Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS METHODS UNIT 3

Section One: Calculator-free

Student Name _____

Teacher Name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (52 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

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Question 1 **(7 marks)**

Let $f(x) = 15 - 4x - 6x^2 - 4x^3 - x^4$.

(a) The curve $y = f(x)$ cuts the horizontal axis at $x = 1$. State, with reasons, whether the function is increasing, decreasing or neither at this point. (2 marks)

(b) Determine $f''(0)$ and use this value to describe the concavity of the curve $y = f(x)$ where it crosses the vertical axis. (2 marks)

(c) Does the curve $y = f(x)$ have any points of inflection? If it does, determine the coordinates of their location. If not, justify your answer. (3 marks)

See next page

Question 2 **(6 marks)**

(a) Determine $f'(-2)$ when $f(x) = 2(3x + 5)^3$. (3 marks)

(b) Determine $g(2)$ when $g'(x) = 12e^{3x-3}$ and $g(1) = 7$. (3 marks)

See next page

Question 3 **(8 marks)**

The function f is defined for $x > 0$ by $f(x) = \frac{e^{4x-1}}{x}$, and $f''(x) = \frac{2(8x^2 - 4x + 1)e^{4x-1}}{x^3}$.

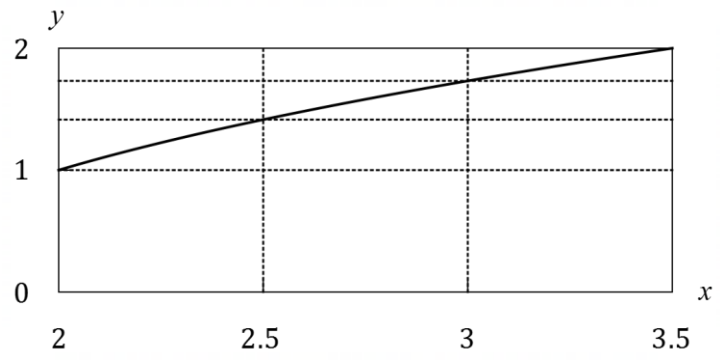
(a) Determine the coordinates and nature of all stationary points of the graph of $y = f(x)$. Justify your answer. (6 marks)

(b) Show that the graph of $y = f(x)$ has no points of inflection. (2 marks)

Question 4 **(8 marks)**

The graph of $y = \sqrt{2x-3}$ between $x = 2$ and $x = 3.5$ is shown at right.

Approximate values for $\sqrt{2}$ and $\sqrt{3}$ are 1.41 and 1.73 respectively.



(a) Use the areas of the rectangles shown to explain why $2.07 < \int_2^{3.5} \sqrt{2x-3}\,dx < 2.57$. (3 marks)

(b) Evaluate $\int_2^{3.5} \sqrt{2x-3}\,dx$. (3 marks)

(c) Evaluate $\int_2^{3.5} \left(1 - \sqrt{2x-3}\right) dx$. (2 marks)

See next page

Question 5 **(8 marks)**

When a seed is randomly selected from a packet and grown, the probability that it yields a white flower is $0.35 = \frac{7}{20}$.

(a) Explain why this context is suitable for modelling with a Bernoulli random variable and state the mean of the Bernoulli distribution. (2 marks)

(b) When several Bernoulli trials are repeated, the total number of successes can be modelled with a binomial random variable provided the trials meet two conditions. Briefly describe these conditions. (2 marks)

(c) Nine seeds are randomly selected and grown. Write an expression for the probability that seven or eight of these seeds will yield a white flower. (2 marks)

See next page

(d) When a gardener wants to be at least 99% certain of obtaining one or more white flowers, the number of seeds n that must be selected and grown will be the solution of the inequality $b^n \leq a$. State, with justification, the value of the constant a and the value of the constant b. (2 marks)

Question 6 **(8 marks)**

Let $f(x) = e^{-2x}(\sin 2x - \cos 2x)$.

(a) Determine $f'(x)$, simplifying your answer. (3 marks)

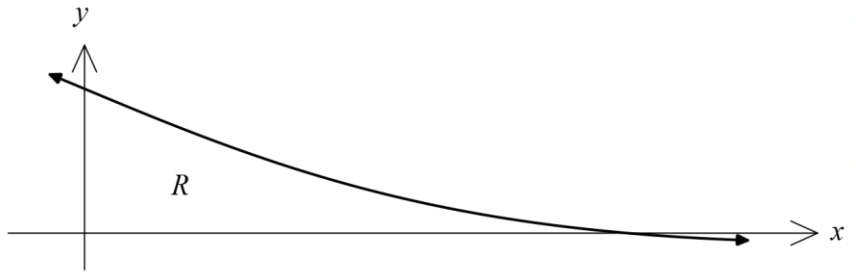
(b) Use **differentiation** and your previous answer to show that

$$\int \left(e^{-2x}\cos 2x\right) dx = \frac{1}{4}e^{-2x}(\sin 2x - \cos 2x) + c,$$

where c is a constant. (2 marks)

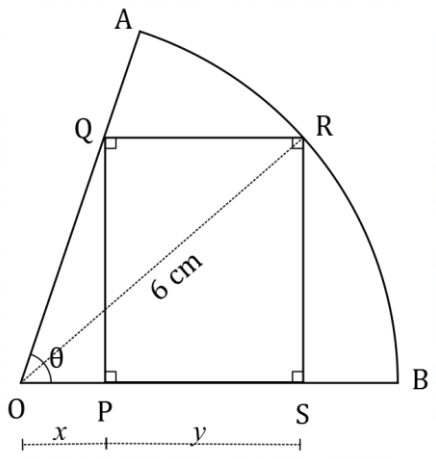
(c) The graph of $y = e^{-2x}\cos 2x$ is shown below. Determine the area of the region R, bounded by the curve, the x-axis and the y-axis.

(3 marks)



Question 7 **(7 marks)**

The diagram shows the vertices of rectangle $PQRS$ lying on sector OAB that subtends an angle of θ in a circle of radius 6 cm, and where $\tan\theta = 4$. Let $OP = x$ cm and $PS = y$ cm.



(a) Show that the perimeter of the rectangle is given by $p = 6x + 4\sqrt{9 - 4x^2}$ cm. (3 marks)

See next page

(b) Determine the maximum perimeter of rectangle $PQRS$. (4 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Markers use only			
Marker	Question	Maximum	Mark
Ms. Tay	1	7	
	2	6	
	3	8	
	4	8	
	5	8	
	6	8	
	7	7	
S1 Total		52	