**Semester Two Examination, 2016**

**Question/Answer Booklet**

# MATHEMATICS APPLICATIONS UNITS 3 AND 4

## Section Two: Calculator-assumed

# SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

## Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer Booklet
Formula Sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|---|---|---|---|---|---|
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 97 | 65 |
| | | | **Total** | 149 | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Booklet.

**See next page**

**Section Two: Calculator-assumed** **65% (97 Marks)**

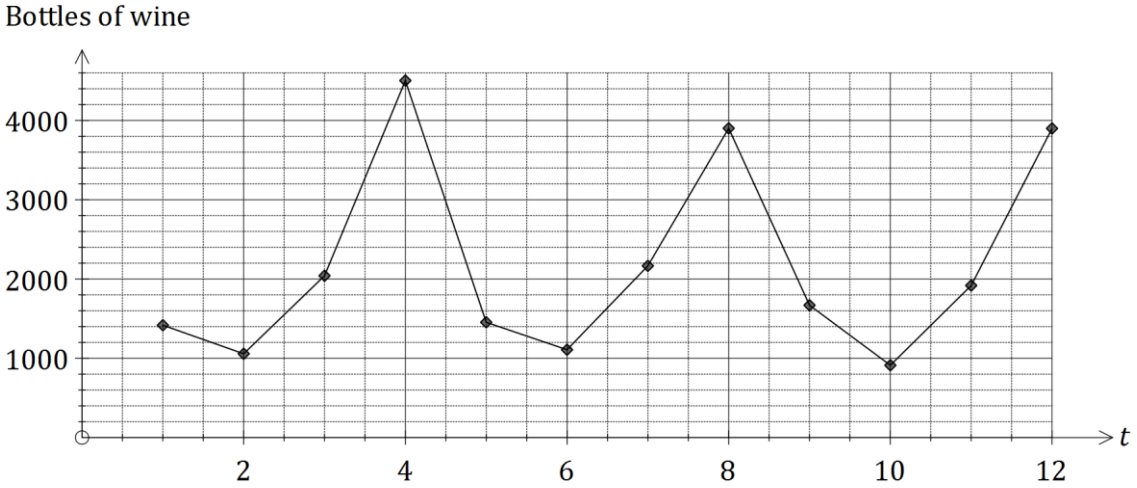
This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

---

**Question 9** **(4 marks)**

The graph and table below show the quarterly sales of bottles of sparkling wine sold by a liquor outlet. The first quarter of 2002 corresponds to $t = 1$, where $t$ is measured in quarters.




| Year | 2002 | | | | 2003 | | | | 2004 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quarter | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| Bottles sold | 1417 | 1057 | 2041 | 4503 | 1454 | 1108 | 2166 | 3903 | 1669 | 912 | 1918 | 3899 |

(a) Determine the value of the centred four-point moving average for the fourth quarter of the year 2003. (2 marks)

| **Solution** |
|---|
| $\dfrac{\frac{1108}{2} + 2166 + 3903 + 1669 + \frac{912}{2}}{4} = 2187$ |
| **Specific behaviours** |
| ✓ shows method involving centering<br>✓ calculates correct value |

(b) Briefly explain:

(i) the purpose of calculating simple moving averages for a time series. (1 mark)

| **Solution** |
|---|
| Smooths time series, making the overall trend of the data easier to see. |
| **Specific behaviours** |
| ✓ makes statement about smoothing data, revealing trend |

(ii) a feature of the graph that suggests the use of a centred four-point moving average is appropriate for this data. (1 mark)

| **Solution** |
|---|
| A regular rise and fall every four points is evident in the graph. |
| **Specific behaviours** |
| ✓ makes statement about repeating pattern every four points |

**See next page**

**Question 10** **(6 marks)**

A survey asked 1660 people whether they believed social classes still exist in Australia. The researchers were interested in whether the belief was dependent on the weekly income of a person. The results are shown in the table below.

| | Weekly income | | | |
|---|---|---|---|---|
| | Under $1000 | $1000 - $1500 | $1500 - $2000 | Over $2000 |
| Yes | 272 | 413 | 494 | 209 |
| No | 27 | 45 | 41 | 22 |
| Don't know | 33 | 40 | 47 | 17 |

(a) Complete the table below, using row or column percentages rounded to the nearest whole number, so that it is possible to determine whether an individual's belief in the existence of social classes is associated with their level of weekly income. (4 marks)

| | Weekly income | | | |
|---|---|---|---|---|
| | Under $1000 | $1000 - $1500 | $1500 - $2000 | Over $2000 |
| Yes | **82** | **83** | **85** | **84** |
| No | **8** | **9** | **7** | **9** |
| Don't know | **10** | **8** | **8** | **7** |

| **Solution** |
|---|
| See table - column totals for first table are 332, 498, 582 and 248 |
| **Specific behaviours** |
| ✓ calculates column totals<br>✓ calculates at least two columns of percentages<br>✓ completes all column percentages<br>✓ rounds consistently to the nearest whole number |

(b) Comment, with reasons, on whether the results of the survey show that the belief that social classes still exist in Australia is dependent on the weekly income of a person. (2 marks)

| **Solution** |
|---|
| No evidence of an association exists between belief and income, as there is little difference in column percentages for each response for the different income groups. |
| **Specific behaviours** |
| ✓ states no evidence from survey<br>✓ supports answer by noting similarities in column percentages |

