Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 1 AND 2

Section One: Calculator-free

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

(a) Solve the equation $4(5-2x)=x-7$ for x. (2 marks)

Solution
$20-8x=x-7$ $27=9x$ $x=3$
Specific behaviours
✓ expands LHS ✓ collects like terms and solves

(b) Ash, Billie and Chris collected a total of 160 cans to recycle. Ash collected twice as many cans as Chris, Chris collected 12 more cans than Billie and Billie collected x cans.

Determine how many cans Billie collected. (3 marks)

Solution
$x+(x+12)+2(x+12)=160$ $4x+36=160$ $4x=124$ $x=\text{Billie's cans}=31$
Specific behaviours
✓ indicates number of cans each has ✓ forms and simplifies equation ✓ correct number

See next page

Question 2 **(5 marks)**

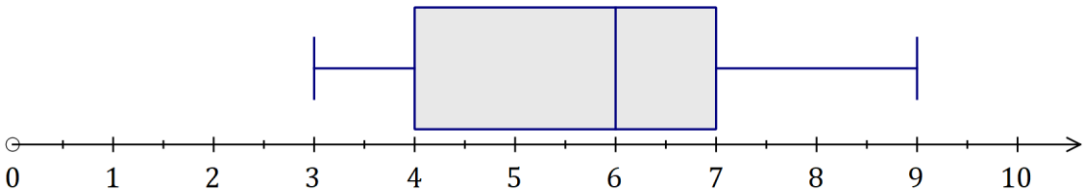
The number of daily absentees at a small school over 15 consecutive days are listed below:

$$7, 6, 4, 5, 6, 6, 3, 9, 7, 6, 7, 6, 4, 5, 4.$$

(a) Determine the five-number summary for this data. (3 marks)

Solution
Ordered: {3, 4, 4} , 4 , {5, 5, 6} , 6 , {6, 6, 6} , 7, {7, 7, 9} Median is 6 Q1 is 4, Q3 is 7 Summary is 3, 4, 6, 7 and 9
Specific behaviours
✓ Min and Max ✓ Median ✓ Q1 and Q3

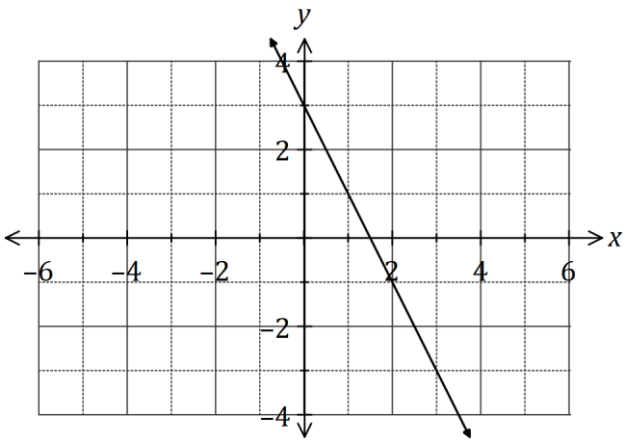
(b) Use the five-number summary to construct a box-plot on the scale below. (2 marks)



Solution
See graph
Specific behaviours
✓ correctly locates five-number summary points ✓ draws box and whiskers

Question 3 **(7 marks)**

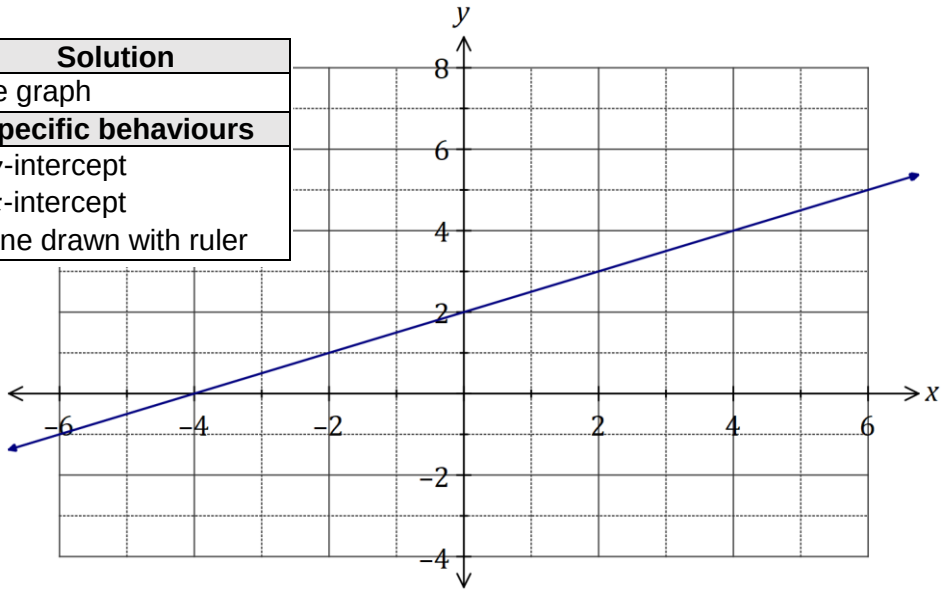
(a) The graph of $y = ax + b$ is shown below. Determine the values of a and b. (2 marks)



