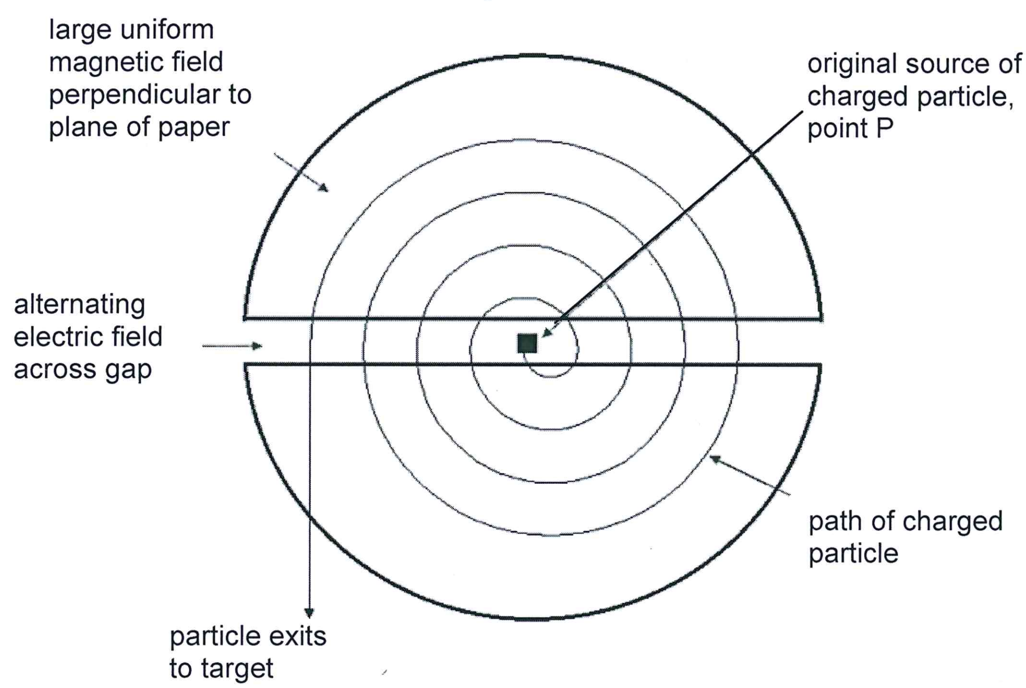
EXAM ANSWERS
Chapter 9.1 - Quantum Principles
Answer 1 : 2010:3:23

(23 marks)

Introduction

A cyclotron is a device in which heavy charged particles such as protons, deuterons (deuterium nuclei, $^{2}_{1}H$) and alpha particles can be accelerated to high energies. The high energy charged particle beam can then be used to study nuclear reactions and can also be used in hospitals to produce short-lived radioisotopes for diagnostic purposes. One such medical cyclotron is located at Sir Charles Gairdner Hospital in Perth. It can accelerate protons and deuterons to very high energies.

Cyclotron



How a cyclotron works

A cyclotron consists of two hollow D-shaped semicircular metal electrodes (called 'dees'), an ion source, an electromagnet and an alternating power supply.

The dees are mounted inside a vacuum chamber that fits between the two flat pole pieces of an electromagnet. The dees are connected to a high frequency alternating voltage supply that provides an alternating electric field across the gap between the dees.

When charged particles are injected at the centre of the dees (point P), they are accelerated by the electric field and then move into a semicircular path inside the hollow space of the dee under the influence of the uniform magnetic field that acts perpendicular to the path of the charged particles. Once inside the dee they are shielded from the electric field and thus do not gain any further energy.

