Semester Two Examination, 2018

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3 AND 4

Section One: Calculator-free

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

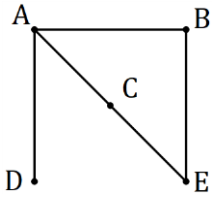
Section One: Calculator-free **35% (51 Marks)**

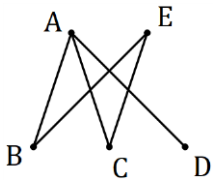
This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(7 marks)**

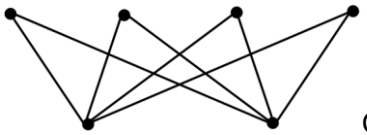
(a) The following bipartite graph shows the subjects studied by three students. Redraw the graph to clearly show the two distinct sets of vertices and hence state which vertices represent the subjects studied. (3 marks)



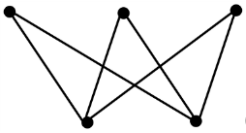
Solution
 Subjects are represented by A and E.
Specific behaviours
✓ clearly shows the two sets of vertices ✓ correct edges ✓ correctly identifies subject vertices

(b) The complete bipartite graph denoted by $K_{p,q}$ has p vertices in one set and q vertices in the other set.

(i) Draw $K_{4,2}$ and state whether the graph is Eulerian, semi-Eulerian or neither. (2 marks)

Solution
 Graph is Eulerian
Specific behaviours
✓ correct graph ✓ correct identification

(ii) Draw $K_{3,2}$ and state whether the graph is Hamiltonian, semi-Hamiltonian or neither. (2 marks)

Solution
 Graph is semi-Hamiltonian
Specific behaviours
✓ correct graph ✓ correct identification

See next page

Question 2 **(9 marks)**

(a) The first two terms of an arithmetic sequence are displayed in the table below.

n	1	2	3	4
T_n	325	352	379	406

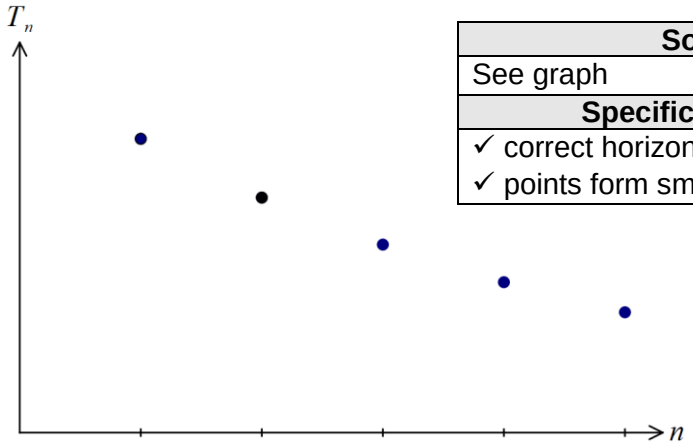
(i) Add the next two terms to the table. (2 marks)

Solution
See table
Specific behaviours
✓ correct difference
✓ both values consistent with difference

(ii) Deduce a rule for the n^{th} term of this sequence. (1 mark)

Solution
$T_n = 325 + (n-1)(27)$
Specific behaviours
✓ correct rule

(b) The first two terms of a geometric sequence are displayed in graphical form below. Plot the likely position of the next three terms of the sequence on the graph. (2 marks)



Solution
See graph
Specific behaviours
✓ correct horizontal spacing
✓ points form smooth, upwards curve

(c) A first-order linear recurrence relation is defined by $T_{n+1} = 0.5T_n + 6,\ T_1 = 30$.

(i) Determine the second and third terms of the sequence. (2 marks)

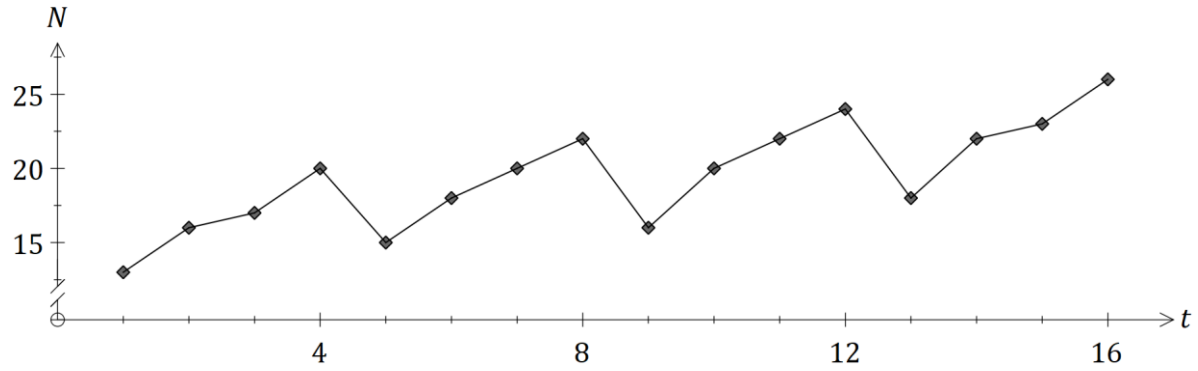
Solution
$T_2 = 0.5(30) + 6 = 21$ $T_3 = 0.5(21) + 6 = 16.5$
Specific behaviours
✓ second term
✓ third term

