Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

MATHEMATICS

3C/3D

Calculator-free

WACE Examination 2015

Marking Key

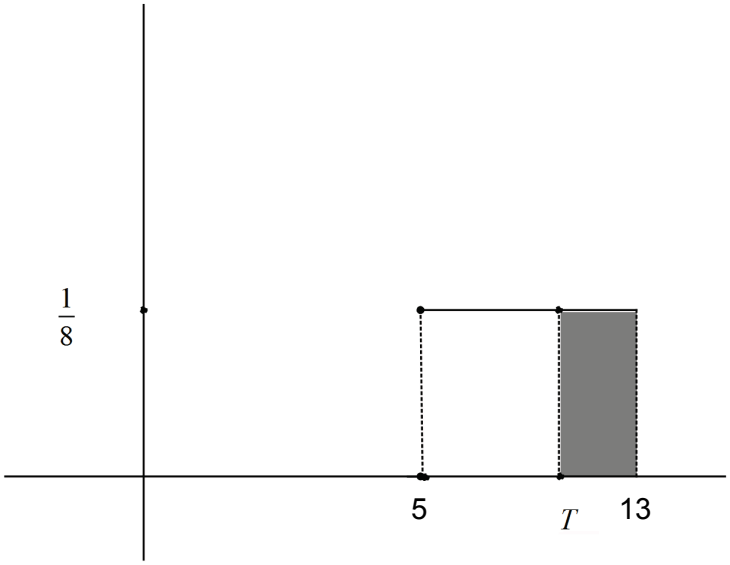
Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

2015/101884

Section One: Calculator Free **$33\frac{1}{3}$% (50 marks)**

Question 1 **(6 marks)**

(a) Jim has observed that 60% of the time the train arrives before T minutes. Determine the value of T. (3 marks)

Solution
 $\frac{1}{8}(13-T)=0.4$ $T=9.8$ Waiting time of 9.8 mins
Specific behaviours
✓ uses a constant value of $\frac{1}{8}$ with pdf ✓ uses areas of rectangles to determine cutoff point ✓ determines 9.8 minutes

(b) Jim has been waiting at the station for eight minutes. What is the probability that he waits less than 10 minutes? (3 marks)

Solution
$\frac{\frac{1}{8}(10-8)}{\frac{1}{8}(13-8)}=\frac{2}{5}$
Specific behaviours
✓ uses conditional probability with areas ✓ determines correct denominator ✓ determines correct numerator

Question 2 **(6 marks)**

(a) Write the above information as three simultaneous equations in terms of x, y and z. (2 marks)

Solution
$2x+4y+4z=72\ldots eq1$
$4x+4y+6z=106\ldots eq2$
$10000x+30000y+50000z=680000 \quad \Rightarrow x+3y+5z=68\ldots eq3$
Specific behaviours
✓ states two correct equations ✓ states three correct equations

(b) Solve the simultaneous equations from part (a) to determine the number of each type of car that can be produced. (4 marks)

Solution
$2x+4y+4z=72\ldots eq1$
$4x+4y+6z=106\ldots eq2$
$10000x+30000y+50000z=680000 \quad \Rightarrow x+3y+5z=68\ldots eq3$
$eq2-2eq1 \Rightarrow -4y-2z=-38$
$eq1-2eq3 \Rightarrow -2y-6z=-64$
$10z=90$
$z=9$
$y=5$
$x=8$
Specific behaviours
✓ eliminates one variable in an equation ✓ determines two equations with one variable eliminated ✓ solves for one variable ✓ solves for all three variables

Question 3 **(5 marks)**

(a) Determine the derivative of e^{e^x}. (2 marks)

Solution
$\frac{d}{dx}\left(e^{e^x}\right) = \frac{d}{du}\left(e^u\right) \times \frac{d}{dx}\left(e^x\right)$ $= e^u \times e^x$ $= e^{e^x} \times e^x$ $= e^{e^x + x}$
Specific behaviours
✓ applies the chain rule correctly ✓ determines correct derivative of e^x

(b) Use your result from part (a) to determine $\int_0^1 e^{e^x + x}\, dx$. (3 marks)

Solution
$\int_0^1 e^{e^x + x}\, dx = \left[e^{e^x} \right]_0^1$ $= e^{e^1} - e^{e^0}$ $= e^e - e^1$ $= e^e - e$
Specific behaviours
✓ determines the correct expression for the integral ✓ substitutes correct values for the two limits for x ✓ calculates the correct value for the integral

