MATHEMATICS APPLICATIONS UNITS 1 AND 2
Section Two:
Calculator-assumed

SOLUTIONS

Student Number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
    *   Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
    *   Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 9 **(5 marks)**

(a) Two full-time secretarial positions were advertised in a newspaper. The first, with a building company, had an annual salary of $42 900 whilst the second, with a real estate company, offered $22 per hour. Both positions were for a 35 hour working week.

Determine which position is better paid, assuming 52 weeks per year. Justify your answer. (2 marks)

$22 \times 35 \times 52 = \$40040$

Building company is better paid.

(b) The same brand of jam can be bought in three sizes: In a 200 g jar for $2.15, a 350g jar for $3.40 or a 550 g jar for $5.40.

(i) Rank the sizes from best to worst value, showing how you made your decision.

(2 marks)

$215 \div 200 = 1.075$
$340 \div 350 \approx 0.97$
$540 \div 550 \approx 0.98$

Hence 340 g jar is best value, then 540 g jar and 200 g jar is worst value.

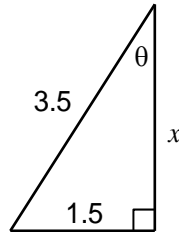(ii) Determine the minimum cost of buying exactly 2 kg of jam. (1 mark)

$3 \times 5.40 + 1 \times 3.40 = \$19.60$

See next page

Question 10 **(7 marks)**

A ladder is 3.5 metres long and is placed on level ground so that it leans against a vertical wall. The foot of the ladder is 1.5 metres from the bottom of the wall.

(a) Draw a neat sketch of the ground, wall and ladder, showing all given measurements.

(1 mark)

(b) Calculate how far up the wall the ladder will reach, correct to one decimal place.

(2 marks)

$$x^2 = 3.5^2 - 1.5^2$$
$$x^2 = 10$$
$$x = 3.162$$
$$\approx 3.2 \text{ m (1 dp)}$$

(c) Calculate the angle between the ladder and the wall at the top of the ladder, correct to the nearest degree. (2 marks)

$$\sin(\theta) = \frac{1.5}{3.5}$$
$$\sin(\theta) = 0.42857$$
$$\theta = 25.38$$
$$= 25° \text{ (nearest degree)}$$

(d) Calculate how far the base of the ladder should be moved from its current location if the ladder is to reach 2.75 metres up the wall. (2 marks)
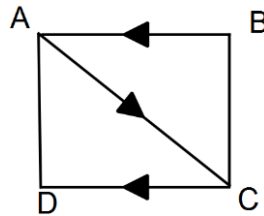
$$d = \sqrt{3.5^2 - 2.75^2}$$
$$= 2.17$$

$$2.17 - 1.5 = 0.67 \text{ m}$$

See next page

Question 11 (6 marks)

The diagram below shows four people (A, B, C and D) in a communication network. The arrows indicate one-way communication, so for example:

- B can send a message directly to A, but A cannot send a message directly to B.

- B and C can both send a message directly to each other.

(a) Complete the table below to show the direct communication links from person to person.

(2 marks)

		To			
		A	B	C	D
From	A	0	0	1	1
	B	1	0	1	0
	C	**0**	**1**	**0**	**1**
	D	**1**	**0**	**0**	**0**

(b) Arrange the information from the table in (a) as the matrix T and calculate T^2. (2 marks)

$$T^2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

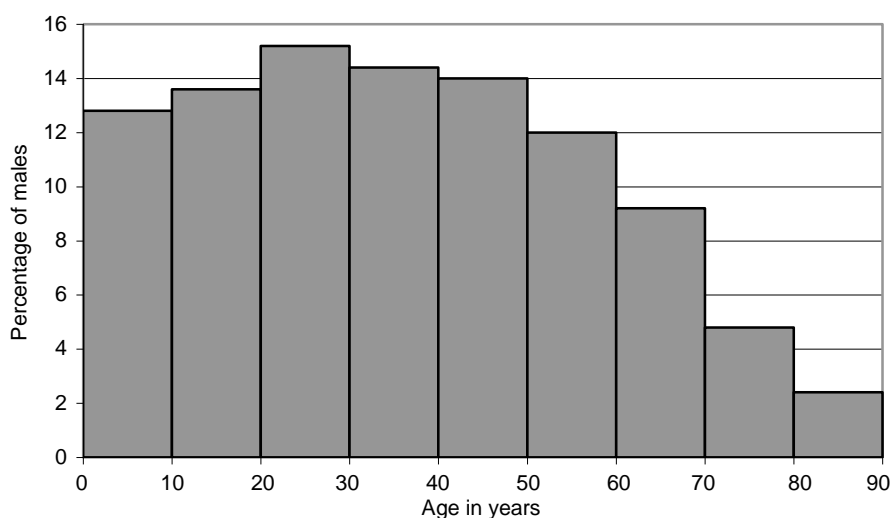(c) In how many ways can C send a message via one other person to

(i) A? (1 mark)

2 ways

(ii) B? (1 mark)

0 ways

See next page

Question 12   **(8 marks)**

The histogram below shows the distribution of ages of the under-90 male population of Australia in the year 2010. The mean and standard deviation of the data shown are 36.6 and 21.7 years respectively.

