



**MATHEMATICS**

**3A/3B**

**Calculator-assumed**

**WACE Examination 2015**

**Marking Key**

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

Section Two: Calculator-assumed

66<sup>2</sup>/<sub>3</sub>% (100 Marks)

Question 7

(9 marks)

- (a) The line  $y = 2x + 7$  is parallel to the tangent to the curve  $y = x^3 + 2x^2 + qx - 6$  at  $x = 1$ . Determine the value of  $q$ . (3 marks)

Solution
$\frac{dy}{dx} = 3x^2 + 4x + q$ $2 = 3 + 4 + q$ $q = -5$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ differentiates <math>y</math> correctly</li> <li>✓ substitutes gradient correctly</li> <li>✓ solves for <math>q</math></li> </ul>

- (b) For the function  $f(x) = (x^2 + 2)(3 - x)$ :
- (i) show the use of the product rule to determine  $f'(x)$  and simplify your answer. (3 marks)

Solution
$f'(x) = 2x(3 - x) + (-1)(x^2 + 2)$ $= -3x^2 + 6x - 2$
Specific behaviours
<ul style="list-style-type: none"> <li>✓✓ correctly shows the use of product rule</li> <li>✓ simplifies</li> </ul>

- (ii) determine the coordinates of the point(s) on the curve of  $f(x)$  where the gradient is 1. (3 marks)

Solution
$-3x^2 + 6x - 2 = 1$ $x = 1 \therefore \text{coordinates are } (1, 6)$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ equates derivative equal to one</li> <li>✓ correctly solves for <math>x</math></li> <li>✓ substitutes and states correct coordinates</li> </ul>

Question 8

(10 marks)

In order to buy a second-hand scooter, Kim obtained a personal loan of \$5000 with monthly repayments of \$440 to be paid at the end of each month. The table below shows the amount owing at the start of each month, the interest payable for that month, the repayment and the amount owing at the end of each month for the first six months.

Month	Amount owing at the start of the month (\$)	Interest (\$)	Repayment (\$)	Amount owing at the end of the month (\$)
1	5000	40	440	4600
2	4600	36.80	440	4196.80
3	4196.80	33.57	440	3790.37
4	3790.37	30.32	440	3380.70
5	3380.70	27.05	440	2967.74
6	2967.74	23.74	440	2551.48

- (a) Calculate the annual interest rate. (2 marks)

Solution
$\frac{40}{5000} \times 100 \times 12 = 9.6\%$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly calculates monthly interest rate</li> <li>✓ correctly converts to annual rate</li> </ul>

- (b) Write a recursive rule to determine the amount owing at the end of each month. (3 marks)

Solution
$A_{n+1} = 1.008A_n - 440, A_0 = 5000$ or $A_n = 1.008A_{n-1} - 440, A_1 = 4600$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly uses monthly interest rate</li> <li>✓ subtracts monthly payment</li> <li>✓ states correct initial value</li> </ul>

- (c) In which month would Kim pay off the loan? (1 mark)

Solution
The twelfth month
Specific behaviours
<ul style="list-style-type: none"> <li>✓ states correct month</li> </ul>

- (d) How much is Kim's final repayment? (2 marks)

Solution
$1.008 \times \$419.71 = \$423.06$ (nearest cent)
Specific behaviours
✓ correctly identifies value at end of 11 <sup>th</sup> month
✓ calculates interest and states final amount

- (e) How much did Kim actually pay for the scooter? (2 marks)

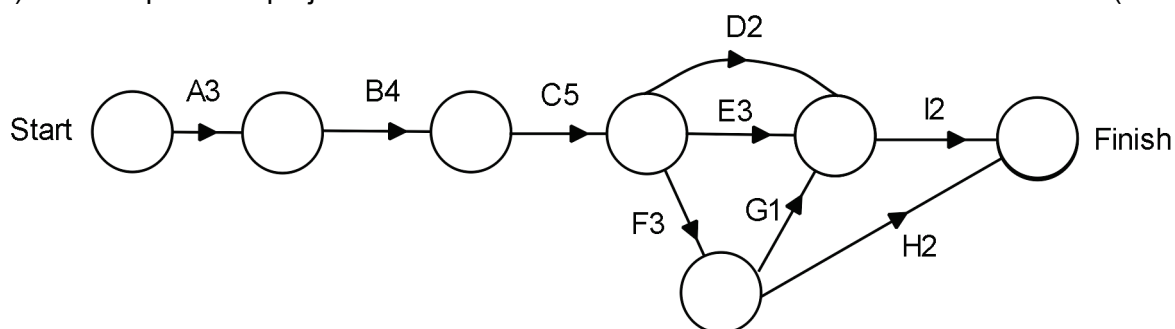
Solution
$11 \times \$440 + \$423.06 = \$5263.06$
Specific behaviours
✓ multiplies repayment by eleven
✓ adds on final payment

**Question 9** (10 marks)

A family decides to add a games room to their house. Details of this project are given in the table below.

