WA Exams Practice Paper E, 2015

Question/Answer Booklet

MATHEMATICS APPLICATIONS UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(3 marks)**

An investor bought 660 shares valued at $2805 and received an annual dividend of 11 cents per share.

(a) Determine the dividend as a percentage of the value of one share. (2 marks)

$2805 \div 660 = 4.25$

$\frac{11}{425} \times 100 = 2.59\%$

(b) The investor could have invested the $2805 in a bank deposit account earning 3.5% p.a. Explain whether this would have been a better option. (1 mark)

Yes, as 3.5% is a better annual return than 2.59%.

Question 9 **(9 marks)**

Part of an electricity account is shown below:

Rate effective at 01/04/09		
648 units @ 13.93 cents per unit	$	90.27
Supply Charge	$	8.43
Domestic Supply Tariff GST @10%	$	9.87
Rate effective at 01/07/09		
589 units @ 16.01 cents per unit	$	94.30
Supply Charge	$	8.82
Domestic Supply Tariff GST @10%	$	10.31
Plus Credit Card Fee For Payment Of $133.85	$	0.85
Credit Card Fee GST @10%	$	0.09
Total	$	222.94

(a) Explain how the figure $10.31 in the account was calculated. (2 marks)

$94.3 + 8.82 = 103.12$

$103.12 \times 10\%$ (GST rate) $= 10.312$ which rounds to 10.31

(b) By what percentage did the price of electricity increase on 01/07/09? (2 marks)

$16.01 - 13.93 = 2.08$

$2.08 \div 13.93 \times 100\% = 14.93\%$ increase.

(c) A transaction fee of 0.7% (incl GST) applies to credit card payments and is shown on the next account. How much would the transaction fee be if this account was paid in full using a credit card? (1 mark)

$222.94 \times 0.7 \div 100 = 1.56058 \approx \$1.56$

See next page

(d) The account was for electricity used from 30 May 09 to 31 Jul 09 inclusive. What was the average daily cost of electricity for this household, excluding credit card fees? (2 marks)

$2 + 30 + 31 = 63$ days

$222.94 - (0.85 + 0.09) = 222.00$

$222.00 \div 63 = 3.5238 \approx \$3.52$ per day.

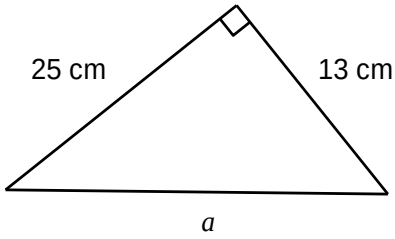
(e) If the Supply Charge is 28.45 cents per day (ex GST) for all households, determine a formula that could be used to calculate the cost of electricity, C in dollars, for a household using a total of n units of electricity for a period of d days after 01/07/09, excluding GST. (2 marks)

$C = 0.2845d + 0.1601n$

Question 10 **(7 marks)**

(a) Determine the value of the unknown in each triangle below.

(i) (2 marks)

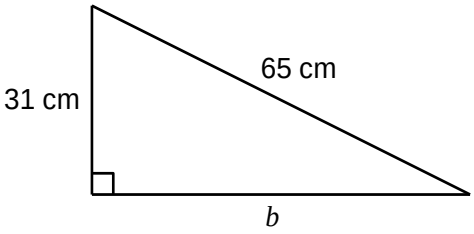


$a^2 = 25^2 + 13^2$

$a = \sqrt{794}$

$= 28.2$ cm

(ii) (2 marks)



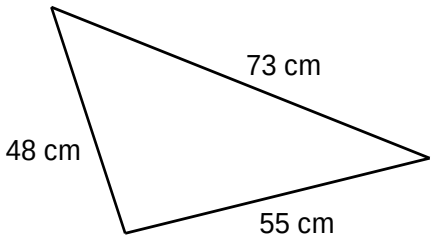
$31^2 + b^2 = 65^2$

$b = \sqrt{3264}$

$= 57.1$ cm

(b) Use Pythagoras' Theorem to determine whether the triangle sketched below, not to scale, is right angled.

