WA Exam Papers
Australian Curriculum Examinations

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WAEP Semester One Examination, 2017

Question/Answer booklet

MATHEMATICS METHODS UNIT 1
Section Two:
Calculator-assumed

SOLUTIONS

Student Number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material with you. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed   65% (98 Marks)
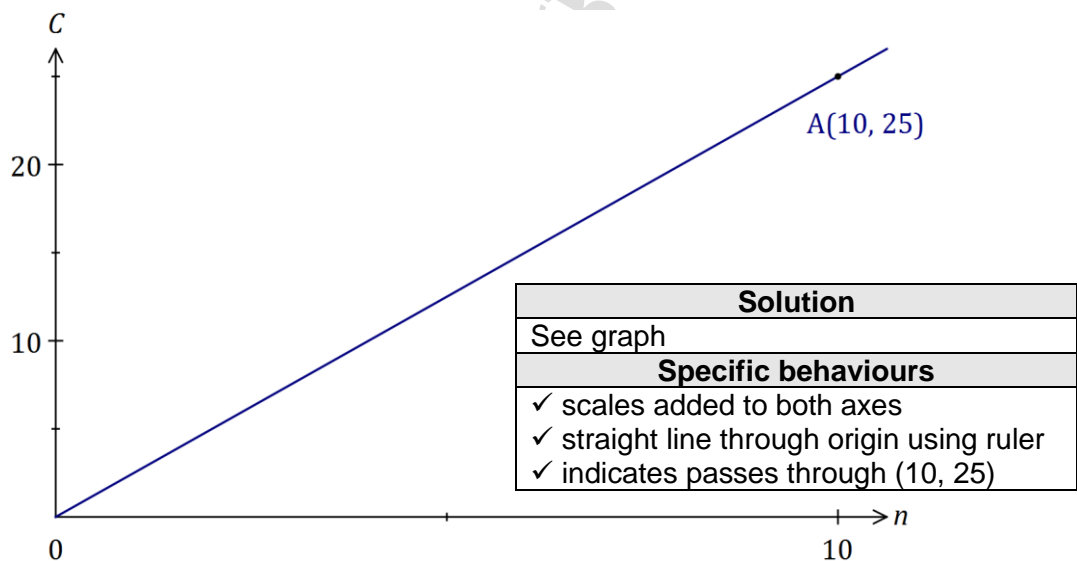
This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8   **(6 marks)**

(a)   The variables C and n are directly proportional to each other, so that when $n = 10$, it is known that $C = 25$. Sketch a graph of the relationship between C and n on the axes below.   (3 marks)

Solution
See graph
Specific behaviours
✓ scales added to both axes
✓ straight line through origin using ruler
✓ indicates passes through (10, 25)

A(10, 25)

(b)   The variables A and n are inversely proportional to each other, so that when $n = 10$, it is known that $A = 60$.

(i)   Write an equation that relates A and n.   (2 marks)

Solution
$$A \propto \frac{1}{n} \Rightarrow A = \frac{k}{n}$$ $$k = 10 \times 60 = 600 \Rightarrow A = \frac{600}{n}$$
Specific behaviours
✓ correct form of equation
✓ determines constant

(ii)   Determine the value of n when $A = 15$.   (1 mark)

Solution
$$n = \frac{600}{15} = 40$$
Specific behaviours
✓ states value

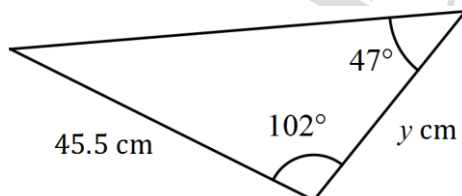S0441-092-4   **See next page**

Question 9 **(9 marks)**

(a) Determine the size, to the nearest degree, of the largest angle in a triangle with sides of lengths 23 cm, 28 cm and 31 cm. (3 marks)
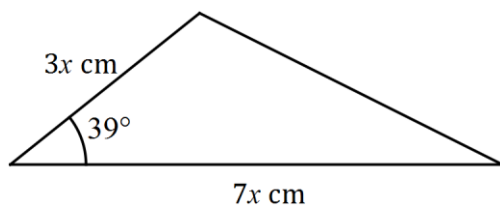
Solution
$\theta = \cos^{-1}\left(\dfrac{23^2 + 28^2 - 31^2}{2 \times 23 \times 28}\right)$ $\theta \approx 74°$
Specific behaviours
✓ shows use of cosine rule   ✓ substitutes correctly   ✓ determines angle

