Exam Answers
Chapter 2·4 – Kepler's Law
Answer 1 2010:2:21

(9 marks)

(a) The radius of the orbit of Venus around the Sun is 1.08×10^{11} m.

(i) Derive an expression that relates the orbital period of Venus to the orbital radius of Venus and the mass of the Sun. (3 marks)

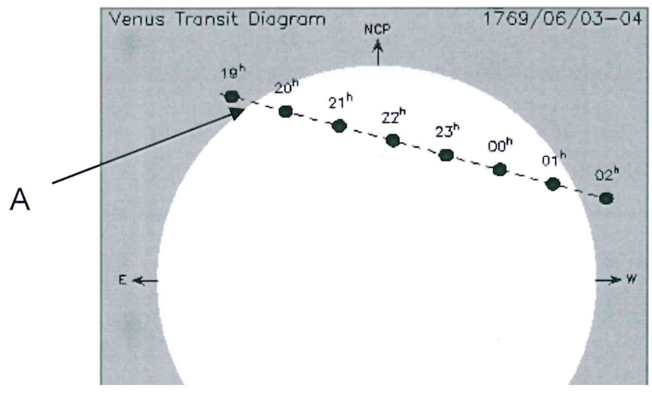
Description	Marks
$F_c = F_g$ $\frac{M_{Venus} \cdot v^2}{r} = G\frac{M_{Sun} \times M_{Venus}}{r^2}$ Where $v = \frac{s}{t} = \frac{2\pi r}{T}$ So $v^2 = \frac{4\pi^2 r^2}{T^2}$ $\frac{M_{Venus} \cdot \frac{4\pi^2 r^2}{T^2}}{r} = G\frac{M_{Sun} \times M_{Venus}}{r^2}$ $\frac{4\pi^2 r^2}{T^2} = G\frac{M_{Sun}}{r}$ $\mathbf{T^2 = \frac{4\pi^2 r^3}{GM_{Sun}}}$ Must derive Kepler's 3rd Law No marks for stating it.	1-3
	Total 3

(ii) Calculate the time in Earth days for Venus to orbit the Sun. (3 marks)

Description	Marks
$T^2 = \frac{4\pi^2 r^3}{GM_{Sun}}$ $T^2 = \frac{4\pi^2 (1.08 \times 10^{11})^3}{(6.67 \times 10^{-11})(1.99 \times 10^{30})}$	1
$T^2 = \frac{4.973 \times 10^{34}}{1.327 \times 10^{20}}$ $T = \sqrt{\frac{4.973}{1.327}} \times 10^7 s$	
$= 1.936 \times 10^7$ s	1
= 224 days	1
	Total 3

The passage of the planet Venus between the Earth and the Sun is a predictable regular occurrence. It is known as the 'transit of Venus'. Captain Cook sailed to Tahiti to measure the time Venus took to cross the Sun's surface.

A is the point at which Venus appears to intersect with the surface of the Sun. This occurs at different times for observers at different positions on the Earth's surface.

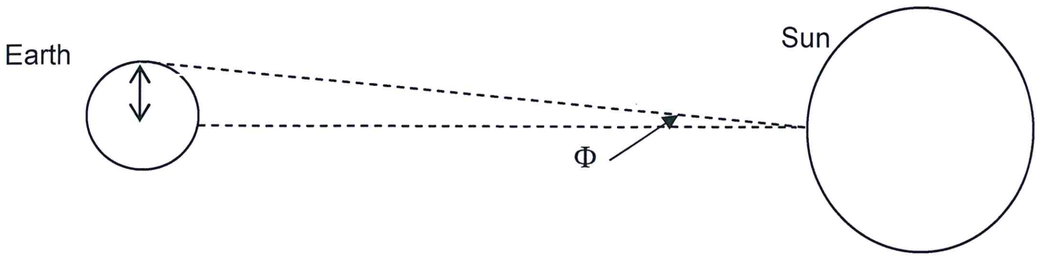


Exam Answers

Chapter 2.4 - Kepler's Law

Answer 1: continued

(b) By measuring the time difference between the occurrence of A at different locations on Earth (Tahiti and California) astronomers were able to measure the solar parallax angle Φ as shown in the diagram below, which is not to scale. In this way the distance from the Earth to the Sun was calculated in 1769 with amazing accuracy.