(ii) In the long-term, the terms of the sequence become very close to k. Determine, with justification, the value of k. (2 marks)

Solution
$k = 0.5k + 6$ $0.5k = 6 \Rightarrow k = 12$
Specific behaviours
✓ forms correct equation using k
✓ correct value of k

Question 3 **(7 marks)**

The graph below shows a time series plot, where N is the number of breakdowns of a printing machine that are recorded during month t.



(a) Describe the seasonality and trend of the time series. (2 marks)

Solution
Cycles of 4 months are evidenced by an increase followed by a decrease. As time goes on, an underlying increasing trend is apparent.
Specific behaviours
✓ indicates seasonality by referring to 4-month cycles ✓ indicates increasing trend

(b) Some of the data is given in the table below.

t	11	12	13	14	15	16
N	22	24	18	22	23	26

(i) Calculate the 4-point centred moving average for $t = 14$. (2 marks)

Solution
$\left(\frac{1}{2} \times 24 + 18 + 22 + 23 + \frac{1}{2} \times 26\right) \div 4 = 88 \div 4 = 22$
Specific behaviours
✓ indicates appropriate method for centring ✓ correct average

(ii) Explain the purpose of calculating simple moving averages for a time series. (2 marks)

Solution
To smooth the data and hence expose the underlying trend.
Specific behaviours
✓ smoothing ✓ exposing trend

(iii) Explain the purpose of centring a 4-point moving average. (1 mark)

Solution
To align the moving average with time.
Specific behaviours
✓ valid reason

See next page

Question 4 **(8 marks)**

(a) Comment, with reasons, on the claim that the graph shown below is simple, planar and satisfies Euler's formula. (3 marks)

Solution
Not simple - contains a loop Is planar - can be drawn with no edges crossing Doesn't satisfy Euler's formula, as it is not connected or $f(2) + v(4) - 2 \neq e(3)$
Specific behaviours
✓ reason for not simple ✓ reason for being planar ✓ reason for not satisfying Euler's

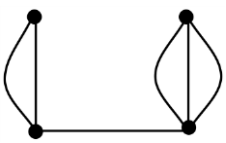
(b) A graph has 6 vertices and 8 edges. Determine the sum of the degrees of the vertices. (1 mark)

Solution
$\text{sum} = 2 \times 8 = 16$
Specific behaviours
✓ correct sum

(c) Draw a semi-Eulerian graph that has 4 vertices and is a tree. (2 marks)

Solution
Example: 
Specific behaviours
✓ semi-Eulerian ✓ tree with 4 vertices

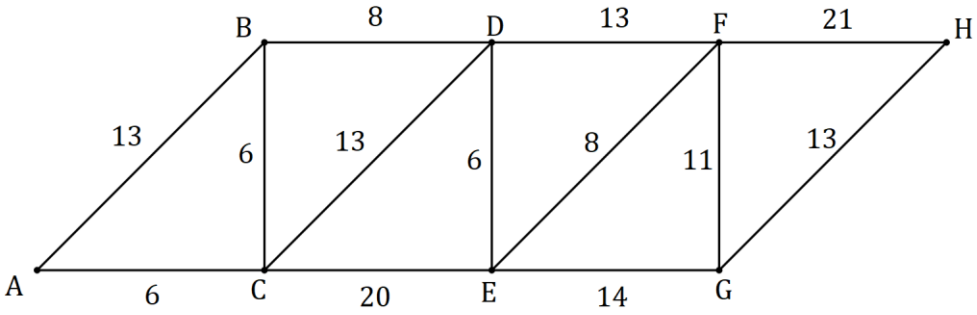
(d) Draw a connected planar graph that has 4 vertices, 4 faces and 1 bridge. (2 marks)

Solution
Example: 
Specific behaviours
✓ connected, 1 bridge ✓ 4 vertices, 4 faces

See next page

Question 5 **(8 marks)**

The vertices in the graph below represent city landmarks and the weights on the edges are the times, in minutes, to travel between adjacent landmarks.



