Western Mathematics Exams

School Name

Half Yearly Examination

2016

Year 10

Advanced Mathematics Course

Solutions

Year 10

WME Solutions
Advanced Mathematics
Half Yearly

Non Calculator

Section 1 Short Answer Section

ANSWERS

No.	WORKING	ANSWER
1.	$(15 - 7) \times 4 = 8 \times 4 = 32$	32
2.	$\frac{19}{20} - \frac{3}{4} = \frac{19}{20} - \frac{15}{20}$ $= \frac{4}{20}$ $= \frac{1}{5}$	$\frac{1}{5}$
3.	$0.64 = \frac{64}{100}$ $= \frac{32}{50}$ $= \frac{16}{25}$	$\frac{16}{25}$
4.	From 4 pm to 11 pm is 7 hours. In 7 hours temp falls $7 \times 2 = 14°$ C Temp at 11 pm = $-3 - 14 = -17°$ C.	$-17°$ C.
5.	Discount = 30% of \$120.00 = 0.3×120 = \$36 Price paid = $120 - 36$ = \$84.00	\$84.00
6.	$4 : 15 = x : 600$ $600 \div 15 = 40$ $x : 600 = 40 \times 4 : 40 \times 15$ $x = 40 \times 4 = 160$ There were 160 tourists	160 tourists

7.	$\theta + 48 + 55 = 180$ (angles on a straight line) $\theta = 180 - 103$ $= 77$	$\theta = 77$
8.	$\angle BDC = 70°$ (supplementary angles) $\angle CBD + 70° + 60° = 180°$(angle sum ΔCBD) $\therefore \angle CBD = 50°$ OR $\angle ABD + 40 + 110 = 180$ (angle sum ΔABD) $\angle ABD = 180 - 150$ $\angle ABD = 30$ $\angle ABC + 40 + 60 = 180$ (angle sum ΔABC) $\angle ABC = 180 - 100$ $\angle ABC = 80$ $\angle CBD = \angle ABC - \angle ABD$ $= 80 - 30$ $= 50°$ OTHER POSSIBLE METHODS	$\angle CBD = 50°$
9.	Perimeter $= 2.4 + 1.8 + 1.6 \times 2 + 1.5 \times 2$ $= 2.4 + 1.8 + 3.2 + 3.0$ $= 10.4$ m	10.4 m
10.	Area $= \frac{1}{2} \times$ Base $\times$ Perp Height $= \frac{1}{2} \times 16 \times 9$ $= 72$ cm^2	72 cm^2
11.	Area Trapezium $= \frac{1.4}{2} \times (0.8 + 1.2)$ $= 0.7 \times 2.0$ $= 1.4$ m^2 Volume of Prism $= 1.4 \times 2.0$ $= 2.8$ m^3	2.8 m^3
12.	$2xy + 3x \times 4y = 2xy + 12xy = 14xy$	$14xy$
13.	$\frac{24x^3y^4}{4x^3y} = 6y^3$	$6y^3$
14.	$10x^2 - 3x(2x - 4y) = 10x^2 - 6x^2 + 12xy$ $= 4x^2 + 12xy$	$4x^2 + 12xy$

15.	$MP = \left(\frac{-6-10}{2}, \frac{7-5}{2}\right)$ $= \left(\frac{-16}{2}, \frac{2}{2}\right)$ $= (-8, 1)$	(-8, 1)
16.	$5.5 \times 10^{-7} = 0.00000055$	0.00000055 metres
17.	$7w + 1 = 2w - 16$ $7w = 2w - 17$ $5w = -17$ $w = -\frac{17}{5}$ $w = -3.4$	$w = -3.4 = -3\frac{2}{5}$
18.	Number of Blue balls $= 50 - 12 - 11 - 7 = 20$ Probability (Blue ball) $= \frac{20}{50} = \frac{2}{5} = 0.4 = 40\%$	Any of $\frac{2}{5} = 0.4 = 40\%$
19.	There are 7 scores out of 20 which are less than 20. Percentage $= \frac{7}{20} \times 100$ $= 35\%$	35%
20.	Sum of the scores = 12 + 61 + 122 + 69 + 167 + 107 = 538 Mean $= \frac{538}{20} = 26\frac{18}{20} = 26\frac{9}{10} = 26.9$	$26\frac{9}{10}$ or 26.9

