Papers written by Australian Maths Software

SEMESTER TWO

YEAR 11

MATHEMATICS METHODS
Units 1 & 2
2016

REVISION 1

Section Two

(Calculator–assumed)

Name: ________________________________

Teacher: ________________________________

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work: 10 minutes

Working time for section: 100 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate
Standard items*:* pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: drawing instruments, templates, notes on up to two unfolded sheets of A4 paper, and up to three calculators approved for use in examinations.

IMPORTANT NOTE TO CANDIDATES
No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor before reading any further.

To be provided by the supervisor
Question/answer booklet for Section Two.
Formula sheet retained from Section One.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	7	7	50	52	35
Section Two Calculator—assumed	**12**	**12**	**100**	**98**	**65**
Total marks				150	

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answer in the Question/Answer booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula Sheet is not to be handed in with your Question/Answer booklet.

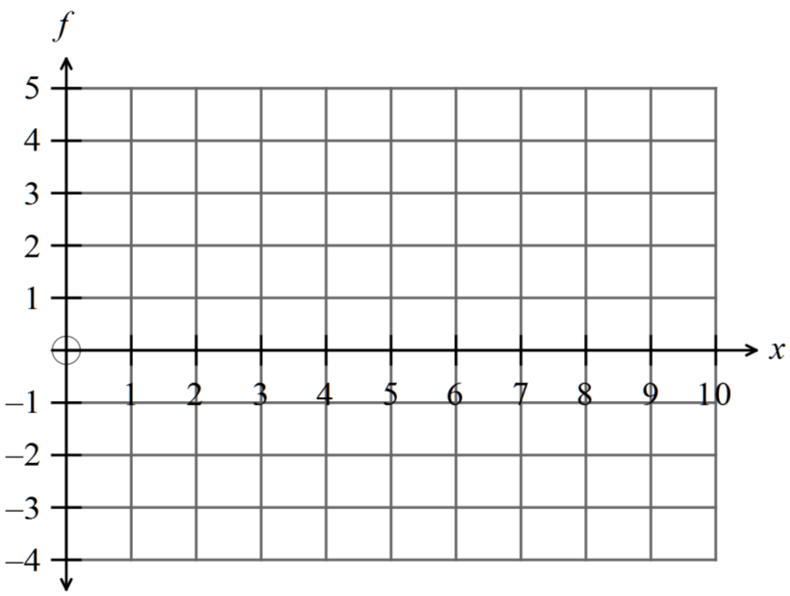
Section Two: Calculator–assumed **98 marks**

This section has **twelve (12)** questions. Attempt **all** questions.
Working time: 100 minutes

Question 8 **(3 marks)**

Graph a function that has the following properties on the set of axes below. (3)

x	0	3	5	7	9
f		5	1	-3	
f'	+	0	-	0	+



Question 9 **(9 marks)**

The number of cats in a suburb is increasing. In 2010 there were only 40 cats in the suburb. A study was made and the number of cats was found to follow the rule $\mathrm{P}(\mathrm{t}) = 40(1.05)^{\mathrm{t}}$ taken from 2010 when $t = 0.$

(a) Write down the expected population of cats as a recursive statement in terms of n where n represents successive years starting in 2010. (2)

(b) Determine the expected population of cats in 2011 and in 2015, (2)

The number of cats in 2015 was counted to be 45.

(c) Comment on the suitability of the model. (1)

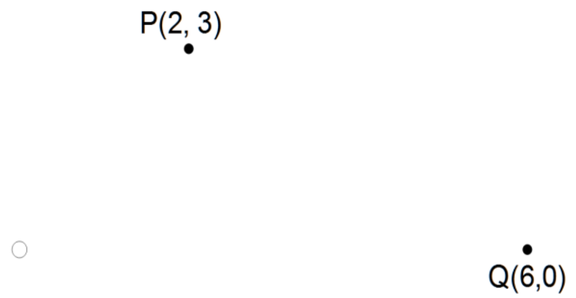
(d) Given the two counts of the population was correct find a new model for the population of the cats that is still exponential. (2)

It was decided if the number of cats exceeded 60, then people living in the suburb would have to have their cats micro-chipped and limited to two per household.

