



**MATHEMATICS**

**3A/3B**

**Calculator-assumed**

**WACE Examination 2013**

**Marking Key**

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

Section Two: Calculator-assumed

(100 Marks)

Question 10

(4 marks)

Prove algebraically that if you add the squares of three consecutive numbers and then subtract 2, you always get a multiple of three.

Solution	
Let $n, n+1, n+2$ be the three consecutive numbers squaring each gives $n^2, n^2 + 2n + 1, n^2 + 4n + 4$ adding and then subtracting 2 gives $3n^2 + 6n + 3$ factorising gives $3(n^2 + 2n + 1)$ which is a multiple of three	
Specific behaviours	
✓	correctly defines variables
✓	correctly squares each term
✓	correctly adds terms and subtracts 2
✓	correctly factorises and gives correct conclusion

Question 11

(8 marks)

(a) In triangle  $ABC$ ,  $c = 7.2$  cm,  $a = 8.4$  cm and  $\angle B = 61^\circ$ .

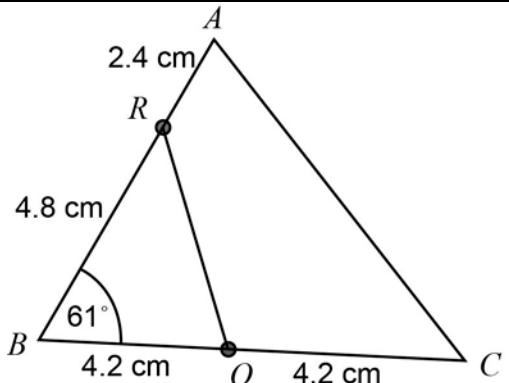
(i) Calculate the length of the side  $AC$ . (1 mark)

Solution	
$AC = 8.0$ cm (to two significant figures) (Accept $AC = 7.98$ cm to two decimal places)	
Specific behaviours	
✓	correctly calculates the side length

(ii) Calculate the size of the angle  $ACB$ . (1 mark)

Solution	
$\angle ACB = 52^\circ$ (to the nearest degree)	
Specific behaviours	
✓	correctly calculates the angle

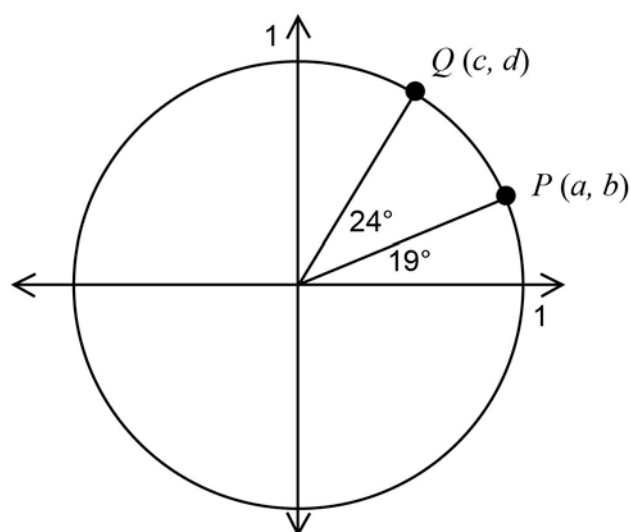
- (iii) If  $Q$  is the midpoint of  $BC$  and  $AR = \frac{1}{2}RB$ , draw a diagram, indicating clearly the lengths of the sides  $AR$ ,  $RB$ ,  $BQ$  and  $QC$ . (2 marks)

Solution	
	
Specific behaviours	
✓	correctly draws $\triangle ABC$ and identifies the position of the points $R$ and $Q$
✓	correctly indicates lengths

- (iv) Calculate the area of the quadrilateral  $ARQC$ . (1 mark)

Solution	
$\text{Area of } \triangle RBQ = \frac{1}{2}(4.8)(4.2)(\sin 61^\circ) = 8.82 \text{ cm}^2$ $\text{Area of } \triangle ABC = \frac{1}{2}(7.2)(8.4)(\sin 61^\circ) = 26.45 \text{ cm}^2$ $\therefore \text{Area of quadrilateral } ARQC = 26.45 - 8.82 = 17.63 \text{ cm}^2$	
Specific behaviours	
✓	correctly calculates area of quadrilateral $ARQC$

- (b) Use the unit circle shown below to determine each of the following, giving your answers in terms of either  $a$ ,  $b$ ,  $c$  or  $d$ .



- (i)  $\sin 19^\circ$  (1 mark)

Solution	
$\sin 19^\circ = b$	
Specific behaviours	
✓	correctly identifies $y$ -coordinate as the sine value

- (ii)  $\cos 43^\circ$  (1 mark)

Solution	
$\cos 43^\circ = c$	
Specific behaviours	
✓	correctly identifies $x$ -coordinate as the cosine value

- (iii)  $\sin 161^\circ$  (1 mark)

Solution	
$\sin 161^\circ = b$	
Specific behaviours	
✓	correctly identifies $y$ -coordinate as the sine value

**Question 12**

**(7 marks)**

Robert buys a car for \$72 000. He pays a deposit of \$12 000 from his savings and borrows the remaining amount from his bank. The interest on the loan is 7.5% per annum and Robert can afford to make monthly repayments of \$1800.

