Neap

Trial Examination 2021

VCE Mathematical Methods Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 18 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2021 VCE Mathematical Methods Units 3&4 Written Examination 2.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

The tangent to the curve $y = x^2 + 2x - 1$ at $x = 1$ has the equation

A. $y = 4x - 2$

B. $y = 2x + 2$

C. $y = -\dfrac{x}{4} + \dfrac{9}{4}$

D. $y = 4x + 2$

E. $y = 4x - 3$

Question 2

The quadratic function $f : [-2, 1) \to R, f(x) = 1 - 2x^2$ has a range equal to

A. $(-7, -1]$

B. $[-7, -1)$

C. $(-7, 1]$

D. $[-7, 1]$

E. R

Question 3

The graph of the cubic function f has a local maximum at $(a, 2)$ and a local minimum at $(b, -3)$, where $a < b$.

If the equation $2f(x) = c$ has exactly one solution, then

A. $c < -6$ or $c > 4$

B. $c < -3$ or $c > 2$

C. $-6 < c < 4$

D. $-3 < c < 2$

E. $-4 < c < 6$

Question 4

Consider the simultaneous linear equations below, where m is a real constant.

$$mx + 3y = 0$$

$$4x + (m+1)y = 5$$

The set of values of m for which the system has a unique solution is

A. R

B. $R \setminus \{-4, 3\}$

C. $R \setminus \{-3, 4\}$

D. $\{-4, 3\}$

E. $\{-3, 4\}$

Question 5

A pet food company produces cans of dog food that are required to contain a minimum of 82 grams of meat. In a random sample of 1000 cans, 927 cans are found to meet this requirement.

If an approximate 95% confidence interval for the proportion of cans that meet this requirement is obtained from this result, the upper bound of this confidence interval is closest to

A. 0.911

B. 0.927

C. 0.943

D. 0.950

E. 0.952

Question 6

Let $f : R \to R,\ f(x) = 1 - 2\sin(3x)$.

The period and the range of f respectively are

A. 2π and $[-1, 1]$.

B. $\dfrac{2\pi}{3}$ and $[-2, 0]$.

C. $\dfrac{2\pi}{3}$ and $[-1, 3]$.

D. 6π and $[-2, 0]$.

E. 6π and $[-1, 3]$.

Question 7

The point $A(-1, 2)$ lies on the graph of the function f. A transformation maps the graph of f to the graph of g, where $g(x) = 2f(x-5)+1$.

The value of $g(4)$ is equal to

A. –11

B. –2

C. 4

D. 5

E. 6

Question 8

If $\int_3^5 f(x)dx = 10$, then $\int_5^3 1-2f(x)dx$ is equal to

A. –32

B. –10

C. 13

D. 17

E. 18

Question 9

A discrete random variable X has the following probability distribution, where m and k are real constants.

x	0	1	2	3
$\Pr(X = x)$	m	k	$k - \frac{1}{4}$	k^2

The maximum value of m is

A. 0

B. $\frac{1}{4}$

C. $\frac{5}{16}$

D. $\frac{1}{2}$

E. $\frac{11}{16}$

Question 10

The average rate of change of the function f with a rule $f(x) = x^3 + 2x$ between $x = a$ and $x = a + 1$ is at a minimum when a is equal to

A. -1

B. $-\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

E. 1

Question 11

Let f be a polynomial function such that $f(a) = a$, $f(-a) = -a$ and $f'(-a) = f'(a) = 0$, where $a > 0$.

Given the range of f is $(-\infty, a]$, the minimum degree of f is

A. 2

B. 3

C. 4

D. 5

E. 6

Question 12

The chance for a particular Olympic archer to successfully hit the bullseye on any given attempt is 75%. The archer is competing in a competition and has four attempts remaining.

The probability that the archer makes exactly three **consecutive** bullseyes is equal to

A. $\frac{27}{256}$

B. $\frac{27}{128}$

C. $\frac{27}{64}$

D. $\frac{135}{256}$

E. $\frac{3}{4}$

Question 13

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \sin(2x) & \text{if } \pi < x < \dfrac{3\pi}{2} \\ 0 & \text{elsewhere} \end{cases}$$

The value of a such that $\Pr(X < a) = \dfrac{1}{4}$ is equal to

A. $\dfrac{9\pi}{8}$

B. $\dfrac{7\pi}{6}$

C. $\dfrac{4\pi}{3}$

D. $\dfrac{11\pi}{8}$

E. $\dfrac{3}{2}$

Question 14

The maximal domain of the function with the rule $f(x) = \dfrac{1}{\sqrt{1-mx}}$, where $m > 0$, is equal to