(a)   Describe the spread of the data shown in the histogram.   (3 marks)

> The distribution is unimodal. The modal age is 20 to 30 years, with just over 15% of males in this group.
>
> The percentage in each group increases from 0 to 30 years and then decreases from 30 to 60 years.
>
> After 60 years, the percentages drop significantly, as is evident in the positive skewness shown in the histogram.

The table below shows the distribution of ages of the under-90 female population of Australia in the year 2010.

Age in years (x)	Percentage of females
$0 \leq x < 10$	12.3
$10 \leq x < 20$	12.7
$20 \leq x < 30$	14.6
$30 \leq x < 40$	14.8
$40 \leq x < 50$	14.2
$50 \leq x < 60$	12.6
$60 \leq x < 70$	9.4
$70 \leq x < 80$	5.7
$80 \leq x < 90$	3.7

(b)   Calculate the mean and standard deviation of the female ages.   (2 marks)

Mean is 38.2 years.

Standard deviation is 22.3 years.

(c)   Comment on any similarities or differences between the male and female age
distributions.   (3 marks)

The mean male age is slightly lower than that of the females
(36.6 compared to 38.2 years), indicating that females tend to
live longer. This is also evident when comparing the
percentages in the 70 to 90 age groups (~7% compared to
9.4%).

The female ages are very slightly more spread than those of
males when comparing the standard deviations (22.3
compared to 21.7 years).

Both distributions show positive skew, as the percentages in
the older age groups decrease.

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Question 13   **(9 marks)**

Amy borrowed $1570 to buy a scooter with a purchase price of $1793.

(a)   What percentage of the purchase price did Amy borrow, correct to 3 significant figures?

(2 marks)

$$1570 \div 1793 \times 100 = 87.562744$$
$$\therefore 87.6\% \quad \text{correct to 3sf}$$

(b)   The loan company charged interest of 14% of the amount borrowed. If Amy repaid the loan and interest in 6 equal amounts, how much was each repayment?   (2 marks)

$$1570 \times 1.14 = 1789.80$$

$$1789.80 \div 6 = \$298.30$$

(c)   Amy expects her scooter to depreciate in value by 15% per year.

(i)   What will its value be after one year, to the nearest ten dollars?   (2 marks)

$$1793 \times 0.85 = 1524.05$$
$$\therefore \$1520 \text{ to nearest ten dollars}$$

(ii)   After how many years will the value of the scooter first be less than $1000?
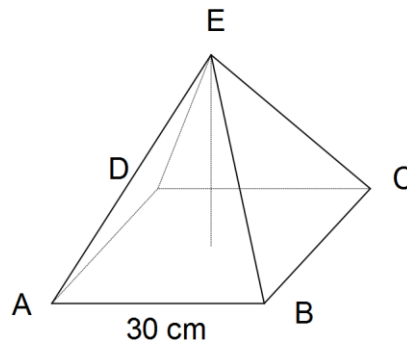
(1 mark)

After 4 years (~$940)

(d)   The purchase price included GST at 10%. How much GST was included in the purchase price?   (2 marks)

$$1793 \div 1.10 = 1630$$

$$1793 - 1630 = 163$$

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Question 14 **(8 marks)**

The diagram shows a square based pyramid, with a perpendicular height of 40 cm and a base length of 30 cm.

(a) Calculate the length AC. (1 mark)

$$\sqrt{30^2 + 30^2} = 42.4 \text{ cm}$$

(b) Determine the length EC. (2 marks)

$$42.4 \div 2 = 21.2$$

$$\sqrt{21.2^2 + 40^2} = 45.3 \text{ cm}$$

(c) M is the midpoint of side BC. Determine the length EM. (2 marks)

$$30 \div 2 = 15$$

$$\sqrt{15^2 + 40^2} = 42.7 \text{ cm}$$

(d) Calculate the total surface area of the pyramid. (3 marks)

One triangular face:
$\frac{1}{2} \times 30 \times 42.7 = 641 \text{ cm}^2$

Square base:
$30 \times 30 = 900 \text{ cm}^2$

TSA:
$900 + 4 \times 641 = 3464 \text{ cm}^2$

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Question 15   **(5 marks)**

(a)   A student on an exchange trip to South Africa changed 600 Australian Dollars into South African Rand when they left Australia.

