Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section One: Calculator-free

SOLUTIONS

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

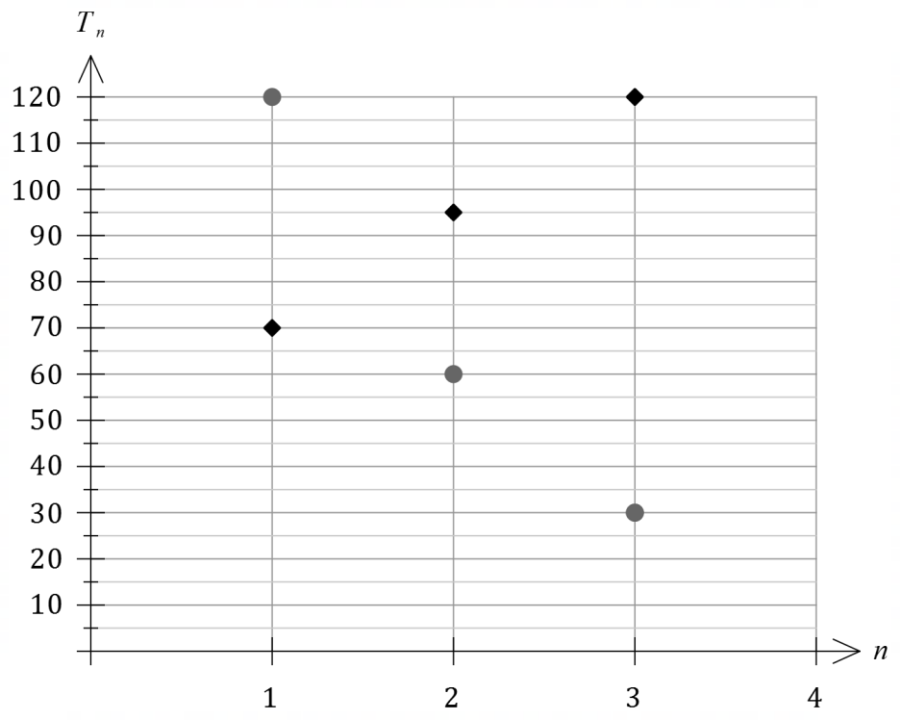
Section One: Calculator-free **35% (52 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

The graph below shows the first few terms of an arithmetic sequence and a geometric sequence.



(a) Deduce a rule for the n^{th} term of the geometric sequence. (2 marks)

Solution
$T_n = 120(0.5)^{n-1}$
Specific behaviours
✓ indicates correct common ratio r
✓ correct rule

(b) Determine the 30^{th} term of the arithmetic sequence. (3 marks)

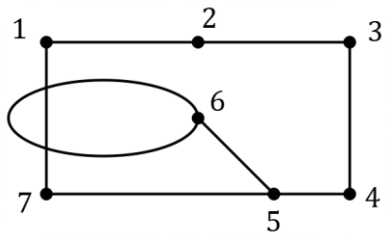
Solution
$T_n = 70 + (n-1)(25)$ $= 45 + 25n$ $T_{30} = 45 + 25 \times 30$ $= 45 + 750$ $= 795$
Specific behaviours
✓ indicates correct common difference d
✓ formulates rule or expression for T_{30}
✓ correct value

See next page

Question 2 **(5 marks)**

The statements in parts (a) to (e) of this question relate to graph G shown at right.

For each statement, state whether it is true or false and support your answer with brief reasons.



