Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 1&2

SOLUTIONS

Section Two: Calculator-assumed

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (99 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

See next page

Question 8 **(7 marks)**

The time in minutes that callers had to wait before their calls were answered at a business are summarised in the following table.

Wait time t	$0 \leq t < 2$	$2 \leq t < 4$	$4 \leq t < 6$	$6 \leq t < 8$	$8 \leq t < 10$
Frequency	4	9	19	22	7

(a) Explain whether the wait times are a discrete or continuous variable. (1 mark)

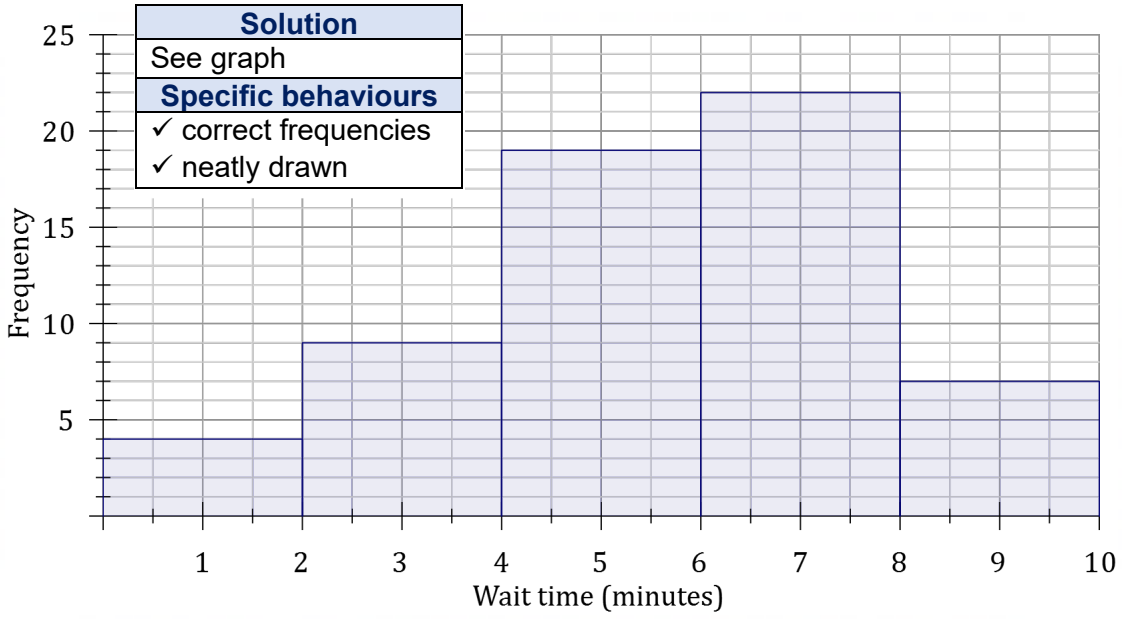
Solution
Continuous - the wait time can take one of an infinite number of values within the intervals.
Specific behaviours
✓ states continuous with brief explanation

(b) Determine the mean and standard deviation of the wait times. (2 marks)

Solution
$\bar{x} = 5.62\ minutes,$ $\sigma_x = 2.13\ minutes.$
Specific behaviours
✓ correct mean ✓ correct standard deviation

(c) Construct a histogram of the wait times on the axes below. (2 marks)

Solution
See graph
Specific behaviours
✓ correct frequencies ✓ neatly drawn



(d) State, with justification, whether the shape of the distribution is symmetric, positively skewed or negatively skewed. (2 marks)

Solution
Negatively skewed – the distribution has a longer tail to the left than to the right.
Specific behaviours
✓ states negatively skewed ✓ justifies answer

Question 9 **(8 marks)**

As a casual waiter at a local restaurant, Kay is paid $\$24.91$ per hour on weekdays, with time-and-a-half paid on weekends and double-time on public holidays. Her hours worked during the last pay period, when Friday was a public holiday, are shown below.