     (i)   When they left, the exchange rate was 1 Australian Dollar to 9.955 SA Rand. How many SA Rand did the student buy?   (1 mark)

$$600 \times 9.955 = 5973 \text{ Rand}$$

     (ii)   During their stay in South Africa, the student spent 4 500 Rand and exchanged the remainder on return to Australia. The exchange rate at that time was 1 Australian Dollar to 10.215 SA Rand. How many Australian Dollars did the student receive?   (2 marks)

$$5973 - 4500 = 1473$$

$$1473 \div 10.215 = \$144.20$$

(b)   Information about a small share portfolio is shown below.

Company	Number of shares	Value of share ($)	Dividend per share (cents)	Dividend per share (%)
Anplow	1250	5.36	18	-
Britter	3500	2.88	-	4.5

Determine the total dividend due for this portfolio.   (2 marks)

$$1250 \times 0.18 = 225$$

$$3500 \times 2.88 \times 4.5\% = 453.6$$

$$225 + 453.6 = \$678.60$$

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Question 16 **(9 marks)**

(a) A solid cube has a volume of 3375 cm^3. Find the length of one edge of this cube.

(2 marks)

$$L^3 = 3375$$
$$L = \sqrt[3]{3375}$$
$$= 15 \text{ cm}$$

(b) A thin wooden stick is placed into an empty cylindrical can of radius 7 cm and height 20 cm. Determine the length of the longest stick that will fit completely inside the can, correct to the nearest cm. (2 marks)

$$l^2 = 14^2 + 20^2$$
$$l^2 = 596$$
$$l = 24.4 \text{ cm}$$
$$\approx 24 \text{ cm}$$

(c) A hemisphere is made by cutting a sphere into two congruent halves. A solid hemisphere of radius 9.8 cm is shown below. (Note: This diagram is not to scale).
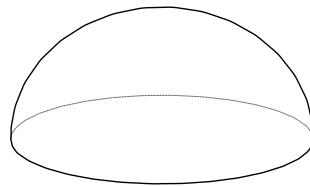
(i) Calculate the volume of this hemisphere. (2 marks)

$$V_{hemisphere} = \frac{4 \times \pi \times 9.8^3}{3} \div 2$$
$$= 1971 \text{ cm}^3$$

(ii) Determine the total surface area of the hemisphere. (3 marks)

$$A_{curved} = 4 \times \pi \times 9.8^2 \div 2$$
$$= 603.44$$
$$A_{circle} = \pi \times 9.8^2$$
$$= 301.72$$
$$TSA = 603.44 + 301.72$$
$$\approx 905 \text{ cm}^2$$

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Question 17   **(7 marks)**

A large group of 3472 students sat an exam. The percentage scores of all students who sat the exam were normally distributed, with a mean of 57.2% and a standard deviation of 12.4%.

(a)   Determine the 0.9 quantile for this distribution.   (1 mark)

$$P(X < k) = 0.9$$
$$k = 73.1\%$$

(b)   If one of the students were chosen at random, were they more likely to have scored between 60 and 70% in the exam or less than 50%?   (3 marks)

$$P(60 < X < 70) = 0.2597$$

$$P(X < 50) = 0.2807$$

More likely to score less than 50%.

(c)   What percentage of students scored within two standard deviations of the mean? (1 mark)

95%

(d)   How many of the 3472 students scored no more than 30% in the exam?   (2 marks)

$$P(X < 30) = 0.01413$$
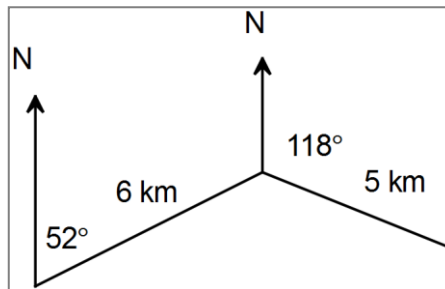
$$0.01413 \times 3472 = 49 \text{ students}$$