	Activity	Time (days)	Immediate Predecessor(s)
<b>A</b>	Lay the foundations	3	-
<b>B</b>	Prepare and pour the concrete floor	4	A
<b>C</b>	Construct the walls	5	B
<b>D</b>	Fit door and window frames	2	C
<b>E</b>	Install electrical fittings	3	C
<b>F</b>	Construct roof	3	C
<b>G</b>	Plaster ceiling	1	F
<b>H</b>	Fit gutters	2	F
<b>I</b>	Paint inside	2	D, E, G

- (a) Complete the project network below. (2 marks)



Solution
See network above
Specific behaviours
✓ draws network in correct order
✓ correctly labels arcs

- (b) Determine the critical path and minimum completion time. (2 marks)

<b>Solution</b>
ABCFGI with a minimum completion time of 18 days
<b>Specific behaviours</b>
✓ correctly states critical path
✓ correctly states minimum completion time

- (c) State the effect on the critical path **and** completion time for each of the following.

- (i) Bricklayers take an extra two days to construct the walls. (2 marks)

<b>Solution</b>
Critical path does not change. Minimum completion time = 20 days
<b>Specific behaviours</b>
✓ correctly states new completion time
✓ critical path stays the same

- (ii) The electrician is held up for three days on another job. (2 marks)

<b>Solution</b>
New critical path is ABCEI. Minimum completion time = 20 days
<b>Specific behaviours</b>
✓ correctly states critical path
✓ correctly states minimum completion time

- (iii) Fitting gutters with downpipes adds an extra two days. (2 marks)

<b>Solution</b>
New critical path is ABCFH. Minimum completion time = 19 days
<b>Specific behaviours</b>
✓ correctly states critical path
✓ correctly states minimum completion time

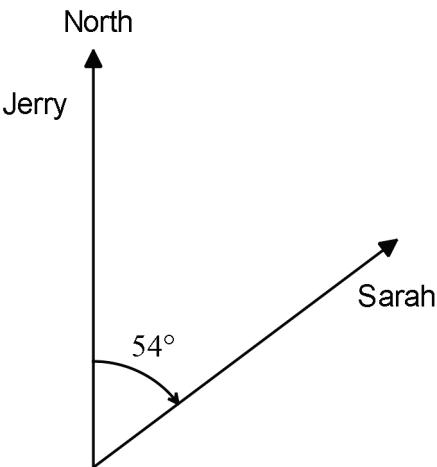
Question 10

(8 marks)

Jerry and Sarah decide to go for a walk. They both set off from the same starting point at 10 am. Jerry heads due north at 1.5 metres per second and Sarah heads on a bearing of 054° at 3.5 kilometres per hour. They both maintain these speeds and directions for the duration of their walk.

- (a) Draw a diagram to represent this information.

(2 marks)

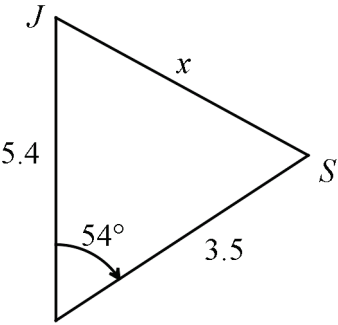
Solution	
	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ correctly shows direction of Jerry</li> <li>✓ correctly shows direction of Sarah in relation to Jerry clockwise</li> </ul>	

- (b) Convert Jerry's speed to kilometres per hour.

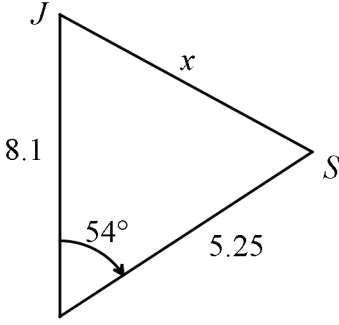
(1 mark)

Solution	
$\frac{1.5}{1000} \times 3600 = 5.4 \text{ km/hr}$	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ correctly converts to kilometres per hour</li> </ul>	

- (c) Calculate the distance (in kilometres) between Jerry and Sarah after they have been walking for one hour. (2 marks)

