Papers written by
Australian Maths
Software

REVISION 3

2016

MATHEMATICS
METHODS

Units 1 & 2

Semester 2
SOLUTIONS

SECTION 1 – Calculator-free

Question 1 **(3 marks)**

A x intercept ✓

B y intercept; local maximum point; turning point ✓

C Local minimum point; turning point ✓

Question 2 **(29 marks)**

(a) $81^{\frac{3}{4}}+10\left(0.001^{\frac{1}{3}}\right)-\sqrt{\frac{1}{16^{-1}}+\frac{1}{9^{-1}}}$

$=\left(3^4\right)^{\frac{3}{4}}+10\left(10^{-3\times\frac{1}{3}}\right)-\sqrt{16+9}$

$=3^3+10^{1-1}-\sqrt{25}$ ✓✓✓

$=27+1-5$

$=23$ ✓

(b) (i) $\left(\frac{1}{3}\right)^{2x+1}=9^3$

$3^{-(2x+1)}=\left(3^2\right)^3$ ✓

$-2x-1=6$ ✓

$2x=-7$

$x=-3.5$ ✓

(ii) $25^x+5^3=6\times5^{x+1}$

$5^{2x}+125=6\times5\times5^x$

Let $y=5^x$

$y^2-30y+125=0$ ✓

$(y-25)(y-5)=0$

$y=25$ or $y=5$

$5^x=5^2$ or $5^x=5$

$x=2$ or $x=1$
✓ ✓

(iii) $x^3+1=x^2+x$

$x^3-x^2+1-x=0$

$x^2(x-1)-1(x-1)=0$ ✓

$(x-1)(x^2-1)=0$

$(x-1)^2(x+1)=0$

$x=1$ (twice) or $x=-1$

✓ ✓

(b) $C=100(0.7)^t$

$35=100(0.7)^t$ ✓

$t=2.94$ ✓

Relief of 2 hours 57 minutes ✓

Question 3 **(6 marks)**

(a) $A_n=\dfrac{1+n}{2}$

$A_1=\dfrac{1+1}{2}=1,\quad A_2=\dfrac{1+2}{2}=1.5,\quad A_3=\dfrac{1+3}{2}=2$ ✓

AP

$A_{n+1}=A_n+0.5,\quad A_1=1$

✓ ✓

(b) (i) $T_{n+1}=2T_n$ with $T_1=6$

$6, 12, 24, 48$ ✓

(ii) GP a = 6, r = 2

$T_n=ar^{n-1}$

$T_n=6\times 2^{n-1}=3\times 2\times 2^{n-1}$ ✓

$T_n=3\times 2^n$ ✓

Question 4 **(4 marks)**

$f(x) = 1 - 4x$

$f(x+h) = \underline{1 - 4(x+h)}$ ✓

By definition

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{(x+h) - x}\right)$$

and

$f(x+h) - f(x) = \underline{\cancel{1} - 4(\cancel{x} + h) - (\cancel{1} - \cancel{4x})}$ ✓

$$\frac{f(x+h) - f(x)}{h} = \frac{-4h}{h}$$ ✓

$= \underline{-4}$

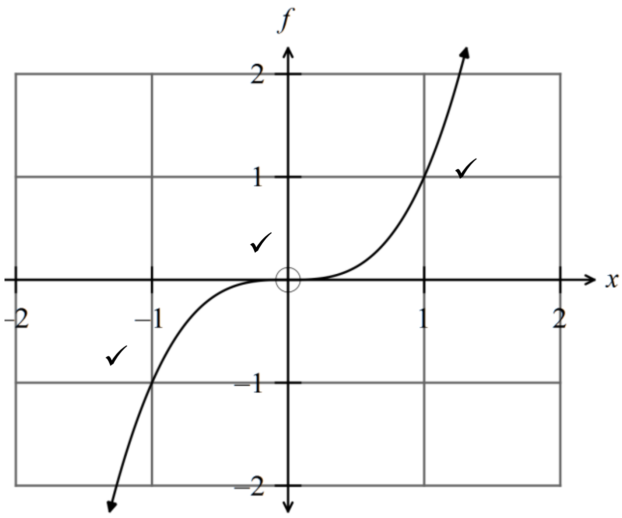
Therefore

$f'(x) = \underline{\lim_{h \to 0}(-4)}$ ✓

$\therefore \; f'(x) = \underline{-4}$

Question 5 **(10 marks)**

(a) (i)