A. $(-\infty, 1]$

B. $(1, \infty)$

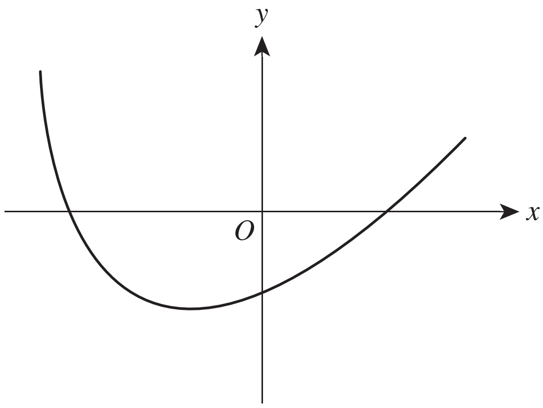
C. $\left(-\infty, \dfrac{1}{m}\right]$

D. $\left(-\infty, \dfrac{1}{m}\right)$

E. $\left(\dfrac{1}{m}, \infty\right)$

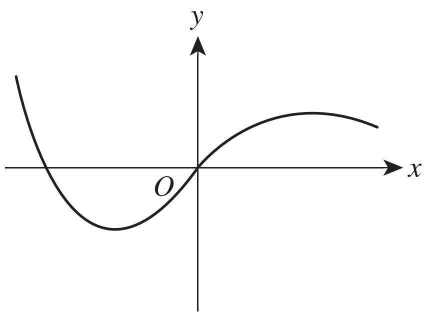
Question 15

The graph of the derivative function f' is shown below.



The graph of the function f could be

A.



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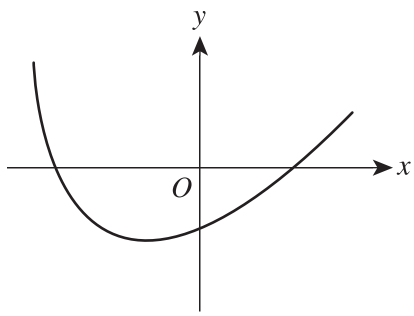

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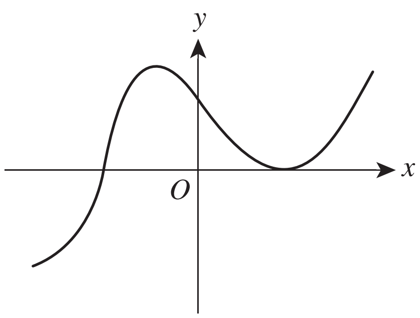

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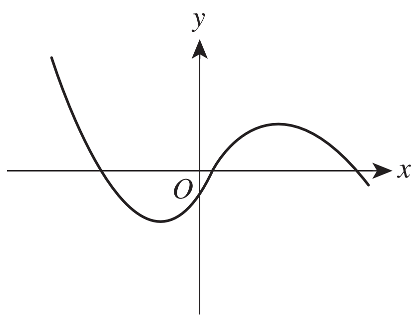
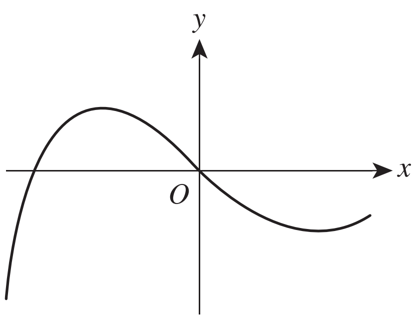


E.



Question 16

The minimum distance from the parabola $y = c - x^2$ to the origin is equal to c when

A. $c > \frac{1}{2}$

B. $c \leq \frac{1}{2}$

C. $c > -\frac{1}{2}$

D. $c > \frac{\sqrt{2}}{2}$

E. $c \in R^+$

Question 17

X is a normally distributed random variable with a mean of 0 and $\Pr(X > 1.5) = 0.1$.