(b) Determine the value of y in the diagram below. (3 marks)

Solution
$180 - 102 - 47 = 31°$ $\dfrac{y}{\sin 31} = \dfrac{45.5}{\sin 47}$ $y = 32.04$
Specific behaviours
✓ determines angle   ✓ shows use of sin rule   ✓ determines value

See next page

(c)   The area of the triangle shown below is 280 cm². Determine the value of x.   (3 marks)

Solution
$0.5 \times 3x \times 7x \times \sin 39 = 280$
$x^2 = 42.37$
$x = 6.51$

Specific behaviours
✓ uses area formula
✓ substitutes correctly
✓ solves for value

Question 10 (6 marks)

From an analysis of the species of fish caught by 107 anglers during a competition, it was found that 67 anglers had caught tailor, 79 anglers had caught herring and 11 had caught neither of these species.

Let set T be the set of anglers who had caught tailor and set H be the set of anglers who had caught herring.

(a) Use set notation to describe the set of anglers who caught tailor but not herring.

(1 mark)

Solution
$T \cap \bar{H}$
Specific behaviours
✓ uses set notation

(b) Determine

(i) $n(H \cap T)$. (2 marks)

Solution
$n(H \cup T) = 107 - 11 = 96$
$96 = 67 + 79 - n(H \cap T) \Rightarrow n(H \cap T) = 50$
Specific behaviours
✓ uses probability rule
✓ determines number

(ii) $n(H \cup \bar{T})$. (1 mark)

Solution
$n(H \cup \bar{T}) = 79 + 11 = 90$
Specific behaviours
✓ determines number

(b) If one angler from the competition is selected at random, determine $P(\bar{H} \mid (H \cup T))$.
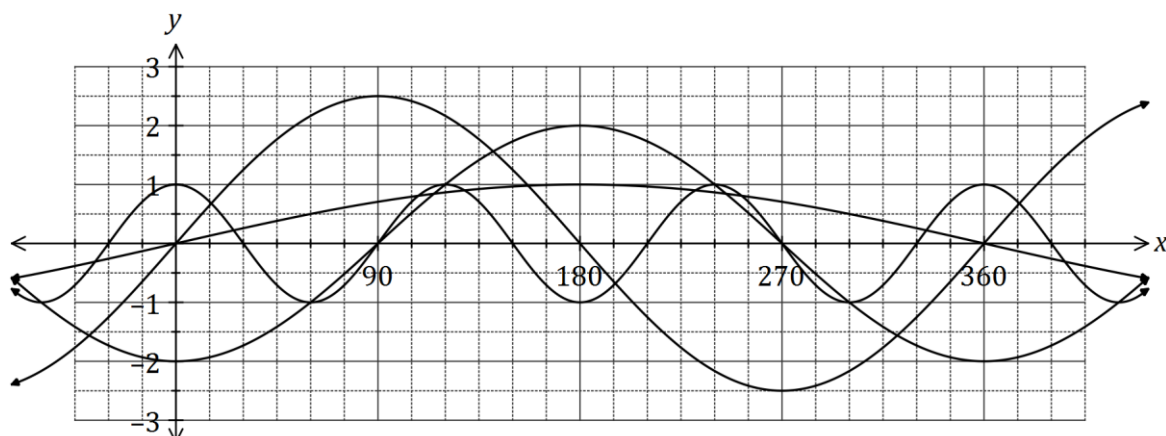
(2 marks)

