



Semester One Examination, 2019

Question/Answer booklet

**MATHEMATICS
APPLICATIONS
UNIT 3**

Section Two:

Calculator-assumed

SOLUTIONS

Student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (98 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

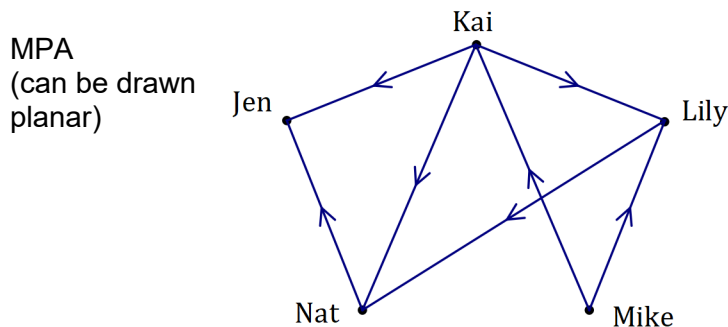
Working time: 100 minutes.

Question 9

(6 marks)

- (a) In a group of five people it was known that Kai was older than Lily, Nat and Jen; Mike was older than Kai and Lily; Nat was older than Jen; and Lily was older than Nat.

- (i) Represent this set of age relationships as a digraph. (2 marks)



Solution
See graph
Specific behaviours
✓ all arcs
✓ all directions correct (as seen here or all the opposite direction)

- (ii) State the number of arcs in the digraph.

Solution
7 arcs
Specific behaviours
✓ correct number

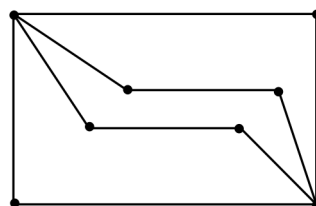
(1 mark)

- (iii) List the five people in order of age, starting with the youngest.

Solution
Jen, Nat, Lily, Kai, Mike
Specific behaviours
✓ correct order

(1 mark)

- (b) Graph H is shown below.



Let t and p be the number of edges in the longest open trail and shortest closed path contained in H respectively. State the values of t and p , given that $t > 0$ and $p > 0$.

(2 marks)

Solution
$t = 9, \quad p = 4$
Specific behaviours
✓ correct value of t
✓ correct value of p

Question 10

(8 marks)

The following data shows the blood haemoglobin (H) levels and packed cell volumes (V) of 10 blood bank donors.

H	11.0	13.3	11.8	14.5	14.8	11.2	13.9	12.2	12.8	11.6
V	0.43	0.47	0.45	0.50	0.51	0.45	0.49	0.46	0.48	0.46

- (a) Graph the data on your calculator and describe features of the graph that suggest the presence of a strong and positive linear association between H and V . (2 marks)

Solution
Points lie very close to a straight line (strong, linear) As one variable increases, the other variable also tends to increase (positive)
Specific behaviours
✓ describes one feature ✓ describes all three features (strong, linear, positive)

- (b) Determine the equation of the least-squares line that models the relationship between H and V , where H is the explanatory variable. (2 marks)

Solution
$V = 0.0175H + 0.248$
Specific behaviours
✓ states equation using given variables ✓ coefficients that round to 0.02 (gradient) and 0.248 (intercept)

- (c) Calculate the correlation coefficient between H and V . (1 mark)

Solution
$r = 0.9598$
Specific behaviours
✓ answer that rounds to 0.960

- (d) What percentage of the variation in V can be explained by the variation in H ? (1 mark)

Solution
92.12%
Specific behaviours
✓ answer that rounds to 92%

- (e) Predict the packed cell volume of a donor with a blood haemoglobin level of 13.4. (1 mark)

Solution
$\hat{V}(13.4) = 0.48$
Specific behaviours
✓ correct prediction that rounds to 0.48 or 0.47

- (f) Describe a potential danger associated with using the least-squares line to predict a packed cell volume from a blood haemoglobin level. (1 mark)

Solution
A prediction that involves extrapolation is dangerous and likely to be unreliable.
Specific behaviours
✓ mentions extrapolation

Question 11

(7 marks)

A company bought and installed a new computer system with an initial value of \$29 400. For accounting purposes, the value of the system decreased by \$2 450 each year.