Day	Thu	Fri	Sat	Sun
Hours worked	6.5	3.5	4	3

(a) Determine the gross wage that Kay earned for her work during this pay period. (3 marks)

Solution
Weekend hours: $(4 + 3) \times 1.5 = 10.5$ Public holiday hours: $3.5 \times 2 = 7$ Gross wage: $(6.5 + 7 + 10.5) \times 24.91 = 24 \times 24.91 = \$597.84$
Specific behaviours
✓ adjusts weekend hours ✓ adjust Friday hours ✓ correct gross wage

The restaurant displays works by local artists and wait staff earn a 5.5% commission on the price of any artwork they sell to customers. On the Sunday of this pay period, Kay sold an acrylic painting priced at $\$780$.

(b) Determine the commission Kay earned by selling the acrylic painting. (1 mark)

Solution
Commission: $0.055 \times 780 = \$42.90$
Specific behaviours
✓ correct commission

(c) Determine the statutory superannuation contribution that Kay's employer must make at a contribution rate of 10.5% for Kay's total earnings this pay period. (2 marks)

Solution
Total earnings: $597.84 + 42.90 = 640.74$ Contribution: $0.105 \times 640.74 = \$67.28$
Specific behaviours
✓ total earnings ✓ correct contribution

(d) Before a recent pay rise, Kay was only paid $\$23.50$ per hour on weekdays. Determine the percentage pay rise that Kay received. (2 marks)

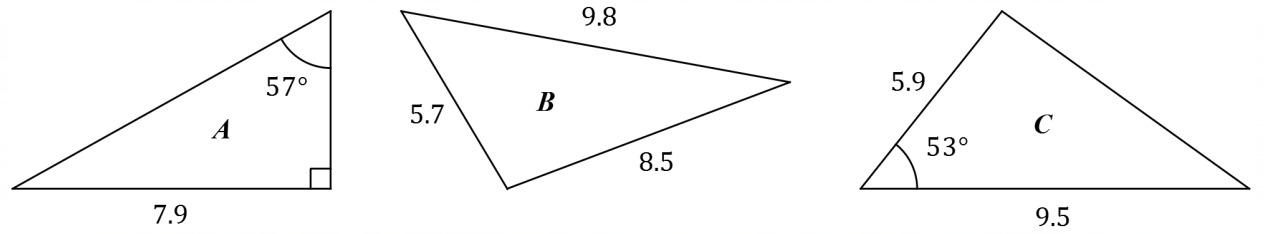
Solution
Pay rise: $24.91 - 23.50 = 1.41$ Percentage: $1.41 \div 23.50 = 0.06 = 6\%$.
Specific behaviours
✓ pay rise ✓ percentage pay rise

See next page

Question 10 **(8 marks)**

A jeweller has designed three triangular pendants A, B and C as shown below, where the dimensions are in centimetres. The pendants are to be cut from a thin sheet of metal.

Determine, with justification, the area of metal required for each.



Solution
Pendant A, using $\frac{1}{2}bh$ $$h = 7.9 \div \tan 57° = 5.13$$ $$A = \frac{1}{2}(7.9)(5.13) = 20.3 \text{ cm}^2$$ Pendant B, using Heron's rule. $$s = (5.7 + 9.8 + 8.5) \div 2 = 12$$ $$A = \sqrt{12(12 - 5.7)(12 - 9.8)(12 - 8.5)} = 24.1 \text{ cm}^2$$ Pendant C, using $\frac{1}{2}ab \sin C$. $$A = \frac{1}{2}(5.9)(9.5)\sin(53°) = 22.4 \text{ cm}^2$$
Specific behaviours
✓ indicates use of trigonometry to calculate second side length in A ✓ correct second side length in A ✓ correct area for pendant A ✓ calculates semi-perimeter of B ✓ shows use of Heron's rule for B ✓ correct area for pendant B ✓ shows use of trig formula for C ✓ correct area for pendant C

See next page

Question 11 **(8 marks)**

A supermarket sells tuna in spring water in three sizes of can, as shown in the table below.

