Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS METHODS UNITS 1&2

Section One: Calculator-free

SOLUTIONS

Student Name ______________________________

Teacher Name ______________________________

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work:	five minutes
Working time:	fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free **35% (52 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

(a) Determine $\frac{d}{dx}\left((3x+2)(x-5)\right)$. (2 marks)

Solution
$\frac{d}{dx}(3x^2-13x-10)=6x-13$
Specific behaviours
✓ expands expression ✓ correctly differentiates

(b) Determine $f'(-1)$ when $f(x)=x^7+3x^4-7x$. (2 marks)

Solution
$f'(x)=7x^6+12x^3-7$ $f(-1)=7(-1)^6+12(-1)^3-7$ $=7-12-7$ $=-12$
Specific behaviours
✓ correctly differentiates ✓ correct value

(c) Determine the instantaneous rate of change of area A when $t=3$ if the area of a region at time t seconds is given by $A=\frac{11}{45}-\frac{7t}{3}+\frac{2t^2}{5}-\frac{t^3}{15}$ cm^2. (2 marks)

Solution
$\frac{dA}{dt}=-\frac{7}{3}+\frac{4t}{5}-\frac{3t^2}{15}$ When $t=3$ $\frac{dA}{dt}=-\frac{7}{3}+\frac{12}{5}-\frac{27}{15}$ $=\frac{-35+36-27}{15}=-\frac{26}{15}\text{ cm}^2/\text{s}$
Specific behaviours
✓ obtains $A'(t)$ ✓ correct rate of change

Question 2 **(7 marks)**

(a) Expand $(x+2)^3$. (2 marks)

Solution
$(x+2)^3 = (x+2)(x^2+4x+4)$ $= x^3+2x^2+4x^2+8x+4x+8$ $= x^3+6x^2+12x+8$ Alternative: $(x+2)^3 = 1\cdot x^3 2^0 + 3\cdot x^2 2^1 + 3\cdot x^1 2^2 + 1\cdot x^0 2^3$ $= x^3+6x^2+12x+8$
Specific behaviours
✓ indicates reasonable attempt to use appropriate method ✓ correct expansion

(b) Solve the equation $\dfrac{x^2+2x}{x^2-2x+4} = 1$. (2 marks)

Solution
$\dfrac{x^2+2x}{x^2-2x+4} = 1$ $x^2+2x = x^2-2x+4$ $4x = 4$ $x = 1$
Specific behaviours
✓ cross multiplies and simplifies ✓ correct value of x

(c) Determine the centre and radius of the circle with equation $x^2+y^2+8y=0$. (3 marks)

Solution
$x^2+y^2+8y=0$ $x^2+(y+4)^2-16=0$ $x^2+(y+4)^2=4^2$ Hence centre is at $(0,-4)$ and radius is 4.
Specific behaviours
✓ completes square ✓ states centre ✓ states radius

Question 3 **(9 marks)**

(a) Solve the equation $4\cos^2\left(\frac{x}{2}\right) - 3 = 0$ for $0 \le x \le 2\pi$. (3 marks)

Solution
$\cos\left(\frac{x}{2}\right) = \pm\frac{\sqrt{3}}{2}$ $\frac{x}{2} = \frac{\pi}{6}, \frac{5\pi}{6}$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$
Specific behaviours
✓ obtains expression for $\cos\frac{x}{2}$ ✓ obtains one correct half-angle ✓ both correct solutions

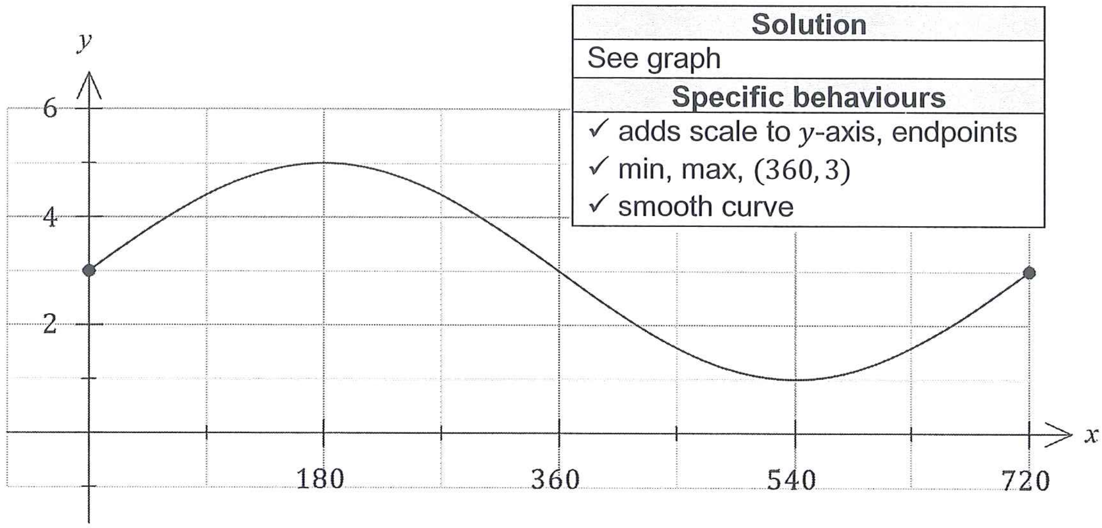
(b) The periodic function f is defined as $f(x) = 2\sin\left(\frac{x}{2}\right) + 3$.

