

Semester Two Examination, 2016

Question/Answer Booklet

MATHEMATICS SPECIALIST UNITS 1 AND 2

Section One: Calculator-free

If required by your examination administrator, please
place your student identification label in this box

Student Number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	13	13	100	98	65
Total				149	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Booklet.

Section One: Calculator-free

35% (51 Marks)

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

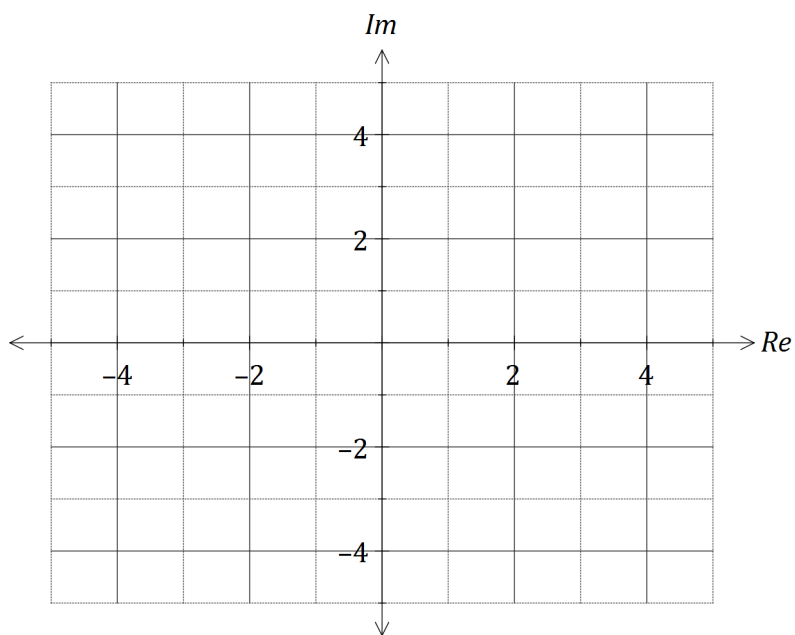
Question 1

(6 marks)

(a) Determine the number of real solutions to the equation $x^2 + x + 1 = 0$. (1 mark)

(b) Determine all complex solutions to the equation $x^2 + 2x + 10 = 0$. (2 marks)

(c) x_1 and x_2 are the complex solutions to the equation $4x^2 = 20x - 41$. If $x_1 = 2.5 + 2i$, plot x_1 , x_2 and $x_1 + x_2$ in the complex plane below. (3 marks)



Question 2**(7 marks)**

Three vectors are given by $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j}$, $\mathbf{b} = \mathbf{i} - 3\mathbf{j}$ and $\mathbf{c} = 3\mathbf{i} + \mathbf{j}$.

Determine

(a) a unit vector \mathbf{d} , parallel to $\mathbf{a} + 2\mathbf{b}$.

(3 marks)

(b) the value(s) of k so that the magnitude of the vector $\mathbf{a} + k\mathbf{b}$ is 4.

(4 marks)

Question 3**(9 marks)**

Consider the matrices $A = \begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 4 & -5 \end{bmatrix}$.

- (a) It is possible to form the product of all four matrices. State the dimensions of the resulting product. (2 marks)

- (b) Determine the matrix $\frac{1}{2}DC$. (2 marks)

- (c) Determine the inverse of matrix A . (2 marks)

- (d) Clearly show use of matrix algebra to solve the system of equations $2x - 3y + 3 = 0$ and $4y = 2x + 2$. (3 marks)

Question 4**(7 marks)**

Let $z_1 = 2 - 2i$ and $z_2 = 3 + i$.

(a) Simplify

(i) $2z_1 - z_2$. (1 mark)

(ii) z_1^3 . (2 marks)

(iii) $\frac{z_1}{z_2}$. (2 marks)

(b) Show that $\bar{z}_1 \times \bar{z}_2 = \overline{z_1 \times z_2}$. (2 marks)

Question 5**(7 marks)**

(a) Solve the equation $\tan\left(\frac{x+25^\circ}{2}\right) = \sqrt{3}$ for $0^\circ \leq x \leq 540^\circ$.

(3 marks)

(b) Prove that $(1 - \cos x)(1 + \sec x) = \sin x \tan x$.

(4 marks)

Question 6**(7 marks)**

- (a) Determine the value(s) of a for which the matrix $\begin{bmatrix} a & a \\ 3 & 2a \end{bmatrix}$ is singular. (2 marks)

- (b) The non-singular matrix B is such that $\begin{bmatrix} -3 & 2 \end{bmatrix} \times B = \begin{bmatrix} 8 & 3 \end{bmatrix}$ and $\begin{bmatrix} 2 & 6 \end{bmatrix} \times B = \begin{bmatrix} 10 & 4 \end{bmatrix}$.

- (i) Use these results to show that $\begin{bmatrix} -1 & 8 \end{bmatrix} \times B = \begin{bmatrix} 18 & 7 \end{bmatrix}$. (2 marks)

- (ii) Determine $\begin{bmatrix} 2 & 1 \end{bmatrix} \times B^{-1}$. (3 marks)

Question 7**(8 marks)**

- (a) Prove that the sum of any three consecutive terms of an arithmetic sequence with first term a and common difference d is always a multiple of three, for $a, d \in \mathbb{N}$. (3 marks)

- (b) Use mathematical induction to prove that $7^{2n-1} + 5$ is always divisible by 12, for $n \in \mathbb{N}$. (5 marks)

Additional working space

Question number: _____

Additional working space

Question number: _____