Question 4 **(5 marks)**

Solve the following inequality.

$$x - 2 \le \frac{4 - 2x}{x + 1}$$

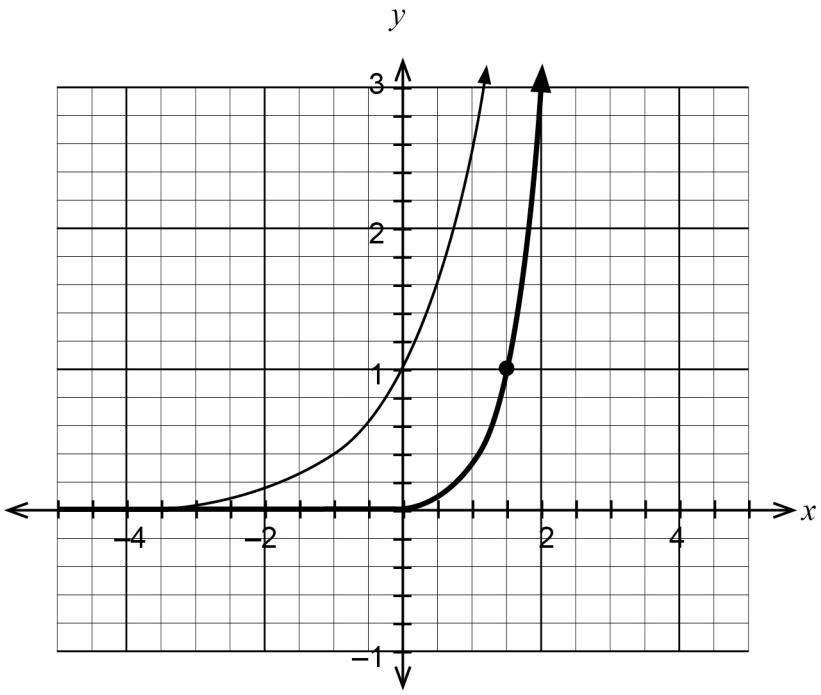
Solution
Multiply both sides by $(x + 1)$ For $x > -1$ (Note x cannot equal -1) $(x+1)(x-2) \le 4 - 2x$ $x^2 - x - 2 \le 4 - 2x$ $x^2 + x - 6 \le 0$ $(x+3)(x-2) \le 0$ $-3 \le x \le 2$ *but* $x > -1$ $-1 < x \le 2$ For $x < -1$ $(x+3)(x-2) \ge 0$ $x \le -3$ Summary $x \le -3,\ -1 < x \le 2$
Specific behaviours
✓ considers case where $x > -1$ ✓ considers case where $x < -1$ ✓ solves for interval $-1 < x \le 2$ ✓ solves for interval $x \le -3$ ✓ excludes $x = -1$

Question 5 **(8 marks)**

(a) Determine the function $f \circ g(x)$ and state its domain and range. (3 marks)

Solution
$f \circ g(x) = e^{2x-3}$ Domain: $x \in R$ Range: $y > 0$
Specific behaviours
✓ states the correct rule for $f \circ g(x)$ ✓ states the correct domain ✓ states the correct range

(b) Sketch $f \circ g(x)$ on the axes above. (2 marks)

Solution
 Function has been translated three units to right and then dilated by factor 0.5 parallel to x axis The point (0,1) has now been translated to (1.5,1)
Specific behaviours
✓ curve translated to the right ✓ shows one well defined point

The function h is defined as $h(x) = -x + a, x \in R$ where a is a constant integer.

