Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section Two: Calculator-assumed

Your name ____

Teacher's name ____

Time allowed for this section

Reading time before commencing work:	ten minutes
Working time:	one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items:	pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

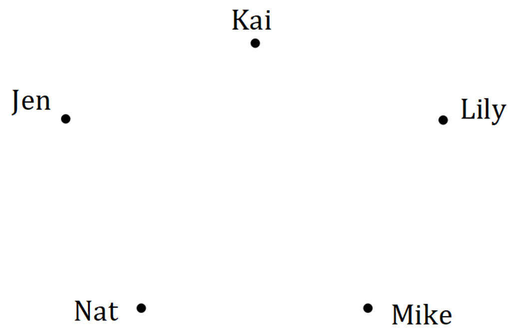
This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

(a) In a group of five people it was known that Kai was older than Lily, Nat and Jen; Mike was older than Kai and Lily; Nat was older than Jen; and Lily was older than Nat.

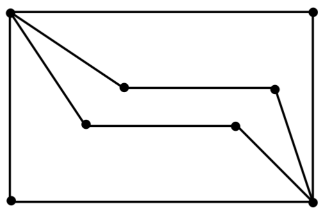
(i) Represent this set of age relationships as a digraph. (2 marks)



(ii) State the number of arcs in the digraph. (1 mark)

(iii) List the five people in order of age, starting with the youngest. (1 mark)

(b) Graph H is shown below.



Let t and p be the number of edges in the longest open trail and shortest closed path contained in H respectively. State the values of t and p, given that $t > 0$ and $p > 0$.

(2 marks)

Question 10 **(8 marks)**

The following data shows the blood haemoglobin (H) levels and packed cell volumes (V) of 10 blood bank donors.

H	11.0	13.3	11.8	14.5	14.8	11.2	13.9	12.2	12.8	11.6
V	0.43	0.47	0.45	0.50	0.51	0.45	0.49	0.46	0.48	0.46

(a) Graph the data on your calculator and describe features of the graph that suggest the presence of a strong and positive linear association between H and V. (2 marks)

(b) Determine the equation of the least-squares line that models the relationship between H and V, where H is the explanatory variable. (2 marks)

(c) Calculate the correlation coefficient between H and V. (1 mark)

(d) What percentage of the variation in V can be explained by the variation in H? (1 mark)

(e) Predict the packed cell volume of a donor with a blood haemoglobin level of 13.4. (1 mark)

(f) Describe a potential danger associated with using the least-squares line to predict a packed cell volume from a blood haemoglobin level. (1 mark)

Question 11 **(7 marks)**

A company bought and installed a new computer system with an initial value of $\$29\,400$. For accounting purposes, the value of the system decreased by $\$2\,450$ each year.

(a) Calculate the value of the system after 3 years. (1 mark)

(b) Determine a recurrence relation for V_n, the value of the system after n years. (2 marks)

(c) Determine

(i) the value of the system after 7 years. (1 mark)

(ii) the number of years for the value of the system to become nothing. (1 mark)

(d) Determine the additional time taken for the system to become worthless if its value decreased by $\$1\,470$ each year instead of $\$2\,450$. (2 marks)

Question 12 **(8 marks)**

A sequence, *K*, is defined as:

$$K_n = 3(2)^n$$

(a) State the first three terms of sequence *K*. (2 marks)

(b) Write sequence *K* using a recursive rule. (2 marks)

A different sequence, *A,* is defined as:

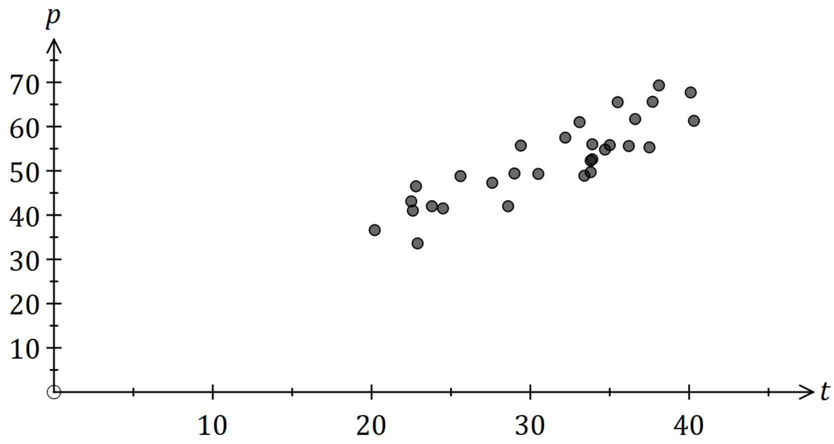
$$A_{n+1} = 0.8\,A_n \qquad where\;\; A_1 = 100$$