Solution
$P(\bar{H} \mid (H \cup T)) = \dfrac{17}{96}$
Specific behaviours
✓ uses $n(H \cup T)$ for denominator
✓ uses $n(T \cap \bar{H})$ for numerator

Question 11   **(6 marks)**

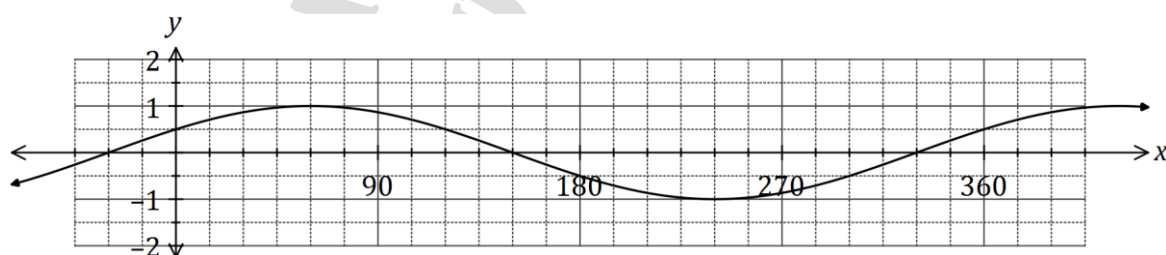(a)   The graphs of $y = a\sin(x)$, $y = \sin(bx)$, $y = d\cos(x)$ and $y = \cos(ex)$ are shown below.

Determine the values of the constants a, b, d and e.   (4 marks)

Solution
$a = 2.5$, $b = 0.5$, $d = -2$, $e = 3$
Specific behaviours
✓✓✓✓ one mark per correct value

(b)   The graph of $y = \cos(x - \alpha)$ is drawn below.

(i)   State the value of the constant α, where $0° < \alpha < 360°$.   (1 mark)

Solution
$\alpha = 60°$
Specific behaviours
✓ states value

(ii)   The graph is also that of $y = \sin(x + \beta)$. State the value of the constant β, where $0° < \beta < 360°$.   (1 mark)

Solution
$\beta = 30°$
Specific behaviours
✓ states value

Question 12 **(8 marks)**

The number of bottles of a drink sold by a store per day was recorded over a period of time. A summary is shown in the table below.

Number of bottles per day	0	1	2	3	4
Frequency	80	50	40	20	10

Based on the above records and assuming sales from day to day are independent

(a)   determine the probability that on any given day in the future

(i)   at least one bottle of the drink is sold. (2 marks)

Solution
$80 + 50 + 40 + 20 + 10 = 200$
$P = 1 - \dfrac{80}{200} = \dfrac{3}{5} = 0.6$

Specific behaviours
✓ uses relative frequencies
✓ determines P

(ii)   exactly three bottles are sold, given that at least one bottle is sold. (2 marks)

Solution
$P = \dfrac{20}{120} = \dfrac{1}{6} \ (\approx 0.167)$

Specific behaviours
✓ denominator
✓ numerator

(b)   determine the probability that a total of two bottles will be sold over any given two-day period. (4 marks)

Solution
Sell (0, 2) or (1, 1) or (2, 0) bottles each day
$P(0,2) = \dfrac{80}{200} \times \dfrac{40}{200} = \dfrac{2}{25} = 0.08$
$P(2,0) = \dfrac{2}{25}$
$P(1,1) = \dfrac{50}{200}^2 = \dfrac{1}{16} = 0.0625$
$P = \dfrac{2}{25} + \dfrac{2}{25} + \dfrac{1}{16} = \dfrac{89}{400} = 0.2225$

Specific behaviours
✓ identifies required events
✓ $P(0,2)$ or $P(2,0)$
✓ $P(1,1)$
✓ determines P