Solution	
 <p>Using cosine rule or otherwise: <math>x = 4.38</math> kilometres after one hour</p>	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ uses correct distances on triangle</li> <li>✓ calculates correct distance <math>JS</math></li> </ul>	

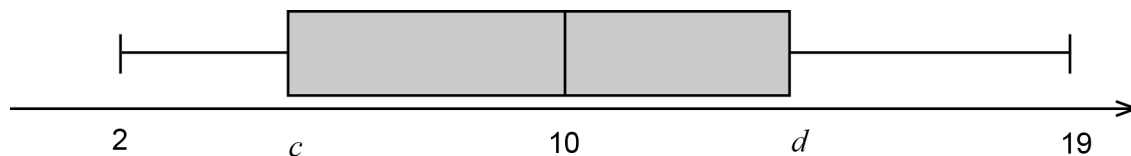
- (d) After a further 30 minutes Sarah injures herself and phones Jerry to assist her. If Jerry jogs at 9.5 kilometres per hour directly towards Sarah, at what time after 10 am will Jerry reach her? (3 marks)

Solution	
 <p>Using cosine rule or otherwise: <math>JS = 6.57</math> km</p> <p>Time = <math>\frac{6.57}{9.5} = 0.69</math> hours = 42 minutes (nearest minute)</p> <p>Therefore time after 10 am is 12:12 pm</p>	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ uses correct distances on triangle and calculates <math>JS</math></li> <li>✓ correctly calculates time</li> <li>✓ correctly states time after 10 am</li> </ul>	

Question 11

(8 marks)

Eight data points generate the boxplot shown below.



The ordered data points are: 2,  $a$ , 7, 8,  $b$ , 12, 16, 19 with an interquartile range of 9.

- (a) Determine the values of  $a$ ,  $b$ ,  $c$  and  $d$ .

(4 marks)

Solution
$\frac{b+8}{2} = 10 \Rightarrow b = 12$ $d = 14 \text{ (UQ)}$ $14 - c = 9 \Rightarrow c = 5$ $\frac{a+7}{2} = 5 \Rightarrow a = 3$
Specific behaviours
✓✓✓✓ correctly determines the values of $a$ , $b$ , $c$ and $d$

- b) Two more data points, 36 and  $-16$  are added to the list.

- (i) Write down the new interquartile range.

(1 mark)

Solution
New IQR = $16 - 3 = 13$
Specific behaviours
✓ correctly calculates new interquartile range

- (ii) Would these two data points be considered as outliers? Show your reasoning.

(3 marks)

Solution
$13 \times 1.5 = 19.5$ $16 + 19.5 = 35.5 < 36 \therefore 36 \text{ is an outlier}$ $3 - 19.5 = -16.5 < -16 \therefore -16 \text{ is not an outlier}$
Specific behaviours
✓ correctly calculates 19.5 boundary ✓ correctly establishes 36 is an outlier ✓ correctly establishes $-16$ is not an outlier



Question 12

(9 marks)

A Fishing, Camping and Four Wheel Drive Show is open Friday, Saturday and Sunday of the same week each year. The attendances for the years 2011 to 2015 are shown in the table below.

Year	Day	Time Period (x)	Attendance	Moving Average (y)	Residual
2011	Friday	1	$C$	–	–
	Saturday	2	$B$	5110.7	$A$
	Sunday	3	6971	5250	1721
2012	Friday	4	4020	5204.3	–1184.3
	Saturday	5	4622	5253.3	–631.3
	Sunday	6	7118	5341	1777
2013	Friday	7	4283	5644.7	–1361.7
	Saturday	8	5533	5742.7	–209.7
	Sunday	9	7412	5985	1427
2014	Friday	10	5010	5973	–963
	Saturday	11	5497	6138.3	–641.3
	Sunday	12	7908	6015.3	1892.7
2015	Friday	13	4641	6270.7	–1629.7
	Saturday	14	6263	6387.7	–124.7
	Sunday	15	8259	–	–

(a) The seasonal component for Saturday is –391.7.

(i) What is the significance of this seasonal component being negative? (1 mark)

Solution
On average Saturday attendances are lower than the general trend <b>or</b> On average Saturday attendances are lower than the moving average figure
Specific behaviours
✓ correctly states a valid reason

(ii) Calculate the value of  $A$  in the table. (2 marks)

Solution
$\frac{A + (-631.3) + (-209.7) + (-641.3) + (-124.7)}{5} = -391.7 \Rightarrow A = -351.5$
Specific behaviours
✓ establishes correct equation ✓ correctly solves for $A$

- (b) Calculate the values of  $B$  and  $C$  in the table. (2 marks)

<b>Solution</b>
$B - 5110.7 = -351.5 \Rightarrow B = 4759.2$ $5110.7 = \frac{C + 4759 + 6971}{3} \Rightarrow C = 3602$
<b>Specific behaviours</b>
✓ correctly calculates the value of $B$ ✓ correctly calculates the value of $C$

- (c) The local council has informed event organisers that once the attendance on any one day exceeds 10 000, a permit to conduct this event the following year at the current location will no longer be approved due to overcrowding. On the basis of the data in the table, predict the day and year when the attendance will first exceed 10 000.