- (a) Calculate the value of the system after 3 years.

(1 mark)

Solution
$29400 - 3 \times 2450 = \$22\,050$
Specific behaviours
✓ correct value

- (b) Determine a recurrence relation for V_n , the value of the system after n years.

(2 marks)

Solution
$V_{n+1} = V_n - 2450, V_0 = 29400$
Specific behaviours
✓ recurrence relation
✓ shows a term of sequence

- (c) Determine

- (i) the value of the system after 7 years.

(1 mark)

Solution
$V_7 = \$12\,250$ (or $29\,400 - 7 \times 2450 = 12\,250$)
Specific behaviours
✓ correct value

- (ii) the number of years for the value of the system to become nothing.

(1 mark)

Solution
$V_n = 0 \Rightarrow n = 12$ (or $29\,400/2450 = 12$)
Specific behaviours
✓ correct time

- (d) Determine the additional time taken for the system to become worthless if its value decreased by \$1 470 each year instead of \$2 450.

(2 marks)

Solution
$V_{n+1} = V_n - 1470, V_0 = 29400$ $V_n = 0 \Rightarrow n = 20$ (or $29\,400/1470 = 20$) $20 - 12 = 8$ additional years
Specific behaviours
✓ new total time
✓ states additional time

Question 12**(8 marks)**A sequence, K , is defined as:

$$K_n = 3(2)^n$$

- (a) State the first three terms of sequence
- K
- .

(2 marks)

Solution
6, 12, 24
Specific behaviours
<ul style="list-style-type: none"> ✓ states first term ✓ correctly states 2nd and 3rd terms

- (b) Write sequence
- K
- using a recursive rule.

(2 marks)

Solution
$K_{n+1} = 2 K_n, \quad K_1 = 6 \text{ (or } K_0 = 3)$
Specific behaviours
<ul style="list-style-type: none"> ✓ recurrence relation ✓ shows a term of sequence

A different sequence, A , is defined as:

$$A_{n+1} = 0.8 A_n \quad \text{where } A_1 = 100$$

- (c) State the first three terms of sequence
- A
- .

(2 marks)

Solution
100, 80, 64
Specific behaviours
<ul style="list-style-type: none"> ✓ states 1st & 2nd terms ✓ states 3rd term

- (d) Determine the smallest value of
- n
- such that
- $K_n > A_n$
- .
-
- Show some working.

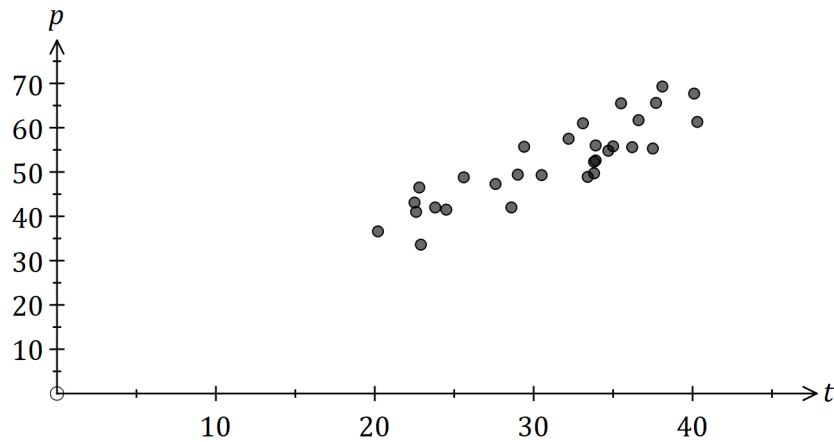
(2 marks)