(a) Graph G has a bridge. (1 mark)

Solution
True - between vertices 5 and 6.
Specific behaviours
✓ correct answer with justification

(b) Graph G is planar. (1 mark)

Solution
True - it could be drawn in the plane so that no two edges cross.
Specific behaviours
✓ correct answer with justification

(c) Graph G has 7 even vertices. (1 mark)

Solution
False - only 5 vertices $(1, 2, 3, 4, 7)$ are even.
Specific behaviours
✓ correct answer with justification

(d) Graph G has a trail of length 9. (1 mark)

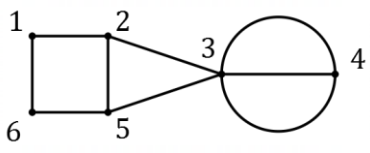
Solution
False - e.g., only 8 edges so maximum possible length of any trail is 8.
Specific behaviours
✓ correct answer with justification

(e) Graph F, shown below, is a subgraph of graph G. (1 mark)

Solution
False - vertices 4 and 7 are adjacent in F but not in G.
Specific behaviours
✓ correct answer with justification

Question 3 **(7 marks)**

(a) Graph G_1 is shown at right.



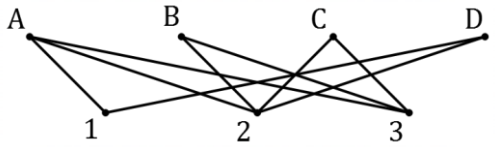
Show that graph G_1 satisfies Euler's formula. (2 marks)

Solution
$v = 6, \quad f = 5, \quad e = 9$ Hence $v + f - e = 6 + 5 - 9 = 2$, as required.
Specific behaviours
✓ clearly indicates correct values for v, f and e ✓ shows that $v + f - e = 2$

(b) A haulage company has three trucks and four drivers. The drivers that are licensed to drive each of the trucks are shown with a tick in the following table.

		Driver			
		A	B	C	D
Truck	1	✓			✓
	2	✓	✓	✓	✓
	3	✓	✓	✓	

(i) Represent the information in the table as bipartite graph G_2. (3 marks)

Solution

Specific behaviours
✓ any graph that is clearly bipartite ✓ all vertices marked with dot labelled ✓ all edges correct

(ii) Graph G_2 can be drawn in the plane. Determine, with justification, the number of faces G_2 has. (2 marks)

Solution
Since G_2 is a connected planar graph, it will satisfy Euler's formula: $v + f - e = 2 \rightarrow 7 + f - 9 = 2 \rightarrow f = 4$ G_2 has 4 faces.
Specific behaviours
✓ suitable reasoning or diagram ✓ correct number of faces, with justification

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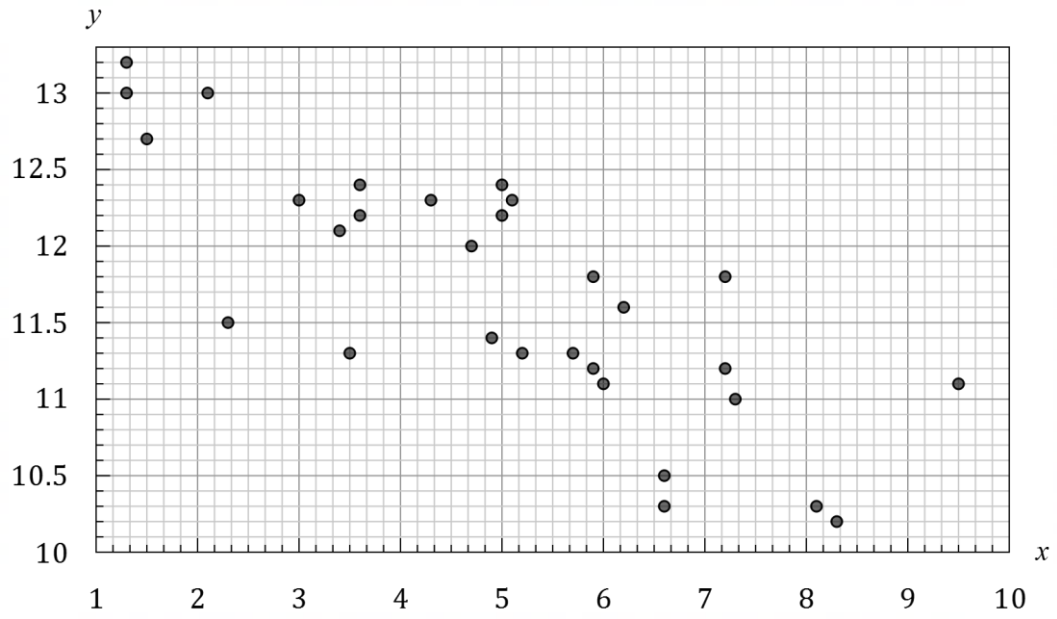
Question 4 **(10 marks)**

Over a period of one week, a group of children were observed and the total time that they slept each day recorded. The scatterplot below shows the age in years and months of each child on the x-axis and their average daily sleep time in hours on the y-axis.