(c) State the first three terms of sequence *A*. (2 marks)

(d) Determine the smallest value of *n* such that $K_n > A_n$.
Show some working. (2 marks)

Question 13 **(7 marks)**

The scatterplot below shows the marks scored by 30 students in their theory (t) and practical (p) exams that were marked out of 60 and 90 marks respectively.



The equation of the least-squares line for the data is $p = 1.38t + 9.2$.

(a) It was found that 77% of the variation in p could be explained by the variation in t. Determine the correlation coefficient r_{tp}. (1 mark)

(b) Interpret the slope of the least-squares line. (2 marks)

(c) Joe and Kai were absent for the practical exam, but it was known that their marks in the theory exam were 28 and 57 respectively. Predict their practical exam marks and explain how reliable each prediction is. (4 marks)

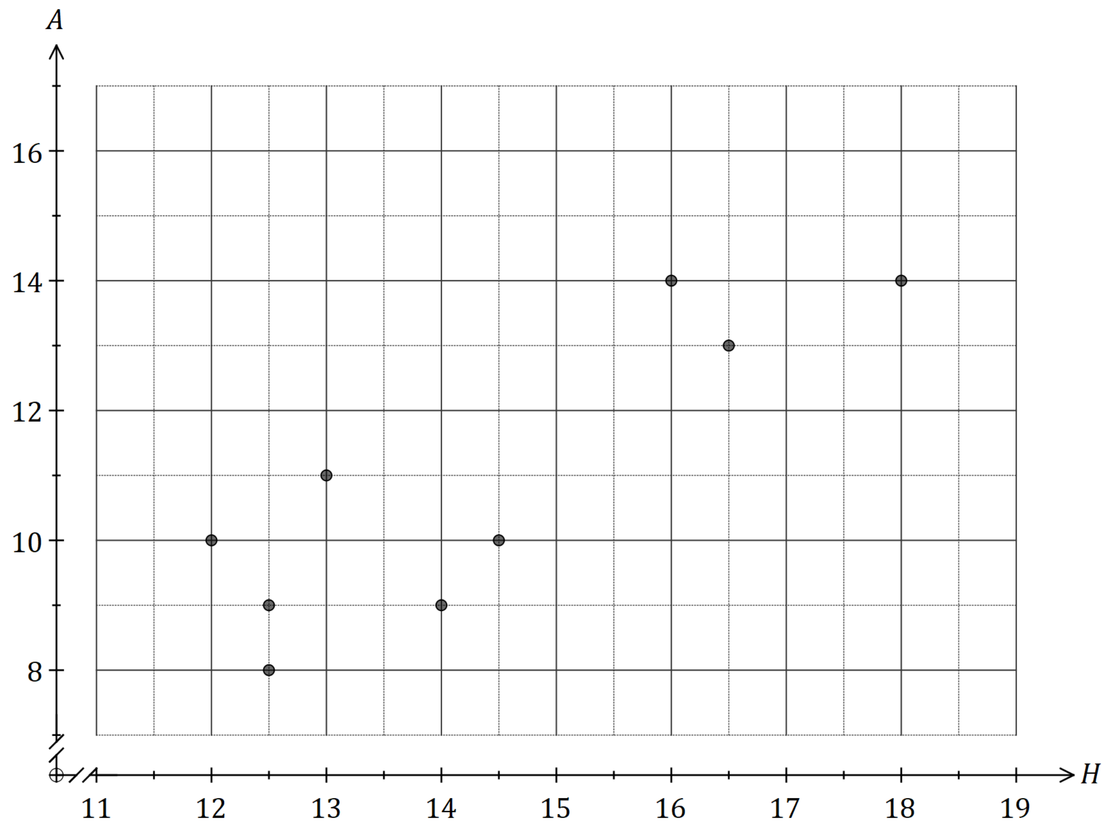
Question 14 **(11 marks)**

A researcher obtained the following data whilst investigating whether it is possible to reliably predict a child's reading ability (A, on a numerical scale of 1 to 20) from their hand span (H, cm).

