Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 1 AND 2

Section Two: Calculator-assumed

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(5 marks)**

The following matrix S shows the number of small sheds (in row 1) and large sheds (in row 2) sold by a company in each of three consecutive months. For example, the element S_{13} represents the number of small sheds sold during the third month.

$$S = \begin{bmatrix} 72 & 68 & 75 \\ 51 & 59 & 38 \end{bmatrix}$$

(a) How many large sheds were sold in the second month? (1 mark)

Solution
59 sheds
Specific behaviours
✓ correct number

(b) Calculate matrix A, where $A = S \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. (1 mark)

Solution
$A = \begin{bmatrix} 215 \\ 148 \end{bmatrix}$
Specific behaviours
✓ correct matrix

(c) Explain what information matrix A shows. (1 mark)

Solution
Number of small and large sheds sold over the 3-month period.
Specific behaviours
✓ clear explanation

Matrix $P = [141 \quad 236]$, where P_{11} and P_{12} represent the profit, in dollars, made by selling a small shed and a large shed respectively.

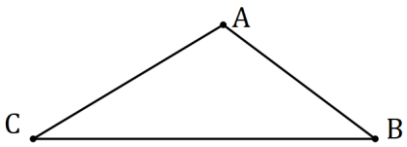
(d) Using matrices A and P, write down a calculation that will result in a matrix showing the total profit from selling all the sheds over the three-month period and state this profit. (2 marks)

Solution
$[141 \quad 236] \times \begin{bmatrix} 215 \\ 148 \end{bmatrix} = [65243]$ Profit is \$65 243
Specific behaviours
✓ product shown in correct order ✓ correct profit

See next page

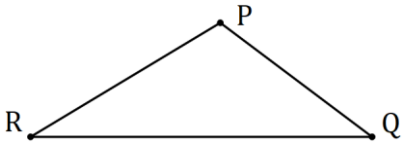
Question 10 **(5 marks)**

(a) Show use of trigonometry to determine the length of side BC in the triangle below, where $\angle BAC = 122°, AB = 58$ cm and $AC = 71$ cm. (2 marks)



Solution
$BC^2 = 58^2 + 71^2 - 2(58)(71)\cos 122$ $BC = 113$ cm
Specific behaviours
✓ substitutes correctly into cosine rule ✓ correct length

(b) Show use of trigonometry to determine the size of angle $\angle PRQ$ in the triangle below, where $\angle QPR = 105°, PR = 45$ cm and $QR = 65$ cm. (3 marks)

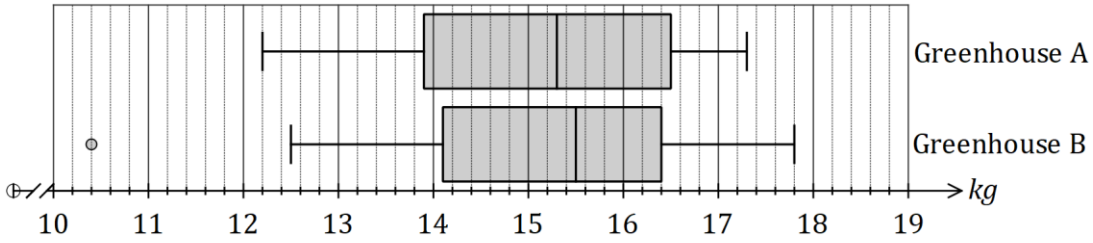


Solution
If $x = \angle PQR$ then $\frac{\sin x}{45} = \frac{\sin 105}{65}$ $x = 42°$ $\angle PQR = 180 - 105 - 42 = 33°$
Specific behaviours
✓ substitutes correctly into sine rule ✓ correct value for $\angle PRQ$ ✓ size of required angle

See next page

Question 11 **(8 marks)**

A hydroponic grower was trialling two different greenhouse systems for growing tomatoes. To compare the systems, the weight of tomatoes produced by each plant in the two greenhouses were recorded. The data is summarised below.