The trend line for moving average against time is  $y = 111.37x + 4825.7$ . (4 marks)

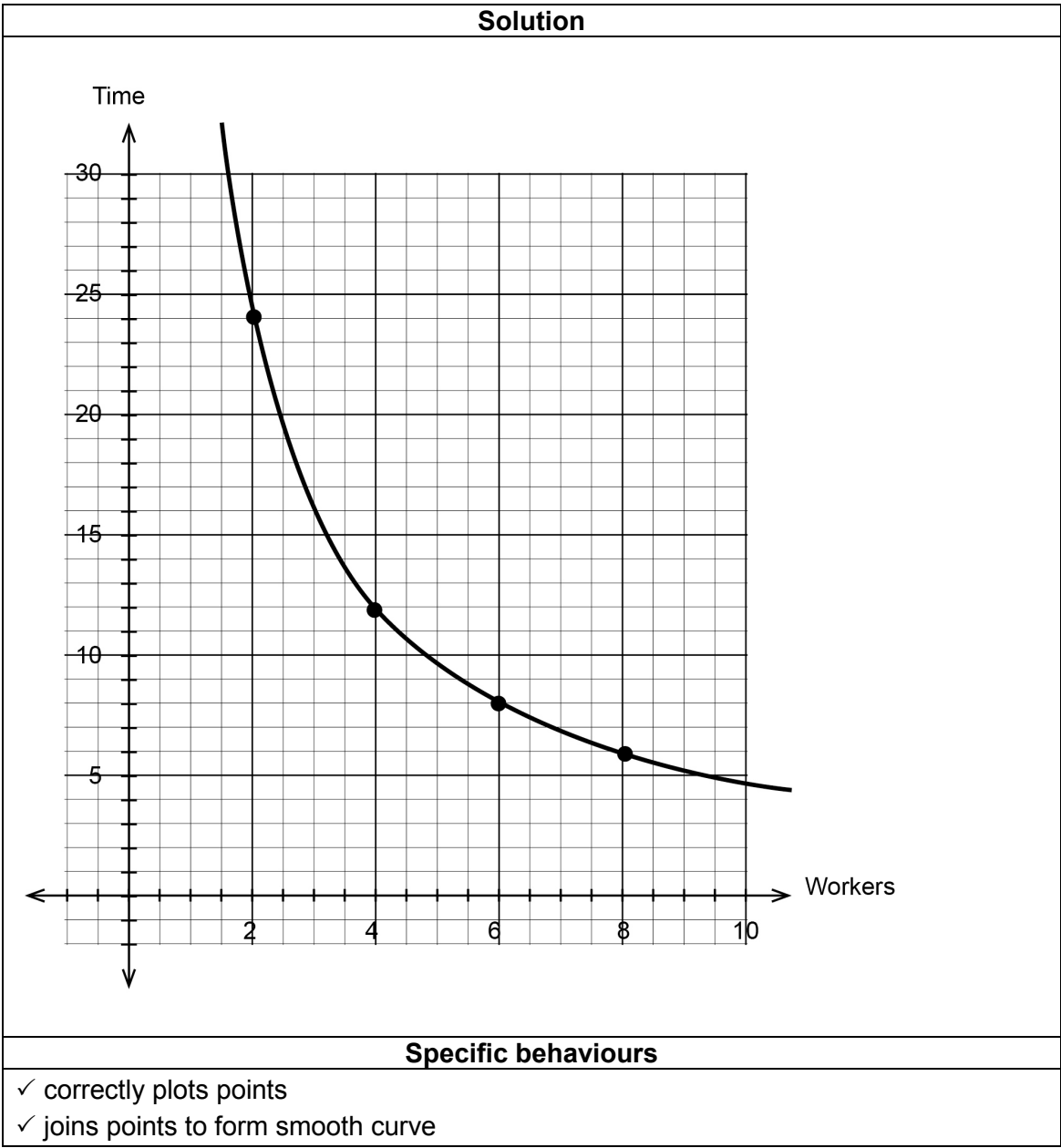
<b>Solution</b>
Sunday seasonal component = 1704 Using the trend line: $10000 = 111.37x + 4825.7 + 1704$ $x = 31.2$ Therefore require $x = 33$ which is a Sunday in the year 2021
<b>Specific behaviours</b>
✓ correctly calculates the Sunday seasonal component ✓ correctly calculates the value of $x$ using the trend line ✓ correctly rounds up and determines the day ✓ correctly determines the year

Question 13 (7 marks)

A shire council required a specific task to be completed and gathered the following information.