Question 13   **(5 marks)**

(a)   Using the unit circle shown, determine the following in terms of a and/or b, given that θ is an acute angle measured in degrees.
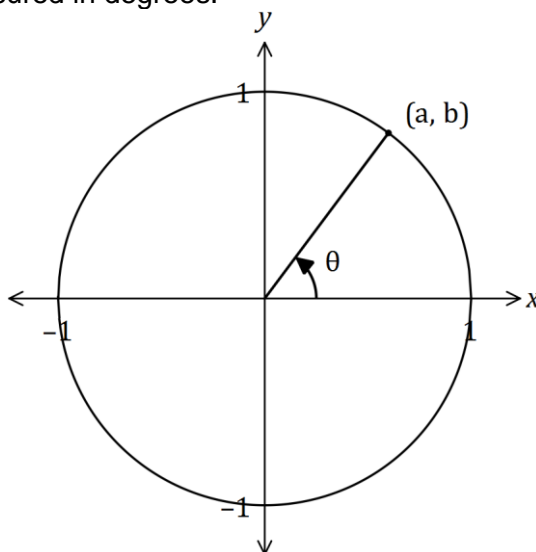
(i) $\sin(\theta)$.   (1 mark)

Solution
(i) b
(ii) $-a$
(iii) $-\dfrac{a}{b}$
Specific behaviours
✓ (i)
✓ (ii)
✓ (iii)

(ii) $\cos(180 - \theta)$.   (1 mark)

(iii) $\tan(90 + \theta)$.   (1 mark)

(b)   Determine x in each of the following cases, where $0 \leq x \leq \frac{\pi}{2}$.

(i) $\sin x = \sin 17\pi$.   (1 mark)

Solution
(i) $x = 0$
(ii) $x = \dfrac{\pi}{6}$
Specific behaviours
✓ (i)
✓ (ii)

(ii) $\cos x = \cos \dfrac{23\pi}{6}$.   (1 mark)

Question 14   **(8 marks)**

(a) The graph of $y = 2x^2 + bx + 16$ has a line of symmetry with equation $x = 3$.
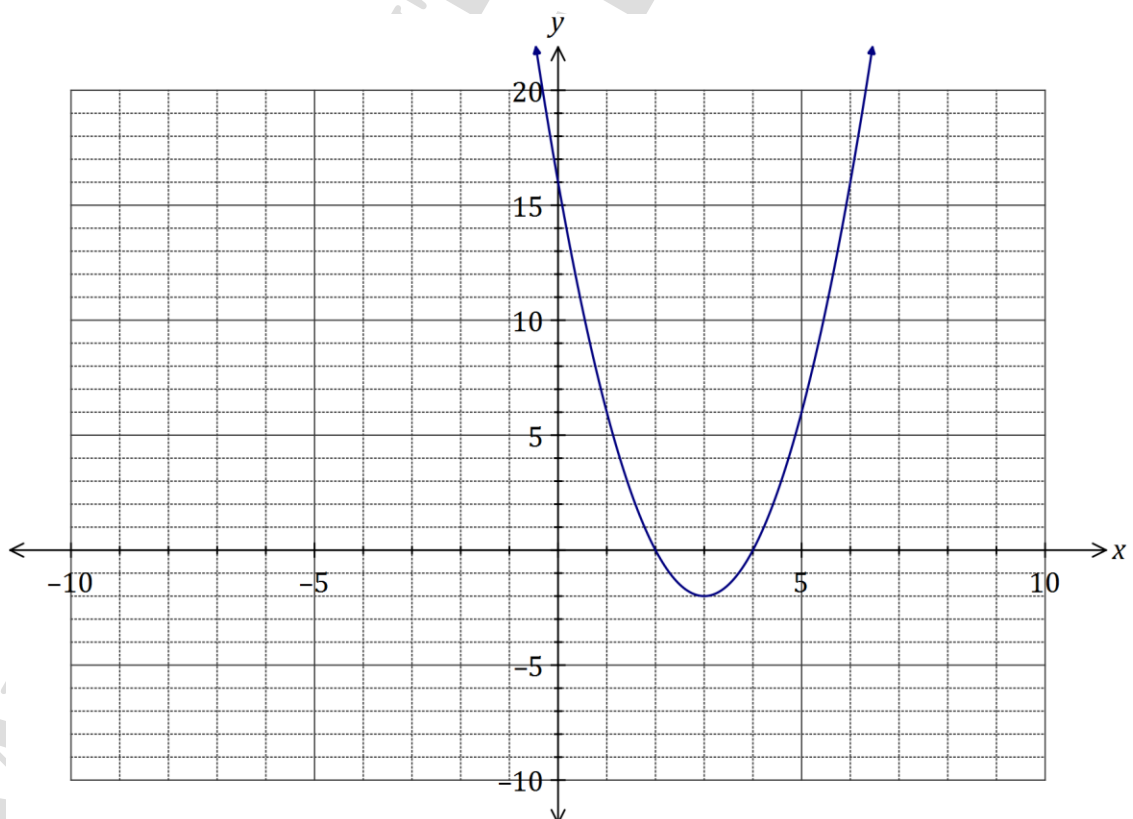
(i) Determine the value of b.   (2 marks)

