

Semester Two Examination, 2016

Question/Answer Booklet

**MATHEMATICS
METHODS
UNITS 1 AND 2**

**Section One:
Calculator-free**

SOLUTIONS

Student Number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
Total				150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Booklet.

Section One: Calculator-free

35% (52 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1

(4 marks)

A box contains a total of 500 marker and highlighter pens of various colours, as shown in the table. Some of the marker pens are permanent and the rest are non-permanent.

	Colour			
Type of pen	Black	Yellow	Pink	Green
Permanent marker	55	83	40	24
Non-permanent marker	45	67	24	12
Highlighter	0	50	46	54

A pen is selected at random from the box. Determine the probability that it is

- (a) a yellow pen.

(1 mark)

Solution
$\frac{83 + 67 + 50}{500} = \frac{200}{500} = \frac{2}{5}$
Specific behaviours
✓ correct probability

- (b) a marker pen.

(1 mark)

Solution
$\frac{500 - (50 + 46 + 54)}{500} = \frac{350}{500} = \frac{7}{10}$
Specific behaviours
✓ correct probability

- (c) a yellow pen or a marker pen.

(1 mark)

Solution
$\frac{200 + 350 - 83 - 67}{500} = \frac{400}{500} = \frac{4}{5}$
Specific behaviours
✓ correct probability

- (d) a green pen, given that it is a highlighter.

(1 mark)

Solution
$\frac{54}{50 + 46 + 54} = \frac{54}{150} = \frac{9}{25} (= 0.36)$
Specific behaviours
✓ correct probability

Question 2

(6 marks)

- (a) Determine
- $f'(x)$
- when
- $f(x) = (x - 5)^2$
- .

(2 marks)

Solution
$f(x) = (x - 5)(x - 5) = x^2 - 10x + 25$ $f'(x) = 2x - 10$
Specific behaviours
✓ expands ✓ differentiates

- (b) Simplify

- (i)
- $\frac{d}{dx}(5x^2 - 4x + 3)$
- .

(1 mark)

Solution
$10x - 4$
Specific behaviours
✓ differentiates all terms

- (ii)
- $\lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h}$
- .

(1 mark)

Solution
$\frac{d}{dx} x^4 = 4x^3$
Specific behaviours
✓ differentiates

- (c) Calculate the gradient of the curve
- $y = 2x^5 - 3x^4$
- where
- $x = -1$
- .

(2 marks)

Solution
$\frac{dy}{dx} = 10x^4 - 12x^3$ $\frac{dy}{dx} = 10 + 12 = 22$
Specific behaviours
✓ differentiates ✓ substitutes and simplifies

Question 3

(8 marks)

- (a) The equations $x^3 + x^2 + ax + b = 0$ and $x^3 - bx^2 - ax + 4 = 0$ both have $x = 2$ as a solution. Determine the values of a and b . (4 marks)

Solution
$8 + 4 + 2a + b = 0 \Rightarrow 2a + b = -12 \dots (1)$ $8 - 4b - 2a + 4 = 0 \Rightarrow 4b + 2a = -12 \dots (2)$ $(2) - (1): 3b = 24 \Rightarrow b = 8$ $(1): 2a + 8 = -12 \Rightarrow a = -10$
Specific behaviours
<ul style="list-style-type: none"> ✓ substitute and simplify first equation ✓ substitute and simplify second equation ✓ solve for first variable ✓ solve for second variable

- (b) The equation $x^3 - x^2 - 14x + 24 = 0$ also has $x = 2$ as a solution. Determine all other solutions to the equation. (4 marks)

Solution
$x^3 - x^2 - 14x + 24 = (x - 2)(x^2 + cx - 12)$ Consider x coefficient: $-14 = -2c - 12 \Rightarrow c = 1$ $x^2 + x - 12 = (x + 4)(x - 3)$ Other solutions are $x = -4$ and $x = 3$
Specific behaviours
<ul style="list-style-type: none"> ✓ uses $(x - 2)$ to start factorisation ✓ obtains quadratic factor ✓ factorises quadratic ✓ states other two solutions

Question 4

(6 marks)

(a) A and B are independent events such that $P(A) = \frac{2}{3}$ and $P(B) = \frac{1}{4}$. Determine

(i) $P(A \cap B)$.

