Papers written by
Australian Maths
Software

REVISION 1

2016

MATHEMATICS
METHODS

Units 1 & 2

Semester 2
SOLUTIONS

SECTION 1 – Calculator-free

Question 1 **(6 marks)**

(a) $f(x) = x^3 + 1.5x^2 - 6x$

$f'(x) = 3x^2 + 3x - 6$ ✓

Turning points where $f'(x) = 0$

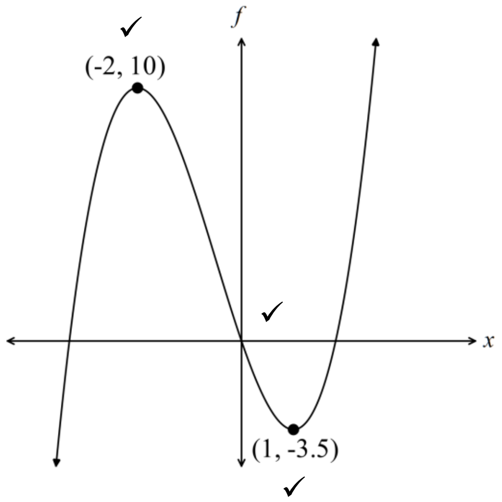
$0 = 3\left(x^2 + x - 2\right)$

$0 = \left(x+2\right)\left(x-1\right)$

$(-2, 10),\ (1, -3.5)$

✓ ✓

(b)



Question 2 **(5 marks)**

(a) $x^2 - 3x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{9 - 4(1)(-4)}}{2}$$

$$x = \frac{3 \pm \sqrt{25}}{2} = \frac{3 \pm 5}{2}$$

$x = 4$ *or* $x = -1$

✓ ✓

(b) $x^2-3x+1=0$

$x^2-3x+\frac{9}{4}=-1+\frac{9}{4}$ ✓

$\left(x-\frac{3}{2}\right)^2=\frac{5}{4}$

$x=\frac{3}{2}\pm\frac{\sqrt{5}}{2}$ ✓

(c) $x^3+x^2-16x+20=0$

$Let\quad P(x)=x^3+x^2-16x+20$

$P(2)=8+4-32+20=0$

$\therefore$ 2 is a root ✓

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -16 & 20 \\ & \downarrow & 2 & 6 & -20 \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

$P(x)=(x-2)(x^2+3x-10)$ ✓

$so\ \ 0=(x-2)(x+5)(x-2)$

$x=2$ (twice) or $x=-5$ ✓

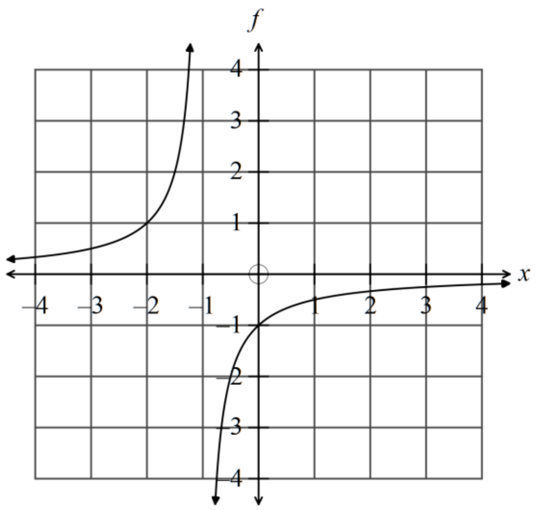
Question 3 **(3 marks)**

$^3C_1\times{}^5C_2=3\times\frac{5!}{3!\,2!}=3\times\frac{5\times4}{2\times1}=30$ ✓ ✓ ✓

