Neap

Trial Examination 2020

VCE Mathematical Methods Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________

Teacher's Name: ______________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 19 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

Let $f: R \to R, f(x) = 2\cos\left(\frac{x}{3}\right) - 1$.

The period and range of f are respectively

A. 2π and $[-2, 2]$.

B. $\frac{2\pi}{3}$ and $[-2, 0]$.

C. $\frac{2\pi}{3}$ and $[-3, 1]$.

D. 6π and $[-2, 0]$

E. 6π and $[-3, 1]$

Question 2

The tangent to the curve $y = x^3 - x + 4$ at $x = 1$ has the equation

A. $y = 2x + 2$

B. $y = -\frac{1}{2}x + \frac{9}{2}$

C. $y = 3x^2 - 1$

D. $y = 4$

E. $y = 2x - 6$

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Question 3

Consider the simultaneous linear equations below, where m is a real constant.

$$(m+2)x + 7y = m + 3$$
$$x + (2m-1)y = 5$$

The set of values of m for which the system has a unique solution is

A. $\left\{-3, \frac{3}{2}\right\}$

B. $R\backslash\left\{-3, \frac{3}{2}\right\}$

C. $R\backslash\{-3\}$

D. $R\backslash\left\{-\frac{3}{2}\right\}$

E. $\left[-3, \frac{3}{2}\right]$

Question 4

The linear function $f: D \to R, f(x) = 3 - 2x$ has a range of $(-1, 5]$.

The domain D is equal to

A. $(-7, 5]$
B. $[-1, 2)$
C. $(0, 3)$
D. $(-7, -5]$
E. $(-1, 2]$

Question 5

A function f has the rule $f(x) = 2x^2 - 5\sqrt{x}$.

The average rate of change of the function f between $x = 1$ and $x = 4$ is

A. $\frac{25}{3}$

B. $\frac{19}{3}$

C. 22

D. $\frac{56}{3}$

E. $\frac{65}{8}$

Question 6

The set of values of k for which $kx^2 - kx + \frac{1}{4} = 0$ has exactly one real solution is

A. $\{-1, 1\}$
B. $\{0, 1\}$
C. $\{1\}$
D. $[0, 1]$
E. $(-\infty, 0) \cup (1, \infty)$

Question 7

A supermarket will only accept avocados from a supplier that weigh between 160 g to 195 g.
The supermarket rejects 10% of avocados for being underweight and 5% for being overweight.

Given that the weight of avocados is normally distributed, which one of the following is closest to the mean, μ, and standard deviation, σ?

A. $\mu = 175$ g and $\sigma = 12$ g
B. $\mu = 187.5$ g and $\sigma = 7$ g
C. $\mu = 187.5$ g and $\sigma = 5$ g
D. $\mu = 175$ g and $\sigma = 144$ g
E. $\mu = 177.5$ g and $\sigma = 10$ g

Question 8

The point $A(1, 3)$ lies on the graph of the function f. A transformation maps the graph of f to the graph of g where $g(x) = 3f(2x - 4) + 1$. The same transformation maps the point A to the point P.

The coordinates of point P are

A. $\left(\frac{5}{2}, 10\right)$
B. $(4, 10)$
C. $\left(\frac{9}{2}, 10\right)$
D. $(4, 8)$
E. $(6, 10)$

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Question 9

Let f and g be two functions such that $f(x+1) = x$ and $g(x+2) = f(x)$.

The function $f(g(x))$ is

A. $x - 4$
B. $x - 3$
C. $x + 1$
D. $x + 2$
E. $x + 4$

Question 10

If $2x + a$ is a factor of $2x^3 - ax^2 - 9x$, where $a \in R^+$, then the value of a is

A. $\frac{1}{2}$
B. 1
C. $\frac{\sqrt{6}}{2}$
D. 2
E. 3

Question 11

The probability of a basketball player successfully making a free throw shot is 80%. The player attempts 5 free throws.