(3 marks)



$48^2 + 55^2 = 5329$

$73^2 = 5329$

Hence $48^2 + 55^2 = 73^2$ and so triangle must be right angled.

See next page

Question 11 **(8 marks)**

Job seekers living at home and under the age of 18 qualify for a government allowance of $226.80 per fortnight, so long as they do not earn more than $143 before tax in that time.

In any fortnight that they do earn more than $143, their allowance will be reduced by 50 cents in the dollar for earnings over $143 and up to $250, and reduced by 60 cents in the dollar for earnings over $250. Once earnings exceed $538.83 the allowance reduces to $0.

Determine the allowance paid to each of the following young job seekers:

(a) Ahn, who earns $220 per fortnight. (2 marks)

$220 - 143 = 77$

$77 \times 0.5 = 38.50$

$226.80 - 38.50 = \$188.30$

(b) Bee, who has a part time job paying $8.35 per hour for 8 hours each week. (2 marks)

$8 \times 2 \times 8.35 = \$133.60$

Gets full allowance of $226.80

(c) Chris, who is paid $19.50 per hour for working three hours a day for five days a week. (2 marks)

$3 \times 5 \times 2 \times 19.50 = \$585.00$

Gets no allowance as income exceeds $538.83

(d) Di, who earns $220 per week. (2 marks)

$2 \times 220 = 440$

$250 - 143 = 107$

$107 \times 0.5 = 53.50$

$440 - 250 = 190$

$190 \times 0.6 = 114$

$226.80 - 53.50 - 114.00 = \$59.30$

See next page

Question 12 **(10 marks)**

As part of their budget, a student was comparing 2 mobile phone plans.

Plan A: Pay \$19 and receive \$100 credit to be used on text messages (25c each) and voice calls (90c per 1-minute block).

Plan B: Pay \$29 and receive \$150 credit to be used on text messages (25c each) and voice calls (85c per 1-minute block).

Both plans charge for voice calls in 1-minute blocks with a 35c flag-fall fee. The flag-fall fee is charged as soon as the person being called answers their phone. Then, if the call is less than 1 minute, a 1-minute block fee is charged. If the call is more than 1 minute but less than 2 minutes, then two 1-minute block fees are charged, and so on.

(a) If the student chose Plan A, never sent text messages and only made short voice calls (less than one-minute),

(i) How many short voice calls could they make? (2 marks)

Cost of call is $0.90 + 0.35 = \$1.25$
Number of calls $= \$100 \div \$1.25 = 80$

(ii) Show that the actual cost to the student per short voice call is 23.75 cents. (1 mark)

Cost per call $= \$19 \div 80$
$= \$0.2375$ or 23.75 cents

(b) Which plan offers the best value for users who only make short (less than one-minute) voice calls? Justify your answer. (3 marks)

Plan B total cost of one short call is \$1.20
Number of calls $= \$150 \div \$1.20 = 125$
Cost per call $= \$29 \div 125 = \$0.232$ or 23.2 cents

Hence Plan B is better value as the real cost per call is less.

See next page

(c) Which plan offers the best value for users who never make voice calls and only send text messages? Justify your answer. (4 marks)

Plan A, make $\$100 \div \$0.25 = 400$ texts.
Real cost per text $= \$19 \div 400$
$= \$0.0475$ or 4.75 cents per text

Plan B, make $\$150 \div \$0.25 = 600$ texts.
Real cost per text $= \$29 \div 600$
$= \$0.048\overline{3}$ or ≈ 4.83 cents per text

Hence Plan A offers the best value as the real cost per text is less than that for Plan B.

See next page

Question 13 **(8 marks)**

(a) The table below, based on figures from the Australian Taxation Office, shows the tax rates for a recent financial year.

