

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section Two: Calculator-assumed

SOLUTIONS

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed**65% (98 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8

(7 marks)

- (a) The monthly units of electricity u consumed by each apartment in a building was strongly associated with the average monthly maximum temperature, T °C. The least-squares line for the variables was $\hat{u} = 88.5 + 2.7T$.

- (i) Predict the units of electricity consumed by an apartment in a month when the average monthly maximum temperature was 33°C. (1 mark)

Solution
$\hat{u} = 88.5 + 2.7(33) = 177.6$ units.
Specific behaviours
✓ correct prediction

- (ii) In a month when the average monthly maximum temperature was 25°C, an apartment consumed 153.4 units of electricity. Calculate the residual for this data point. (2 marks)

Solution
$\hat{u} = 88.5 + 2.7(25) = 156$
Residual: $u - \hat{u} = 153.4 - 156 = -2.6$ units.
Specific behaviours
✓ indicates correct \hat{u}
✓ correct residual

- (b) In a government study, the correlation coefficient for the association between age and superannuation balance for employed adults was found to be 0.755. What percentage of the variation in superannuation balance for employed adults is unexplained by their variation in age? (2 marks)

Solution
$r^2 = 0.755^2 = 0.57$
Since 57% of the variation is explained, then 43% is unexplained.
Specific behaviours
✓ calculates coefficient of determination
✓ correct percentage

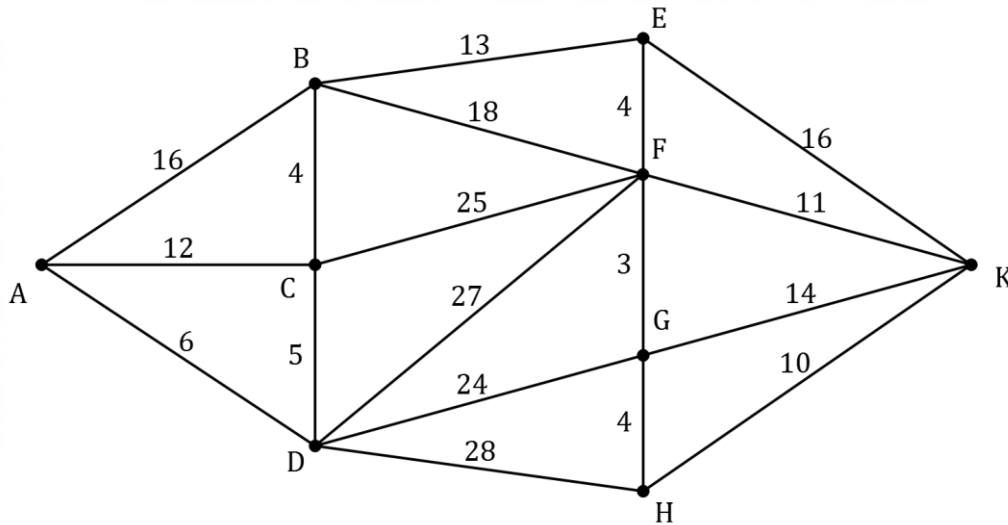
- (c) After measuring the age and hearing acuity of a group of pensioners, a researcher observed a negative linear association between the variables and found that 61% of the variation in hearing acuity can be explained by the variation in age. Determine the correlation coefficient for the association. (2 marks)

Solution
$r^2 = 0.61 \rightarrow r = \pm\sqrt{0.61} = \pm 0.781$
Since association is negative, then $r = -0.781$.
Specific behaviours
✓ indicates square root of coefficient of determination
✓ correct correlation coefficient

Question 9

(7 marks)

The graph below represents a network of distribution centres. Each edge weight is the cost in dollars to transport a parcel between adjacent centres (the vertices).