Solution										
$3(2)^n > 100(0.8)^{n-1}$										
$n > 4.07 \quad \therefore n = 5$										
or										
	<table><tr><th>n</th><th>K_n</th><th>A_n</th></tr><tr><td>4</td><td>48</td><td>51.2</td></tr><tr><td>5</td><td>96</td><td>40.96</td></tr></table>	n	K_n	A_n	4	48	51.2	5	96	40.96
n	K_n	A_n								
4	48	51.2								
5	96	40.96								
	$\therefore n = 5$									
Specific behaviours										
✓ shows working										
✓ correct value of n										

Question 13

(7 marks)

The scatterplot below shows the marks scored by 30 students in their theory (t) and practical (p) exams that were marked out of 60 and 90 marks respectively.



The equation of the least-squares line for the data is $p = 1.38t + 9.2$.

- (a) It was found that 77% of the variation in p could be explained by the variation in t . Determine the correlation coefficient r_{tp} . (1 mark)

Solution
$r_{tp} = \sqrt{0.77} = 0.877$
Specific behaviours
✓ correct r to at least 2dp

- (b) Interpret the slope of the least-squares line. (2 marks)

Solution
For every extra theory mark the corresponding practical mark tends to increase by 1.38. [Note: increases may be expressed as percentages ($1/60 = 1.6\%$ $1.38/90 = 1.53\%$)]
Specific behaviours
✓ identifies increase in context ✓ uses 1.38

- (c) Joe and Kai were absent for the practical exam, but it was known that their marks in the theory exam were 28 and 57 respectively. Predict their practical exam marks and explain how reliable each prediction is. (4 marks)

Solution
Joe: $p(28) = 48$. Joe's predicted score of 48 is reliable as correlation is strong and it is interpolated. Kai: $p(57) = 88$. Kai's predicted score of 88 is unreliable as despite strong correlation it is extrapolated.
Specific behaviours
✓ prediction for Joe ✓ comments on strong correlation AND interpolation to justify Joe's is reliable ✓ prediction for Kai ✓ uses extrapolation to justify Kai's is unreliable

Question 14

(11 marks)

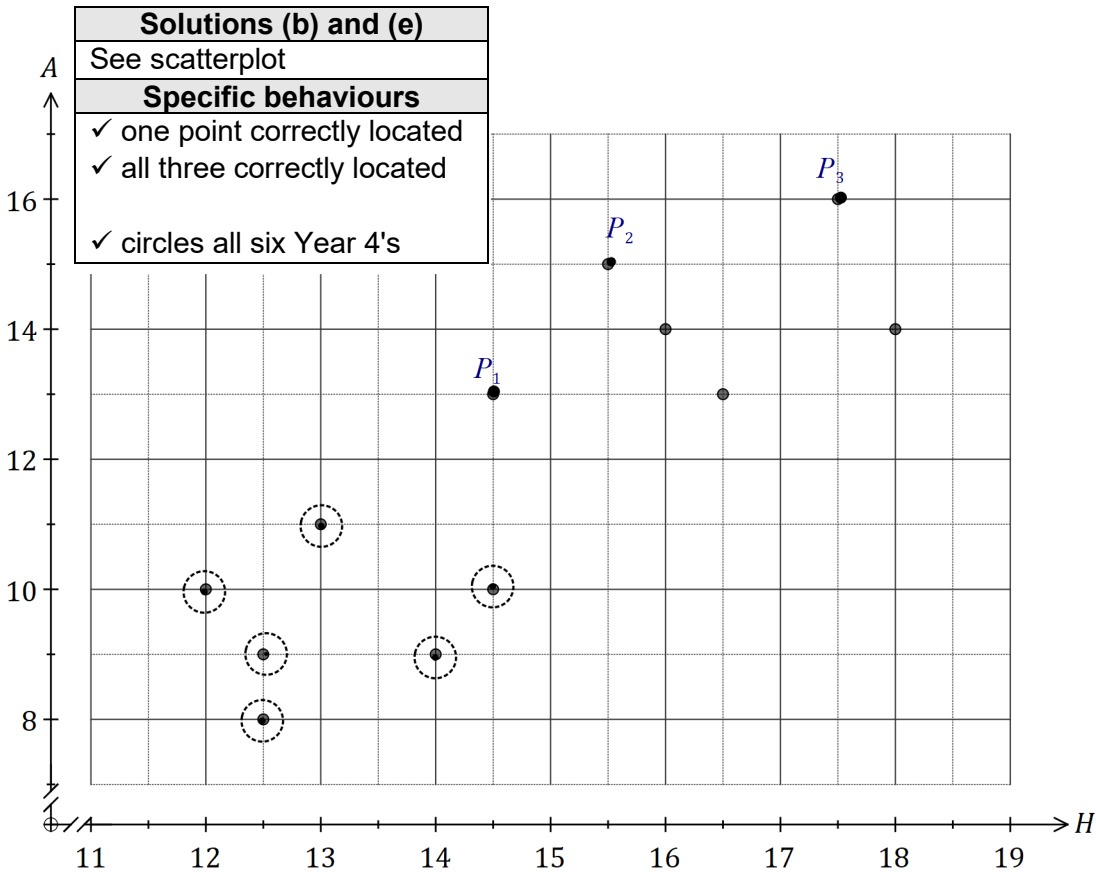
A researcher obtained the following data whilst investigating whether it is possible to reliably predict a child's reading ability (A , on a numerical scale of 1 to 20) from their hand span (H , cm).

