Neap

Trial Examination 2020

VCE Specialist Mathematics Units 3&4

Written Examination 2

Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

1	A	B	C	**D**	E
2	A	B	C	**D**	E
3	A	**B**	C	D	E
4	A	B	**C**	D	E
5	A	B	C	D	**E**
6	A	B	**C**	D	E
7	A	**B**	C	D	E
8	**A**	B	C	D	E
9	**A**	B	C	D	E
10	A	B	C	D	**E**
11	A	**B**	C	D	E
12	A	**B**	C	D	E
13	A	B	C	**D**	E
14	**A**	B	C	D	E
15	A	B	C	D	**E**
16	A	B	C	**D**	E
17	**A**	B	C	D	E
18	A	B	**C**	D	E
19	A	B	**C**	D	E
20	A	B	C	D	**E**

Question 1 **D**

The graph of $y = \dfrac{1}{x^2 + x - 2}$ has a horizontal asymptote with equation $y = 0$ since $y \to 0$ as $x \to \pm\infty$.

The graph has vertical asymptotes when the denominator equals zero.

$$\begin{aligned} x^2 + x - 2 &= 0 \\ (x+2)(x-1) &= 0 \\ x &= -2,\ 1 \end{aligned}$$

Question 2 **D**

The maximal domain of f satisfies $-1 \le 4x - 1 \le 1$.

$0 \le 4x \le 2$

$0 \le x \le \dfrac{1}{2}$

The range can be found by considering the vertical translation of $\dfrac{\pi}{3}$.

The range of arccos(x) is $[0, \pi]$.

Hence by adding the factor of $\dfrac{\pi}{3}$, the range of f is $\left[\dfrac{\pi}{3}, \dfrac{4\pi}{3}\right]$.

Question 3 **B**

Using $\cos(x) = \pm\sqrt{1 - \sin^2(x)}$ gives:

$$\begin{aligned} \cos(x) &= \pm\sqrt{1 - \left(-\frac{1}{3}\right)^2} \\ &= \pm\frac{2\sqrt{2}}{3} \end{aligned}$$

cos(x) is negative in the third quadrant and so cos(x) = $-\dfrac{2\sqrt{2}}{3}$.

Using $\cot(x) = \dfrac{\cos(x)}{\sin(x)}$ gives:

$$\cot(x) = \frac{-\frac{2\sqrt{2}}{3}}{-\frac{1}{3}}$$

Hence $\cot(x) = 2\sqrt{2}$.

Alternatively, use $\cot^2(x) = \operatorname{cosec}^2(x) - 1$ where $\operatorname{cosec}(x) = -3$.

Question 4 **C**

There is a repeated linear factor in the denominator, so the partial fraction form is $\dfrac{A}{x-1} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$.

Use of an expand command on a CAS gives $\dfrac{4}{x-1} + \dfrac{16}{(x-2)^2}$. Note that $B = 0$.

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Question 5 **E**

Since $v = iu$, then $OU \perp OV$.

Hence $\angle UOV = 90°$.

Let A be the area of triangle OUV.

$$A = \frac{1}{2}|u||v|$$
$$= \frac{1}{2}|2 + 3i|^2$$
$$= \frac{13}{2}$$

Question 6 **C**

$$\frac{w}{z} = \frac{ux + vy}{x^2 + y^2} + \left(\frac{vx - uy}{x^2 + y^2}\right)i$$

$$\mathrm{Re}\left(\frac{w}{z}\right) = \frac{\mathrm{Re}(w)}{\mathrm{Re}(z)} \Rightarrow \frac{ux + vy}{x^2 + y^2} = \frac{u}{x}$$

Method 1:

Solving $\frac{ux + vy}{x^2 + y^2} = \frac{u}{x}$ for y gives $y = 0$ or $y = \frac{vx}{u}$.

$y = \frac{vx}{u} \Rightarrow vx - uy = 0$.

Hence $\mathrm{Im}(z) = 0$ or $\mathrm{Im}\left(\frac{w}{z}\right) = 0$.