(e) Using the new model, determine if this action would need to be done by 2020. (2)

Question 10 **(5 marks)**

Consider the points in the diagram below



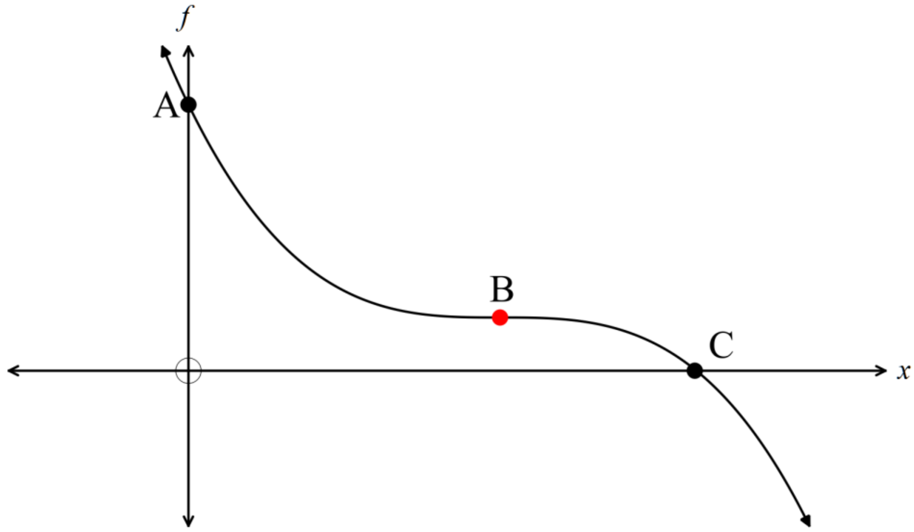
(a) Show that point Q(6, 0) is exactly 5 units away from P. (1)

(b) Find the point R such that QR is perpendicular to PQ, and is exactly 5 units from Q and has a positive y ordinate. (2)

(c) Determine the coordinates of S given PQRS is a square. (2)

Question 11 **(19 marks)**

(a) The diagram below shows the function $f(x) = -(x-3)^3 + 2 = -x^3 + 9x^2 - 27x + 29$



(i) Find the coordinates of point A. (1)

(ii) Find the equation of the normal at point B where the gradient is equal to zero. (3)

(iii) Use your calculator to determine the coordinates of point C. (1)

(b) The position of a particle is given by $x = t^3 - 9t$ where x is in metres and t is in seconds for $t \geq 0$.