Because the dees are connected to an alternating voltage supply, the charged particles are accelerated by the electric field each time they cross the gap, increasing their energy by a small amount qV. Therefore their speed increases and they move into larger and larger path radii. If the charged particles do not arrive at the gap when the polarity is correct, they will fall out of synchronisation and the beam will be lost. So for the satisfactory operation of the cyclotron, the frequency of the alternating voltage must be equal to the orbital or cyclotron frequency of the charged particles. This condition is valid only when the speed of the charged particles is much less than the speed of light. At higher particle speeds (above about 10% of the speed of light) the frequency of the circulating particle decreases steadily due to relativistic effects. Thus the particle goes out of step with the frequency of the oscillator and its energy stops increasing.

In the normal operation of the cyclotron, when the charged particles reach the outside perimeters of the dees, they are deflected by the electric field of an ejector plate and strike the outside target.

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Charged particle data

Type of charged particle	Mass of charged particle (kg)	Charge of the particle (coulombs)	$\frac{q}{m}$
electron	9.11×10^{-31}	1.60×10^{-19}	
proton	1.67×10^{-27}	1.60×10^{-19}	
deuteron	3.34×10^{-27}	1.60×10^{-19}	

(a) What provides the centripetal force that acts on the charged particle? (1 mark)

Description	Marks
The magnetic force	1
	Total 1

(b) Operation of the cyclotron is based on the principle that frequency of revolution is independent of the speed of charged particles and the radius of the circular path. Use the equations given in the Formulae and Constants Sheet to show that frequency, f is given by $f = \frac{qB}{2\pi m}$. (4 marks)

Description	Marks
$v = 2\pi r / t = 2\pi r f$ $f = \frac{v}{2\pi r}$	1
$\frac{mv^2}{r} = q v B$ $r = \frac{m v}{Bq}$ $\frac{v}{r} = \frac{B q}{m}$	1-2
combine: $f = \frac{B q}{2\pi m}$	1
	Total 4

(c) Suppose a cyclotron with a dee radius of 53.0 cm is tuned to accelerate protons at an oscillator frequency of 12.0 MHz. Calculate the strength of the magnetic field needed to accelerate deuterons with the same frequency. (3 marks)

Description	Marks
$f = \frac{qB}{2\pi m}$ $B = \frac{2\pi(12 \times 10^{6}\,\text{Hz})(3.34 \times 10^{-27}\,\text{kg})}{(1.60 \times 10^{-19}\,\text{C})}$ $= 1.6$ T	1-3
	Total 3

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(d) A conventional cyclotron begins to fail beyond a proton energy of 50 MeV.

(i) Explain why is this so. (2 marks)

Description	Marks
At higher energies the speed of the charged particle is **relativistic** (approaching the speed of light) and its **mass cannot be considered as constant**	1-2
	Total 2