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Question 18 **(9 marks)**

(a) A boat sails on a bearing of 052⁰ for 6 km and then turns and sails on a bearing of 118⁰ for a further 5 km.
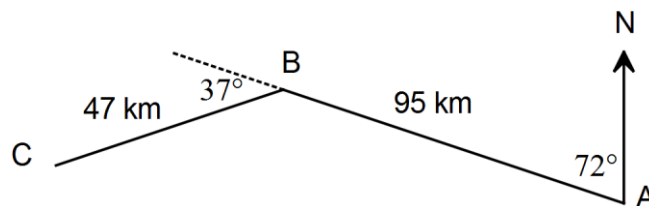
(i) Sketch a diagram of the course of this boat. (2 marks)

(ii) By what angle did the boat turn through when it changed bearing from 052⁰ to 118⁰? (1 mark)

$$118 - 52 = 66°$$

(b) A plane left an airport at A and flew for 95 km on a bearing of N72W to point B, before turning 37⁰ to the left and continuing for another 47 km to point C, as shown below.

(i) Determine the straight line distance of C from A. (3 marks)

$$180 - 37 = 143$$
$$AC^2 = 47^2 + 95^2 - 2 \times 7 \times 95 \cos 143$$
$$AC = 135.5 \text{ km}$$

(ii) Determine the compass bearing of A from C. (3 marks)

$$\frac{135.5}{\sin 143} = \frac{47}{\sin BAC}$$
$$\angle BAC = 12°$$
$$12 + 72 = 84°$$
Bearing is S84E

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Question 19   **(11 marks)**

HDL, a courier company, deliver packages the same day within the CBD of a city. They charge $25 to deliver a package up to 4 kg in weight and then a further $10 per 2 kg or part thereof, up to a maximum weight of 15 kg.

(a)   Complete the table below.   **(2 marks)**

Package weight (kg)	0 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14	14 - 15
Delivery charge ($)	25	**35**	**45**	**55**	**65**	**75**	85

(b)   A business has five packages to send within the CBD, two weighing 1.5 kg, two weighing 7 kg and one that weighs 12.5 kg. Calculate how much will they be charged.   **(2 marks)**
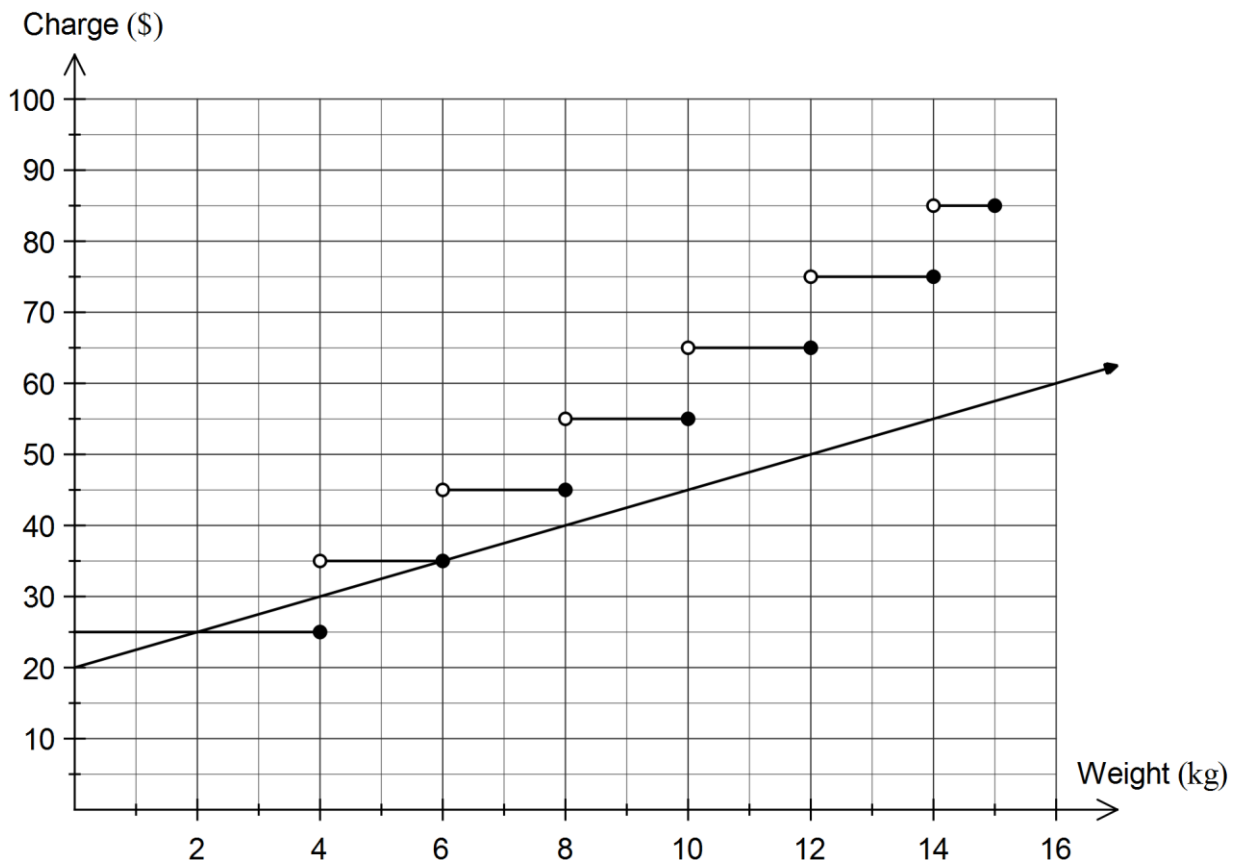
$2 \times 25 = 50$

