# **MATHEMATICAL METHODS**

# Written examinations 1 and 2

# FORMULA SHEET

## **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

MATH METH

## **Mathematical Methods Formulas**

2

### Mensuration

area of a trapezium:  $\frac{1}{2}(a+b)h$  volume of a pyramid:  $\frac{1}{3}Ah$  curved surface area of a cylinder:  $2\pi rh$  volume of a sphere:  $\frac{4}{3}\pi r^3$  volume of a cylinder:  $\pi r^2 h$  area of a triangle:  $\frac{1}{2}bc\sin A$  volume of a cone:

### Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \text{ for } x > 0$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = c$$

#### **Statistics and Probability**

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\text{mean:} \quad \mu = E(X)$$

$$\text{variance:} \quad \text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$$

| Discrete distributions |   |                      |  |  |  |  |  |  |  |  |
|------------------------|---|----------------------|--|--|--|--|--|--|--|--|
|                        | Pr(X = x)   | Pr(X = x) mean $v$   |  |  |  |  |  |  |  |  |
| general                | p(x)  | $\mu = \sum x  p(x)$ | $\sigma^2 = \sum (x - \mu)^2 p(x)$                                   |  |  |  |  |  |  |  |
|                        |   |                      | $= \sum x^2 p(x) - \mu^2$  |  |  |  |  |  |  |  |
| binomial               | ${}^{n}C_{x} p^{x}(1-p)^{n-x}$  | np                   | np(1-p)  |  |  |  |  |  |  |  |
| hypergeometric         | $\frac{{}^{D}C_{x}{}^{N-D}C_{n-x}}{{}^{N}C_{n}}$  | $n\frac{D}{N}$       | $n\frac{D}{N}\left(1-\frac{D}{N}\right)\left(\frac{N-n}{N-1}\right)$ |  |  |  |  |  |  |  |
| Continuous distribut   | ions  |                      |  |  |  |  |  |  |  |  |
| normal                 | If X is distributed N( $\mu$ , $\sigma^2$ ) and $Z = \frac{X - \mu}{\sigma}$ , then Z is distributed N(0, 1). |                      |  |  |  |  |  |  |  |  |