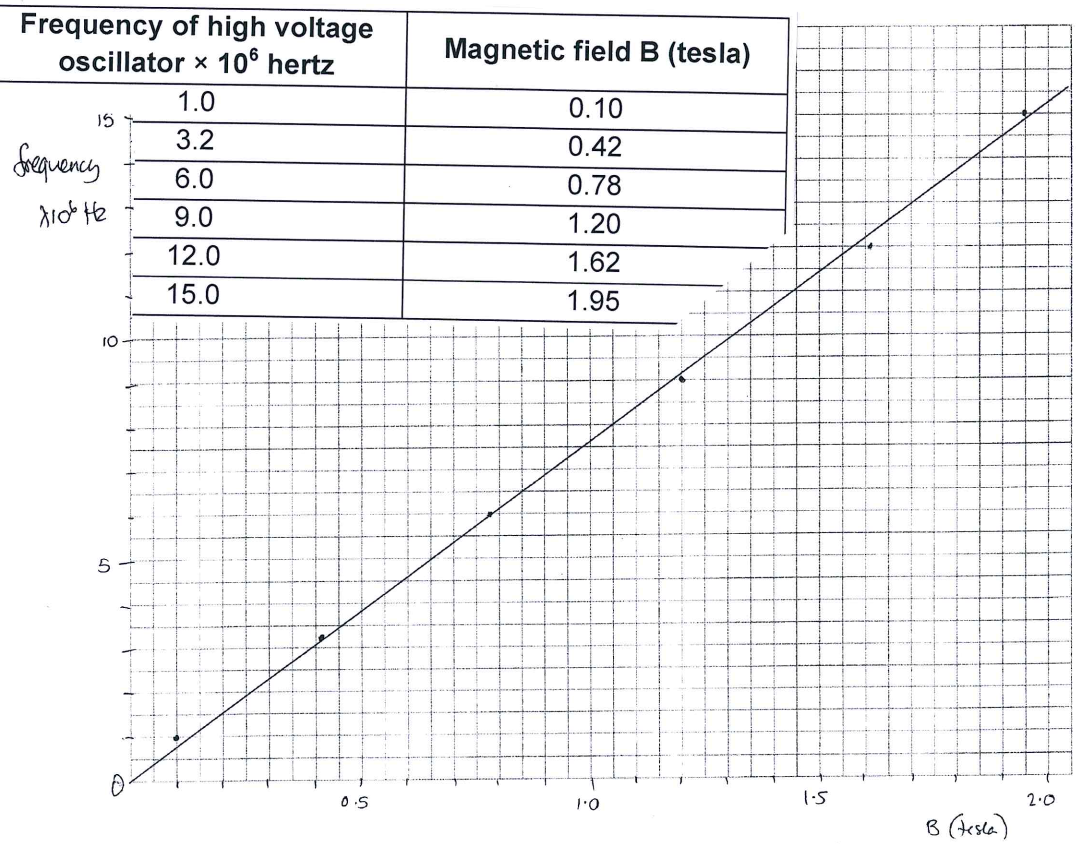
(ii) At what electron energy will the same cyclotron begin to fail? (2 marks)

Description	Marks
Assume same limiting electron velocity as proton as this will give same proportional increase in mass. Then limiting energy is given by For electron $E_{ke} = \frac{1}{2} m_e v^2 = q_e V$ For proton $E_{kp} = \frac{1}{2} m_p v^2 = q_p V$ Since v is the same for both then $\frac{q_e V}{q_p V} = \frac{m_e}{m_p}$ Thus $q_e V = \frac{m_e}{m_p} \times q_p V = \frac{9.11 \times 10^{-31}}{1.67 \times 10^{-27}} \times 50\text{MeV} = 27\text{keV}$ or assume a reasonable relativistic value for v (0.1c or greater) or calculate v and determine E_{ke} $E_{ke} = \frac{1}{2} m_e (3 \times 10^7)^2$ $E_{ke} = \frac{1}{2} (9.11 \times 10^{-31})(3 \times 10^7)^2$ $E_{ke} = 40.95 \times 10^{-17}\text{J} = \frac{40.95 \times 10^{-17}}{1.6 \times 10^{-19}}\text{eV}$ =2.56 keV	1-2
	Total 2

(e) An unknown particle was tested and gave the following values of high voltage oscillator frequency and the corresponding magnetic field:

(i) Using the graph paper on the next page, plot a straight line graph with magnetic field on the *x*-axis and frequency on the *y*-axis. (3 marks)

Frequency of high voltage oscillator × 10^6 hertz	Magnetic field B (tesla)
1.0	0.10
3.2	0.42
6.0	0.78
9.0	1.20
12.0	1.62
15.0	1.95



Description	Marks
Points plotted, axes labelled, line of best fit drawn.	1–3
	Total 3

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(ii) Calculate the gradient of this graph. (3 marks)

Description	Marks
Gradient calculated from graph line (note: if just use data from table then zero) Value about 7.6×10^{6} C kg^{-1} (accept 7.3×10^{6} C kg^{-1} to 7.9×10^{6} C kg^{-1}). 1 mark for value and 1 mark for unit	1 1–2
	Total 3

(iii) Use the gradient to find the ratio $\frac{\text{charge on the particle}}{\text{mass of particle}}$ for the unknown particle. (3 marks)

(If you could not complete (ii), use a gradient of magnitude 2.9×10^{10}.)

