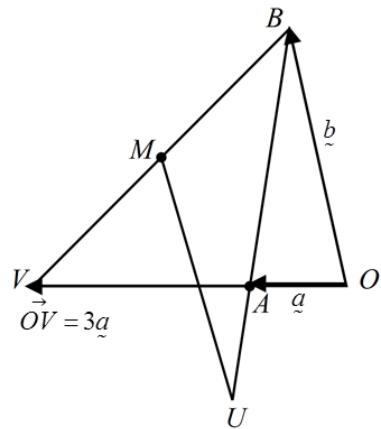
2b.

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = 1 \quad 1M$$

$$\underline{m} \cdot \underline{\hat{r}} = -6x + 2y + 3z = 0 \text{ and } \underline{n} \cdot \underline{\hat{r}} = 2x - 3y + 6z = 0 \quad 1M$$

Solve the three equations simultaneously for x, y and z

$$x = \frac{3}{7}, y = \frac{6}{7}, z = \frac{2}{7} \text{ or } x = -\frac{3}{7}, y = -\frac{6}{7}, z = -\frac{2}{7} \quad 1A$$

2c.**2c.i.**

$$\vec{BA} = \underline{a} - \underline{b} \quad 1A$$

2c.ii.

$$\vec{BM} = \frac{1}{2}(3\underline{a} - \underline{b}) \quad 1A$$

2d.

$$\begin{aligned} \vec{MU} &= \vec{MB} + \vec{BU} \\ &= -\vec{BM} + p \vec{BA} \\ &= -\frac{1}{2}(3\underline{a} - \underline{b}) + p(\underline{a} - \underline{b}) \\ &= \left(p - \frac{3}{2}\right)\underline{a} + \left(\frac{1}{2} - p\right)\underline{b} \end{aligned} \quad 1A$$

2e.Let N be the midpoint of the line segment OV , which is where MU intersects OV .Need to prove that \vec{NM} can be expressed in terms of \underline{b} .

$$\begin{aligned}\vec{NV} &= \frac{1}{2} \vec{OV} = \frac{3}{2} \vec{a} \\ \vec{NM} &= \vec{NV} + \vec{VM} \\ &= \vec{NV} - \vec{BM} \\ &= \frac{3}{2} \vec{a} - \left(\frac{3}{2} \vec{a} - \frac{1}{2} \vec{b} \right) \\ \vec{NM} &= \frac{1}{2} \vec{b} \text{ and } \vec{OB} = \vec{b}\end{aligned}$$

1M

Therefore line segment OB is parallel to NM , and consequently also parallel to MU .

2f.

$\vec{NM} = \frac{1}{2} \vec{b}$, therefore \vec{MU} can also be expressed in terms of \vec{b} only (i.e. \vec{MU} parallel to $\vec{OB} = \vec{b}$).

$$\text{However, from part d., } \vec{MU} = \left(p - \frac{3}{2} \right) \vec{a} + \left(\frac{1}{2} - p \right) \vec{b}.$$

$$\text{Therefore } \left(p - \frac{3}{2} \right) \vec{a} = \vec{0}, \text{ and } p - \frac{3}{2} = 0$$

1M

$$p = \frac{3}{2}$$

1A

Question 3

3a.

$$x = \cos(t) \text{ and } y = \sin(t)$$

Using the identity $\cos^2(t) + \sin^2(t) = 1$,

$$x^2 + y^2 = 1$$

This is the equation of a circle of unit radius, as required

1M

3b.

K is the area of the sector such that the arc length LM is t and the angle subtended at the centre is t^c .
1M

$$K = \frac{t}{2\pi} \times (\pi \times 1^2)$$

$$K = \frac{t}{2}$$

1A

3c.

$$x = \frac{1}{2}(e^t + e^{-t}) \text{ and } y = \frac{1}{2}(e^t - e^{-t})$$

$$\text{LHS} = x^2 - y^2$$

$$= \left(\frac{1}{2}(e^t + e^{-t}) \right)^2 - \left(\frac{1}{2}(e^t - e^{-t}) \right)^2$$

1M

$$= \frac{1}{4} \left((e^{2t} + 2 + e^{-2t}) - (e^{2t} - 2 + e^{-2t}) \right)$$

$$= \frac{1}{4}(4) = 1 = \text{RHS, as required}$$
1M

3d.Area of triangle ONQ

$$A + B = \frac{1}{2} \left(\frac{1}{2} (e^t + e^{-t}) \times \frac{1}{2} (e^t - e^{-t}) \right)$$

$$A + B = \frac{e^{2t} - e^{-2t}}{8}$$
1A

Given that $A = \frac{e^{-2t} (e^{4t} - 4te^{2t} - 1)}{8}$,

$$B = \frac{e^{2t} - e^{-2t}}{8} - \frac{e^{-2t} (e^{4t} - 4te^{2t} - 1)}{8}$$
1M

$$B = \frac{4t}{8} = \frac{t}{2}$$
1A

Note the analogous results of area K for the circle with equation $x^2 + y^2 = 1$ and area B for the hyperbola with equation $x^2 - y^2 = 1$.