(1 mark)

Solution
$\frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$
Specific behaviours
✓ calculates probability

(ii) $P(B|A)$.

(1 mark)

Solution
$\frac{1}{4}$
Specific behaviours
✓ writes $P(B)$

(iii) $P(A \cup B)$.

(2 marks)

Solution
$\frac{2}{3} + \frac{1}{4} - \frac{1}{6} = \frac{8+3-2}{12} = \frac{9}{12} = \frac{3}{4}$
Specific behaviours
✓ uses probability law
✓ calculates probability

(b) A number is selected at random from the set of positive integers. Event P occurs when the number is odd, event Q occurs when the number is a multiple of five and event R occurs when the number is a perfect square. Determine the smallest number that belongs to the following sets:

(i) $\bar{P} \cap (Q \cup R)$.

(1 mark)

Solution
Even and either MF or PS: 4
Specific behaviours
✓ writes number

(ii) $\bar{P} \cap Q \cap R$.

(1 mark)

Solution
Even and MF and PS: 100
Specific behaviours
✓ writes number

Question 5

(9 marks)

Solve the following equations for x :

(a) $(x - 11)^2 - 49 = 0.$

(2 marks)

Solution
$x - 11 = \pm 7$ $x = 4, 18$
Specific behaviours
<ul style="list-style-type: none"> ✓ adjusts equation and takes square root ✓ states both solutions

(b) $27^{x+1} = 9^{1-x}.$

(3 marks)

Solution
$3^{3(x+1)} = 3^{2(1-x)}$ $3x + 3 = 2 - 2x$ $x = -\frac{1}{5}$
Specific behaviours
<ul style="list-style-type: none"> ✓ writes both sides as powers of 3 ✓ equates indices ✓ solves

(c) $\sin^2 x - \cos^2 x = \frac{1}{2}, 0 \leq x \leq 360^\circ.$

(4 marks)

Solution
$\sin^2 x - (1 - \sin^2 x) = \frac{1}{2}$ $\sin^2 x = \frac{3}{4}$ $\sin x = \frac{\sqrt{3}}{2} \Rightarrow x = 60, 120$ $\sin x = -\frac{\sqrt{3}}{2} \Rightarrow x = 240, 300$ $x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$
Specific behaviours
<ul style="list-style-type: none"> ✓ uses Pythagorean identity and simplifies ✓ shows two possible values for $\sin x$ ✓ determines first two solutions ✓ determines another two solutions

Question 6

(5 marks)

- (a) The expression $(2x - 1)^3$ can be expanded to give $8x^3 + ax^2 + 6x - 1$. Show that the value of a is -12 . (2 marks)

Solution
$(2x - 1)^3 = \binom{3}{3} (2x)^3 (-1)^0 + \binom{3}{2} (2x)^2 (-1)^1 + \dots$ $= 8x^3 + 3 \times 4x^2 \times (-1) + \dots$ $a = 3 \times 4 \times -1 = -12$
Specific behaviours
✓ shows expansion ✓ shows how x^2 coefficient derived <i>NB ✓✓ if expands correctly into quadratic, etc</i>

- (b) Using the result from (a), or otherwise, determine $f(x)$ if $f'(x) = (2x - 1)^3$ and $f(1) = 5$. (3 marks)

Solution
$f(x) = \frac{8x^4}{4} - \frac{12x^3}{3} + \frac{6x^2}{2} - x + c$ $= 2x^4 - 4x^3 + 3x^2 - x + c$ $f(1) = 2 - 4 + 3 - 1 + c = 5$ $c = 5$ $f(x) = 2x^4 - 4x^3 + 3x^2 - x + 5$
Specific behaviours
✓ antidifferentiates ✓ substitutes $x = 1$ ✓ evaluates c and clearly writes $f(x)$

Question 7

(7 marks)

The first three terms, in order, of geometric sequence are $x - 5$, $x - 1$ and $2x + 4$.

(a) Explain why $(x - 1)(x - 1) = (x - 5)(2x + 4)$.

(2 marks)

Solution
Ratio of terms: $r = \frac{T_2}{T_1} = \frac{T_3}{T_2} \Rightarrow \frac{x - 1}{x - 5} = \frac{2x + 4}{x - 1}$ Cross multiply: $(x - 1)^2 = (x - 5)(2x + 4)$
Specific behaviours
✓ uses ratio of terms ✓ uses cross multiplication

(b) Determine the value(s) of x .

