MATHEMATICS APPLICATIONS UNITS 1 AND 2

Section One:
Calculator-free

SOLUTIONS

Student Number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1.　The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2.　Write your answers in this Question/Answer Booklet.

3.　You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4.　Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5.　**Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6.　It is recommended that you **do not use pencil**, except in diagrams.

7.　The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

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Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(4 marks)**

(a) Solve the equation $3(2x-1)+x-2=23$. (2 marks)

$$6x-3+x-2=23$$
$$7x=28$$
$$x=4$$

(b) Ann has saved x dollars, Ben has saved twice as much as Ann and Cia has saved $40 less than Ben. Ann, Ben and Cia have saved a total of $410.

Determine how much Ann has saved. (2 marks)

$$x+2x+2x-40=410$$
$$5x=450$$
$$x=90$$

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Question 2   **(7 marks)**

(a) Lily and Said planned to collect data on the number of hours that the 30 students in their Year 11 class spent watching TV on a weekday night.

(i) Lily created a table as shown at right and planned to ask each student which group they belonged to.

Time (hours)	Tally
0 - 1	
1 - 2	
2 or more	

Describe one advantage and one disadvantage of her method.   (2 marks)

> D: Not clear into which group a response of 1 hour, etc, would go.
> D: Not enough groups to get meaningful data to graph.
> A: is quick and easy to carry out.
> Etc

(ii) Said decided to ask each student and simply make a list of all their times.

Describe one advantage or disadvantage of his method.   (1 mark)

> D: More time consuming for follow up work.
> A: Can group data later and still keep raw data.
> Etc

(b) Another student collected the resting pulse rates of her class and her data is listed below.

57 72 83 51 73 86 70 62 55 51 53 67 69 52 51 61

(i) Construct a stem and leaf plot of this data.   (2 marks)

> 5 | 7 1 5 1 3 2 1
> 6 | 2 7 9 1
> 7 | 2 3 0
> 8 | 3 6

(ii) Use the stem and leaf plot to describe the modality and shape of this data.   (2 marks)

> Data is uni modal.
> (more pulse rates in the 50 – 59 stem than any other)
>
> Shape is positively skewed.
> (data is tailing off in the higher stems)

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Question 3   **(9 marks)**

Consider the matrices $A = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 & -1 \\ 3 & -2 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 3 & 1 \\ -1 & -2 \end{bmatrix}$.

(a)   State the size of

   (i)   the square matrix   (1 mark)

   > D is a 2 by 2 matrix

   (ii)   the row matrix   (1 mark)

   > B is a 1 by 2 matrix

(b)   Calculate

   (i) $D - 3I$, where I is the identity matrix.   (2 marks)

   > $$\begin{bmatrix} 3 & 1 \\ -1 & -2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -5 \end{bmatrix}$$

   (ii) DC (2 marks)

   > $$\begin{bmatrix} 3 & 1 \\ -1 & -2 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & -1 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & -2 & -2 \\ -8 & 4 & -1 \end{bmatrix}$$

(c)   Explain why the product AC cannot be calculated.   (1 mark)