Question 4 **(14 marks)**

(a) Parabolic, concave up, turning point at (-2 ,2). ✓✓

(b) $f(x)=-\frac{1}{x+1}$

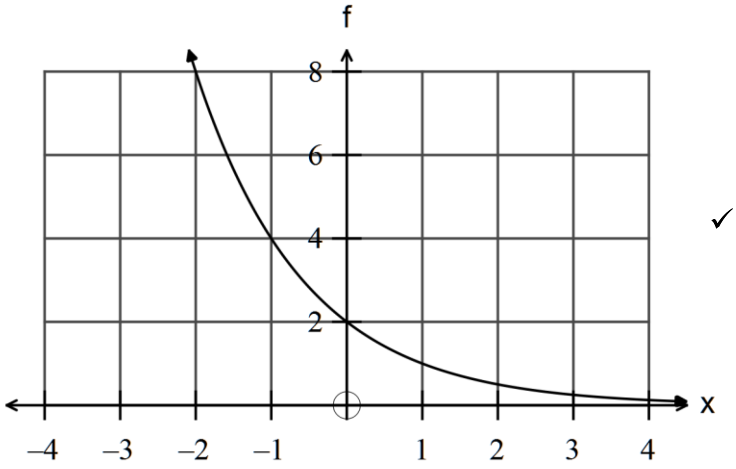


✓✓

(c) The relationship represents a circle with centre (-2, 1) and radius 2. ✓✓

(d) $y=(x-1)(x-2)(x+3)$ ✓✓✓ -1/error

(e) $f(x)=2^{1-x}$



✓✓

(f) $f(x)=3-3^{x}$ ✓✓✓

Question 5 **(10 marks)**

(a) $\dfrac{2\times 8^{\frac{1}{3}}-16^{0.25}}{81^{-\frac{3}{4}}}$

$$=\frac{2\times\left(2^3\right)^{\frac{1}{3}}-\left(2^4\right)^{0.25}}{\left(3^4\right)^{-\frac{3}{4}}} \quad ✓$$

$$=\frac{2\times 2-2}{3^{-3}}$$

$$=2\times 27$$

$$=54 \quad ✓$$

(b) (i) $\dfrac{4^{1-2x}}{8^{x}}=16$

$$\frac{2^{2(1-2x)}}{2^{3x}}=2^4$$

$$2^{2-4x-3x}=2^4 \quad ✓$$

$$\therefore 2-7x=4$$

$$7x=-2$$

$$x=-2/7 \quad ✓$$

(ii) $3^{2x} - 12\left(3^x\right) + 3^3 = 0$

Let $y = 3^x$

$y^2 - 12y + 27 = 0$ ✓

$(y-3)(y-9) = 0$

$y = 3 \; or \;\; y = 9$

$3^x = 3 \; or \;\; 3^x = 9$

$x = 1 \;\; or \;\; x = 2$

✓ ✓

(c) $\dfrac{a \times \sqrt{c}}{b} = \dfrac{\overset{2}{\cancel{4}.6} \times 10^2 \times \sqrt{4 \times 10^{-4}}}{\cancel{2}.3 \times 10^{-2}} = 2 \times 10^{2+2} \times 2 \times 10^{-2} = 400$

✓ ✓

(d) 6.1×10^9 ✓

Question 6 **(6 marks)**

(a) (i) $A_n = 1 + 3n$ $\quad 4, 7, 10, 13$ ✓✓

(ii) $A_{n+1} = A_n + 3, \quad A_1 = 4$ ✓✓

(b) $3, 6, 12, ...GP \quad a = 3, \;\; r = 2$ ✓

$T_n = ar^{n-1}$ ✓

$T_n = 3 \times 2^{n-1}$

Question 7 **(7 marks)**

(a) $sin\left(\dfrac{\pi}{6}\right) + sin^2\left(\dfrac{\pi}{6}\right) + sin^3\left(\dfrac{\pi}{6}\right) + ...$

$= \dfrac{1}{2} + \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{2}\right)^3 ...$ ✓ $\quad S_\infty = \dfrac{a}{1-r}$

$= \dfrac{\frac{1}{2}}{1 - \frac{1}{2}}$ ✓

$= 1$ ✓

(b) (i) $P(A \cup B) = P(A) + P(B - P(A \cap B)$ ✓

$= 0.3 + 0.5 - 0.2$

$= 0.6$ ✓

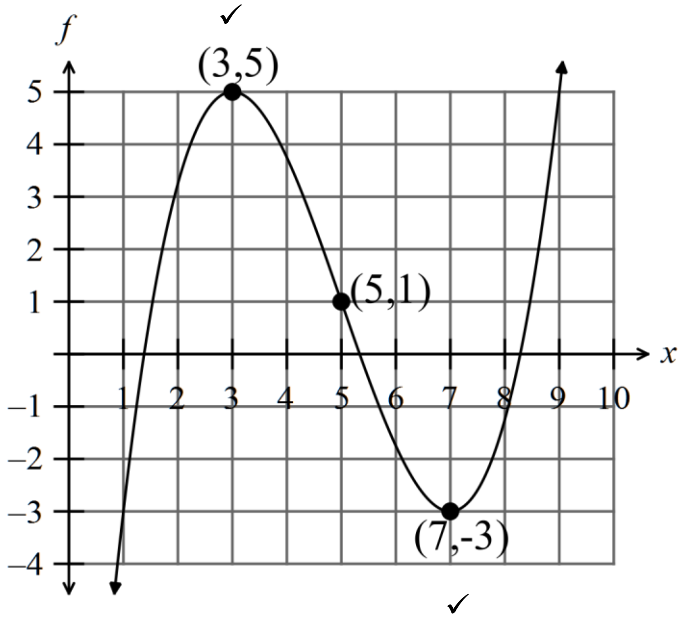
(ii) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.2}{0.5} = \dfrac{2}{5}$