Method 2:

$$ux^2 + vxy = ux^2 + uy^2$$
$$y(vx - uy) = 0$$
$$y = 0 \text{ or } vx - uy = 0$$

Hence $\mathrm{Im}(z) = 0$ or $\mathrm{Im}\left(\frac{w}{z}\right) = 0$.

Question 7 **B**

$P(z) = (z - 3)(z - 3i)(z + 3i)$ so $z + 3i$ is a linear factor of $P(z)$.

$(z^2 + 9)$ is a quadratic factor and so **A** is incorrect.

3 and $3i$ are roots of the equation $P(z) = 0$ and are not linear factors of $P(z)$.

Hence **D** and **E** are incorrect.

Question 8 A

$$\text{Arg}\left(\frac{z-4}{i}\right) = \text{Arg}(1+i)$$

$$\text{Arg}(z-4) - \text{Arg}(i) = \text{Arg}(1+i)$$

$$\text{Arg}(z-4) = \text{Arg}(1+i) + \text{Arg}(i)$$

$$\text{Arg}(z-4) = \frac{\pi}{4} + \frac{\pi}{2}$$

$$= \frac{3\pi}{4}$$

Let $z = x + yi$ and so $z - 4 = (x-4) + yi$.

$$\frac{y}{x-4} = \tan\left(\frac{3\pi}{4}\right)$$

$$= -1$$

So $y = 4 - x, x < 4$.

Question 9 A

Let $u = \sqrt{x}$.

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow 2\frac{du}{dx} = \frac{1}{\sqrt{x}}$$

$$\int \frac{\sin(\sqrt{x})}{\sqrt{x}}dx = 2\int \sin(u)\frac{du}{dx}dx$$

$$= 2\int \sin(u)du$$

Question 10 E

$$2\frac{dy}{dx} + \arctan(e^x) = \sin(x) \Rightarrow \frac{dy}{dx} = \frac{1}{2}(\sin(x) - \arctan(e^x))$$

The initial point is (0, 1). That is, $a = 0$ and $b = 1$.

Euler's method using a step size of 0.2 gives:

$a = 0$ $\quad f(a) = f(0) = \frac{1}{2}(\sin(0) - \arctan(1)) = -\frac{\pi}{8}$

$x_1 = 0.2$ $\quad f(x_1) = f(0.2) = \frac{1}{2}(\sin(0.2) - \arctan(e^{0.2}))$

Using $y_{n+1} = y_n + hf(x_n)$:

$$y_1 = b + hf(a)$$

$$= 1 - \frac{\pi}{40}$$

$$y_2 = y_1 + hf(x_1)$$

$$= \left(1 - \frac{\pi}{40}\right) + 0.1(\sin(0.2) - \arctan(e^{0.2}))$$

Question 11 B

Differentiating with respect to x gives:

$$-4\sin(y)\frac{dy}{dx} = -2\cos(x)$$

So $\dfrac{dy}{dx} = \dfrac{\cos(x)}{2\sin(y)}$.

At $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$, $\dfrac{dy}{dx} = \dfrac{\cos\left(\frac{\pi}{6}\right)}{2\sin\left(\frac{\pi}{3}\right)} = \dfrac{1}{2}$.

The equation of the tangent to C at the point $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ is $y - \frac{\pi}{3} = \frac{1}{2}\left(x - \frac{\pi}{6}\right)$.

Solving $y - \frac{\pi}{3} = \frac{1}{2}\left(x - \frac{\pi}{6}\right)$ for x with $y = 0$ gives $x = -\frac{\pi}{2}$.

Question 12 B

A is correct as the gradient is positive for $a < x < c$.

At $x = b$, the gradient has its maximum value, which is non-zero. Hence the graph of $y = f(x)$ has a non-stationary point of inflection, not a stationary point of inflection. Hence **B** is incorrect.

The gradient is decreasing for $x > b$, so **C** is correct.

