Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS

3A/3B

Calculator-free

WACE Examination 2012

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

When examiners design an examination, they develop provisional marking keys that can be reviewed at a marking key ratification meeting and modified as necessary in the light of candidate responses.

Section One: Calculator-free **(50 Marks)**

Question 1 **(8 marks)**

One hundred people were categorised on the basis of hair colour and gender as follows:

	Brunettes	**Blondes**	**Redheads**	**Total**
Men	26	***10***	4	40
Women	***32***	21	***7***	***60***
Total	58	***31***	***11***	***100***

(a) Complete the two-way table above. (2 marks)

Solution
See table above
Specific behaviours
✓ completes at least three (3) correct entries ✓ completes all entries correctly

(b) What is the probability that a person selected at random from the group is

(i) a blonde man? (1 mark)

Solution
$\frac{10}{100}=\frac{1}{10}$
Specific behaviours
✓ calculates correct probability

(ii) a redhead? (1 mark)

Solution
$\frac{11}{100}$
Specific behaviours
✓ calculates correct probability

(iii) a brunette or a woman? (2 marks)

Solution
$\dfrac{58+60-32}{100}=\dfrac{86}{100}=\dfrac{43}{50}$
Specific behaviours
✓ calculates the probability of being brunette or woman ✓ correctly calculates probability by subtracting P (brunette ∩ woman)

(iv) a man, given that their hair colour is brunette? (2 marks)

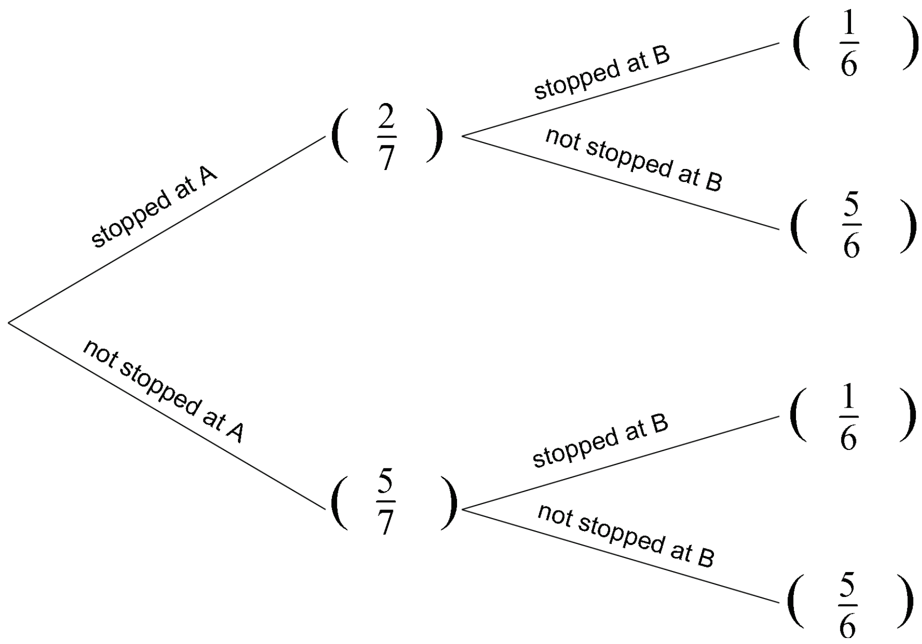
Solution
$\dfrac{26}{58}=\dfrac{13}{29}$
Specific behaviours
✓ calculates reduced sample space for conditional probability (58) ✓ determines correct probability

Question 2 **(7 marks)**

A teacher travelling to school has to pass through two sets of traffic lights A and B that operate independently of each other. The probabilities that he will be stopped at these lights are $\frac{2}{7}$ and $\frac{1}{6}$ respectively, with corresponding delays of 1 minute and 3 minutes.

Without these delays his journey takes 30 minutes.

