MATHEMATICS METHODS

MAWA Semester 2 (Units 3 and 4) Examination 2017

Calculator-free

Marking Key

The release date for this exam and marking scheme is

- **the end of week 1 of term 4, 2017**

Section One: Calculator-free (52 Marks)

Question 1(a)

Solution	
$k + 2k + 0.5k + 0.5k + k = 1$ $$5k = 1$$ $$k = \frac{1}{5}$$	
Marking key/mathematical behaviours	Marks
• correctly solves for k.	1

Question 1(b)

Solution	
No, This is not a discrete probability distribution as $P(5)$ has a negative value and probabilities need to positive.	
Marking key/mathematical behaviours	Marks
• recognises the properties of discrete probability distributions	1
• relates above property back to the question to justify answer	1

Question 1(c)

Solution	
$$\mu = np = 90$$ $$\therefore \sigma^2 = 6^2 = 36$$ $$\sigma^2 = np(1-p)$$ $$36 = 90(1-p) \quad \text{Expand and solve for } p$$ $$\therefore p = 0.6$$ $$\frac{6}{10}n = 90 \quad \text{Sub value of } p \text{ and solve for } n.$$	
Marking key/mathematical behaviours	Marks
• determines variance	1
• solves for parameter, p	1
• solves for parameter, n	1

Question 2(a)

Solution	
A census involves **every member** of the **population** being tested/questioned/investigated	
Marking key/mathematical behaviours	**Marks**
• indicates the need to include every member of the population	1

Question 2(b)

Solution	
There would be no stoves left to sell as all of them would have broken down.	
Marking key/mathematical behaviours	**Marks**
• indicates that there would be no items left for sale (no marks for cheaper or quicker)	1

Question 2(c)

Solution	
Use the unique serial numbers to select a random sample or similar	
Marking key/mathematical behaviours	**Marks**
• indicates use of a suitable random selection method (based on serial numbers or other method)	1

Question 2(d)

Solution	
Using the list of the serial numbers, select every 400th stove	
Marking key/mathematical behaviours	**Marks**
• indicates use of a suitable selection method	1

Question 3(a) (i)

Solution	
$f'(x) = \dfrac{(2 + \cos x) \times 1 - (-\sin x)x}{(2 + \cos x)^2} = \dfrac{2 + \cos x + x \sin x}{(2 + \cos x)^2}$	
Marking key/mathematical behaviours	**Marks**
• correct use of the quotient rule	1
• differentiates correctly	1

Question 3(a) (ii)

Solution	
$f'(x) = (3 + 2\cos 2x)e^{3x + \sin 2x}$	
Marking key/mathematical behaviours	**Marks**
• correct use of the chain rule	1
• differentiates correctly	1

Question 3(b)

Solution	
Differentiating both sides: $f''(x)f(x) + f'(x)^2 = 0$	
so $\quad f''(x) = -\dfrac{f'(x)^2}{f(x)}$	
But $f'(x) = \dfrac{1}{f(x)}$ and so $f''(x) = -\dfrac{1}{f(x)^3}$ i.e. $f''(x)f(x)^3 = -1$	
Marking key/mathematical behaviours	Marks
• uses product rule correctly	1
• obtains correct expression for $f''(x)$	1
• replaces $f'(x)$ with $\dfrac{1}{f(x)}$	1

Question 4(a)(i)

Solution	
Approximately 200 samples are involved	
Marking key/mathematical behaviours	Marks
• States the number of samples (allow 190 to 210)	1

Question 4(a)(ii)

Solution	
Find the mean of the sample proportions, (from the graph) ≈ 0.4 May use sample proportion as an estimate of the population proportion	
Marking key/mathematical behaviours	Marks
• identifies sample proportion as 0.4 (by reference to the graph or calculation)	1
• uses the sample proportion as an estimate for the population proportion	1

Question 4(b)