For $x < a$, $f'(x) < 0$. At $x = a$, $f'(x) = 0$. For $x > a$ (x just greater than a), $f'(x) > 0$. So the gradient changes from negative to positive. Hence there is a local minimum on the graph of $y = f(x)$ at $x = a$. Hence **D** is correct.

For $x < c$ (x just less than c), $f'(x) > 0$. At $x = c$, $f'(x) = 0$. For $x > c$, $f'(x) < 0$. So the gradient changes from positive to negative. Hence there is a local maximum on the graph of $y = f(x)$ at $x = c$. Hence **E** is correct.

Question 13 D

Solving $x^2\sin(x) - 2 = 0$ for x with $-8 \le x \le 8$ gives 5 solutions.

Hence there are 5 points of inflection.

Question 14 A

If $y = e^{-x}\sin(x)$, then $\dfrac{dy}{dx} = e^{-x}(\cos(x) - \sin(x))$ and $\dfrac{d^2y}{dx^2} = -2e^{-x}\cos(x)$.

Testing each alternative in turn by substituting y, $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ into the LHS reveals that **A** is correct.

$$\begin{aligned}\text{LHS} &= -2e^{-x}\cos(x) + 2e^{-x}(\cos(x) - \sin(x)) + 2e^{-x}\sin(x)\\ &= 0\end{aligned}$$

Question 15 **E**

Method 1:

$S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$

$\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$

$8 = 12x \times \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{2}{3x}$

Now, $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$.

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$

$= 3x^2 \times \frac{2}{3x}$

$= 2x$

As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$.

Method 2:

As $x = V^{\frac{1}{3}}$ and $S = 6x^2$, then $S = 6V^{\frac{2}{3}}$.

$\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ and so $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$.

$\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dV}{dS}$

$= 8 \times \frac{1}{4}V^{\frac{1}{3}}$

$= 2V^{\frac{1}{3}}$

Question 16 D

The particle reaches its maximum velocity at $t = 4$.

When $t = 4$:

$$\begin{aligned} v &= 3 \times 4 - \frac{3}{8} \times 4^2 \\ &= 6 \end{aligned}$$

Hence **A** is correct.

Since $a = \frac{dv}{dt}$, the gradient of the velocity–time graph gives the acceleration at time t.

For $4 \le t \le T$, the gradient is zero, so the acceleration is zero. Hence **B** is correct.

$$\frac{dv}{dt} = 3 - \frac{3}{4}t$$

At $t = 2$:

$$\begin{aligned} \frac{dv}{dt} &= 3 - \frac{3}{4} \times 2 \\ &= 1.5 \end{aligned}$$

Hence **C** is correct.

For a velocity–time graph, the signed area between times t_1 and t_2 represents the particle's displacement between these two times. For $4 \le t \le T$, the area under the graph is given by $6(T - 4)$ metres.

Alternatively:

$$\begin{aligned} d &= \int_4^T 6\,dt \\ &= 6\Big[t\Big]_4^T \\ &= 6(T - 4) \end{aligned}$$

Hence **D** is incorrect.

$$\int_0^4 \left(3t - \frac{3}{8}t^2\right) dt = 16$$

Hence **E** is correct.

Question 17 A

For linearly dependent vectors we can express $\underset{\sim}{c}$ as a linear combination of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

That is, $\underset{\sim}{c} = m\underset{\sim}{a} + n\underset{\sim}{b}$, where m and n are not both zero.

$p\underset{\sim}{i} + q\underset{\sim}{j} = (m+n)\underset{\sim}{i} + n\underset{\sim}{j} + (m+3n)\underset{\sim}{k}$

Equating coefficients gives the system of equations $m + n = p$, $n = q$ and $m + 3n = 0$.

Solving this system of equations gives either $q = n$ and $p = -2n$ or $q = -\frac{m}{3}$ and $p = \frac{2m}{3}$.

So $\frac{p}{q} = -2$.