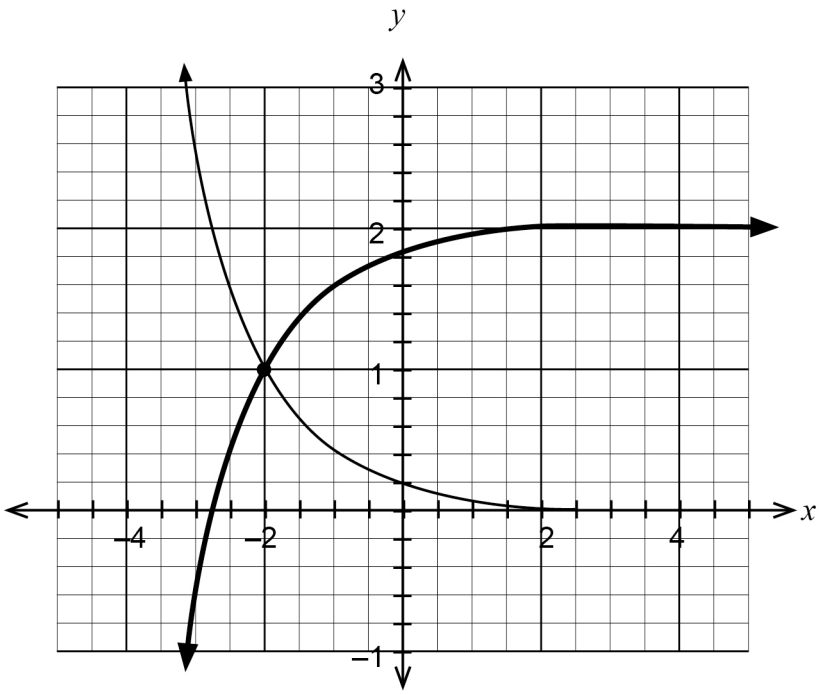
The function $f \circ h(x)$ is drawn below and includes the point $(-2, 1)$.

(c) Determine the value of a. (1 mark)

Solution
$f \circ h(x)$ is a translation of $f(x)$ two units to the right and then a reflection in y axis, so $a = -2$
Specific behaviours
✓ states correct value for a

(d) On the axes above, sketch the function $y = -f \circ h(x) + 2$. (2 marks)

Solution



Specific behaviours

✓ reflection in the x axis
✓ translation of two units up

Question 6 **(5 marks)**

(a) Determine a function $f(x)$ that satisfies all of the above properties. (3 marks)

(Hint: consider the derivative of $f(x)$.)

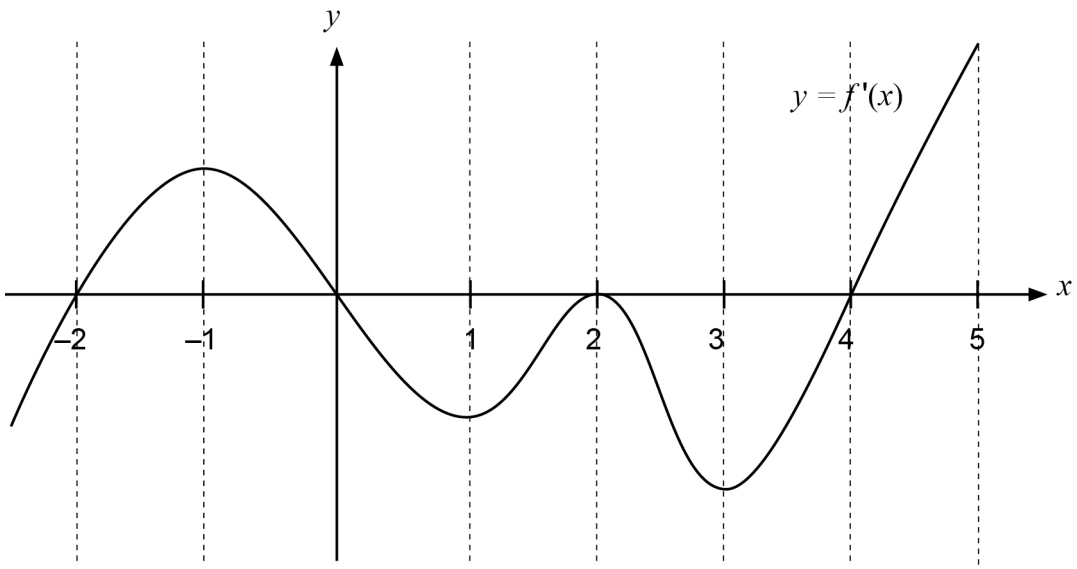
Solution
$f'(x) = \frac{d}{dx}\int_{-\infty}^{x} f(t)\,dt = f(x)$ $f(x) = A_0 e^x$ $f(0) = 1 \Rightarrow A_0 = 1$ $f(x) = e^x$
Specific behaviours
✓ obtains $f'(x)$ ✓ identifies differential equation $y' = ky$ and states value of constant ✓ states $f(x)$

(b) Is the function $f(x)$ above unique? Justify your answer. (2 marks)

Solution
The function is unique as there is only one solution to the differential equation
Specific behaviours
✓ states that function is unique ✓ states that there is only one solution to differential equation

Question 7 **(9 marks)**

The figure below shows the graph of the derivative f' of a function f.