Solution
$a = -2$ $b = 3$
Specific behaviours
✓ correct value for a ✓ correct value for b

(b) Draw the graph of the line $y = \frac{1}{2}x + 2$ on the axes below. (3 marks)

Solution
See graph
Specific behaviours
✓ y-intercept ✓ x-intercept ✓ line drawn with ruler



(c) Determine the gradient of the line $3x - 2y = 12$. (2 marks)

Solution
$2y = 3x - 12$ $y = 1.5x - 6$ $\text{Gradient} = 1.5$
Specific behaviours
✓ rearranges into $y =$ form ✓ states gradient

See next page

Question 4 **(8 marks)**

(a) The power P consumed by a device can be calculated using the formula $P = I^2R + \sqrt{2R}$.

Calculate the value of P when

(i) $I = 5$ and $R = 8$. (2 marks)

Solution
$P = 5^2 \times 8 + \sqrt{2 \times 8}$ $P = 25 \times 8 + \sqrt{16}$ $P = 200 + 4 = 204$
Specific behaviours
✓ substitutes correctly, simplifying I^2 and $2R$ ✓ evaluates correctly

(ii) $I = 0.5$ and $R = 50$. (2 marks)

Solution
$P = 0.5^2 \times 50 + \sqrt{2 \times 50}$ $P = 0.25 \times 50 + \sqrt{100}$ $P = 12.5 + 10 = 22.5$
Specific behaviours
✓ substitutes correctly, simplifying I^2 and $2R$ ✓ evaluates correctly

(b) The variable D is related to diameters d_1, d_2 and d_3 by the formula $D = \frac{d_1}{4} + \frac{2}{d_2 - d_3}$.

Calculate the value of D when

(i) $d_1 = 2$, $d_2 = 9$ and $d_3 = 1$. (2 marks)

Solution
$D = \frac{2}{4} + \frac{2}{9-1} = \frac{2}{4} + \frac{2}{8}$ $D = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$
Specific behaviours
✓ substitutes correctly, simplifying $d_2 - d_3$ ✓ evaluates correctly

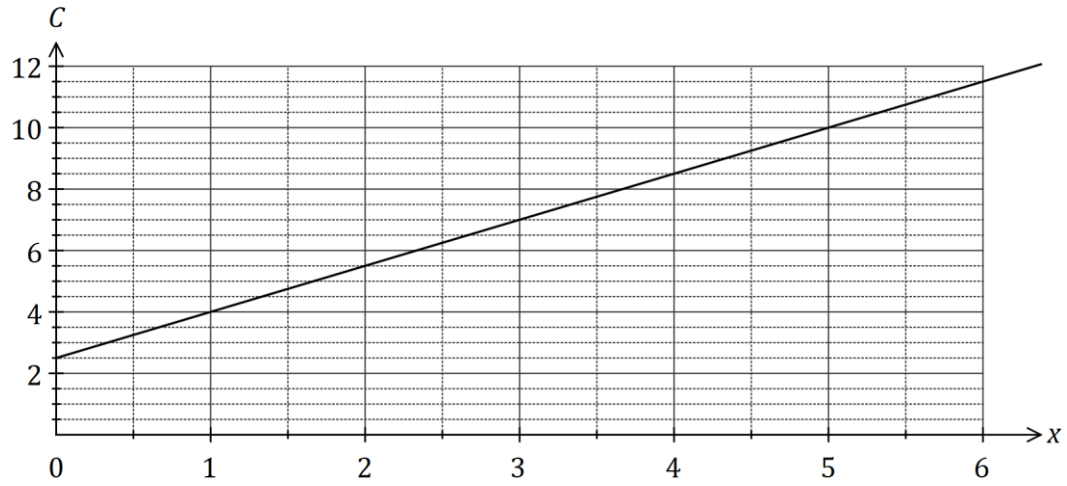
(ii) $d_1 = 18$, $d_2 = 3.5$ and $d_3 = 4$. (2 marks)