✓ ✓

SECTION 2 – Calculator-assumed

Question 8 **(3 marks)**

✓



✓ general shape

✓

Question 9 **(9 marks)**

(a) $P_{n+1} = 1.05 \times P_n, \quad P_1 = 40$ ✓✓

(b) 2011 $P_1 = 1.05 \times 40 = 42$ ✓

2015 $P_5 = 40 \times 1.05^5 \approx 51$ ✓

(c) The model is not perfect as it suggest 51 cats and there are only 45. ✓
The ratio should be less than 1.05.

(d) 2010 $t = 0 \quad P = 40$

2015 $t = 5 \quad P = 45$

$P(t) = 40(r)^t$

$45 = 40(r)^5$

$r = 1.0238$ ✓

$P(t) = 40(1.0238)^t$ ✓

(e) 2020 t = 10

$P(10) = 40(1.0238)^{10} \approx 51$ ✓

No, the number of expected cats in 2020 is 51, so the action need not be taken. ✓

Question 10 **(5 marks)**

(a) $PQ^2 = (6-2)^2 + (0-3)^2$

$PQ^2 = 25$ ✓

$PQ = 5$

(b) P to Q is 4 units left and 3 units down..

R will be 3 units right and 4 units up.

i.e. Q(6, 0) $\longrightarrow$ R(6+3, 0+4)

R(9,4) ✓ ✓

Check $RQ^2 = (3)^2 + (4)^2$ $RQ = 5$

$m_{RQ} = \frac{4}{3}, \quad m_{PQ} = -\frac{3}{4}$

(c) Likewise

S will be 3 units right and 4 units up from P.

i.e. P(2, 3) $\longrightarrow$ S(2+3, 3+4)

S(5,7) ✓ ✓

Question 11 **(19 marks)**

(a) $f(x) = -(x-3)^3 + 2 = -x^3 + 9x^2 - 27x + 29$

(i) $f(0) = 29$ $A(0, 29)$ ✓

(ii) $f'(x) = -3x^2 + 18x - 27$ ✓

$f'(x) = 0$ $-3\left(x^2 - 6x + 9\right) = 0$

$\left(x-3\right)^2 = 0$

$B(3,2)$ ✓ Equation of the normal is $x = 3$ ✓

(iii) C(4.26, 0) ✓

(b) $x = t^3 - 9t$

(i) $v = \frac{dx}{dt} = 3t^2 - 9$ ✓

(ii) $At\ t = 3, \quad x = 0m,$ ✓ $v = 18m/s$ ✓

(iii) Particle changes direction when the velocity equals zero. ✓

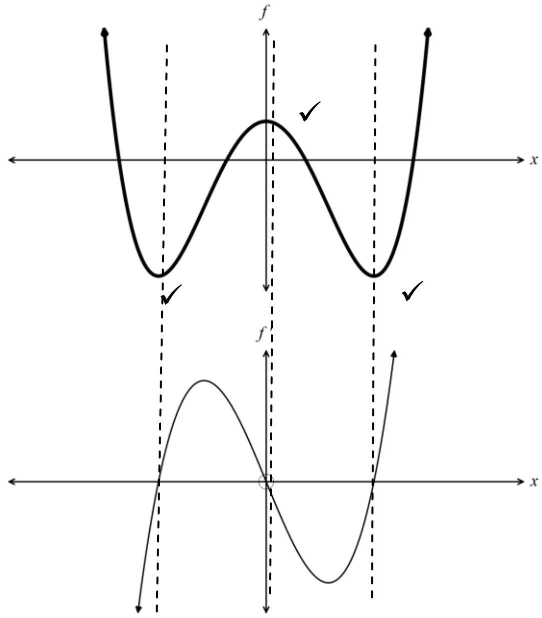
$3t^2 - 9 = 0$

$t^2 = 3$

$t = \sqrt{3} \quad as \quad t \geq 0$ ✓

$x_{\sqrt{3}} = \left(\sqrt{3}\right)^3 - 9\sqrt{3} = -6\sqrt{3}\ m$ ✓✓

(c) (i)



✓ shape and concavity

(ii) The turning points of f occur where f ' = 0. There are three of those. ✓
Where f ' < 0, then the gradient of f is negative. ✓
Where f ' > 0, then the gradient of f is positive. ✓
If f '=0, if the gradient just before the point is –ve and just after +ve, then the point in a minimum turning point.
If f '=0, if the gradient just before the point is +ve and just after -ve, then the point in a maximum turning point.