- (a) Determine the minimum cost to transport a parcel from A to K and state the path that should be used to achieve this minimum. (3 marks)

Solution
Minimum distance each vertex is from A : $A0, B15, C11, D6, E28, F32, G30, H34, K43$
Hence minimum cost is \$43 using path $A - D - C - B - E - F - K$.
Specific behaviours
✓ evidence of method (listing of trials on graph, etc.) ✓ correct minimum cost ✓ lists the path which gives the cost stated

- (b) A new route is proposed between centres C and G which will reduce the minimum cost to transport a parcel from A to K by \$7. Determine the cost to transport a parcel between centres C and G . (2 marks)

Solution
$AC = 11, GK = 14$ $11 + CG + 14 = 43 - 7$ $CG = 36 - 25 = 11$ Hence cost is \$11.
Specific behaviours
✓ indicates appropriate method ✓ correct cost

- (c) A parcel is transported along a route that is a cycle of 3 edges in the graph. Determine the maximum possible transport cost and describe the corresponding cycle. (2 marks)

Solution
Cycle for maximum cost is $CDFC$ and cost is \$57. (Also $CFDC, FCDF, \dots$ etc)
Specific behaviours
✓ correctly describes a cycle ✓ correct cost

Question 10**(8 marks)**

A random sample of adults who were not working and not seeking work were recently asked for the main reason that they were not looking for work. The responses, categorised by the sex of the adult and their main reason, are summarised in the table below.

Reason	Male	Female
Education	143	116
Family considerations	33	117
Other	84	37

- (a) How many adults gave a response?

(1 mark)

Solution
$143 + 33 + 84 + 116 + 117 + 37 = 530$ adults.
Specific behaviours
✓ correct total

- (b) What percentage of the females gave education as their main reason?

(2 marks)

Solution
$116 + 117 + 37 = 270$
$116 \div 270 = 43\%$
Specific behaviours
✓ indicates use of correct figures
✓ any percentage that rounds to 43%

- (c) Complete the following table showing column percentages for the above data, rounding entries to the nearest whole number.

(2 marks)

Solution		
Reason	Male (%)	Female (%)
Education	55	43
Family considerations	13	43
Other	32	14
Specific behaviours		
✓ both columns add to 100		
✓ both columns correct		

- (d) Discuss whether the data from the survey suggests the presence of an association between the variables, sex and reason, giving an example to support your analysis.

(3 marks)

Solution
The data does suggest an association exists between the variables as the pairs of percentages in each row (i.e., for each reason) are quite different for males and females. For example, 55% of Males compared to 43% of Females, gave education as their main reason.
Specific behaviours
✓ states association exists
✓ explains using difference in percentages across reason categories
✓ provides example comparing Males and Females for one response.

See next page

Question 11

(8 marks)

The cooling system for a mobile cool room has just been turned on. The temperature T_n °C inside the cool room, n hours later, is modelled by the linear recurrence relation