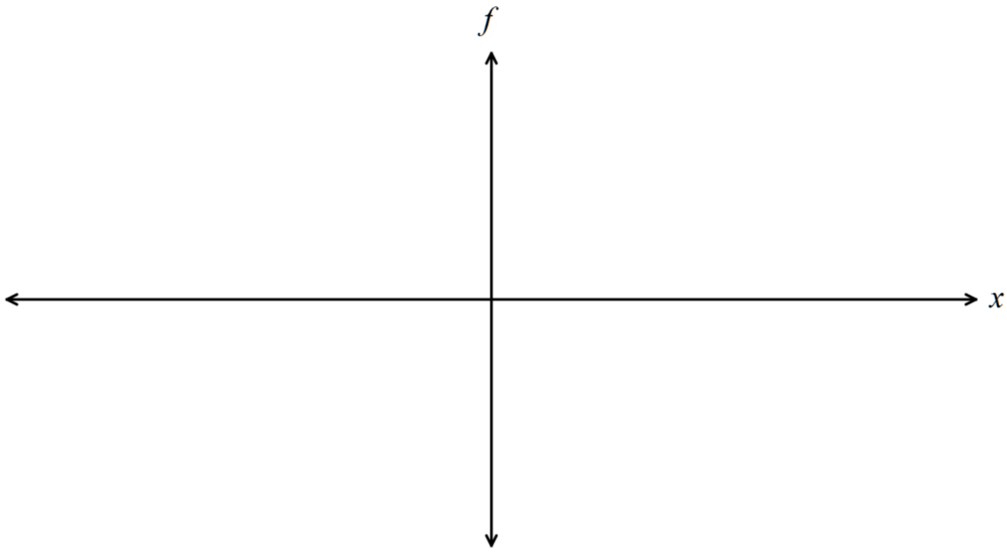
Find

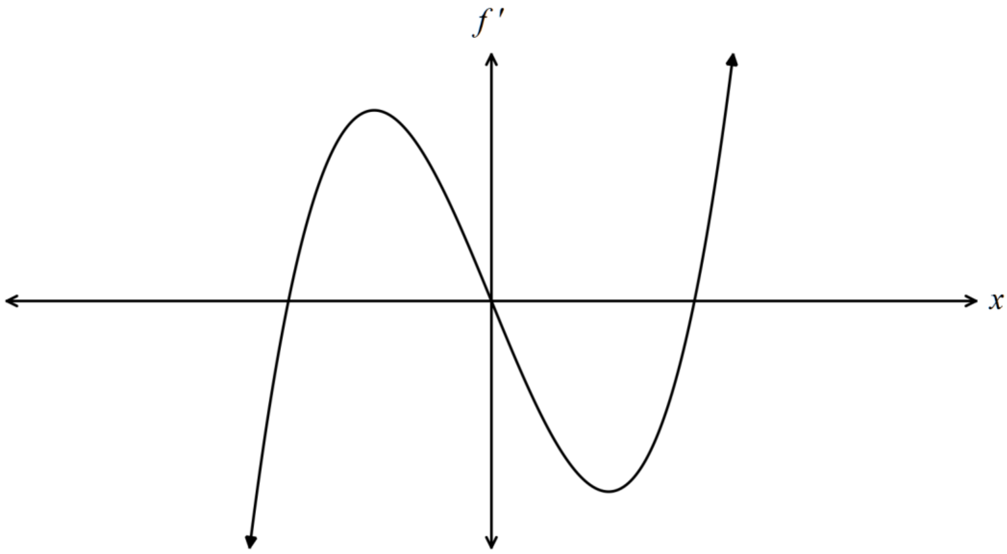
(i) the expression for velocity. (1)

(ii) the position and velocity at $t = 3$. (2)

(iii) Determine when the particle changes direction and the position of the particle at that time. (4)

(c) (i) Given the graph of the function $y = f\,'(x)$, sketch the function $y = f(t)$. (4)

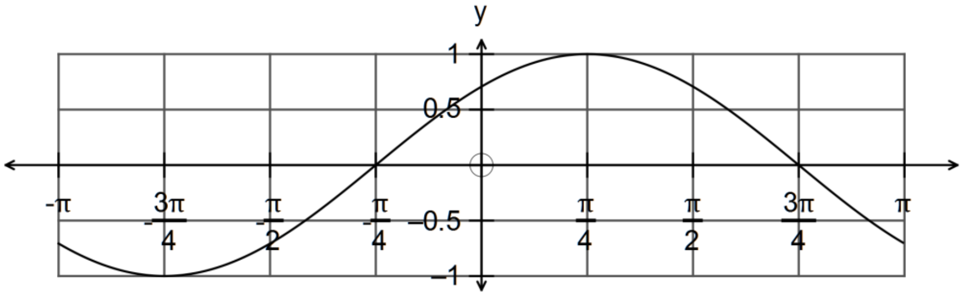




(ii) Comment on the relationship between the two functions as shown on the graphs. (3)

Question 12 **(14 marks)**

(a) Determine the equation for the trigonometric graph below. (2)



(b) Solve for x

(i) $cos(3x) = -1$ for $-90^0 \leq x \leq 90^0$ (2)

(ii) $tan\left(x + \frac{\pi}{6}\right) = \sqrt{3}$ for $0 \leq x \leq \frac{\pi}{2}$ (2)