The variance of X is closest to

A. 1.17

B. 1.28

C. 1.37

D. 1.92

E. 3.67

Question 18

For $y = \sqrt{\frac{1}{f(x)}}$, $\frac{dy}{dx}$ is equal to

A. $\frac{-f'(x)}{2\sqrt{f(x)}}$

B. $\frac{-f'(x)}{\left[\sqrt{f(x)}\right]^3}$

C. $-\frac{1}{2}\left[\frac{1}{\sqrt{f(x)}}\right]^3$

D. $\frac{-f'(x)}{2\left[\sqrt{f(x)}\right]^3}$

E. $\frac{-1}{2\sqrt{f'(x)}}$

Question 19

Consider the function g, where $g(x) = \sin(a^x)$ and a is a real number greater than 1.

If g is a one-to-one function, the domain of g could be

A. R^+

B. $\left(-\infty, \log_a\left(\frac{\pi}{2}\right)\right)$

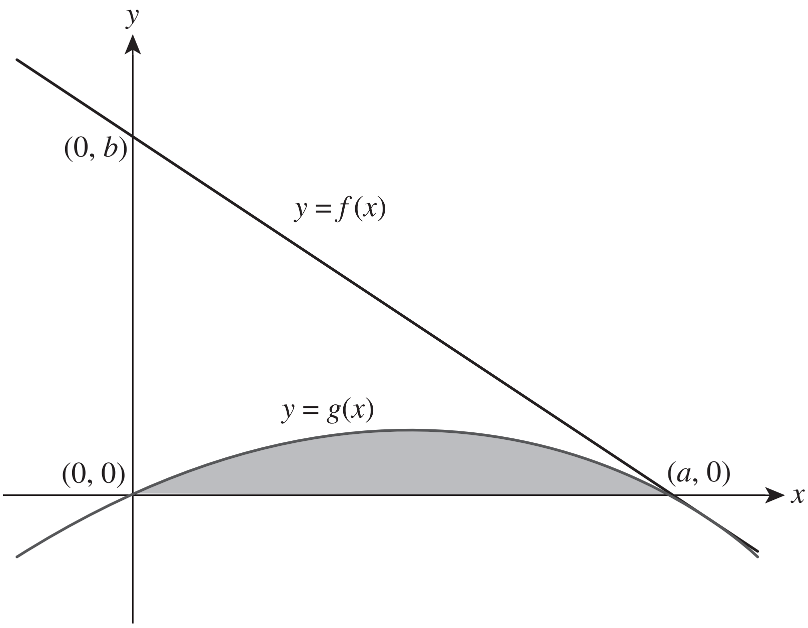
C. $\left(\log_a\left(\frac{\pi}{2}\right), \infty\right)$

D. $(-\infty, \log_a(\pi))$

E. $(\log_a(\pi), \infty)$

Question 20

The graph below shows a linear function f and a quadratic function g, where $g(x) \le f(x)$ for $x \in R$.



Given that $f(a) = g(a)$ and the equation $f(x) = g(x)$ has one solution, the area of the shaded region can be expressed as

A. $\frac{ab}{6}$

B. $\frac{a^3}{6}$

C. ab

D. $\frac{ab}{2}$

E. $\frac{ab}{12}$

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

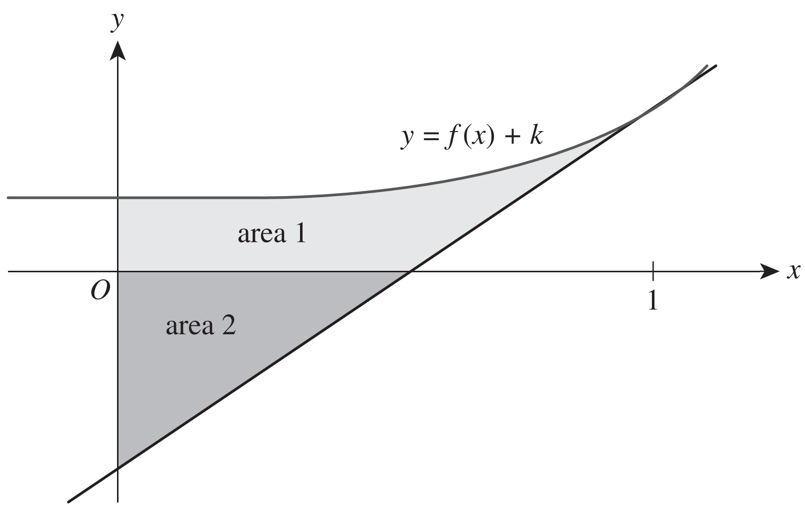
Question 1 (10 marks)

Consider the function f with the rule $f(x) = x^3$.