Can size	Small	Medium	Large
Net weight (g)	95	185	425
Price per can ($)	1.10	2.10	4.65

(a) Calculate the cost in cents per gram for each size of can and hence explain why the large can represents the best value for money. (3 marks)

Solution
Small: $110 \div 95 = 1.16$ c/g Medium: $210 \div 185 = 1.14$ c/g Large: $465 \div 425 = 1.09$ c/g The large can is the best value for money as it has the lowest unit price of all three sizes.
Specific behaviours
✓ one correct unit price ✓ all correct unit prices ✓ correct explanation

The net weight is the sum of the weight of the tuna and the spring water in the can. The information labels on the small, medium and large cans state that $75\%, 70\%$ and 65% of the net weights respectively are tuna.

(b) Show that the medium size can contains 129.5 g of tuna. (1 mark)

Solution
$185 \times 0.70 = 129.5$ g
Specific behaviours
✓ shows correct calculation

(c) Determine the cost in cents per gram of tuna for each size of can. (3 marks)

Solution
Contents: Small is $95 \times 0.75 = 71.25$ g and large is $425 \times 0.65 = 276.25$ g. Unit costs: Small: $110 \div 71.25 = 1.54$ c/g Medium: $210 \div 129.5 = 1.62$ c/g Large: $465 \div 276.25 = 1.68$ c/g
Specific behaviours
✓ correct tuna contents of other cans ✓ one correct unit price ✓ all correct unit prices

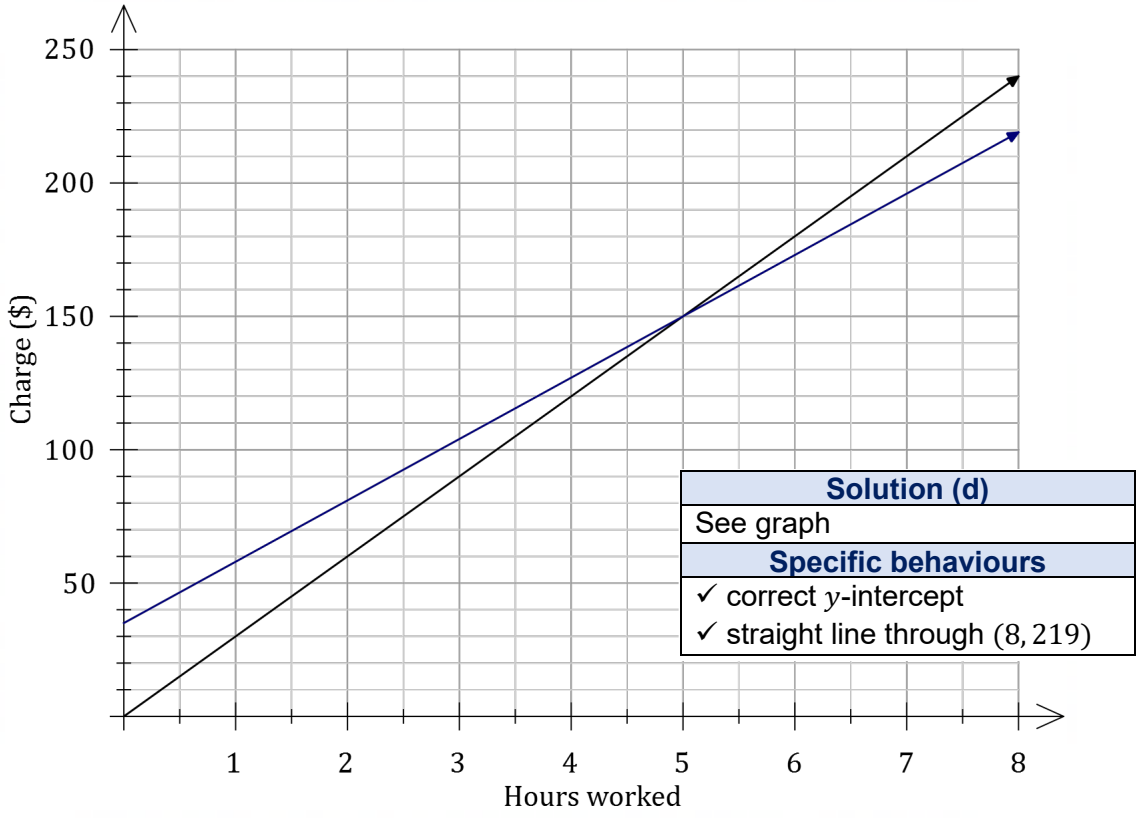
(d) Comment on whether the large can really represents the best value for money. (1 mark)

Solution
No – the large can is now the worst value when the actual weight of tuna is considered.
Specific behaviours
✓ sensible comment

See next page

Question 12 **(9 marks)**

A new business complex is comparing the costs charged by Dee and Faye to carry out general maintenance once per week. Dee will charge an attendance fee of $\$35$ plus $\$23$ per hour. The cost of using Faye is shown on the graph below.