**See next page**

**Question 11** **(8 marks)**

A reducing balance loan of \$6 590.24 was used to buy a second-hand motorbike. At the end of each month, interest of 0.85% was added to the loan before a repayment of \$580 was made. The table below shows the balance of the loan for the first few months.

| Month $(n)$ | Balance at start of month $(T_n)$ | Interest | Repayment | Balance carried forward to start of next month |
|---|---|---|---|---|
| 1 | 6 590.24 | 56.02 | 580.00 | 6 066.26 |
| 2 | 6 066.26 | 51.56 | 580.00 | 5 537.82 |
| 3 | 5 537.82 | ***A*** | 580.00 | ***B*** |

(a) Write a recurrence relation to model the loan balance in the second column of the table. (2 marks)

| **Solution** |
|---|
| $T_{n+1} = T_n \times 1.0085 - 580, \quad T_1 = 6590.24$ |
| **Specific behaviours** |
| ✓ writes relation<br>✓ includes initial term |

(b) Determine the values of $A$ and $B$ in the table. (2 marks)

| **Solution** |
|---|
| $A = 5537.82 \times 0.0085 = \$47.07$<br>$B = 5537.82 + 47.07 - 580 = \$5004.89$ |
| **Specific behaviours** |
| ✓ calculates $A$<br>✓ calculates $B$ |

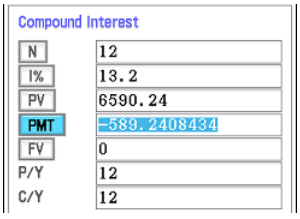
(c) Determine how many repayments are needed to reduce the balance to zero and show that the total amount of interest paid is \$369.76. (2 marks)

| **Solution** |
|---|
| 12 repayments.<br>$I = 12 \times 580 - 6590.24 = 6960 - 6590.24 = \$369.76$ |
| **Specific behaviours** |
| ✓ states 12 repayments<br>✓ shows interest calculation |

(d) Determine the new monthly repayment for the above loan, if the only change to the loan structure is:

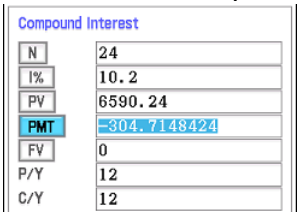
(i) the interest rate increased to 1.1% per month. (1 mark)

| **Solution** |
|---|
| \$589.24 |
| **Specific behaviours** |
| ✓ states repayment |




(ii) the interest rate remained at 0.85% per month but the loan was repaid over two years. (1 mark)

| **Solution** |
|---|
| \$304.71 |
| **Specific behaviours** |
| ✓ states repayment |




**See next page**

**Question 12** **(7 marks)**

A report on the expected population growth of eleven Asian cities between 2015 and 2030 included the following statistics.

| City | Country | 2015 Pop (millions) $x$ | 2030 Pop (millions) $y$ |
|---|---|---|---|
| Tokyo | Japan | 38 | 37 |
| Delhi | India | 26 | 36 |
| Shanghai | China | 24 | 31 |
| Mumbai | India | 21 | 28 |
| Dhaka | Bangladesh | 18 | 27 |
| Calcutta | India | 15 | 19 |
| Manila | Philippines | 13 | 17 |
| Tianjin | China | 11 | 15 |
| Jakarta | Indonesia | 10 | 14 |
| Bangkok | Thailand | 9 | 12 |
| Chengdu | China | 8 | 10 |

(a) Calculate the correlation coefficient between $x$ and $y$. (1 mark)

| **Solution** |
|---|
| $r = 0.945$ |
| **Specific behaviours** |
| ✓ states coefficient to at least 2 dp |

(b) Re-calculate the correlation coefficient between $x$ and $y$ with the data point for Tokyo removed and explain why it has increased. (2 marks)

| **Solution** |
|---|
| $r = 0.991$<br>$r$ has increased since the remaining points lie much closer to a straight line. |
| **Specific behaviours** |
| ✓ states new coefficient<br>✓ reasonable attempt at explanation |

(c) Excluding the data point for Tokyo, determine the equation of the least-squares line to model the relationship between $x$ and $y$. (2 marks)

| **Solution** |
|---|
| $\hat{y} = 1.371x - 0.353$ |
| **Specific behaviours** |
| ✓ states equation<br>✓ rounds coefficients to no more than 3 dp |

(d) Use the least-squares line to predict the population of Guangzhou (China) in 2030, given that it's 2015 population was 12 million, and comment on the reliability of your prediction. (2 marks)

| **Solution** |
|---|
| $\hat{y} = 1.371(12) - 0.353 \approx 16.1$ million<br>Very reliable prediction as no extrapolation involved and correlation is very strong. |
| **Specific behaviours** |
| ✓ makes prediction, rounding sensibly<br>✓ states very reliable with at least one reason |