The spreadsheet below shows the balance and interest of the loan for the first 6 months and the last 3 months.

Month	Opening balance at the start of each month	Interest
1	60 000.00	375.00
2	58 575.00	366.09
3	57 141.09	357.13
4	<b>A</b>	<b>B</b>
5	54 246.34	339.04
6	52 758.38	330.89
.	.	.
.	.	.
.	.	.
36	4 442.52	27.77
37	2 670.28	16.69
38	886.97	

- (a) Calculate the values of A and B, correct to two decimal places. (2 marks)

<b>Solution</b>	
$A = 57\,141.09 + 357.13 - 1800 = \$55\,698.22$ (\$55 698.23 from the spreadsheet)	
$B = 55\,698.23 \times 0.075 \div 12 = \$348.11$	
<b>Specific Behaviours</b>	
✓	correctly calculates the value of A
✓	correctly calculates the value of B

- (b) One of the opening balances for the first 6 months is incorrect. Identify which entry is incorrect and state the correct value of the balance. (2 marks)

<b>Solution</b>	
\$52 758.38 in the sixth month is incorrect. The correct value is \$52 785.38.	
<b>Specific behaviours</b>	
✓	identifies the incorrect entry
✓	states the correct value

- (c) How long will it take for the amount owing to fall below \$10 000? (1 mark)

<b>Solution</b>	
After the 32 <sup>nd</sup> payment or at the beginning of the 33 <sup>rd</sup> month	
<b>Specific behaviours</b>	
✓	identifies the correct time

- (d) Calculate the amount of Robert's final payment. (1 mark)

Solution
$886.9704 + 5.5436 = \$892.51$
Specific behaviours
✓ correctly calculates the final payment

- (e) What is the total amount of interest that Robert has paid? (1 mark)

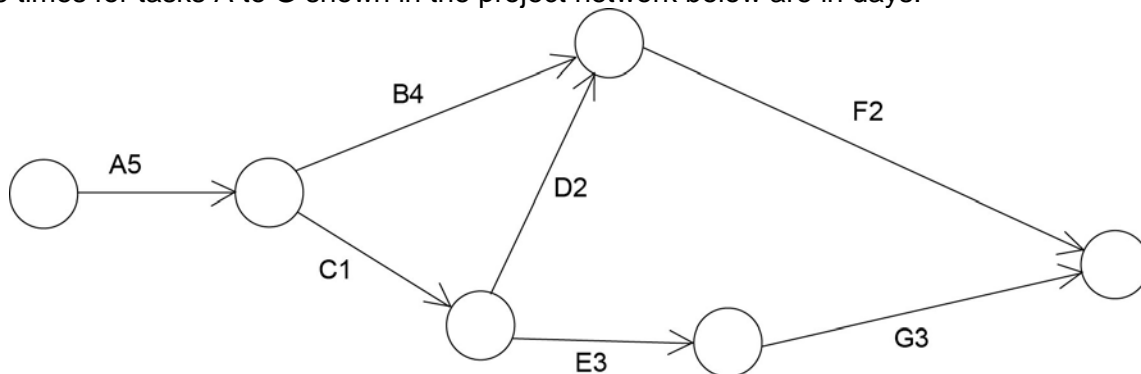
Solution
$(37 \times 1800 + 892.514) - 60\,000 = \$7492.51$
Specific behaviours
✓ correctly calculates the interest

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0.5 1 2 B A [ ]											

Question 13

(6 marks)

The times for tasks A to G shown in the project network below are in days.



- (a) Determine the critical path and the minimum completion time.

(2 marks)

Solution	
<p>Critical path: ACEG Minimum completion time: 12 days</p>	
Specific behaviours	
✓	correctly states critical path
✓	correctly states minimum completion time

- (b) What is the minimum amount of time two people would take to complete all of the tasks if only one person can be allocated to each task at any one time? (2 marks)

Solution	
$5 + 4 + 4 = 13$ days	
Specific behaviours	
✓	identifies areas where double-up can take place
✓	states correct answer

- (c) Under what condition(s) would it be possible to complete all of the tasks in the minimum completion time found in part (a)? (2 marks)

Solution	
The job can be completed in 12 days only if a third person is used so that tasks B and D are completed at the same time.	
Specific behaviours	
✓	identifies that a third person is required
✓	identifies that jobs B and D are to be completed concurrently

Question 14

(6 marks)

- (a) Given that  $B$  is inversely proportional to  $c$ , i.e.  $B = \frac{k}{c}$ , where  $k$  is a constant, describe the effect on  $B$  if

- (i)  $c$  is doubled. (1 mark)

Solution	
$c \rightarrow 2c \Rightarrow B = \frac{k}{2c} = \frac{1}{2} \left( \frac{k}{c} \right) \Rightarrow$ , therefore $B$ is halved	
Specific Behaviours	
✓	correctly concludes that $B$ is halved

- (ii)  $c$  is halved. (1 mark)

Solution	
$c \rightarrow \frac{1}{2}c \Rightarrow B = \frac{k}{\frac{1}{2}c} = 2 \left( \frac{k}{c} \right)$ , therefore $B$ is doubled	
Specific Behaviours	
✓	correctly concludes that $B$ is doubled

- (b) Boyle's Law states that the volume ( $V$ ) of a given mass of gas at a fixed temperature varies inversely as the pressure ( $P$ ), where  $V$  is in litres (L) and  $P$  is in Pascals (Pa).