(a) For what values of x does f have a local maximum or minimum? (2 marks)

Solution
f has a local maximum at $x = 0$ f has a local minimum at $x = -2$ and $x = 4$
Specific behaviours
✓ determines local minimums ✓ determines local maximum and does **not** state $x = 2$ as a local point

(b) For what values of x does f have an inflection point? (2 marks)

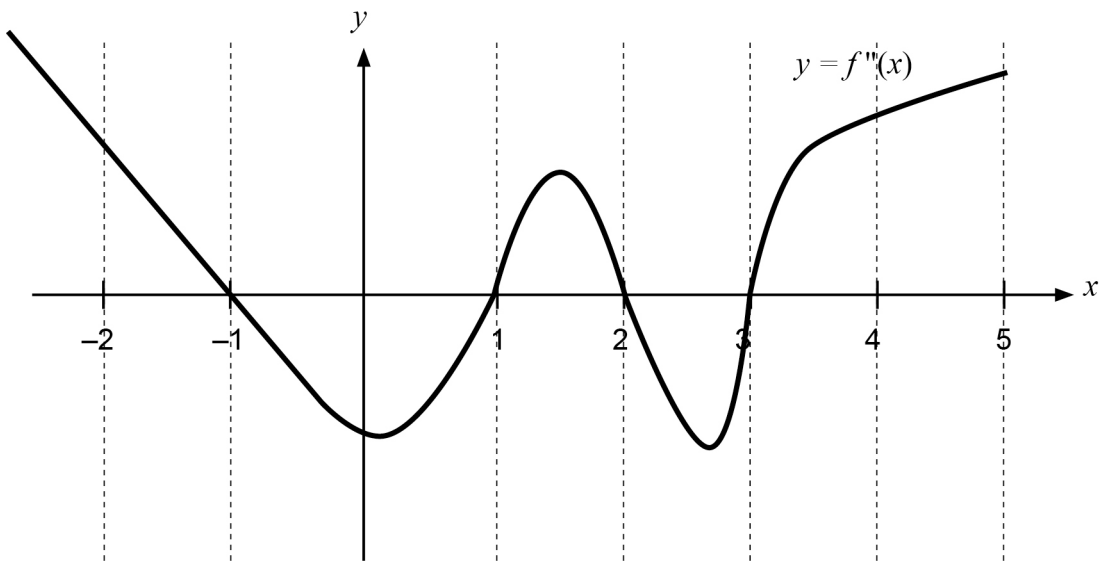
Solution
f has inflection points at $x = -1$, $x = 1$, $x = 2$ and $x = 3$
Specific behaviours
✓ determines three inflection points ✓ determines all inflection points

(c) Does f have a horizontal point of inflection? Explain. (2 marks)

Solution
At $x = 2$ we have a stationary point $f'(x) = 0$ with the same sign for $f'(x)$ either side Alternative solution: $f'(x) = 0 = f''(x)$ which in the context presented implies a horizontal point of inflection
Specific behaviours
✓ states $x = 2$ only ✓ states reasoning

(d) On the axis below, sketch the graph of f''. (3 marks)