✓ general shape

(ii) $y = x^3$ ✓✓

(b) (i) $m = 0$ ✓ $P(0,-1)$ ✓

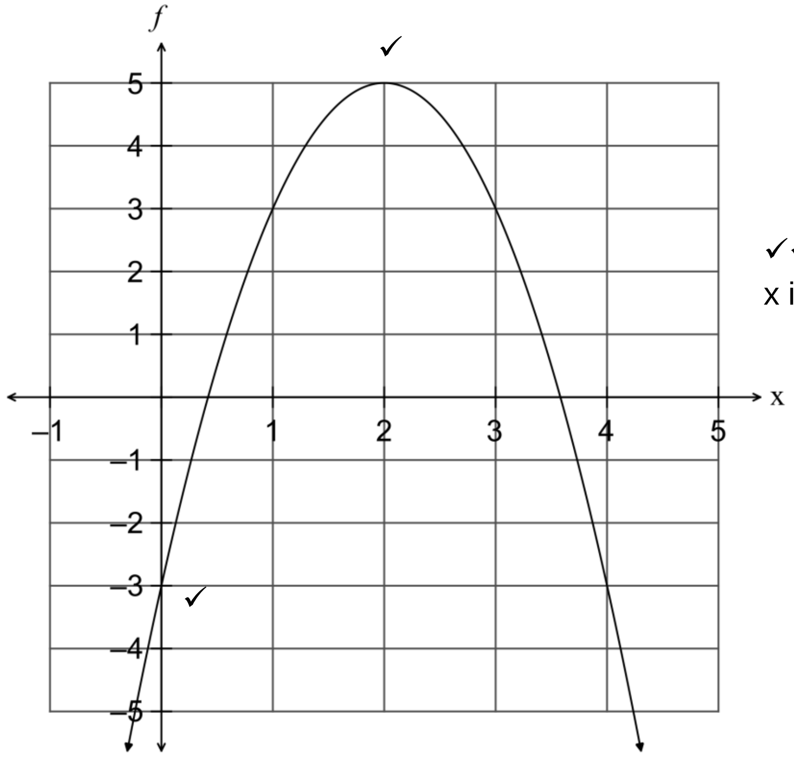
(ii) $(2,1)$ ✓ *or* $(-2,1)$

(iii) B is an x intercept. ✓

Question 6 **(13 marks)**

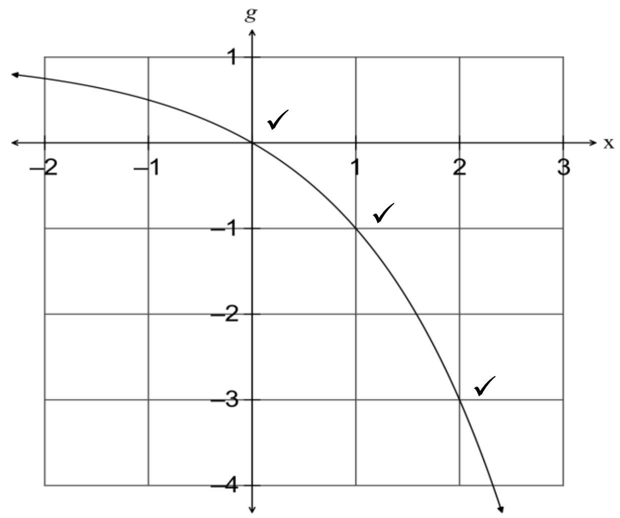
(a) $y = (x-1)^2(x+1)$ ✓✓ ✓

(b) $f(x) = -2(x-2)^2 + 5$



✓✓ concavity, x intercepts

(c) $g(x) = 1 - 2^x$



(d) $y = 3^{x+1}$ ✓✓✓

SECTION 2 – Calculator-assumed

Question 7 **(10 marks)**

(a) 2017 $\$35\,000 \times 1.05 = \$36\,750$ ✓

2018 $\$35\,000 \times 1.05^2 = \$38\,587.50$ ✓

(b) A GP with a = 35 000, r =1.05 ✓✓

(c) $35\,000 + 35\,000(1.05) + 35\,000(1.05)^2 + ... + 35\,000(1.05)^{10}$ ✓

$= 35\,000\left(\dfrac{(1-1.05\wedge 10)}{1-1.05}\right)$ ✓

$= \$440\,226.24$ ✓

(d) $401235.78 = 35\,000\left(\dfrac{\left(1-(1+r)^{10}\right)}{1-(1+r)}\right)$ ✓✓