The equation of the least-squares line is $\hat{y} = 13.2 - 0.3x$.

(a) How many children aged 7 were in the group? (1 mark)

Solution
3 children
Specific behaviours
✓ correct number

(b) Name the response variable. (1 mark)

Solution
Average daily **sleep** time.
Specific behaviours
✓ correct variable

(c) Describe the linear association between the variables in terms of direction and strength. (2 marks)

Solution
Association is negative and moderate to strong.
Specific behaviours
✓ direction ✓ strength (accept moderate or strong, $r = -0.8$)

(d) Interpret the slope of the least-squares line in the context of this question. (2 marks)

Solution
For every extra year of age, the average daily sleep of a child is observed to decrease by 0.3 hours.
Specific behaviours
✓ identifies as age increases sleep decreases ✓ quantifies change using value of slope in context

(e) Predict the average daily sleep time of a child who has just had their fourth birthday. (2 marks)

Solution
$\hat{y} = 13.2 - 0.3 \times 4$ $= 13.2 - 1.2$ $= 12.0 \text{ h}$
Specific behaviours
✓ substitutes into least-squares line ✓ correct prediction

(f) A paediatrician looked at the data and said, " as children age, it causes them to need less sleep". Comment on this statement. (2 marks)

Solution
The observed association between the two variables is not a sufficient condition to establish a causal relationship and so the statement is not true.
Specific behaviours
✓ states statement is not true ✓ takes issue with claim of a causal relationship

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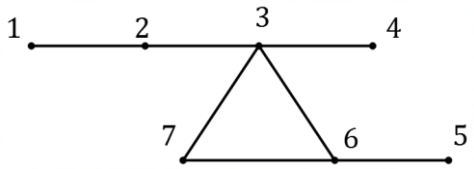
Question 5 **(10 marks)**

A farm has seven buildings spread out over an area of land. There are paved roads between buildings 1 and 2, 2 and 3, 3 and 4, 3 and 6, 3 and 7, 5 and 6, and 6 and 7.

(a) Construct graph G to represent this network of buildings and paved roads. (2 marks)

Solution

Specific behaviours
✓ correct number of labelled vertices ✓ correct graph

(b) Determine the length of

(i) the longest trail in graph G. (1 mark)

Solution
Length is 6. (e.g., 1-2-3-6-7-3-4)
Specific behaviours
✓ correct length

(ii) the shortest, closed walk in graph G that visits all seven vertices. (1 mark)

Solution
Length is 11. (e.g., 1-2-3-4-3-6-5-6-7-3-2-1)
Specific behaviours
✓ correct length

(c) State, with justification, whether graph G is a Hamiltonian graph. (3 marks)

Solution
No. The graph does not contain a path which passes through every vertex only once and finishes at the starting vertex.
Specific behaviours
✓ states no ✓ correct justification for every vertex ✓ correct justification for cycle

(d) A new paved road is planned between two different buildings. With the addition of this road, graph G will be a semi-Hamiltonian graph. Determine the possible pairs of buildings that the road could be built between. (3 marks)

Solution
For a Hamiltonian path to exist, add an edge between any of the following pairs of vertices: 1-4, 1-5, 1-7, 4-2, 4-5, 4-7, 5-2. There are 7 possible pairs of buildings the road can run between.
Specific behaviours
✓ lists one pair of vertices so that G has a Hamiltonian path ✓ lists at least 5 possible distinct pairs ✓ lists all 7 possible pairs

See next page

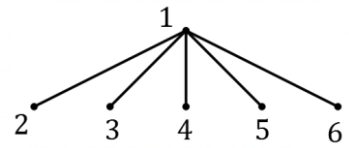
Question 6 **(7 marks)**

A complete bipartite graph with n vertices in one group and m vertices in the other group will have nm edges.