3e.

Solve for t , $\frac{d}{dt} \left(\frac{1}{2} (e^t + e^{-t}) \right) = \frac{3}{4}$

$$t = \log_e(2)$$
1M

Substitute $t = \log_e(2)$ in x and y

$$x = \frac{5}{4} \text{ and } y = \frac{3}{4}$$

The cartesian coordinates are $\left(\frac{5}{4}, \frac{3}{4} \right)$

1A

The left screenshot shows the input and output of a CAS system for solving the derivative equation. It shows the derivative of $\frac{1}{2}(e^t + e^{-t})$ with respect to t set equal to $\frac{3}{4}$, resulting in $t = \ln(2)$. Below it, the CAS calculates the value of $\frac{1}{2}(e^t + e^{-t})$ at $t = \ln(2)$ and $\frac{1}{2}(e^t - e^{-t})$ at $t = \ln(2)$, both resulting in $\frac{5}{4}$ and $\frac{3}{4}$ respectively.

The right screenshot shows the input and output of a CAS system for solving the derivative equation with the condition $x = \ln(2)$. It shows the derivative of $\frac{e^x + e^{-x}}{2}$ with respect to x set equal to $\frac{3}{4}$, resulting in $\frac{e^x - e^{-x}}{2} |_{x=\ln(2)}$. Below it, the CAS calculates the value of $\frac{e^x - e^{-x}}{2}$ at $x = \ln(2)$, resulting in $\frac{3}{4}$.

3f.

At $t = a$ the gradient of the normal is $-\frac{3}{5}$, therefore

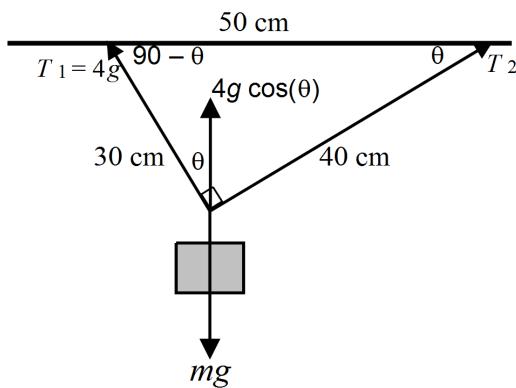
$$\frac{dy}{dx} = \frac{5}{3}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$
1M

Solve for t , $\frac{dy}{dt} \times \frac{dt}{dx} = \frac{5}{3}$

$$a = \log_e(2)$$

1A

Question 4**4a.**Resolving forces parallel to the $4g$ tension force.

$$4g = mg \cos(\theta) \quad 1M$$

$$4 = m \times \frac{40}{50}$$

$$m = 5 \quad 1A$$

4b.Resolving forces perpendicular to the $4g$ tension force.

$$T = 5g \sin(\theta)$$

$$T = 5g \times \frac{30}{50} = 3g \quad 1A$$

Alternatively,

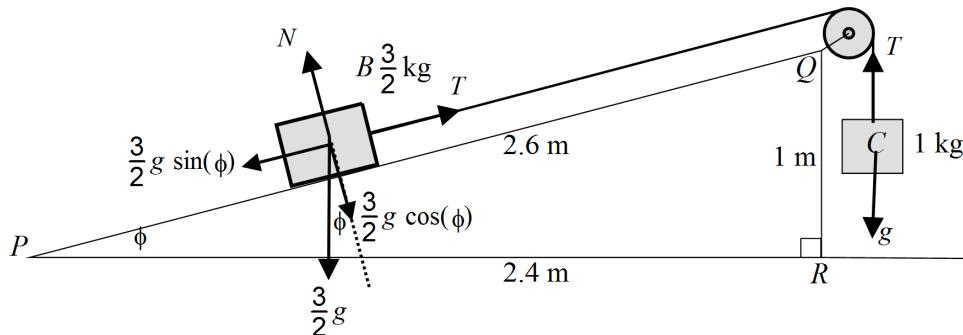
$$mg = \frac{T_1}{\cos(\theta)} = \frac{T_2}{\sin(\theta)}$$

$$T_2 = T_1 \tan(\theta)$$

$$T_2 = 4g \times \frac{30}{40} = 3g$$

4c.