- (i) Write an equation using  $k$  as the constant of proportionality to represent the relationship between  $V$  and  $P$ . (1 mark)

Solution	
$V = \frac{k}{P}$	
Specific behaviours	
✓	correctly formulates the equation

- (ii) Given that  $V = 3.4$  L when  $P = 9.6$  Pa, calculate the pressure of the gas when the volume is 12.7 L. (3 marks)

Solution	
$3.4 = \frac{k}{9.6} \Rightarrow k = 32.64$	
$12.7 = \frac{32.64}{P} \Rightarrow P = 2.57$ Pa	
Specific behaviours	
✓	correctly calculates $k$
✓	correctly substitutes into equation
✓	correctly evaluates $P$

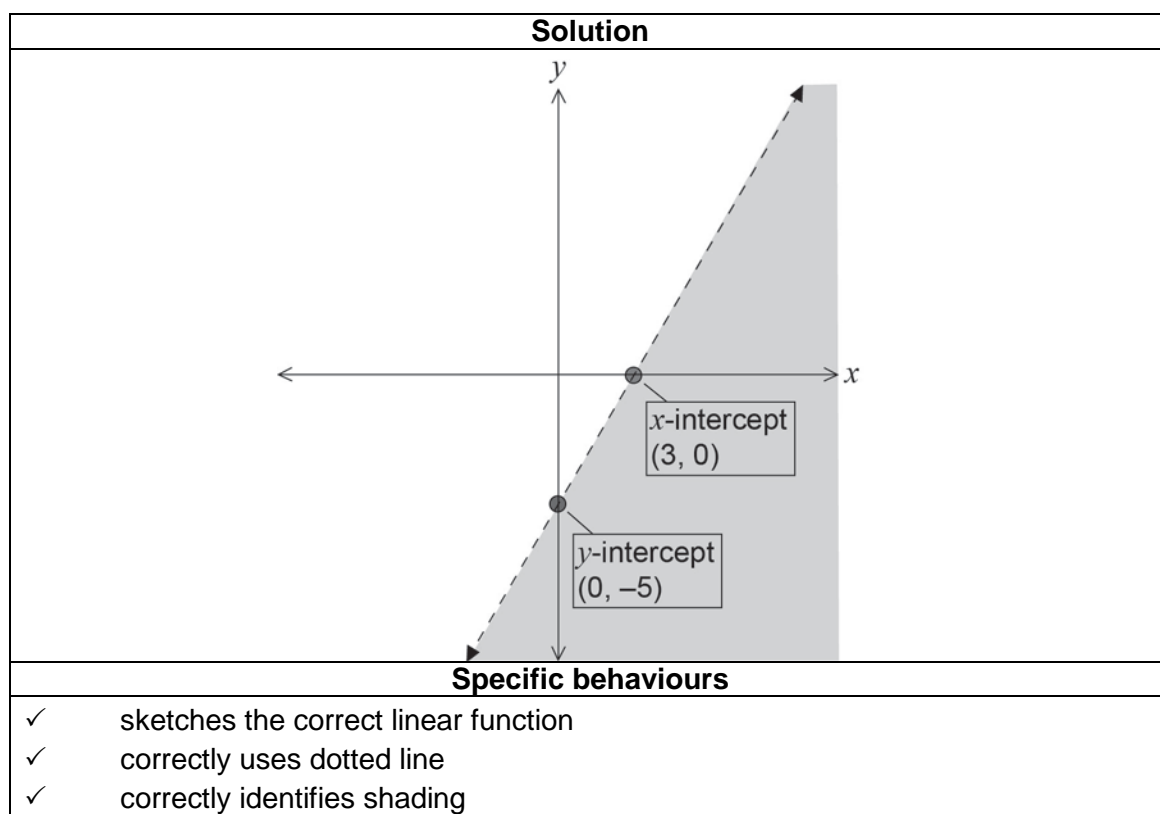


Question 15

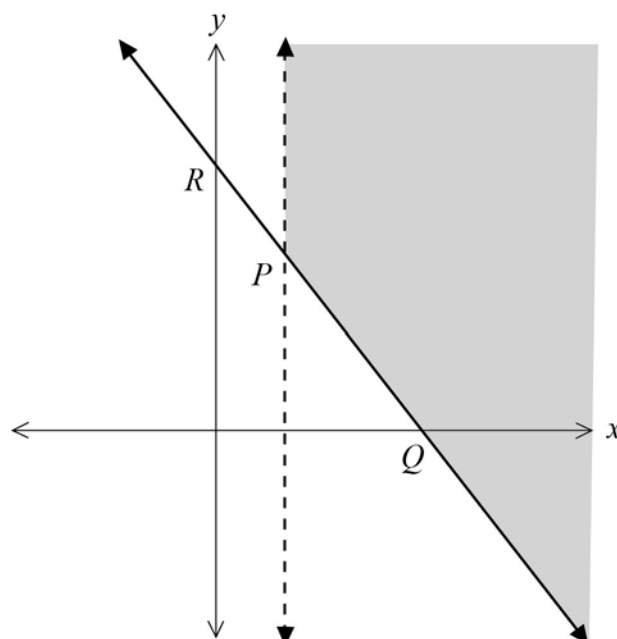
(6 marks)

(a) Sketch the inequality  $5x - 3y > 15$  on the axes below.