Question 18 C

$$\begin{aligned}\overrightarrow{XZ} &= \overrightarrow{XY} + \overrightarrow{YZ} \\ &= \left|\overrightarrow{XY}\right|\underset{\sim}{i} + \left|\overrightarrow{YZ}\right|\cos(\alpha)\underset{\sim}{i} - \left|\overrightarrow{YZ}\right|\sin(\alpha)\underset{\sim}{j} \\ &= 300\underset{\sim}{i} + 600\cos(\alpha)\underset{\sim}{i} - 600\sin(\alpha)\underset{\sim}{j} \\ &= 300(1 + 2\cos(\alpha))\underset{\sim}{i} - 600\sin(\alpha)\underset{\sim}{j}\end{aligned}$$

Question 19 C

The initial momentum, p_i, is mU kg m s^{-1}. The final momentum, p_f, is mV kg m s^{-1}. The change in momentum is $\Delta p = p_f - p_i = mV - mU$.

Hence $\Delta p = m(V - U)$.

Alternatively:

$\Delta p = m\Delta v$, where Δv is the change in velocity.

$\Delta p = m(V - U)$

Question 20 E

$$\begin{aligned}\sum \underset{\sim}{F} &= \underset{\sim}{F}_1 + \underset{\sim}{F}_2 \\ &= (p + r)\underset{\sim}{i} + (q + s)\underset{\sim}{j}\end{aligned}$$

$$\left|\sum \underset{\sim}{F}\right| = \left|\underset{\sim}{F}_1 + \underset{\sim}{F}_2\right|$$

$$m\left|\underset{\sim}{a}\right| = \sqrt{(p + r)^2 + (q + s)^2}$$

Therefore, $\left|\underset{\sim}{a}\right| = \frac{\sqrt{(p + r)^2 + (q + s)^2}}{m}$.

SECTION B

Question 1 (8 marks)

a. $\overrightarrow{OR} = \overrightarrow{OA} + \overrightarrow{AR}$ and so $\overrightarrow{OR} = \underset{\sim}{a} + m\overrightarrow{AB}$. M1

$\overrightarrow{OR} = \underset{\sim}{a} + m(\overrightarrow{AO} + \overrightarrow{OB})$ and so $\overrightarrow{OR} = \underset{\sim}{a} + m(-\underset{\sim}{a} + \underset{\sim}{b})$. A1

$\overrightarrow{OR} = \underset{\sim}{a} - m\underset{\sim}{a} + m\underset{\sim}{b}$ and so $\overrightarrow{OR} = (1 - m)\underset{\sim}{a} + m\underset{\sim}{b}$.

b. $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$ and so $\overrightarrow{OR} = \overrightarrow{OP} + n\overrightarrow{PQ}$. M1

$\overrightarrow{OR} = \overrightarrow{OP} + n(\overrightarrow{PO} + \overrightarrow{OQ})$

So $\overrightarrow{OR} = \frac{3}{5}\underset{\sim}{a} + n\left(-\frac{3}{5}\underset{\sim}{a} + 3\underset{\sim}{b}\right)$. A1

$\overrightarrow{OR} = \frac{3}{5}\underset{\sim}{a} - \frac{3}{5}n\underset{\sim}{a} + 3n\underset{\sim}{b}$ and so $\overrightarrow{OR} = \frac{3}{5}(1 - n)\underset{\sim}{a} + 3n\underset{\sim}{b}$.

c. $\overrightarrow{OR} = (1 - m)\underset{\sim}{a} + m\underset{\sim}{b}$ and $\overrightarrow{OR} = \frac{3}{5}(1 - n)\underset{\sim}{a} + 3n\underset{\sim}{b}$.

So $1 - m = \frac{3}{5}(1 - n)$ and $m = 3n$. A1

Using CAS, attempt to solve these two equations for m and n. M1

$m = \frac{1}{2}$ and $n = \frac{1}{6}$ A1

d. $\overrightarrow{PR} = \frac{1}{6}\overrightarrow{PQ}$

$PR : PQ = 1 : 6$ A1

Note: Consequential on ***Question 1c.***

Question 2 (13 marks)

a. **Method 1:**

Parametric equations are $x = \cos(t)$ and $y = \frac{1}{2}\sin(2t)$.