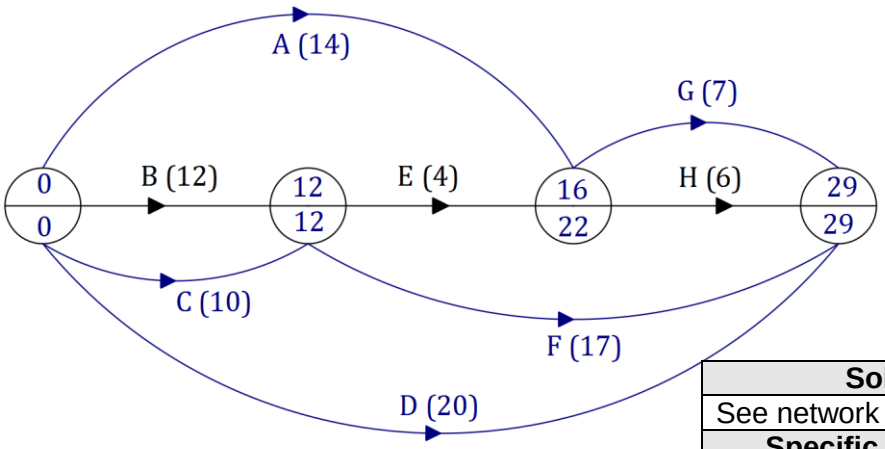
**See next page**

**Question 13** **(8 marks)**

The tasks involved in a project, their immediate predecessors and duration, are shown below.

| Task | Duration (minutes) | Predecessor(s) | Earliest Start Time | Latest Start Time | Float Time |
|---|---|---|---|---|---|
| A | 14 | - | 0 | 2 | 2 |
| B | 12 | - | 0 | 0 | 0 |
| C | 10 | - | 0 | 2 | 2 |
| D | 20 | - | 0 | 9 | 9 |
| E | 4 | B, C | 12 | 18 | 6 |
| F | 17 | B, C | 12 | 12 | 0 |
| G | 7 | A, E | 16 | 22 | 4 |
| H | 6 | A, E | 16 | 23 | 5 |

(a) Use the information above to complete the project network below. (3 marks)




| Solution |
|---|
| See network |
| **Specific behaviours** |
| ✓ at least 3 tasks added |
| ✓ all tasks added |
| ✓ labels and direction |

(b) Use forward and backward scanning to determine the earliest start time, latest start time and float time for each task, writing all values in the table above. (3 marks)

| Solution |
|---|
| See table |
| **Specific behaviours** |
| ✓ correct EST's, ✓ correct LST's, ✓ correct floats |

(c) List the tasks on the critical path for this project and state the minimum completion time. (2 marks)

| Solution |
|---|
| B and F - 29 minutes |
| **Specific behaviours** |
| ✓ identifies tasks on CP<br>✓ states MCT |

**See next page**

**Question 14** **(8 marks)**

A car scrapyard had a stockpile of 2 500 old tyres. A recycling program was introduced to reduce the number of tyres in the stockpile by 30% each week, but at the end of each week another 120 tyres were added to the pile from newly scrapped cars.

The number of tyres, $T_n$, in the stockpile at the start of week $n$ can be modelled by the recursive rule

$$T_{n+1} = T_n \times 0.7 + 120, \qquad T_1 = 2500$$

(a) Explain the significance of the 0.7 multiplier in the rule. (1 mark)

| Solution |
|---|
| 30% of tyres are removed each week, so 70% (0.7) remain. |
| **Specific behaviours** |
| ✓ reasonable explanation |

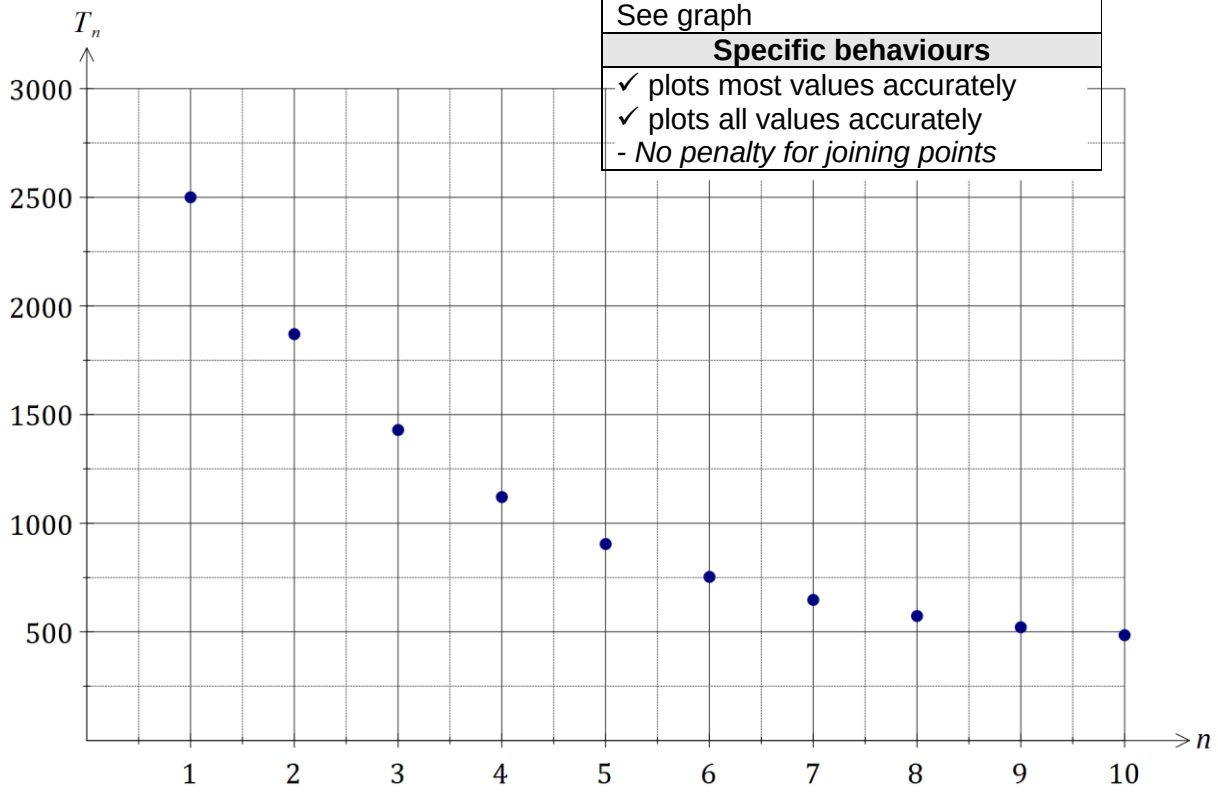
(b) Complete the table below, rounding the missing numbers of tyres to the nearest whole number. (2 marks)