(a) Ignoring the outlier, compare the range of weights produced by plants in greenhouse A with that of greenhouse B. (2 marks)

Solution
$A: 17.3 - 12.2 = 5.1, \quad B: 17.8 - 12.5 = 5.3$ The range of A is smaller than that of B.
Specific behaviours
✓ indicates both ranges ✓ comparison of ranges

(b) State and use the interquartile ranges to compare the spread of weights produced by plants in greenhouse A with that of greenhouse B. (2 marks)

Solution
$A: 16.5 - 13.9 = 2.6, \quad B: 16.4 - 14.1 = 2.3$ The spread of weights in A is larger than the spread of weights in B.
Specific behaviours
✓ indicates both IQRs ✓ comparison of spreads

(c) Using the result of a relevant calculation, explain why one of the weights in greenhouse B was identified as an outlier. (2 marks)

Solution
$14.1 - 1.5 \times 2.3 = 10.65$ The weight of 10.4 is an outlier as it is below 10.65.
Specific behaviours
✓ calculates $LQ - 1.5 \times IQR$ ✓ states weight of outlier and that it is below cut-off

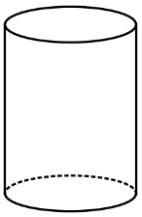
(d) Explain whether there is evidence to support the conjecture that the system in one greenhouse produces a larger crop of tomatoes than the other. (2 marks)

Solution
Yes, since the median weight from B exceeds that from A and the plant producing the largest crop was in greenhouse B.
Specific behaviours
✓ states there is evidence ✓ valid explanation

See next page

Question 12 **(7 marks)**

Soup is sold in cylindrical tins that have an internal diameter of $7.6\ \text{cm}$ and a height of $13.6\ \text{cm}$.



(a) Calculate the internal surface area of the can. (3 marks)

Solution
$r = 7.6 \div 2 = 3.8$ $A = 2\pi \times 3.8^2 + 2\pi \times 13.6$ $= 415\ \text{cm}^2$
Specific behaviours
✓ indicates correct radius ✓ substitutes into formula ✓ evaluates

(b) Calculate the capacity of the can in millilitres. ($1\ \text{mL} = 1\ \text{cm}^3$) (2 marks)

Solution
$V = \pi \times 3.8^2 \times 13.6$ $= 617\ \text{mL}$
Specific behaviours
✓ substitutes into formula ✓ evaluates

(c) Before the cans are sealed, they are stood on their circular end and filled with $540\ \text{mL}$ of soup. Determine the depth of soup in the can. (2 marks)

Solution
$540 = \pi \times 3.8^2 \times h$ $h = 11.9\ \text{cm}$
Specific behaviours
✓ substitutes into formula ✓ solves for h

See next page

Question 13 **(7 marks)**

Individual use coffee bags are packed in boxes of 8, 18 or 28. Customers can buy cartons containing 4, 5 or 6 boxes, as shown in the following table.

Carton	Carton price ($)	Boxes per carton	Coffee bags per box
A	39.20	4	28
B	17.76	6	8
C	30.60	5	18

(a) A customer orders a total of 16 cartons, comprising 7 of type A, 4 of type B and the rest of type C. Calculate the cost of this order, given that orders of more than $\$150$ qualify for a 25% discount. (3 marks)

Solution
$7 \times 39.20 = 274.40$ $4 \times 17.76 = 71.04$ $5 \times 30.60 = 153.00$ $\text{Total} = 498.44$ $498.44 \times 0.75 = \$373.83$
Specific behaviours
✓ cost of A's and B's ✓ cost of C's and total cost ✓ correct cost with discount

(b) Determine the cost of one coffee bag in each type of carton and hence list the carton types from best to worst value in terms of the price per coffee bag. (4 marks)

Solution
A: $4 \times 28 = 112$ $39.20 \div 112 = \$0.35$ B: $6 \times 8 = 48, 17.76 \div 48 = \$0.37$ C: $5 \times 18 = 90, 30.60 \div 90 = \$0.34$ Best to worst: C, A, B
Specific behaviours
✓ calculates number of bags in a carton ✓ calculates unit cost for same carton ✓ calculates all unit costs ✓ correct ranking (based on seen unit costs)

See next page

Question 14 **(11 marks)**

As part of an investigation into youth fitness, a researcher collected the sit-and-reach (SR) measurements of 250 students. The data is summarised in the table below.