(a) Complete the tree diagram, entering the appropriate probabilities in the given brackets. (2 marks)



Solution
See diagram above
Specific behaviours
✓ correctly completes first stage ✓ correctly completes second stage

(b) Determine the probability that

(i) the journey takes no more than 30 minutes. (1 mark)

Solution
$\frac{5}{7} \times \frac{5}{6} = \frac{25}{42}$
Specific behaviours
✓ calculates correct probability

(ii) the teacher encounters just one delay. (2 marks)

Solution
$\frac{2}{7} \times \frac{5}{6} + \frac{5}{7} \times \frac{1}{6} = \frac{15}{42}$
Specific behaviours
✓ calculates probability for stopping at A and not at B ✓ calculates probability for stopping at B and not at A

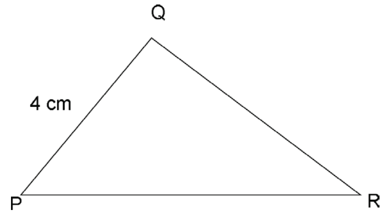
(c) One morning the teacher has only 32 minutes to reach school on time. Determine the probability that he will be late. (2 marks)

Solution
$\frac{2}{7} \times \frac{1}{6} + \frac{5}{7} \times \frac{1}{6} = \frac{1}{6}$ OR stopping at B is only way to be late, therefore = $\frac{1}{6}$
Specific behaviours
✓ calculates probability for stopping at A and stopping at B ✓ calculates probability for not stopping at A and stopping at B

Question 3 **(6 marks)**

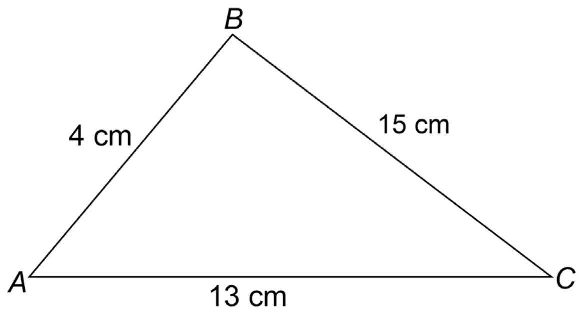
(a) In triangle PQR , the length of the side PQ is 4 cm, $\sin R = 0.4$ and $\sin Q = 0.3$.

Determine the exact length of the side PR. (3 marks)

Solution
$\dfrac{PR}{\sin Q} = \dfrac{4}{\sin R}$ $PR = \dfrac{4 \times 0.3}{0.4}$ $PR = \dfrac{1.2}{0.4} = 3$ cm 
Specific behaviours
✓ uses sine rule correctly ✓ rearranges for PR correctly ✓ calculates the value of PR correctly

(b) In triangle ABC, $a = 15$ cm, $b = 13$ cm and $c = 4$ cm.

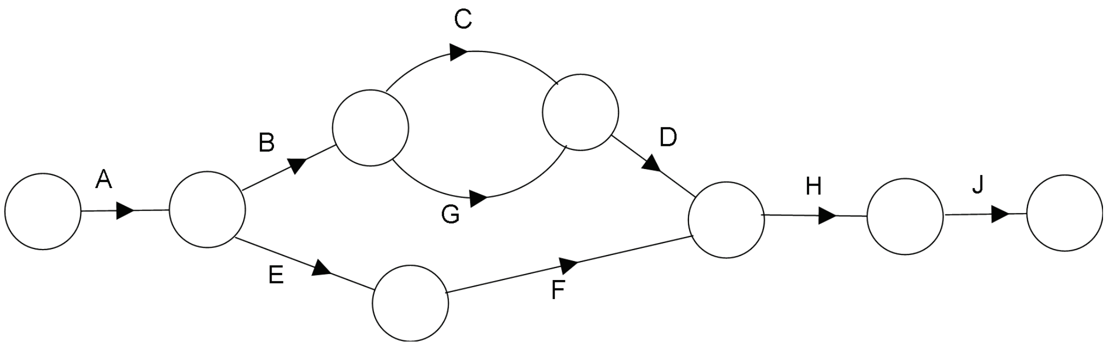
Given that the area of the triangle is 24 cm^2 determine the value of $\sin B$. (3 marks)

Solution
$A = \dfrac{1}{2} ac \sin B$ $24 = \dfrac{1}{2} 15 \times 4 \sin B$ $48 = 15 \times 4 \sin B$ $\dfrac{48}{60} = \sin B$ $\dfrac{12}{15} = \sin B$ 
Specific behaviours
✓ uses the formula for area of a triangle ✓ substitutes correctly into formula ✓ evaluates $\sin B$