$r = 3\%$ ✓

Question 8 **(13 marks)**

(a) (i) $S_n = \dfrac{n}{2}\left(2a + (n-1)d\right)$

$S_{52} = \dfrac{52}{2}\left(2 \times 10 + 51 \times 0.5\right)$ ✓✓

$S_{52} = \$1183$ ✓

(ii) $n = 7 \times 52 + 1$

$n = 365$ ✓

$T_{365} = 10 + 364 \times 0.5$ ✓

$T_{365} = 192$ ✓

(iii) $a = \$192, d = 0.50, n = 52$ ✓

$S_n = \dfrac{n}{2}\left(2a + (n-1)d\right)$

$S_{52} = \dfrac{52}{2}\left(2 \times 192 + 51 \times 0.5\right)$ ✓

$S_{52} = \$10\,645$ ✓

(b) $1-\frac{1}{\sqrt{2}}+\frac{1}{2}-\frac{1}{2\sqrt{2}}+........$

$S_\infty=\frac{a}{1-r},\quad a=1,\ r=-\frac{1}{\sqrt{2}}$ ✓

$S_\infty=\frac{1}{1-\left(-\frac{1}{\sqrt{2}}\right)}$ ✓

$=\frac{1}{\frac{\sqrt{2}+1}{\sqrt{2}}}$

$=\frac{\sqrt{2}}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}$ ✓

$=\sqrt{2}\,\frac{\left(\sqrt{2}-1\right)}{2-1}$

$S_\infty=\sqrt{2}\left(\sqrt{2}-1\right)$ ✓

Question 9 **(7 marks)**

$m_{AB}=\frac{3}{3}=1$

$m_{DC}=\frac{3}{4}$ ✓

$\therefore\ AB$ is not parallel to DC, so ABCD is not a parallelogram, i.e. not a square, rectangle or rhombus. ✓✓

$AB=\sqrt{(-3)^2+(-3)^2}=\sqrt{18}$

$AD=\sqrt{(-3)^2+3^2}=\sqrt{18}$ ✓

$\therefore\ AB=AD$ ✓

$BC=\sqrt{(-4)^2+3^2}=5$

$DC=\sqrt{(-4)^2+(-3)^2}=5$

$\therefore BC=DC$ ✓

$\therefore\ ABCD$ is a kite. ✓

Question 10 **(8 marks)**

(a) $y = x^4 - 16x^2$.