Child	B	C	D	E	F	G	J	K	L	M	N	P
H	13.0	12.5	16.0	17.5	15.5	16.5	14.5	18.0	12.0	14.0	14.5	12.5
A	11	8	14	16	15	13	10	14	10	9	13	9

- (a) State the explanatory variable for this investigation. (1 mark)

Solution
Hand span
Specific behaviours
✓ correct variable

- (b) Add the three missing data points to the scatterplot below. (2 marks)



- (c) Determine the correlation coefficient between the two variables. (1 mark)

Solution
$r = 0.838$
Specific behaviours
✓ correct value (at least 2dp)

- (d) Using the scatterplot from (b) and the correlation coefficient from (c), the researcher was satisfied that a linear association existed between A and H . Explain why they reached this conclusion. (2 marks)

Solution
The points lie close to a straight line, indicating linear form.
The correlation coefficient is close to 1, indicating a strong linear association.
Specific behaviours
<ul style="list-style-type: none"> ✓ explains linear form ✓ refers to strong correlation

The researcher then discovered that the children labelled B, C, J, L, M and P were all in Year 3 and the remainder in Year 6.

- (e) Circle the Year 3 children on the graph. (1 mark)
- (f) Calculate the correlation coefficient between A and H for the Year 3 children only. (1 mark)

Solution
$r = 0.147$
Specific behaviours
✓ correct value (at least 2dp)

- (g) Identify a non-causal explanation for the conclusion reached by the researcher in (d) and explain how this new information affects that conclusion. (3 marks)

Solution
A non-causal explanation is that the year group (or age) of a child is a confounding variable.
The new information indicates there is no association as within a year group the correlation is very weak.
Specific behaviours
<ul style="list-style-type: none"> ✓ identifies year group (or age) as a confounding variable ✓ states there is no association ✓ uses weak correlation to justify no association

Question 15

(7 marks)

A water tank is initially empty. At the start of each hour, 150 L of water is quickly poured into the tank but during the following hour, 20% of all the water in the tank leaks out.

This situation can be modelled by the recurrence relation $V_{n+1} = 0.8V_n + 150$, $V_0 = 150$, where V_n is the volume of water in the tank, in litres, at the start of the n^{th} hour.