Question 4 **(7 marks)**

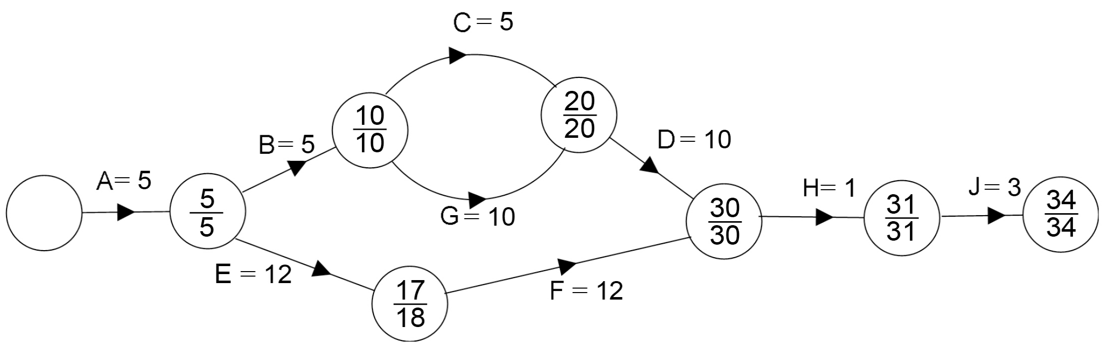
A small airline running a shuttle service between the city and a large rural town wishes to minimise the time taken to 'turn the aircraft around' (the turnaround time) at each destination in order to maximise the number of flights each day. The table below lists all activities that must be carried out each time the aircraft lands in order for it to be ready for the next take-off.

	Activity	Time (minutes)
A	Engage gate	5
B	Passengers disembark	5
C	Service cabin	5
D	Passengers board	10
E	Unload cargo	12
F	Load cargo	12
G	Service toilets	10
H	Disengage gate	1
J	Push aircraft from gate position	3



(a) By completing the project network, state the critical path and the minimum turnaround time. (3 marks)

Solution



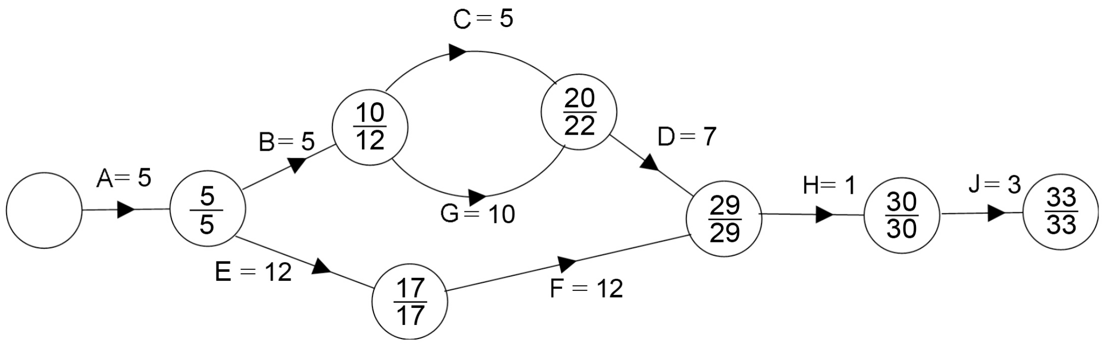
Critical path is ABGDHJ
Minimum turnaround time = 34 minutes

Specific behaviours

✓ completes project network
✓ states the correct critical path
✓ states the minimum turnaround time

(b) If passenger boarding time could be reduced by three minutes, what would the minimum turnaround time become? State the new critical path. (2 marks)

Solution



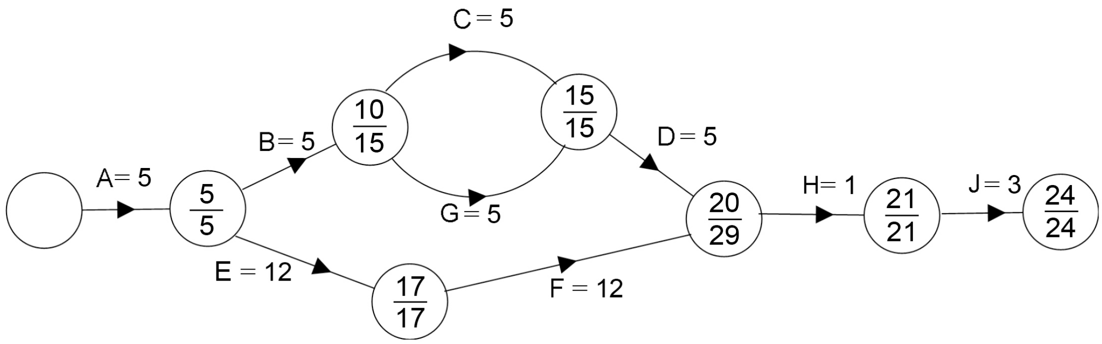
The new minimum turnaround time would be 33 minutes and critical path is AEFHJ

