Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS METHODS UNIT 3

Section Two: Calculator-assumed

SOLUTIONS

Student Name________________________________

Teacher Name________________________________

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work:	ten minutes
Working time:	one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

SN115-195-4

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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Question 8 **(8 marks)**

A small body moving in a straight line has an initial velocity of 15 cm/s as it leaves point P. The acceleration of the body at time t seconds is $6 - 1.5t$ cm^2/s, $t \geq 0$.

(a) Determine the displacement of the body relative to P after 2 seconds. (4 marks)

Solution
$v = \int 6 - 1.5t\,dt = 6t - 0.75t^2 + c$ $t = 0, v = 15 \Rightarrow c = 15$ $v(t) = 6t - 0.75t^2 + 15$ $x(2) = \int_0^2 v(t)\,dt \quad \text{OR} \quad x(t) = 3t^2 - 0.25t^3 + 15t$ $= 40 \text{ cm}$
Specific behaviours
✓ antidifferentiates acceleration, with constant   ✓ obtains expression for velocity   ✓ integral for change in displacement OR displacement function   ✓ correct displacement

(b) Determine the maximum velocity of the body. (2 marks)

Solution
$a = 0 \Rightarrow t = 4$ $v(4) = 27 \text{ cm/s}$
Specific behaviours
✓ indicates time   ✓ correct maximum velocity

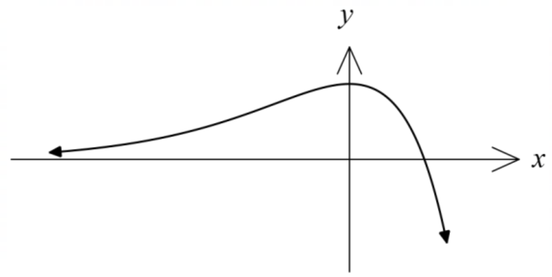
(c) Determine the maximum displacement of the body relative to P. (2 marks)

Solution
$v = 0 \Rightarrow t = 10$ $x(10) = \int_0^{10} v(t)\,dt = 200 \text{ cm}$
Specific behaviours
✓ indicates time   ✓ correct maximum displacement

Question 9 **(7 marks)**

Let $f(x) = (4 - x)e^{0.25x}$.

The graph of $y = f(x)$ is shown at right.



Use calculus to determine the coordinates of the stationary point, the point of inflection and to justify that the stationary point is a local maximum.

Solution
$$f'(x) = -\frac{xe^{0.25x}}{4}$$ $$f'(x) = 0 \text{ when } -\frac{xe^{0.25x}}{4} = 0 \Rightarrow x = 0, \text{and } f(0) = 4.$$ $$f''(x) = -\frac{xe^{0.25x} + 4e^{0.25x}}{16}$$ $$f''(0) = -{}^{1}/_{4}, \text{ and so } f''(0) < 0, \text{curve concave down.}$$ Hence the stationary point is a local maximum and is located at $(0, 4)$. $$f''(x) = 0 \text{ when } -\frac{xe^{0.25x} + 4e^{0.25x}}{16} = 0 \Rightarrow x = -4$$ $$f(-4) = {}^{8}/_{e} \quad (\approx 2.94)$$ Hence the point of inflection is at $(-4, {}^{8}/_{e})$.
Specific behaviours
✓ obtains first derivative ✓ sets first derivative equal to zero and solves for x ✓ obtains second derivative ✓ shows second derivative at stationary point is less than zero ✓ concludes stationary point is a maximum and states coordinates ✓ sets second derivative equal to zero and solves for x ✓ states coordinates of point of inflection

Question 10 **(8 marks)**

Due to the natural variability in the size, density and so on of fruit and syrup used in canning peaches, a cannery purposely overfills the cans it produces. However, some cans still end up being underweight and the probability p that a canning machine produces such a can is known to be constant. Random samples of n cans are taken from the machine and the number X of underweight cans in each sample is recorded. The mean and standard deviation of X are 2.16 and 1.44 respectively.

