Neap

Trial Examination 2020

VCE Specialist Mathematics Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 12 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2020 VCE Specialist Mathematics Units 3&4 Written Examination 1.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (2 marks)

Consider the function $f(x) = \arctan\left(\frac{2}{x}\right)$, where $x \neq 0$.

Find an expression for $f'(x)$. Give your answer in the form $\frac{a}{x^2 + b}$, where $a, b \in Z$.

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Question 2 (3 marks)

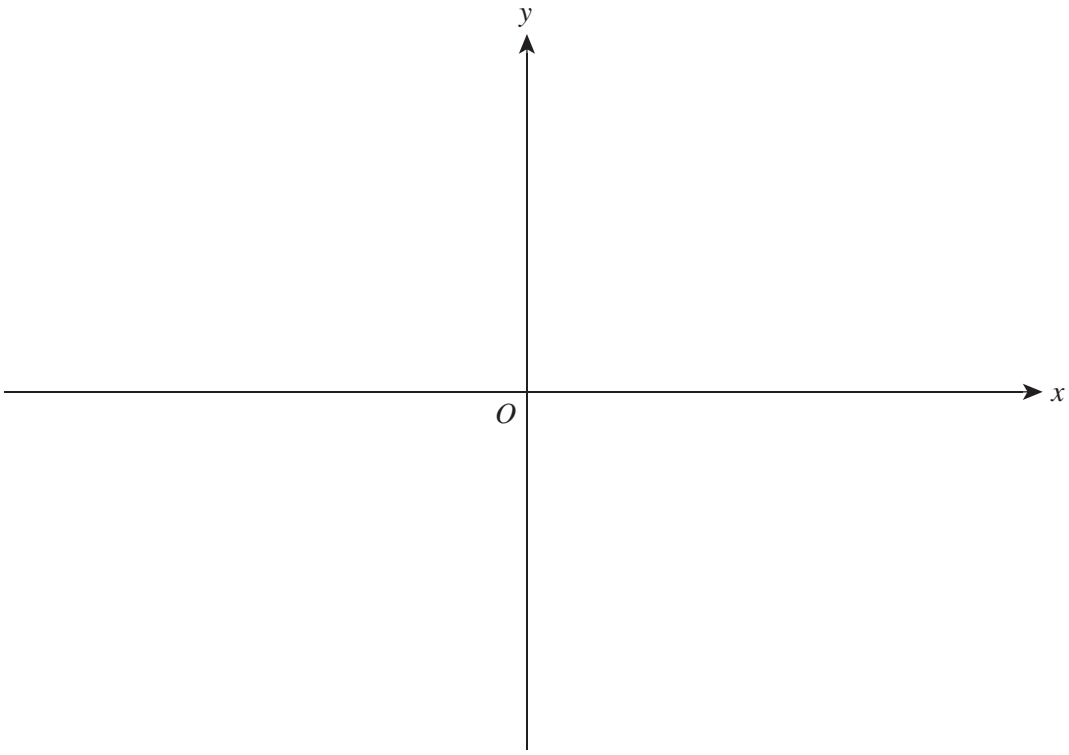
Consider the equation $z^2 - (1 - 2i)z + 1 + 5i = 0$. One of the roots of the equation is $-1 + i$.

a. State why the conjugate root theorem **cannot** be applied to find the other root of the equation. 1 mark

b. Find the other root of the equation, expressing your answer in cartesian form. 2 marks

Question 3 (4 marks)

Sketch the graph of $y = \dfrac{x^2 - x + 2}{x - 2}$ on the axes below. Clearly label any intercepts and stationary points with their the coordinates, and any asymptotes with their equations.



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Question 4 (3 marks)

Find the set of values of x for which $|2x + 1| < |x|$.

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Question 5 (5 marks)

Relative to a fixed origin O, the point A has position vector $\underset{\sim}{a} = 8\underset{\sim}{i} - 6\underset{\sim}{j} + 5\underset{\sim}{k}$ and the point B has position vector $\underset{\sim}{b} = t\underset{\sim}{i} + t\underset{\sim}{j} + 2t\underset{\sim}{k}$, where $t \in R$, $t \geq 0$.