(3 marks)



- (b) Determine the inequalities that define the shaded region shown on the axes below given that  $P$ ,  $Q$  and  $R$  have coordinates  $(2, 5)$ ,  $(6, 0)$  and  $(0, 7.5)$  respectively. (3 marks)



Solution	
Vertical inequality: $x > 2$	
Gradient of $PQ$ is $\frac{0-5}{6-2} = -1.25$	
Therefore equation of oblique line is $y = -1.25x + 7.5$ since $y$ intercept is given (or by CAS).	
The inequality is then $y \geq -1.25x + 7.5$	
Specific behaviours	
✓	correctly identifies vertical inequality
✓	correctly writes the equation of oblique line
✓	correctly identifies the oblique inequality

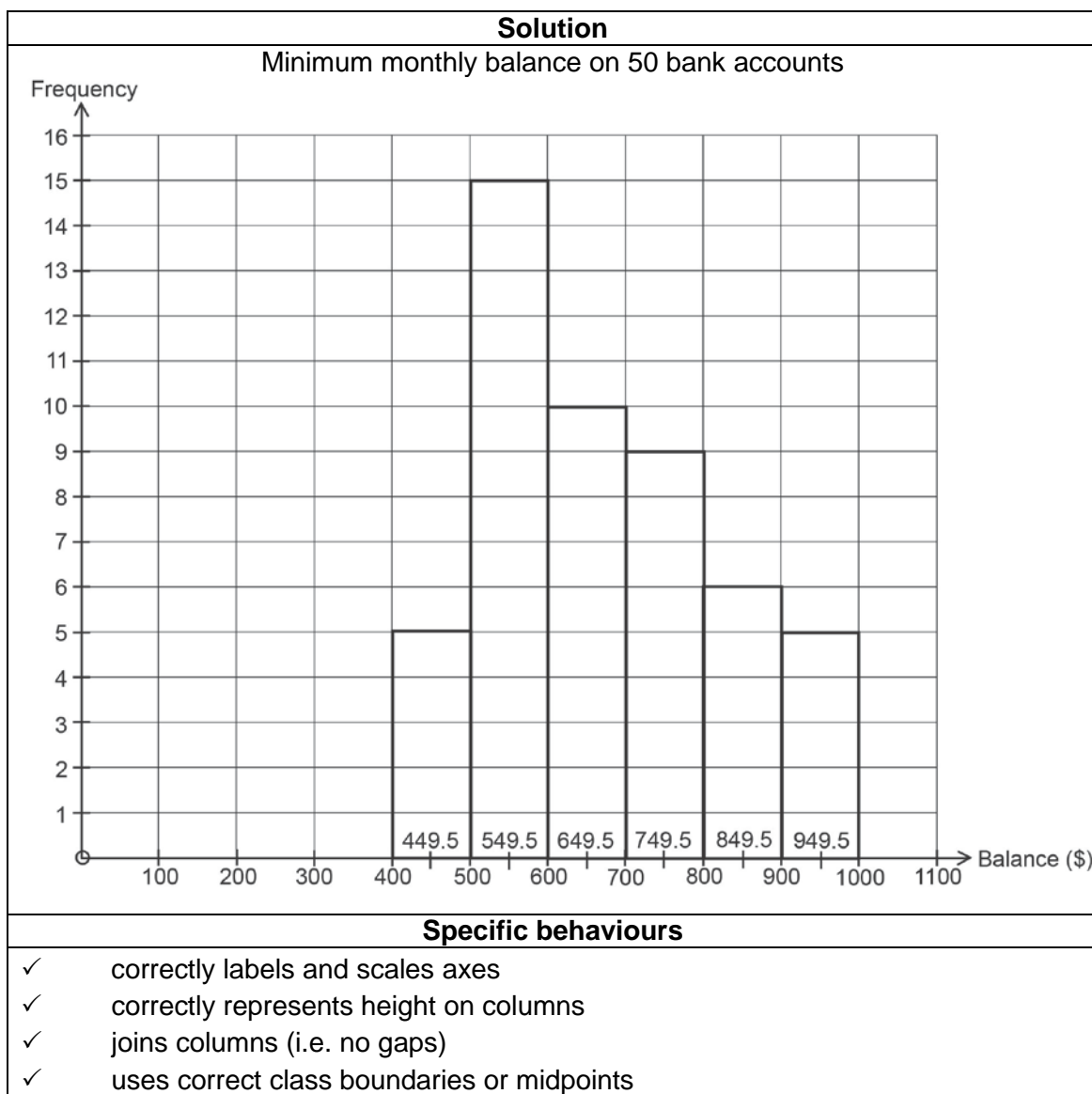
**Question 16**

**(9 marks)**

A survey of 50 bank accounts, all in credit, revealed the following distribution of the minimum monthly balances (rounded to the nearest dollar).