- (a) Complete the table below, giving volumes to the nearest litre. (2 marks)

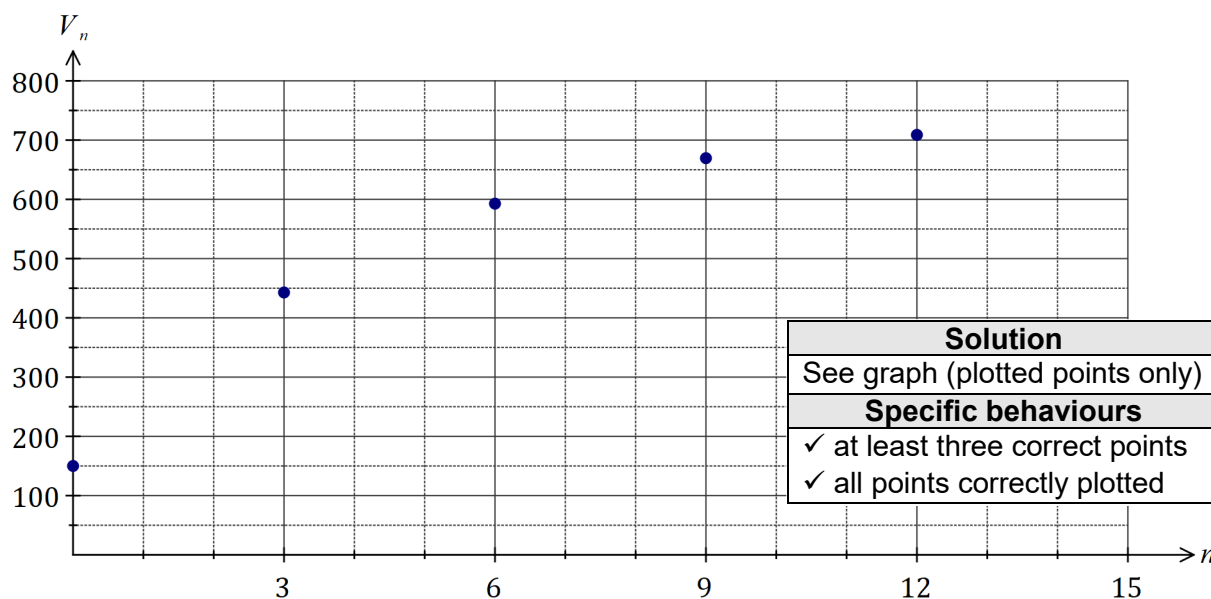
n	0	3	6	9	12
V_n	150	443	593	669	

Solution	
See table	
Specific behaviours	
✓ at least two correct values	
✓ all correct values	

- (b) At the start of which hour does the tank first hold at least 745 L? (1 mark)

Solution
$V_n \geq 745 \Rightarrow n = 22$ At the start of the 22 nd hour.
Specific behaviours
✓ correct number

- (c) Plot the points from the table on the axes below. (2 marks)



- (d) The tank has a maximum capacity of 800 L. If possible, determine the least number of hours since filling commenced that the tank will start to overflow. If not possible, explain why not. (2 marks)

Solution
Not possible as the amount of water in the tank will never exceed 750 L.
Specific behaviours
✓ states not possible
✓ uses value of steady-state maximum

Question 16

(9 marks)

A study categorized the weight of hospitalised children as obese, overweight, normal or underweight. The numbers of children in each category are shown by gender in the table below.

	Obese	Overweight	Normal	Underweight
Male	16	71	145	13
Female	23	39	161	27

- (a) An overweight child is randomly chosen from the study. If possible, explain whether they are more likely to be a boy or a girl. If not possible, explain your reasoning. (2 marks)

Solution
More likely to be a boy, as the number of overweight boys is almost twice the number of overweight girls.
Specific behaviours
<ul style="list-style-type: none"> ✓ chooses boys ✓ explanation that compares numbers

- (b) What percentage of the boys in the study were classified as underweight? (2 marks)