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $T_n$ | 2400 | 1800 | 1380 | 1086 | 880 | 736 | 635 | 565 | 515 | 481 |

| Solution |
|---|
| See table |
| **Specific behaviours** |
| ✓ at least two values correct |
| ✓ all values correct and rounded as required |

(c) Graph the number of tyres in the stockpile at the start of each week on the axes below. (2 marks)

| Solution |
|---|
| See graph |
| **Specific behaviours** |
| ✓ plots most values accurately |
| ✓ plots all values accurately |
| *- No penalty for joining points* |




**See next page**

(d) Use the graph to comment on how the number of tyres in the stockpile is changing. (1 mark)

| **Solution** |
| --- |
| The number of tyres decreased rapidly to start with but is levelling out. |
| **Specific behaviours** |
| ✓ makes valid comment using the shape of their graph. |

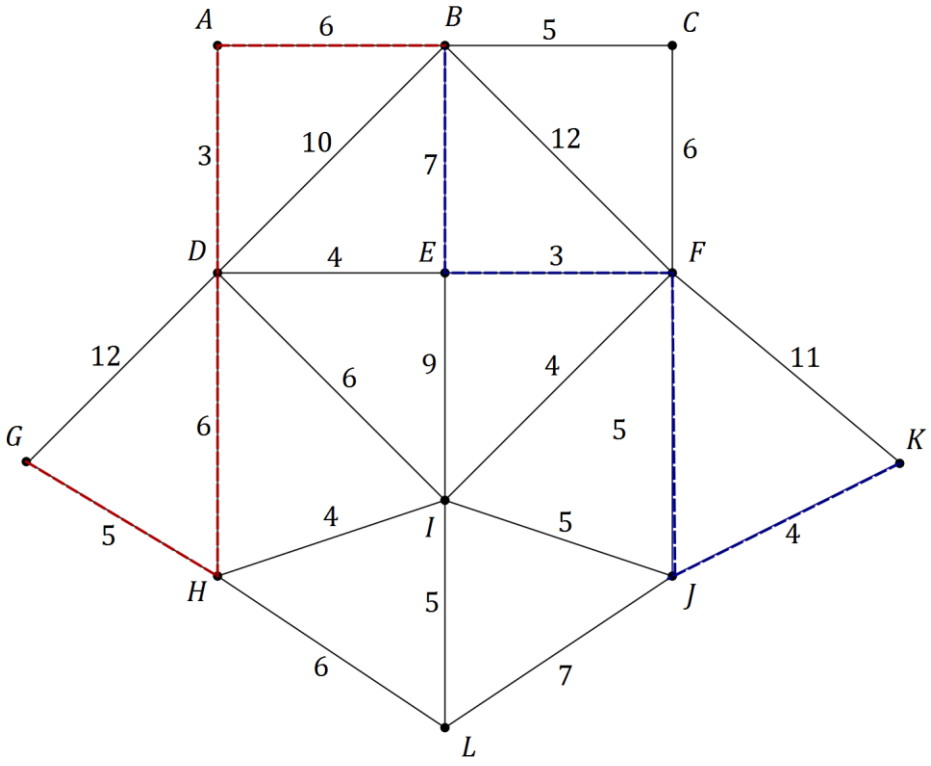
(e) According to the model, how many tyres will the stockpile contain after many years have passed? (2 marks)

| **Solution** |
| --- |
| $x = 0.7x + 120 \Rightarrow 0.3x = 120 \Rightarrow x = 400$ |
| **Specific behaviours** |
| ✓ Sets up a steady state equation<br>✓ solves equation for long term steady state value |

**See next page**

**Question 15** **(10 marks)**

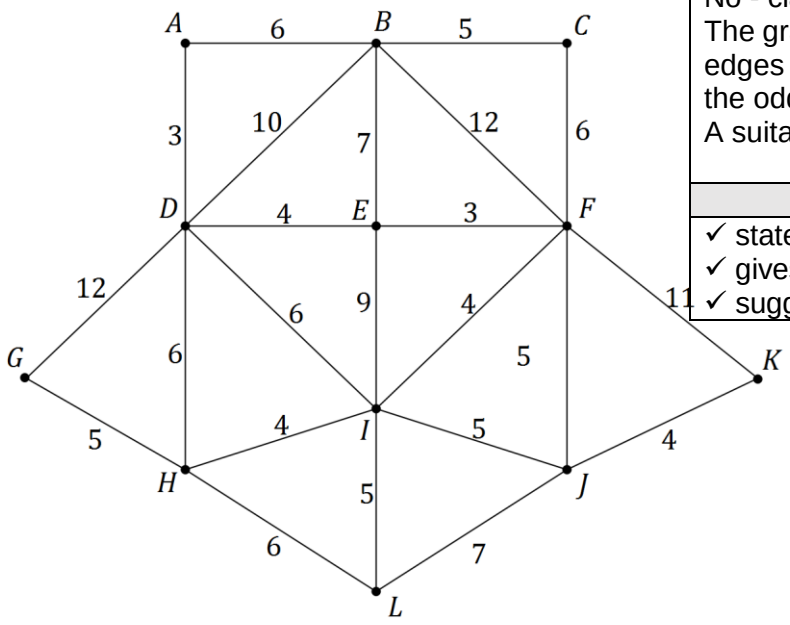
The network below shows the times, in minutes, taken by taxis to travel along roads between junctions $A$, $B$, $C$, ..., $L$.




