Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 1

Section One: Calculator-free

SOLUTIONS

Your name _____

Teacher's name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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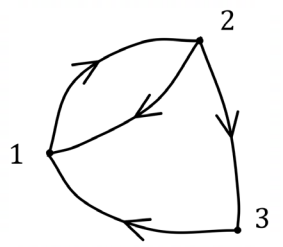
Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

Three people who share a computer network have each forgotten their own password but can remember at least one other person's password. In the following diagram, the directed lines indicate that the person at the start of the line remembers the password of the person at the end of the line. For example, person 3 remembers the password of person 1.



(a) Create a square matrix M where the entry $M_{i,j}$ is 0 if person i does not remember the password of person j, and is 1 if they do. (3 marks)

Solution
$M = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
Specific behaviours
✓ 3×3 matrix with just zeros and ones ✓ leading diagonal all zeros ✓ correct matrix

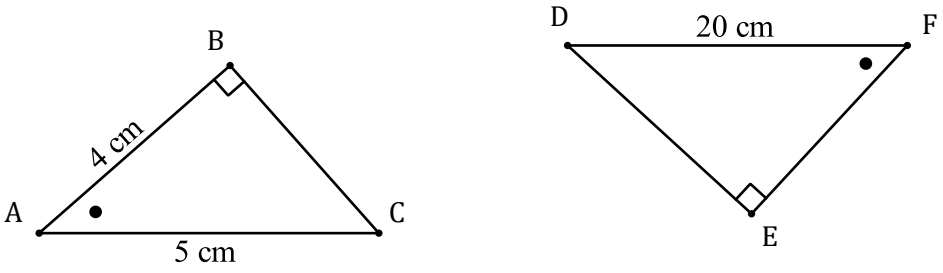
(b) Determine M^2. (2 marks)

Solution
$M^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ Or fills matrix using two-step knowledge of password. $M^2 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
Specific behaviours
✓ indicates appropriate method ✓ correct matrix

See next page

Question 2 **(5 marks)**

The diagram below, not drawn to scale, shows two similar right triangles.



(a) Calculate the scale factor for the larger triangle relative to the smaller. (1 mark)

Solution
$20 \div 5 = 4$ The scale factor is 4.
Specific behaviours
✓ correct scale factor

(b) Determine the length of side BC, the length of side DE and the length of side EF. (3 marks)

Solution
$BC = \sqrt{5^2 - 4^2} = 3 \text{ cm}$ $DE = 3 \times 4 = 12 \text{ cm}$ $EF = 4 \times 4 = 16 \text{ cm}$
Specific behaviours
✓ calculates BC ✓ uses scale factor to calculate DE ✓ uses scale factor to calculate DF

(c) Calculate how many times greater the area of the large triangle is compared to the area of the small triangle. (1 mark)

Solution
$4^2 = 16$ The area is 16 times larger.
Specific behaviours
✓ squares linear scale factor

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Question 3 **(5 marks)**

(a) Describe the type (using row, column, square, zero or identity) of each of the following matrices:

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$. (1 mark)

Solution
Zero
Specific behaviours
✓ correct type

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. (1 mark)

Solution
Square
Specific behaviours
✓ correct type

(iii) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$. (1 mark)

Solution
Column
Specific behaviours
✓ correct type

(b) State the size of the matrix $[1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0]$ in the form $m \times n$. (1 mark)

Solution
1×6
Specific behaviours
✓ correct size

(c) Write the 3×3 identity matrix I. (1 mark)

Solution
$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Specific behaviours
✓ correct matrix

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Question 4 **(5 marks)**

(a) Calculate the value of $p + q \times r$ when $p = 4, q = 6$ and $r = 2$. (1 mark)

Solution
$4 + 6 \times 2 = 4 + 12 = 16$
Specific behaviours
✓ correct value

(b) Use the formula $y = (x + 1)(x - 2)$ to calculate y when $x = 1.5$. (2 marks)

Solution
$y = (2.5)(-0.5)$ $= -1.25$
Specific behaviours
✓ evaluates terms in brackets ✓ correct value