$y = \sin(t)\cos(t)$ and so $y^2 = \sin^2(t)\cos^2(t)$. A1

$y^2 = \cos^2(t)(1 - \cos^2(t))$

Substituting $\cos^2(t) = x^2$ into $y^2 = \cos^2(t)(1 - \cos^2(t))$ gives M1
$y^2 = x^2(1 - x^2)$ as required.

Method 2:

Parametric equations are $x = \cos(t)$ and $y = \frac{1}{2}\sin(2t)$.

Substituting $\cos^2(t) = x^2$ into $\cos(2t) = 2\cos^2(t) - 1$ gives $\cos(2t) = 2x^2 - 1$. A1

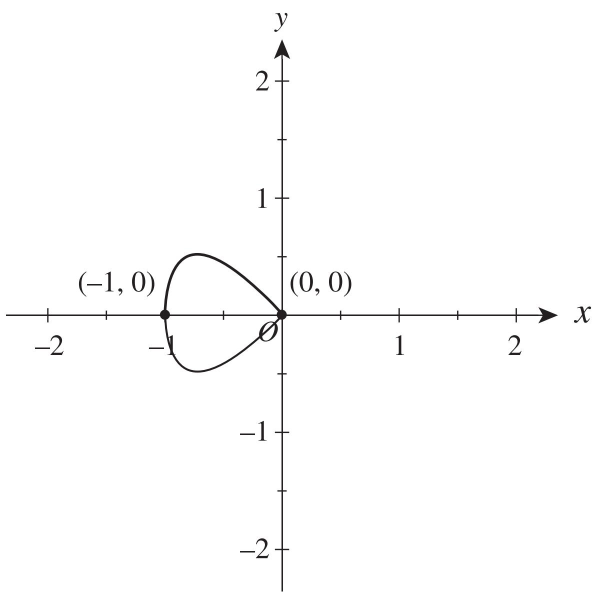
$\sin(2t) = 2y$

Substituting $\cos(2t) = 2x^2 - 1$ and $\sin(2t) = 2y$ into $\sin^2(2t) + \cos^2(2t) = 1$ gives
$4y^2 + (2x^2 - 1)^2 = 1$. M1

$$\begin{aligned} 4y^2 &= 1 - (4x^4 - 4x^2 + 1) \\ &= 4x^2 - 4x^4 \end{aligned}$$

So $y^2 = x^2(1 - x^2)$ as required.

b.



upper branch with correct shape and location A1
lower branch with correct shape and location A1
correct intercepts (–1, 0) *and* (0, 0) A1

c. Let V be the volume of the solid formed.

$$V = \pi\int_{-1}^{0} y^2 dx = \pi\int_{-1}^{0} x^2(1 - x^2)dx$$ M1

$$= \pi\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_{-1}^{0}$$

$$= \frac{2\pi}{15} \text{ (cubic units)}$$ A1

d. **i.** When $t = \frac{2\pi}{3}$, $x = \cos\left(\frac{2\pi}{3}\right)$ and $y = \frac{1}{2}\sin\left(\frac{4\pi}{3}\right)$.

So point P has coordinates $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{4}\right)$. M1

Using a CAS or otherwise, the equation of the normal to C at P

is $y = -\sqrt{3}x - \frac{3\sqrt{3}}{4}$. A1

ii. **Method 1:**

The upper branch of C has the equation $y = -x\sqrt{1 - x^2}$. A1

Solving $-x\sqrt{1 - x^2} = -\sqrt{3}x - \frac{3\sqrt{3}}{4}$ for x gives M1

$x = -0.9378\ \ldots$ A1

So point Q has coordinates $(-0.938, 0.325)$ (correct to three decimal places). A1

Method 2:

Substituting $x = \cos(t)$ and $y = \frac{1}{2}\sin(2t)$ into $y = -\sqrt{3}x - \frac{3\sqrt{3}}{4}$ gives

$\frac{1}{2}\sin(2t) = -\sqrt{3}\cos(t) - \frac{3\sqrt{3}}{4}$.