$\frac{dy}{dx} = 4x^3 - 32x$

$If \ \frac{dy}{dx} = 0, \quad 0 = 4x^3 - 32x$

$0 = 4x\left(x^2 - 8\right)$

$x = 0, \ \ x = \pm 2\sqrt{2}$

$(0,0), \left(2\sqrt{2}, -64\right), \left(-2\sqrt{2}, 64\right)$

✓ ✓ ✓

(b) $At \ \ x = 2, \ \frac{dy}{dx} = 4x^3 - 32x$

$\frac{dy}{dx} = -32$

$y = -32x + c$

$(2, -48) \quad -48 = -64 + c$

$c = 16$

$y = -32x + 16$

✓ ✓

(c) $\left(0.83, -10.51\right), \ \left(-4.83, 170.51\right)$

✓ ✓

Question 11 **(14 marks)**

(a) (i) $x = t^2 - 4t + 4$ for $t \geq 0$

$At \ \ t = 0, \ \ x = 4$ ✓

(ii) $\frac{dx}{dt} = 2t - 4$ ✓

(iii) $64 = t^2 - 4t + 4$

$t^2 - 4t - 60 = 0$

$(t - 10)(t + 6) = 0$ ✓

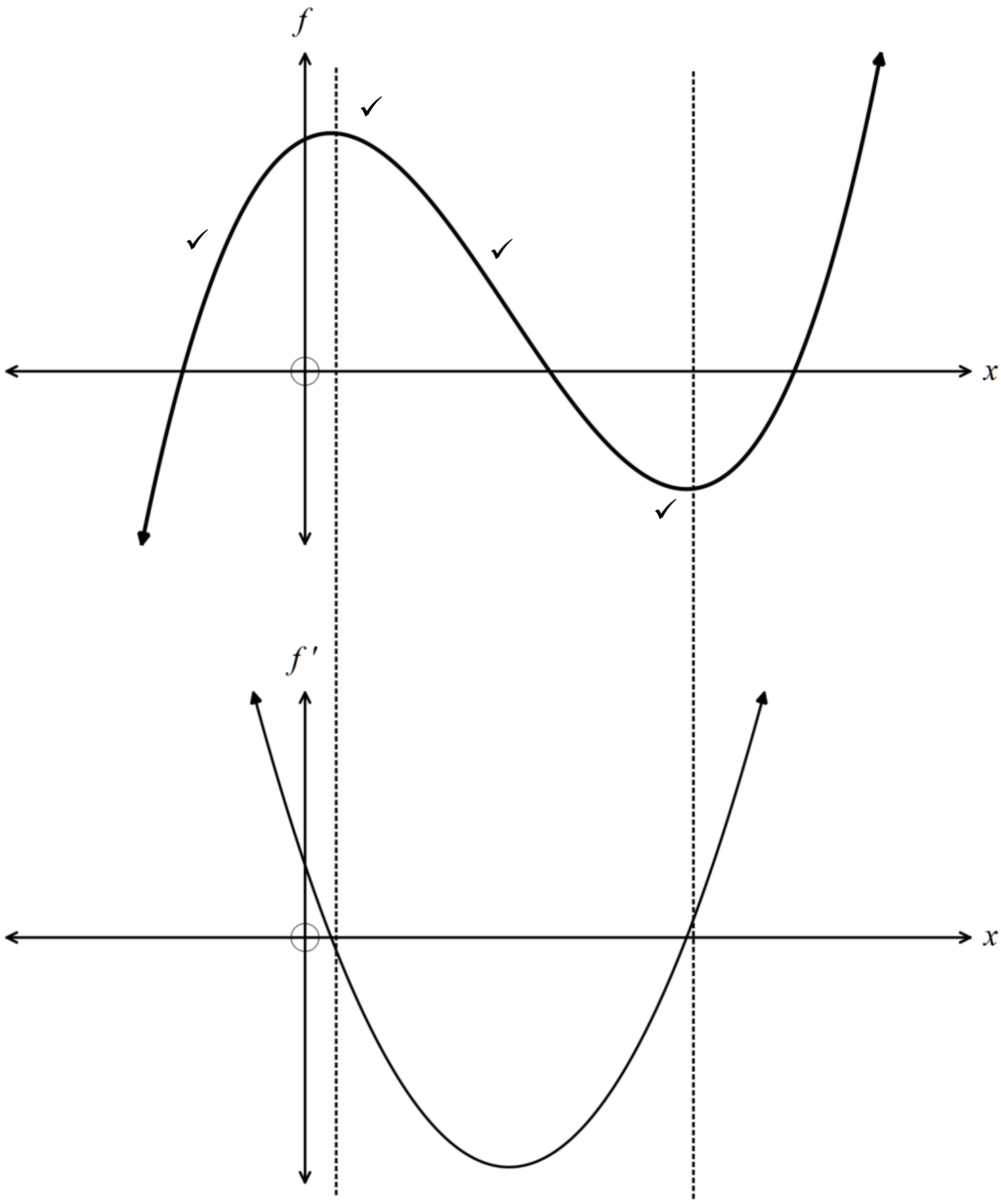
$t = 10 \ \ or \ \ t = -6 \ \ but \ \ t \geq 0$ ✓