The maximum gradient occurs where f ' is maximum. Likewise minimum.

Question 12 **(14 marks)**

(a) $y = cos\left(x - \frac{\pi}{4}\right)$ ✓✓

(b) (i) $cos(3x) = -1$ for $-90^0 \le x \le 90^0$

$3x = 180^0 \pm n360^0$ ✓

$x = 60^0 \pm n120^0$

$x = 60^0 \quad or \quad x = -60^0$
✓

(ii) $tan\left(x + \frac{\pi}{6}\right) = \sqrt{3}$ for $0 \le x \le \frac{\pi}{2}$

$x + \frac{\pi}{6} = \frac{\pi}{3} \pm n\pi$ ✓

$x = \frac{\pi}{6} \pm n\pi$

$x = \frac{\pi}{6}$ ✓

(c) $l = r\theta \quad \Rightarrow \quad 11 = 8\theta \quad \Rightarrow \quad \theta = \frac{11}{8}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}8^2 \times \frac{11}{8}$

$A = \frac{88}{2}$

$A = 44\,cm^2$ ✓

(d) Let $y = -\frac{\pi}{2}$

$cos\left(x - \frac{\pi}{2}\right) = cos(x)cos\left(-\frac{\pi}{2}\right) - sin(x)sin\left(-\frac{\pi}{2}\right)$

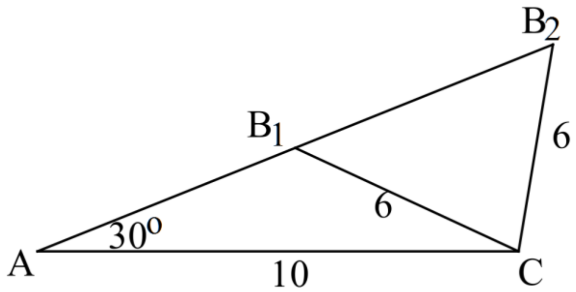
✓ $cos\left(\frac{\pi}{2} - x\right) = 0 - sin(x)\left(-sin\left(\frac{\pi}{2}\right)\right)$ $cos(-\beta) = cos(\beta)$ and $sin(-\beta) = -sin(\beta)$

✓

$cos\left(\frac{\pi}{2} - x\right) = -sin(x) \times (-1)$

$cos\left(\frac{\pi}{2} - x\right) = sin(x)$

(e)



$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\sin B = 10 \times \frac{\sin 30^o}{6}$$

$$\sin B = \frac{5}{6}$$ ✓

$B = 56.44^o$ *or* $B = 180^o - 56.44^o = 123.56^o$

$C = 93.56^o$ ✓ $C = 26.44^o$ ✓

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$c = \dfrac{6}{\sin 30^o} \times \sin 93.56^o$ $c = \dfrac{6}{\sin 30^o} \times \sin 26.48^o$

$c = 11.997$ $c = 5.344$

$AB \approx 12$ ✓ *OR* $AB = 5.34$ ✓

Question 13 **(5 marks)**

(a) $f(x) = x^3 - 9x$

$f'(x) = 3x^2 - 9$ ✓

$f'(3) = 27 - 9 = 18$ ✓

$y = 18x + c$

$(3,0) \Rightarrow 0 = 54 + c \quad \rightarrow c = -54$

$y = 18x - 54$ ✓

(b) $f'(x) = 3x^2 - 9$

$f'(-3) = 27 - 9 = 18$ ✓

Therefore the tangent at (-3, 0) is parallel to the tangent at (3, 0). ✓

Question 14 **(8 marks)**

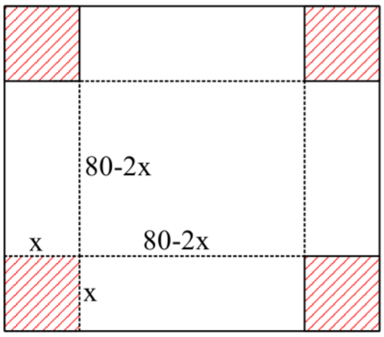
(a) (i) $g'(x) = -2x + 15x^4$ ✓✓

(ii) $f(x) = (2x+1)^2 = 4x^2 + 4x + 1$ ✓

$f'(x) = 8x + 4$ ✓

(b) By calculator x = 0.39794 ✓✓✓

Question 15 **(6 marks)**



Let the side of the little square be x.