The probability that the player successfully makes at least 3 consecutive free throws to is equal to

A. 0.02048
B. 0.06144
C. 0.512
D. 0.7168
E. 0.8

Question 12

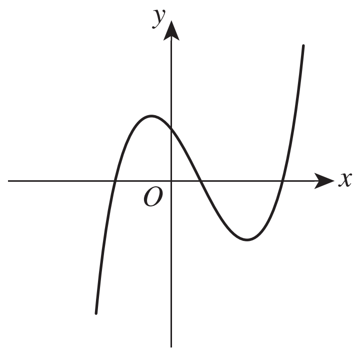
Two events, A and B, are independent where $\Pr(A \cap B) = 0.24$ and $\Pr(A \cup B) = 0.76$.

If $\Pr(A) > \Pr(B)$, then $\Pr(A)$ is equal to

A. 0.28
B. 0.40
C. 0.48
D. 0.50
E. 0.60

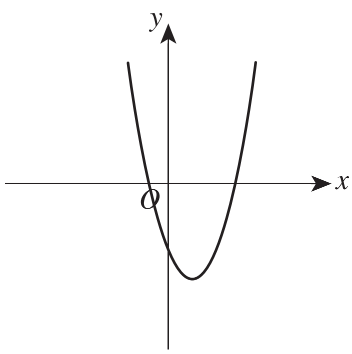
Question 13

The graph of the derivative function f' is shown below.



The graph of the function f could be

A.



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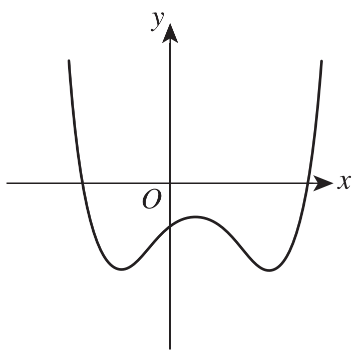

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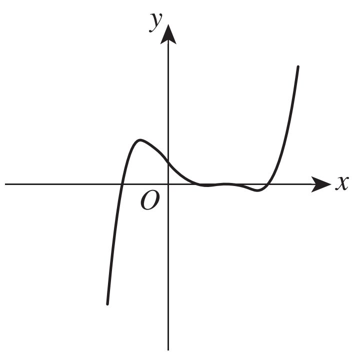

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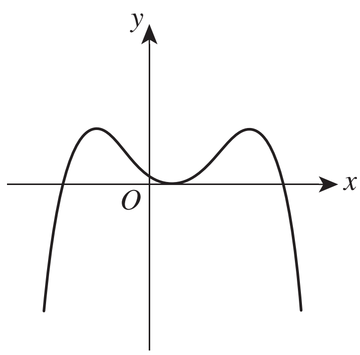
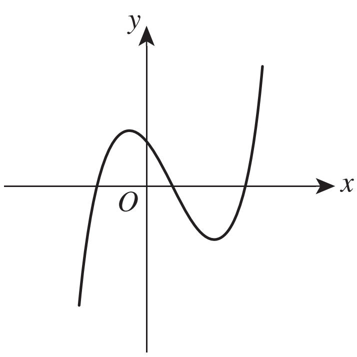


E.



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Question 14

The maximal domain of the function with the rule $f(x) = \dfrac{1}{\log_e(2-x)}$ is

A. $R\backslash\{2\}$

B. $(-\infty, 2)$

C. $(-\infty, 2)\backslash\{1\}$

D. $(2, \infty)$

E. $R\backslash\{1\}$

Question 15

If X is a normally distributed random variable with a mean of 0 and $\Pr(X > 1.5) = 0.1$, then the variance of X is closest to

A. 1.17

B. 1.28

C. 1.37

D. 1.92

E. 3.67

Question 16

If $g : R \to R, g(x) = x^3 - 3x^2 + 3x + 1$, then which one of the following is true?

A. The graph of g intersects the graph of g^{-1} at exactly 3 distinct points.

B. The graph of g intersects the graph of g^{-1} at exactly 2 distinct points.

C. The graph of g intersects the graph of g^{-1} at exactly 1 distinct points.

D. The graph of g does not intersect the graph of g^{-1}.

E. g does not have an inverse.

Question 17

The function f has the property $f(x) + 2f(y) = (2x + y)f(xy)$.