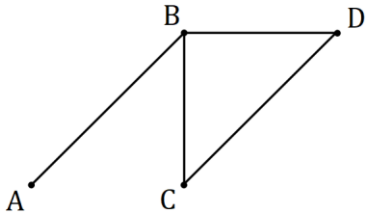
(a) Determine the shortest time to travel from A to H. (3 marks)

Solution
$A - C - D - E - G - H = 52$ mins
Specific behaviours
✓ lists a path with correct time
✓ lists another path with correct time
✓ indicates the correct shortest time

(b) The travel times from A to B and from G to H both increase by 4 minutes. Explain how these changes affect your answer to (a). (2 marks)

Solution
AB increase has no effect as edge not used. GH changes path to $A - C - D - F - H$ and increases time by 1 min (to 53 mins).
Specific behaviours
✓ indicates no effect for AB
✓ indicates time increase for GH

(c) Construct the adjacency matrix M for the subgraph shown below, using column and row headings in the order A, B, C, D. (2 marks)



Solution
$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$
Specific behaviours
✓ symmetrical entries
✓ all entries correct

(d) If matrix $N = M^3$, determine the value of $N_{1,1}$. (1 mark)

Solution
$N_{1,1} = 0$
(Since no walk from A to A along 3 edges)
Specific behaviours
✓ correct value

See next page

Question 6 **(6 marks)**

Three trucks, selected from a choice of four, are to be used to carry sand from a quarry to three building sites. The table below shows the weight of sand that each truck can carry to each site per day.

	Truck 1	Truck 2	Truck 3	Truck 4
Site A	43	45	44	44
Site B	39	41	39	38
Site C	43	42	46	47

Use the Hungarian algorithm to show that the maximum amount of sand that can be transported to the three sites is 132 tonnes per day and state the required allocation of trucks to achieve this maximum.

Solution

Subtract from maximum (47) and add dummy row:

	1	2	3	4
A	4	2	3	3
B	8	6	8	9
C	4	5	1	0
-	0	0	0	0

Reduce rows 1 & 2:

	1	2	3	4
A	2	0	1	1
B	2	0	2	3
C	4	5	1	0
-	0	0	0	0

Cover with 3 lines (shaded above) - smallest uncovered number is 1.

Reduce again (subtract 1 from uncovered, add to twice covered):

	1	2	3	4
A	1	0	0	1
B	1	0	1	3
C	3	5	0	0
-	0	1	0	1

Make assignment (shaded above):
Truck 1 - not used
Truck 2 - to site B (41)
Truck 3 - to site A (44)
Truck 4 - to site C (47)

Total tonnage: $41 + 44 + 47 = 132$ tonnes

Specific behaviours

✓ subtracts from maximum
✓ adds dummy row
✓ reduces rows
✓ covers with 3 lines
✓ reduces again
✓ states assignment, showing values that make total

See next page

Question 7 **(6 marks)**

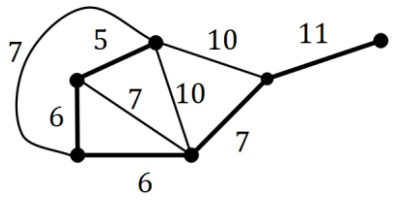
A connected graph has 6 vertices and 9 edges. The vertices represent towns and the edges represent roads between the towns. The lengths of the edges, in kilometres, are:

$$5 \quad 6 \quad 6 \quad 7 \quad 7 \quad 7 \quad 10 \quad 10 \quad 11$$

(a) Determine the minimum possible length of a Hamiltonian cycle for such a graph. (2 marks)

Solution
$l = 5 + 6 + 6 + 7 + 7 + 7 = 38$ km
Specific behaviours
✓ indicates use of shortest six edges
✓ correct total

(b) Draw a possible graph, given that that it is also simple and has a minimum spanning tree of length 35 km. Clearly label the edge lengths and highlight the minimum spanning tree. (4 marks)

Solution
Example (MST can be $5, 6, 6, 7, 11$ or $5, 6, 7, 7, 10$)

Specific behaviours
✓ no multiple edges or loops
✓ connected graph with $v = 6$ and $e = 9$
✓ labels and highlighted MST
✓ length of MST is 35

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____