Solution
$x = -\dfrac{b}{2a} \Rightarrow b = -3 \times 2 \times 2 = -12$
Specific behaviours
✓ uses line of symmetry
✓ value of b

(ii) Draw the graph of the parabola on the axes below.   (3 marks)

Solution
See graph
Specific behaviours
✓ turning point
✓ three axes intercepts
✓ smooth curve

See next page

(b) One of the solutions to the equation $2x^3 + 21x^2 + cx - 495 = 0$ is $x = 5$. Determine the value of c and all other solutions. (3 marks)

Solution
Using CAS, when $x = 5, 5c + 280 = 0 \Rightarrow c = -56$
Use CAS to solve $2x^3 + 21x^2 - 56x - 495 = 0$
$x = -11, x = -4.5$ and $x = 5$

Specific behaviours
✓ substitutes $x = 5$
✓ determines c
✓ states other two solutions

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Question 15 **(8 marks)**

In a school survey of 197 students in Year 11 and Year 12, it was observed that 75 of the 96 Year 12 students studied a science subject and that 10 students in Year 11 did not study a science subject.

(a) If one student is selected at random from those surveyed, determine the probability that

(i) they were in Year 11. (1 mark)

Solution
$\frac{197-96}{197}=\frac{101}{197}$ (≈ 0.513)
Specific behaviours
✓ states probability

(ii) they studied a science subject. (1 mark)

Solution
$\frac{75+(101-10)}{197}=\frac{166}{197}$ (≈ 0.843)
Specific behaviours
✓ states probability

(iii) they were in Year 12 or studied a science subject. (2 marks)

Solution
$\frac{166+21}{197}=\frac{187}{197}$ (≈ 0.949)
Specific behaviours
✓ uses union correctly
✓ states probability

(iv) they studied a science subject given that they were in Year 11. (2 marks)

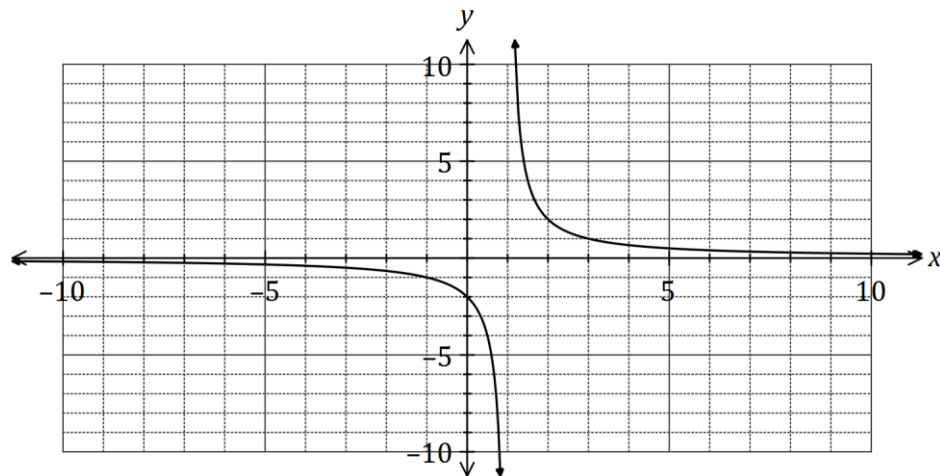| Solution |
|---|
| $\frac{91}{101}$ (≈ 0.901) |
| **Specific behaviours** |
| ✓ uses condition |
| ✓ states probability |

