**MATHEMATICS
METHODS
UNIT 1**
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction
fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that
you do not have any unauthorised notes or other items of a non-personal nature in the
examination room. If you have any unauthorised material with you, hand it to the supervisor
before reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(7 marks)**

Four points have coordinates A(-1, 2), B(4, 3), C(5, 1) and D(1, -2).

(a) Determine the coordinates of the mid-point of A and B. (1 mark)

> (1.5, 2.5)

(b) Determine the coordinates of the y-intercept of the line passing through A and D. (1 mark)

> (0, 0)

(c) Determine the gradient of the line passing through A and D. (1 mark)

> $$m = \frac{2-(-2)}{-1-(1)} = \frac{4}{-2} = -2$$

(d) Determine the equation of the line passing through B that is parallel to the line $y = 3x - 1$. (2 marks)

> $$y - 3 = 3(x - 4)$$
> $$y = 3x - 9$$

(e) The equation of the line through BC is $y = -2x + 11$. Determine the equation of the line passing through A that is perpendicular to this line. (2 marks)

> $$y - 2 = \tfrac{1}{2}(x - (-1))$$
> $$y = \tfrac{1}{2}x + \tfrac{5}{2}$$

Question 2   **(8 marks)**

(a)   Find $f(-1)$ if $f(x) = 2(x-1)(x-2)^2$.   (1 mark)

$$f(-1) = 2(-2)(-3)^2 = -36$$

(b)   Solve the following equations

      (i) $(x-3)(2x+3) = 0$.   (1 mark)

$$x = 3, \ x = -\tfrac{3}{2}$$

      (ii) $3x^2 = 12x + 15$.   (2 marks)

$$x^2 - 4x - 5 = 0$$
$$(x+1)(x-5) = 0$$
$$x = -1, \ x = 5$$

      (iii) $16x^2 - 9 = 0$.   (2 marks)

$$(4x+3)(4x-3) = 0$$
$$x = \pm\tfrac{3}{4}$$

(c)   Determine the coordinates of the turning point of $y = x^2 + 6x - 2$.   (2 marks)

$$y = (x+3)^2 - 11$$

$$(-3, -11)$$

Question 3 **(3 marks)**

(a) Solve $\sin x = -\frac{1}{2}$ for $0 \le x \le 360°$. (1 mark)

$$210°, \ 330°$$

(b) Show that the exact value of $\sin\left(\dfrac{7\pi}{12}\right)$ is $\dfrac{\sqrt{2}\left(\sqrt{3}+1\right)}{4}$. (2 marks)
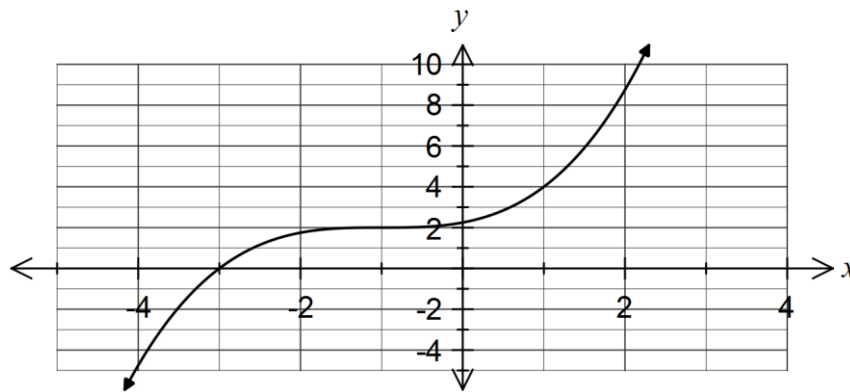
$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$$

$$= \sin\left(\frac{\pi}{3}\right) \cdot \cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{3}\right) \cdot \sin\left(\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} + \frac{1}{2} \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}\left(\sqrt{3}+1\right)}{4}$$

Question 4 **(11 marks)**

(a) The graph below shows the function $y = a(x-b)^3 + c$.