$$T_{n+1} = 0.68T_n + 1.12, \quad T_0 = 19.$$

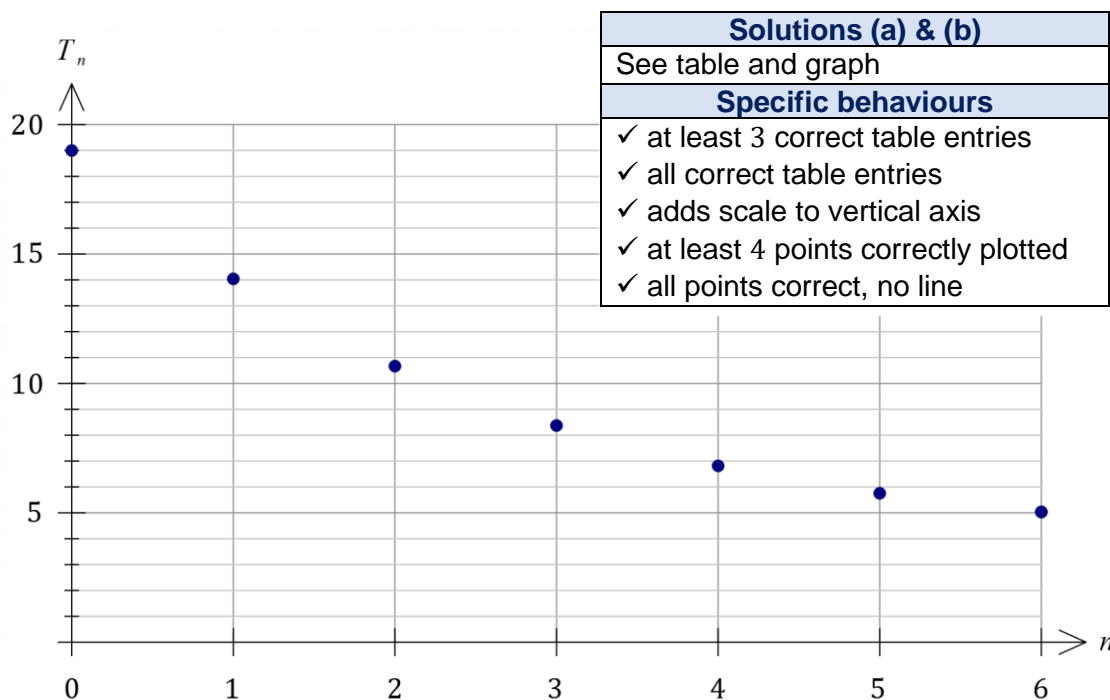
(a) Complete the table of temperatures below.

(2 marks)

n	0	1	2	3	4	5	6
T_n (°C)	19.0	14.0	10.7	8.4	6.8	5.8	5.0

(b) Add a scale to the vertical axis below and then plot the temperature inside the cool room every hour.

(3 marks)



(c) After how many hours does the model predict that the temperature inside the cool room will first reach within 0.2° of its steady state? Justify your answer.

(3 marks)

Solution
Using either the sequence or solving $T_n = 0.68T_n + 1.12$ the steady state temperature is 3.5° C. Hence, to be within 0.2° the temperature must be 3.7° C or below. From the sequence, $T_{11} = 3.72$ and $T_{12} = 3.65$ and so the cool room will first reach required temperature after 12 hours.
Specific behaviours
✓ indicates steady state temperature ✓ states correct number of hours ✓ valid justification