   > Number of columns in A ≠ Number of rows in C

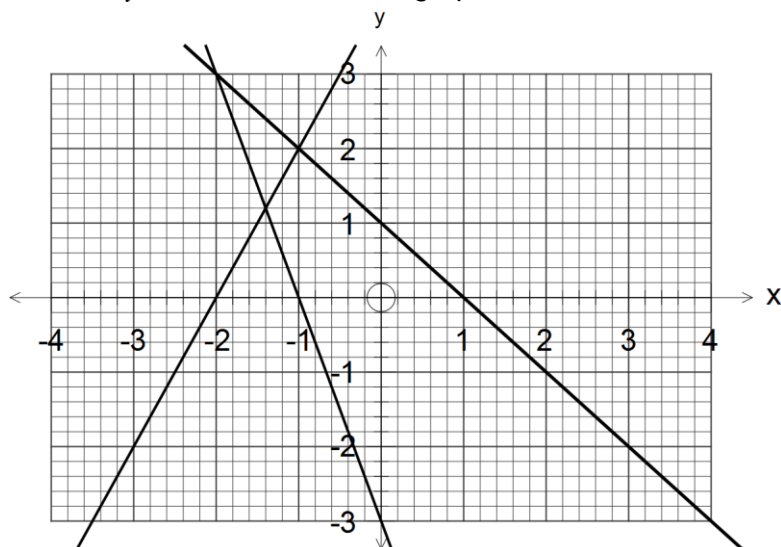(d)   If $\begin{bmatrix} p & q \end{bmatrix} \times A = \begin{bmatrix} 3p \end{bmatrix}$, determine the value q.   (2 marks)

   > $$\begin{bmatrix} p & q \end{bmatrix} \times \begin{bmatrix} 3 \\ -3 \end{bmatrix} = \begin{bmatrix} 3p \end{bmatrix}$$
   > $$3p - 3q = 3p$$
   > $$-3q = 0$$
   > $$q = 0$$

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Question 4   **(6 marks)**

Two lines, $3x + y = -3$ and $y = 2x + 4$, have been graphed on the axes below.

(a)   Use the graph to solve the simultaneous equations $3x + y = -3$ and $y = 2x + 4$ as accurately as you can. Briefly explain the method you used.   (2 marks)

> Read the coordinates of the point where the two lines intersect.
> Solution is
> $x = -1.4$
> $y = 1.2$

(b)   Draw the line $x + y = 1$ on the above axes.   (2 marks)

(c)   Use the graph to solve the simultaneous equations $3x + y = -3$ and $x + y = 1$.   (2 marks)

> Solution is
> $x = -1$
> $y = 2$

See next page

Question 5 **(8 marks)**

(a) The temperature, T ^{0}C, of v litres of water in a hot water urn t minutes after it is switched on is given by the formula $T = \dfrac{10t}{v} + 28$.

(i) Calculate the temperature of 2 litres of water after 12 minutes. (2 marks)

$$T = \frac{10 \times 12}{2} + 28$$
$$= 60 + 28$$
$$= 88°C$$

(ii) Determine how long the urn has been switched on, if 5 litres of water has reached a temperature of 40ºC. (2 marks)

$$40 = \frac{10t}{5} + 28$$
$$12 = \frac{10t}{5}$$
$$60 = 10t$$
$$t = 6 \text{ minutes}$$

(b) From a height h metres above sea level, the approximate distance, d kilometres, that an observer can see to the horizon is given by $d = 8\sqrt{\dfrac{h}{5}}$.

(i) Calculate the distance an observer 20 meters above sea level can see. (2 marks)

$$d = 8 \times \sqrt{\frac{20}{5}}$$
$$= 8 \times \sqrt{4}$$
$$= 8 \times 2$$
$$= 16 \text{ km}$$

(ii) A person can see 24 km to the horizon. Determine their height above sea level.
(2 marks)

$$24 = 8 \times \sqrt{\frac{h}{5}}$$
$$3 = \sqrt{\frac{h}{5}}$$
$$9 = \frac{h}{5}$$
$$h = 45 \text{ m}$$

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Question 6 **(5 marks)**

(a) A straight line has equation $3x + 2y = 11$. Determine the slope of this line. (1 mark)