Solution (b)
$\frac{13}{245} = 5.3\%$
Specific behaviours
<ul style="list-style-type: none"> ✓ correct fraction ✓ correct percentage

Solution (c)
See table
Specific behaviours
<ul style="list-style-type: none"> ✓ two entries correct ✓ one row correct ✓ all entries correct

- (c) Complete the table of **row** percentages below to the nearest whole number. (3 marks)

(%)	Obese	Overweight	Normal	Underweight
Male	7	29	59	5
Female	9	16	64	11

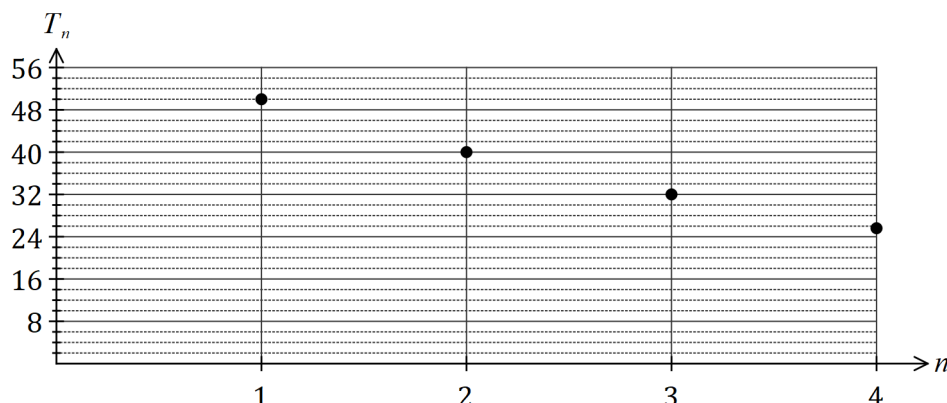
- (d) Does the table of row percentages suggest the presence of an association between the categorical variables? Justify your answer. (2 marks)

Solution
Yes, as the percentages in each column are quite different. For example, underweight females (11%) is more than twice the percent of underweight males (5%).
Specific behaviours
<ul style="list-style-type: none"> ✓ yes to association ✓ uses actual values from the table to indicate differences in column percentages to justify answer

Question 17

(6 marks)

A piledriver is hammering a pile into the ground. The graph below shows the distance T_n (in cm) the pile moves into the ground on the n^{th} hit of the piledriver.



The values of T_n form a geometric sequence.

- (a) Use information from the graph to determine the common ratio for the sequence. (1 mark)

Solution
$r = 32 \div 40 = 0.8$
Specific behaviours
✓ correct ratio

- (b) Write a recurrence relation to generate the values of T_n . (2 marks)

Solution
$T_{n+1} = T_n \times 0.8, \quad T_1 = 50$
Specific behaviours
✓ recurrence relation
✓ states value of T_1

- (c) Write the n^{th} term rule for the values of T_n . (1 mark)

Solution
$T_n = 50(0.8)^{n-1}$
Specific behaviours
✓ correct rule

- (d) Determine

- (i) the distance the pile moves into the ground on the seventh hit of the piledriver. (1 mark)

Solution
$T_7 = 13.1 \text{ cm}$
Specific behaviours
✓ correct distance that rounds to 13 cm

- (ii) on which hit the pile first moves less than one mm into the ground. (1 mark)

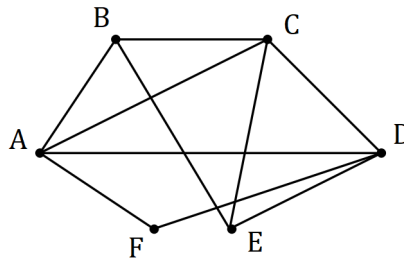
Solution
$T_n < 0.1 \Rightarrow n = 29$
Specific behaviours
✓ correct value

See next page

Question 18

(7 marks)

Each vertex on the graph below represents an airport and an edge between two airports indicates that an airline has a direct flight, in both directions, between the airports.



- (a) Redraw the graph to clearly show that it is planar.