Which one of the following is a possible rule for the function f?

A. $2x$

B. $2x^2$

C. $\sqrt{x}$

D. $\dfrac{1}{x}$

E. $\dfrac{1}{x^2}$

Question 18

If $\int_3^8 f(x)\,dx = 10$ and $\int_{10}^8 f(x) = 4$, then $\int_3^{10} f(x) + 1\,dx$ is equal to

A. 6

B. 7

C. 13

D. 14

E. 21

Question 19

The minimum distance from the parabola $y = x^2 - 4$ to the origin is

A. 2

B. $\frac{\sqrt{14}}{2}$

C. $\frac{\sqrt{15}}{2}$

D. $\sqrt{2}$

E. 4

Question 20

Let n be a positive even integer and let $f(x) = n^{n-1}x^n\log_e(nx)$.

The number of stationary points of f is

A. 0

B. 1

C. 2

D. n

E. $n - 1$

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided. Write using blue or black pen.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

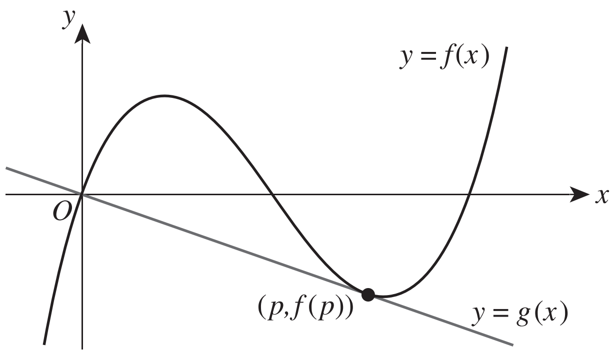
Question 1 (7 marks)

Consider the following functions, where a is a positive constant:

$$f : R \to R,\ f(x) = x^3 - 6x^2 + 8x$$

$$g : R \to R,\ g(x) = -ax$$

Part of the graphs of f and g are shown in the diagram below.



The line $y = g(x)$ is tangent to the graph of f at $x = p$ where $p > 0$.

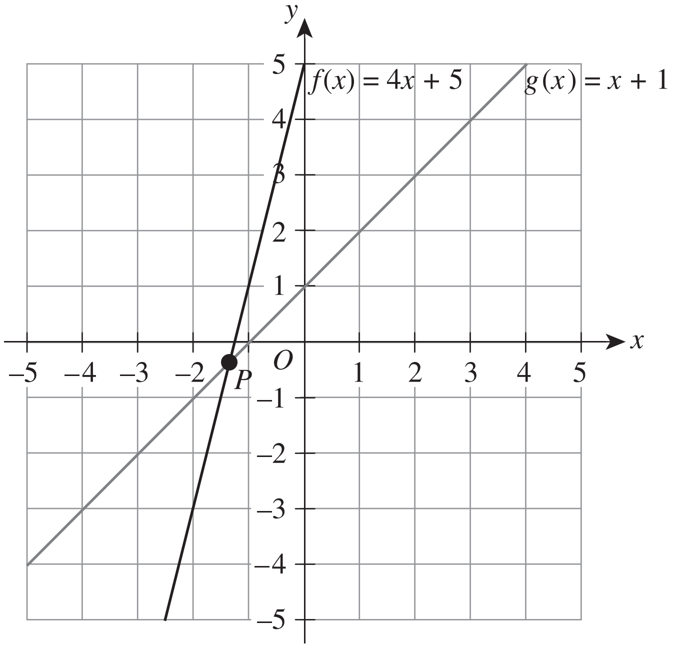
a. Find the coordinates of the x-intercepts of the graph of $y = f(x)$. 1 mark

b. Show that $a = 1$ and $p = 3$. 3 marks

c. Find the maximum vertical distance between the graphs of f and g over the interval $x \in [0, 3]$. 3 marks

Question 2 (18 marks)

Part of the graphs of the functions f and g are shown below where $f : R \to R, f(x) = 4x + 5$ and $g : R \to R, g(x) = x + 1$.