Child	B	C	D	E	F	G	J	K	L	M	N	P
H	13.0	12.5	16.0	17.5	15.5	16.5	14.5	18.0	12.0	14.0	14.5	12.5
A	11	8	14	16	15	13	10	14	10	9	13	9

(a) State the explanatory variable for this investigation. (1 mark)

(b) Add the three missing data points to the scatterplot below. (2 marks)



(c) Determine the correlation coefficient between the two variables. (1 mark)

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(d) Using the scatterplot from (b) and the correlation coefficient from (c), the researcher was satisfied that a linear association existed between A and H. Explain why they reached this conclusion. (2 marks)

The researcher then discovered that the children labelled B, C, J, L, M and P were all in Year 3 and the remainder in Year 6.

(e) Circle the Year 3 children on the graph. (1 mark)

(f) Calculate the correlation coefficient between A and H for the Year 3 children only. (1 mark)

(g) Identify a non-causal explanation for the conclusion reached by the researcher in (d) and explain how this new information affects that conclusion. (3 marks)

Question 15 **(7 marks)**

A water tank is initially empty. At the start of each hour, 150 L of water is quickly poured into the tank but during the following hour, 20% of all the water in the tank leaks out.

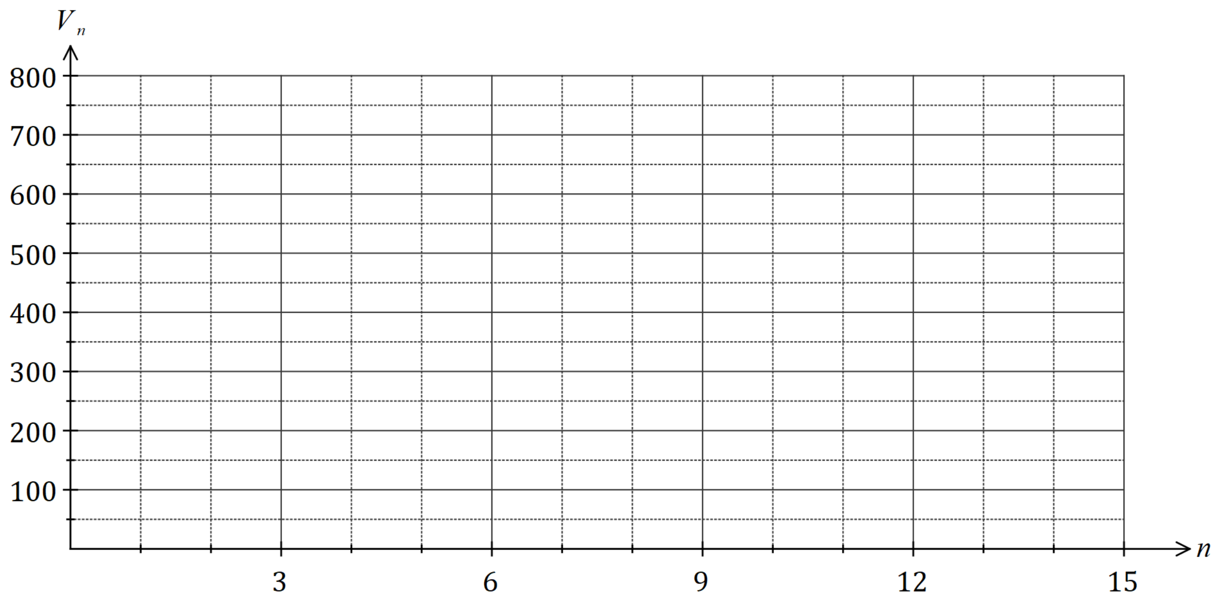
This situation can be modelled by the recurrence relation $V_{n+1} = 0.8V_n + 150,\ V_0 = 150$, where V_n is the volume of water in the tank, in litres, at the start of the n^{th} hour.

(a) Complete the table below, giving volumes to the nearest litre. (2 marks)

n	0	3	6	9	12
V_n					709

(b) At the start of which hour does the tank first hold at least 745 L? (1 mark)

(c) Plot the points from the table on the axes below. (2 marks)



(d) The tank has a maximum capacity of 800 L. If possible, determine the least number of hours since filling commenced that the tank will start to overflow. If not possible, explain why not. (2 marks)

Question 16 **(9 marks)**

A study categorized the weight of hospitalised children as obese, overweight, normal or underweight. The numbers of children in each category are shown by gender in the table below.

