Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 1&2

Section One: Calculator-free

SOLUTIONS

Your name _____

Teacher's name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

A fruit picker is paid 40 cents for each kilogram of cherries they pick, and a box of cherries contains 15 kg of the fruit.

(a) Determine, in dollars, how much a picker is paid to pick one box of cherries. (2 marks)

Solution
$40 \div 100 = \$0.40$ $0.40 \times 15 = \$6.00$
Specific behaviours
✓ indicates correct calculation(s) ✓ correctly evaluates

(b) Determine how much a picker earns in a day when they pick 24 boxes of cherries. (1 mark)

Solution
$6 \times 24 = 6 \times 20 + 6 \times 4$ $= 120 + 24$ $= \$144$
Specific behaviours
✓ correctly evaluates

(c) A picker worked for five days and earned a total of $\$480$. Calculate how many kilograms of cherries they picked in this time. (2 marks)

Solution
$480 \div 0.4 = \frac{4800}{4}$ $= 1200 \text{ kg}$
Specific behaviours
✓ indicates correct calculation (alternative methods can be used) ✓ correct number of kilograms

Question 2 **(6 marks)**

(a) The surface area S of an object is given by $S = 2(wh + lh + wl)$.
Calculate the value of S when $h = 8$ cm, $l = 4$ cm and $w = 0.5$ cm. (2 marks)

Solution
$S = 2(0.5 \times 8 + 4 \times 8 + 0.5 \times 4)$ $= 2(4 + 32 + 2)$ $= 2 \times 38$ $= 76 \text{ cm}^2$
Specific behaviours
✓ simplifies three terms in brackets ✓ sums terms and doubles

(b) The quantity Δ is calculated using the formula $\Delta = \sqrt{b^2 - 4ac}$.
Calculate the value of Δ when $a = 2, b = 7$ and $c = 3$. (2 marks)

Solution
$\Delta = \sqrt{7^2 - 4 \times 2 \times 3}$ $= \sqrt{49 - 24}$ $= \sqrt{25}$ $= 5$
Specific behaviours
✓ simplifies two terms in roots ✓ calculates difference and takes root

(c) Given that $a^2 - b^2 = (a + b)(a - b)$, evaluate $42^2 - 32^2$. (2 marks)

Solution
$42^2 - 32^2 = (42 + 32)(42 - 32)$ $= (74)(10)$ $= 740$
Specific behaviours
✓ expresses as sum and difference ✓ correct value

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Question 3 **(6 marks)**

The weights in kilograms of the 10 members of netball squad A are shown below. The standard deviation of these weights is 7.9 kg.

$$54, 66, 55, 65, 78, 62, 75, 58, 57, 70.$$

(a) Determine the mean weight of the members of squad A. (2 marks)

Solution
$\Sigma x = 640$ $\bar{x} = 640 \div 10 = 64 \text{ kg}$
Specific behaviours
✓ correctly adds the weights ✓ calculates mean

Netball squad B also has 10 members. The mean and standard deviation of the weights of the members of this squad are 68 and 4.5 kg respectively.

(b) Write a statement, with reasoning, that compares the variation in weights of the members of the two squads. (2 marks)

Solution
The standard deviation for squad A is greater than squad B, since $7.9 > 4.5$. Hence there is more variation in weight amongst the members of squad A compared to squad B.
Specific behaviours
✓ compares standard deviations ✓ indicates greater sd means greater variation (more spread out)

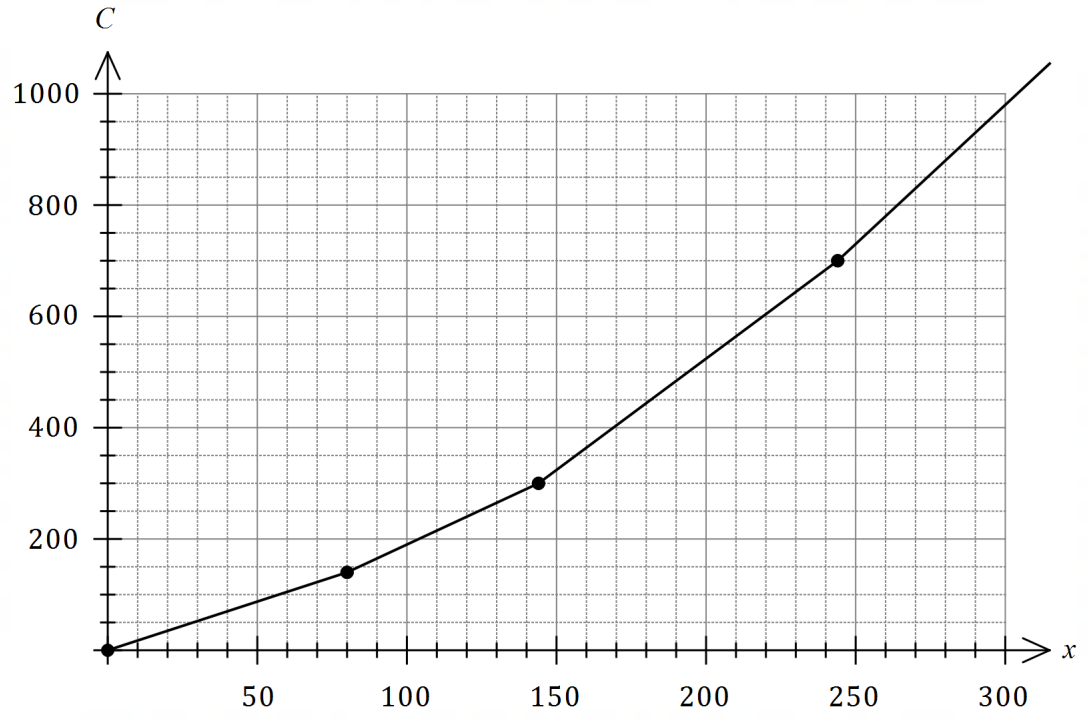
(c) Write a statement, with reasoning, that compares the weights of the members of the two squads. (2 marks)