$At \ t = 10, \ v = 16\,ms^{-1}$ ✓

(iv) $At \ \ v = 2, \ 2 = 2t - 4 \qquad t = 3$ ✓

$At \ \ t = 3, \ \ x = 9 - 12 + 4 = 1$ ✓

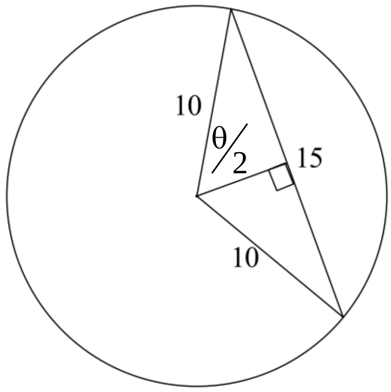
(b) (i)

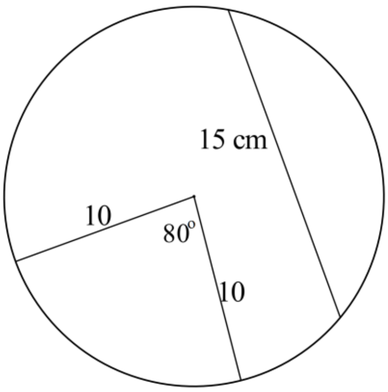


(ii) There is a turning point on f where f '(x) = 0, i.e. there are two turning points.
If f' > 0 just before f '(x) = 0, followed by f ' < 0, then there is a maximum turning point.
If f ' < 0 just before f '(x) = 0, followed by f ' > 0, then there is a minimum turning point.
If f ' > 0, the gradient of f is positive.
If f '< 0, the gradient of f is negative. ✓✓✓

Question 12 **(13 marks)**

(a)





$$\sin\left(\frac{\theta}{2}\right) = \frac{7.5}{10}$$

$$\theta = 1.696124158 \quad ✓$$

$$Area_{segment} = \frac{1}{2}r^2\left(\theta - \sin(\theta)\right)$$

$$Area_{segment} = \frac{1}{2}10^2\left(1.696124158 - \sin(1.696124158)\right)$$

$$Area_{segment} \approx 35.2\,cm^2 \quad ✓$$

$$Area_{sector} = \frac{1}{2}r^2\theta$$

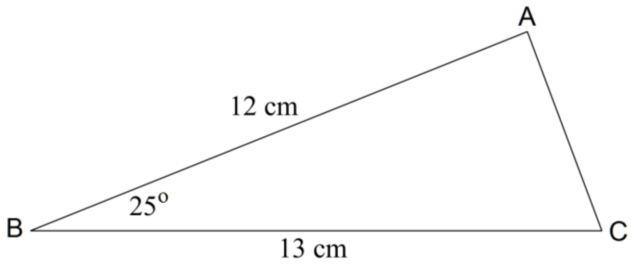
$$Area_{sector} = \frac{1}{2}10^2 \times \left(\frac{80^o}{180^o}\pi\right)$$

$$Area_{sector} \approx 69.8\ cm^2 \quad ✓$$

The sector has the bigger area. ✓

(b) $y = \tan\left(x - \frac{\pi}{3}\right)$ ✓✓

(c)



$$b^2 = a^2 + c^2 - 2ac\cos(ABC)$$

$$b^2 = 13^2 + 12^2 - 2\times 13\times 12\cos\left(25^o\right) \quad ✓ \qquad (= 30.23197)$$

$$b \approx 5.5\,cm \quad ✓$$

(d) $sin\left(2x-\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$ for $0\le x\le\pi$

$\left(2x-\frac{\pi}{4}\right)=-\frac{\pi}{4}+n2\pi$ *or* $\left(2x-\frac{\pi}{4}\right)=-\frac{3\pi}{4}+n2\pi$

$2x=n2\pi$ $\quad$ $2x=-\frac{\pi}{2}+n2\pi$

$x=n\pi$ $\quad$ $x=-\frac{\pi}{4}+n\pi$

$x=0,\pi$ ✓ ✓ $\quad$ $x=\frac{3\pi}{4}$ ✓

(e) $sin(x+y)=sin(x)cos(y)+cos(x)sin(y)$

Put $y=\frac{\pi}{2}$ ✓

$sin\left(x+\frac{\pi}{2}\right)=\cancel{sin(x)cos\left(\frac{\pi}{2}\right)}+cos(x)sin\left(\frac{\pi}{2}\right)$