(1 mark)

Solution
Specific behaviours
✓ correct equivalent, no crossed edges

- (b) Demonstrate that the graph satisfies Euler's formula.

(2 marks)

Solution
$v + f - e = 6 + 6 - 10 = 2$
Specific behaviours
✓ correct count of v , f and e ✓ substitutes into formula and simplifies correctly

In order to check in-flight catering quality, an airline manager plans to leave airport A , travel on at least one flight between the 10 pairs of airports and then return to A . The manager does not use any other mode of transport between airports.

- (c) Determine the minimum number of flights the manager must take and list, in order, the airports visited.

(2 marks)

Solution
$A - F - D - A - C - D - E - C - B - E - B - A = 11 \text{ flights}$
Specific behaviours
✓ correct number ✓ correct list (many exist)

- (d) Another manager, based at a different airport, claimed he could carry out the quality check in fewer flights by starting at his airport and finishing at another airport. Comment on this claim.

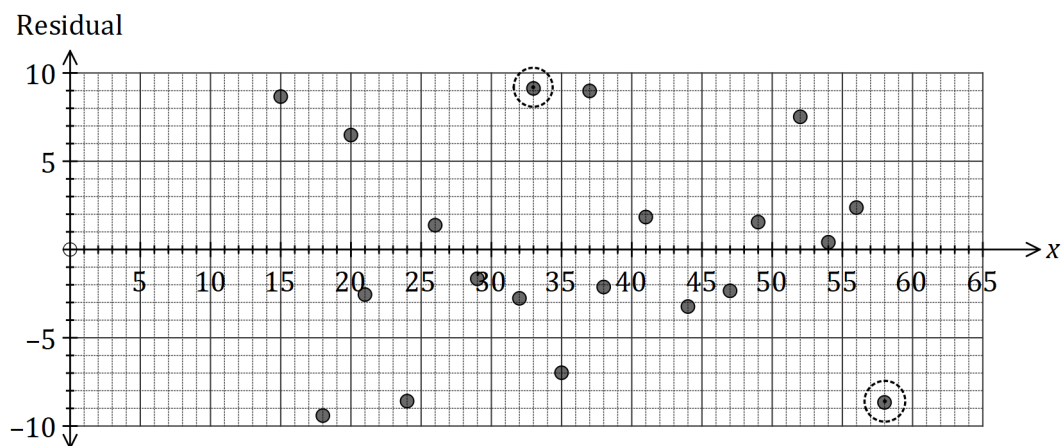
(2 marks)

Solution
Claim is true if manager based at B or E . Graph is semi-Eulerian and starting from these airports only 10 flights required.
Specific behaviours
✓ agrees with claim conditionally ✓ states correct number of flights

Question 19**(6 marks)**

A statistician wants to check whether a linear model is appropriate for a bivariate data set they are analysing. The least-squares line to model the linear relationship is $y = 1.54x - 13.9$ and the correlation coefficient between the variables is very strong.

The residual plot using the linear model is shown below for all but two of the data points.



- (a) Calculate the residuals for the missing points (33, 46) and (58, 66.5) and plot them on the graph above. (4 marks)

Solution
$y(33) = 36.92$ and $46 - 36.92 = 9.08$
$y(58) = 75.42$ and $66.5 - 75.42 = -8.92$
Specific behaviours
<ul style="list-style-type: none"> ✓ calculates both predicted values ✓ residual for one point ✓ residual for second point ✓ accurately plots both residuals

- (b) Use the residual plot to explain whether fitting a linear model to the data is appropriate. (2 marks)

Solution
Linear model is appropriate as no pattern is evident in the residuals.
Specific behaviours
<ul style="list-style-type: none"> ✓ states linear model is appropriate ✓ refers to no pattern evident in residuals

Question 20

(8 marks)

- (a) An investor has \$2 520 in an account. One month later, and at the start of each subsequent month, a deposit of \$95 is added to the account. Interest, calculated as 0.38% of the balance at the start of the month, is added to the account just before each deposit is made.