Solution
The mean of squad A is lower than squad B, since $64 < 68$. Hence the members of squad A tend to be lighter compared to the members of squad B.
Specific behaviours
✓ compares means ✓ indicates members of one squad tend to be lighter

See next page

Question 4 **(6 marks)**

The annual cost of water, $\$C$, varies with x, the total number of kilolitres of water used by a household in a year, as shown in the graph below.



(a) Determine the cost of water for a household that used 100 kL in a year. (1 mark)

Solution
Any cost between $\$180$ and $\$200$
Specific behaviours
✓ cost within given range

(b) Calculate the slope of the line between $(144, 300)$ and $(244, 700)$ and interpret, in context, this slope. (3 marks)

Solution
$m = \frac{700 - 300}{244 - 144} = \frac{400}{100} = 4$ Each kilolitre of water in this interval costs $\$4$.
Specific behaviours
✓ appropriate use of slope formula ✓ correct slope ✓ reasonable interpretation ($4/kL)

(c) Describe how the slope of the piecewise graph changes as the yearly household water use increases and suggest, in context, a reason for this change. (2 marks)

Solution
As water use increases, so the slope of each part of the graph increases. Reason: To discourage excessive use, households that use the most water will pay more per kL than households who use less water.
Specific behaviours
✓ describes change ✓ any plausible reason

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Question 5 **(7 marks)**

Let $\boldsymbol{P} = \begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix}$ and $\boldsymbol{Q} = \begin{bmatrix} 2 & 0 \\ -1 & 4 \end{bmatrix}$.

(a) Express $4\boldsymbol{Q} - \boldsymbol{P}$ as a single matrix. (2 marks)

Solution
$4\boldsymbol{Q} - \boldsymbol{P} = 4\begin{bmatrix} 2 & 0 \\ -1 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix}$ $= \begin{bmatrix} 8 & 0 \\ -4 & 16 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix}$ $= \begin{bmatrix} 8 & -3 \\ -3 & 14 \end{bmatrix}$
Specific behaviours
✓ correct multiple ✓ correct difference

(b) Calculate the matrix $\boldsymbol{P} \times \boldsymbol{Q}$. (2 marks)

Solution
$\boldsymbol{P} \times \boldsymbol{Q} = \begin{bmatrix} 0 & 3 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ -1 & 4 \end{bmatrix}$ $= \begin{bmatrix} -3 & 12 \\ -4 & 8 \end{bmatrix}$
Specific behaviours
✓ at least two elements correct ✓ correct result

(c) Determine the value of a, the value of b and the value of c given the matrix equation

$$\begin{bmatrix} 4 & 1 & -4 \\ 0 & 2 & 5 \end{bmatrix} + a\begin{bmatrix} -1 & -2 & 2 \\ 1 & b & -1 \end{bmatrix} = \begin{bmatrix} 7 & 7 & c \\ -3 & -1 & 8 \end{bmatrix}.$$

(3 marks)

Solution
Using top left elements: $4 - a = 7 \rightarrow a = -3$ Using bottom centre elements: $2 - 3b = -1 \rightarrow 3b = 3 \rightarrow b = 1$ Using top right elements: $-4 - 3(2) = c \rightarrow c = -10$ $a = -3, \quad b = 1, \quad c = -10$
Specific behaviours
✓ value of a ✓ value of b ✓ value of c

See next page

Question 6 **(7 marks)**

(a) The area of a trapezium can be calculated using the formula shown below.

$$A = \frac{h}{2}(a + b)$$

Determine the value of a when $A = 50, h = 5$ and $b = 8$. (3 marks)

Solution
$50 = \frac{5}{2}(a + 8)$ $100 = 5(a + 8)$ $20 = a + 8$ $a = 12$
Specific behaviours
✓ substitutes ✓ one correct step to simplify ✓ correct value

(b) To hire a scooter from company A, a customer is charged a fee of $\$9$ plus $\$3$ per hour, but to hire one from company B, they are charged a fee of $\$5$ plus $\$5$ per hour.