WME Solutions

Year 10 *Advanced Mathematics*

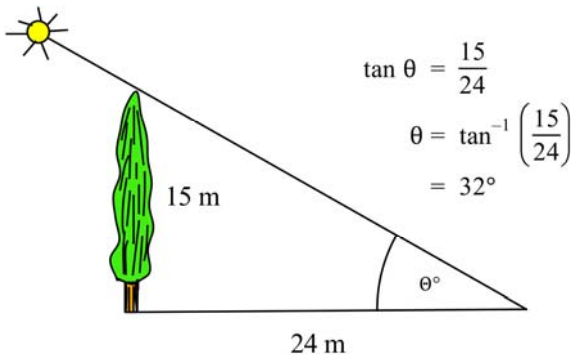
Half Yearly

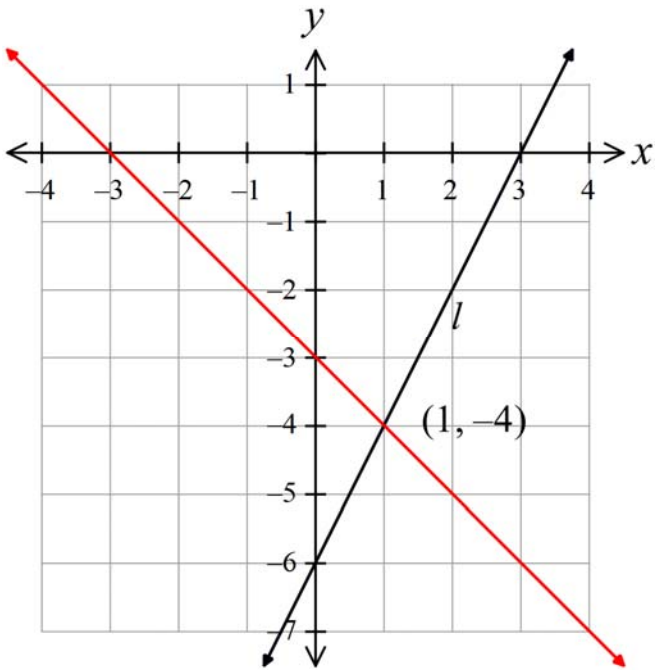
Calculator Allowed

Section 2 **Part A** Multiple Choice Section

ANSWERS

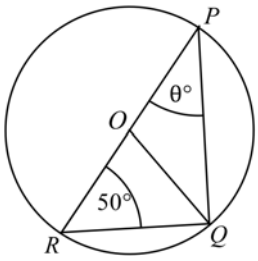
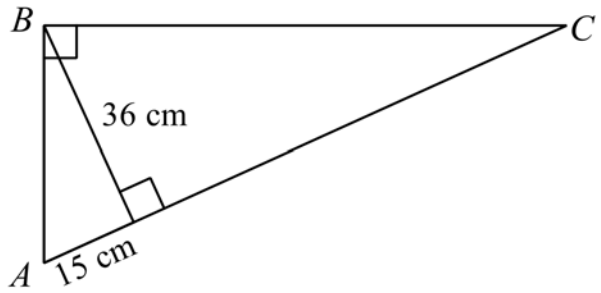
No.	WORKING	ANSWER
21.	$\text{Pay} = 48 \times 36 + 48 \times 4 \times 1.5$ $= \$1728 + \$288$ $= \$2016$	C
22.	$\text{Amount Paid} = \$120 + \$45 \times 24$ $= \$120 + \$1080$ $= \$1200$ $\text{Interest} = 1200 - 960$ $= \$240.00$	B
23.	$S = \frac{D}{T}$ $T = \frac{D}{S}$ $= \frac{2400}{640}$ $= 3.75$ $=$ 3 hours and 45 min	D
24.	In ΔABC and ΔABD AB is common $\angle ABC = \angle BAD = 90°$ (angles in a rectangle) $BC = AD$ (opposite sides of rectangle) $\Delta ABC \equiv \Delta ABD$ (SAS)	C

25.	$\angle PNM = \angle KPN = 40°$ (alt angles on $\parallel$ lines) $\angle NPM = \angle NMP$ (base $\angle$ isosceles Δ) $2 \times \angle NMP + 40 = 180$ (angle sum Δ) $2 \times \angle NMP = 140$ $\angle NMP = 70$ $\angle PML + 70 = 90$ (adjacent angles in rectangle) $\angle PML = x = 20$	A
26.	The sum of the exterior angles of a convex polygon = 360° Since the polygon is regular, the exterior angles are all equal. $\therefore \ 10 \times \angle HFG = 360°$ $\angle HFG = \frac{360°}{10} = 36°$	C
27.	Cylinder: radius = 4 cm, height = 12 cm Volume = $\pi r^2 h$ $= \pi \times 4^2 \times 12$ $= 603.1858 \text{ cm}^3$ $\approx$ 600 ml	B
28.	Area = $\frac{1}{2} \times 120 \times 104 \times 2 + 120 \times 200 \times 3$ $= 12\,480 + 72\,000$ $= 84\,480 \text{ cm}^2$ $= 8.4 \text{ m}^2$	D
29.	$\sin 26° = \frac{b}{12.4}$ $b = 12.4 \times \sin 26°$ $= 5.4$ km	A
30.	 $\tan \theta = \frac{15}{24}$ $\theta = \tan^{-1}\left(\frac{15}{24}\right)$ $= 32°$	A