(a) A taxi is required at junction $B$, and the two taxis currently available are at $G$ and $K$. Determine which taxi will reach junction $B$ first, justifying your answer by stating the minimum travel time and route for each taxi. (5 marks)

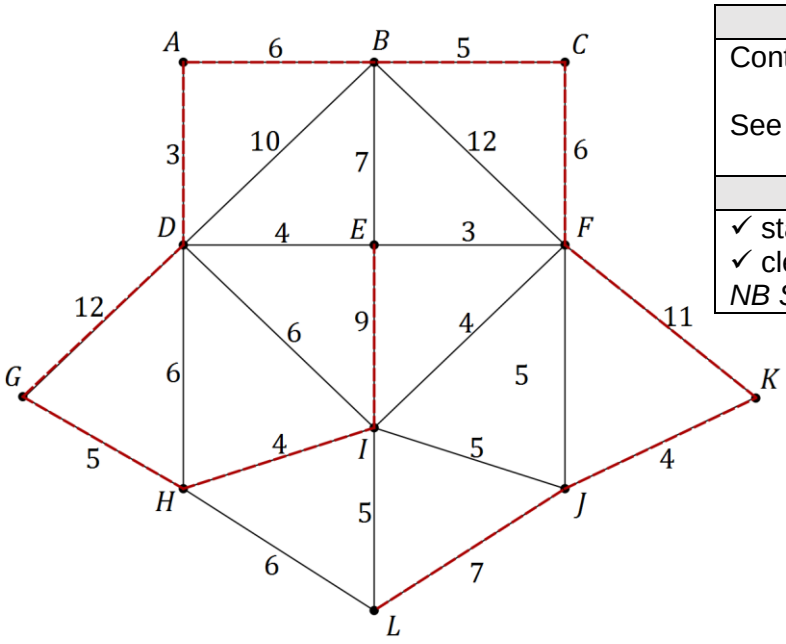
| Solution |
|---|
| Taxi from G: G-H-D-A-B = 20 minutes<br><br>Taxi from K: K-J-F-E-B = 19 minutes<br><br>Taxi from G will reach B first. |
| **Specific behaviours** |
| ✓ states taxi from K arrives first<br>✓ lists, in order, route from G<br>✓ states total time from G<br>✓ lists, in order, route from K<br>✓ states total time from K |

(b) A taxi driver told another driver that it was possible to drive along all the roads shown on the network just once, starting from $A$. Was the driver's claim correct? If yes, show a possible route on the copy of the network below and state its length. If no, explain why and suggest, if possible, a different starting point from which such a route would be possible. (3 marks)




| Solution |
|---|
| No - claim was not correct.<br>The graph is semi-Eulerian. To travel along all edges just once, the starting point must be one of the odd vertices, which A is not.<br>A suitable starting point would be either B or L. |
| **Specific behaviours** |
| ✓ states claim not correct<br>✓ gives reasonable explanation<br>✓ suggests suitable starting point |

(c) Does the network contain a Hamiltonian cycle, Hamiltonian path or neither? If the latter, explain why. If a Hamiltonian cycle or path exists, clearly show it on the copy of the network below. (2 marks)

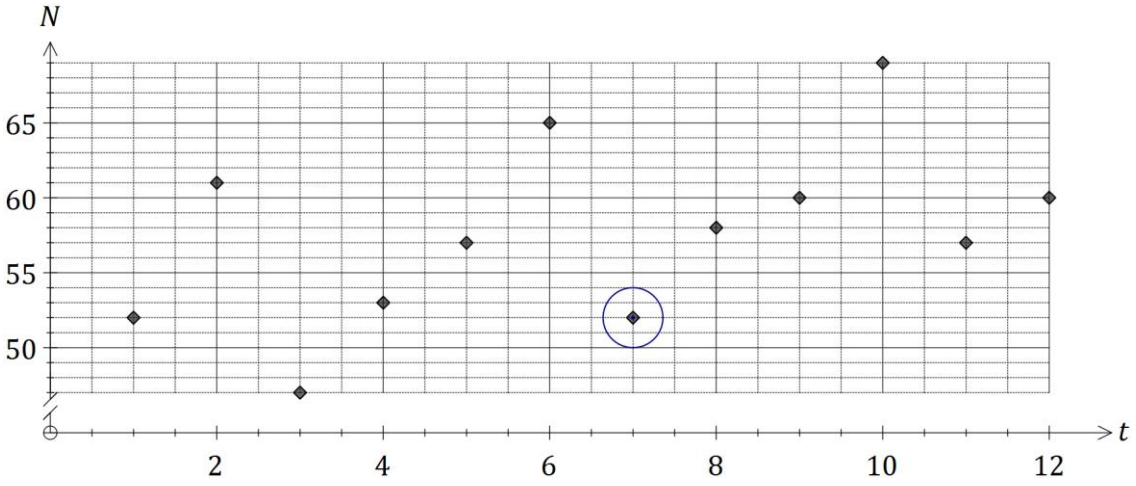



| Solution |
|---|
| Contains a Hamiltonian path.<br><br>See network for an example. |
| **Specific behaviours** |
| ✓ states Hamiltonian path<br>✓ clearly shows path on diagram<br>*NB Several variations exist* |