(a) X is a discrete random variable. Explain why it is **discrete** and **random**. (2 marks)

Solution
Discrete: it can only take integer values between 0 and n (or a countable number of distinct values). Random: the number of underweight cans in a sample is not predictable, or by chance (or its possible values are numerical outcomes resulting from a random process).
Specific behaviours
✓ explains discrete ✓ explains random (Do not accept 'because sampling is random', etc)

(b) Name the distribution of X and determine the value of n and the value of p. (3 marks)

Solution
Distribution of X is binomial. $$np = 2.16$$ $$np(1-p) = 1.44^2$$ Solving simultaneously gives $n = 54$ and $p = \frac{1}{25} = 0.04$.
Specific behaviours
✓ states binomial distribution ✓ forms simultaneous equations ✓ value of p and value of n

(c) Another canning machine produces an underweight can with a probability of 0.015. Determine the probability that when a random sample of 30 cans from this machine are weighed

(i) exactly one of the cans is underweight. (2 marks)

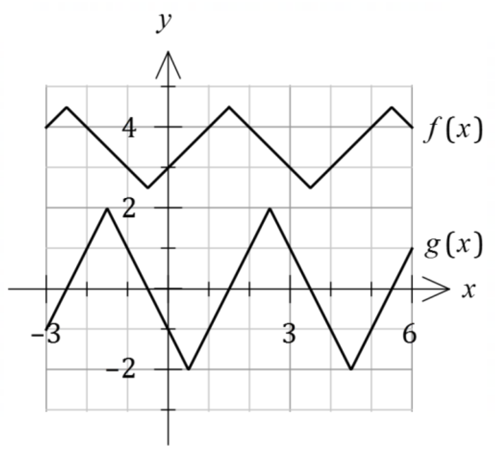
Solution
Let be the number of underweight cans, so that $Y \sim B(30, 0.015)$. $P(Y = 1) = 0.2903$.
Specific behaviours
✓ states distribution ✓ correct probability

(ii) more than one of the cans are underweight. (1 mark)

Solution
$P(Y \geq 2) = 0.0742$
Specific behaviours
✓ correct probability

Question 11 **(8 marks)**

The graphs of the continuous functions $y = f(x)$ and $y = g(x)$ are shown at right.



(a) Evaluate the derivative of $f(x)g(x)$ at $x = -2$. (2 marks)

Solution
$\frac{d}{dx}\left(f(x)g(x)\right)_{x=-2} = f'(-2)g(-2) + f(-2)g'(-2)$ $= (-1)(1) + (4)(2)$ $= 7$
Specific behaviours
✓ indicates correct values of $f'(-2)$ and $g'(-2)$ ✓ correctly evaluates derivative

(b) Evaluate the derivative of $f\left(g(x)\right)$ at $x = 5$. (3 marks)

Solution
$\frac{d}{dx}f\left(g(x)\right)_{x=5} = f'\left(g(5)\right) \times g'(5)$ $= f'(-1) \times g'(5)$ $= -1 \times 2$ $= -2$
Specific behaviours
✓ indicates correct application of chain rule ✓ indicates correct value of $f'\left(g(5)\right)$ ✓ correctly evaluates derivative

(c) Evaluate the derivative of $\frac{g'(x)}{f(x)}$ at $x = 0$. (3 marks)

Solution
$\frac{d}{dx}\left(\frac{g'(x)}{f(x)}\right)_{x=0} = \frac{g''(0)f(0) - g'(0)f'(0)}{\left(f(0)\right)^2}$ $= \frac{(0)(3) - (-2)(1)}{3^2}$ $= {}^{2}/_{9}$
Specific behaviours
✓ indicates correct application of quotient rule ✓ indicates correct value of $g''(0)$ ✓ correctly evaluates derivative

Question 12 **(8 marks)**

A bag contains four black and four red balls. Two balls are drawn at random and in succession from the bag. At the first draw, if the ball is black it is replaced in the bag, otherwise the ball is not replaced. Let X be the number of black balls drawn.

