Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section One: Calculator-free

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

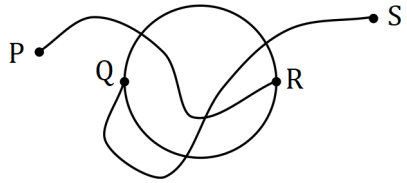
Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

Graph G is shown below.



(a) Calculate the sum of the degrees of the vertices of G. (1 mark)

Solution
$\text{Sum} = 1 + 3 + 3 + 1 = 8$
Specific behaviours
✓ correct sum

(b) State whether the following statements are true or false, briefly explaining your answer in each case.

(i) G is a simple graph. (1 mark)

Solution
False - multiple edges between Q and R
Specific behaviours
✓ correct response and reason

(ii) G contains a bridge. (1 mark)

Solution
True - edge PR (or QS) is a bridge
Specific behaviours
✓ correct response and reason

(iii) G is a planar graph. (1 mark)

Solution
True - it could be drawn (in plane) with no edges crossing
Specific behaviours
✓ correct response and reason

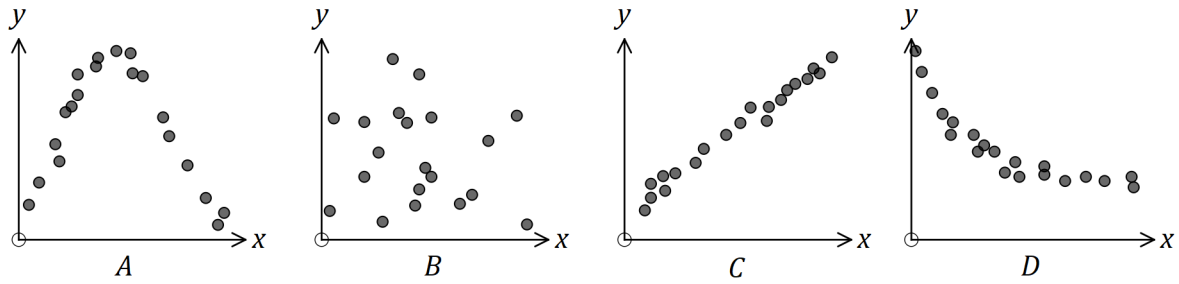
(iv) G satisfies Euler's formula. (1 mark)

Solution
True - it is a connected planar graph (or shows Euler works with v=4 f=2 e=4)
Specific behaviours
✓ correct response and reason

See next page

Question 2 **(6 marks)**

Consider the following four scatterplots A, B, C and D.



(a) Identify a scatterplot that suggests a non-linear relationship exists between the variables x and y. Justify your choice. (2 marks)

Solution
A or D. The points display an obvious pattern / lie along a curved line / etc, etc.
Specific behaviours
✓ correct choice ✓ justification

(b) Identify a scatterplot that suggests a linear relationship exists between the variables x and y. Justify your choice and state the direction of the association. (2 marks)

Solution
C. The points lie very close to a straight line. The direction is positive.
Specific behaviours
✓ correct choice with justification ✓ correct direction

(c) Identify a scatterplot that suggests no relationship exists between the variables x and y. Justify your choice. (2 marks)

Solution
B. The points appear randomly scattered on the graph.
Specific behaviours
✓ correct choice ✓ justification

Question 3 **(7 marks)**

(a) If $A_{n+1} = \frac{1}{2}A_n,\ A_1 = 48$ and $B_{n+1} = 2B_n + 3,\ B_1 = 3.5$ determine $B_5 - A_5$. (3 marks)

Solution
$A: 48, 24, 12, 6, 3 \Rightarrow A_5 = 3$ $B: 3.5, 10, 23, 49, 101 \Rightarrow B_5 = 101$ $B_5 - A_5 = 101 - 3 = 98$
Specific behaviours
✓ value of A_5 ✓ value of B_5 ✓ correct difference

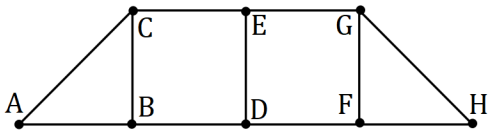
(b) Deduce a rule for the n^{th} term of the geometric sequence that has $T_2 = 12$ and $T_3 = 4$ and hence or otherwise determine T_6. (4 marks)

Solution
$r = \frac{4}{12} = \frac{1}{3}$ $a = 12 \div \frac{1}{3} = 36$ $T_n = 36\left(\frac{1}{3}\right)^{n-1}$ $T_6 = 4 \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{4}{27}$
Specific behaviours
✓ correct ratio ✓ correct first term ✓ correct rule in required form ✓ correct term

See next page

Question 4 **(6 marks)**

Graph P is shown below.