$2 \times 45 = 90$

$1 \times 75 = 75$

$50 + 90 + 75 = \$215$

(c)   Complete the graph below to show the delivery charges for packages of up to 15 kg.   **(3 marks)**

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Another courier company, FexEd, deliver packages within the CBD for $2.50 per kilogram plus a pick up fee of $20.

(d) How much would FexEd charge to deliver a 10 kg parcel? (1 mark)

$$10 \times 2.50 + 20 = \$45$$

(e) Add a line to the graph above to show the delivery charges for FexEd. (2 marks)

(f) Briefly explain which company is the most economical to deliver single packages of various weights. (1 mark)

For a single package weighing between 2 and 4 kg, DHL will be the cheapest. For all other weights, FexEd will be the cheapest.

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Question 20   **(8 marks)**

A young person qualifies for youth allowance of $272.80 per fortnight, so long as he does not earn more than $415 before tax in that time.

In any fortnight that he does earns more than $415, his allowance will be reduced by 50 cents in the dollar for earnings over $415 and up to $498, and reduced by 60 cents in the dollar for earnings over $498.

The young person has a part time job that pays $14.55 per hour.

(a)   Calculate his fortnightly youth allowance if he works for 12 hours each week. Justify your answer.   (2 marks)

> Earnings in fortnight: $14.55 \times 12 \times 2 = \$349.20$
>
> Paid full allowance of $272.80.

(b)   Calculate his fortnightly youth allowance if he works for 40 hours every two weeks.
                                                                                   (3 marks)

> $14.55 \times 40 = 582$
>
> $582 - 498 = 84$
> $84 \times 0.6 = 50.40$
>
> $498 - 415 = 83$
> $83 \times 0.5 = 41.50$
>
> $272.80 - 50.40 - 41.50 = \$180.90$

(c)   Determine, to the nearest hour, how many hours the young person can work in a fortnight before they are no longer paid any youth allowance.   (3 marks)

> $272.80 - 41.50 = 231.30$
>
> $231.30 \div 0.60 = 385.50$
>
> $385.50 + 498 = 883.50$
>
> $883.50 \div 14.55 = 60.72$
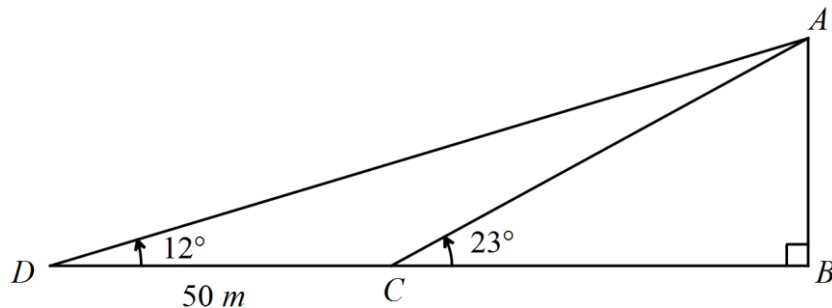> ≈ 61 hours per fortnight

Question 21 **(6 marks)**

A small boat on a lake noticed that the angle of elevation of the top of a tall building on the shore was 12⁰. It then motored directly towards the building for 50 metres, at which point the angle of elevation had increased to 23⁰.

(a)   Draw a sketch of this situation. (1 mark)

(b)   Show that the straight line distance of the boat from the top of the building is 54.5 metres.
(3 marks)

$$\angle DAC = 23 - 12$$
$$= 11$$

$$\frac{AC}{\sin 11} = \frac{50}{\sin 12}$$

$$AC = 54.48$$
$$\approx 54.5 \text{ metres}$$

(c)   Determine the height of the building, to the nearest metre. (2 marks)

$$54.5 \times \sin 23 = 21.3$$
$$\approx 21 \text{ metres}$$

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____