Solution
$D = \frac{18}{4} + \frac{2}{3.5-4} = \frac{18}{4} + \frac{2}{-0.5}$ $D = 4.5 - 4 = 0.5$
Specific behaviours
✓ substitutes correctly, simplifying $d_2 - d_3$ ✓ evaluates correctly

See next page

Question 5 **(6 marks)**

The graph below shows the cost C, in dollars, of taking a journey of x km with a taxi company.



(a) State the cost of taking a 4 km journey. (1 mark)

Solution
$8.50
Specific behaviours
✓ correct cost

(b) How much more expensive is a 5 km journey compared to one of 4 km? (1 mark)

Solution
$10 - 8.50 = \$1.50$
Specific behaviours
✓ correct difference

(c) State and interpret, in context, the value of the vertical axis intercept of the graph. (2 marks)

Solution
$2.50, a fixed cost payable regardless of distance travelled.
Specific behaviours
✓ value, ✓ interprets as fixed cost, flag-fall, etc

(d) State and interpret, in context, the value of the gradient of the graph. (2 marks)

Solution
$m = 1.5$, the cost per kilometre of a journey is $1.50
Specific behaviours
✓ value, ✓ interprets as cost per kilometre

See next page

Question 6 **(6 marks)**

The heights of 5-year old boys are normally distributed with a mean of 109 cm and a standard deviation of 4 cm.

(a) Use the $68\%, 95\%, 99.7\%$ rule to determine the approximate percentage of 5-year old boys that can be expected to have heights between

(i) 97 and 121 cm. (2 marks)

Solution
109 ± 12 $\pm 12 \div 4 = \pm 3$ sd's $\Rightarrow 99.7\%$
Specific behaviours
✓ indicates ± 3 sd from mean ✓ correct percent

(ii) 109 and 113 cm. (2 marks)

Solution
$113 - 109 = 4$ $4 \div 4 = 1$ sd $\Rightarrow 68 \div 2 = 34\%$
Specific behaviours
✓ indicates 1 sd above mean ✓ correct percent

(b) Nurses at a health clinic measure the heights of children and refer anyone with a height that is more than two standard deviations from the mean of the child's age group to a doctor.

The last eight boys, aged 5, who attended the clinic had the following heights:

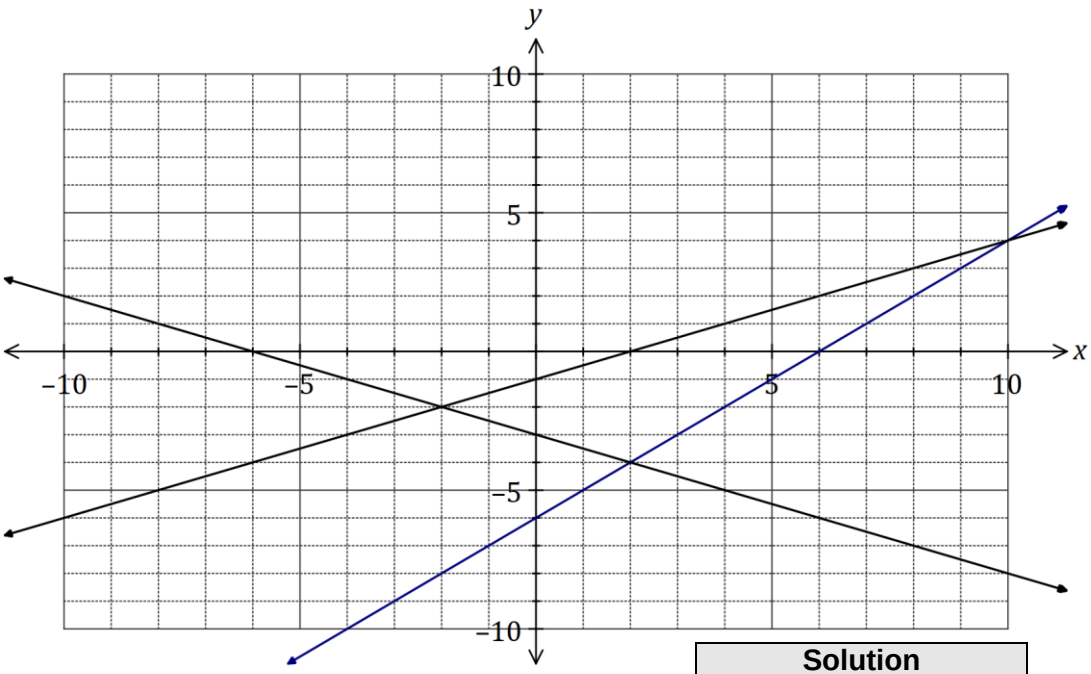
$$111 \quad 108 \quad 104 \quad 118 \quad 112 \quad 103 \quad 116 \quad 109 \text{ cm.}$$

