MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

TSSM
Creating VCE Success

2009 Trial Examination

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
11	11	40
		Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 12 pages.
- Working space is provided throughout the book.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. If $f(x) = \dfrac{3\cos(2x)}{\sin(2x)}$ show that: $f'(x) = \dfrac{-6}{\sin^2(2x)}$.

2 marks

b. Hence, find: $\displaystyle\int\left(\dfrac{2}{\sin^2(2x)} + 1\right)dx$.

2 marks

Question 2

a. If: $f(x) = -3\log_e(x+2) - 1$, show that the graph of $y = f(x)$ has a y intercept $\left(0, \log_e\left(\dfrac{1}{8e}\right)\right)$

.

_____

_____

_____

_____

_____

_____

<div align="right">2 marks</div>

b. Find the inverse function: $f^{-1}(x)$.

_____

_____
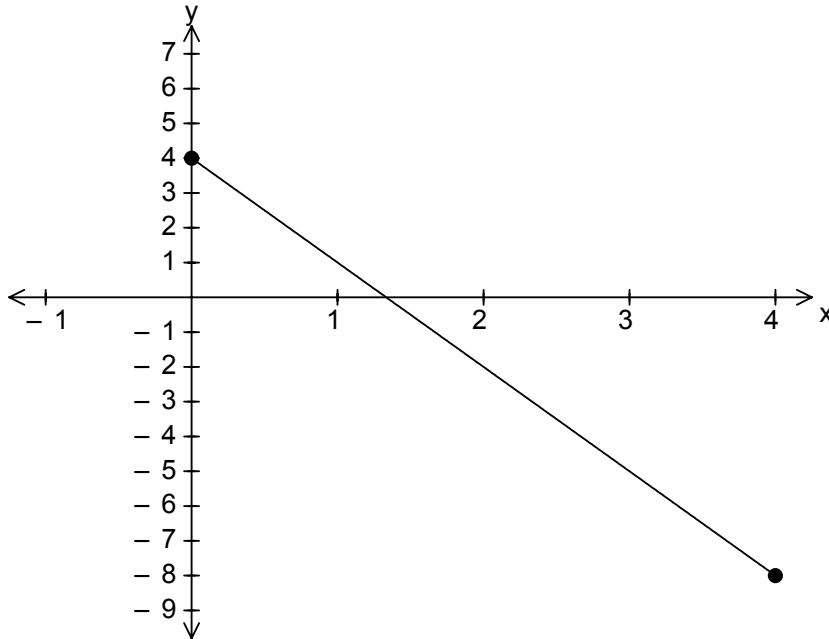
_____

_____

_____

_____

<div align="right">2 marks</div>

TURN OVER

Question 3

The graph of the function $g(x) = 4 - 3x, x \in [0,4]$ is shown on the axes below.

a. Sketch the graph of $h(x) = |g(x)| - 2$, for $x \in [0,4]$ on the same set of axes.

Show end points and axial intercepts.

_____

_____

2 marks

b. State the domain for the derivative function $h'(x)$.

_____

_____

1 mark

Question 4

a. Solve the equation $\sqrt{3}\tan\left(\dfrac{x}{2}\right)+1=0$, for $\left[-3\pi,2\pi\right]$.