Solution (d)
See graph
Specific behaviours
✓ correct y-intercept
✓ straight line through $(8, 219)$

(a) Use the graph to state how much Faye charges per hour. (1 mark)

Solution
$\$30$ per hour.
Specific behaviours
✓ correct charge

(b) Show that for 4 hours work, Dee will cost $\$7$ more than Faye. (2 marks)

Solution
Faye: $4 \times \$30 = \$120$. Dee: $\$35 + 4 \times \$23 = \$127$.
Hence Dee will cost $\$127 - \$120 = \$7$ more.
Specific behaviours
✓ correct cost for Faye
✓ correct cost for Dee

See next page

(c) Complete the table below to show how much Dee will charge. (2 marks)

Hours worked	2	4	6	8
Dee's charge ($)	**81**	**127**	**173**	**219**

Solution
See table
Specific behaviours
✓ at least 2 correct
✓ all correct

(d) Add a line to the graph on the previous page to represent the cost of using Dee for the maintenance. (2 marks)

The business complex has a budget of $180 per week for the maintenance.

(e) State, with reasoning, which person you would recommend is given the work. (2 marks)

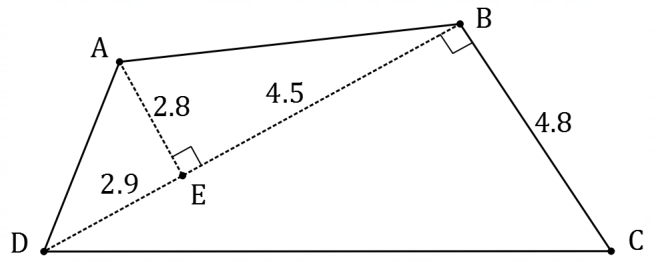
Solution
Employ Dee. For $180, Faye will work for 6 hours but Dee will work 6.3hrs which is longer. (or 0.3 x 60 = 18 min longer).
Specific behaviours
✓ correct person
✓ compares time worked

See next page

Question 13 **(9 marks)**

The plan at right, not drawn to scale, shows garden bed $ABCD$, with all measurements in metres.

Point E lies on the diagonal from from B to D so that $DE = 2.9$ m, $AE = 2.8$ m and $BE = 4.5$ m.



(a) Determine the area of the garden bed. (3 marks)

Solution
$BD = 2.9 + 4.5 = 7.4.$ $\Delta BCD = 4.8 \times 7.4 \div 2 = 17.76$ $\Delta BAD = 2.8 \times 7.4 \div 2 = 10.36$ $\text{Area} = 17.76 + 10.36 = 28.12 \text{ m}^2$
Specific behaviours
✓ indicates correct composite areas using right triangles ✓ correct area for one triangle ✓ correct area of garden bed

A budget of $\$420$ is allocated by the owner to buy mulch for the garden bed, and mulch is available in 0.035 m^3 bags that cost $\$12$ each.

(b) Determine the volume of mulch that can be bought using the budgeted amount. (2 marks)

Solution
$420 \div 12 = 35$ bags. $35 \times 0.035 = 1.225 \text{ m}^3$
Specific behaviours
✓ correct number of bags ✓ calculates volume in bags

(c) Determine the depth of the mulch once it has been spread uniformly over the garden bed, giving your answer to the nearest millimetre. (2 marks)

Solution
$1.225 \div 28.12 = 0.0436 \text{ m}$ $0.0436 \times 1000 = 43.6 \text{ mm}$ Depth of mulch will be 44 mm (or 4.4 cm or 0.044 m).
Specific behaviours
✓ calculates depth in metres ✓ correct dept, to nearest millimetre