a. **i.** Find the coordinates of the point of intersection, P. 1 mark

ii. Find the distance of P from the origin. 1 mark

iii. Find the size of the acute angle between the graphs of f and g at P. Give your answer to the nearest degree. 1 mark

A transformation $T_1 : R^2 \to R^2$ maps g to a new function h, where T_1 is given by the following:

$$T_1\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix}, \text{ where } a \in R^+$$

b. **i.** Describe the transformations that map the graph of $y = g(x)$ to $y = h(x)$. 1 mark

ii. Hence, or otherwise, find the rule of the function $h(x)$ in terms of a. 2 marks

c. State the value(s) of a for which there is a unique solution to the equation $f(x) = h(x)$. 2 marks

d. Find the coordinates of the point of intersection of the graphs of $y = f(x)$ and $y = h(x)$ in terms of a. 2 marks

e. Find the value of a for which the distance between the point of intersection of the graphs of $y = f(x)$ and $y = h(x)$ and the origin is a minimum. 3 marks

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Consider the function $p : [-1, 1] \rightarrow R, p(x) = x + 1$.

A transformation $T_2 : R^2 \rightarrow R^2$ maps p to a new function q where T_2 is given by the following:

$$T_2\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix}, \text{ where } a \in R^-$$

f. **i.** State the domain of q. 1 mark

ii. Find the value(s) of a for which there is a unique solution to the equation $q(x) = f(x)$. 3 marks

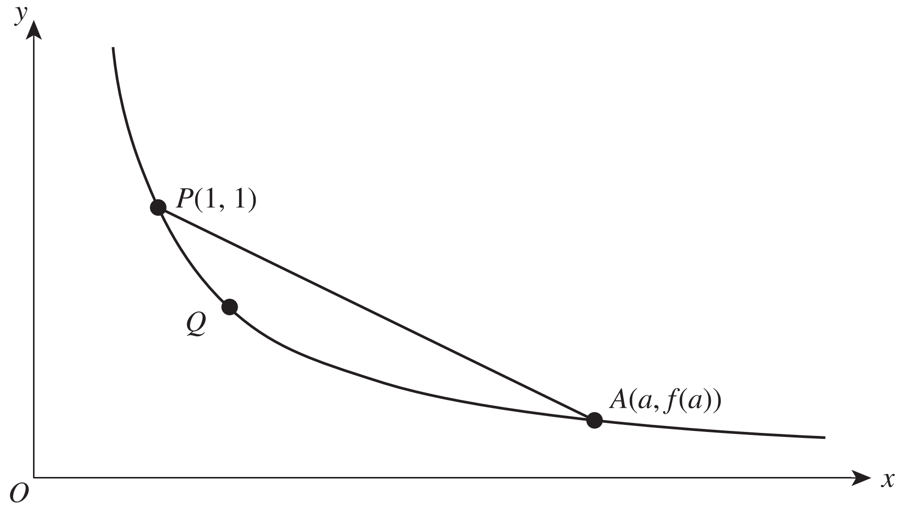
iii. The range of possible distances between the point of intersection of $y = f(x)$ and $y = q(x)$ and the origin is given by the set $(m_1, m_2]$.

State the value of m_1. 1 mark

Question 3 (13 marks)

The diagram below shows part of the graph of the function $f: R^+ \to R, f(x) = \frac{1}{x}$.

The line segment PA is drawn from the point $P(1, 1)$ to the point $A(a, f(a))$, where $a > 1$. The point Q lies on the graph of $f(x)$ between P and A.



a. **i.** Find the average rate of change of f between P and A in terms of a. 1 mark

ii. If the tangent to the graph of f at the point Q has a gradient equal to the average rate of change of f between P and A, find the coordinates of the point Q in terms of a. 2 marks

b. **i.** Find $\int_1^e f(x)dx$. 1 mark

ii. If $0 < b < 1$, find the exact value of b such that $\int_b^1 f(x)dx = 1$. 1 mark