**See next page**

**Question 16** **(10 marks)**

The number of orders for new homes that a building company received each quarter over the past three years is shown in the graph and table below.




| Year | Quarter | Time period ($t$) | Number of orders ($N$) | Yearly mean | Percent of yearly mean |
|---|---|---|---|---|---|
| 2013 | 1 | 1 | 52 | $\boldsymbol{A}$ | 97.7% |
| | 2 | 2 | 61 | | 114.5% |
| | 3 | 3 | 47 | | 88.3% |
| | 4 | 4 | 53 | | 99.5% |
| 2014 | 1 | 5 | 57 | 58.00 | 98.3% |
| | 2 | 6 | 65 | | 112.0% |
| | 3 | 7 | $\boldsymbol{B}$ | | $\boldsymbol{C}$ |
| | 4 | 8 | 58 | | 100.0% |
| 2015 | 1 | 9 | 60 | 61.50 | $\boldsymbol{D}$ |
| | 2 | 10 | 69 | | 112.2% |
| | 3 | 11 | 57 | | 92.7% |
| | 4 | 12 | 60 | | 97.5% |

(a) Calculate the values of the entries $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ in the table. (4 marks)

| **Solution** |
|---|
| $A = \frac{52+61+47+53}{4} = \frac{213}{4} = 53.25$<br><br>$\frac{57+65+B+58}{4} = 58 \Rightarrow B = 52$<br><br>$C = \frac{52}{58} \times 100 = 89.7\%$<br><br>$D = \frac{60}{61.5} \times 100 = 97.6\%$ |
| **Specific behaviours** |
| ✓ calculates A, ✓ calculates B<br>✓ calculates C, ✓ calculates D<br>** May use sum of year %'s = 400 for C, D* |

**See next page**

(b) Plot the number of orders received in the third quarter of 2014 on the graph. (1 mark)

| **Solution** |
|---|
| See graph |
| **Specific behaviours** |
| ✓ plots point |

(c) Calculate the second quarter seasonal index and use it to deseasonalise the sales for the second quarter of 2015. (2 marks)

| **Solution** |
|---|
| Index: $\frac{1.145+1.120+1.122}{3} = 1.129$<br><br>Deseasonalised sales: $\frac{69}{1.129} = 61.1$ |
| **Specific behaviours** |
| ✓ calculate index<br>✓ deseasonalise number |

(d) The trend in the deseasonalised number of homes, $n$, can be modelled by the line with equation $n = 0.935t + 51.509$. Forecast the actual number of homes that are likely to be ordered in the second quarter of 2016 if the above seasonality and trends continue. (3 marks)

| **Solution** |
|---|
| Trend: $n = 0.935(14) + 51.509 = 64.599$<br><br>Seasonal adjustment: $64.599 \times 1.129 = 72.93$<br><br>Expect 73 homes to be ordered |
| **Specific behaviours** |
| ✓ calculates correct trend value<br>✓ makes seasonal adjustment<br>✓ rounds to nearest whole number |

**See next page**

**Question 17** **(7 marks)**

An investor purchased a set of old coins for \$12 900, expecting the value of the set to increase by the 5.5% each year.

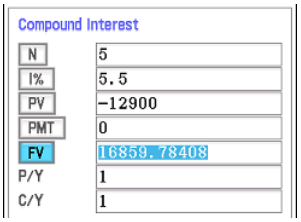
(a) A recurrence relation that can be used to model the value, $V_n$, of the set of coins $n$ years after they were bought is $V_{n+1} = V_n \times r,\ V_o = a$.

(i) State the values of $r$ and $a$ in this relation. (2 marks)

| Solution |
|---|
| $r = 1.055$ and $a = 12900$ |
| **Specific behaviours** |
| ✓ states $r$<br>✓ states $a$ |

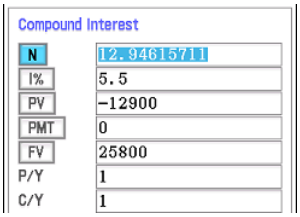
(ii) Use the relation to calculate the expected value of the set of coins after five years, giving your answer to the nearest one hundred dollars. (2 marks)

| Solution |
|---|
| Exactly \$16 859.78 which rounds to \$16 900 to nearest \$. |
| **Specific behaviours** |
| ✓ calculates value<br>✓ rounds to nearest hundred dollars |



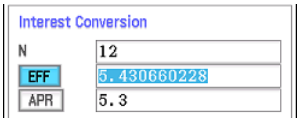

(iii) How many years will it take for the value of the set of coins to double? (1 mark)

| Solution |
|---|
| 13 years. |
| **Specific behaviours** |
| ✓ states whole number of years |




(b) The investor could have left the \$12 900 in a savings account paying interest of 5.3% per annum, compounded monthly. Determine the effective annual interest rate of this account and comment on whether the savings account would have been a better option for the investor. (2 marks)

| Solution |
|---|
| $R_E = 5.43\%$<br><br>Not a better option, as $5.43\%$ is less than the expected annual increase of the coins. |
| **Specific behaviours** |
| ✓ states effective rate as a percentage<br>✓ states not better option, with reason. |




**See next page**

**Question 18** **(7 marks)**

The table below shows the number of people who cycled to work ($n$, in hundreds) and the average commute time for motorists ($t$, in minutes) for a small city over a number of years.