Taxable income	***Tax on this income***
$1 – $6,000	Nil
$6,001 – $34,000	15c for each $1 over $6,000
$34,001 – $80,000	$4,200 plus 30c for each $1 over $34,000
$80,001 – $180,000	$18,000 plus 40c for each $1 over $80,000
$180,001 and over	$58,000 plus 45c for each $1 over $180,000

(i) Calculate the income tax payable by a person with a taxable income of $64 440. (2 marks)

$64440 - 34000 = 30440$

$30440 \times 0.30 = 9132$

$4200 + 9132 = \$13332$

(ii) If the income tax payable by a person amounted to $1875, how much was their taxable income? (2 marks)

Choose 2nd row of tax table, as tax < $4200

$1875 \div 0.15 = 12500$

$12500 + 6000 = 18500$

(b) A bank is offering two different types of personal loan for any customer who borrows between $500 and $10 000. One loan is based on a flat rate of 9.9%pa whilst the other loan adds 8.5% interest annually on a compounding basis.

A customer plans to borrow $4000 and repay the loan and interest in full after five years. Which of the two loans do you recommend they use? Justify your answer. (4 marks)

Using simple interest

$I = \dfrac{4000 \times 9.9 \times 5}{100} = \$1980$ so must repay $5980

Using compound interest

$A = 4000 \times \left(1 + \dfrac{8.5}{100}\right)^5 = \$6014.63$ to repay

Hence choose simple interest loan, as the total to repay after five years is less than the compound interest loan.

See next page

Question 14 **(9 marks)**

A manufacturer makes two products R and S, that each need a certain number of component parts A, B, C and D as shown in the matrix P below.

$$\begin{array}{c} \\ R \\ S \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 2 & 1 & 3 & 2 \\ 2 & 3 & 2 & 3 \end{bmatrix} \end{array}$$

The cost of parts A, B, C and D are \$27, \$21, \$24 and \$32 respectively.

(a) Write this information as either a row or column matrix, whichever can form a product with matrix P. (1 mark)

$$\begin{bmatrix} 27 \\ 21 \\ 24 \\ 32 \end{bmatrix}$$

(b) Use your calculator to determine the product of your answer in (a) with matrix P. (2 marks)

$$\begin{bmatrix} 211 \\ 261 \end{bmatrix}$$

(c) Briefly explain what information your answer to (b) displays. (1 mark)

Top row shows the total cost of parts for product R and the bottom row cost of parts for product S.

(d) The cost of part C is decreased by \$5 and part D is increased by \$7. What effect does this have on the cost of manufacturing each product? (2 marks)

New total cost matrix is $\begin{bmatrix} 210 \\ 272 \end{bmatrix}$.

Total cost of R has decreased by \$1 whilst total cost of S has increased by \$11.

(e) Write down a suitable matrix product to determine the number of each of the components A, B, C and D required to make 215 of product P and 335 of product Q, and evaluate this product. (3 marks)

$$\begin{bmatrix} 215 & 335 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 & 2 \\ 2 & 3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1100 & 1220 & 1315 & 1435 \end{bmatrix}$$

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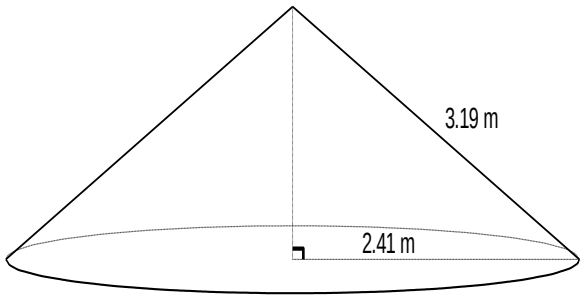
Question 15 **(7 marks)**

A pile of wheat is in the shape of cone with a sloping side of length 3.19 m and a base radius of 2.41 m as shown.