Number of workers	Time to complete the task (days)
2	24
4	12
6	8
8	6

- (a) Plot the data on the following set of axes and join the points with a smooth curve. (2 marks)



- (b) (i) State, in words, the relationship between the number of workers and the time taken to complete the task. (1 mark)

Solution
As the number of workers increase the time taken to do the task decreases Inverse proportion
Specific behaviours
✓ states the correct word description

- (ii) State the mathematical relationship between the number of workers and the time taken to complete the task. (1 mark)

Solution
Inverse proportion
Specific behaviours
✓ correctly recognises inverse proportion

- (c) If  $w$  represents the number of workers and  $d$  represents the number of days required to complete the task, determine a mathematical equation for  $w$  in terms of  $d$ . (2 marks)

Solution
$w = \frac{48}{d}$
Specific behaviours
✓ calculates correct constant ✓ writes equation in the correct order i.e. $w$ in terms of $d$

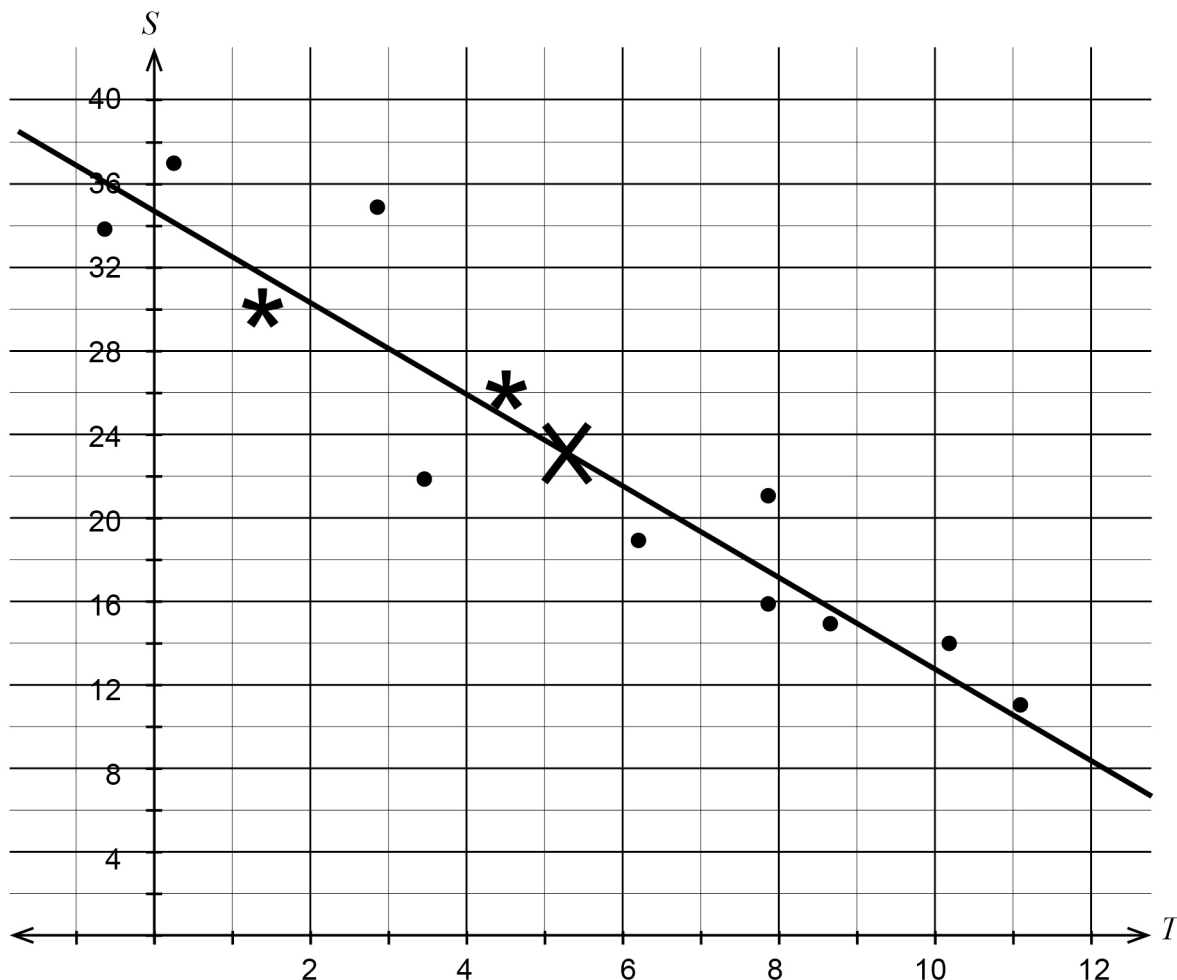
- (d) What is the minimum number of workers required to finish the task in less than 14 days? (1 mark)