Balance	\$400–\$499	\$500–\$599	\$600–\$699	\$700–\$799	\$800–\$899	\$900–\$999
Frequency	5	15	10	9	6	5

- (a) Draw a frequency histogram on the grid below to represent the bank balance data.(4 marks)



- (b) Determine the median class and the modal class for the data.

**(2 marks)**

<b>Solution</b>	
Median class is \$600–\$699	
Modal class is \$500–\$599	
<b>Specific behaviours</b>	
✓	correctly identifies median class
✓	correctly identifies modal class

- (c) Complete the table below and use it to estimate the mean and standard deviation for the minimum monthly balance. (3 marks)

Balance	\$400–\$499	\$500–\$599	\$600–\$699	\$700–\$799	\$800–\$899	\$900–\$999
Midpoint	\$449.50	\$549.50	\$649.50	\$749.50	\$849.50	\$949.50
Frequency	5	15	10	9	6	5

Solution	
Estimate for the mean is \$671.50	
Estimate for the standard deviation is \$148.72	
Specific behaviours	
✓	correctly determines midpoints in the table
✓	correctly determines the mean
✓	correctly determines the standard deviation

Question 17

(14 marks)

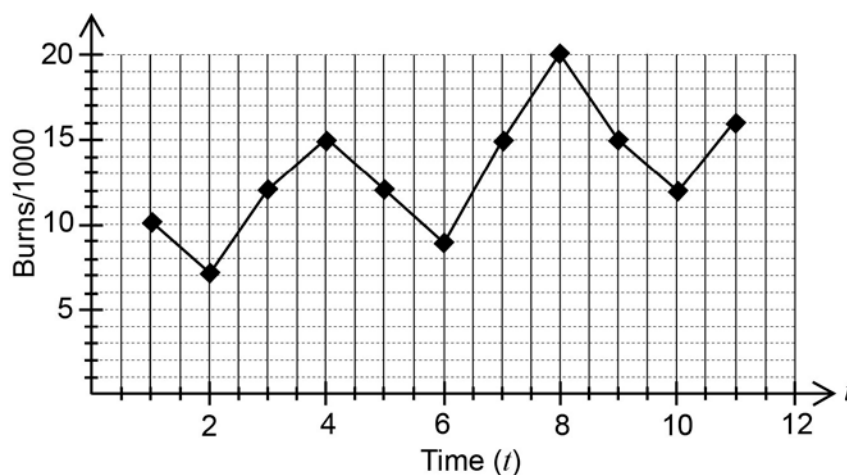
The rate of sunburn casualties per 1000 people was collected over a period of three years by the Sandy Beach Surf Club. The data are presented in the table below.

Year	Quarter	Time ( <i>t</i> )	Number of burns per 1000 people	Four-point centred moving averages ( <i>y</i> )	Residuals
2010	Mar	1	10		
	Jun	2	7		
	Sep	3	12	11.25	0.75
	Dec	4	15	<i>A</i>	<i>C</i>
2011	Mar	5	12	12.375	−0.375
	Jun	6	9	13.375	−4.375
	Sep	7	15	14.375	0.625
	Dec	8	20	15.125	4.875
2012	Mar	9	15	15.625	−0.625
	Jun	10	12	15.625	−3.625
	Sep	11	16		
	Dec	12	<i>B</i>		

- (a) Calculate the value of the missing entries marked by *A*, *B* and *C*. (4 marks)

Solution	
$\frac{3.5 + 12 + 15 + 12 + 4.5}{4} = 11.75 = A$	
$C = 15 - A = 3.25$	
$\frac{10 + 15 + 12 + 16 + 0.5B}{4} = 15.625 \Rightarrow B = 19$	
Specific behaviours	
✓	calculates <i>A</i> correctly
✓	calculates <i>C</i> correctly
✓	correctly formulates an equation involving <i>B</i>
✓	solves for <i>B</i> correctly

The graph below shows the number of burns per 1000 people plotted against time.



- (b) How does the graph support the choice of a four-point centred moving average? (1 mark)

The graph has a cycle of four.	
Specific behaviours	
✓	states correct reason

- (c) (i) Using time,  $t$  and the four-point centred moving averages,  $y$ , determine the equation of the regression line  $y = at + b$ , stating  $a$  and  $b$  correct to two decimal places.

(2 marks)

Solution	
$y = 0.71t + 9.10$	
Specific behaviours	
✓	determines correct equation
✓	states $a$ and $b$ correct to two decimal places

- (ii) State the correlation coefficient  $r_{ty}$ . (1 mark)

Solution	
$r = 0.98$	
Specific behaviours	
✓	states correct correlation coefficient

- (d) Using the regression equation together with the seasonal component for March, predict the number of burn cases per 1000 people for March 2014 to the nearest whole number. (3 marks)

Solution
$\frac{-0.375 + (-0.625)}{2} = -0.5$ is the seasonal component for March $t = 17 \Rightarrow y = 21.17$ prediction = $21.17 - 0.5 = 20.67 \approx 20$ burn cases (Accept 20 or 21)
Specific behaviours
✓ correctly calculates the seasonal component for March ✓ calculates $y$ correctly ✓ correctly adds the seasonal component and rounds to the nearest whole number