a. Find the equation of the tangent at the point where $x = 1$. 1 mark

b. Find the area bound by $y = f(x)$, the tangent line at $x = 1$ and the x-axis. 3 marks

The graph of $y = f(x) + k$, where $k \in R^+$, and the tangent line at the point where $x = 1$ are sketched below.



c. Find the coordinates of the x-intercept of the tangent line in terms of k. 2 marks

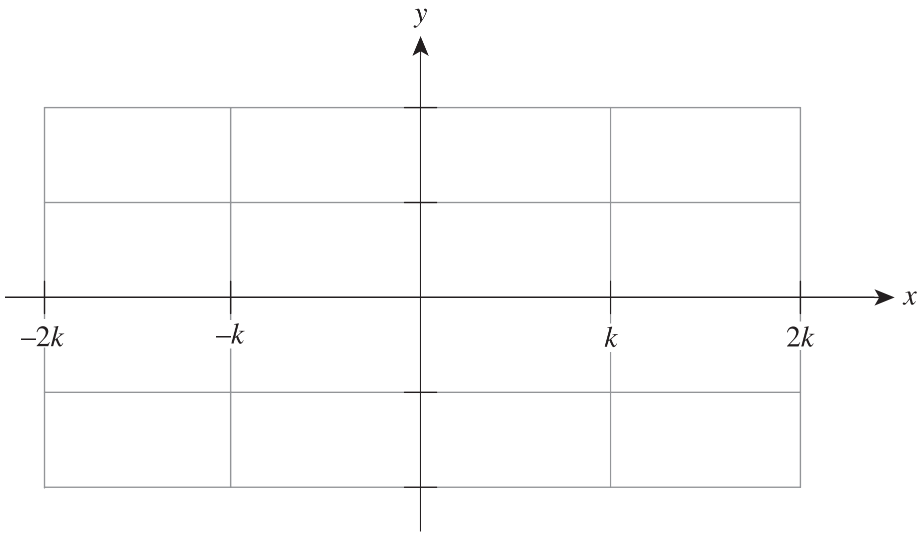
d. Area 1 is the area bound by the curve $y = f(x) + k$, the tangent line at $x = 1$ and the coordinate axes. Area 2 is the area bound by the tangent line at $x = 1$ and the coordinate axes.

Find the value of k so that area 1 = area 2. 4 marks

Question 2 (6 marks)

Consider the function with the rule $f(x) = (x + k)(x - k)^2$, where $k \in R^+$.

a. Graph $y = f(x)$ on the set of axes below, showing the coordinates of all intercepts and turning points in terms of k where appropriate. 3 marks



b. Consider the equation $x^2 - k^2 = \frac{1}{x - k}$.

i. Show that this equation can be expressed as $(x + k)(x - k)^2 = 1$. 1 mark

ii. For what value(s) of k does the equation $x^2 - k^2 = \frac{1}{x - k}$ have exactly two solutions? 2 marks

Question 3 (17 marks)

Sheepdog trials are held at a large country show. The trials involve the dogs completing a circuit of obstacles as quickly as possible. The two main sheepdog breeds that compete are border collies and kelpies.

The finishing time for border collies is normally distributed with a mean of 280 seconds and a standard deviation of 24 seconds.

a. Find the probability that the finishing time for a randomly selected border collie will be between 250 and 300 seconds. Give your answer correct to three decimal places. 1 mark

It is known that 35% of the border collies competing are purebred. The remaining 65% of the border collies are crossbred.

b. A sheepdog is considered elite if it completes the course in less than 240 seconds.

Find the probability that a randomly selected border collie is both purebred and elite. Give your answer correct to three decimal places. 2 marks

c. **i.** Find the probability that a random sample of 20 border collies will contain exactly 6 purebred border collies. Give your answer correct to three decimal places. 2 marks

ii. For a random sample of 20 border collies, $\hat{P}$ is the random variable that represents the proportion of border collies that are purebred.

Find the probability that $\hat{P}$ is greater than 33%. Give your answer correct to three decimal places. 2 marks

iii. For a random sample of n border collies, $\hat{P}_n$ is the random variable that represents the proportion of border collies who are purebred.