$but\ \ sin\left(\frac{\pi}{2}\right)=1$ ✓

$\therefore sin\left(x+\frac{\pi}{2}\right)=cos(x)$

Question 13 **(12 marks)**

(a) $y=2+10x-x^2$

(i) $f'(x)=10-2x$ ✓

$f'(2)=6$ ✓

(ii) $\frac{f(2.01)-f(2)}{0.01}=\frac{18.0599-18}{0.01}=\frac{0.0599}{0.01}=5.99$ ✓ ✓ ✓

(ii) The answers are similar because $\frac{f(x+h)-f(x)}{h}$ approximates the slope at a point by using the gradient of a very small interval close to the given x value. ✓✓

(b) (i) $g'(y)=20y-9y^2$ ✓

(ii) $g'(x)=\frac{1}{5}-\frac{12x}{5}$ ✓✓

Question 14 **(17 marks)**

(a) $(1+2x)^4 = 1+8x+24x^2+32x^3+16x^4$ ✓

The coefficient of x^3 is 32. ✓

(b) (i) A and Q ✓

$P(A \cap Q) = 0.10$

$P(A) = 0.5 \qquad P(Q) = 0.2$ ✓

$P(A) \times P(Q) = 0.5 \times 0.2 = 0.10$ ✓

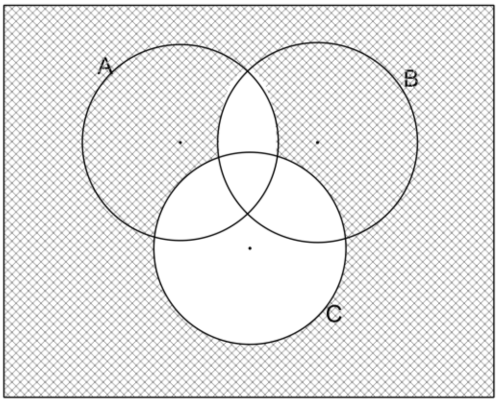
$= P(A \cap Q)$

Therefore the events are independent.

(ii) B and R because $P(B \cap R) = 0$ ✓

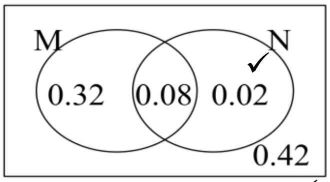
(iii) $\mathrm{P}\left((\mathrm{A} \cup \mathrm{Q}) \cap \overline{\mathrm{R}}\right) = 0.3$ ✓✓

(c) $\overline{(A \cap B) \cup C}$



✓✓✓ -1/error

(d) (i)



$\mathrm{P}(\mathrm{N}) = 0.02 + 0.08$ ✓

$\mathrm{P}(\mathrm{N}) = 0.10$ ✓

(ii) $\mathrm{P}(\mathrm{M} \mid \overline{\mathrm{N}}) = \dfrac{0.32}{0.42 + 0.32} = \dfrac{0.32}{0.74} = \dfrac{32}{74}$ ✓ ✓

Question 15 **(6 marks)**

$A = (4 - x)y$

$A = (4 - x)x^2$

$A = 4x^2 - x^3$ ✓

For maximum area $\dfrac{dA}{dx} = 0$ ✓

$\dfrac{dA}{dx} = 8x - 3x^2$ ✓

$If\ \dfrac{dA}{dx} = 0, \quad 0 = 8x - 3x^2$

$= x(8 - 3x)$

$x \neq 0 \quad x = \dfrac{8}{3}$ ✓

Test for maximum

x	1	$\dfrac{8}{3}$	3
$\dfrac{dA}{dx}$	$+$	0	$-$ ✓

Therefore maximum

$If\ \ x = \dfrac{8}{3},\ y = ?$

$y = \left(\dfrac{8}{3}\right)^2 = \dfrac{64}{9} = 7\dfrac{1}{9}$

$\therefore\ P\left(2\dfrac{2}{3}, 7\dfrac{1}{9}\right)$

Therefore the dimensions of the maximum sized rectangle are $2\dfrac{2}{3} \times 7\dfrac{1}{9}$. ✓

End of solutions