Peta hires three scooters from company B and Rho hires four scooters from company A. If they both hire the scooters for the same length of time, x hours, and their bills are the same, use the above information to write an equation and hence determine the value of x. (4 marks)

Solution
$3(5 + 5x) = 4(9 + 3x)$ $15 + 15x = 36 + 12x$ $3x = 21$ $x = 7$
Specific behaviours
✓ one correct expression for cost ✓ equates both cost expressions ✓ expands at least one side correctly ✓ correct value of x

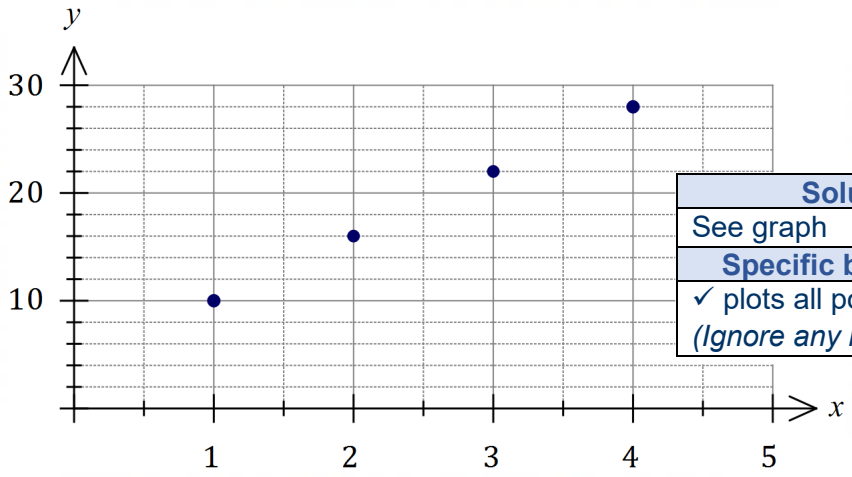
See next page

Question 7 **(7 marks)**

Water is pumped into a storage tank. The table below shows the depth of water, y cm, in the tank after the pump has been running for x minutes.

x	1	2	3	4
y	10	16	22	28

(a) Plot these points on the axes below. (1 mark)



Solution
See graph
Specific behaviours
✓ plots all points *(Ignore any line)*

(b) What feature of the graph in part (a) suggests a linear formula relates x and y? (1 mark)

Solution
All points on the graph lie in a straight line, or reasonable answer.
Specific behaviours

(c) The formula $y = ax + b$ relates the depth of water to the time. Determine the value of the constant a and the value of the constant b. (2 marks)

Solution
Gradient: $a = 6$ Axis-intercept: $b = 4$
Specific behaviours
✓ value of a ✓ value of b

(d) Determine the depth of water in the tank after 9 minutes. (1 mark)

Solution
$y = 6(9) + 4 = 54 + 4 = 58$ cm
Specific behaviours
✓ correct depth using values from (c)

(e) Determine the time required for the depth of water in the tank to reach 124 cm. (2 marks)

Solution
$124 = 6x + 4$ $120 = 6x$ $x = \frac{120}{6} = 20$ min
Specific behaviours
✓ substitutes into relation ✓ solves for x

See next page

Question 8 **(8 marks)**

A farmer grew two varieties of potato, Exton and Sebago. The time for each crop to mature is normally distributed with the times shown in the following table.

Variety	Exton	Sebago
Mean time for crop to mature (days)	118	112
Standard deviation of time for crop to mature (days)	3	4

(a) Use deviations from the mean (standard scores) to explain which of the following would be more unusual: a crop of **Exton** maturing in 116 days; or a crop of **Sebago** maturing in 116 days. (3 marks)

Solution

$$SS_E = \frac{116 - 118}{3} = -\frac{2}{3}$$

$$SS_S = \frac{116 - 112}{4} = \frac{4}{4} = 1$$

Sebago maturing in 116 days is more unusual as its standard score is furthest from zero.

Specific behaviours

✓ one correct standard score
✓ states Sebago is more unusual
✓ explanation using standard scores

(b) Use the 68%, 95%, 99.7% rule to determine the approximate probability that

(i) a crop of **Sebago** takes at least 112 days to mature. (1 mark)

Solution

$$P(\text{greater than mean}) = 50\% = 0.5$$

Specific behaviours

✓ correct probability

(ii) a crop of **Exton** takes between 112 and 124 days to mature. (2 marks)

Solution

$$SS = \frac{112 - 118}{3} = -2, \qquad SS = \frac{124 - 118}{3} = +2$$

$$P(\text{within 2 sd of mean}) = 95\% = 0.95$$

Specific behaviours

✓ indicates within 2 sd of mean
✓ correct probability (full marks if answer only)

(iii) a crop of **Sebago** matures in less than 108 days. (2 marks)

Solution

$$SS = \frac{108 - 112}{4} = -1$$

$$P(\text{within 1 sd of mean}) = 68\%$$

$$P = \frac{1 - 0.68}{2} = 0.16$$

Specific behaviours

✓ indicates standard score for days
✓ correct probability (full marks if answer only)

End of questions

Supplementary page

Question number: _____