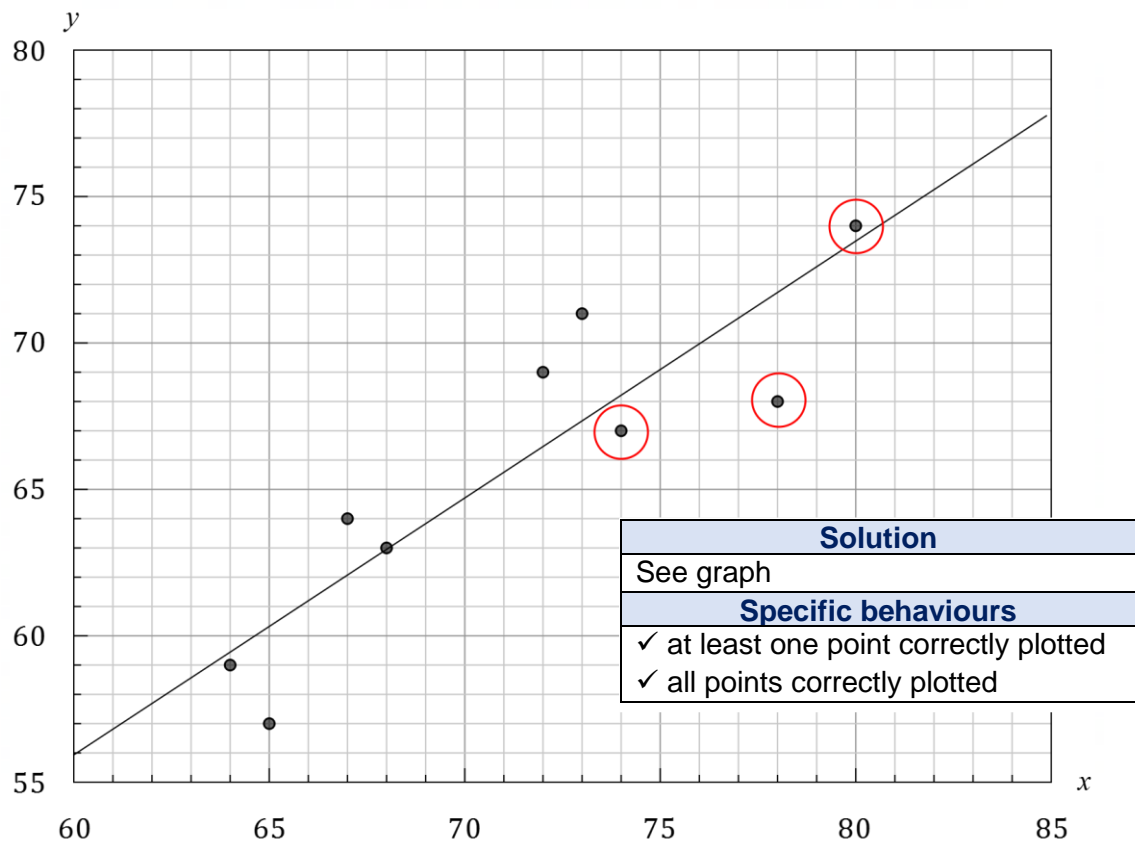
Question 12

(13 marks)

The table below shows the life expectancy, in years, of females and males in nine countries in Oceania.

Country	Female (x)	Male (y)
Federated States of Micronesia	72	69
Kiribati	64	59
Marshall Islands	73	71
Nauru	65	57
New Caledonia	80	74
Palau	78	68
Papua New Guinea	68	63
Solomon Islands	74	67
Tuvalu	67	64

- (a) On the scatterplot below, plot the three missing data points from the table. (2 marks)



- (b) Determine the coefficient of determination between the variables and interpret its value in the context of the question. (2 marks)

Solution
$r^2 = 0.7947 \approx 0.79$
79% of the variation in the male life expectancy can be explained by the variation in the female life expectancy.
Specific behaviours
✓ correct coefficient (to at least 2 dp)
✓ correct interpretation

- (c) State the correlation coefficient between the variables and use its value to comment on the strength of the linear association between female and male life expectancy for these countries. (2 marks)

Solution
$r = 0.891$
The linear association between the variables is strong.
Specific behaviours
<ul style="list-style-type: none"> ✓ correct value of r ✓ states association is strong

- (d) Determine the equation of the least-squares line to model the relationship between the variables and draw this line on the scatterplot. (3 marks)

Solution
$y = 0.877x + 3.294$
Specific behaviours
<ul style="list-style-type: none"> ✓ correct equation (coefficients to at least 2 dp) ✓ any reasonable line of best fit ✓ ruled lined through (60, 56) & (85, 78)

- (e) The life expectancy of a female from Fiji is 70. Predict, to the nearest year, the life expectancy of a male from the same country and comment on any factors that affect the validity of your prediction. (2 marks)

Solution
$\hat{y}(70) = 64.7 \approx 65$
With strong correlation and the age lying within the range of data, it is reasonable to assume the prediction is valid.
Specific behaviours
<ul style="list-style-type: none"> ✓ prediction ✓ notes valid, with both reasons

- (f) The life expectancy of a female Australian is 86. Predict, to the nearest year, the life expectancy of a male Australian and comment on any factors that affect the validity of your prediction. (2 marks)

Solution
$\hat{y}(86) = 78.7 \approx 79$
Despite the strong correlation, this prediction involves extrapolation and should be treated with caution.
Specific behaviours
<ul style="list-style-type: none"> ✓ prediction ✓ notes dangers of extrapolation

Question 13

(7 marks)

Anika had a bank account that paid no interest. At the start of the year her account balance was \$3570, and at the end of the first week and every week thereafter she withdrew \$85.