(a) Determine $P(X = 2)$. (1 mark)

Solution
$P(X = 2) = \frac{4}{8} \times \frac{4}{8} = \frac{1}{4}$
Specific behaviours
✓ correct probability

(b) Use exact values to complete the probability distribution table for X below. (3 marks)

x	$P(X = x)$
0	$\frac{3}{14}$
1	$\frac{15}{28}$
2	$\frac{1}{4}$

Solution
$P(X = 0) = \frac{4}{8} \times \frac{3}{7} = \frac{3}{14}$ $P(X = 1) = 1 - \frac{1}{4} - \frac{3}{14} = \frac{15}{28}$
Specific behaviours
✓ calculates $P(X = 0)$ ✓ calculates $P(X = 1)$ ✓ completes table using correct exact values

(c) Determine the mean and variance of X. (4 marks)

Solution
$E(X) = 0 \times \frac{3}{14} + 1 \times \frac{15}{28} + 2 \times \frac{1}{4} = \frac{29}{28} \quad (\approx 1.036)$ $\mathrm{Var}(X) = \left(0 - \frac{29}{28}\right)^2 \times \frac{3}{14} + \left(1 - \frac{29}{28}\right)^2 \times \frac{15}{28} + \left(2 - \frac{29}{28}\right)^2 \times \frac{1}{4} = \frac{363}{784} \quad (\approx 0.463)$ Alternatively, $\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = \left(0^2 \times \frac{3}{14} + 1^2 \times \frac{15}{28} + 2^2 \times \frac{1}{4}\right) - \left(\frac{29}{28}\right)^2 = \frac{363}{784} (\approx 0.463)$
Specific behaviours
✓ expression for mean   ✓ correct mean   ✓ expression for variance   ✓ correct variance

Question 13 **(8 marks)**

The following table shows the probability distribution of a discrete random variable X, where k is a constant.

x	-1	1	2	4
$P(X=x)$	$3k$	0.2	$10k^2$	0.4

(a) Determine the value of k. (3 marks)

Solution
$3k+0.2+10k^2+0.4=1$ $10k^2+3k-0.4=0$ $k=\frac{1}{10}=0.1\ \ (k \geq$
Specific behaviours
✓ indicates sum of probabilities is 1 ✓ forms equation ✓ solves and states single value of k

(b) Determine $E(X)$. (2 marks)

Solution
$E(X)=(-1)(0.3)+(1)(0.2)+(2)(0.1)+(4)(0.4)$ $=\frac{17}{10}=1.7$
Specific behaviours
✓ indicates correct method ✓ correct expected value

(c) Given that $\mathrm{Var}(X)=4.41$, determine the following for the discrete random variable Y:

(i) $E(Y)$ when $Y=3-2X$. (1 mark)

Solution
$E(Y)=3-2E(X)=3-2(1.7)=-\frac{2}{5}=-0.4$
Specific behaviours
✓ correct value

(ii) $\mathrm{Var}(Y)$ when $Y=\frac{X}{2}-4$. (1 mark)

Solution
$\mathrm{Var}(Y)=\left(\frac{1}{2}\right)^2\mathrm{Var}(X)=\frac{1}{4}(4.41)=\frac{441}{400}=1.1025$
Specific behaviours
✓ correct value

(iii) The standard deviation of Y when $Y=9X+8$. (1 mark)

Solution
$\sigma_Y=9\sqrt{\mathrm{Var}(X)}=9(2.1)=\frac{189}{10}=18.9$
Specific behaviours
✓ correct value

Question 14 **(9 marks)**

A full water tank takes 38 seconds to empty. The volume V litres of water in the tank, t seconds after emptying began, is changing at a rate given by

$$\frac{dV}{dt} = \sqrt[3]{9t+1} - 7, \qquad 0 \le t \le 38.$$

(a) Determine the initial rate of change of volume. (1 mark)