SR measurement (cm)	Number of students
$18 < x \leq 22$	5
$22 < x \leq 26$	10
$26 < x \leq 30$	32
$30 < x \leq 34$	68
$34 < x \leq 38$	90
$38 < x \leq 42$	36
$42 < x \leq 46$	9

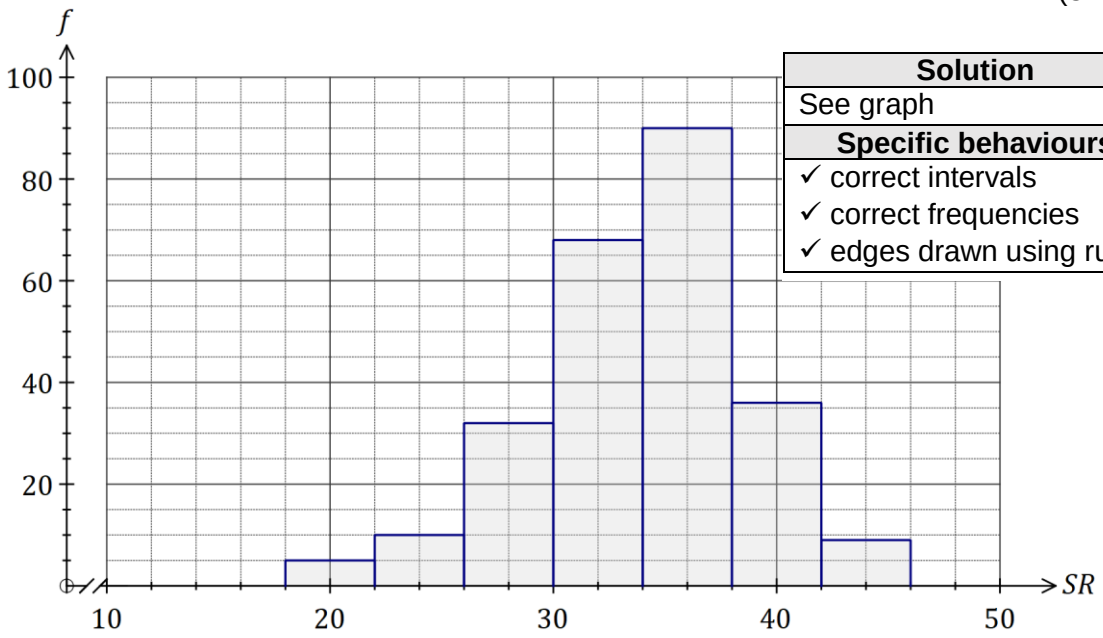
(a) Use the mid-point of each class interval to determine the mean and standard deviation of the sit-and-reach measurements. (2 marks)

Solution
$\bar{x} = 33.95$ cm, $sd = 4.88$ cm
Specific behaviours
✓ correct mean
✓ correct sd

(b) Explain why it was necessary to use the mid-point of each class interval to determine the statistics in (a). (1 mark)

Solution
Data has been grouped; No access to raw data; Best estimate for students in each interval; etc, etc
Specific behaviours
✓ any reasonable explanation

(c) Draw a histogram on the axes below to display the distribution of SR measurements. (3 marks)



Solution
See graph
Specific behaviours
✓ correct intervals
✓ correct frequencies
✓ edges drawn using ruler

See next page

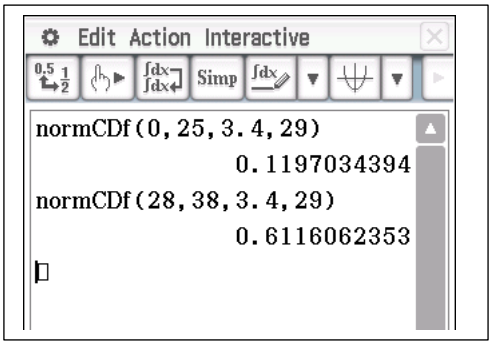
(d) Use features of the histogram to describe the distribution of SR measurements for this group of students. (3 marks)