(a) Calculate the area of wheat exposed to the air (that is, the curved surface area of the pile of wheat, but not the circular base). (1 mark)

$$A = \pi \times 2.41 \times 3.19$$
$$= 24.15 \text{ m}^2$$

(b) Calculate the height of the pile of wheat, giving your answer in metres to two decimal places. (2 marks)

$$3.19^2 - 2.41^2 = 4.368$$
$$\sqrt{4.368} = 2.09 \text{ m}$$

(c) Calculate the volume of wheat in the pile, correct to one decimal place. (2 marks)

$$V = \frac{\pi \times 2.41^2 \times 2.09}{3}$$
$$= 12.7 \text{ m}^3$$

(d) After a truck delivered another load of wheat, the volume of the cone increased to 29.5 m^3 and the height increased to 2.75 m. Calculate the new radius of the base of the cone. (2 marks)

$$29.5 = \frac{\pi \times r^2 \times 2.75}{3}$$
$$r = 3.2 \text{ m}$$

See next page

Question 16 **(8 marks)**

Information about three companies listed on the Australian share market is shown below.

Company	ASX Code	Market value of share ($)	Earnings per share (cents)	Price-to-earnings ratio	Annual dividend (cents)	Percentage dividend (%)
Hunter	HHL	1.81	14	**A**	13.5	**B**
iiNet	IIN	8.40	**C**	21.5	22.1	2.63
JB HiFi	JBH	15.59	126	12.4	**D**	5.28

(a) Determine the values of A, B, C and D in the table above. (4 marks)

$181 \div 14 = A \Rightarrow A = 12.9$

$13.5 \div 181 = B \Rightarrow B = 7.46\%$

$840 \div C = 21.5 \Rightarrow C = 39$

$D \div 15.59 = 5.28\% \Rightarrow D = 82.3$

(b) An investor held a portfolio consisting of 5000 HHL shares, 1000 IIN shares and 500 JBH shares. Determine

(i) the total value of this portfolio. (2 marks)

$5000 \times 1.81 = 9050$
$1000 \times 8.40 = 8400$
$500 \times 15.59 = 7795$

$\text{Total} = \$25245$

(ii) the total annual dividend paid for this portfolio. (2 marks)

$5000 \times 0.135 = 675$
$1000 \times 0.221 = 221$
$500 \times 0.823 = 411.50$

$\text{Total} = \$1307.50$

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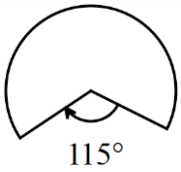
Question 17 **(11 marks)**

(a) A sector of a circle of radius 28 cm is shown below.



(i) Determine the area of the sector. (2 marks)

$360 - 115 = 245$

$\frac{\pi \times 28^2 \times 245}{360} = 1676 \text{ cm}^2$

(ii) Determine the perimeter of the sector. (2 marks)

$\frac{2 \times \pi \times 28 \times 245}{360} = 119.7$

$119.7 + 28 + 28 = 175.7 \text{ cm}$

(b) Another sector of a circle of radius 21.5 cm has an area of 335 cm^2. Determine the perimeter of this sector. (3 marks)

$335 = \frac{\pi \times 21.5^2 \times \theta}{360}$

$\theta = 83°$

$\frac{2 \times \pi \times 21.5 \times 83}{360} = 31.1$

$P = 21.5 + 21.5 + 31.1$

$= 74.1 \text{ cm}$

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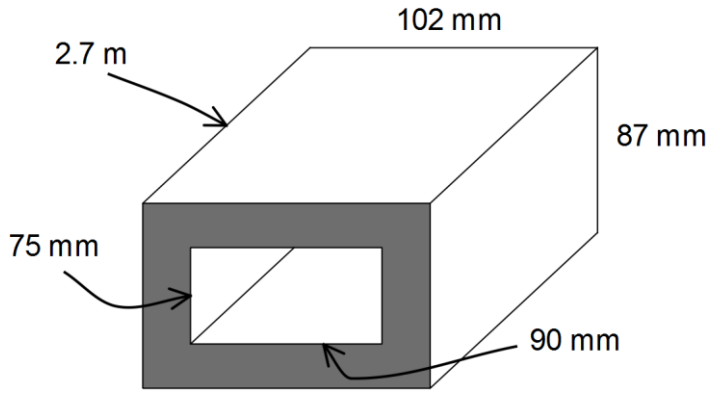
(c) A 2.7 m length of steel beam has dimensions as shown in the sketch below (not to scale).