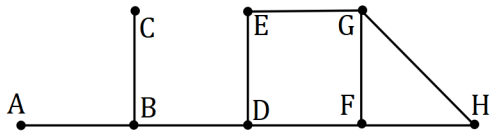
(a) Explain why P is Hamiltonian. (2 marks)

Solution
The graph contains a **cycle (or closed path)** that passes through **all vertices**.
Specific behaviours
✓ path that passes through all vertices ✓ starts and finishes at same vertex (cycle) [If the word PATH or CYCLE is not mentioned somewhere in answer need to describe a path, so also indicate no repeated vertices or edges]

(b) A single edge is to be removed from P so that it is no longer Hamiltonian. Name a suitable edge and state how many other edges you could have chosen. (2 marks)

Solution
Edge AB. 7 other edges to choose. [$AC, BD, CE, DF, EG, FH, GH$] *(NB Not inner edges BC, DE or FG)*
Specific behaviours
✓ names any edge on perimeter ✓ correct number of alternatives

(c) Draw a connected subgraph of P that has 8 vertices, 9 edges and is neither Hamiltonian nor semi-Hamiltonian. (2 marks)

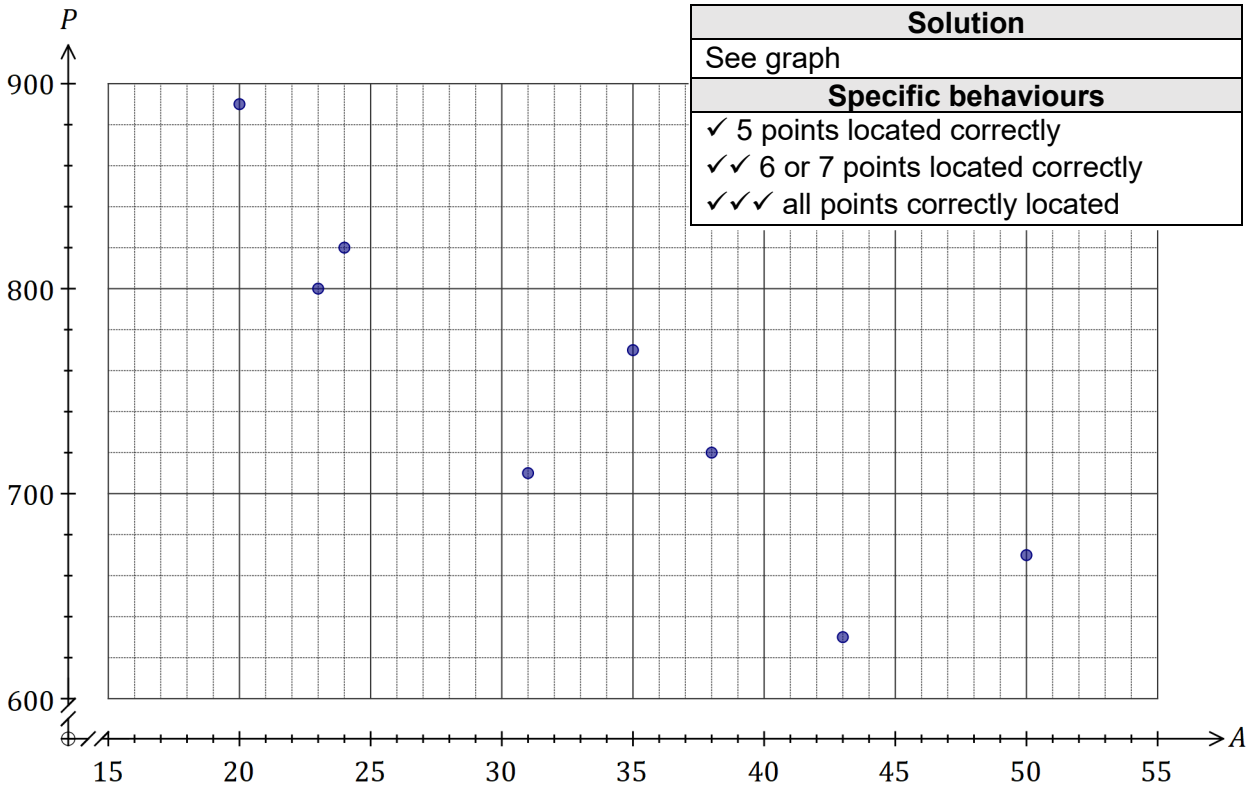
Solution

Specific behaviours
✓ removes 2 edges from P (but still connected with vertices labelled) ✓ graph is neither Hamiltonian nor semi-Hamiltonian

Question 5 **(8 marks)**

The age in years, A, for eight randomly chosen drivers was recorded, together with their car insurance premium P to the nearest ten dollars. The data is shown in the table below.

Age (Years) A	50	20	31	35	38	24	43	23
Premium ($) P	670	890	710	770	720	820	630	800