Solution
-The dataset is negatively skewed (skewed left). -The dataset is unimodal, with $34 - 38$ the modal group. -The bulk of measurements lie between 26 and 42 cm, with just a handful outside this interval. -The frequencies increase up to the modal group and then decrease. -There is no indication of any outliers. -Etc etc
Specific behaviours
✓ describes shape ✓ describes modality ✓ describes one other feature

(e) In a previous investigation, the researcher found that the SR measurements for an older group of people were normally distributed with a mean of 29 cm and a standard deviation of 3.4 cm. Determine the percentage of people in this older group who had an SR measurement

(i) less than 25 cm. (1 mark)

Solution
$P(X < 25) = 12\%$
Specific behaviours
✓ correct percentage



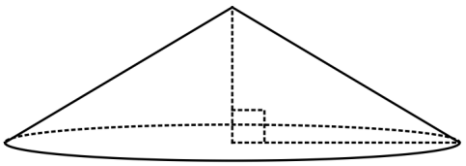
(ii) between 28 and 38 cm. (1 mark)

Solution
$P(28 < X < 38) = 61\%$
Specific behaviours
✓ correct percentage

See next page

Question 15 **(8 marks)**

The roof of a circular building has the shape of a right-circular cone with a base radius of $5.6\ \text{m}$ and a perpendicular height of $3.3\ \text{m}$.



(a) Building regulations require that one air vent is required for every $30\ \text{m}^3$ of space (or part) in the roof. Determine the number of air vents required for the roof shown. (3 marks)

Solution
$V = (\pi \times 5.6^2 \times 3.3) \div 3$ $= 108.4\ \text{m}^3$ $108.4 \div 30 = 3.6$ Need 4 air vents
Specific behaviours
✓ substitutes correctly ✓ correct volume ✓ states required number of air vents

(b) The curved surface of the roof is to be tiled (excluding the circular base). The tile that the builder has chosen has a coverage rate of 16 tiles per square metre and can be bought in packs of 124. Determine the number of packs the builder must order. (5 marks)

Solution
$s = \sqrt{5.6^2 + 3.3^2} = 6.5\ \text{m}$ $A = \pi \times 5.6 \times 6.5$ $= 114.4\ \text{m}^2$ $\text{Tiles} = 114.4 \times 16 = 1830$ $\text{Packs} = 1830 \div 124 = 14.7$ Order 15 packs
Specific behaviours
✓ calculates slant height ✓ uses formula for curved surface area ✓ calculates surface area ✓ calculates number of tiles ✓ calculates number of packs

See next page

Question 16 **(8 marks)**

(a) The statistical investigation process is a cyclical process that begins with the need to solve a real-world problem and aims to reflect the way statisticians work. Step P is one of the four steps in the cyclical process and involves the design and implementation of a plan to collect or obtain appropriate data.

(i) Describe the key elements of the step immediately after P in the cyclical process. (2 marks)

Solution
Select and apply appropriate graphical or numerical techniques to analyse the data.
Specific behaviours
✓ indicates analysis ✓ indicates use of graph or calculation

(ii) Describe the key elements of the step immediately before P in the cyclical process. (2 marks)

Solution
Clarify the problem and formulate one or more questions that can be answered with data.
Specific behaviours
✓ indicates clarify/identify problem ✓ indicates formulation of question(s)

(b) A student was carrying out a statistical investigation involving dogs.

(i) Describe an example of a categorical variable the student could investigate and list two different responses that could be recorded. (2 marks)

Solution
Type of breed: Poodle, terrier Type of coat: Silky, wiry Etc etc.
Specific behaviours
✓ valid example relating to dogs ✓ two different and valid responses

(ii) Describe an example of a continuous numerical variable the student could investigate and list two different responses that could be recorded. (2 marks)

Solution
Weight: 8.5 kg, 7.2 kg Length of tail: 5 cm, 56 cm. Etc etc.
Specific behaviours
✓ valid example relating to dogs ✓ two different and valid responses with units

See next page

Question 17 **(9 marks)**

A second-hand car yard paid $\$33\,500$ for a vehicle and later sold it to a customer for $\$26\,930$.