(c) Use the formula $s = \frac{1}{2}at^2 - ut$ to calculate s when $a = 5.6, t = 10$ and $u = 12.5$. (2 marks)

Solution
$s = \frac{1}{2} \times 5.6 \times 10^2 - 12.5 \times 10$ $= 2.8 \times 100 - 125$ $= 280 - 125$ $= 155$
Specific behaviours
✓ substitutes and starts to simplify ✓ correct value

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Question 5 **(7 marks)**

A young person has drawn up a budget for their weekly income of $\$800$. Some of their income is allocated to rent, living expenses, loan repayments, phone, and the rest to savings.

	A	B
1	Weekly budget	
2	Rent	$\$235.00$
3	Debt repayments	
4	Living expenses	$\$290.00$
5	Phone	$\$65.00$
6	Savings	
7	**Total**	$\mathbf{\$800.00}$

The young person allocates 15% of their weekly income to debt repayments.

(a) Determine the budget allocation for debt repayments. (2 marks)

Solution

$10\% = 80 \rightarrow 5\% = 40 \rightarrow 15\% = 120$

Allocation is $\$120$ per week.

Specific behaviours

✓ indicates use of an appropriate method
✓ correct allocation

(b) The young person has been saving for a holiday and needs to save another $\$720$ to make the booking. If the above budget is maintained, determine the number of weeks it will take them to save this amount. (3 marks)

Solution

$235 + 120 + 290 + 65 = 710$

Weekly savings: $800 - 710 = \$90$

Number of weeks will be $\frac{720}{90} = \frac{72}{9} = 8$.

Will take 8 weeks to save the amount.

Specific behaviours

✓ indicates weekly savings
✓ indicates correct method for time to save
✓ correct number of weeks

(c) If the budget allocations for rent and phone increase by $\$7$ and $\$3$ respectively, what effect will this have on your answer to part (b)? (2 marks)

Solution

Will save $7 + 3 = \$10$ less, so $\$80$ per week.
Number of weeks will be $720 \div 8 = 9$ weeks.
Will change the time to 9 weeks (or increase the time by 1 week).

Specific behaviours

✓ indicates correct savings figure
✓ correctly describes effect on previous answer

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Question 6 **(10 marks)**

One weekend, a company operated three boats to run harbour trips.

On Saturday, boats D, E and F carried $48, 63$ and 52 adults, and $28, 42$ and 22 children, respectively.

(a) Represent this information in a 2×3 matrix X, with adults in the top row, and boats in alphabetical order. (1 mark)

Solution
$X = \begin{bmatrix} 48 & 63 & 52 \\ 28 & 42 & 22 \end{bmatrix}$
Specific behaviours
✓ correct matrix

On Sunday, a total of 82 adults and 88 children took trips. Boats D and E carried an equal number of adults and boat E carried 18 children. Compared to Saturday, boat F took 34 fewer adults and 6 more children.

(b) Represent the Sunday passenger numbers in matrix Y, using the same row and column labels as matrix X. (3 marks)

Solution
Boat F: $52 - 34 = 18$ adults and $22 + 6 = 28$ children. Children in boat D: $88 - 18 - 28 = 42$. Split remaining $82 - 18 = 64$ adults, $64 \div 2 = 32$ in boats D and E. $Y = \begin{bmatrix} 32 & 32 & 18 \\ 42 & 18 & 28 \end{bmatrix}$
Specific behaviours
✓ column D ✓ column E ✓ column F

(c) Calculate matrix T, where $T = X + Y$ and explain what information it represents. (2 marks)

Solution
$T = \begin{bmatrix} 48 & 63 & 52 \\ 28 & 42 & 22 \end{bmatrix} + \begin{bmatrix} 32 & 32 & 18 \\ 42 & 18 & 28 \end{bmatrix} = \begin{bmatrix} 80 & 95 & 70 \\ 70 & 60 & 50 \end{bmatrix}$ T shows the number of adults and children carried by boats D, E and F over the weekend.
Specific behaviours
✓ correct matrix T ✓ correct explanation

The price per trip was $\$10$ per adult and $\$5$ per child.