Solving $\frac{1}{2}\sin(2t) = -\sqrt{3}\cos(t) - \frac{3\sqrt{3}}{4}$ for t gives M1

$t = 2.0944\ldots$ or $t = 3.4959\ldots$ A1

$t = 2.0944\ldots$ corresponds to $t = \frac{2\pi}{3}$ and so the normal meets C again at

$\left(\cos(3.4959\ldots), \frac{1}{2}\sin(2(3.4959\ldots))\right)$. A1

So point Q has coordinates $(-0.938, 0.325)$ (correct to three decimal places). A1

Note: Consequential on ***Question 2d.i.***

Question 3 (11 marks)

a. **i.** LHS: $r\text{cis}\left(\frac{\theta}{2}\right)\left(\text{cis}\left(\frac{\theta}{2}\right)-\text{cis}\left(-\frac{\theta}{2}\right)\right)=r\text{cis}\left(\frac{\theta}{2}+\frac{\theta}{2}\right)-r\text{cis}\left(\frac{\theta}{2}-\frac{\theta}{2}\right)$ M1

$\text{cis}\left(\frac{\theta}{2}+\frac{\theta}{2}\right)=\text{cis}(\theta)$ and $\text{cis}\left(\frac{\theta}{2}-\frac{\theta}{2}\right)=\text{cis}(0)=1$ A1

So $r\text{cis}\left(\frac{\theta}{2}\right)\left(\text{cis}\left(\frac{\theta}{2}\right)-\text{cis}\left(-\frac{\theta}{2}\right)\right)=r\text{cis}(\theta)-r$.

ii. LHS: $\frac{z}{z-r}=\frac{r\text{cis}(\theta)}{r\text{cis}(\theta)-r}$

$$=\frac{r\text{cis}(\theta)}{r\text{cis}\left(\frac{\theta}{2}\right)\left(\text{cis}\left(\frac{\theta}{2}\right)-\text{cis}\left(-\frac{\theta}{2}\right)\right)}$$ M1

$$=\frac{\cos\left(\frac{\theta}{2}\right)+i\sin\left(\frac{\theta}{2}\right)}{2i\sin\left(\frac{\theta}{2}\right)}$$ A1

$$=\frac{1}{2}+\frac{1}{2i}\cot\left(\frac{\theta}{2}\right)$$ A1

As $\frac{1}{2i}=-\frac{1}{2}i$, $\frac{z}{z-r}=\frac{1}{2}-\frac{1}{2}i\cot\left(\frac{\theta}{2}\right)$.

b. Using CAS to solve $z^2-2z+4=0$ for z gives:

$z=2\text{cis}\left(-\frac{\pi}{3}\right)$ or $z=2\text{cis}\left(\frac{\pi}{3}\right)$. A1

c. $\left(\frac{2w}{w-1}\right)^2-2\left(\frac{2w}{w-1}\right)+4=0$ M1

Comparing $z^2-2z+4=0$ with the above equation, the transformation from z to w is $z=\frac{2w}{w-1}$. A1

d. **Method 1:**

Using **part c.**, solving $z = \dfrac{2w}{w-1}$ for w gives $w = \dfrac{z}{z-2}$. M1

From **part a.ii.** and **part b.**, $r = 2$ and $\theta = \pm\dfrac{\pi}{3}$. A1

Using $w = \dfrac{1}{2} - \dfrac{1}{2}i\cot\left(\dfrac{\theta}{2}\right)$ gives:

$w = \dfrac{1}{2} - \dfrac{1}{2}i\cot\left(-\dfrac{\pi}{6}\right)$, $w = \dfrac{1}{2} - \dfrac{1}{2}i\cot\left(\dfrac{\pi}{6}\right)$ A1