Solution
$\frac{dV}{dt} = \sqrt[3]{9(0)+1} - 7 = -6\ \text{L/s}$
Specific behaviours
✓ correct rate of change

(b) Use the increments formula to estimate the volume of water that empties from the tank during the first one-third of a second. (2 marks)

Solution
$\delta V \approx \frac{dV}{dt}\delta t$ $\approx -6 \times {}^{1}/_{3} \approx -2$ An estimated 2 L empties from the tank.
Specific behaviours
✓ shows use of the increments formula   ✓ correct estimate

(c) Determine the initial volume of water in the tank. (3 marks)

Solution	Alternative Solutions
$\Delta V = \int_0^{38} \sqrt[3]{9t+1} - 7\,dt$ $= -66$ Hence tank initially contained 66 L.	$V = \int \sqrt[3]{9t+1} - 7\,dt = \frac{1}{12}(9t+1)^{\frac{4}{3}} - 7t + c$ But when $t = 38, V = 0$ and so $c = {}^{791}/_{12}$ $V(0) = 66$ L
Specific behaviours	**Specific behaviors**
✓ writes correct integral ✓ evaluates total change ✓ states correct initial volume	✓ correct anti derivative ✓ correct value for c ✓ states correct initial volume

(d) Determine the time, to the nearest 0.01 second, when the tank is half full. (3 marks)

<table>
<tr><th>Solution</th><th>Alternative Solution</th></tr>
<tr><td>$V = \int \sqrt[3]{9t+1} - 7\,dt = \frac{1}{12}(9t+1)^{\frac{4}{3}} - 7t + c$
But when $t = 0, V = 66$ and so $c = {}^{791}/_{12} = 61.91\overline{6}$

$\frac{1}{12}(9t+1)^{\frac{4}{3}} - 7t + \frac{791}{12} = \frac{66}{2}$
$t = 8.85\text{ s}$</td><td>$33 = -\int_0^T \sqrt[3]{9t+1} - 7\,dt$
$T = 8.85$s</td></tr>
<tr><th>Specific behaviours</th><th>Specific behaviours</th></tr>
<tr><td>✓ obtains antiderivative
✓ evaluates constant of integration
✓ solves for time</td><td>✓ writes correct integral
✓ forms correct equation
✓ solves for time</td></tr>
</table>

Question 15 **(8 marks)**

The concentration of a drug in the plasma of a monkey, C micrograms per litre, t hours after being administered, can be modelled by $C = C_0e^{kt}$, where C_0 and k are constants. Each dose of the drug increases the existing concentration by $310\ \mu\text{g/L}$, and the concentration of the drug is known to halve every 3 hours and 40 minutes.

A monkey, with no existing trace of the drug, was administered a first dose at $8:45$ am.

(a) Use the model to determine the rate of change of concentration of the drug in the monkey's plasma later that day at $12:25$ pm. (4 marks)

Solution
$0.5 = e^{3.\overline{6}k} \rightarrow k = -0.189$ $\dfrac{dC}{dt} = kC_0e^{kt} = kC$ At $12:25$ pm, $t = 3\text{h }40\text{m}$ and so $C = 310 \div 2 = 155$. $\dfrac{dC}{dt} = -0.189(155)$ $= -29.3\ \mu\text{g/L/h}$
Specific behaviours
✓ correctly forms equation for k using half life   ✓ solves for k ✓ indicates either expression for rate of change   ✓ correctly calculates rate of change

An additional dose is administered every time the concentration falls to $110\ \mu\text{g/L}$.