Determine the values of a, b and c. (3 marks)

$$a = \tfrac{1}{4}, \; b = -1, \; c = 2$$

(b) Expand $(2x-1)(x+1)^2$. (2 marks)

$$(2x-1)(x^2 + 2x + 1) = 2x^3 + 3x^2 - 1$$

(c) Solve

(i) $(3x+2)(2x-1)(x+5) = 0$. (1 mark)

$$x = -\tfrac{2}{3}, \; x = \tfrac{1}{2}, \; x = -5$$

(ii) $(x^2 + 2x - 15)(2x+5) = 0$. (2 marks)

$$(x+5)(x-3)(2x+5) = 0$$
$$x = -5, \; x = 3, \; x = -\tfrac{5}{2}$$

(iii) $x^3 + 4x^2 - 7x - 10 = 0$. (3 marks)
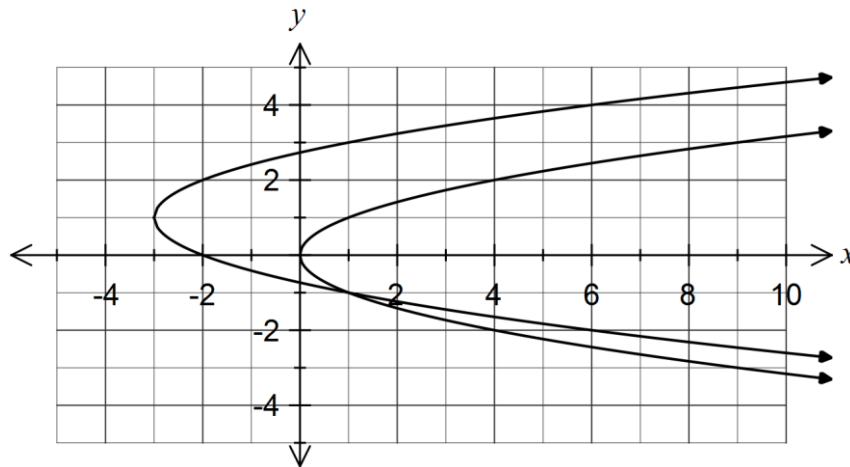
$$f(-1) = 0 \;\Rightarrow\; (x+1)(x^2 + 3x - 10) = 0$$
$$(x+1)(x-2)(x+5) = 0$$
$$x = -5, \; x = -1, \; x = 2$$

See next page

Question 5 **(6 marks)**

(a) The graph of $y^2 = x$ is shown below.

(i) Explain how the graph shows that y is not a function of x. (1 mark)

A vertical line drawn for $x > 0$ cuts the graph in more than one place – hence not a function.

(ii) Add the graph of $(y-1)^2 = x+3$ to the axes above. (2 marks)

(b) Determine the radius and centre of the circle given by $x^2 + y^2 = 14y - 12x - 76$. (3 marks)

$$x^2 + 12x + y^2 - 14y = -76$$
$$(x+6)^2 + (y-7)^2 = -76 + 6^2 + 7^2$$
$$= 9$$
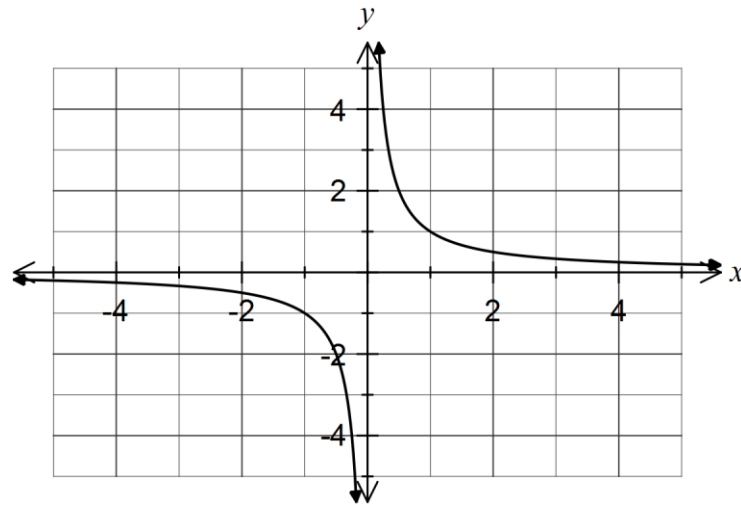$$= 3^2$$

Centre at (-6, 7) and radius 3 units.