(b) Without calculating any further probabilities, is there any indication that studying a science subject is independent of Year? Justify your answer. (2 marks)

Solution
No indication, as P(Study Science)≈ 0.84 but P(Study Science | Year 11)≈ 0.90 which is not the same, indicating that choice of science is NOT independent of year.
Specific behaviours
✓ draws conclusion
✓ uses reasoning based on existing probabilities

Question 16 **(8 marks)**

The graph of the function defined by $f(x) = \dfrac{a}{x+b}$ is shown below.

(a)   Determine the values of a and b. (2 marks)

Solution
Using vertical asymptote, $b = -1$ Using y-intercept, $a = 2$
Specific behaviours
✓ value of a, ✓ value of b

(b)   State the domain and range of $f(x)$. (2 marks)

Solution
$D_f = \{x : x \in \mathbb{R}, x \neq 1\}$ $R_f = \{y : y \in \mathbb{R}, y \neq 0\}$
Specific behaviours
✓ indicates $x \neq 1$, ✓ indicates $y \neq 0$

(c)   Determine the equations of the asymptotes of the graph of $y = f(2x)$. (2 marks)

Solution
Vertical asymptote: $x = \dfrac{1}{2}$, horizontal asymptote: $y = 0$
Specific behaviours
✓ vertical asymptote, ✓ horizontal asymptote.

(d)   Describe the transformation required on the graph of $y = f(x)$ to obtain the graph of

(i) $y = f(x + 8)$. (1 mark)

Solution
Translate 8 units to the left.
Specific behaviours
✓ description

(ii) $y = \frac{1}{2}f(x)$. (1 mark)

Solution
Dilate vertically by scale factor $\dfrac{1}{2}$
Specific behaviours
✓ description

Question 17 **(9 marks)**

An assortment box contains 16 different chocolates, 7 of which have soft centres and the remainder have hard centres. A selection of 5 chocolates is to be made from the box.

(a) Determine the number of different selections that can be made. (2 marks)

Solution
$\binom{16}{5} = 4368$ selections
Specific behaviours
✓ uses combination notation
✓ evaluates

(b) Determine the number of different selections that can be made if the chocolates must all have hard centres. (2 marks)

Solution
$\binom{16 - 7}{5} = \binom{9}{5} = 126$ selections
Specific behaviours
✓ uses combination notation
✓ evaluates

(c) Determine the probability that a randomly chosen selection has

(i) two more soft centred chocolates than hard centred chocolates. (1 mark)

Solution
$P = 0$ (not possible)
Specific behaviours
✓ states numerical probability

(ii) no more than one soft centred chocolate. (4 marks)

Solution
$P(\text{soft} = 0) = \binom{7}{0} \times \binom{9}{5} = 126$
$P(\text{soft} = 1) = \binom{7}{1} \times \binom{9}{4} = 7 \times 126 = 882$
$P = \dfrac{126 + 882}{4368} = \dfrac{3}{13} \ (\approx 0.231)$
Specific behaviours
✓ identifies required cases
✓ $P(s = 0)$
✓ $P(s = 1)$
✓ determines probability