The account balance after n deposits is T_n , and can be modelled by the recurrence relation $T_{n+1} = 1.0038T_n + 95$, $T_0 = 2520$.

- (i) Determine the balance in the account after 7 deposits have been made. (1 mark)

Solution
$T_7 = \$3\,260.43$
Specific behaviours
✓ correct amount to the nearest cent

- (ii) After how many deposits does the balance of the account first exceed \$5 000 and what is the balance of the account at that time. (2 marks)

Solution
$T_n > 5000 \Rightarrow n = 23$
$T_{23} = \$5\,028.51$
Specific behaviours
✓ correct month; ✓ correct amount to the nearest cent

- (b) The investor also has \$385 in another account. One week later, and at the start of each subsequent week, a deposit of \$9.75 is added to the account. Interest, calculated as 0.072% of the balance at the start of the week, is added to the account just before each deposit is made.

- (i) Write a recurrence relation to model the balance of this account after n deposits. (3 marks)

Solution
$T_{n+1} = 1.00072T_n + 9.75$, $T_0 = 385$
Specific behaviours
✓ correct multiplier; ✓ correct addition; ✓ correct initial term

- (ii) Determine the balance in this account after 52 deposits have been made. (1 mark)

Solution
$T_{52} = \$916.10$
Specific behaviours
✓ correct amount

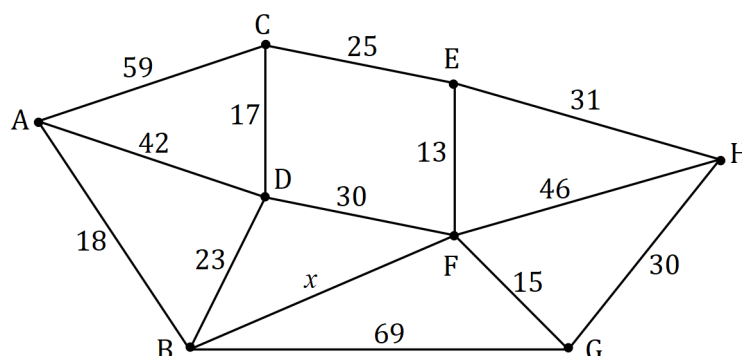
- (iii) By considering the total deposits made, or otherwise, determine the total interest added to this account after 52 deposits have been made. (1 mark)

Solution
Deposits = $52 \times 9.75 = 507.00$ Interest = $916.10 - 385 - 507 = \$24.10$
Specific behaviours
✓ correct amount

Question 21

(8 marks)

The vertices below represent 8 computers in a network and the weights on each edge represent the time, in milliseconds, for a signal to be sent directly between connected computers.



- (a) Given that $x = 55$, determine the path required and the time taken to send a signal in the least time between

- (i) B and G.

(2 marks)

Solution	
$B - D - F - G$ in $23 + 30 + 15 = 68$ ms	
Specific behaviours	
✓ correct path; ✓ correct time for stated path	

- (ii) A and F.

(2 marks)

Solution	
$A - B - D - F$ in $18 + 23 + 30 = 71$ ms	
Specific behaviours	
✓ correct path; ✓ correct time for stated path	

- (iii) A and H.

(2 marks)

Solution	
$A - B - D - C - E - H$ in $18 + 23 + 17 + 25 + 31 = 114$ ms	
Specific behaviours	
✓ correct path; ✓ correct time for stated path	

- (b) Determine the largest value of x , to the nearest millisecond, to ensure that the fastest route to send a signal between A and H will pass through F. Justify your answer.

(2 marks)

Solution	
$A - B - F - E - H$ in $18 + x + 13 + 31 = 62 + x$	
$62 + x < 114$ and so largest value of $x = 51$	
Specific behaviours	
✓ explanation	
✓ correct value of x	

Supplementary page

Question number: _____

Supplementary page

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