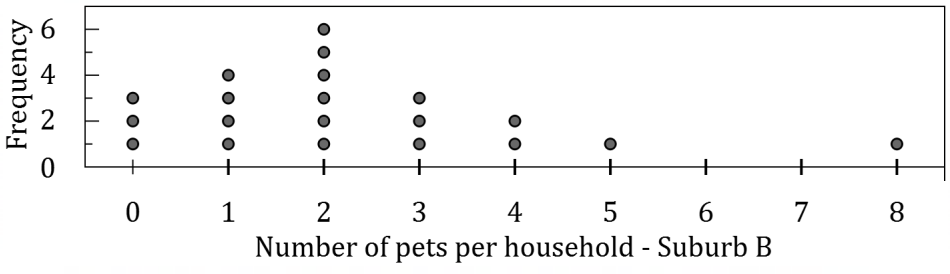
(d) A fence between corners A and B of the garden bed is to be replaced. Determine the length of this fence. (2 marks)

Solution
$AB = \sqrt{2.8^2 + 4.5^2} = 5.3 \text{ m}$
Specific behaviours
✓ indicates correct use of Pythagoras' Theorem ✓ correct length

See next page

Question 14 **(7 marks)**

Following a survey of pet ownership in 20 randomly chosen households in city Suburb A, the mean and standard deviation was found to be 2.55 and 1.56 pets per household respectively. The dot plot below shows the results of a similar survey from households in city Suburb B.



(a) Calculate the mean and standard deviation of the data for Suburb B. (2 marks)

Solution
$\bar{x} = 2.3, \quad \sigma_x = 1.87$
Specific behaviours
✓ correct mean ✓ correct standard deviation

(b) State, with justification, in which of the two suburbs pet ownership is highest. (1 mark)

Solution
Suburb A: it's mean of 2.55 is greater than mean of 2.3 for Suburb B.
Specific behaviours
✓ correct suburb, with justification

(c) Use the $\text{Q3} + 1.5 \times \text{IQR}$ criteria to show that Suburb B contains a possible outlier. (2 marks)

Solution
$\text{Q3} + 1.5 \times \text{IQR} = 3 + 1.5 \times (3 - 1) = 3 + 1.5 \times 2 = 6.$ Since one household had 8 pets and this exceeds 6, this household is classified as a possible outlier.
Specific behaviours
✓ calculate correct upper limit $\text{Q3} + 1.5 \times \text{IQR}$ ✓ Show that 8>6, so identify that 8 is an outlier

(d) Due to the likelihood of an outlier in one of the data sets, it can be argued that there are better statistics than the mean and standard deviation to compare the location and spread of the two groups. Name these two statistics and briefly explain why they are better. (2 marks)

Solution
Alternatives are the median and interquartile range. They are more representative statistics as they are less affected by outliers.
Specific behaviours
✓ names median and explains. ✓names IQR and explains.

See next page

Question 15 **(11 marks)**

Ron won a lottery prize of $\$21\,879$ and decided to invest the whole amount for eight months. He deposited one-third of the prize into a savings account, another third into a term deposit and bought shares in a logistics company with the remainder.

(a) Determine the interest that Ron earned from the savings account, given that it paid simple interest of 1.8% per annum. (3 marks)

Solution
Amount deposited: $21\,879 \div 3 = 7293$ Simple interest for 4 months: $I = 7293 \times \frac{1.8}{100} \times \frac{8}{12}$ $= \$87.52$
Specific behaviours
✓ calculates amount deposited ✓ correct expression for simple interest ✓ correct amount of interest

(b) Determine the interest that Ron earned from the term deposit, given that it paid interest of 3.1% per annum compounded monthly. (3 marks)

Solution
Future value of investment: $A = 7293\left(1 + \frac{3.1}{12 \times 100}\right)^{8}$ $= 7445.09$ Interest: $I = 7445.09 - 7293 = \$152.09$
Specific behaviours
✓ correct expression for future value ✓ correct future value ✓ correct amount of interest

See next page

(c) Shares in the logistics company cost Ron $\$5.10$ each and the company paid a special dividend of 12 cents per share to shareholders whilst Ron owned them.

Determine the profit that Ron made from his logistics shares if he sold them for $\$5.36$ at the end of the eight-month period, ignoring all transaction costs and brokerage fees.