(a) Calculate the percentage loss made on the sale. (2 marks)

Solution
$33500 - 26930 = 6570$ $\frac{6570}{33500} \times 100 = 19.6\%\ \text{loss}$
Specific behaviours
✓ calculates loss ✓ correct percentage

(b) The customer paid a deposit of $\$2\,930$ and took out a loan for the remainder of the price with an interest rate of 10.9% per annum. Calculate the simple interest on the loan for the first month. (3 marks)

Solution
$26930 - 2930 = 24000$ $24000 \times 0.109 = 2616$ $2616 \div 12 = \$218$
Specific behaviours
✓ calculates principle ✓ interest for one year ✓ interest for one month

(c) The price the customer paid included 10% GST. Calculate the amount of GST included in the price. (2 marks)

Solution
$\frac{26930}{11} = \$2\,448.18$
Specific behaviours
✓ correct method ✓ amount of GST

(d) The standard premium to insure the vehicle was $\$2\,388$, but the customer was offered a discount of 35% for not having made any claims over the past five years. Determine the premium after the discount was applied. (2 marks)

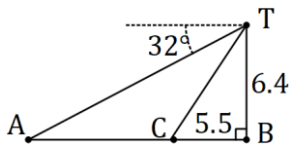
Solution
$2388 \times 0.65 = \$1\,552.20$
Specific behaviours
✓ correct method ✓ correct premium

See next page

Question 18 **(8 marks)**

The top of a vertical pole T stands 6.4 m above a surrounding level playing field. The angle of depression from T to a small animal at A is $32°$. The animal leaves A, moves directly towards the base of the pole B and stops at C (before reaching B). The distance BC is 5.5 m.

(a) Sketch a diagram to show the above information. (2 marks)

Solution

Specific behaviours
✓ vertical TB with angle of depression to A ✓ lengths TB and BC, making a right-angle

(b) Calculate the line of sight distance from the top of the pole to A. (2 marks)

Solution
$AT = 6.4 \div \sin 32$ $= 12.1$ m
Specific behaviours
✓ uses sine ratio ✓ correct distance

(c) Determine the angle of depression from T to C. (2 marks)

Solution
$\angle TCB = \tan^{-1}\frac{6.4}{5.5}$ $= 49.3°$
Specific behaviours
✓ uses tangent ratio ✓ correct angle (to nearest degree)

(d) Calculate the distance travelled by the animal from A to C. (2 marks)

Solution
$AB = 6.4 \div \tan 32 = 10.2$ m $AC = 10.2 - 5.5 = 4.7$ m
Specific behaviours
✓ calculates AB ✓ calculates AC

See next page

Question 19 **(7 marks)**

The wind chill index I is a measure of how quickly a person exposed to a wind will lose heat. It is calculated using the formula below, where v is the speed of the wind in metres per second and T is the air temperature in degrees Celsius.

$$I = \left(10\sqrt{v} - v + 10.2\right)(34 - T)$$

(a) Determine I when the air temperature is $-3°C$ and there is a wind of $12\ \text{m/s}$ blowing. (2 marks)

Solution
$I = (10\sqrt{12} - 12 + 10.2)(34 - -3)$ $= 1215$
Specific behaviours
✓ correct substitution ✓ evaluates

(b) Calculate the change in the wind chill index when the air temperature is $-8°C$ and the strength of the wind decreases from $22\ \text{m/s}$ to $8\ \text{m/s}$. (3 marks)

Solution
$I = (10\sqrt{22} - 22 + 10.2)(34 - -8)$ $= 1474$ $I = (10\sqrt{8} - 8 + 10.2)(34 - -8)$ $= 1280$ $\text{Decrease} = 1474 - 1280 = 194$
Specific behaviours
✓ first value correct ✓ second value correct ✓ indicates decrease and states amount

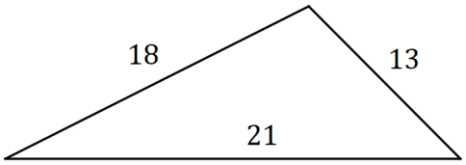
(c) A person is likely to suffer from frostbite when the wind chill index reaches $1\,500$. At what temperature will this happen, on a day when the wind has a speed of $25\ \text{m/s}$? (2 marks)

Solution
$1500 = (10\sqrt{25} - 25 + 10.2)(34 - T)$ $1500 = 35.2(34 - T)$ $T = 34 - \dfrac{1500}{35.2}$ $= -8.6°C$
Specific behaviours
✓ substitutes and simplifies ✓ solves for temperature

See next page

Question 20 **(7 marks)**

A model of a triangular gable has measurements shown below, in centimetres.