(a) Construct a scatterplot of this data on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ 5 points located correctly
✓✓ 6 or 7 points located correctly
✓✓✓ all points correctly located

(b) Use features of the scatterplot to fully describe the association that exists between age and premium. (3 marks)

Solution
There is a moderate to strong, negative, linear association between age and premium.
Specific behaviours
✓ mentions strength
✓ mentions form
✓ mentions direction

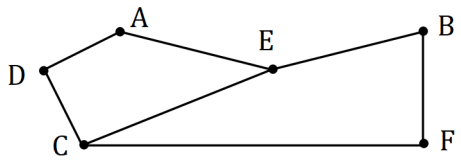
(c) A student looked at the scatterplot and claimed that getting older causes your insurance premium to decrease. Comment on this claim. (2 marks)

Solution
The claim is wrong in assuming that a causal relationship exists.
Specific behaviours
✓ claim is false
✓ association does not imply causation

See next page

Question 6 **(6 marks)**

(a) Graph G_1 is shown below.



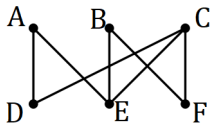
(i) Complete the adjacency matrix for G_1. (2 marks)

	A	B	C	D	E	F
A	0	0	0	1	1	0
B	0	0	0	0	1	1
C	0	0	0	1	1	1
D	1	0	1	0	0	0
E	1	1	1	0	0	0
F	0	1	1	0	0	0

Solution
See matrix
Specific behaviours
✓ two 3×3 blocks of zeroes
✓ correct matrix

(ii) Redraw G_1 to clearly show that it is bipartite. (2 marks)

Solution



Specific behaviours

✓ two distinct sets of vertices
✓ all seven edges

(b) The adjacency matrix for graph G_2 is shown below. Show that G_2 is also bipartite by listing the two distinct groups of vertices. (2 marks)