Specific behaviours

✓ states correct turnaround time
✓ correctly states the new critical path

(c) The original toilet servicing and passenger boarding times are each reduced to five minutes. How much time in total would need to be reduced from cargo loading and unloading before this change would make a difference to the minimum turnaround time? (2 marks)

Solution



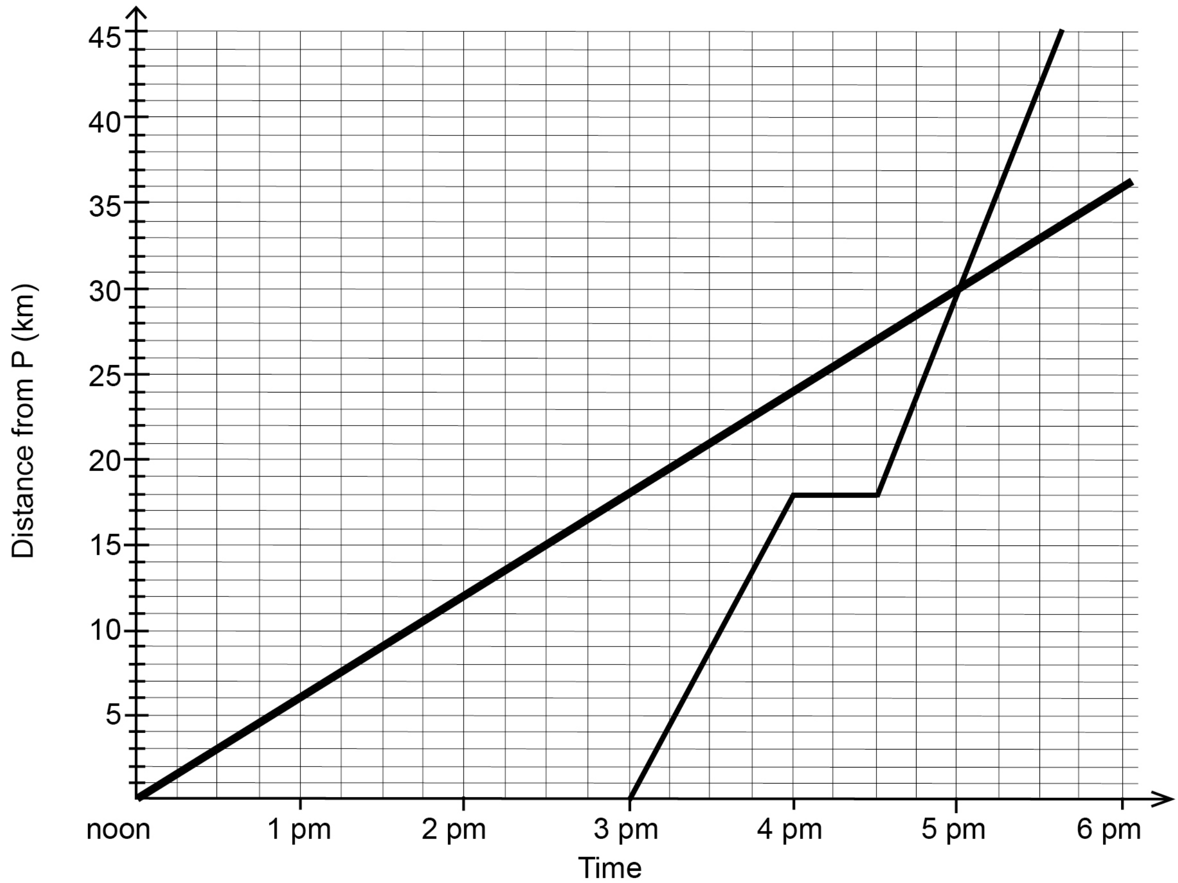
More than 9 minutes is needed

Specific behaviours

✓ states that the time needed is 9 minutes
✓ states that more than 9 minutes are needed to make the change

Question 5 **(5 marks)**

Starting at noon from a point P, a man A walks along a road at a steady 6 km h $^{-1}$. At 3 pm a cyclist B leaves point P and cycles at a steady 18 km h $^{-1}$ in pursuit of A. After travelling for an hour, B has a tyre puncture which delays him for half an hour. He then continues at 24 km h $^{-1}$.