- (e) Comment on the reliability of your prediction from part (d). (2 marks)

Solution
Unreliable since it is extrapolation
Specific behaviours
✓ correctly states it is unreliable ✓ states correct reason

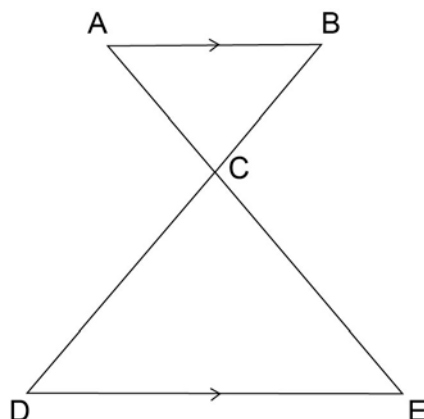
- (f) Suggest **one** factor that could have affected the reliability of your prediction. (1 mark)

Solution
March was particularly cold, or March was particularly hot
Specific behaviours
✓ correctly gives any reasonable answer

Question 18

(6 marks)

For the given diagram AB is parallel to DE.



- (a) At each step of the proof below, the statement and reason are provided. Complete the table below. (Note: At each step of the proof, either the statement or the reason may be false, but not both.) (4 marks)

Solution			
Step	Statement	Reason	True/False
1	$\angle BCA = \angle DCE$	Vertically opposite angles	True
2	$\angle BAC = \angle DEC$	Alternate angles	True
3	$\angle ABD = \angle EDB$	Corresponding angles	False
4	$\triangle ABC$ is congruent to $\triangle EDC$	Angles in each triangle are equal	False
Specific behaviours			
✓	states the correct conclusion for step 1		
✓	states the correct conclusion for step 2		
✓	states the correct conclusion for step 3		
✓	states the correct conclusion for step 4		

- (b) For each false step in the table above, rewrite the correct statement(s) or reason(s) in the table below. (2 marks)

Solution		
Step	Statement	Reason
1		
2		
3		Alternate angles
4	$\triangle ABC$ is similar to $\triangle EDC$	
Specific behaviours		
✓	rewrites correct reason for step 3	
✓	rewrites the correct statement for step 4	

Question 19

(6 marks)

Four relatives, Rosalind, Adam, Derek and Jennifer, have been left \$45 500 in the will of their great uncle. The condition of the will is that Adam receives  $\frac{2}{3}$  as much as Rosalind, Derek

receives  $\frac{1}{4}$  as much as Adam, and Jennifer receives  $\frac{3}{8}$  as much as Derek.

Let  $x$  be the amount Rosalind receives.

- (a) Write the amount that each person receives in terms of  $x$ . Leave your answers as fractions. (3 marks)

Solution	
Adam: $\frac{2}{3}x$	Derek: $\frac{1}{4}\left(\frac{2}{3}x\right) = \frac{1}{6}x$ Jennifer: $\frac{3}{8}\left(\frac{1}{6}x\right) = \frac{1}{16}x$
Specific behaviours	
✓	correctly states an algebraic expression for Adam's share
✓	correctly states an algebraic expression for Derek's share
✓	correctly states an algebraic expression for Jennifer's share

- (b) Write an equation, solve it for  $x$  and hence state the amount, in dollars, each person receives. (3 marks)