Description	Marks
q/ m from experiment = $2\pi f/B = 2\pi \times$ gradient = 4.8×10^{7} C kg^{-1} (accept 4.5×10^{7} C kg^{-1} to 5.0×10^{7} C kg^{-1}). Or if using gradient = 2.9×10^{10} q/ m from experiment = $2\pi f/B = 2\pi \times$ gradient = 1.8×10^{11} C kg^{-1}	1–3
	Total 3

(iv) Circle the unknown particle involved and justify your selection. (2 marks)

(I) electron (II) proton (III) neutron (IV) deuteron

Description	Marks
deuteron q/m of deuteron from supplied data = $\frac{1.6 \times 10^{-19}}{3.34 \times 10^{-27}}$ = 4.79×10^{7} C kg^{-1} Or if using gradient = 2.9×10^{10}, **electron** q/m of electron from supplied data = $\frac{1.6 \times 10^{-19}}{9.91 \times 10^{-31}}$ = 1.76×10^{11} C kg^{-1}	1–2
	Total 2

Answer 2 2011:3:22

(19 marks)

Muons and Relativity

Muons are subatomic particles that were discovered in 1936 by researchers studying cosmic radiation. The researchers noticed some particles whose paths in a magnetic field curved in a direction indicating negative charge, with path curvature indicating a mass between a proton mass and an electron mass.

Researchers first thought these particles were hadrons (heavy particles made of quarks). Hadrons such as protons and neutrons consist of three quarks and are called baryons. The new particles were thought to be mesons, that is, hadrons containing two quarks. Hadrons may emit either a neutrino or an antineutrino when they decay.

Further investigation showed that muons emit both a neutrino and an antineutrino when they decay, indicating that muons are leptons – fundamental particles that are not made of quarks. The most familiar lepton is the electron. Muon decay can be summarised as

muon → electron + neutrino + antineutrino

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Most naturally-occurring muons are created when cosmic rays collide with atoms in the upper atmosphere, approximately 10 km above the Earth. A muon has a rest mass of $\frac{106\ \text{MeV}}{c^2}$, a charge of -1 and an average lifetime of 2.2×10^{-6} s.

(a) The table below contains information about some subatomic particles. Complete the last column of the table by writing baryon, meson or lepton to indicate the group of particles to which the individual particle belongs. (4 marks)

Particle	Quark structure	Decay products	Baryon, meson or lepton
Lambda	charm, up, down	proton, pion, kaon	
Tau		tau neutrino, electron, electron anti-neutrino	
Kaon+	strange, charm	muon and muon neutrino	
Xi	up, strange, strange	lambda and pion	

Description	Marks
Baryon	1
lepton	1
meson	1
baryon	1
	Total 4

(b) Muons travel at almost the speed of light. Calculate the average distance that a muon created in the upper atmosphere would travel before it decayed. Assume that its speed is equal to c and that there are no relativistic effects. (2 marks)

Description	Marks
$s = v/t = 3 \times 10^8 \times 2.2 \times 10^{-6}$	1
$s = 6.6 \times 10^2$ m [0.66 km]	1
	Total 2

(c) Muons created by cosmic rays in the upper atmosphere can be detected by detectors on the Earth's surface. This means that the muons have travelled much further than expected. An explanation of this phenomenon involves the effects of relativity.
Explain how relativity affects the muons and enables them to travel over a greater distance than that calculated in (b). (3 marks)

Description	Marks
Time measured on Earth depends on the speed of the observer and the object being viewed	1
As the speed increases the time [dilation] increases (either frame reference of muon or Earth is valid) (correct length contractional , so valid)	1
Muons last longer since they travel so fast	1
	Total 3

(d) Express the rest mass of a muon in kilograms, and compare this to the rest mass of a proton. (3 marks)