Question 18 **(8 marks)**

Over a 24-hour period, the depth of water, d metres, in a harbour at time t hours after midnight was given by

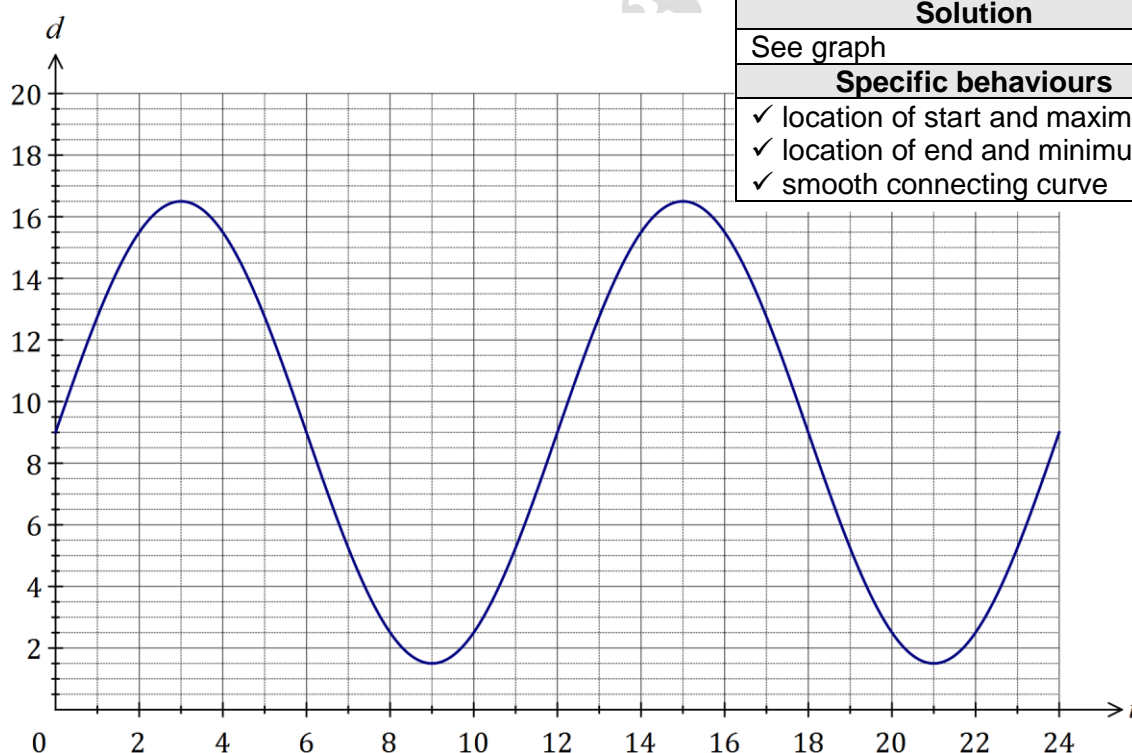$$d = 7.5 \sin\left(\frac{\pi t}{6}\right) + 9, \qquad 0 \le t \le 24.$$

(a) Determine the depth of water at 11 pm. (1 mark)

Solution
$d = 5.25$ m

Specific behaviours
✓ states depth

(b) Draw the graph of the water depth on the axes below. (3 marks)

Solution
See graph

Specific behaviours
✓ location of start and maximums
✓ location of end and minimums
✓ smooth connecting curve

(c) At what time, in hours and minutes, did the depth of water first exceed 15 metres?

(2 marks)

Solution
$t = 1.771$ - at 1:46 am

Specific behaviours
✓ time as decimal
✓ time in h and m

(d) Determine the fraction of the 24-hour period during which the depth of water exceeded 12.75 m. (2 marks)

Solution
Between 1 am and 5 am in first 12 h.
$\frac{4}{12} = \frac{1}{3}$

Specific behaviours
✓ determines time interval
✓ states fraction

See next page

Question 19 **(8 marks)**

For two events A and B, $P(A) = x$, $P(B) = 0.45$ and $P(A \cap \bar{B}) = 0.15$.

