Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 1&2

Section One: Calculator-free

SOLUTIONS

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

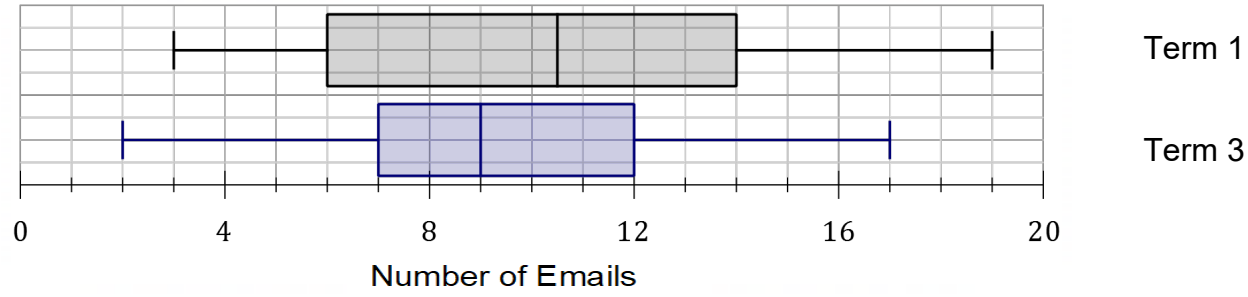
Section One: Calculator-free **35% (51 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(7 marks)**

The distribution of the number of emails received each day by a school receptionist during the first 10 days of Term One is shown in the box plot below.



(a) State the median and inter-quartile range of the data shown in the box plot. (2 marks)

Solution
Median of the data is 10.5 and IQR is $14 - 6 = 8$.
Specific behaviours
✓ correct median ✓ correct IQR

The number of emails received by the same receptionist on each of the first 10 days of Term Three were $4, 11, 7, 8, 2, 17, 10, 7, 12$ and 16.

(b) Construct a box plot for the Term Three data on the same axis as Term One. (3 marks)

Solution
Ordered data is $2, 4, 7, 7, 8, 10, 11, 12, 16, 17$ and so the five points for the box plot are $2, 7, 9, 12, 17$.
Specific behaviours
✓ draws median ✓ draws box ✓ draws whiskers

(c) Compare the data sets for Term 1 and Term 3, using the median and interquartile range to justify your answer. (2 marks)

Solution
The median decreasing from 10.5 to 9 indicates that the daily numbers of phishing emails received fell from Term One to Term Three. The interquartile range decreasing from 8 to 5 indicates that there was less variation in the daily numbers of emails received in Term Three compared to Term One.
Specific behaviours
✓ associates median with decrease in numbers from term one to term 3. ✓ associates IQR with decrease in variation/spread from term one to term 3.

See next page

Question 2 **(6 marks)**

On leaving a store, customers were asked to rate the quality of service that they received on a scale of 1 to 4, where 1 is excellent and 4 is poor.

(a) Explain why customer satisfaction rating is classified as a categorical and ordinal variable. (2 marks)

Solution
The variable is categorical as the values it can take on are qualitative categories (ratings from 1 to 4), and ordinal as there is a natural order to the categories.
Specific behaviours
✓ links categorical to values that are categories ✓ links ordinal to natural order of categories

The last 21 customer satisfaction ratings are listed below:

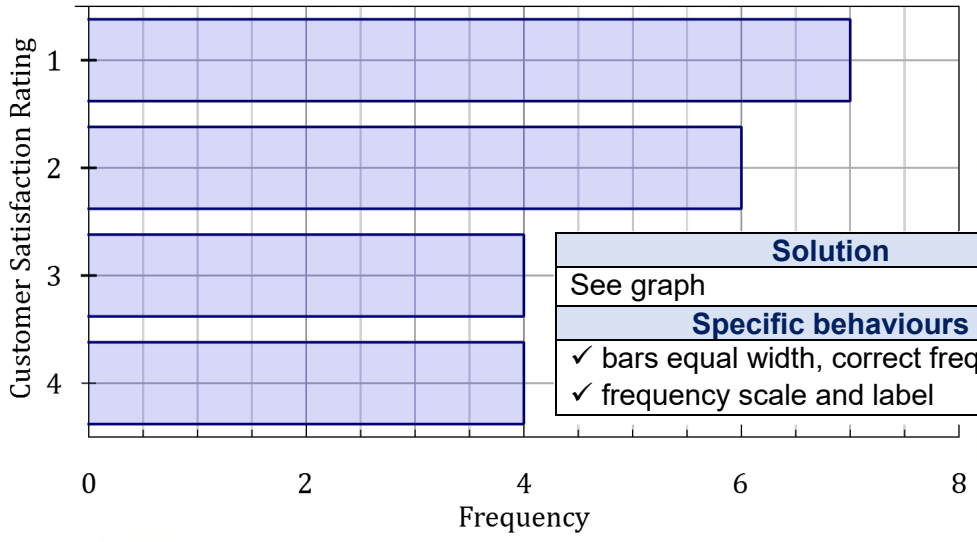
$$1 \quad 2 \quad 3 \quad 3 \quad 2 \quad 2 \quad 1 \quad 1 \quad 1 \quad 4 \quad 4 \quad 1 \quad 4 \quad 2 \quad 1 \quad 3 \quad 3 \quad 2 \quad 2 \quad 4 \quad 1$$