Description	Marks
$106\ \frac{\text{MeV}}{\text{C}^2}$ to mass $= \frac{106 \times 10^6 \times 1.6 \times 10^{-19}}{(3 \times 10^8)^2}$	1
$= 1.88 \times 10^{-28}$ kg	1
Proton to muon ~ 9x larger	1
	Total 3

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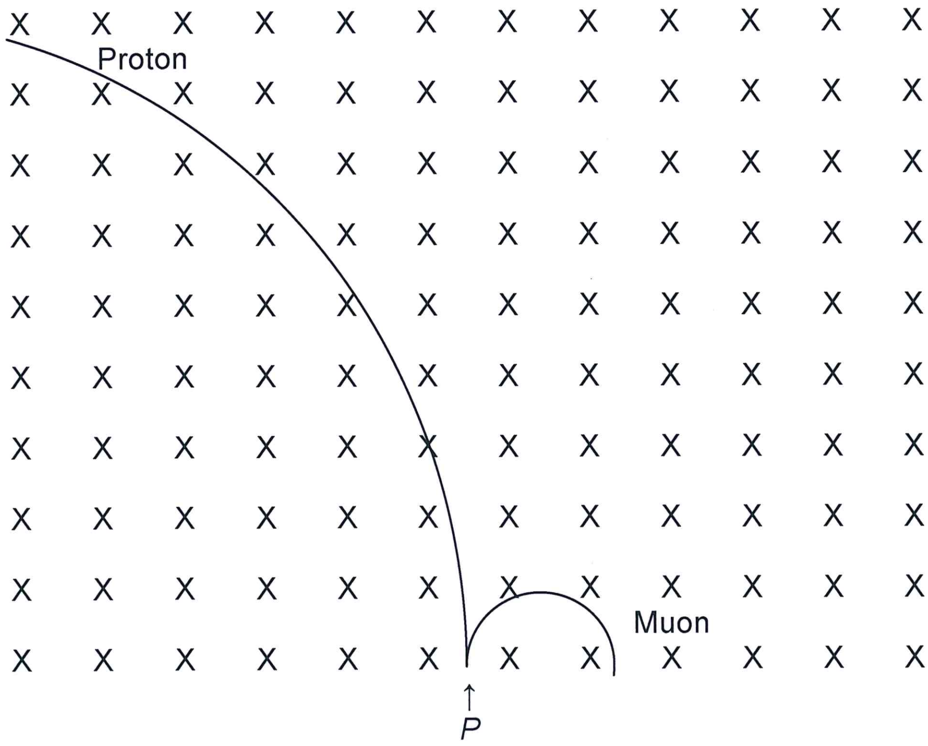
(e) On the diagram below sketch and label two lines representing the paths you would expect a proton and a muon to follow in the given magnetic field. Assume both particles are injected into the field at P with the same velocity. (3 marks)



Description	Marks
The curve with the larger radius moves to the left = proton	1
The smaller curve moves to the right = muon	1
Scale of curve ~1/9th the size	1
	Total 3

(f) Injecting and directing a charged particle using magnetic and electric fields is a commonly-used phenomenon. It is used in old (cathode ray tube) television technology as well as in high technology applications such as the CERN Large Hadron Collider.

Using formulae from your Formulae and Constants Sheet, show the derivation of the formula below that determines a particle's velocity from its mass (m) and charge (q), having been accelerated through a potential difference (V). You must show all steps. (4 marks)

$$v = \sqrt{\frac{2Vq}{m}}$$

Description		Marks
Accelerated through Electric field E or over a potential difference V		1
F = Eq or Vq/d = ma	W = Vq = E_k	1
a = Vq/md and $v^2 = u^2 + 2as$, where u = 0	E_k = ½mv^2 - ½mu^2, where u = 0	1
v^2 = 2Vqs/md (s = d) so v^2 = 2Vq/m $v = \sqrt{\frac{2Vq}{m}}$	v^2 = 2Vq/m $v = \sqrt{\frac{2Vq}{m}}$	1
		Total 4