SPECIALIST MATHEMATHICS

Written examination 1

FORMULA SHEET

Directions to students

Remove this formula sheet during reading time.

This formula sheet is provided as a reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{\left(x-h\right)^2}{a^2} + \frac{\left(y-k\right)^2}{b^2} = 1$ hyperbola: $\frac{\left(x-h\right)^2}{a^2} - \frac{\left(y-k\right)^2}{b^2} = 1$

Circular (trigometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$
 $\cot^2(x) + 1 = \csc^2(x)$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$cos(2x) = cos^{2}(x) - sin^{2}(x) = 2cos^{2}(x) - 1 = 1 - 2sin^{2}(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$$

function	sin ⁻¹	\cos^{-1}	tan ⁻¹
domain	[-1, 1]	[-1, 1]	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$\big[0,\pi\big]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$z = x + yi = r(\cos\theta + i\sin\theta) = r\cos\theta$$

$$|z| = \sqrt{x^2 + y^2} = r \qquad -\pi < \text{Arg } z \le \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2) \qquad \qquad \frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

 $z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e(x) + c$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$$

$$\int \sec^2(ax)dx = \frac{1}{a}\tan(ax) + c$$

$$\int \frac{d}{dx} \left(\sin^{-1}(x) \right) = \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \, a > 0$$

$$\int \frac{d}{dx} (\cos^{-1}(x)) = \frac{1}{\sqrt{1 - x^2}} dx = \cos^{-1}(\frac{x}{a}) + c, \, a > 0$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:
$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

quotient rule:
$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method: If
$$\frac{dy}{dx} = f(x)$$
, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration:
$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

constant (uniform) acceleration:
$$v = u + at$$
 $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$ $s = \frac{1}{2}(u + v)t$

Vectors in two and three dimensions

$$\underline{\mathbf{r}} = x\underline{\mathbf{i}} + y\underline{\mathbf{j}} + z\underline{\mathbf{k}}$$

$$\left|\underline{\mathbf{r}}\right| = \sqrt{x^2 + y^2 + z^2} = r$$

$$r_{1\bullet}r_{2} = r_{1}r_{2}\cos\theta = x_{1}x_{2} + y_{1}y_{2} + z_{1}z_{2}$$

$$\dot{\underline{\mathbf{r}}} = \frac{d\underline{\mathbf{r}}}{dt} = \frac{dx}{dt}\,\dot{\mathbf{i}} + \frac{dy}{dt}\,\dot{\underline{\mathbf{j}}} + \frac{dz}{dt}\,\dot{\underline{\mathbf{k}}}$$

Mechanics

momentum: p = mv

equation of motion: R = ma

friction: $F \le \mu N$

END OF FORMULA SHEET