(3 marks)

Solution
$x^2 - 2x + 1 = 2x^2 - 6x - 20$ $0 = x^2 - 4x - 21$ $(x - 7)(x + 3) = 0$ $x = 7, -3$
Specific behaviours
✓ expands both sides ✓ re-arranges equal to zero ✓ factorises and states both solutions

(c) Determine all possible values for the fourth term of the sequence.

(2 marks)

Solution
$x = 7$, terms are 2, 6, 18, $18 \times 3 = 54$ $x = -3$, terms are $-8, -4, -2, -2 \times \frac{1}{2} = -1$ Fourth term is either 54 or -1
Specific behaviours
✓ determines first possibility ✓ determines second possibility

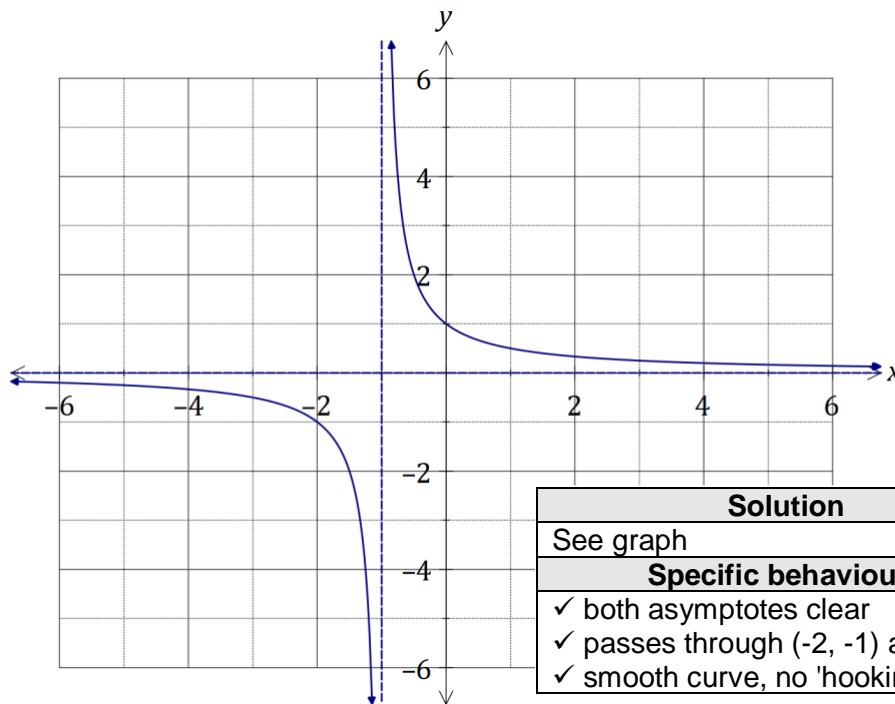
Question 8

(7 marks)

Let $f(x) = \frac{1}{x+1}$, $x \neq -1$.

(a) Sketch the graph of $y = f(x)$ on the axes below.

(3 marks)

**Solution**

See graph

Specific behaviours

- ✓ both asymptotes clear
- ✓ passes through (-2, -1) and (0, 1)
- ✓ smooth curve, no 'hooking'

(b) Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h \rightarrow 0$ to determine the slope of $f(x)$ when $x = 2$. (4 marks)

Solution

$$\begin{aligned}
 m &= \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} \text{ with } x = 2 \\
 &= \frac{\frac{1}{2+h+1} - \frac{1}{2+1}}{h} = \frac{\frac{1}{3+h} - \frac{1}{3}}{h} \\
 &= \frac{\frac{3 - 3 - h}{h \times 3(3+h)}}{h} \\
 &= \frac{-1}{9 + 3h} \\
 &= -\frac{1}{9} \text{ as } h \rightarrow 0
 \end{aligned}$$

Specific behaviours

- ✓ substitutes and simplifies quotient
- ✓ adds fractions in numerator and simplifies
- ✓ cancels h in quotient
- ✓ substitutes $h = 0$ to obtain slope

Additional working space

Question number: _____