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c. **i.** Express the area of the region bounded by the line segment PA, the x-axis, the line $x = 1$, and the line $x = a$ in terms of a. 2 marks

ii. Find the value of a for which this area is equal to 1. 2 marks

iii. Hence, show that this value of $a < e$. 1 mark

d. Find m and k such that $\int_{k}^{m} f(kx)dx = \frac{1}{k}$ and $\int_{m-1}^{k} f(kx)dx = \frac{1}{k}$. 3 marks

Question 4 (10 marks)

Two contestants, Aaron and Bethany, are competing as a team in a quiz show where each contestant answers a set of five multiple-choice questions. Each question has five possible outcomes (A, B, C, D and E), of which only one is correct.

a. Aaron decides to guess the answer to each of his five questions, so that he randomly chooses A, B, C, D, or E. Let the random variable X be the number of questions that Aaron correctly answers.

i. What is the probability that Aaron will answer none of the questions correctly? 1 mark

ii. Hence, find the probability that Aaron will answer at least 3 questions correctly, given that he answers at least 1 correctly. Give your answer correct to four decimal places. 2 marks

b. The probability that Bethany will answer any question correctly, independently of her answer to any other question is $p\,(p > 0)$. Let the random variable Y be the number of questions that Bethany correctly answers.

Given that $\Pr(Y > 3) = 11\Pr(Y = 5)$, show that the value of p is $\frac{1}{3}$. 2 marks

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c. The total score for the team is found by adding the number of questions Aaron answered correctly to the number of questions that Bethany answered correctly.

Find the probability that their total score was less than 2. Give your answer correct to four decimal places. 2 marks

d. The random variable T represents the time taken for the contestants to answer the five questions. T is normally distributed with a mean of 100 seconds and a standard deviation of σ. For Bethany, $\Pr(Y \geq 1) = \Pr(T \geq 91)$.

Calculate the value of σ, correct to four decimal places. 3 marks

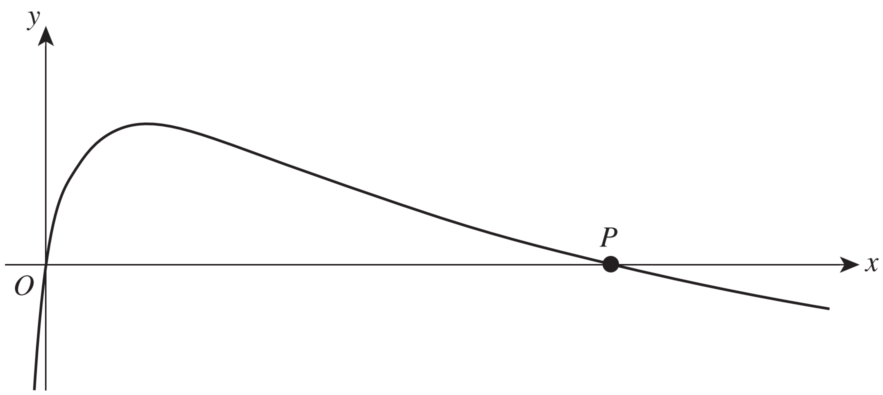
Question 5 (12 marks)

Consider the function with a rule given by $f(x) = \sin(\log_e(x + 1))$.

a. **i.** State the maximal domain of f. 1 mark

ii. State the range of the graph of $y = f(x)$. 1 mark

Part of the graph of $y = f(x)$ is shown below, with the first positive x-intercept indicated by the point P.



b. Show that the coordinates of P are given by $(e^{\pi} - 1, 0)$. 2 marks

Let $g(x) = f(x - 1)$.

c. **i.** Show that $g(x) = \sin(\log_e(x))$. 1 mark

ii. Find the coordinates of the two x-intercepts of the graph of $y = g(x)$ between $x = 0.5$ and $x = 30$. 1 mark

d. Differentiate $x(\sin(\log_e(x)) - \cos(\log_e(x)))$ and hence show that

$$\int g(x)dx = \frac{x}{2}(\sin(\log_e(x)) - \cos(\log_e(x))).$$

3 marks

e. Find the area bounded by the curve of $y = f(x)$ and the x-axis between the origin and P. 3 marks

END OF QUESTION AND ANSWER BOOKLET