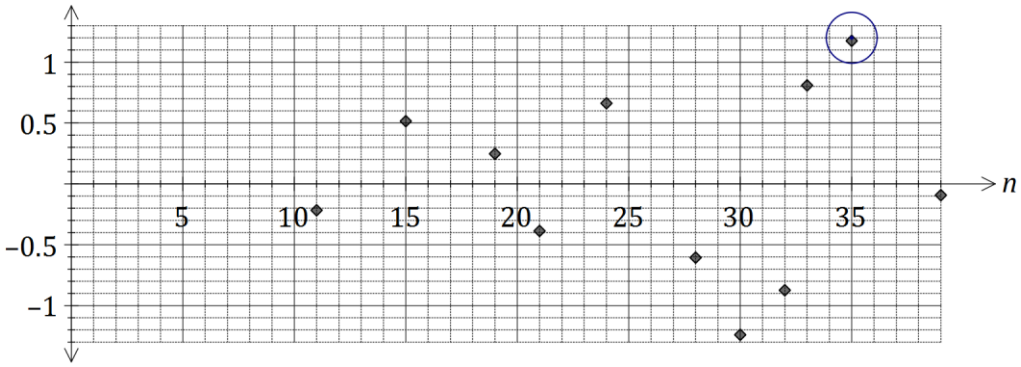
| Year | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ (00's) | 11 | 15 | 19 | 21 | 24 | 28 | 30 | 32 | 33 | 35 | 39 |
| $t$ (m) | 25 | 27 | 28 | 28 | 30 | 30 | 30 | 31 | 33 | 34 | 34 |

The least-squares line that models the linear relationship between the variables is $t = 0.317n + 21.73$ and the correlation coefficient is $0.966$.

(a) What percentage of the variation in average commute times can be explained by the variation in the number of cyclists? (1 mark)

| Solution |
|---|
| $0.966^2 \approx 0.93$ so 93% |
| **Specific behaviours** |
| ✓ calculates correct percentage |

The residual plot for the linear model is shown below.




(b) Calculate the missing residual for 2010 and plot it on the graph above. (3 marks)

| Solution |
|---|
| $\hat{t} = 0.317 \times 35 + 21.73 = 32.825$; $34 - 32.825 = 1.175$<br>See circled point on graph |
| **Specific behaviours** |
| ✓ calculates $\hat{t}$<br>✓ calculates residual<br>✓ plots point (35, 1.175) |

(c) Use the residual plot to explain whether fitting a linear model to the data is appropriate. (2 marks)

| Solution |
|---|
| Linear model is appropriate as there are no patterns evident with the displayed residuals. |
| **Specific behaviours** |
| ✓ states yes<br>✓ states reason |

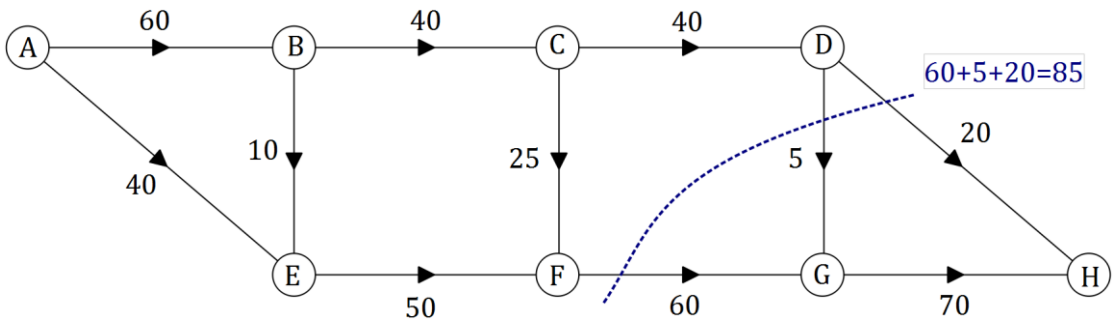
(d) Can the above data be used to conclude that the variables $n$ and $t$ are causally related? Briefly explain your answer. (1 mark)

| Solution |
|---|
| No. It is likely there is confounding due to a common response to another variable such as population size. |
| **Specific behaviours** |
| ✓ states no with sensible reason. |

**See next page**

**Question 19** **(7 marks)**

In a loan application office, incoming loans are reviewed at desk $A$ and then move through various checks carried out at desks $B$, $C$, $D$, $E$, $F$ and $G$ before final approval is made at desk $H$. The weighted digraph below shows the maximum number of loans that can pass between desks during one working day.