- (a) Calculate the balance of Anika's account after 2 weeks.

(1 mark)

Solution
$A_2 = \$3570 - 2 \times \$85 = \$3400$
Specific behaviours
✓ correct balance

Let the balance in Anika's account at the end of the n^{th} week be A_n .

- (b) Deduce a rule for A_n and hence determine the balance of Anika's account after 11 weeks.

(2 marks)

Solution
$A_n = 3570 - 85n$
$A_{11} = 3570 - 85 \times 11$ $= \$2635$
Specific behaviours
✓ correct rule (accept recursive or n^{th} term)
✓ correct balance

- (c) For how many weeks was Anika able to withdraw \$85?

(1 mark)

Solution
42 weeks, since $A_{42} = 0$.
Specific behaviours
✓ correct number of weeks

In the same year, Anika's friend Brodie had a similar account. The balance B_n of his account at the end of the n^{th} week was given by the recurrence relation $B_{n+1} = B_n - 70$, $B_0 = 3090$.

- (d) Determine the balance of Brodie's account after 11 weeks.

(1 mark)

Solution
$B_{11} = \$2320$
Specific behaviours
✓ correct balance

- (e) At the end of one week during the year, the balance of Anika's account was identical to that of Brodie's. Determine which week this was and the balance of both accounts at that time.

(2 marks)

Solution
$A_{32} = B_{32} = 850$
At the end of the 32 nd week, when their balances were \$850.
Specific behaviours
✓ correct week
✓ correct balance

Question 14

(7 marks)

A business bought an articulated dump truck valued at \$85 000. The value of the truck depreciated by 29% each year.

- (a) By how much did the value of the truck depreciate during the first year and what was its value one year after it was bought? (2 marks)

Solution
$0.29 \times \$85\,000 = \$24\,650$ $V_1 = \$85\,000 - \$24\,650 = \$60\,350$
Specific behaviours
✓ depreciation amount ✓ value after one year

- (b) Deduce a recursive rule for V_n , the value of the truck after n years. (2 marks)

Solution
$V_{n+1} = 0.71V_n, \quad V_0 = 85\,000$
Specific behaviours
✓ indicates correct multiplier ✓ correct rule with initial term

- (c) Calculate the value of the truck after 5 years. (1 mark)

Solution
$V_5 = \$15\,335.95$
Specific behaviours
✓ correct value (accept reasonable rounding)

- (d) During which year does the value of the truck first depreciate by less than \$1000? Justify your answer. (2 marks)

Solution		
n	V_n	$V_n - V_{n+1}$
9	3897.12	$V_9 - V_{10} = 1130$
10	2766.96	$V_{10} - V_{11} = 802$
11	1964.54	
By observing terms of the sequence, the annual depreciation will first be less than \$1000 during the 11 th year.		
Specific behaviours		
✓ indicates appropriate reasoning ✓ correct year		

Question 15

(10 marks)

An industrial chemist varied the amount of accelerant (a grams) used when making an epoxy resin and recorded the time taken (t seconds) for the resin to set. The results are shown below.

a	5.0	6.0	7.0	7.5	8.0	9.0	10.5	11.5	13.0	14.0
t	18.8	17.5	17.4	14.8	13.9	13.6	12.8	11.5	11.6	10.9

The chemist suspected that a linear association might exist between the variables and calculated the correlation coefficient $r_{at} = -0.94$.

- (a) After seeing this value of the correlation coefficient, the chemist said to their assistant "it looks like there is a strong and negative linear association between the variables". Explain this interpretation of the coefficient. (2 marks)

Solution
Since r is close to -1 , the strength of the linear association is strong.
Since $r < 0$, the direction of the linear association is negative.
Specific behaviours
✓ explains strong
✓ explains negative

The chemist also noted that the least-squares line for the data was $\hat{t} = 22.27 - 0.873a$ and used it to calculate nine residuals for the linear model as shown below, rounded to one decimal place.