Method 2:

$$4w^2 - 4w(w-1) + 4(w-1)^2 = 0$$

$$4w^2 - 4w^2 + 4w + 4w^2 - 8w + 4 = 0$$

$$w^2 - w + 1 = 0 \quad \text{M1}$$

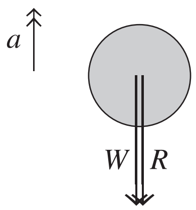
$$w = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2}$$

$$= \frac{1}{2} \pm \frac{1}{2}i\sqrt{3} \quad \text{A1}$$

$$= \frac{1}{2} - \frac{1}{2}i\cot\left(-\frac{\pi}{6}\right),\ w = \frac{1}{2} - \frac{1}{2}i\cot\left(\frac{\pi}{6}\right) \quad \text{A1}$$

Question 4 (15 marks)

a. During upward motion, the two forces acting downwards are weight, *W* newtons, and air resistance, *R* newtons.



correct forces of magnitude W and R newtons both acting downwards A1

b. $0.1\dfrac{dv}{dt} = -0.1g - 0.02v$ A1

$$\frac{dv}{dt} = -g - 0.2v$$

$$= -0.2(5g + v)$$

c. $\displaystyle\int \frac{dv}{5g+v} = \int -0.2\,dt$ M1

$\log_e|5g + v| = -0.2t + c$ A1

$5g + v = Ae^{-0.2t}$ (where $A = e^c$)

When $t = 0$, $v = 5g$ and so $A = 10g$. M1

$5g + v = 10ge^{-0.2t} \Rightarrow v = 5g(2e^{-0.2t} - 1)$ A1

d. At maximum height, $v = 0$. That is, $2e^{-0.2t} - 1 = 0$.

Attempting to solve $2e^{-0.2t} - 1 = 0$ for t. M1

$$2e^{-0.2t} - 1 = 0$$
$$e^{-0.2t} = \frac{1}{2}$$
$$t = -5\log_e\left(\frac{1}{2}\right) \text{ or } t = 5\log_e(2)$$ A1

$$H = \int_0^{5\log_e(2)} 5g(2e^{-0.2t} - 1)dt$$ A1

$$= 25g(1 - \log_e(2))$$ A1

e. For the ball's downward motion, $v < 0$ and so $-0.02v > 0 (R > 0)$. That is, R opposes the ball's motion. The same initial conditions apply and so the differential equation applies throughout the motion. A1

f. **i.** $\int_T^0 5g(2e^{-0.2t} - 1)dt \ (= 0)$ M1

$50ge^{-0.2T} + 5gT - 50g \ (= 0)$ A1

Taking out $5g$ as a common factor from $50ge^{-0.2T} + 5gT - 50g = 0$ and then dividing through by 10. M1

So $\frac{T}{10} + e^{-0.2T} = 1$.

ii. Solving $\frac{T}{10} + e^{-0.2T} = 1$ for T gives $T = 7.97$ (s) (correct to two decimal places). A1

Question 5 (13 marks)

a. The equations of motion for each mass are:

Mass A: $T - 10 - 2g\sin(\alpha) = 2a$ (1)

Mass B: $3g - T = 3a$ (2) A1

(1) + (2) gives $3g - 10 - 2g\left(\frac{3}{5}\right) = 5a$, where $\sin(\alpha) = \frac{3}{5}$. M1

So $a = \frac{9g}{25} - 2$ (m s^{-2}).

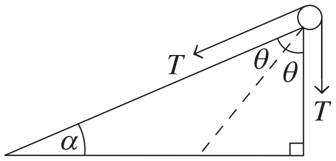
b. From (2), $3g - T = 3a$ and so $T = 3g - 3a$.