	Obese	Overweight	Normal	Underweight
Male	16	71	145	13
Female	23	39	161	27

(a) An overweight child is randomly chosen from the study. If possible, explain whether they are more likely to be a boy or a girl. If not possible, explain your reasoning. (2 marks)

(b) What percentage of the boys in the study were classified as underweight? (2 marks)

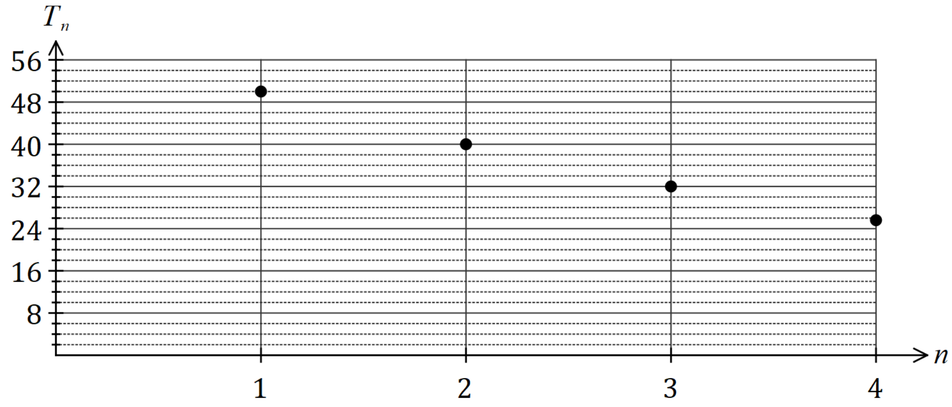
(c) Complete the table of **row** percentages below to the nearest whole number. (3 marks)

(%)	Obese	Overweight	Normal	Underweight
Male	7			
Female				11

(d) Does the table of row percentages suggest the presence of an association between the categorical variables? Justify your answer. (2 marks)

Question 17 **(6 marks)**

A piledriver is hammering a pile into the ground. The graph below shows the distance T_n (in cm) the pile moves into the ground on the n^{th} hit of the piledriver.



The values of T_n form a geometric sequence.

(a) Use information from the graph to determine the common ratio for the sequence. (1 mark)

(b) Write a recurrence relation to generate the values of T_n. (2 marks)

(c) Write the n^{th} term rule for the values of T_n. (1 mark)

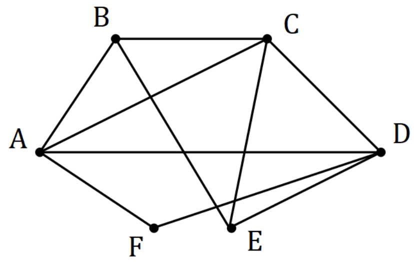
(d) Determine

(i) the distance the pile moves into the ground on the seventh hit of the piledriver. (1 mark)

(ii) on which hit the pile first moves less than one mm into the ground. (1 mark)

Question 18 **(7 marks)**

Each vertex on the graph below represents an airport and an edge between two airports indicates that an airline has a direct flight, in both directions, between the airports.



(a) Redraw the graph to clearly show that it is planar. (1 mark)

(b) Demonstrate that the graph satisfies Euler's formula. (2 marks)

In order to check in-flight catering quality, an airline manager plans to leave airport A, travel on at least one flight between the 10 pairs of airports and then return to A. The manager does not use any other mode of transport between airports.

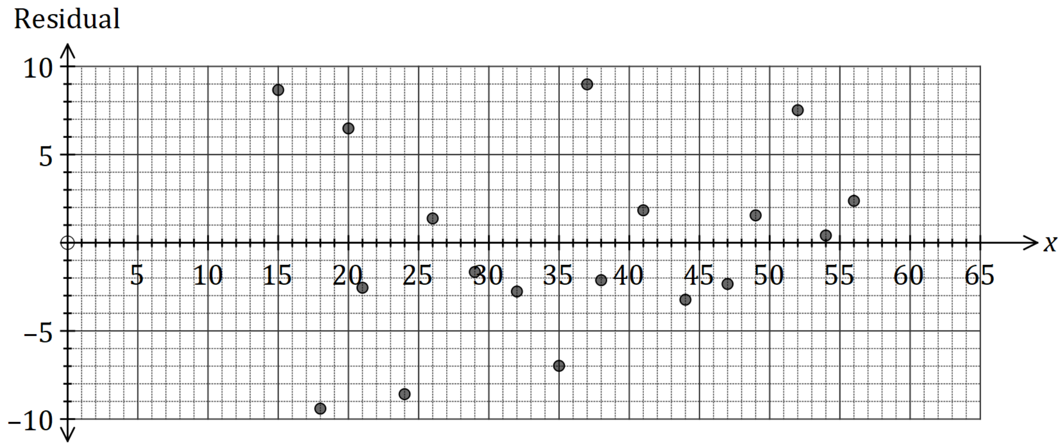
(c) Determine the minimum number of flights the manager must take and list, in order, the airports visited. (2 marks)