Solution
4
Specific behaviours
✓ correctly calculates the number of workers required

Question 14

(12 marks)

Over a 10-day period last winter, a company that supplies home gas heating appliances noted from its records the minimum overnight temperature ( $T^\circ$  Celsius) and the number of service calls ( $S$ ) on each day. This information is shown on the scatter plot below.



- (a) The least squares regression line for this data is  $S = -2.2T + 34.9$ . Sketch this line on the scatter plot. (2 marks)

Solution	
See graph above	
Specific behaviours	
✓ the line passes through correct $y$ -intercept	
✓ the line has an equal number of data points above and below	

- (b) The data for days 11 and 12 are (1.4, 30) and (4.6, 26) respectively. Plot these two points on the scatter plot. (2 marks)

Solution
See graph above
Specific behaviours
✓✓ correctly plots each point

- (c) The correlation coefficient  $r_{TS}$  for the first 10 days is  $-0.93$ . What is the effect on the value of  $r_{TS}$  with the inclusion of the two extra data points? Does  $r_{TS}$  increase, decrease or stay the same? Justify your answer. (2 marks)

Solution
Decrease, closer to $-1$ since the points are close to the line
Specific behaviours
✓ correctly identifies a decrease
✓ gives a correct justification

- (d) Comment briefly on what the correlation coefficient,  $r_{TS}$  for this data set is actually measuring. (2 marks)

Solution
The strength of the linear relationship
Specific behaviours
✓ correctly comments on the strength
✓ correctly mentions the word 'linear relationship'

- (e) Use the regression line to predict the number of service calls for an overnight temperature of  $11.4^{\circ}\text{C}$ . Comment on the reliability of your prediction. (3 marks)

Solution
Using the regression line, $S = 9.82 \approx 10$ service calls
Reliable since the correlation is strong and the prediction is close to known data
Specific behaviours
✓ calculates the correct number of service calls
✓ correctly states prediction is reliable
✓ correctly mentions strong correlation and/or prediction is close to known data

- (f) Estimate and mark clearly on the scatter plot the position of the point  $(\bar{T}, \bar{S})$  for the 12 plotted points. (1 mark)

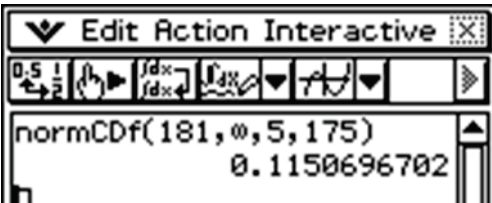
Solution
See graph above (large cross)
Specific behaviours
✓ correctly plots point on line about half way

Question 15

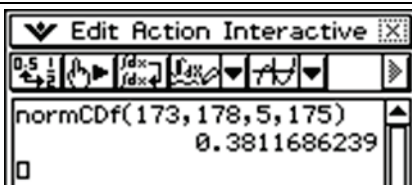
(8 marks)

- (a) The heights of the Year 12 cohort at a particular school were normally distributed, with a mean height of 175 cm and a standard deviation of 5 cm.

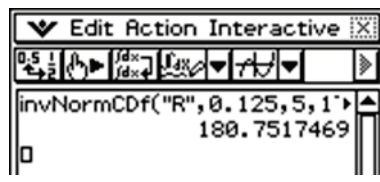
- (i) Determine the percentage of students that were at least 181 cm tall. (2 marks)

Solution	
Using CAS Probability is 0.115 Therefore percentage is 11.5%	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ calculates correct probability</li> <li>✓ states correct percentage</li> </ul>	

- (ii) What is the probability that a Year 12 student selected at random will be between 173 cm and 178 cm tall? (1 mark)

Solution	
Using CAS Probability is 0.381	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ calculates correct probability</li> </ul>	

- (iii) Calculate the maximum height of the middle 75% of Year 12 students, correct to three significant figures. (3 marks)