a. Show that $\left|\overrightarrow{AB}\right| = \sqrt{6t^2 - 24t + 125}$. 2 marks

b. Find the minimum distance between points A and B. 3 marks

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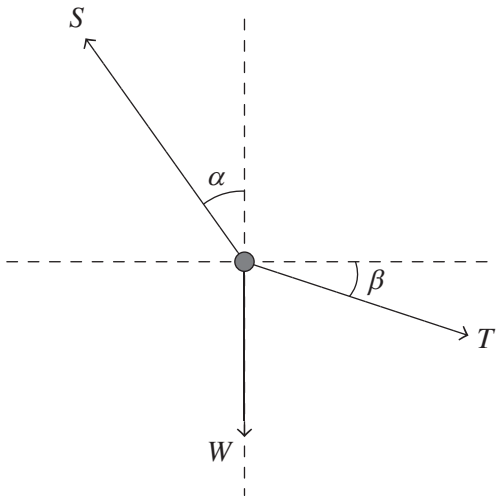
Question 6 (4 marks)

A particle is held in equilibrium by three coplanar forces of magnitudes *S, T* and *W* newtons. The force of magnitude *S* newtons makes an angle α with the vertical, and the force of magnitude *T* newtons makes an angle β with the horizontal, as shown in the diagram below.



Show that $T = \dfrac{W\tan(\alpha)}{\cos(\beta) - \sin(\beta)\tan(\alpha)}$.

Question 7 (4 marks)

A curve C has the equation $y^2 = 2x - (x + y)^2$.

Find the equations of the tangents to C that are parallel to the x-axis.

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Question 8 (4 marks)

A curve is defined by the parametric equations $x = t^2 - 2\log_e(t)$ and $y = 4t - 4$, where $1 \leq t \leq 2$.

Find the length of the curve, expressing your answer in the form $a + b\log_e(2)$, where $a, b \in Z$.

Question 9 (4 marks)

Solve the equation $\arctan(2x) + \arctan(x) = \arctan(3)$, $x \in R$.

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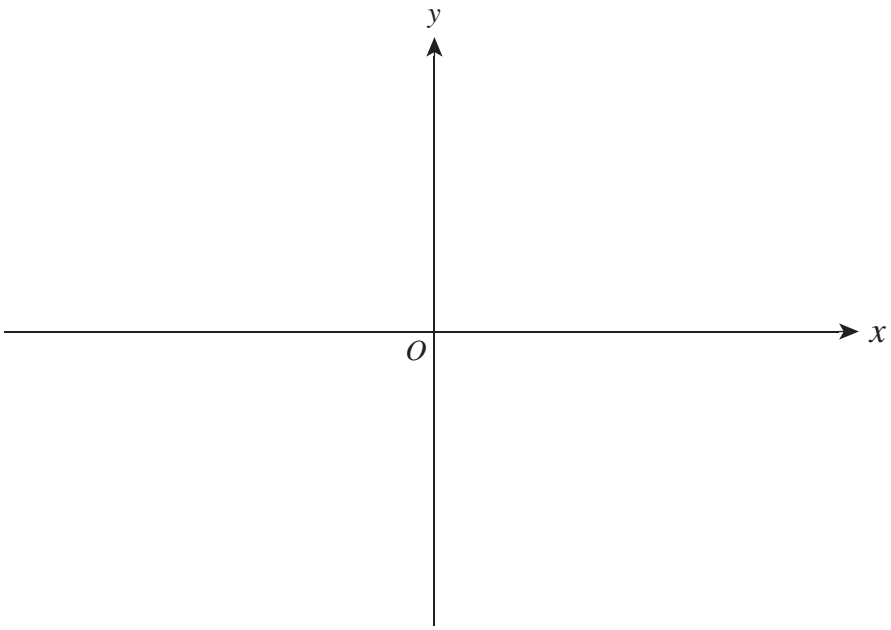
Question 10 (7 marks)

A particle P moves such that at time t its position vector $\underset{\sim}{\mathrm{r}}(t)$ from a fixed origin is given by $\underset{\sim}{\mathrm{r}}(t) = e^t\cos(t)\underset{\sim}{\mathrm{i}} + e^t\sin(t)\underset{\sim}{\mathrm{j}}$, $0 \le t \le \pi$. The velocity vector of particle P is denoted by $\dot{\underset{\sim}{\mathrm{r}}}(t)$.

a. Show that $\underset{\sim}{\mathrm{r}}(t)$ always makes an angle of $\frac{\pi}{4}$ with $\dot{\underset{\sim}{\mathrm{r}}}(t)$. 4 marks

b. Sketch the path of the particle P for $0 \leq t \leq \pi$ on the axes below. Label the particle's initial and final positions with their coordinates and indicate its initial direction of motion. 3 marks



END OF QUESTION AND ANSWER BOOKLET

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