Part of B's journey has been drawn on the graph above.

(a) Draw the travel graphs that represent the journey for both A and B. (2 marks)

Solution
See diagram above
Specific behaviours
✓ draws A's journey correctly ✓ completes B's journey correctly

(b) Determine the time when B catches up with A. (1 mark)

Solution
5 pm (from graph)
Specific behaviours
✓ correctly determines time

(c) Determine the time at which a motorist C travelling at a steady 60 km h $^{-1}$ must have left P if he overtakes A and B at the same moment that B catches up with A. (2 marks)

Solution
C is travelling at 60 km per hour, so would travel 30 km in half an hour. Thus C would have to leave P at 4.30 pm.
Specific behaviours
✓ correctly calculates distance travelled by C in half an hour ✓ states the correct time

Question 6 **(12 marks)**

(a) For the cubic function $y = 2x^3 + kx^2 + c$, where k and c are constants, $\frac{dy}{dx} = 4$ at the point $(-2,8)$.

By calculating the values of k and c, determine an equation for y in terms of x. (6 marks)

Solution
$y = 2x^3 + kx^2 + c$ $\therefore \frac{dy}{dx} = 6x^2 + 2kx \Rightarrow 4 = 6(-2)^2 + 2k(-2)$ $\therefore 4 = 24 - 4k \quad \Rightarrow k = 5$ Hence $y = 2x^3 + 5x^2 + c \Rightarrow 8 = 2(-2)^3 + 5(-2)^2 + c$ $8 = -16 + 20 + c \Rightarrow c = 4$ $\therefore \quad y = 2x^3 + 5x^2 + 4$
Specific behaviours
✓ correctly determines the first derivative ✓ correctly substitutes gradient and $x = -2$ into equation ✓ correctly solves equation for k ✓ correctly substitutes into cubic equation using the point $(-2, 8)$ ✓ correctly solves the equation for c ✓ states an equation for y in terms of x

(b) For another cubic function $y = 2x^3 - 3x^2 - 4$, determine the coordinates of the local maximum point. (6 marks)

Solution
$\therefore \dfrac{dy}{dx} = 6x^2 - 6x = 0$ when $6x(x-1) = 0$ $\therefore x = 0 \quad or \quad 1$ Since y gets large and negative as x gets large and negative, the maximum turning point must lie to the left of the minimum turning point, therefore, $x = 0$ must be the x-coordinate of the maximum turning point. The coordinates of the maximum turning point are $(0, -4)$
Specific behaviours
✓ correctly determines the first derivative ✓ equates the first derivative to zero ✓ correctly factorises the equation ✓ correctly solves equation for both values of x ✓ correctly determines which value of x would give a maximum ✓ correctly states the coordinates of the maximum turning point

Question 7 **(5 marks)**

(a) A recursive sequence is defined by $T_n = T_{n-1} + T_{n-2} + 2$ with $T_1 = 2$ and $T_2 = -2$.

(i) Determine the values of T_3 and T_4 . (2 marks)

(ii) Paul states: *'The sequence will result in a positive two (2) when n is an odd number and will result in a negative two (–2) when n is an even number'.*

Comment on Paul's conjecture. (1 mark)

Solution
(i) $T_1 = 2$ $T_2 = -2$ $T_3 = -2 + 2 + 2 = 2$ $T_4 = 2 + (-2) + 2 = 2$
(ii) This conjecture is false as when $n = 4$ the value obtained is positive
Specific behaviours
✓ calculates T_3 correctly ✓ calculates T_4 correctly ✓ states that the conjecture is false

(b) Pauline states: *'If a, b and c are real numbers such that $(a+b)^2 = c^2$ then $a+b = c$'*.
Provide a counter example to show that Pauline's conjecture is false. (2 marks)

Solution
One possible counter example is $a=-1,\ b=-1,\ c=2$ If $a=-1,\ b=-1$ and $c=2$ Then $(a+b)^2 = ((-1)+(-1))^2 = (-2)^2 = 4$ But $a+b=(-1)+(-1)=-2$ $\neq c\ (=2)$ So conjecture is false.
Specific behaviours
✓ determines values for a, b and c that make $(a+b)^2 = c^2$ true ✓ determines values for a, b and c that make $(a+b) = -c$