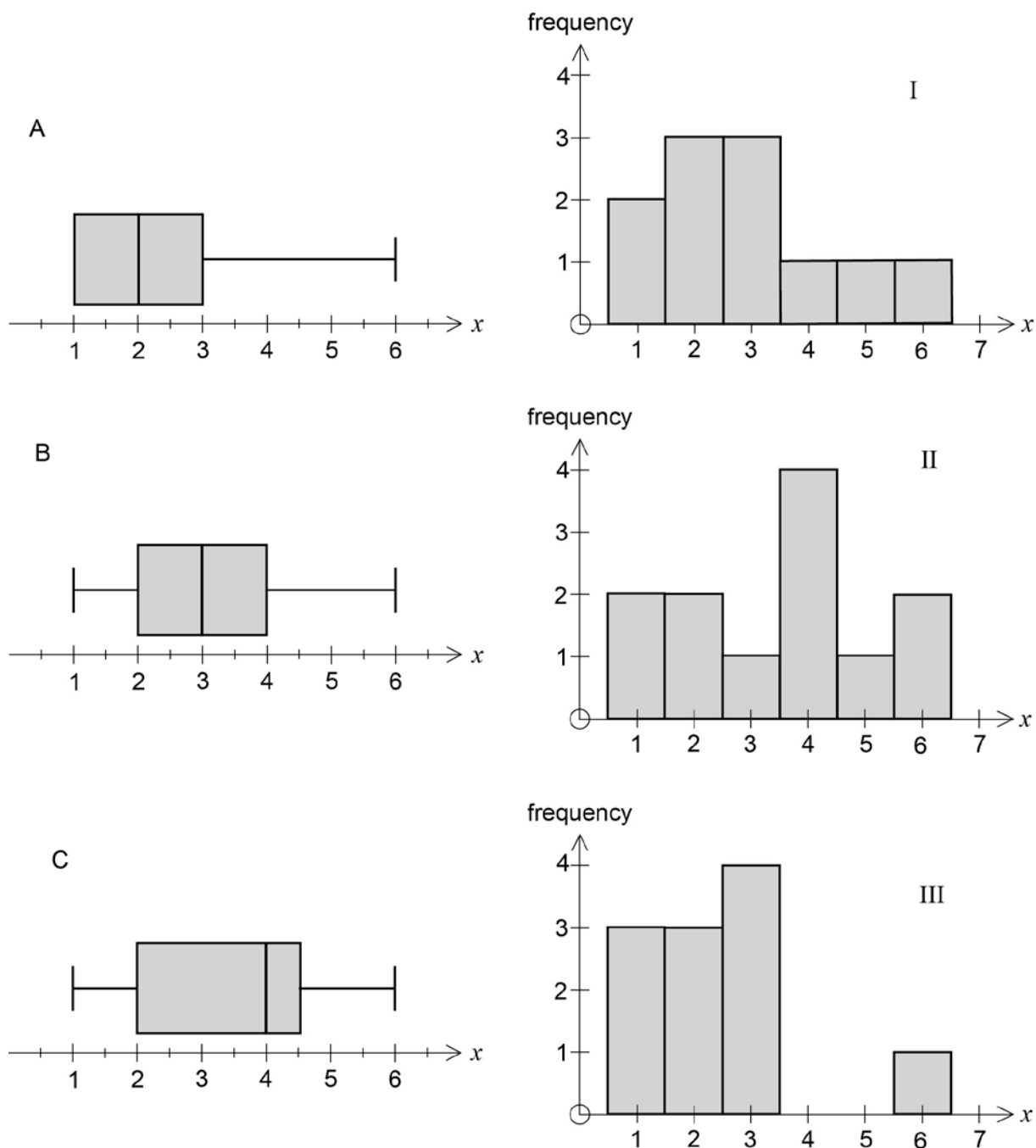
Solution	
$x + \frac{2}{3}x + \frac{1}{6}x + \frac{1}{16}x = 45\,500 \Rightarrow x = 24\,000$	
Rosalind: \$24 000    Adam: \$16 000    Derek: \$4000    Jennifer: \$1500	
Specific Behaviours	
✓	correctly formulates an equation
✓	correctly solves for $x$
✓	correctly states the amount each person receives



Question 20

(10 marks)

- (a) (i) Three sets of data were used to create the histograms frequency and boxplots shown below. Match each frequency histogram with its corresponding boxplot. (3 marks)

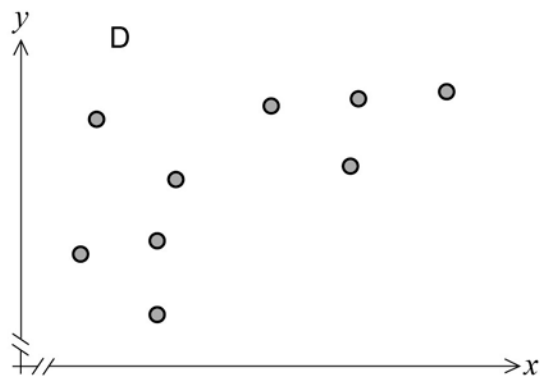
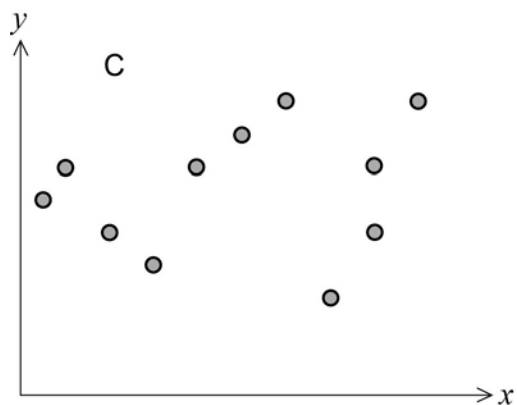
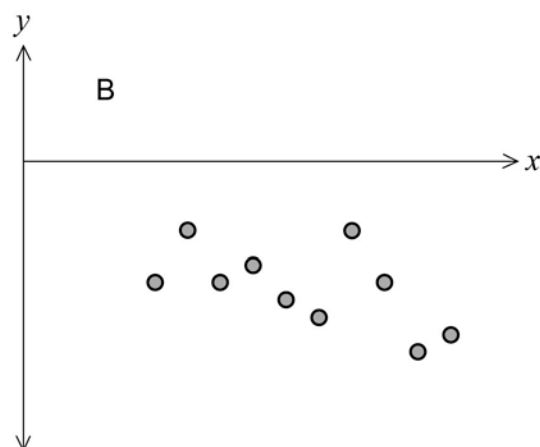
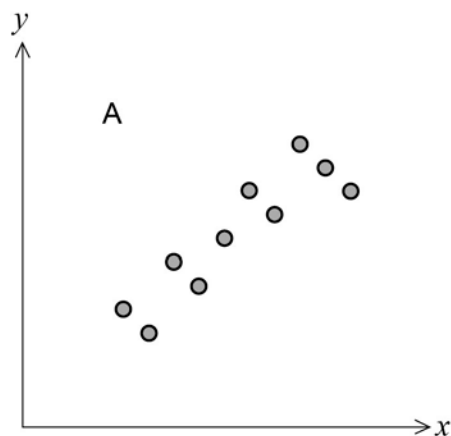