Solution	
• Survey is restricted to listeners of one particular station and therefore not representative of the population • Survey is using a self-selection model and this indicates bias • Timing may exclude some groups of people • Access to a telephone is presumed • People could respond more than once • Nature of the question means football fans may be more likely to respond	
Marking key/mathematical behaviours	Marks
• Lists one possibility	1
• Lists a second possibility	1

Question 5

Solution
Note that:

$$y = 1 - 4x^2 \Rightarrow y' = -8x$$

$$\therefore \int \frac{10x}{1-4x^2}dx \quad = -\frac{10}{8}\int \frac{-8x}{1-4x^2}dx$$

$$= -\frac{5}{4}\ln|1-4x^2| + c$$

Marking key/mathematical behaviours	Marks
• Recognises $\frac{f'(x)}{f(x)}$	1
• Determines (or uses) derivative of denominator	1
• Uses constants to achieve numerator of -8x	1
• Finds integral accurately	1

Question 6(a)

Solution	
Since $5^{-2} = \frac{1}{25}$ we have $\log_5\left(\frac{x+24y}{25}\right) = \ln_5(x+y)$ (#)	
i.e. $\frac{x+24y}{25} = x+y$ and hence $y = -24x$	
Marking key/mathematical behaviours	Marks
• Obtains equation (#) or equivalent	1
• Obtains correct answer	1

Question 6(b)(i)

Solution
If $a > b$ then $\log_a b < 0$ i.e. negative

Marking key/mathematical behaviours	Marks
• obtains correct answer	1

Question 6(b)(ii)

Solution
If $a \neq b$ then $\log_a b$ and $\log_b a$ have opposite signs (#) and are unequal.
So $a = b$ if $\log_a b = \log_b a$

Marking key/mathematical behaviours	Marks
• deduces that $\log_a b$ and $\log_b a$ have opposite signs if $a \neq b$	1
• complete proof correctly	1

Question 7(a)

Solution
$$A = -\int_1^4 \left(-\frac{8}{\sqrt{x}}\right) dx$$ $$= \int_1^4 \frac{8}{\sqrt{x}} \, dx$$

Marking key/mathematical behaviours	Marks
• sets up integral in either form	1

Question 7(b)

Solution
$$A = \int_1^4 \frac{8}{\sqrt{x}} \, dx$$ $$= \left[16\sqrt{x}\right]_1^4$$ $$= 16(2) - 16$$ $$= 16 \text{ square units}$$

Marking key/mathematical behaviours	Marks
• integrates correctly	1
• correctly evaluates the integral	1

Question 7(c)

Solution
$$\int_1^a -\frac{8}{\sqrt{x}} \, dx = -25$$ $$\Rightarrow -16\sqrt{a} + 16 = -25$$ $$-16\sqrt{a} = -41$$ $$a = \frac{41^2}{16^2}$$

Marking key/mathematical behaviours	Marks
• correctly states the equation to be solved	1
• solves for a	1

Question 8(a)

Solution
$$\int_0^c \frac{x^3}{4}\,dx$$ $$=\frac{1}{4}\left[\frac{x^4}{4}\right]_0^c$$ $$\frac{c^4}{16}=1$$ $$c^4=16$$ $$c=\pm 2$$ As -2 is outside of the lower boundary $$c=2$$

Marking key/mathematical behaviours	Marks
• uses the correct integral	1
• equates the pdf to one and solves for c	1
• Clearly states why $c=-2$ has been rejected	1

Question 8(b)

Solution
$$E(x)=\int_{-\infty}^{\infty} xw(x)\,dx$$ $$=\int_{12}^{32} x\left(\tfrac{1}{20}\right)dx$$ $$=\left[\tfrac{1}{20}\times\tfrac{x^2}{2}\right]_{12}^{32}$$ $$=\tfrac{1}{40}(32^2-12^2)$$ $$=22$$ $$\therefore E(x)=22$$