Substituting $a = \frac{9g}{25} - 2$ into $T = 3g - 3a$ gives:

$$T = 3g - 3\left(\frac{9g}{25} - 2\right)$$ A1

$$= \frac{48g}{25} + 6 \text{ (N)}$$

c. From the following diagram:



$\theta = \dfrac{90° - \alpha}{2} = \dfrac{90° - \tan^{-1}\left(\frac{3}{4}\right)}{2}$ M1

So $\theta = 26.6°$ to the vertical (correct to one decimal place). A1

Alternatively, $\theta = 26.6°$ to the plane (correct to one decimal place).

The magnitude of F exerted by the string on P is given by $F = 2T\cos(\theta)$ where $T = 24.816$ (N) and $\theta = 26.6°$. M1

So $F = 44.4$ (N) (correct to one decimal place). A1

d. First consider the motion of mass A under constant acceleration for the first 2 seconds.

Either:

$\dfrac{dv}{dt} = \dfrac{9g}{25} - 2 \Rightarrow v = \left(\dfrac{9g}{25} - 2\right)t + C$

When $t = 0$, $v = 0$ and so $C = 0$.

So $v = \left(\dfrac{9g}{25} - 2\right)t$. M1

$\dfrac{ds}{dt} = \left(\dfrac{9g}{25} - 2\right)t \Rightarrow s = \dfrac{1}{2}\left(\dfrac{9g}{25} - 2\right)t^2 + D$

When $t = 0$, $s = 0$ and so $D = 0$.

When $t = 2$, $s = 2\left(\dfrac{9g}{25} - 2\right)$ (= 3.056) (m). A1

Award M1 A1 *for using a CAS to solve* $\dfrac{d^2s}{dt^2} = \dfrac{9g}{25} - 2$ *with* $v = 0$, $s = 0$ *when* $t = 0$ *directly and then substituting* $t = 2$ *into* $s = \dfrac{1}{2}\left(\dfrac{9g}{25} - 2\right)t^2$ *to obtain* $s = 2\left(\dfrac{9g}{25} - 2\right)$ (= 3.056) (*m*).

Or:

Use of $v = u + at$ with $u = 0$, $a = \dfrac{9g}{25} - 2$ and $t = 2$ to obtain $v = 2\left(\dfrac{9g}{25} - 2\right)$. M1

Use of $s = \left(\dfrac{u+v}{2}\right)t$ with $u = 0$, $v = 2\left(\dfrac{9g}{25} - 2\right)$ and $t = 2$ to obtain $s = 2\left(\dfrac{9g}{25} - 2\right)$ (= 3.056) (m). A1

Then:

Now consider the motion of mass A at the instant the string breaks. There is no longer any tension in the string and so the new acceleration (deceleration) needs to be calculated.

The new equation of motion for mass A is $-10 - 2g\sin(\alpha) = 2a$.

$-10 - 2g\left(\frac{3}{5}\right) = 2a$ where $\sin(\alpha) = \frac{3}{5}$.

So $a = -\frac{3g}{5} - 5$ $(= -10.88)$ (m s^{-2}). M1

Either:

$\frac{d}{ds}\left(\frac{1}{2}v^2\right) = -\frac{3g}{5} - 5 \Rightarrow \frac{1}{2}v^2 = \left(-\frac{3g}{5} - 5\right)s + C$

When $s = 0$, $v = 2\left(\frac{9g}{25} - 2\right) = (3.056)$ and so $C = 2\left(\frac{9g}{25} - 2\right)^2 = (4.6695\ldots)$. M1

Solving for s we obtain $s = 0.429$ (m). A1

Or:

Use of $v^2 = u^2 + 2as$ with $u = 2\left(\frac{9g}{25} - 2\right) = 3.056$, $a = -\frac{3g}{5} - 5 = -10.88$ and $v = 0$. M1

Solving for s we obtain $s = 0.429$ (m). A1

Then:

Hence the total distance travelled by mass A up the plane is 3.49 (m) (correct to two decimal places). A1

Note: Award a maximum of 6 marks. Due to the various acceptable methods, more working is shown in the suggested solution than is expected or required from a student.

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