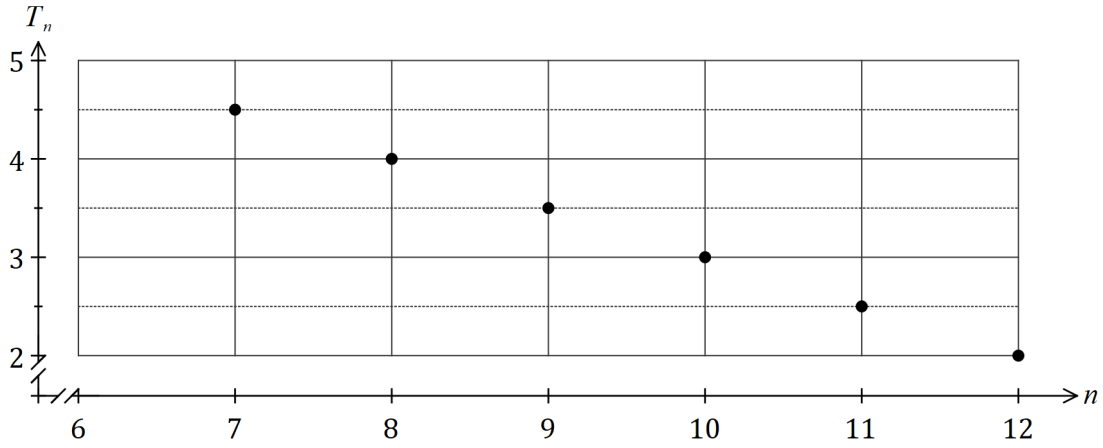
	L	M	N	P	Q	R
L	0	0	1	0	1	1
M	0	0	0	1	0	0
N	1	0	0	1	0	0
P	0	1	1	0	1	0
Q	1	0	0	1	0	0
R	1	0	0	0	0	0

Solution
One group contains $\{L, P\}$ and the other contains $\{M, N, Q, R\}$
Specific behaviours
✓ six vertices listed in two groups with only one miss placed vertex
✓ both groups correct

See next page

Question 7 **(7 marks)**

Some of the terms of a sequence are shown in the graph below.



(a) State the name given to this type of sequence and explain the feature of the graph that supports your answer. (2 marks)

Solution
Arithmetic. The points of the sequence lie in a straight line.
Specific behaviours
✓ states arithmetic ✓ uses linear nature

(b) Determine

(i) T_{13}. (1 mark)

Solution
$T_{13} = 2 - 0.5 = 1.5$
Specific behaviours
✓ correct value

(ii) T_1. (1 mark)

Solution
$T_1 = 4.5 + (6 \times 0.5) = 7.5$
Specific behaviours
✓ correct value

(c) Determine a rule for the n^{th} term of this sequence in the form $T_n = an + b$, clearly showing the value of the constant a and the value of the constant b. (2 marks)

Solution
$T_n = 7.5 + (n-1) \times (-0.5)$ $= 7.5 - 0.5n + 0.5$ $= -0.5n + 8$
Specific behaviours
✓ correctly substitutes into general term rule ✓ correctly simplifies

(d) Determine n given that $T_n = -92$. (1 mark)

Solution
$-0.5n + 8 = -92 \Rightarrow -0.5n = -100 \Rightarrow n = 200$
Specific behaviours
✓ correct value

See next page

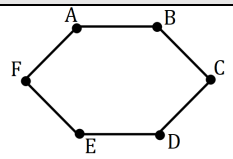
Question 8 **(7 marks)**

(a) Briefly describe how to draw a graph to show that it is planar. (1 mark)

Solution
Ensure that no two edges cross.
Specific behaviours
✓ states no edges should cross

A connected planar graph G has $2x - 2$ vertices and $3x - 6$ edges.

(b) Draw a possible graph for G when $x = 4$ that illustrates your answer to (a). (2 marks)

Solution
 MPA
Specific behaviours
✓ graph with 6 vertices and 6 edges ✓ correct possibility with 2 faces and no edges crossing

(c) Determine the number of faces of graph G in terms of x. (2 marks)

Solution
$f + v = e + 2$ $f + 2x - 2 = 3x - 6 + 2$ $f = x - 2$
Specific behaviours
✓ substitutes into Euler's relation ✓ correct expression

(d) Explain why it is not possible that

(i) $x = 2.5$. (1 mark)

Solution
$e = 3(2.5) - 6 = 1.5$, but must be a whole number of edges or $f = 2.5 - 2 = 0.5$, but must be a whole number of faces
Specific behaviours
✓ explanation using edges or faces

(ii) $x = 2$. (1 mark)

Solution
$f = 2 - 2 = 0$, but must have at least one face. or $e = 3(2) - 6 = 0$ & $v = 2(2) - 2 = 2$ so not connected
Specific behaviours
✓ explanation using no faces or if says no edges need to also state not connected due to 2 vertices

End of questions

Supplementary page

Question number: _____