Calculate the Earth – Sun distance in kilometres if the solar parallax angle $\Phi = 0.00250°$. (3 marks)

Description	Marks
Tan $\phi = \frac{R_E}{R_{E-S}} = \frac{6.38 \times 10^6}{r_{E-S}}$	1
So $r_{E-S} = \frac{6.38 \times 10^6}{\tan 0.0025}$ = 146 million km (1.46×10^{11} m)	1-2
	Total 3

Answer 2 2011:2:17

(12 marks)

The planet Jupiter has a mass of 1.90×10^{27} kg, a radius of 71 500 km and many moons.

The closest moon, Metis, has a mass of 9.56×10^{16} kg and a mean orbital radius of 1.28×10^5 km. Metis has an average planetary radius of 21.5 km.

(a) Calculate the gravitational force of attraction between Jupiter and Metis. (3 marks)

Description	Marks
$F_g = \frac{GM_J m_M}{r^2}$	1
$F_g = \frac{6.67 \times 10^{-11} \times 1.90 \times 10^{27} \times 9.56 \times 10^{16}}{(1.28 \times 10^8)^2}$	1
F = 7.39×10^{17} N	1
	Total 3

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Chapter 2·4 Kepler's Law
Answer 2: continued

(b) Calculate the time it takes in hours for Metis to orbit around Jupiter. (4 marks)

Description	Marks
$F_c = \frac{mv^2}{r}$ and v =2πr/t Substitute for v in F_c $F_c = \frac{m\left(\frac{2\pi r}{t}\right)^2}{r} = \frac{4\pi^2 mr}{t^2}$	1
Rearrange for t and substitute values in $t = \sqrt{\frac{4\pi^2 mr}{F_c}} = \sqrt{\frac{4\pi^2 \times 9.56\times10^{16} \times 1.28\times10^8}{7.39\times10^{17}}}$ Note: if Kepler's Law recalled correctly and all values are shown correctly but calculated answer is incorrect, 3 marks out of 4.	1
t = 25568 seconds or 2.56 × 10^4 s	1
Convert to hours 25568 s /3600 = 7.10 hours	1
	Total 4

(c) Calculate the magnitude and direction of the net gravitational force acting on a 1.00 kg mass resting on the surface of Metis that faces Jupiter. (5 marks)

Description	Marks
(1 mark for two forces) $F_{gnet} = \frac{GM_J 1kg}{r_J^{\ 2}} - \frac{GM_M 1kg}{r_M^{\ 2}}$ where towards Jupiter is positive	1
(1 mark for numbers in) $F_{gnet} = \frac{6.67\times10^{-11} \times 1.90\times10^{27} \times 1}{(1.28\times10^8 - 21.5\times10^3)^2} - \frac{6.67\times10^{-11} \times 9.56\times10^{16} \times 1}{(21.5\times10^3)^2}$	1
(1 mark for realising opposite directions) $F_{g\ net}$ = 7.736-0.0138	1
(1 mark for answer with units) $F_{g\ net}$ = 7.72 N	1
(1 mark for direction given somewhere) Towards Jupiter	1
	Total 5

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Answer 3: 2014:1:9

(5 marks)

Use the information given in the Formulae and Data Booklet to calculate the orbital period, in seconds, of the Moon around the Earth.

Description	Marks
$F_c = F_g$ so $v^2 / r = G M_E / r^2$ $T^2 = 4 \pi^2 r^3 / G M_E$	1–2
Uses correct values M_M = ... 7.35×10^{22} kg M_E = ... 5.97×10^{24} kg Mean Earth-Moon distance.................... $r = 3.84 \times 10^8$ m Universal gravitational constant.............. $G = 6.67 \times 10^{-11}$ N m² kg⁻²	1
$T^2 = 4 \times \pi^2 \times (3.84 \times 10^8)^3 / (6.67 \times 10^{-11} \times 5.97 \times 10^{24})$	1
$T = 2\ 369\ 335$ s $= 2.37 \times 10^6$ s	1
Total	**5**