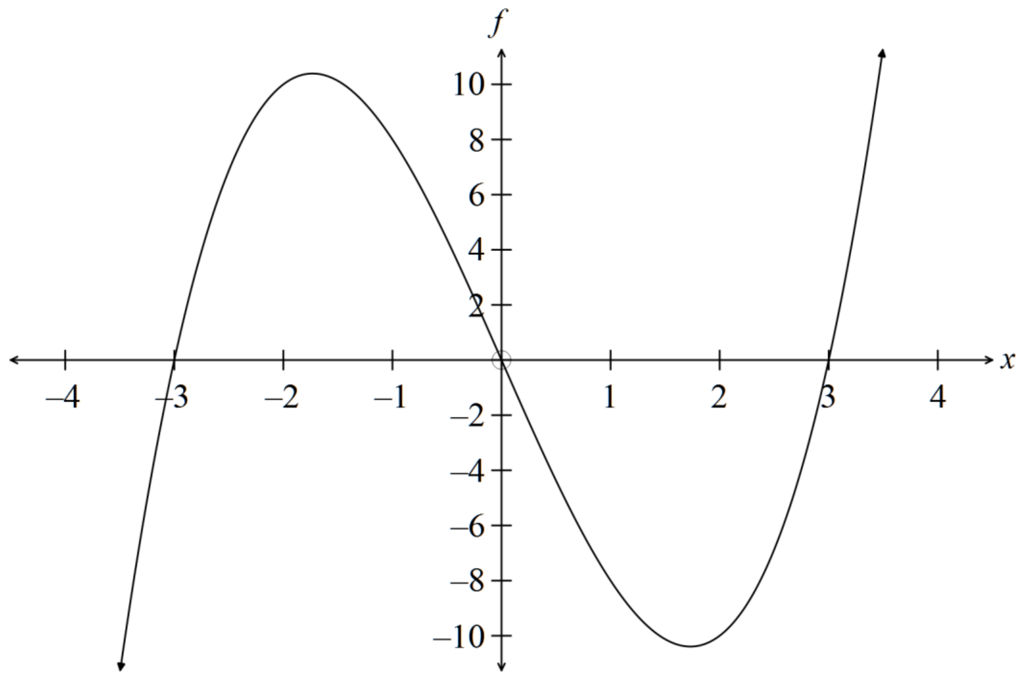
(c) A circle has a radius of 8 cm.
Find the area of a sector that has an arc length of 11 cm. (1)

(d) Given $cos(x+y)=cos(x)cos(y)-sin(x)sin(y)$ show how to develop the formula $cos\left(x-\frac{\pi}{2}\right)=sin(x)$. (2)

(e) ABC is a triangle with AC = 10 cm, BC = 6 cm and angle BAC is 30^{o}.
Find the length of BA. (4)

Question 13 **(5 marks)**

Consider the function $f\left(x\right)=x^3-9x$ as shown below.



(a) Find the equation of the tangent at the point (3, 0). (3)

(b) Show that the tangent at (-3, 0) is parallel to the tangent at (3, 0). (2)

Question 14 **(8 marks)**

(a) Find the derivatives of the functions below

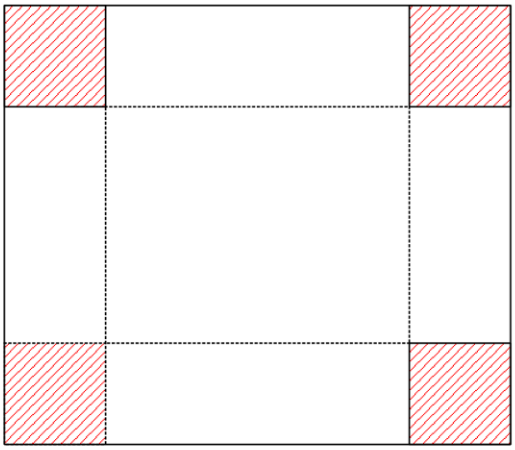
(i) $g(x) = 4 - x^2 + 3x^5$ (2)

(ii) $f(x) = (2x+1)^2$ (3)

(b) Solve the equation $2^{x+1} = 5^{1-x}$ (3)

Question 15 **(6 marks)**

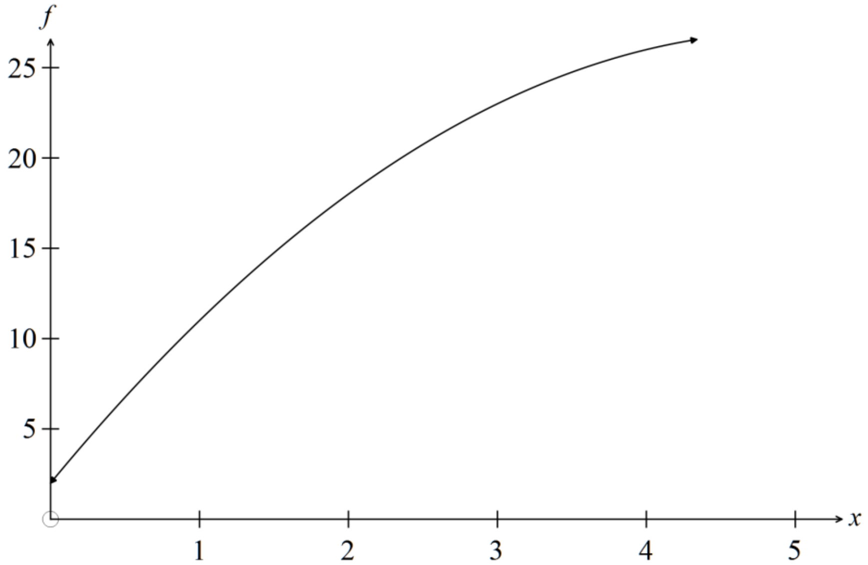
A <u>square</u> piece of cardboard with sides equal to 80 cm, is to be made into an open topped box by cutting out a little square from each corner and folding up the sides.