3 MATH METH

Table 1 Normal distribution – cdf

|     | 0      | 1      | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---|---|----|----|----|----|----|----|----|
| 0.0 | .5000  | .5040  | .5080  | .5120  | .5160  | .5199  | .5239  | .5279  | .5319  | .5359  | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | .5398  | .5438  | .5478  | .5517  | .5557  | .5596  | .5636  | .5675  | .5714  | .5753  | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 35 |
| 0.2 | .5793  | .5832  | .5871  | .5910  | .5948  | .5987  | .6026  | .6064  | .6103  | .6141  | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | .6179  | .6217  | .6255  | .6293  | .6331  | .6368  | .6406  | .6443  | .6480  | .6517  | 4 | 8 | 11 | 15 | 19 | 23 | 26 | 30 | 34 |
| 0.4 | .6554  | .6591  | .6628  | .6664  | .6700  | .6736  | .6772  | .6808  | .6844  | .6879  | 4 | 7 | 11 | 14 | 18 | 22 | 25 | 29 | 32 |
|     |        |        |        |        |        |        |        |        |        |        |   |   |    |    |    |    |    |    |    |
| 0.5 | .6915  | .6950  | .6985  | .7019  | .7054  | .7088  | .7123  | .7157  | .7190  | .7224  | 3 | 7 | 10 | 14 | 17 | 21 | 24 | 27 | 31 |
| 0.6 | .7257  | .7291  | .7324  | .7357  | .7389  | .7422  | .7454  | .7486  | .7517  | .7549  | 3 | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | .7580  | .7611  | .7642  | .7673  | .7703  | .7734  | .7764  | .7793  | .7823  | .7852  | 3 | 6 | 9  | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | .7881  | .7910  | .7939  | .7967  | .7995  | .8023  | .8051  | .8078  | .8106  | .8133  | 3 | 6 | 8  | 11 | 14 | 17 | 19 | 22 | 25 |
| 0.9 | .8159  | .8186  | .8212  | .8238  | .8264  | .8289  | .8315  | .8340  | .8365  | .8389  | 3 | 5 | 8  | 10 | 13 | 15 | 18 | 20 | 23 |
|     |        |        |        |        |        |        |        |        |        |        |   |   |    |    |    |    |    |    |    |
| 1.0 | .8413  | .8438  | .8461  | .8485  | .8508  | .8531  | .8554  | .8577  | .8599  | .8621  | 2 | 5 | 7  | 9  | 12 | 14 | 16 | 18 | 21 |
| 1.1 | .8643  | .8665  | .8686  | .8708  | .8729  | .8749  | .8770  | .8790  | .8810  | .8830  | 2 | 4 | 6  | 8  | 10 | 12 | 14 | 16 | 19 |
| 1.2 | .8849  | .8869  | .8888  | .8907  | .8925  | .8944  | .8962  | .8980  | .8997  | .9015  | 2 | 4 | 6  | 7  | 9  | 11 | 13 | 15 | 16 |
| 1.3 | .9032  | .9049  | .9066  | .9082  | .9099  | .9115  | .9131  | .9147  | .9162  | .9177  | 2 | 3 | 5  | 6  | 8  | 10 | 11 | 13 | 14 |
| 1.4 | .9192  | .9207  | .9222  | .9236  | .9251  | .9265  | .9279  | .9292  | .9306  | .9319  | 1 | 3 | 4  | 6  | 7  | 8  | 10 | 11 | 13 |
|     |        |        |        |        |        |        |        |        |        |        |   |   |    |    |    |    |    |    |    |
| 1.5 | .9332  | .9345  | .9357  | .9370  | .9382  | .9394  | .9406  | .9418  | .9429  | .9441  | 1 | 2 | 4  | 5  | 6  | 7  | 8  | 10 | 11 |
| 1.6 | .9452  | .9463  | .9474  | .9484  | .9495  | .9505  | .9515  | .9525  | .9535  | .9545  | 1 | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 1.7 | .9554  | .9564  | .9573  | .9582  | .9591  | .9599  | .9608  | .9616  | .9625  | .9633  | 1 | 2 | 3  | 3  | 4  | 5  | 6  | 7  | 8  |
| 1.8 | .9641  | .9649  | .9656  | .9664  | .9671  | .9678  | .9686  | .9693  | .9699  | .9706  | 1 | 1 | 2  | 3  | 4  | 4  | 5  | 6  | 6  |
| 1.9 | .9713  | .9719  | .9726  | .9732  | .9738  | .9744  | .9750  | .9756  | .9761  | .9767  | 1 | 1 | 2  | 2  | 3  | 4  | 4  | 5  | 5  |
|     |        |        |        |        |        |        |        |        |        |        |   |   |    |    |    |    |    |    |    |
| 2.0 | .9772  | .9778  | .9783  | .9788  | .9793  | .9798  | .9803  | .9808  | .9812  | .9817  | 0 | 1 | 1  | 2  | 2  | 3  | 3  | 4  | 4  |
| 2.1 | .9821  | .9826  | .9830  | .9834  | .9838  | .9842  | .9846  | .9850  | .9854  | .9857  | 0 | 1 | 1  | 2  | 2  | 2  | 3  | 3  | 4  |
| 2.2 | .9861  | .9864  | .9868  | .9871  | .9875  | .9878  | .9881  | .9884  | .9887  | .9890  | 0 | 1 | 1  | 1  | 2  | 2  | 2  | 3  | 3  |
| 2.3 | .9893  | .9896  | .9898  | .9901  | .9904  | .9906  | .9909  | .9911  | .9913  | .9916  | 0 | 1 | 1  | 1  | 1  | 2  | 2  | 2  | 2  |
| 2.4 | .9918  | .9920  | .9922  | .9925  | .9927  | .9929  | .9931  | .9932  | .9934  | .9936  | 0 | 0 | 1  | 1  | 1  | 1  | 1  | 2  | 2  |
|     |        |        |        |        |        |        |        |        |        |        |   |   |    |    |    |    |    |    |    |
| 2.5 | .9938  | .9940  | .9941  | .9943  | .9945  | .9946  | .9948  | .9949  | .9951  | .9952  | 0 | 0 | 0  | 1  | 1  | 1  | 1  | 1  | 1  |
| 2.6 | .9953  | .9955  | .9956  | .9957  | .9959  | .9960  | .9961  | .9962  | .9963  | .9964  | 0 | 0 | 0  | 0  | 1  | 1  | 1  | 1  | 1  |
| 2.7 | .9965  | .9966  | .9967  | .9968  | .9969  | .9970  | .9971  | .9972  | .9973  | .9974  | 0 | 0 | 0  | 0  | 0  | 1  | 1  | 1  | 1  |
| 2.8 | .9974  | .9975  | .9976  | .9977  | .9977  | .9978  | .9979  | .9979  | .9980  | .9981  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 1  | 1  |
| 2.9 | .9981  | .9982  | .9982  | .9983  | .9984  | .9984  | .9985  | .9985  | .9986  | .9986  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
|     |        |        |        |        |        |        |        |        |        |        |   |   |    |    |    |    |    |    |    |
| 3.0 | .9987  | .9987  | .9987  | .9988  | .9988  | .9989  | .9989  | .9989  | .9990  | .9990  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.1 | .9990  | .9991  | .9991  | .9991  | .9992  | .9992  | .9992  | .9992  | .9993  | .9993  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.2 | .9993  | .9993  | .9994  | .9994  | .9994  | .9994  | .9994  | .9995  | .9995  | .9995  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.3 | .9995  | .9995  | .9995  | .9996  | .9996  | .9996  | .9996  | .9996  | .9996  | .9997  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.4 | .9997  | .9997  | .9997  | .9997  | .9997  | .9997  | .9997  | .9997  | .9997  | .9998  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
|     |        |        |        |        |        |        |        |        |        |        |   |   |    |    |    |    |    |    |    |
| 3.5 | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | .9998  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.6 | .9998  | .9998  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.7 | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.8 | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | .9999  | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3.9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
|     |        |        |        |        |        |        |        |        |        |        | 1 |   |    |    |    |    |    |    |    |