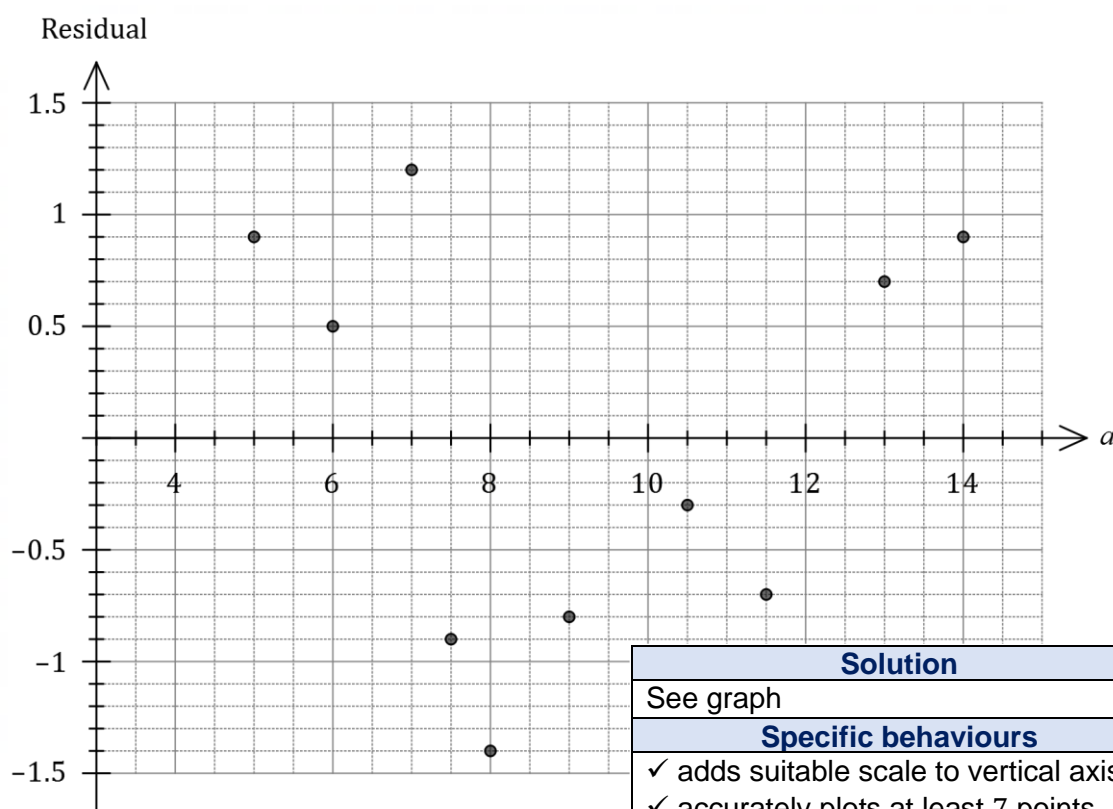
a	5.0	6.0	7.0	7.5	8.0	9.0	10.5	11.5	13.0	14.0
Residual	0.9	0.5	1.2	-0.9	-1.4	-0.8	-0.3	-0.7	0.7	0.9

- (b) Show how the residual of -0.3 was calculated and determine the residual associated with 7.5 grams of accelerant. (3 marks)

Solution
$\hat{t}(10.5) = 22.27 - 0.873(10.5) = 13.1, \quad 12.8 - 13.1 = -0.3$
$\hat{t}(7.5) = 22.27 - 0.873(7.5) = 15.7, \quad 14.8 - 15.7 = -0.9$
Specific behaviours
✓ calculates $\hat{t}(10.5)$ and subtracts from 12.8
✓ shows calculation for $\hat{t}(7.5)$
✓ calculates missing residual

- (c) Construct a residual plot for the data on the axes below.