(a) Explain the meaning of the term **complete** for such a graph. (1 mark)

Solution
Complete: Every vertex of one group is connected to every vertex of the other group.
Specific behaviours
✓ reasonable explanation

(b) Draw a complete bipartite graph with 5 edges. The graph should clearly be bipartite with its vertices labelled $1, 2, 3, \ldots$ and so on. (2 marks)

Solution

Specific behaviours
✓ bipartite graph with required number of edges ✓ labelled vertices

(c) Construct the adjacency matrix for the graph you drew in part (b). (2 marks)

Solution
$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
Specific behaviours
✓ 6×6 matrix (or table) filled with 0's and 1's. ✓ correct matrix for graph in part (b) (mpa)

(d) Determine, with justification, the minimum number of vertices that a complete bipartite graph with 24 edges can have. (2 marks)

Solution
With 24 edges, the numbers of vertices in each set will be $(1, 24) \rightarrow v = 25$ or $(2, 12) \rightarrow v = 14$ or $(3, 8) \rightarrow v = 11$ or $(4, 6) \rightarrow v = 10$ Hence minimum number of vertices is 10.
Specific behaviours
✓ correct minimum number ✓ justifies answer by comparing to at least one other possibility

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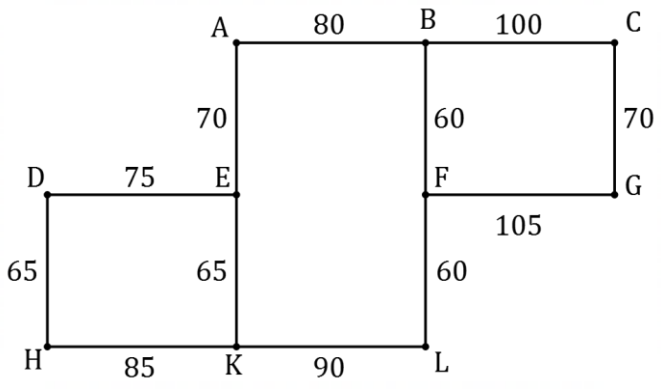
Question 7 **(8 marks)**

The graph below represents a town centre, with street corners shown as vertices, streets as edges and the length, in metres, of each street as an edge weight.

The total length of all streets in the town centre is 925 m.



(a) List, in order, the vertices that lie on the shortest path from H to C and state the length of this path in metres. (3 marks)

Solution
$H - D - E - A - B - C = 390$ m
Specific behaviours
✓ any path with correct length ✓ lists vertices on shortest path ✓ states length of shortest path

(b) Is the graph Eulerian? Justify your answer. (2 marks)

Solution
No - for the graph to be Eulerian there must be no odd vertices, but this graph contains four.
Specific behaviours
✓ states no ✓ states there must be no odd vertices

(c) A cleaning team must make their way along every street in the town centre, starting and finishing at the same street corner. Determine a possible walk they can travel, the minimum distance they must walk and explain how they can achieve this minimum. (3 marks)

Solution
The cleaning team can start at any vertex and travel edges BF and EK twice. $H - D - E - A - B - C - G - \boldsymbol{F} - \boldsymbol{B} - \boldsymbol{F} - L - \boldsymbol{K} - \boldsymbol{E} - \boldsymbol{K} - H$ Hence the minimum distance is the sum of the weights plus the two extra edges: $d = 925 + 65 + 60 = 1050$ metres.
Specific behaviours
✓ identifies a possible walk that meets requirements (mpa) ✓ calculates a distance, supported by a possible walk ✓ states correct minimum with reasons

End of questions

Supplementary page

Question number: _____