Determine the value of x under each of the following conditions:

(a) $P(A \cap B) = 0.1$. (1 mark)

Solution
Note that $P(A \cap B) = x - 0.15$
$x = 0.15 + 0.1 = 0.25$
Specific behaviours
✓ determines value

(b) $P(\bar{A} \cap B) = 0.1$. (1 mark)

Solution
$x - 0.15 + 0.1 = 0.45$
$x = 0.5$
Specific behaviours
✓ determines value

(c) A and B are mutually exclusive. (2 marks)

Solution
$x - 0.15 = 0$
$x = 0.15$
Specific behaviours
✓ uses $A \cap B = \emptyset$
✓ determines value

(d) $P(B \mid A) = 0.75$. (2 marks)

Solution
$\dfrac{x - 0.15}{x} = 0.75$
$x = 0.6$
Specific behaviours
✓ writes equation
✓ solves equation

(e) A and B are independent. (2 marks)

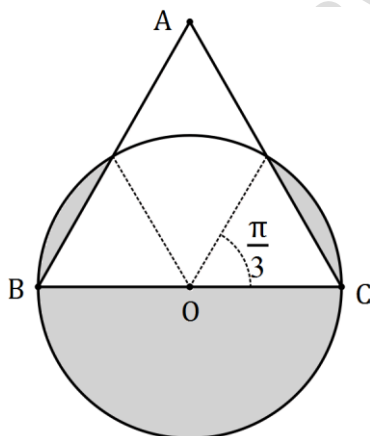| Solution |
|---|
| $x \times 0.45 = x - 0.15$ |
| $x = \dfrac{3}{11} \ (\approx 0.273)$ |
| **Specific behaviours** |
| ✓ use independence rule to write equation |
| ✓ solves equation |

Question 20 **(9 marks)**

(a)   A running track has circular ends of radius 100 m and two straight, parallel sides PQ and RS that are both 120 m long, as shown below. Determine, to the nearest metre, the total length of the track. (4 marks)
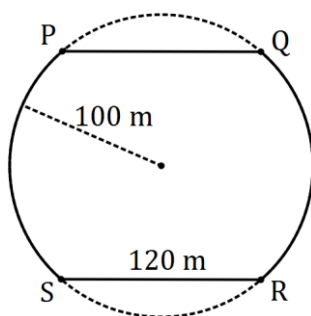
Solution
$\angle POS = 2 \times \cos^{-1}\dfrac{60}{100}$ $\angle POS = 2 \times 0.9273 = 1.8546$ arc $PS = 100 \times 1.8546 = 185.46$ Total $2 \times (185.46 + 120) = 610.92 \approx 611$ m

Specific behaviours
✓ method to find arc angle   ✓ determines arc angle   ✓ arc length   ✓ total length

(b)   The diagram shows a circle with centre O and diameter BC, and an equilateral triangle ABC. Determine the exact fraction of the area of the circle that lies outside the triangle. (5 marks)

Solution
Segment angle $= \dfrac{\pi}{3}$ Area segments $= 2 \times \dfrac{1}{2}r^2\left(\dfrac{\pi}{3} - \sin\dfrac{\pi}{3}\right)$ $= r^2\left(\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2}\right)$ Outside area $= \dfrac{\pi r^2}{2} + r^2\left(\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2}\right)$ Fraction $= \dfrac{\dfrac{\pi r^2}{2} + r^2\left(\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2}\right)}{\pi r^2}$ $= \dfrac{5}{6} - \dfrac{\sqrt{3}}{2\pi} = \dfrac{5\pi - 3\sqrt{3}}{6\pi}$

Specific behaviours
✓ determines segment angle   ✓ determines segment area   ✓ determines total outside area   ✓ expresses outside as fraction of whole   ✓ simplifies (CAS) as exact value

End of questions

Additional working space

Question number: _____