_____
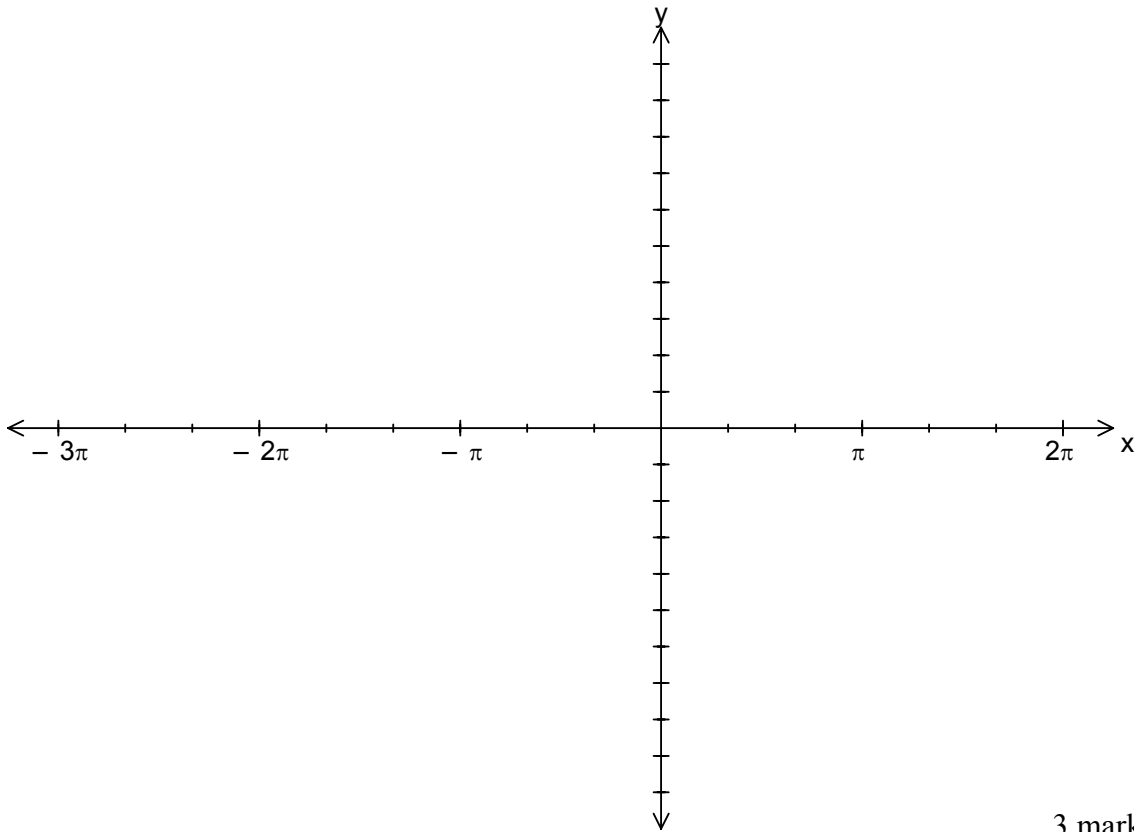
_____

_____

_____

_____

_____

<div align="right">2 marks</div>

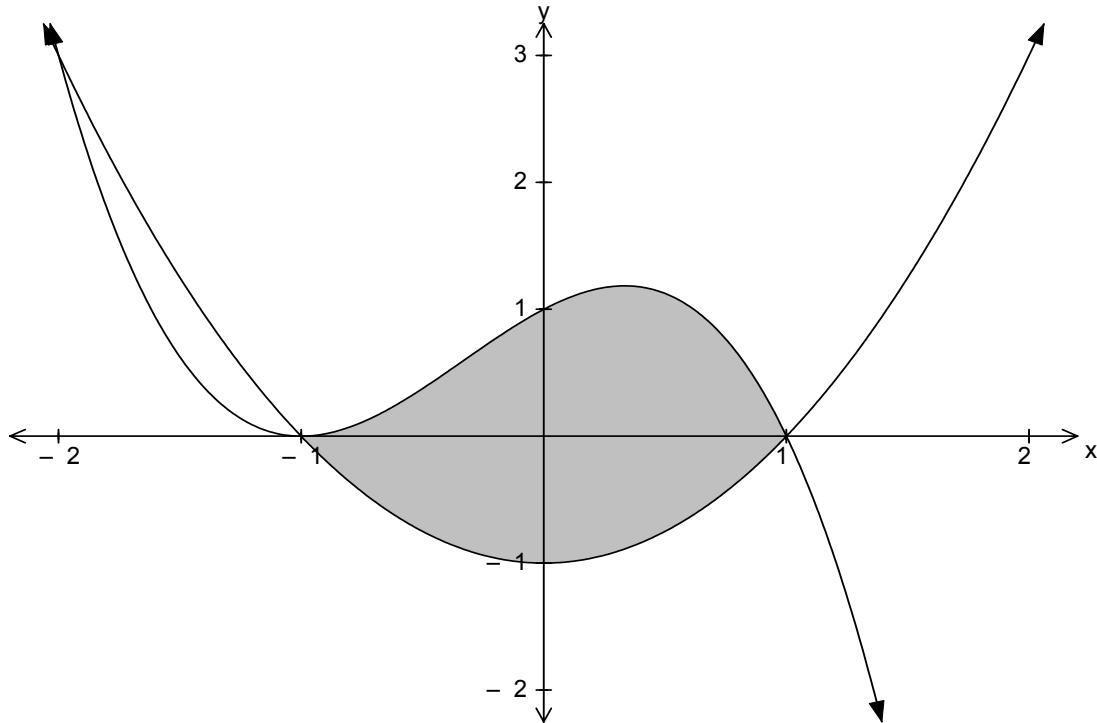**b.** Hence sketch: $y=\sqrt{3}\tan\left(\dfrac{x}{2}\right)+1$, for $\left[-3\pi,2\pi\right]$, showing all axial intercepts and asymptotes.