Determine the weight of the beam if 1 cm^3 of steel has a weight of 7.8 g, giving your answer to the nearest kg. (4 marks)

Area of cross-section:

$$10.2 \times 8.7 - 9.0 \times 7.5 = 88.74 - 67.5$$
$$= 21.24 \text{ cm}^2$$

Volume of steel:

$$21.24 \times 270 = 5734.8 \text{ cm}^3$$

Weight of steel:

$$5734.8 \times 7.8 = 44731.44 \text{ g}$$

$$44731.44 \div 1000 = 44.7 \text{ kg}$$
$$\approx 45 \text{ kg}$$

See next page

Question 18 **(9 marks)**

Four students have just met to form a team, but not all members of the team know each other's mobile phone number:

- Amy only knows Ben and Cai's numbers
- Ben only knows Dara's number
- Cai only knows Amy and Ben's numbers
- Dara only knows Ben and Cai's numbers

(a) Create the four by four matrix M, to show the number of one-stage paths for each person to send a text message by mobile phone to another person. (3 marks)

		Can text this person			
		A	B	C	D
This person	A	0	1	1	0
	B	0	0	0	1
	C	1	1	0	0
	D	0	1	1	0

(b) Calculate M^2. (2 marks)

$$M^2 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

(c) Explain whether or not Ben can get a text message to Amy via another team member by referring to an element of M^2. (2 marks)

No, as $m^2_{2,1} = 0$, which means that there are no ways for Ben to send a text to Amy via two-stages.

(d) Calculate M^3 and explain what the significance of the number in the second row and first column of M^3 means. (2 marks)

$$M^3 = \begin{bmatrix} 0 & 2 & 2 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \\ 0 & 2 & 2 & 1 \end{bmatrix} \qquad m^3_{2,1} = 1$$

The 1 means that there is one way that Ben can send a text message to Amy via three-stages (two other people).

See next page

Question 19 **(9 marks)**

Consider the solid prism of length 30 cm shown in figure 1. The prism has a square cross-section with a quarter of a circle removed from the top right corner, as shown in figure 2. The square has a side length of 8 cm and the radius of the circle is 4 cm.

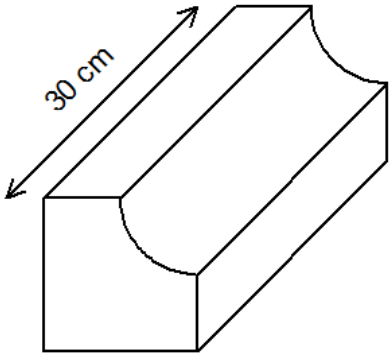


Figure 1

Not to scale

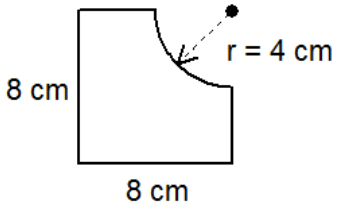


Figure 2

(a) Show that the area of the cross-section shown in Figure 2 is 51.43 cm^2, rounded to two decimal places. (3 marks)

Square: $8 \times 8 = 64$

Quadrant: $\pi \times 4^2 \div 4 = 12.57$

Area $= 64 - 12.57 = 51.43$ cm^2

(b) Calculate the volume of the solid prism shown in Figure 1. (1 mark)

Volume $= 51.43 \times 30 = 1543$ cm^3

(c) Calculate the total surface area of the prism shown in Figure 1. (5 marks)

Base and LHS: $8 \times 30 \times 2 = 480$ Top and RHS: $4 \times 30 \times 2 = 240$

Front and Back: $51.43 \times 2 = 102.86$

Curved Area: $\frac{2\pi rh}{4} = \frac{2 \times \pi \times 4 \times 30}{4} = 188.50$

Total SA $= 480 + 240 + 102.86 + 188.50$

$= 1011.36$

≈ 1011 cm^2

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____