Solution	
Using CAS right tail = 12.5% gives maximum height of 180.75 cm which is 181 cm (three significant figures)	
Specific behaviours	
<ul style="list-style-type: none"> <li>✓ correctly uses right tail of 12.5%</li> <li>✓ calculates maximum height</li> <li>✓ rounds correctly to three significant figures</li> </ul>	

- (b) At another school, 54 Year 11 students had heights between one and two standard deviations above the mean. Given that the heights were normally distributed, calculate how many students were in Year 11. (2 marks)

<b>Solution</b>
$95\% - 68\% = 27\% \div 2 = 13.5\%$ 54 students represent 13.5% of the Year 12 cohort Therefore 400 students would be 100%
<b>Specific behaviours</b>
✓ correctly calculates percentage of students between one and two standard deviations above the mean ✓ correctly calculates Year 12 cohort



Question 16

(8 marks)

(a) The farmer is restricted to 4000 metres of fencing.

(i) Using this information, write an equation involving  $x$  and  $y$ .

(2 marks)

Solution
$4x + 6y = 4000$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly identifies coefficients of <math>x</math> and <math>y</math></li> <li>✓ states correct equation</li> </ul>

(ii) Rewrite this equation for  $y$  in terms of  $x$ .

(1 mark)

Solution
$y = \frac{4000}{6} - \frac{4x}{6} = \frac{2000}{3} - \frac{2x}{3}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly rewrites equation from (a) (i)</li> </ul>

(b) (i) Show that the area of each paddock is given by  $A = \frac{2000x}{3} - \frac{2x^2}{3}$ .

(2 marks)

Solution
$A = xy$ $= x \left( \frac{2000}{3} - \frac{2x}{3} \right) = \frac{2000x}{3} - \frac{2x^2}{3}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly writes an expression for the area of the paddock</li> <li>✓ correctly substitutes to get the required area formula</li> </ul>

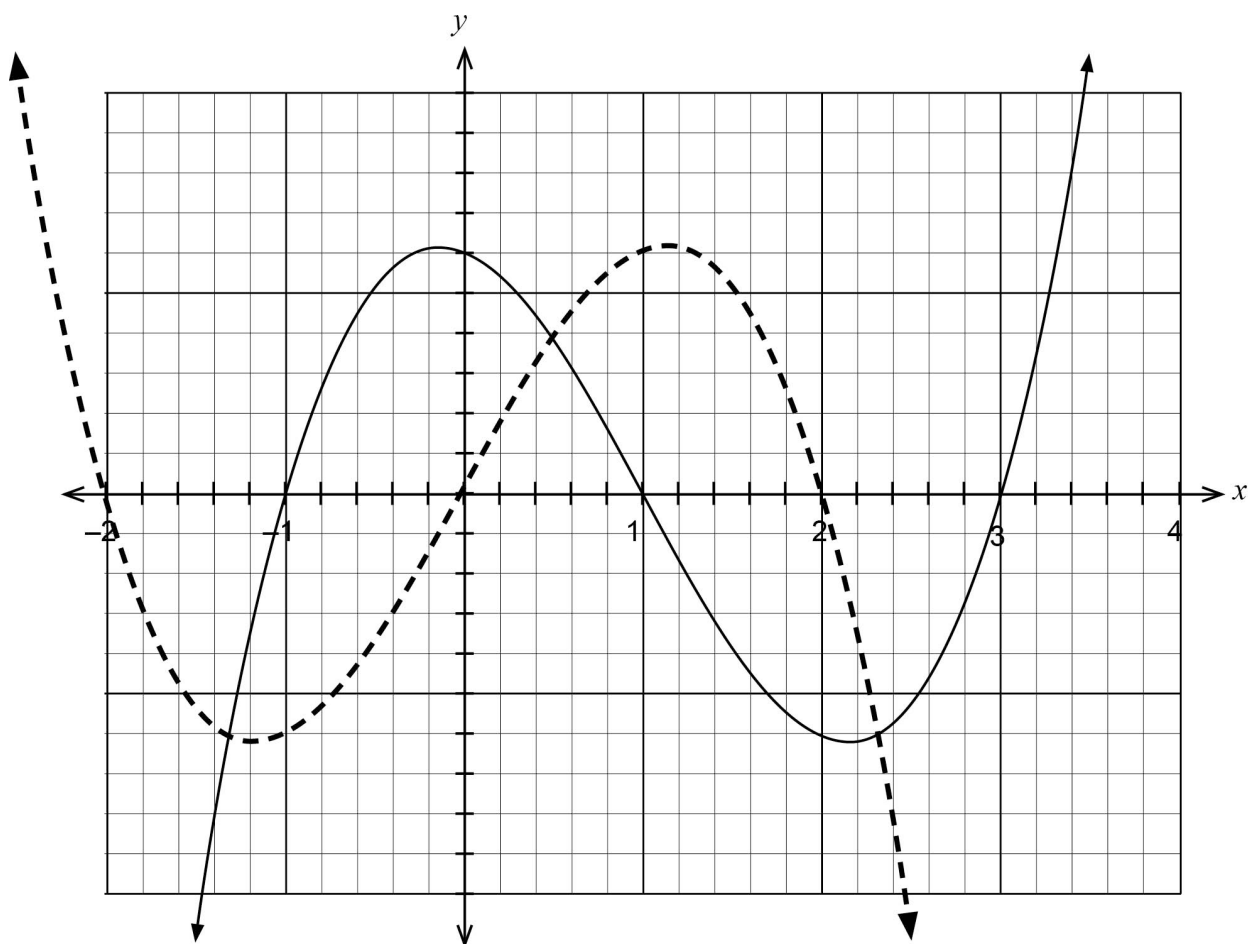
- (ii) Use calculus techniques to determine the largest possible area of each of the three paddocks, and state the dimensions of each paddock. (3 marks)