(d) Another manager, based at a different airport, claimed he could carry out the quality check in fewer flights by starting at his airport and finishing at another airport. Comment on this claim. (2 marks)

Question 19 **(6 marks)**

A statistician wants to check whether a linear model is appropriate for a bivariate data set they are analysing. The least-squares line to model the linear relationship is $y = 1.54x - 13.9$ and the correlation coefficient between the variables is very strong.

The residual plot using the linear model is shown below for all but two of the data points.



(a) Calculate the residuals for the missing points $(33, 46)$ and $(58, 66.5)$ and plot them on the graph above. (4 marks)

(b) Use the residual plot to explain whether fitting a linear model to the data is appropriate. (2 marks)

Question 20 **(8 marks)**

(a) An investor has $\$2\ 520$ in an account. One month later, and at the start of each subsequent month, a deposit of $\$95$ is added to the account. Interest, calculated as 0.38% of the balance at the start of the month, is added to the account just before each deposit is made.

The account balance after n deposits is T_n, and can be modelled by the recurrence relation $T_{n+1} = 1.0038T_n + 95,\ T_0 = 2520.$

(i) Determine the balance in the account after 7 deposits have been made. (1 mark)

(ii) After how many deposits does the balance of the account first exceed $\$5\ 000$ and what is the balance of the account at that time. (2 marks)

(b) The investor also has $\$385$ in another account. One week later, and at the start of each subsequent week, a deposit of $\$9.75$ is added to the account. Interest, calculated as 0.072% of the balance at the start of the week, is added to the account just before each deposit is made.

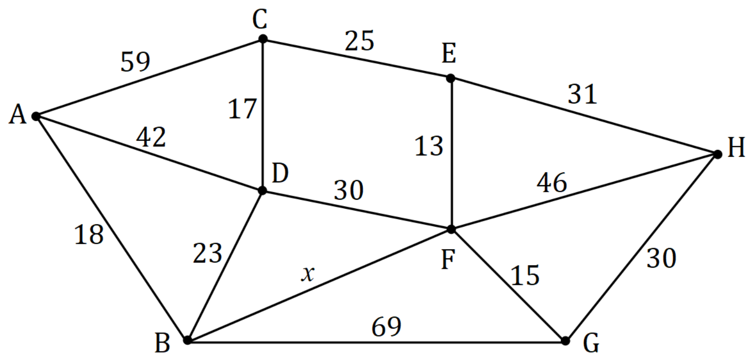
(i) Write a recurrence relation to model the balance of this account after n deposits. (3 marks)

(ii) Determine the balance in this account after 52 deposits have been made. (1 mark)

(iii) By considering the total deposits made, or otherwise, determine the total interest added to this account after 52 deposits have been made. (1 mark)

Question 21 **(8 marks)**

The vertices below represent 8 computers in a network and the weights on each edge represent the time, in milliseconds, for a signal to be sent directly between connected computers.



(a) Given that $x = 55$, determine the path required and the time taken to send a signal in the least time between

(i) B and G. (2 marks)

(ii) A and F. (2 marks)

(iii) A and H. (2 marks)

(b) Determine the largest value of x, to the nearest millisecond, to ensure that the fastest route to send a signal between A and H will pass through F. Justify your answer.
(2 marks)

End of questions

Supplementary page

Question number: _________

Supplementary page

Question number: _________

Supplementary page

Question number: _________

	Markers use only		
MARKER	Question	Maximum	Mark
Mr Lee	9	6	
	10	8	
	11	7	
Mr Galbraith	12	8	
	13	7	
	14	11	
Ms Thompson	15	7	
	16	9	
	17	6	
Ms Sun	18	7	
	19	6	
	20	8	
	21	8	
	S2 Total	98	