(a) Determine, showing systematic working, the maximum number of loan applications that can be processed through the office in any one day. (3 marks)

| Solution (A) |
|---|
| ABCDH - 20<br>ABCDGH - 5<br>ABCFGH - 15<br>ABEFGH - 10<br>AEFGH - 35<br>Total - 85 loan applications per day |
| **Specific behaviours** |
| ✓ at least three paths with correct flow contribution<br>✓ all paths with correct flow contribution<br>✓ states maximum flow |

| Solution (B) |
|---|
| See min cut on diagram<br>Total - 85 loan applications per day |
| **Specific behaviours** |
| ✓ shows cuts with values<br>✓ shows min cut<br>✓ states maximum flow |

(b) Between which two desks does the largest spare capacity to move loan applications exist, and what is the spare capacity? (2 marks)

| Solution |
|---|
| Between desks C and D - spare capacity of 15 applications per day. |
| **Specific behaviours** |
| ✓ states correct desks<br>✓ states spare capacity |

(c) An internal review identified that more loan applications could be processed in a day by increasing the number currently transferred between just two of the desks. Which were the two desks identified and how many additional loan applications could be processed each day? (2 marks)

| Solution |
|---|
| Between F and G - an increase of 5 loan applications per day. |
| **Specific behaviours** |
| ✓ states desks<br>✓ states increase |

**See next page**

**Question 20** **(7 marks)**

The value, $V_n$ in hundreds of dollars, of a large computer system, $n$ months after being purchased, can be modelled by a geometric sequence. The first few values are shown in the table below.

| Month, $n$ | 0 | 1 | 2 |
|---|---|---|---|
| Value, $V_n$ (hundreds of dollars) | 800 | 760 | 722 |

(a) Determine the percentage decrease in the value of the system each month. (2 marks)

| **Solution** |
|---|
| $r = \frac{760}{800} = 0.95.$<br>Hence $100\% - 95\% = 5\%$ decrease per month. |
| **Specific behaviours** |
| ✓ calculates ratio of two consecutive terms<br>✓ states percentage decrease |

(b) Use the information in the table to deduce a non-recursive rule for $V_n$ in terms of $n$. (2 marks)

| **Solution** |
|---|
| $V_n = 800(0.95)^n$ |
| **Specific behaviours** |
| ✓ uses first term and ratio from (a) in rule for n$^{th}$ term<br>✓ determines rule shown or equivalent |

(c) Determine the decrease in value of the system one year after purchase. (2 marks)

| **Solution** |
|---|
| $V_{12} = 800(0.95)^{12} \approx 432.3$<br>Decrease is $800 - 432.3 = 367.7$, so a decrease of \$36 770 |
| **Specific behaviours** |
| ✓ calculates value after 12 months<br>✓ determines loss, clearly stating units |

(d) State, with reasons, whether the dollar decrease in value of the system during the second year will be the same, more, or less than the decrease over the first year. (1 mark)

| **Solution** |
|---|
| Less - as smaller initial value to depreciate at the start of second year. |
| **Specific behaviours** |
| ✓ correct response with valid reason or calculation shown |

**See next page**

**Question 21** **(8 marks)**

An annuity is set up with an initial sum of \$200 000 and guaranteed interest of 4.2% per annum, compounded monthly.

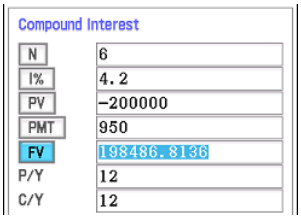
(a) If regular monthly payments of \$950 are withdrawn at the end of each month, just after interest is added, the balance of the annuity, $A_n$, after $n$ withdrawals, will be given by the recurrence relation $A_{n+1} = 1.0035 \times A_n - d,\ A_0 = 200000$.

(i) Explain the relevance of the figure $1.0035$ in this relation and state the value of $d$. (2 marks)

| Solution |
|---|
| $1.0035 = 1 + \frac{4.2}{12 \times 100}$ - adds monthly interest, and $d = 950$ |
| **Specific behaviours** |
| ✓ explains $1.0035$ is adjusted interest rate<br>✓ states $d$ |

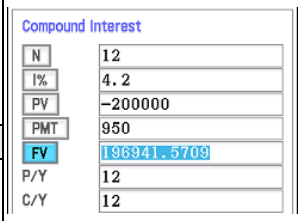
(ii) State the value of the annuity after six withdrawals. (1 mark)

| Solution |
|---|
| \$198 486.81 |
| **Specific behaviours** |
| ✓ states correct value |




(iii) Calculate the amount the total interest accrued by the annuity just after the 12th withdrawal. (3 marks)

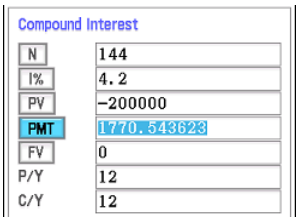
| Solution |
|---|
| $200000 - 12 \times 950 = 200000 - 11400 = 188600$<br>$A_{12} = 196941.57$<br>$I = 196941.57 - 188600 = \$8\ 341.57$ |
| **Specific behaviours** |
| ✓ determines value of annuity without interest<br>✓ determines value of annuity with interest<br>✓ calculates interest |




(b) Determine the maximum monthly withdrawal that can be made if the annuity is to last

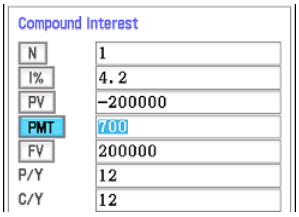
(i) for exactly 12 years. (1 mark)

| Solution |
|---|
| \$1 770.54 |
| **Specific behaviours** |
| ✓ states withdrawal |




(ii) forever, that is, be a perpetuity. (1 mark)

| Solution |
|---|
| \$700 |
| **Specific behaviours** |
| ✓ states withdrawal |




**End of questions**

**Additional working space**

Question number: _____