(3 marks)

Solution
Number of shares bought: $n = 7293 \div 5.10 = 1430$ Rons profit per share (dividend plus increase in value): $p = 0.12 + (5.36 - 5.10) = 0.38$ Total profit: $P = 0.38 \times 1430 = \$543.40$
Specific behaviours
✓ calculates number of shares bought ✓ calculates profit per share (can be done in 1 or 2 calcs) ✓ calculates total profit from shares

(d) Determine the percentage gain that Ron made on his lottery prize through these three investments at the end of the eight-month period. (2 marks)

Solution
Total gain: $87.52 + 152.09 + 543.40 = \$783.01$ Percentage gain: $\frac{783.01}{21\,879} = 3.58\%$
Specific behaviours
✓ calculates total gain ✓ calculates percentage gain

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Question 16 **(7 marks)**

The marks of 1600 students who sat a math exam were normally distributed with a mean of 63.5 and standard deviation of 8.8.

(a) What percentage of the students are expected to have an exam mark within two standard deviations of the mean? (1 mark)

Solution
95%
Specific behaviours
✓ correct percentage

(b) Determine the probability that a randomly chosen student who sat the exam

(i) had a mark that was no more than 66. (1 mark)

Solution
$P(X \leq 66) = 0.612$
Specific behaviours
✓ correct probability

(ii) had a mark that was at least one-and-a-half standard deviations above the mean. (1 mark)

Solution
$P(Z > 1.5) = 0.067$
Specific behaviours
✓ correct probability

(c) Determine the standard score for Sue, who had an exam mark of 69. (2 marks)

Solution
$z = \dfrac{69 - 63.5}{8.8}$ $= 0.625$
Specific behaviours
✓ indicates correct calculation ✓ correct standard score

(d) In Sue's previous math exam that had a mean of 62.7 and a standard deviation of 7.0, Sue also had a mark of 69. However, her teacher said her performance had worsened in this latest exam despite her achieving the same mark. Explain, with mathematical justification, the reasoning behind her teacher's statement. (2 marks)

Solution
$z = \dfrac{69 - 62.7}{7.0}$ $= 0.9$ Her standard score has decreased from 0.9 to 0.625 and hence her mark has worsened relative to the group.
Specific behaviours
✓ calculates standard score ✓ explanation comparing standard scores

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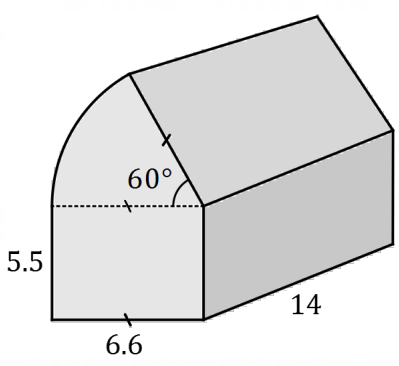
Question 17 **(8 marks)**

The cross section of the prism shown in the diagram consists of a $60°$ sector of a circle of radius 6.6 cm sitting atop a rectangle of width 6.6 cm and height 5.5 cm.

The length of the prism is 14 cm.

The diagram is not drawn to scale.



(a) Determine the area of the sector. (2 marks)

Solution
$A = \pi \times 6.6^2 \times \frac{60}{360} = 22.8 \text{ cm}^2$
Specific behaviours
✓ indicates correct method
✓ correct area of sector

(b) Determine the volume of the prism. (2 marks)

Solution
$A = 5.5 \times 6.6 + 22.8 = 36.3 + 22.8 = 59.1$ $V = 59.1 \times 14 = 827.5 \text{ cm}^3$
Specific behaviours
✓ calculates total cross-sectional area
✓ correct volume of prism

(c) Determine the total surface area of the prism to the nearest square centimetre. (4 marks)

Solution
Curved surface area: $A = 2\pi(6.6)(14) \times \frac{60}{360} = 96.8$ Rectangles: $A = 2(5.5 \times 14) + 2(6.6 \times 14) = 338.8$ TSA is two ends plus curved SA plus rectangles: $A = 2 \times 59.1 + 96.8 + 338.8 = 553.78 = 554 \text{ cm}^2$
Specific behaviours
✓ expression for arc length of curved surface
✓ correct curved surface area
✓ areas of rectangles
✓ correct total surface area

See next page

Question 18 **(9 marks)**

(a) A scale model of a prism has a triangular cross-section with an area of 28 cm^2 and a height of 16 cm.