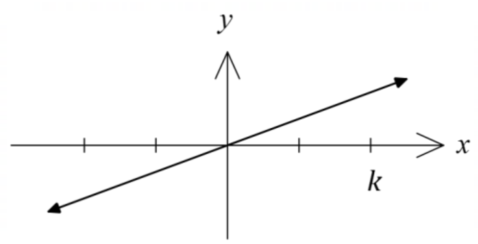
(b) Determine the expected time of day, to the nearest minute, that the third dose will be administered to the monkey. (4 marks)

Solution
Time until second dose is given: $310e^{-0.189t} = 110 \rightarrow t = 5.481$.   New $C_0 = 110 + 310 = 420 \rightarrow C = 420e^{-0.189t}$.   Time from second to third dose: $420e^{-0.189t} = 110 \rightarrow t = 7.087$.   Total time: $T = 5.481 + 7.087 = 12.568 = 12\text{h }34\text{m}$. Hence third dose will be given at $8:45 + 12\text{h }34\text{m} = 9:19$ pm.
Specific behaviours
✓ time until second dose administered   ✓ indicates new equation for concentration   ✓ time between second and third doses   ✓ correct time of day

Question 16 **(7 marks)**

(a) Consider the function $f(x) = mx$, where m is a constant. The graph of $y = f(x)$ is shown at right, k is a constant and

$$\int_0^k f(x)\,dx = 4.$$



Determine the value of

(i) $\displaystyle\int_{-k}^{k} f(x)\,dx.$ (1 mark)

Solution
$\displaystyle\int_{-k}^{k} f(x)\,dx = \int_{-k}^{0} f(x)\,dx + \int_0^k f(x)\,dx = -4 + 4 = 0$
Specific behaviours
✓ correct value

(ii) $\displaystyle\int_{-k}^{0} 2f(x+k)\,dx.$ (2 marks)

Solution
$\displaystyle\int_{-k}^{0} 2f(x+k)\,dx = 2\int_0^k f(x)\,dx = 2(4) = 8$
Specific behaviours
✓ uses linearity to move constant outside integral ✓ correct value

(b) The polynomial function $g(x)$ is such that $\displaystyle\int_{-2}^{5} g(x)\,dx = 10.$

Determine the value of $\displaystyle\int_{-2}^{2} (2 - g(x))\,dx + \int_2^5 (2x - g(x))\,dx.$ (4 marks)

Solution
$\displaystyle I = \int_{-2}^{2} \big(2 - g(x)\big)\,dx + \int_2^5 \big(2x - g(x)\big)\,dx$ $\displaystyle = \int_{-2}^{2} (2)\,dx - \int_{-2}^{2} \big(g(x)\big)\,dx + \int_2^5 (2x)\,dx - \int_2^5 \big(g(x)\big)\,dx$ $\displaystyle = [2x]_{-2}^{2} - \int_{-2}^{5} \big(g(x)\big)\,dx + [x^2]_2^5$ $= (4+4) - 10 + (25-4)$ $= 19$
Specific behaviours
✓ uses linearity to obtain four integrals ✓ uses additivity to combine integrals of $g(x)$ ✓ evaluates $2x$ integral correctly ✓ correct value

Question 17 **(10 marks)**

A machine learning model is being developed to recognise a pathogen in medical images. The performance of the model is stable and the results of the last 250 runs of the machine are shown in the table below.

		Model recognises a pathogen in image	
		Yes	No
Image contains a pathogen	Yes	154	21
	No	6	69

(a) Determine the probability that the model recognises a pathogen in a randomly selected image that contains a pathogen. (1 mark)

Solution
$p = \dfrac{154}{154+21} = \dfrac{154}{175} = \dfrac{22}{25} = 0.88$
Specific behaviours
✓ correct probability

(b) The model is used to check 9 randomly selected images. Determine the probability that it returns exactly one incorrect result. (3 marks)

Solution
Probability of an incorrect result, p: $p = \dfrac{6+21}{250} = \dfrac{27}{250} = 0.108$ Let X be number of incorrect results, then $X \sim B(9, 0.108)$. $P(X = 1) = 0.3896$
Specific behaviours
✓ calculates probability of incorrect result   ✓ states distribution is binomial, with parameters   ✓ calculates probability