The sides of triangle PQR are in the ratio of the pythagorean triple (5, 12, 13).



$$\underline{R = ma} \text{ for block } B: T - \frac{3}{2}g \sin(\phi) = \frac{3}{2}a \quad (\text{equation 1})$$

$$\underline{R = ma} \text{ for } C: g - T = a \quad (\text{equation 2}) \quad 1\text{M}$$

Adding equations 1 and 2

$$g - \frac{3}{2}g \times \frac{5}{13} = \frac{5}{2}a$$

$$\frac{5}{2}a = g - \frac{15}{26}g$$

$$\frac{5}{2}a = \frac{11}{26}g \quad 1\text{M}$$

$$a = \frac{11}{65}g \text{ ms}^{-2}$$

4d.i.

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2 \times \frac{11 \times 9.8}{65} \times 1 \quad 1\text{M}$$

$$v = 1.82 \text{ ms}^{-1} \quad 1\text{A}$$

4d.ii.

The acceleration of B just after the string becomes slack:

$$\underline{R = ma}$$

$$\frac{3}{2}a = -\frac{3}{2}g \times \sin(\phi)$$

$$a = -9.8 \times \frac{5}{13}$$

$$a = -3.77 \text{ ms}^{-2}$$

The magnitude of the acceleration is 3.77 ms^{-2} 1A

4d.iii.

Find the distance travelled up the plane by B after the string becomes slack

$$v^2 = u^2 + 2as$$

$$0^2 = (1.824\dots)^2 + 2 \times (-3.769\dots)s \quad 1\text{M}$$

$$s = 0.44 \text{ m}$$

Total distance that B travels up the plane = 1.44 metres 1A

Question 5**5a.**

$$v = u + at$$

$$v_0 = 0 + g \times 10$$

$$v_0 = 10g = 98 \text{ ms}^{-1}$$

1A

5b.

$$v(6) = (98 - 4)e^{-\frac{9.8 \times 6}{4}} + 4$$

$$v(6) = 4.0 \text{ ms}^{-1} \quad 1A$$

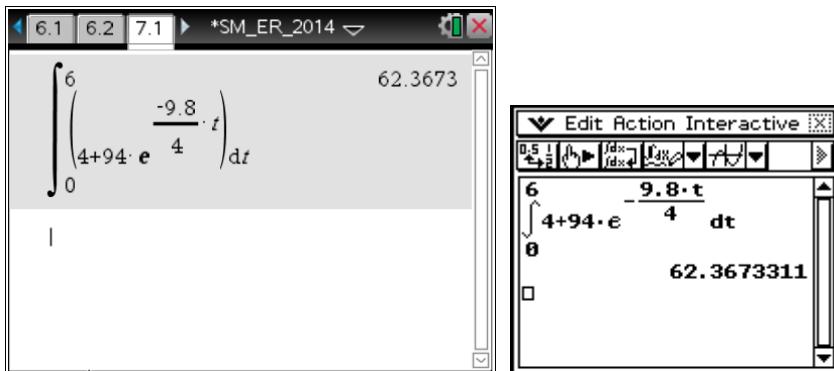
This shows that Lin hit the ground at terminal velocity, i.e. as $t \rightarrow \infty$, $(v_0 - 4)e^{-\frac{gt}{4}} \rightarrow 0$ and $v(t) \rightarrow 4$.

5c.

$$\frac{dx}{dt} = (v_0 - 4)e^{-\frac{gt}{4}} + 4$$

$$x = \int_0^6 \left(94e^{-\frac{9.8t}{4}} + 4 \right) dt \quad 1M$$

$$x = 62.4 \text{ m} \quad 1A$$

**5d.**

Distance travelled during free fall:

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + 4.9 \times 10^2 = 490 \text{ m} \quad 1A$$

Total distance travelled from balloon to the ground:

$$490 + 62.4 = 552.4 \text{ m} \quad 1A$$

5e.

$$m \frac{dv}{dt} = mg - kv$$

$$\int dt = -m \int \frac{dv}{kv - mg} \quad 1M$$

$$t = -\frac{m}{k} \log_e(|kv - mg|) + c$$

When $t = 0, v = u$, therefore

$$c = \frac{m}{k} \log_e (|ku - mg|) \quad 1M$$

Therefore,

$$t = -\frac{m}{k} \log_e \left(\left| \frac{kv - mg}{ku - mg} \right| \right)$$

$$kv = (ku - mg) \times e^{-\frac{k}{m}t} + mg$$

$$v = \frac{mg}{k} + \left(u - \frac{mg}{k} \right) \times e^{-\frac{k}{m}t}, \text{ as required} \quad 1M$$

5f.

$$\text{As } t \rightarrow \infty, \left(\frac{mg}{k} + \left(u - \frac{mg}{k} \right) \times e^{-\frac{k}{m}t} \right) \rightarrow 4.2$$

$$\text{Therefore, } \frac{mg}{k} = 4.2$$

$$k = \frac{90 \times 9.8}{4.2} = 210 \quad 1A$$

END OF SECTION 2 SOLUTIONS