(b) Organise the ratings by completing the following frequency table. (2 marks)

Rating	1	2	3	4
Frequency	**7**	**6**	**4**	**4**

Solution
See table
Specific behaviours
✓ frequencies add to 21 ✓ correct frequencies

(c) Construct a bar chart to display the ratings on the grid below. (2 marks)



Solution
See graph
Specific behaviours
✓ bars equal width, correct frequencies ✓ frequency scale and label

Question 3 **(7 marks)**

(a) Three equations are shown below. Solve the linear equation. (3 marks)

$$3(1-2x)=2x^2-2, \qquad 3(x-2)=2^x-2, \qquad 2(x-2)=3(1-2x).$$

Solution
$2(x-2)=3(1-2x)$ $2x-4=3-6x$ $8x=7$ $x=\frac{7}{8}$
Specific behaviours
✓ chooses correct equation ✓ correctly expands both sides ✓ solves for correct value of x

(b) Use the formula $y-y_1=m(x-x_1)$ to determine the value of x when $y=-1, y_1=-6$, $m=2$ and $x_1=4$. (2 marks)

Solution
$-1-(-6)=2(x-4)$ $5=2x-8$ $2x=13$ $x=\frac{13}{2}=6.5$
Specific behaviours
✓ correctly substitutes, simplifying LHS ✓ solves for correct value of x

(c) The volume V of a frustrum of height h that has base areas of A and B is given by

$$V=\frac{h}{3}\left(A+\sqrt{AB}+B\right).$$

Determine the height of a frustrum with a volume of 26 cm^3 and base areas A and B of 4 cm^2 and 25 cm^2 respectively. (2 marks)

Solution
$26=\frac{h}{3}\times\left(4+\sqrt{4\times 25}+25\right)$ $26=\frac{39h}{3}$ $26=13h$ $h=2$ cm
Specific behaviours
✓ substitutes and simplifies bracketed term ✓ correct height

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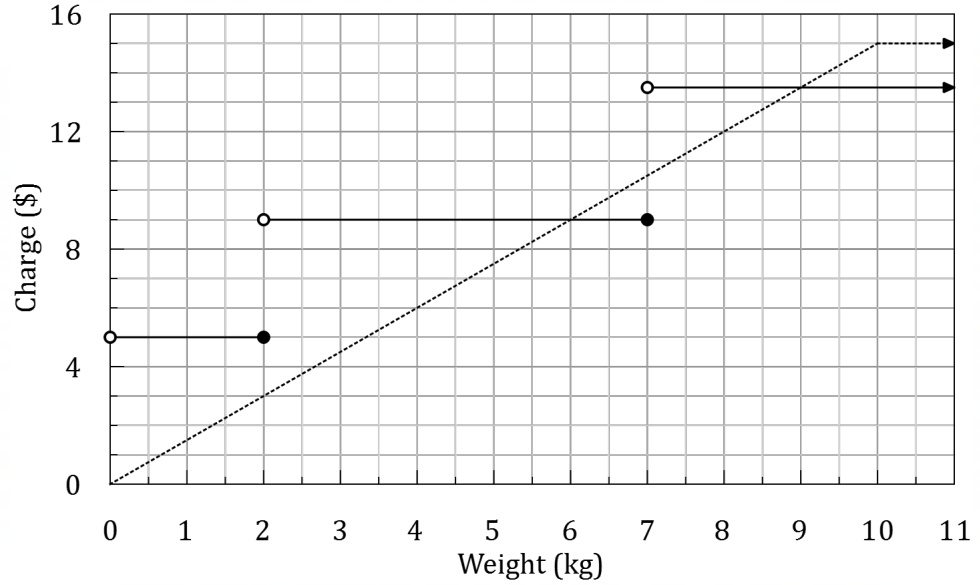
Question 4 **(7 marks)**

The graph below shows the charges made by Company A (dotted line) and Company B (solid lines) to deliver packages based on their weight.