$$2y = 11 - 3x$$
$$y = 5.5 - 1.5x$$
Slope is -1.5

(b) Another line has equation $x - 3y = 15$. Determine
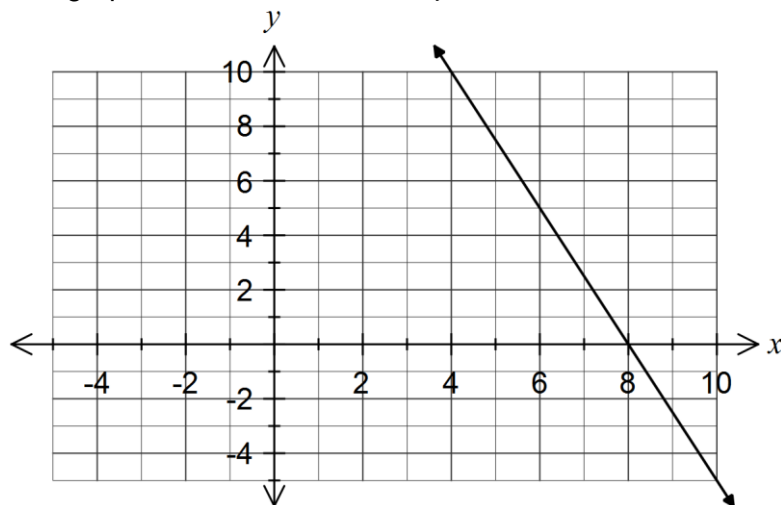
(i) the y-intercept of this line. (1 mark)

$$x = 0 \implies y = -5$$

Intercept is -1.5

(ii) the x-intercept of this line. (1 mark)

$$y = 0 \implies x = 15$$

Intercept is 15

(c) The graph below shows the line $y = mx + c$.

Determine the values of m and c. (2 marks)

$$m = -2.5$$

$$c = 20$$
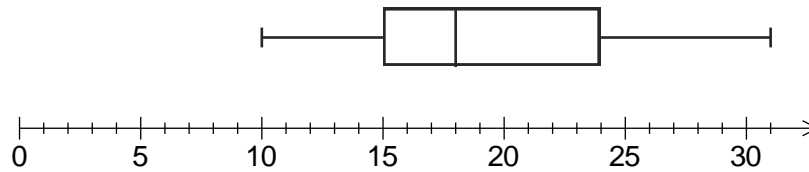
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Question 7 **(7 marks)**

The boxplot below shows the distribution of the number of times a production machine broke down per month in a factory over a period of years.

(a) State the median, range and interquartile range for this distribution. (3 marks)

Median is 18

Range is 21

IQR is 9

(b) One month the machine broke down an unusual number of times. This number, n, was excluded from the box and whisker plot above as it was considered to be an outlier.

(i) Determine the range of possible values for n. (3 marks)

$1.5 \times IQR = 13.5$
$15 - 13.5 = 1.5$
$24 + 13.5 = 37.5$

$0 \le n \le 1$ or $n \ge 38$

(ii) How would the standard deviation of the number of breakdowns per month change if the outlier was included in the data set? (1 mark)

SD would increase.

See next page

Question 8 **(6 marks)**

(a) A vertical pole standing on level field casts a shadow that extends 6 metres from the base of the pole. At the same time, the shadow of a 20 cm tall ruler is 30 cm. Determine the height of the pole. (2 marks)

$$\frac{h}{6} = \frac{20}{30}$$

$$h = \frac{2}{3} \times 6$$

$$h = 4 \text{ m}$$

(b) A map has a scale of 1:10 000. On the map, the length of a route between two points is 152 mm. Determine the actual length of the route in kilometres. (2 marks)

$$d = \frac{152 \times 10000}{1000 \times 1000}$$

$$= 1.52 \text{ km}$$

(c) The length of a rectangular room shown on the plan of a building is 17 cm. The actual length of the room is 8.5 metres. The area of the room on the plan is 200 cm². Determine the area of the actual room, in square metres. (2 marks)

$$17 \text{ cm} : 8.5 \text{ m}$$
$$2 \text{ cm} : 1 \text{ m}$$
$$1 \text{ cm} : \tfrac{1}{2} \text{ m}$$

$$200 \text{ cm}^2 = 200 \times \tfrac{1}{2} \times \tfrac{1}{2}$$
$$= 50 \text{ m}^2$$

End of questions

Additional working space

Question number: _____