Solution	
A matches with III	
B matches with I	
C matches with II	
Specific behaviours	
✓	correctly identifies III
✓	correctly identifies I
✓	correctly identifies II

- (ii) Using the boxplot labelled A, calculate the smallest positive integer that would be considered an outlier. (3 marks)

<b>Solution</b>	
<p>IQR = 2  Outlier is more than <math>UQ + 1.5 \times IQR</math>  i.e. <math>3 + 1.5 \times 2 = 6</math>, therefore outlier is 7</p>	
<b>Specific behaviours</b>	
✓	correctly states IQR
✓	correctly calculates 6 as bound above which outliers occur
✓	correctly states outlier is 7

(b) Below are the scatterplots for four different sets of bivariate data.



Six possible correlation coefficients for the scatterplots are: 0.2,  $-0.9$ , 0.6, 1.3,  $-0.6$ , and 0.9.

Match each scatterplot with one correlation coefficient.

(4 marks)

Solution	
A matches with 0.9	
B matches with $-0.6$	
C matches with 0.2	
D matches with 0.6	
Specific behaviours	
✓	correctly identifies 0.9 for A
✓	correctly identifies $-0.6$ for B
✓	correctly identifies 0.2 for C
✓	correctly identifies 0.6 for D

Question 21

(9 marks)

A square based pyramid is constructed such that the sum of the perimeter of the base and the perpendicular height is 48 cm.

- (a) If the side length of the base of the pyramid is  $x$  cm and the perpendicular height of the pyramid is  $h$  cm, write an equation connecting the two variables. (1 mark)

Solution	
$4x + h = 48$	
Specific Behaviours	
✓	formulates correct equation

- (b) Using the equation from part (a), show that the volume ( $V$ ) of the pyramid is given by the equation  $V = \frac{4}{3}x^2(12 - x) \text{ cm}^3$ . (2 marks)

Solution	
$V = \frac{1}{3}x^2h = \frac{1}{3}x^2(48 - 4x) = \frac{4}{3}x^2(12 - x)$	
Specific Behaviours	
✓	correctly substitutes for $h$
✓	correctly factorises equation

- (c) Show the use of the product rule to determine  $\frac{dV}{dx}$ . (Do not simplify your answer.) (2 marks)

Solution	
$\frac{dV}{dx} = \frac{8}{3}x(12 - x) + \frac{4}{3}x^2(-1)$	
Specific Behaviours	
✓	states correct first term
✓	states correct second term

- (d) Using calculus techniques, determine the dimensions of this pyramid for maximum volume and state this volume. (4 marks)

Solution	
$4x(8 - x) = 0$ when $x = 0$ or $8$ using $x = 8, h = 16$ $\therefore$ base length = 8 cm and perpendicular height = 16 cm $V_{\max} = \frac{1024}{3} \text{ cm}^3$	
Specific behaviours	
✓	correctly equates $\frac{dV}{dx} = 0$
✓	correctly solves for $x$
✓	correctly calculates height
✓	correctly calculates volume

Question 22

(9 marks)

In England, motor vehicle licence plates consist of two letters followed by two digits followed by three letters (e.g. VW45PRT).

(a) How many different licence plates are possible if

- (i) there is no restriction on the number of times each letter and each digit is used? (2 marks)

Solution
$26^5 \times 10^2 = 11\,881\,376\,000$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ expresses the number of ways of arranging the letters</li> <li>✓ expresses the number of ways of arranging the digits and states correct answer</li> </ul>

- (ii) the first two letters must be vowels and no letter and no digit may be repeated? (2 marks)

Solution
$5 \times 4 \times 10 \times 9 \times 24 \times 23 \times 22 = 21\,859\,200$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly identifies restriction on first two letters</li> <li>✓ correctly identifies restriction for no repeats</li> </ul>

(b) In a batch of licence plates only the digits 1, 2, 3 and 4 and the letters A, B, C, D, E and F have been used. If one licence plate is chosen at random from the batch and no digit or letter may be repeated, determine the following probabilities. Leave your answers as fractions.

- (i) The licence plate is DF21BAE. (1 mark)

Solution
$\frac{1}{6 \times 5 \times 4 \times 3 \times 4 \times 3 \times 2} = \frac{1}{8640}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ states correct probability</li> </ul>

- (ii) The letters A and B are next to each other. (2 marks)

Solution
$\frac{(2 \times 1 \times 4 \times 3 \times 4 \times 3 \times 2) \times 3}{8640} = \frac{1728}{8640} = \frac{1}{5}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ applies restriction to calculate correct numerator</li> <li>✓ states correct probability</li> </ul>

- (iii) The last three letters are B, D and F, given that the two-digit number is 21.  
(2 marks)

Solution
$\frac{3 \times 2 \times 1 \times 3 \times 2 \times 1}{6 \times 5 \times 1 \times 4 \times 3 \times 2} = \frac{36}{720} = \frac{1}{20}$
Specific behaviours
<ul style="list-style-type: none"><li>✓ identifies restriction for numerator</li><li>✓ identifies restriction for denominator</li></ul>

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