(a) Determine the charge made by each company to deliver a 7 kg package. (2 marks)

Solution
Company A charge \$10.50 and Company B charge \$9.
Specific behaviours
✓ states Company A and charge ✓ states Company B and charge

(b) Give an example of the weight of a package for which both companies charge the same amount. (1 mark)

Solution
6 kg or 9 kg.
Specific behaviours
✓ one correct weight

(c) State the rate per kilogram charged by Company A. (1 mark)

Solution
\$1.50 per kg.
Specific behaviours
✓ correct rate

(d) A business has 50 identical 5.2 kg packages that need to be delivered separately. State which company is the cheaper of the two to deliver these packages and determine what the total charge w (3 marks)

Solution
Company A is cheaper. They charge $5.2 \times \$1.5 = \$7.80$ per package. Total cost is $50 \times \$7.80 = \$390$.
Specific behaviours
✓ chooses cheaper company ✓ indicates cost per package ✓ calculates total cost

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Question 5 **(7 marks)**

(a) The number N of insects in a population after t days when the temperature is T °C can be modelled by the formula $N = (T - 13.6) \times 10^{0.5t}$. Determine, as a whole number, the value of N after 8 days when the temperature is 15.7°C. (2 marks)

Solution
$N = (15.7 - 13.6) \times 10^{0.5\times 8}$ $= 2.1 \times 10^4$ $= 21\,000$
Specific behaviours
✓ substitutes and correctly simplifies to $a \times 10^b$ ✓ correct value

(b) The following spreadsheet is used to calculate a diversity index I for insects.
The number n of different insect species caught at three monitoring stations is entered into cells $B2, C2$ and $D2$ of the spreadsheet and then other cells automatically update.

	A	B	C	D	E	F
1		Station 1	Station 2	Station 3	Total	
2	n	1	4	5	**10**	
3	$n(n+1)$	2	20	**30**	**52**	Index I
4					**0.52**	**0.48**

The formulas in various cells are $D3 = D2 \times (D2 + 1)$ $E2 = B2 + C2 + D2$

$E3 = B3 + C3 + D3$ $E4 = E3 \div (E2 \times E2)$ $F4 = 1 - E4$.

(i) Enter the missing value in each of cells $D3, E2, E3, E4$ and $F4$. (3 marks)

Solution
$D3 = 5 \times 6 = 30,$ $E2 = 1 + 4 + 5 = 10,$ $E3 = 2 + 20 + 30 = 52$ $E4 = 52 \div (10 \times 10) = 0.52,$ $F4 = 1 - 0.52 = 0.48$
Specific behaviours
✓ correct value for $D3$ ✓ correct values for $E2, E3$ ✓ correct values for $E4, F4$

After the next catch of insects, the value of n at Station 3 dropped from 5 to 0.

(ii) Determine, with justification, whether the diversity index I increases or decreases. (2 marks)

Solution
$E2 = 5,$ $E3 = 22,$ $E4 = 22 \div 25 = 88 \div 100 = 0.88,$ $F4 = I = 0.12.$ The value of I decreases from 0.48 to 0.12.
Specific behaviours
✓ correctly recalculates value of I ✓ states index decreases, with justification

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Question 6 **(10 marks)**

Jack, Kurt and Liv are friends who work every Tuesday and Wednesday for a charity, selling magazine subscriptions and prize draw tickets.

On Tuesday Jack sold 13 subscriptions and 8 tickets, Kurt sold 12 subscriptions and 7 tickets, and Liv sold 8 subscriptions and 11 tickets.

(a) Copy and complete the matrix $\mathbf{T} = \begin{bmatrix} \_ & 12 & \_ \\ 8 & \_ & \_ \end{bmatrix}$ to show the Tuesday sales information for the three friends. (1 mark)

Solution
$\mathbf{T} = \begin{bmatrix} 13 & 12 & 8 \\ 8 & 7 & 11 \end{bmatrix}$
Specific behaviours
✓ correct matrix

On Wednesday the three friends sold a total of 36 subscriptions and 23 tickets, with Jack selling 7 tickets, Kurt selling 15 subscriptions and 6 tickets, and Liv selling 9 subscriptions.

(b) Deduce the number of subscriptions sold by Jack, the number of tickets sold by Liv and hence organise the sales information for Wednesday in matrix $\mathbf{W}$, using the same row and column headings as in matrix $\mathbf{T}$. (1 mark)

Solution
Subscriptions: $36 - 15 - 9 = 12$. Tickets: $23 - 7 - 6 = 10$. $\mathbf{W} = \begin{bmatrix} 12 & 15 & 9 \\ 7 & 6 & 10 \end{bmatrix}$
Specific behaviours
✓ correct matrix

Let matrix $\mathbf{A} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and matrix $\mathbf{B} = (\mathbf{T} + \mathbf{W}) \times \mathbf{A}$.