(i) State the amplitude and period of f. (2 marks)

Solution
Amplitude is 2 and period is $360° \div 0.5 = 720° = 4\pi$.
Specific behaviours
✓ correct amplitude ✓ correct period

(ii) Sketch the graph of $y = f(x)$ on the axes below over the domain $0 \le x \le 720°$. (3 marks)

Solution
See graph
Specific behaviours
✓ adds scale to y-axis, endpoints ✓ min, max, $(360, 3)$ ✓ smooth curve



(iii) State the range of f. (1 mark)

Solution
$R_f = \{y: y \in \mathbb{R}, 1 \le y \le 5\}$
Specific behaviours
✓ correct restriction for y

Question 4 **(7 marks)**

(a) Determine $f(2)$ in simplified form when $f(x) = 9^{0.5-x}$. (2 marks)

Solution
$f(2) = 9^{0.5-2}$ $= 9^{-\frac{3}{2}}$ $= 3^{-3}$ $= \frac{1}{27}$
Specific behaviours
✓ simplifies to correct power of 3 ✓ correct value

(b) Determine the value of $a^2 \div b$ in scientific notation when $a = 8 \times 10^{-3}$ and $b = 1.6 \times 10^{-2}$. (2 marks)

Solution
$\frac{a^2}{b} = \frac{(8 \times 10^{-3})^2}{1.8 \times 10^{-2}}$ $= \frac{64 \times 10^{-6}}{1.6 \times 10^{-2}}$ $= 40 \times 10^{-4}$ $= 4 \times 10^{-3}$
Specific behaviours
✓ correct value of a^2 ✓ correct value in scientific notation

(c) Solve the equation $\sqrt{10\,000^x} = 0.1\sqrt{10}$. (3 marks)

Solution
$\sqrt{10\,000^x} = 0.1\sqrt{10}$ $\sqrt{10^{4x}} = \frac{\sqrt{10}}{10}$ $10^{2x} = 10^{-\frac{1}{2}}$ $2x = -\frac{1}{2}$ $x = -\frac{1}{4}$
Specific behaviours
✓ expresses LHS as power of 10 ✓ expresses RHS as power of 10 ✓ correct value of x

Alternative Solution
$\sqrt{10\,000^x} = \frac{1}{10}\sqrt{10}$ $10000^x = \frac{10}{100}$ $10^{4x} = 10^{-1}$ $4x = -1$ $x = -\frac{1}{4}$
Specific behaviours
✓ squares both sides ✓ expresses both sides as powers of 10 ✓ correct value of x

Question 5 **(7 marks)**

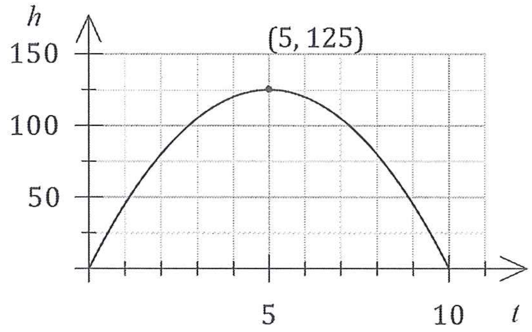
(a) Given $\frac{dh}{dt} = 12t^3 - 12t - 1$ and $h = 30$ when $t = 2$, determine the value of h when $t = 1$.

(3 marks)

Solution
$h = \frac{12t^4}{4} - \frac{12t^2}{2} - t + c$ $= 3t^4 - 6t^2 - t + c$ $30 = 3(2)^4 - 6(2)^2 - 2 + c \Rightarrow c = 8$ $h(1) = 3(1)^4 - 6(1)^2 - 1 + 8$ $= 3 - 6 - 1 + 8$ $= 4$
Specific behaviours
✓ correctly antidifferentiates ✓ evaluates constant c ✓ calculates value of h

(b) The height h metres above the ground of a small body t seconds after it is projected vertically upwards is shown in the position-time graph.

Given that $h = at^2 + bt$, where a and b are constants, determine the speed of the body when $t = 3.8$.



(4 marks)

Solution	Alternate Solution
Using roots: $h = at(t - 10)$ Using point: $125 = 5a(5 - 10) \Rightarrow a = -5$ Velocity: $h = -5t^2 + 50t$ $\frac{dh}{dt} = -10t + 50$ Speed: $h'(3.8) = -10(3.8) + 50 = 12\text{ m/s}.$	Using $(5,125)$: $125 = 25a + 5b$ Using $(10,0)$: $0 = 100a + 10b$ Solve simultaneously: $a = -5, b = 50$ $h = -5t^2 + 50t$ $\frac{dh}{dt} = -10t + 50$ Speed: $h'(3.8) = -10(3.8) + 50 = 12\text{ m/s}.$
Specific behaviours	**Specific behaviours**
✓ uses roots or turning point to obtain expression for h in terms of a and t ✓ obtains expression for h in terms of t ✓ correctly differentiates ✓ correct speed	✓ forms 2 equations ✓ obtains expression for h in terms of t ✓ correctly differentiates ✓ correct speed

Question 6 **(7 marks)**

Let $f(x) = \frac{1}{4}(x-4)(x-8)$.