(3 marks)



Solution
See graph
Specific behaviours
✓ adds suitable scale to vertical axis
✓ accurately plots at least 7 points
✓ accurately plots all points

- (d) Does the residual plot support the chemist's suspicions that a linear model fits the data? Explain your answer.

(2 marks)

Solution
The residual plot does not support the linear model as a pattern is evident in the residuals.
Specific behaviours
✓ states plot does not support linear model
✓ states pattern evident in residuals

Question 16

(6 marks)

The mass of a small puppy was measured as 625 g when it was one week old. A week later its mass had increased by 50 g.

- (a) Assuming that the weekly mass of the puppy can be modelled by an arithmetic sequence, predict the mass of the puppy when it is 8 weeks old. (2 marks)

Solution
$T_n = 625 + 50(n - 1)$ $T_8 = 625 + 50 \times (8 - 1)$ $= 975 \text{ g}$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates appropriate method ✓ correct mass

- (b) Assuming that the weekly mass of the puppy can be modelled by a geometric sequence, predict the mass of the puppy when it is 8 weeks old. (3 marks)

Solution
$r = \frac{625 + 50}{625} = 1.08$ $T_n = 625(1.08)^{n-1}$ $T_8 = 625(1.08)^{8-1}$ $= 1071 \text{ g}$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates common ratio ✓ indicates appropriate method ✓ correct mass

- (c) Comment on the usefulness of these models as the puppy grows over the next few years. (1 mark)

Solution
Not very useful, since both models have the mass of the puppy increasing forever, yet all dogs reach their adult weight after a year or two.
Specific behaviours
✓ sensible comment that notes models eventually not useful

Question 17

(8 marks)

The annual number of mobile phone subscriptions and new cars sold in Queensland, as collated by a researcher, are shown in the table below.

Year	2013	2014	2015	2016	2017	2018	2019
Subscriptions (s , in millions)	6.18	6.31	6.43	6.58	6.72	6.82	6.95
New cars (c , in thousands)	241	246	252	257	260	266	268

The researcher wanted to identify whether new car sales in Queensland could be predicted from mobile phone subscriptions.

- (a) Quantify the strength of the linear association between the variables s and c . (1 mark)

Solution
$r = 0.993$
Specific behaviours
✓ calculates r

- (b) Determine the equation of the least-squares line that can be used to predict c from s . (2 marks)

Solution
$\hat{c} = 35.58s + 21.95$
Specific behaviours
✓ correct coefficients
✓ uses given variables

- (c) Use the least-squares line to predict the number of new car sales in another Australian state that had 6.8 million mobile phone subscriptions, and comment, with reasons, on the validity of your prediction. (3 marks)

Solution
$\hat{c} = 35.58(6.8) + 21.95 = 264$ Sales predicted to be 264 thousand cars. This prediction is not valid as the line is derived from sales of cars and smartphones in Queensland. In another state, it is unlikely that the variables will have the same association as in Queensland.
Specific behaviours
✓ correct prediction in the thousands ✓ states prediction not valid for another state ✓ supplies reason that prediction not valid

- (d) Describe a possible non-causal explanation for the observed association between mobile phone subscriptions and new cars sold, justifying your choice. (2 marks)

Solution
The association is likely due to a common response to a third variable - the population of the state. As the population increases, so there will be more people to buy cars and mobile phone subscriptions.
Specific behaviours
✓ identifies a confounding variable such as population (<i>do not accept time</i>) ✓ explains common response to confounding variable

Question 18

(9 marks)

The adjacency matrix for the connected planar graph G is $\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$.

(a) Determine, with justification, the number of faces that G has.