(c) Calculate matrix $\mathbf{B}$ and explain what information it represents. (3 marks)

Solution
$\mathbf{T} + \mathbf{W} = \begin{bmatrix} 13 & 12 & 8 \\ 8 & 7 & 11 \end{bmatrix} + \begin{bmatrix} 12 & 15 & 9 \\ 7 & 6 & 10 \end{bmatrix} = \begin{bmatrix} 25 & 27 & 17 \\ 15 & 13 & 21 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 25 & 27 & 17 \\ 15 & 13 & 21 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 69 \\ 49 \end{bmatrix}$ Matrix $\mathbf{B}$ represents the total number of subscriptions and tickets sold by the friends over the two days.
Specific behaviours
✓ correctly adds matrices ✓ correctly multiplies matrices ✓ correct explanation of result

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All workers for this charity are paid a commission of $\$10$ for each magazine subscription sold and $\$6$ for each prize draw ticket sold.

Matrix $\mathbf{C}$ represents the commission earned by each of the three friends on Tuesday.

(d) Use matrix $\mathbf{T}$ and another matrix to write a matrix calculation for matrix $\mathbf{C}$ and hence determine the matrix $\mathbf{C}$. (2 marks)

Solution
$\mathbf{C} = [10 \quad 6]\begin{bmatrix}13 & 12 & 8\\ 8 & 7 & 11\end{bmatrix}$ $= [178 \quad 162 \quad 146]$
Specific behaviours
✓ shows correct matrix multiplication ✓ correct result

Each worker is also paid a flat rate of $\$80$ per day worked.

Matrix $\mathbf{E}$ represents the total amount earned (commission plus flat rate) by each of the three friends on Wednesday.

(e) Write a matrix calculation using matrix $\mathbf{W}$ and two other matrices, the result of which is matrix $\mathbf{E}$. (1 mark)

Solution
$\mathbf{E} = [10 \quad 6]\begin{bmatrix}12 & 15 & 9\\ 7 & 6 & 10\end{bmatrix} + [80 \quad 80 \quad 80]$
Specific behaviours
✓ shows correct matrix multiplication and addition

(f) Determine matrix $\mathbf{E}$ and hence state the difference in pay between the highest and lowest paid of the friends on Wednesday. (2 marks)

Solution
$\mathbf{E} = [10 \quad 6]\begin{bmatrix}12 & 15 & 9\\ 7 & 6 & 10\end{bmatrix} + [80 \quad 80 \quad 80]$ $= [162 \quad 186 \quad 150] + [80 \quad 80 \quad 80]$ $= [242 \quad 266 \quad 230]$ Difference in pay is $266 - 230 = \$36$.
Specific behaviours
✓ calculates matrix ✓ calculates difference

See next page

Question 7 **(7 marks)**

(a) A reader is shopping for books. The book *Can Fish Count* costs $\$21$ more than the book *Diving Deep*. Let x dollars be the cost of *Can Fish Count*.

(i) Write an expression involving x for the cost of *Diving Deep*. (1 mark)

Solution
$x - 21$
Specific behaviours
✓ correct expression

(ii) The two books together cost $\$89$. Write an equation involving x and solve it to determine the cost of the book *Can Fish Count*. (2 marks)

Solution
$x + x - 21 = 89$ $2x = 110$ $x = 55$
Specific behaviours
✓ writes equation ✓ solves equation

(b) At a mobile food truck, an order for 3 burgers and 4 drinks came to $\$59.50$ whilst a smaller order for 1 burger and 2 drinks came to $\$23.50$. Let x dollars be the cost of one burger and y dollars be the cost of one drink.

(i) Write two simultaneous equations involving x and y to represent the cost of the two orders. (1 mark)

Solution
$3x + 4y = 59.5$ $x + 2y = 23.5$
Specific behaviours
✓ two correct equations

(ii) Solve the simultaneous equations to determine the cost of a burger and the cost of a drink. (3 marks)

Solution
Subtract double the second equation from the first: $3x + 4y = 59.5$ $2x + 4y = 47$ $x = 12.5$ $2y = 23.5 - 12.5 = 11$ $y = 5.5$ Cost of a burger is $\$12.50$ and cost of a drink is $\$5.50$.
Specific behaviours
✓ indicates use of an appropriate method ✓ correct cost of a burger ✓ correct cost of a drink

End of questions

Supplementary page

Question number: _____