(i) Determine the volume of the scale model. (1 mark)

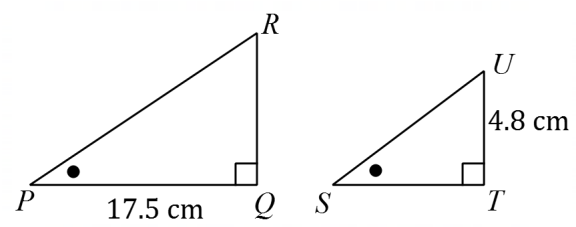
Solution
$V = 28 \times 16 = 448 \text{ cm}^3$
Specific behaviours
✓ correct volume

(ii) The height of the actual prism is 120 cm. Determine the area of the cross-section of the actual prism. (3 marks)

Solution
Length scale factor $sf = 120 \div 16 = 7.5$ Area scale factor $asf = 7.5^2 = 56.25$ Area of cross-section $A = 28 \times 56.25 = 1575 \text{ cm}^2$
Specific behaviours
✓ length scale factor ✓ area scale factor ✓ correct cross-sectional area

(b) Two similar triangles are shown at right, not drawn to scale.

The length of side PQ is 17.5 cm, the length of side UT is 4.8 cm, and the area of triangle PQR is 52.5 cm^2.



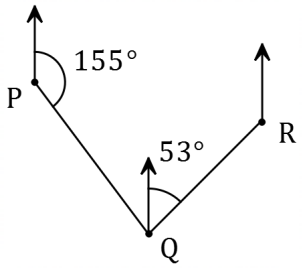
Determine the length of side SU. (5 marks)

Solution
Using the area of ΔPQR: $\frac{1}{2}(17.5)(QR) = 52.5$ $QR = 6$ Length scale factor: $sf = 4.8 \div 6 = 0.8$ Length of ST: $ST = 17.5 \times 0.8 = 14$ Length of SU: $SU = \sqrt{14^2 + 4.8^2} = 14.8 \text{ cm}$
Specific behaviours
✓ area equation for QR ✓ solves for QR ✓ calculates length scale factor ✓ calculates ST ✓ calculates SU

See next page

Question 19 **(8 marks)**

A small robot moves at a speed of 0.8 metres per second and is currently at point P on level ground. The robot moves for 12 seconds on a bearing of $155°$ to point Q and then turns and moves for a further 9 seconds on a bearing of $053°$ to point R, as shown below.



(a) Show that the size of $\angle PQR = 78°$. (1 mark)

Solution
$\angle PQR$ is the sum of the supplement of $155°$, which is $25°$, and $53°$.
Specific behaviours
✓ explanation or calculation

(b) Show use of trigonometry to determine the distance of P from R. (3 marks)

Solution
$PQ = 0.8 \times 12 = 9.6$ m, $\quad QR = 0.8 \times 9 = 7.2$ m $PR^2 = 9.6^2 + 7.2^2 - 2(9.6)(7.2)\cos 78°$ $PR = \sqrt{115.26} = 10.7$ m
Specific behaviours
✓ calculates lengths PQ and QR ✓ expression for distance using cosine rule ✓ correct distance

(c) Show use of trigonometry to determine the size of angle $\angle QPR$ to the nearest degree. (2 marks)

Solution
$\dfrac{\sin \angle QPR}{7.2} = \dfrac{\sin 78°}{10.7}$ $\angle QPR = 40.995° = 41°$
Specific behaviours
✓ expression for angle using the sine rule ✓ correct angle

(d) The robot must return directly to P from R. Determine the bearing it should move in and the time it will take. (2 marks)

Solution
Bearing $P \rightarrow R$ is $155° - 41° = 114°$ and so bearing $R \rightarrow P$ is $180° + 114° = 294°$. Time is $10.7 \div 0.8 = 13.4$ s.
Specific behaviours
✓ correct bearing ✓ correct time

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____