Solution
$\frac{dA}{dx} = \frac{2000}{3} - \frac{4x}{3} = 0 \text{ when } x = 500$ <p>when <math>x = 500</math> m, <math>y = 333.3</math> m giving a maximum area of 166 666.7 square metres</p>
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly differentiates and equates to zero</li> <li>✓ correctly solves for <math>x</math> and <math>y</math></li> <li>✓ correctly calculates area</li> </ul>

Question 17

(7 marks)

The cubic function  $y = f(x)$  is shown below.



- (a) The function is in the form  $f(x) = a(x-1)(x+b)(x+c)$ . If the coordinates of the  $y$ -intercept are  $(0, 6)$ , determine the values of  $a$ ,  $b$  and  $c$ . (3 marks)

Solution
$b = 1, c = -3$ Using the $y$ -intercept: $6 = a(-1)(1)(-3) \Rightarrow a = 2$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly determines the value of <math>b</math></li> <li>✓ correctly determines the value of <math>c</math></li> <li>✓ correctly determines the value of <math>a</math></li> </ul>

- (b) Given the domain  $-1 \leq x \leq 3$ , for what values of  $x$  is the function concave down, given that  $(1, 0)$  is a point of inflection? (2 marks)

Solution
for $-1 \leq x < 1$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly determines the boundaries</li> <li>✓ does not include the end points at <math>x = 1</math></li> </ul>

- (c) The function is transformed to produce the function  $g(x) = -f(x+1)$ . Sketch  $g(x)$  on the same set of axes as  $f(x)$ . (2 marks)

Solution
See graph above (dashed line)
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly identifies the reflection in the <math>x</math>-axis</li> <li>✓ correctly identifies the translation of one unit to the left</li> </ul>

Question 18

(4 marks)

Prove that one more than  $(n+1)^2 - (n-1)^2$  is always odd, where  $n$  is a positive integer.

Solution
$(n+1)^2 - (n-1)^2$ $= n^2 + 2n + 1 - n^2 + 2n - 1$ $= 4n \quad \therefore \text{one more is } 4n + 1$ $4n \text{ is even since it has a factor of 2, therefore } 4n + 1 \text{ is odd as required}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ correctly expands expression</li> <li>✓ correctly simplifies</li> <li>✓ correctly adds one to get <math>4n + 1</math></li> <li>✓ correctly states <math>4n</math> has a factor of 2 therefore even</li> </ul>

This document – apart from any third party copyright material contained in it – may be freely copied, or communicated on an intranet, for non-commercial purposes in educational institutions, provided that it is not changed and that the School Curriculum and Standards Authority is acknowledged as the copyright owner, and that the Authority's moral rights are not infringed.

Copying or communication for any other purpose can be done only within the terms of the Copyright Act 1968 or with prior written permission of the School Curriculum and Standards Authority. Copying or communication of any third party copyright material can be done only within the terms of the Copyright Act 1968 or with permission of the copyright owners.

Any content in this document that has been derived from the Australian Curriculum may be used under the terms of the Creative Commons Attribution-NonCommercial 3.0 Australia licence.

*Published by the School Curriculum and Standards Authority of Western Australia*  
303 Sevenoaks Street  
CANNINGTON WA 6107