Find the least value of n for which $\Pr\left(P \geq \dfrac{2}{n}\right) > 0.96$. 2 marks

A random sample of 52 kelpies is taken, with a proportion of them being purebred kelpies. The approximate confidence interval for the proportion is (0.1184, 0.3047).

d. **i.** Determine the sample proportion used in the calculation of this confidence interval. 1 mark

ii. Find the level of confidence used to calculate this confidence interval. Give your answer correct to the nearest percentage. 2 marks

e. The age of all dogs competing in the sheepdog trials is given by the probability density function A with the rule

$$Q(t)=\begin{cases}\dfrac{mt^2}{e^t} & t\geq 0\\ 0 & t<0\end{cases}$$

where t is given in years and m is a real constant.

i. Differentiate $\left(t^2+2t+2\right)e^{-t}$ and hence show that the value of m is equal to $\frac{1}{2}$. 3 marks

ii. Find the probability that the age of a randomly selected sheepdog is greater than the mean age. 2 marks

Question 4 (7 marks)

Let $f(x) = e^{2x} - 2e^{x}$.

a. Find the coordinates of any stationary points of *f*. 2 marks

Let $g : [a, \infty) \to R,\ g(x) = e^{2x} - 2e^{x}$.

b. Find the minimum value of *a* for which the inverse function g^{-1} exists. 1 mark

c. Find the rule and the domain of the inverse function g^{-1}. 3 marks

d. Find the coordinates of the point of intersection between *g* and g^{-1}. Give your answer correct to two decimal places. 1 mark

Question 5 (20 marks)

Consider the functions $f: R \to R,\ f(x) = (x-2)(x-1)$ and $g: R \to R^+,\ g(x) = \log_e(x)$.

a. **i.** State the domain and range of $g(f(x))$. 2 marks

ii. Find $\dfrac{d\left[g(f(x))\right]}{dx}$ and hence explain why the graph of $y = f(g(x))$ has no stationary points. 2 marks

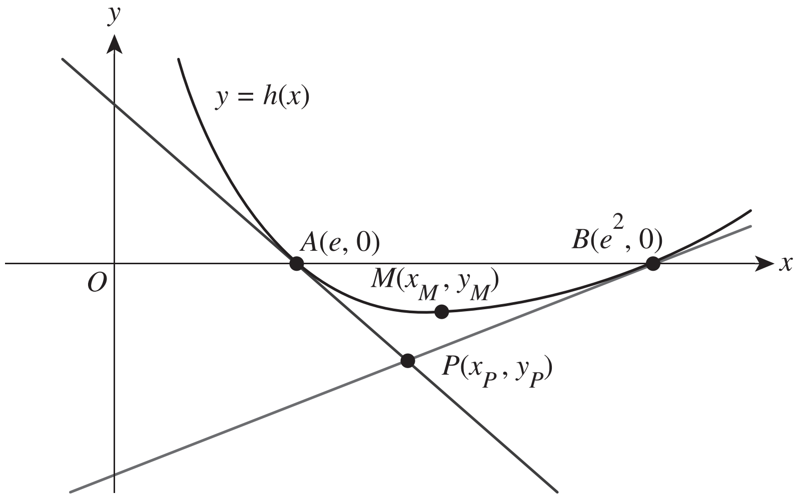
Let $h(x) = f(g(x))$.

b. **i.** Show that the rule for h can be expressed as $h(x) = \left(\log_e(x) - \dfrac{3}{2}\right)^2 - \dfrac{1}{4}$. 2 marks

ii. Hence, find the range of $h(x)$. 1 mark

 VCE_Maths_Methods_34_Ex2_QB

Part of the graph of $y = h(x)$ is shown below. The points A and B are the x-intercepts of the graph of $y = h(x)$ and $M(x_M, y_M)$ is the turning point. Tangent lines to the graph of $y = h(x)$ at the points A and B intersect at the point $P(x_P, y_P)$.



c. **i.** Find $h'(x)$. 1 mark

ii. Hence, find x_M. 1 mark

d. Show that $x_P = \dfrac{2e^2}{e+1}$. 2 marks

e. Prove that $x_P < x_M$. 2 marks

f. **i.** Find the angle $\angle APB$. Give your answer in the form $\tan^{-1}(\alpha) + \tan^{-1}(\beta)$. 2 marks

ii. Prove that $\angle APB$ is an obtuse angle. 1 mark

g. Find the area bound by the graph of $y = h(x)$ and the tangent lines at the points A and B.

Give your answer in the form $\dfrac{\alpha e^3 + \beta e^2 + \gamma e}{2e+2}$, where $\alpha,\ \beta,\ \gamma \in Z^+$. 4 marks

END OF QUESTION AND ANSWER BOOKLET