See next page

Question 6 **(4 marks)**

The graph of the function $g(x) = \dfrac{1}{x}$ is shown below.

(a) State the domain of $g(x-2)$. (1 mark)

$$\{x : x \in \mathbb{R}, \ x \neq 2\}$$

(b) State the range of $g(0.5x)$. (1 mark)

$$\{y : y \in \mathbb{R}, \ y \neq 0\}$$

(c) The graph of $y = g(x)$ is dilated by a scale factor of $\frac{1}{2}$ parallel to the x-axis. Determine the equation of the transformed graph. (1 mark)
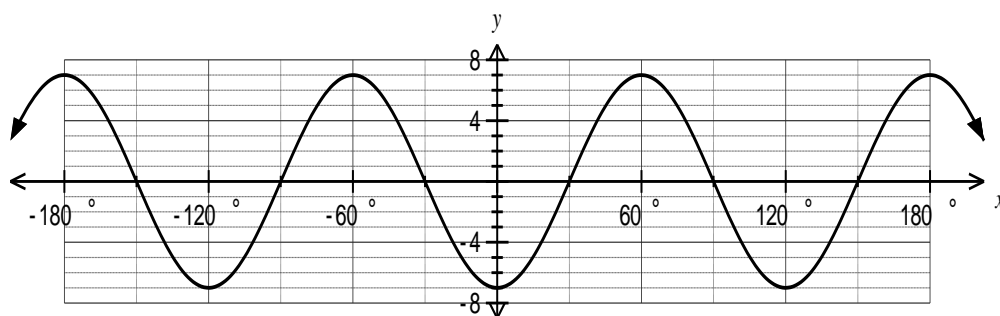
$$y = g(2x) = \frac{1}{2x}$$

(d) Comment on the behaviour of $g(x) - 1$ as $x \to \infty$. (1 mark)

$$y \to -1$$

See next page

Question 7 **(7 marks)**

(a)   The graph of the trigonometric function $y = f(x)$ is shown.

(i)   State the amplitude of $f(x)$.   (1 mark)

7

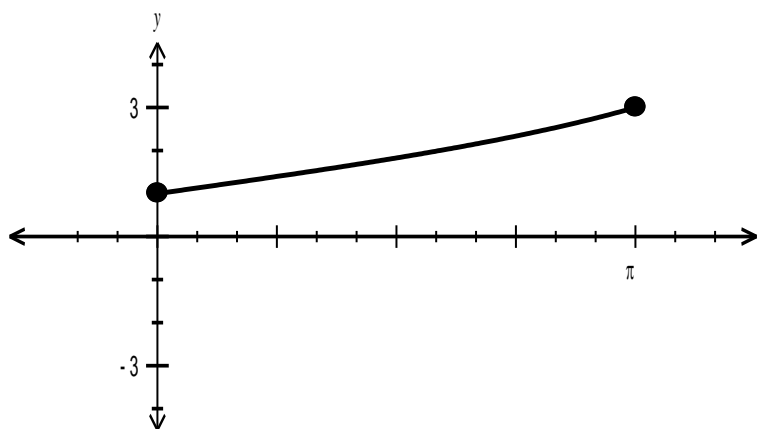(ii)   State the period of $f(x)$.   (1 mark)

120°

(iii)   State the equation of the trigonometric function $f(x)$.   (2 marks)

$$y = -7\cos(3x)$$

(b)   On the axes below, sketch the graph of $y = 1 + 2\tan\left(\dfrac{x}{4}\right)$ for $0 \le x \le \pi$.   (3 marks)

See next page

Question 8 **(6 marks)**

(a)   Expand $(x-1)^3$. (2 marks)

$$(x-1)^3 = 1(x)^3 + 3(x)^2(-1) + 3(x)(-1)^2 + 1(-1)^3$$
$$= x^3 - 3x^2 + 3x - 1$$

(b)   Determine the coefficient of the x^3 term in the expansion of $(2x+3)^6$. (4 marks)

$$(2x+3)^6 = \ldots + 20(2x)^3(3)^3 + \ldots$$

$$20 \times 8 \times 27 = 20 \times 216$$
$$= 4320$$

End of questions

Additional working space

Question number: _____