Solution

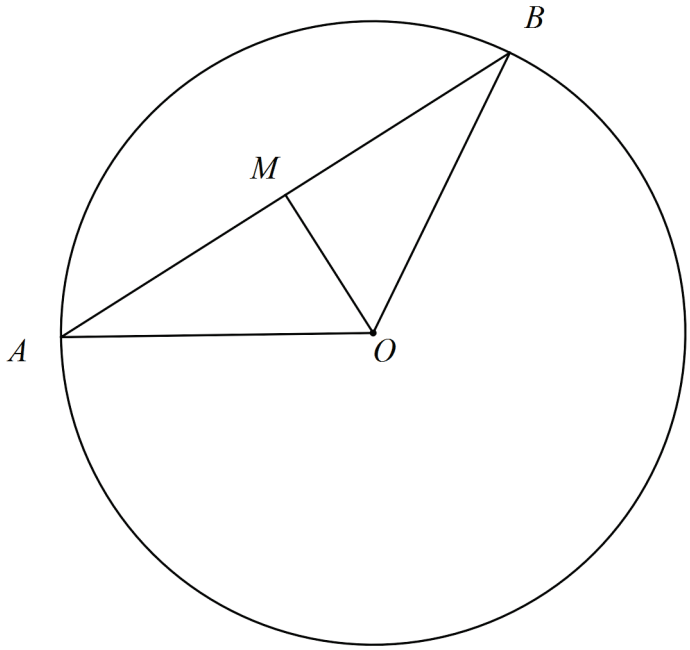


Specific behaviours

✓ shows all four x intercepts
✓ shows three, out of four, turning points in correct intervals
✓ shows a negative y intercept

Question 8 **(6 marks)**

Consider the chord $\overline{AB}$ in a circle with centre $O,$ as in the diagram below.

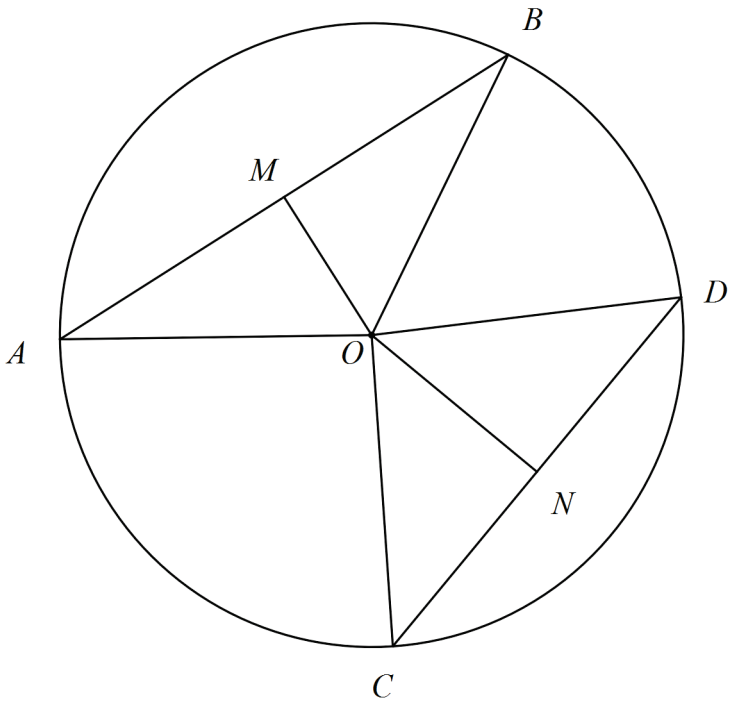


(a) Prove that if M is the midpoint of the chord $\overline{AB}$, then the line segment $\overline{OM}$ is perpendicular to $\overline{AB}$. (3 marks)

Solution
$\Delta AMO \cong \Delta BMO$ due to SSS test S: $\overline{OA} = \overline{OB}$ as they are both radii S: $\overline{AM} = \overline{MB}$ as M is the midpoint S: $\overline{OM}$ is a common side to both triangles $\angle AMO$ corresponds to $\angle BMO$ and are supplementary, therefore both right angles
Specific behaviours
✓ states congruent triangles ✓ shows congruence through SSS test ✓ shows that $\angle AMO$ and $\angle BMO$ are right angles

(b) Hence, or otherwise, prove that if two chords of a circle are of the same length, then both chords are equidistant from the centre. (3 marks)

Solution



Consider chord $\overline{CD}$ of equal length to $\overline{AB}$ with N, M midpoints

$\Delta AMO \cong \Delta CNO$ due to RHS test

R: $\overline{OM}, \overline{ON}$ perpendicular to chords as from Part (a) above

H: $\overline{OA} = \overline{OC}$ as both are radii

S: $\overline{AM} = \overline{CN}$ as they are half chords of equal length

Side-lengths $\overline{OM}$, $\overline{ON}$ are corresponding sides hence equal, therefore chords are equidistant from centre

Specific behaviours

✓ states congruent triangles
✓ shows test for congruence
✓ shows that midpoints of both chords are equidistant from centre

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