(d) Represent the price information in matrix P that can be meaningfully multiplied by matrix T, calculate PT and explain what information it represents. (4 marks)

Solution
$P = \begin{bmatrix} 10 & 5 \end{bmatrix}$ $PT = \begin{bmatrix} 10 & 5 \end{bmatrix} \begin{bmatrix} 80 & 95 & 70 \\ 70 & 60 & 50 \end{bmatrix} = \begin{bmatrix} 1150 & 1250 & 950 \end{bmatrix}$ Matrix PT shows the total passenger fares collected over the weekend by boats D, E and F.
Specific behaviours
✓ correct matrix P ✓ one correct entry in PT ✓ correct product ✓ correct explanation

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Question 7 **(8 marks)**

Consider the following matrices:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 5 \\ 5 & -3 \end{bmatrix} \quad C = \begin{bmatrix} -2 & 1 \\ 0 & 2 \end{bmatrix} \quad D = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad E = \begin{bmatrix} 0 & 2 \end{bmatrix} \quad F = \begin{bmatrix} 2 & -1 \end{bmatrix}.$$

(a) Calculate, where possible, the following. If not possible, give a reason why.

(i) $A + C$. (1 mark)

Solution
$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}$
Specific behaviours
✓ calculates result

(ii) $D \times B$. (1 mark)

Solution
Not possible, as number of columns in D (1) is not the same as the number of rows in B (2).
Specific behaviours
✓ explanation using matrix dimensions

(iii) $9F - 7E$. (2 marks)

Solution
$9[2 \quad -1] - 7[0 \quad 2] = [18 \quad -9] - [0 \quad 14] = [18 \quad -23]$
Specific behaviours
✓ calculates multiples ✓ calculates difference

(iv) $B \times C$. (2 marks)

Solution
$B \times C = \begin{bmatrix} 4 & 5 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -8 & 14 \\ -10 & -1 \end{bmatrix}$
Specific behaviours
✓ at least two elements correct ✓ correct result

(b) Calculate A^6. (2 marks)

Solution
$A^2 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$ $A^3 = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 1 \end{bmatrix}$ Pattern for $A_{1,1}$ continues $2, 4, 8, 16, 32, 64, \ldots$ $A^6 = \begin{bmatrix} 64 & 0 \\ 0 & 1 \end{bmatrix}$
Specific behaviours
✓ calculates A^2 ✓ correct A^6

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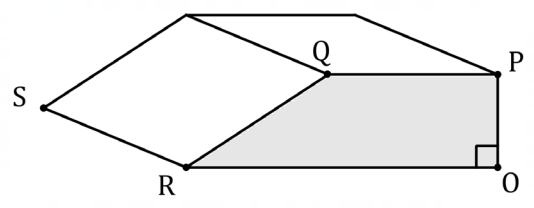
Question 8 **(7 marks)**

The diagram below, not drawn to scale, shows a prism with trapezoidal cross-section $OPQR$, where $OP = 5$ cm, $PQ = 8$ cm, $QR = 13$ cm, $OR = 20$ cm and $RS = 10$ cm.



Solution
$A = \frac{1}{2} \times (8 + 20) \times 5$ $= 14 \times 5 = 70 \text{ cm}^2$
Specific behaviours
✓ indicates appropriate method ✓ calculates area

(b) Calculate the volume of the prism. (2 marks)

Solution
$V = 70 \times 10$ $= 700 \text{ cm}^3$
Specific behaviours
✓ correctly uses answer from (a) ✓ calculates volume

(c) Determine the total surface area of the prism. (3 marks)

Solution
Top, bottom, back and sloping face: $A = 10 \times (5 + 8 + 13 + 20)$ $= 10 \times 46 = 460$ Two ends: $A = 2 \times 70 = 140$ Total: $A = 460 + 140 = 600 \text{ cm}^2$
Specific behaviours
✓ indicates appropriate method for four rectangles ✓ includes both trapezoidal faces ✓ calculates total surface area

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____