Determine, with justification, how many of these boys were referred. (2 marks)

Solution
Not referred if height between 101 and 117 cm. So, **one** boy referred (118 cm).
Specific behaviours
✓ correct number ✓ justification (eg standard score or states bounds)

Question 7 **(7 marks)**

(a) The lines with equations $2y + 2 = x$ and $2y + x = -6$ are shown on the axes below.



(i) Draw the line $y = x - 6$ on the axes above. (2 marks)

Solution
See graph
Specific behaviours
✓ y-intercept
✓ x-intercept

(ii) Solve the simultaneous equations $y = x - 6$ and $2y + 2 = x$. (2 marks)

Solution
From graph, $x = 10, y = 4$
Specific behaviours
✓ indicates appropriate method
✓ correct values of x and y

(b) Solve the simultaneous equations $x + y = 7$ and $2y + x = -6$. (3 marks)

Solution
$(2y + x = -6)$ $-(x + y = 7)$ $y = -13$ $x - 13 = 7$ $x = 20$
Specific behaviours
✓ indicates use of substitution or elimination
✓ solves for y
✓ solves for x

See next page

Question 8 **(8 marks)**

A group of friends who frequently travelled abroad exchanged foreign currency between themselves using the conversion table below. For example, members of the group could exchange 100 dollars for 20 dinars or 50 kroner for 10 dollars.

Country/Currency	Australian/Dollar			
	10	20	50	100
Denmark/Krone	50	100	250	500
Japan/Yen	800	1 600	4 000	8 000
Kuwait/Dinar	2	4	10	20
Thailand/Baht	250	500	1 250	2 500

(a) How many baht can be exchanged for 30 dollars? (1 mark)

Solution
$\$30 = \$20 + \$10 = 500 + 250 = 750$ baht
Specific behaviours
✓ correct amount

(b) How many dollars can be exchanged for $20\,000$ yen? (2 marks)

Solution
$20000 = 8000 + 8000 + 4000$ $= 100 + 100 + 50$ $= 250$ dollars
Specific behaviours
✓ appropriate method ✓ correct amount

(c) Before travelling to Kuwait, one of the group exchanged 490 dollars for the local currency. How many dinars did they receive? (2 marks)

Solution
$490 = 5 \times 100 - 10$ $= 5 \times 20 - 2$ $= 98$ dinars
Specific behaviours
✓ appropriate method ✓ correct amount

(d) Another member of the group exchanged 500 dollars for a mixture of $1\,350$ kroner and some yen. Determine the amount of Japanese currency they received. (3 marks)

Solution
$1350 = 500 + 500 + 250 + 100$ $= \$100 + \$100 + \$50 + \$20$ $= \$270$ $\$500 - \$270 = \$230$ $\$230 = \$100 + \$100 + \$20 + \$10$ $= 8000 + 8000 + 1600 + 800$ $= 18\,400$ yen
Specific behaviours
✓ converts kroner to dollars ✓ calculates remaining dollars ✓ converts dollars to yen

End of questions

Supplementary page

Question number: _____