<div align="right">3 marks</div>
<div align="right">**TURN OVER**</div>

Question 5

The graphs of $f(x) = (x-1)(x+1)$ and $g(x) = (1-x)(x+1)^2$ are shown below.

Find the exact area enclosed by the curves $f(x)$ and $g(x)$ for $x \in [-1,1]$.

_____

_____

_____

_____

_____

_____

<div align="right">3 marks</div>

Question 6

If $f(x) = \dfrac{1}{\sqrt{x}}$, use the approximation formula $f(x+h) \approx f(x) + h.f'(x)$ to find the approximate value of $\dfrac{1}{\sqrt{4.1}}$. Give your answer as a **fraction**.

_____

_____

_____

_____

_____

<div align="right">3 marks</div>

Question 7

A spinner is numbered 1 to 5. The number X on the spinner is a discrete random variable, with a probability distribution given by:

x	1	2	3	4	5
Pr(X=x)	0.35	0.25	0.1	0.15	0.15

a. What is the median of X?

_____

_____

<div align="right">1 mark</div>

b. Find the probability of spinning 5, given that the number is greater than 2.

_____

_____

_____

<div align="right">1 mark</div>
<div align="right">**TURN OVER**</div>

Question 8

Tom is a keen windsurfer and he has noticed that the conditions at Sandy Beach follow a pattern. If it is windy on one day, the probability that it will be windy on the next day is 0.8. If it is still on one day, the probability of it being windy on the next day is 0.5.

a. If it is windy on Thursday, find the probability that it will be windy on Saturday **and** Sunday, correct to 3 decimal places.

_____

_____

_____

_____

_____

_____

_____

_____

2 marks

b. If it is still on Friday, find the probability that it will be windy on **at least** one day over the weekend, correct to 3 decimal places.

_____

_____

_____

_____

_____

_____

_____

_____

2 marks

Question 9

A continuous random variable X has a probability density function defined by

$$f(x) = \begin{cases} \dfrac{x}{2} & 1 \le x \le 2 \\ k & 2 < x \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

a. Use calculus to show that: $k = \dfrac{1}{8}$.

_____

_____

_____

_____

_____

_____

2 marks

b. Find Pr $(X \le 3)$.

_____

_____

_____

_____

_____

1 mark

TURN OVER

Question 10

The heights of boys in Year 12 are normally distributed with a mean of 175 cm and a standard deviation of 10 cm. It is known that Pr $(Z > 0.5) = 0.3$.

a. Using the information above, find the probability that a boy selected at random is greater than 170 cm.

_____

_____

_____

_____

_____

2 marks

b. Find the probability that a randomly selected boy is more than 180cm, given that his height is above the mean.
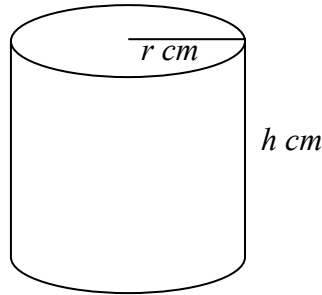
_____

_____

_____

_____

_____

1 mark

Question 11

Joan is baking a nut loaf in a cylindrical tin that **doesn't** have a lid.

a. If she uses $200\pi \ cm^2$ of paper to line the base and sides of the tin, show that: $h = \dfrac{100}{r} - \dfrac{r}{2}$.

_____

_____

_____

_____

_____

_____

<div align="right">2 marks</div>

b. Show that the volume of the tin is given by: $V = 100\pi r - \dfrac{\pi r^3}{2}$.

_____

_____

_____

_____

_____

<div align="right">2 marks</div>
<div align="right">**TURN OVER**</div>

c. Use calculus to show that the maximum volume of the tin will be when $r = \dfrac{10\sqrt{6}}{3} cm$.

_____

_____

_____

_____

_____

_____

<div align="right">2 marks</div>

END OF QUESTION AND ANSWER BOOK