(a) Determine the equation of the tangent to the curve $y = f(x)$ when $x = 0$. (4 marks)

Solution
$f(x) = \frac{1}{4}(x^2 - 12x + 32) = \frac{x^2}{4} - 3x + 8$ $f'(x) = \frac{x}{2} - 3$ $f'(0) = -3, \quad f(0) = 8$ $y - 8 = -3(x - 0)$ $y = -3x + 8$
Specific behaviours
✓ expands polynomial   ✓ obtains derivative   ✓ calculates slope of tangent and y-coordinate   ✓ correct equation of tangent

(b) The tangent to the curve $y = f(x)$ at $(2, 3)$ is perpendicular to a different tangent to the same curve at point P. Determine the equation of the tangent at P. (3 marks)

Solution
Let slope at P be m: $m = -\frac{1}{f'(2)} = -\frac{1}{-2} = \frac{1}{2}$ Then require $f'(x) = m$: $\frac{x}{2} - 3 = \frac{1}{2}, \quad \frac{x}{2} = \frac{7}{2}, \quad x = 7$ y-coordinate of P: $y = \frac{1}{4}(3)(-1) = -\frac{3}{4}$ Tangent: $y - \left(-\frac{3}{4}\right) = \frac{1}{2}(x - 7)$ $y = \frac{1}{2}x - \frac{17}{4}$
Specific behaviours
✓ obtains slope at P ✓ obtains coordinates of P ✓ correct equation of tangent

Question 7 **(9 marks)**

Let $f(x) = x^3 - 3x + 2$.

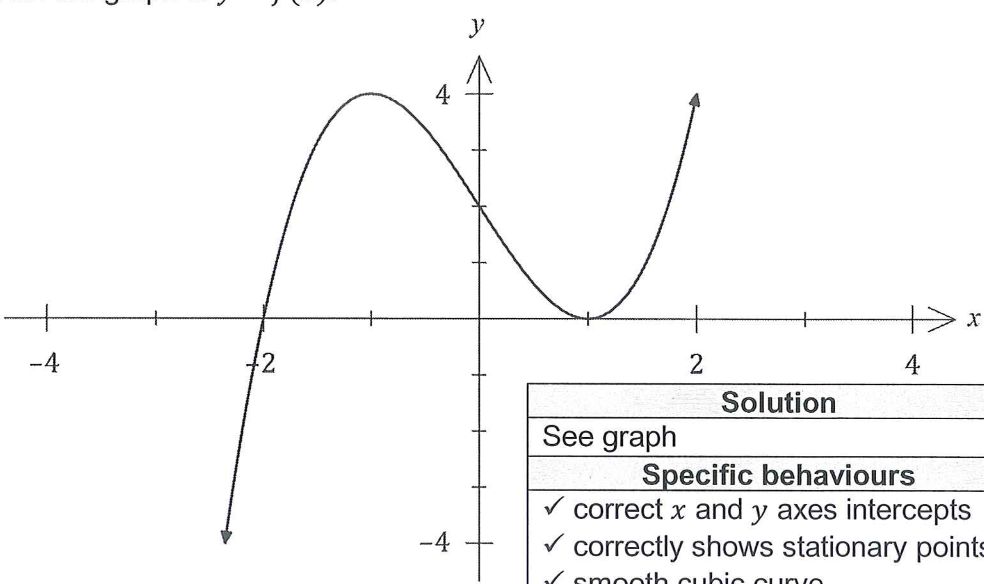
(a) Show that $f(-2) = 0$ and hence factorise $f(x)$. (3 marks)

Solution
$f(-2) = -8 + 6 + 2 = 0$ $f(x) = (x + 2)(x^2 + ax + 1)$ Using x^2 terms, $a + 2 = 0 \Rightarrow a = -2$. $f(x) = (x + 2)(x^2 - 2x + 1)$ $= (x + 2)(x - 1)(x - 1)$
Specific behaviours
✓ shows sum of three terms is zero ✓ expresses f as product of linear and quadratic factor ✓ determines all linear factors

(b) Determine the location of the stationary points of the curve $y = f(x)$. (3 marks)

Solution
$f'(x) = 3x^2 - 3$ $f'(x) = 0 \Rightarrow 3(x^2 - 1) = 3(x + 1)(x - 1) = 0 \Rightarrow x = \pm 1$ $f(1) = 0, \qquad f(-1) = 4$ Stationary points at $(1, 0)$ and $(-1, 4)$.
Specific behaviours
✓ correct derivative of f ✓ indicates that $f'(x) = 0$ ✓ correct coordinates of stationary points

(c) Sketch the graph of $y = f(x)$. (3 marks)



Solution
See graph
Specific behaviours
✓ correct x and y axes intercepts ✓ correctly shows stationary points ✓ smooth cubic curve

End of questions

Supplementary page

Question number: _________