Find the dimensions of the square so that the volume of the box is a maximised. (6)

Question 16 **(6 marks)**

Consider the function $f(x) = 2 + 10x - x^2$ as shown in the diagram below.



(a) Determine the average rate of change of the function on the interval $1 \leq x \leq 4$. (3)

(b) Determine the point in the interval where the instantaneous rate of change is equal to the average rate of change of the function. (2)

(c) Given $\frac{\delta y}{\delta x}$ can be used to represent the average rate of change on an interval, explain why $\frac{\delta y}{\delta x}$ is greater on the interval $1 \leq x \leq 2$ than on the interval $3 \leq x \leq 4$. (1)

Question 17 **(9 marks)**

Jenny is planning a hike on the Bibbulmun Track. She plans to take it easy at the beginning and expects to be able to walk two kilometres more each day until she reaches a rate of 28 km per day. She then plans to continue at 28 km a day until she reaches her destination.

The total distance she plans to walk is 250 kms.
Jenny plans to start her walk just walking 10 km per day.

(a) On what day will Jenny first walk 28 km? (3)

(b) How long will it take Jenny to reach her destination? (4)

Jenny's friend Steve plans to walk the same track at the same time, but he plans to walk a steady 20 km each day

(c) Who arrives first? (2)

Question 18 **(5 marks)**

Given the definition $f\,'(x)=\lim\limits_{h\to 0}\left(\dfrac{f\left(x+h\right)-f(x)}{\left(x+h\right)-x}\right)$, determine the derivative of the function $f(x)=3x^2$ using first principles. (5)

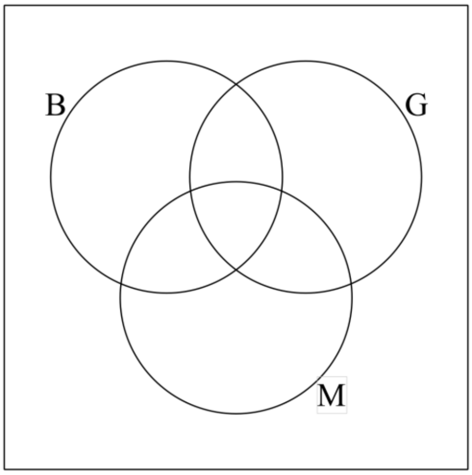
Question 19 **(9 marks)**

(a) A year group of 70 students of whom 30 are boys, were checked to see eye colour and hair colour.
Let B represent those with “black hair”, G represent those with “green eyes” and M represents the males in the group.

(i) Place the data

$$n(M) = 18$$
$$n(G) = 15$$
$$n(B) = 30$$
$$n(M \cap G \cap B) = 3$$
$$n(G \cap B) = 6$$
$$n(B \cap M) = 14$$
$$n(\overline{M} \cap G \cap \overline{B}) = 5$$

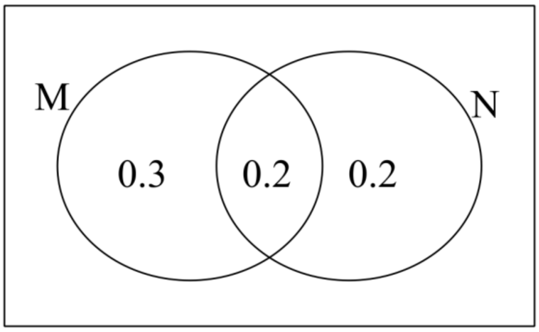
on the Venn diagram below. (3)



(ii) Find $n(G \cap M)$ (1)

(iii) Find $n(\overline{M \cup B \cup G})$ (1)

(b) Given the data in the Venn diagram below



Determine with reasons whether

(i) M and N are independent. (3)

(ii) M and N are mutually exclusive. (1)

END OF SECTION TWO