(c) The model is used to check 30 randomly selected images. Determine the probability that it returns at least 26 correct results. (3 marks)

Solution
Probability of a correct result, p: $$p = 1 - \frac{27}{250} = \frac{223}{250} = 0.892$$ Let X be number of correct results, then $X \sim B(30, 0.892)$. $$P(X \geq 26) = 0.7817$$
Specific behaviours
✓ calculates probability of correct result   ✓ states distribution is binomial, with parameters   ✓ calculates probability

(d) The model is repeatedly used to check batches of 50 randomly selected images that do not contain a pathogen. Determine the mean and standard deviation of the probability distribution for the number of correct results the model produces. (3 marks)

Solution
$$p = \frac{69}{6+69} = \frac{69}{75} = \frac{23}{25} = 0.92$$ $$\mu = np = 50 \times \frac{23}{25} = 46$$ $$\sigma = \sqrt{np(1-p)} = \sqrt{46 \times \frac{2}{25}} = \frac{2\sqrt{23}}{5} \approx 1.918$$
Specific behaviours
✓ calculates conditional probability   ✓ calculates mean   ✓ calculates standard deviation

Question 18 **(10 marks)**

A small body moves in a straight line with velocity v cm/s at time t s given by

$$v(t) = 11 + 4\sin\left(\frac{\pi t}{10}\right) - 6\sin\left(\frac{\pi t}{5}\right), \qquad t \geq 0.$$

(a) By viewing the graph of the velocity function on your calculator, or otherwise, state the minimum velocity of the body for $t \geq 0$ to the nearest 0.01 cm/s, and hence explain why the distance travelled by the body in any interval of time will always be the same as the change in displacement of the body. (2 marks)

Solution
$v_{MIN} = 2.02 \text{ cm/s}$ Distance travelled same as change in displacement as the velocity is always positive.
Specific behaviours
✓ states minimum velocity ✓ explanation

(b) Determine the distance travelled by the body between $t = 0$ and $t = 20$. (2 marks)

Solution
$d = \int_0^{20} v(t)\, dt$ $= 220 \text{ cm}$
Specific behaviours
✓ writes correct integral ✓ correct distance

The distance travelled (x cm) by the body in any 10 second interval from $t = \theta$ to $t = \theta + 10$ is given by the function $x(\theta) = a + b\cos\left(\frac{\pi\theta}{10}\right)$.

(c) Determine the value of the constant a and the value of the constant b. (2 marks)

Solution
$x(\theta) = \int_\theta^{\theta+10} v(t)\, dt$ $= 110 + \frac{80}{\pi}\cos\left(\frac{\pi\theta}{10}\right)$ Hence $a = 110$ and $b = \frac{80}{\pi}$.
Specific behaviours
✓ writes integral ✓ uses result to state both values

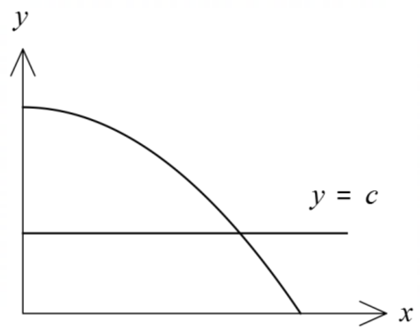
(d) During the first 35 seconds, there is an 10 second interval in which the distance travelled by the body is a minimum. Using calculus methods, determine when this interval occurs and justify that the distance is a minimum. (4 marks)

Solution
$x'(\theta) = -8\sin\left(\frac{\pi\theta}{10}\right)$ $x'(\theta) = 0 \text{ when } \theta = 0, 10, 20$ $x''(\theta) = -\frac{4\pi}{5}\cos\left(\frac{\pi\theta}{10}\right)$ $x''(0) = -\frac{4\pi}{5}, \quad x''(10) = \frac{4\pi}{5}$ Hence when the interval starts at $\theta = 10$ seconds, the distance is a minimum since at this time the first derivative of the distance function is zero and the second derivative is positive.
Specific behaviours
✓ obtains derivative and equates to zero ✓ indicates times when derivative is zero (0, 10 and 20) ✓ uses second derivative to identify first minimum ✓ states correct start time

Question 19 **(7 marks)**

The line $y = c$ divides the area in the first quadrant under the curve $y = 64 - x^2$ into two equal halves, as shown in the diagram.