Marking key/mathematical behaviours	Marks
• Substitutes into the correct formula	1
• Integrates correctly and arrives at the required answer	1

Question 8(c)

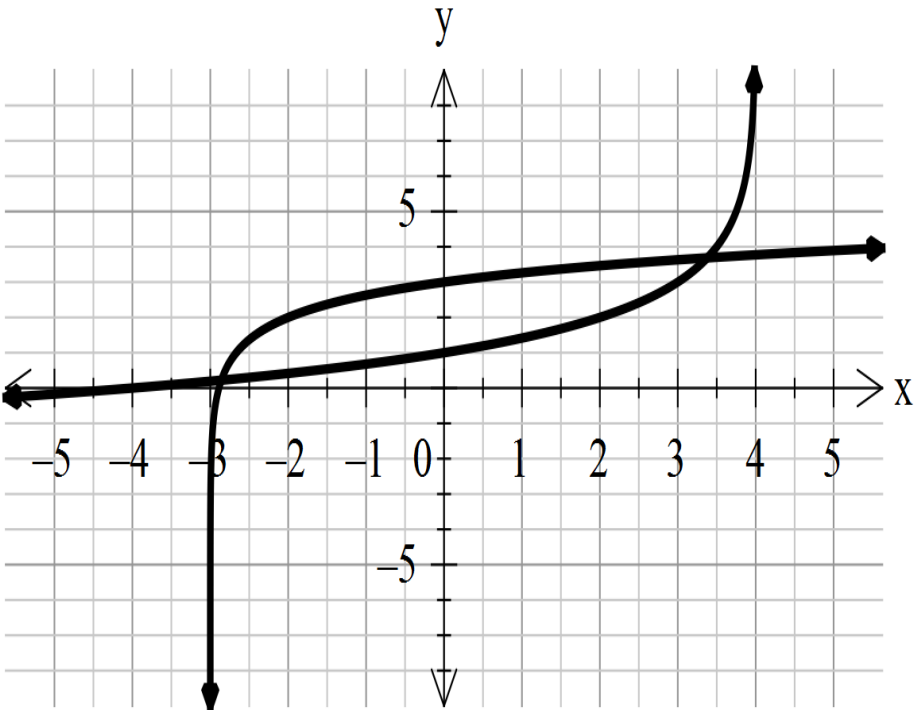| Solution	
(i) From the graph, read the relative frequencies for the parking times (0,30] mins and (30, 60] mins ➔ Pr (at most 60 minutes) = 0.38 + 0.42 = 0.8. (ii) relative frequency of vehicles parked between 1 and 1.5hrs = 0.14. 200 x 0.14 = 28 vehicles. (iii) Answers will vary. One possible answer: The data shows that 80% of the vehicles are parked for at most 60 minutes. Businesses that would tailor to this time length could be a nearby café or a mini mall with a few necessities stores (ie. Grocery store, bakery, pharmacy, clothing shop). (iv) Answers will vary. One possible answer: On which day of the week was the data collected?	
Marking key/mathematical behaviours	**Marks**
• sums the two relative frequencies required • multiplies the relative frequency by the total number of parked cars • links to a type of business requiring less than 1 hour of parking • links question to day of the week or other plausible variable.	1 1 1 1

Question 9(a)

Solution	
Since the graph has a vertical asymptote at $x = -3$, $b = 3$ Since the point $(-2,2)$ lies on the graph, $2 = \log_a(-2+3)+c$, i.e. $c = 2$ Since the point $(0,3)$ lies on the graph, $3 = \log_a + 2$, i.e. $\log_a(3) = 1$ and hence $a = 3$	
Marking key/mathematical behaviours	**Marks**
• evaluates a b and c correctly	1, 1, 1

Question 9(b)

Solution

Marking key/mathematical behaviours	Marks
• asymptote at $x = 4$	1
• y intercept at $(0,1)$	1
• x intercept at $(-4,0)$	1
• graph of standard logarithm function reflected in both axes	1