(3 marks)

Solution
<p>G has 5 faces.</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ draws G, correct v and e ✓ draws G in the plane ✓ states number of faces

Alternative Solution
<p>Vertices $v = 5$ (number of matrix rows) and edges $e = 8$ (sum of elements above matrix diagonal).</p> <p>G is connected planar graph, so using Euler's formula then $5 + f - 8 = 2$ and so $f = 2 + 8 - 5 = 5$. G has 5 faces.</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ states number of vertices and edges ✓ uses Euler's formula ✓ states number of faces

(b) Use elements from the adjacency matrix to explain why G is a simple graph.

(3 marks)

Solution
<p>Elements on the leading diagonal are all 0 and so there are no loops.</p> <p>All other elements in the matrix are 0 or 1 and so there are no multiple edges.</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ states no loops and no multiple edges ✓ uses diagonal elements to justify no loops ✓ uses other elements to justify no multiple edges

- (c) Ore's theorem states that a simple graph with n vertices is Hamiltonian if, for every pair of distinct vertices V_a and V_b which are not adjacent, the sum of the degrees of V_a and V_b is greater than or equal to n . Use Ore's theorem to show that G is Hamiltonian. (3 marks)

Solution
<p>G has two pairs of vertices that are not adjacent: V_1 and V_4 with degrees 3 and 2 respectively. V_2 and V_4 with degrees 3 and 2 respectively.</p> <p>For each pair, $3 + 2 = 5$ and 5 is equal to n, the number of vertices, and so G is Hamiltonian.</p> <p><i>(NB Using adjacency matrix, non-adjacent pairs identified by 0 elements not on leading diagonal, and degree is sum of elements in row.)</i></p>
Specific behaviours
<ul style="list-style-type: none"> ✓ identifies one pair of non-adjacent vertices and states their degrees ✓ identifies two pairs of non-adjacent vertices and states their degrees ✓ shows the sum is equal to n and states conclusion

Question 19

(8 marks)

Joe plans to invest \$64 000 in an account that pays interest of 0.45% per month. At the end of each month, just after interest is added to the account, he will withdraw \$220. The balance of his account, a_n , after n withdrawals can be modelled by the recurrence relation

$$a_{n+1} = 1.0045a_n - 220, \quad a_0 = 64\,000.$$

- (a) Determine the balance of the account after 12 withdrawals have been made and describe how the balance has changed since the account was opened. (2 marks)

Solution
$a_{12} = \$64\,836.50$ Change in balance = $\$64\,836.50 - \$64\,000 = \$836.50$ Account balance has increased by \$836.50.
Specific behaviours
✓ calculates a_{12} ✓ states balance has increased and amount of increase

- (b) Calculate the total withdrawn from the account after 12 withdrawals, and hence show that the total interest paid into the account over this time is \$3476.50. (2 marks)

Solution
Total withdrawn: $12 \times \$220 = \2640 . Hence $I = \$836.50 + \$2640 = \$3476.50$.
Specific behaviours
✓ calculates total withdrawn ✓ shows how to derive total interest

- (c) The balance of Joe's account will first exceed \$66 000 after the k^{th} withdrawal. Determine the value of k and state the balance of the account at this time. (2 marks)

Solution
$k = 28, \quad a_{28} = \$66\,024.31$
Specific behaviours
✓ value of k ✓ value of a_k

- (d) If, after making the 12th withdrawal, Joe then changed the amount he withdrew each month to \$295, how would this change the way the account balance grew in the future? Justify your answer. (2 marks)

Solution
$a_{n+1} = 1.0045a_n - 295, \quad a_0 = 64\,836.50$ For this new sequence, the account balance doesn't grow but starts to decrease, as seen by $a_1 = \$64\,833.27$, which is \$3.23 less than a_0 . OR the account balance decreases as the amount taken out exceeds the interest 0.45% of $64\,836.50 = 291.76$ which is less than the withdrawal of 295.
Specific behaviours
✓ shows modified recursive rule OR states the balance decreases because the amount taken out > interest ✓ uses terms of the new sequence to explain the balance now decreases OR shows the interest to be less than the withdrawal

Supplementary page

Question number: _____