Determine, with reasoning, the value of c.



<table>
<tr><th>Solution</th><th>Alternative solution</th></tr>
<tr><td>

Let the curve and line intersect when $x = a$, so that $c = 64 - a^2$.

Area above line is area between curve and line:

$$A_A = \int_0^a (64 - x^2) - (64 - a^2)\,dx = \frac{2a^3}{3}$$

Area below line is rectangle plus area under curve:

$$A_B = a(64 - a^2) + \int_a^8 (64 - x^2)\,dx = 64a - a^3 + \frac{a^3}{3} - 64a + \frac{1024}{3} = \frac{1024}{3} - \frac{2a^3}{3}$$

Require $A_A = A_B$ and so

$$\frac{2a^3}{3} = \frac{1024}{3} - \frac{2a^3}{3}$$
$$a = 4\sqrt[3]{4}$$

Hence $c = 64 - \left(4\sqrt[3]{4}\right)^2 = 64 - 32\sqrt[3]{2} \approx 23.683$.

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At the intercept, $64 - x^2 = c$

$$x = \sqrt{64 - c}$$

Area above line is area between curve and line:

$$A_A = \int_0^{\sqrt{64-c}} (64 - x^2)\,dx - c\sqrt{64 - c} = \left[64x - \frac{x^3}{3}\right]_0^{\sqrt{64-c}} - c\sqrt{64 - c} = \frac{2(64 - c)\sqrt{64 - c}}{3}$$

Area below line is rectangle plus area under curve:

$$A_B = \int_{\sqrt{64-c}}^8 (64 - x^2)\,dx + c\sqrt{64 - c} = \left[64x - \frac{x^3}{3}\right]_{\sqrt{64-c}}^8 + c\sqrt{64 - c} = \frac{2((c - 64)\sqrt{64 - c} + 512)}{3}$$

Require $A_A = A_B$,

Hence $c = 64 - \left(4\sqrt[3]{4}\right)^2 = 64 - 32\sqrt[3]{2} \approx 23.683$

</td></tr>
<tr><th>Specific behaviours</th><th>Specific behaviours</th></tr>
<tr><td>

✓ expresses c in terms of x-coordinate of intersection
✓ writes integral for upper area
✓ evaluates and simplifies integral
✓ writes expression for lower area
✓ evaluates and simplifies integral
✓ equates expressions and solves for a
✓ substitutes to obtain c

</td><td>

✓ expresses x coordinate of the intersection in terms of c.
✓ writes integral for upper area
✓ evaluates and simplifies integral
✓ writes expression for lower area
✓ evaluates and simplifies integral
✓ equates expressions for both areas
✓ solves equation to obtain c

</td></tr>
</table>

End of questions

Question 19 Alternative Solution 2
Total area under curve: $$A=\int_0^8 (64-x^2)\,dx=\frac{1024}{3}$$ Half of the area $=\frac{1024}{3}\div 2=\frac{512}{3}$ or $170.67\ u^2$ $\frac{512}{3}=\int_0^{\sqrt{64-c}}(64-x^2)-c\,dx$ (or any area of the half section) $$=\frac{-2(c-64)\sqrt{64-c}}{3}$$ Hence $c=64-32\sqrt[3]{2}\approx 23.683$
Specific behaviours
✓ writes correct integral for the whole area   ✓ evaluates the correct whole area   ✓ evaluates half of the area   ✓ writes expression for any half area (lower or upper section)   ✓ evaluates and simplifies integral   ✓ equates expressions for half areas   ✓ solves equation to obtain c