(a) Use Heron's rule to determine the area of the model of the gable. (3 marks)

Solution
$s = \frac{18 + 13 + 21}{2} = 26$ $A = \sqrt{26(26 - 18)(26 - 13)(26 - 21)}$ $= 116.3 \text{ cm}^2$
Specific behaviours
✓ calculates semi-perimeter ✓ substitutes into Heron's rule ✓ correct area

The model was drawn to a scale using measurements taken from a building, where the length of the shortest side of the gable was 5.2 m.

(b) Calculate the scale factor used to draw the model. (1 mark)

Solution
$5.2 \times \frac{100}{13} = 40 \Rightarrow \text{SF} = 1:40$
Specific behaviours
✓ correct scale factor

(c) The gable on the building requires repainting, at a cost of $\$21$ per square metre. Determine the cost of repainting the gable, to the nearest dollar. (3 marks)

Solution
$\text{Area} = 116.3 \times (40 \div 100)^2$ $= 116.3 \times 0.16$ $= 18.6 \text{ m}^2$ $\text{Cost} = 18.6 \times 21 \approx \$391$
Specific behaviours
✓ indicates area scale factor ✓ building area ✓ calculates cost, rounding

See next page

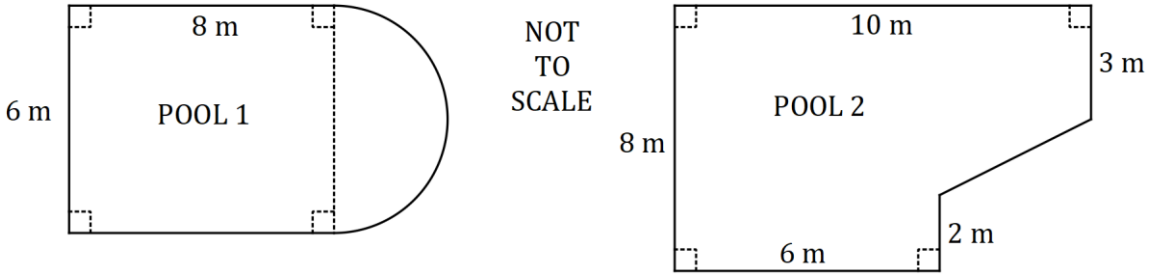
Question 21 **(8 marks)**

When working out the cost of building an in-ground swimming pool, a contractor calculates a shape factor k to use in a spreadsheet, where

$$k = \frac{P^2}{4A}.$$

P is the perimeter of the pool in metres and A is the area of the pool in square metres.

Pool 1 is rectangular with a semi-circular end and pool 2 is rectangular with a cut-out as shown.



Determine, with justification, which of the pools shown has the larger shape factor k.

Solution
Pool 1 $A_1 = 6 \times 8 + \frac{1}{2} \times \pi \times 3^2 = 62.14$ $P_1 = 8 + 6 + 8 + \frac{1}{2} \times 2 \times \pi \times 3 = 31.42$ $k_1 = \frac{31.42^2}{4 \times 62.14} = 3.97$ Pool 2 $A_2 = 8 \times 10 - 2 \times 4 - \frac{1}{2} \times 4 \times 3 = 66$ $P_2 = 3 + 10 + 8 + 6 + 2 + x = 29 + x$ $x = \sqrt{4^2 + 3^2} = 5$ $P_2 = 34$ $k_2 = \frac{34^2}{4 \times 66} = 4.38$ Pool 2 has the larger shape factor.
Specific behaviours
✓ area of pool 1 ✓ perimeter of pool 1 ✓ k for pool 1 ✓ area of pool 2 ✓ indicates missing length in pool 2 ✓ perimeter of pool 2 ✓ k for pool 2 ✓ identifies pool with larger k

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

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