$V = (80-2x)(80-2x)x$

$V = 4x^3 - 320x^2 + 6400x$ ✓

For maximum volume $\dfrac{dV}{dx} = 0$ ✓

$\dfrac{dV}{dx} = 12x^2 - 640x + 6400$ ✓

$If\ \dfrac{dV}{dx} = 0,\ x = 40 \quad or \quad x = 13.\dot{3}$

$But\ \ x \neq 40,\ \ so\ \ x = 13.\dot{3}$ ✓

Test for maximum

x	0	$13.\dot{3}$	14
$\dfrac{dA}{dx}$	+	0	−

✓

Therefore maximum

Therefore the dimensions of the square are $13.\dot{3}$x$13.\dot{3}$ cm for maximum volume. ✓

Question 16 **(6 marks)**

(a) Average rate of change= $\dfrac{f(4)-f(1)}{4-1}=\dfrac{26-11}{3}=5$ ✓ ✓ ✓

(b) $f'(x)=10-2x$ ✓ $10-2x=5$ $x=2.5$ $P(2.5,20.75)$ ✓

(c) The chord on the interval $1\le x\le 2$ has a greater gradient than the chord on the interval $3\le x\le 4$ ✓

Question 17 **(9 marks)**

(a) 10,12,14,…28

$a=10,\ d=2$ ✓
$T_n=a+(n-1)d$
$28=10+(n-1)2$ ✓
$18=(n-1)2$
$n=10$ ✓

Jenny will walk 28 km on the 10th day.

(b) Need S_{10} and then she walks 28 km per day.

$S_n=\dfrac{n}{2}(a+l)$
$S_{10}=\dfrac{10}{2}(10+28)$ ✓
$S_{10}=190$ ✓

She goes 190 in 10 days and there is 60 kms more to walk. ✓

$\dfrac{60}{28}=2+\dfrac{4}{28}$ so she needs 3 extra days.

Jenny takes 13 days. ✓

(c) $\dfrac{250}{20}=12.5$ ✓

Steve needs 12,5 days i.e. he arrives on the 13th day.

They arrive on the same day (but Jenny gets there earlier than Steve as she has less distance to travel on the 13th day). ✓

Question 18 **(5 marks)**

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{(x+h) - x}\right)$$

$$f(x) = 3x^2$$

$$f(x+h) = 3(x+h)^2$$

$$= 3(x^2 + 2xh + h^2) \quad \checkmark$$

By definition

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{(x+h) - x}\right)$$

$$f(x+h) - f(x) = 3(x^2 + 2xh + h^2) - (3x^2)$$

$$= 6xh + 3h^2 \quad \checkmark$$

$$\frac{f(x+h) - f(x)}{h} = \frac{h(6x + 3h)}{h}$$

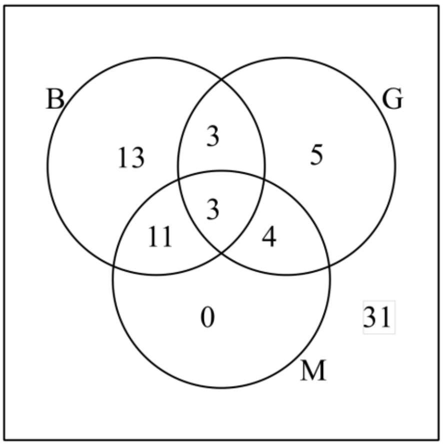
$$= 6x + 3h \quad \checkmark$$

$$\therefore \quad f'(x) = \lim_{h \to 0}(6x + 3h) \quad \checkmark$$

$$\therefore \quad f'(x) = 6x \quad \checkmark$$

Question 19 **(9 marks)**

(a) (i)



(ii) $n(G \cap M) = 7$ ✓

(iii) $n(\overline{M \cup B \cup G}) = 31$ ✓

(b) (i) If M and N are independent, then

$P(M \cap N) = P(M) \times P(N)$ ✓

$P(M) \times P(N) = 0.5 \times 0.4 = 0.20$

$P(M \cap N) = 0.2$ ✓

Therefore events M and N are independent. ✓

OR

If M and N are independent, then

$P(M) = P(M \mid N)$ ✓

$P(M) = 0.5$

$P(M \mid N) = \frac{0.2}{0.4} = 0.5$ ✓

Therefore events M and N are independent. ✓

(iii) If events M and N are mutually exclusive then $P(M \cap N) = 0$

but $P(M \cap N) = 0.2 \neq 0$ so the events are NOT mutually exclusive. ✓

End of solutions