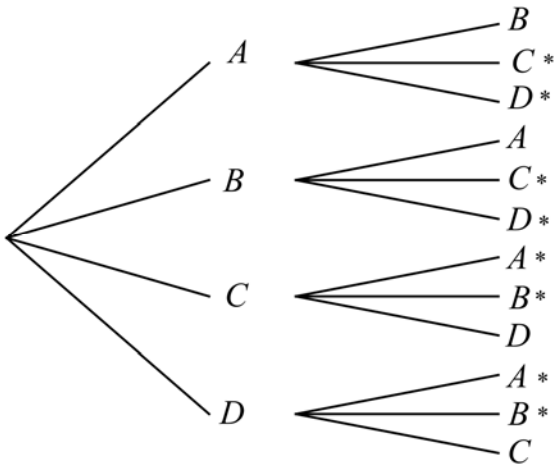
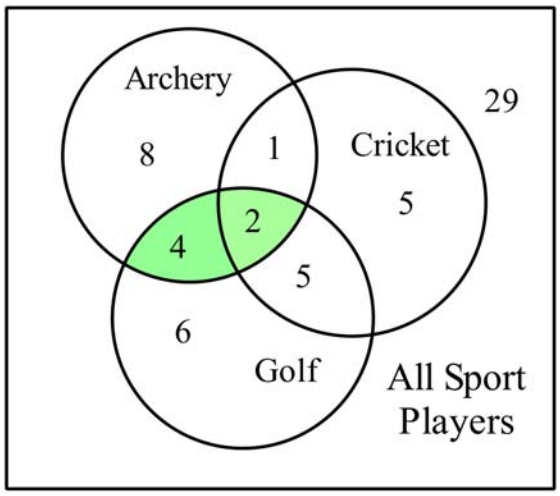
31.	There are 80 books altogether Total Crime and Thriller = $17 + 23 = 40$ Probability (Crime or Thriller) = $\frac{40}{80} = \frac{1}{2}$	C
32.	$12m^2n^3 - 16m^2n = 4m^2n(3n^2 - 4)$.	B
33.	 Find the gradient of l $m = \frac{6}{3} = 2$ Find the y intercept $b = -6$ Equation is $y = mx + b$ $y = 2x - 6$	D
34.	Draw the graph of $y = -x - 3$ y-intercept = -3, so mark -3 on y axis Gradient = -1, so go across 1 and down 1 to obtain a second point, and draw the line as shown on graph above. Point of intersection is (1, -4)	B

35.	$(2x^3y^{-2})^3 = 2^3 \times x^{3\times 3} \times y^{-2\times 3}$ $= 8x^9y^{-6}$ $= \frac{8x^9}{y^6}$	B
36.	Open circle on -3 indicate $<$ and closed on 2 indicates $\leq$ $-3 < x \leq 2$	A
37.	It is bimodal as two scores occurred four times which is more than any other score.	A
38.	Median = average of 9th and 10th scores = $\frac{5+6}{2} = 5.5$ Mean = $\frac{6+3+16+18+28+16}{18} = \frac{87}{18} = 4.8\dot{3}$ Mode = 4 and 7 Range is from 2 to 8, ie 6 Statement D is inaccurate as the most common (mode) is both 4 and 7, not just 7.	D
39.	There are 18 data scores altogether, and 4 of these represent groups of four. Sector size = $\frac{4}{18} \times 360 = 80°$	D

40.	Mean : Since two scores greater than the mean are added, the mean will change. Median: Since the mode is 14 with 3 scores, and the median is 11, the most number of 11's possible is 2. Since there were an even number of scores, the median came from two 11's or two other scores with an average of 11. When two scores greater than 11 are added, the median will move to the right, and hence will change. Mode: Mode is 14 which occurs 3 times, and since the highest score is 9 (minimum) + 6 (range) = 15 there were no 16's previously, so now there are two 16's, so 14 remains the mode. Range : The range was 6, going from 9 to 15, so adding 2 16's makes it 7, so it changes. Only the mode doesn't change.	C
41.	$\begin{aligned} A &= 2400(1.06)^3 \\ &= \$2858.4384 \\ \text{Interest} &= 2858.4384 - 2400 \\ &= 458.4384 \\ &= \$458.44 \end{aligned}$	C
42.	$\begin{aligned} SV &= 32500(1-0.085)^4 \\ &= 32500(0.915)^4 \\ &= \$22780.7352703125 \\ &= \$22\,780 \end{aligned}$	B
43.	$a^{-\frac{2}{3}} = \frac{1}{a^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{a^2}}$	A
44.	$\begin{aligned} \sqrt{24} + \sqrt{54} &= \sqrt{4} \times \sqrt{6} + \sqrt{9} \times \sqrt{6} \\ &= 2\sqrt{6} + 3\sqrt{6} \\ &= 5\sqrt{6} \end{aligned}$	A
45.	$\begin{aligned} \frac{6-2\sqrt{5}}{\sqrt{5}} &= \frac{6-2\sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{6\sqrt{5}-2\sqrt{25}}{\sqrt{25}} \\ &= \frac{6\sqrt{5}-10}{5} \end{aligned}$	D

46.	In $\Delta\ OQR$ $OR = OQ$ (equal radii) $\therefore \quad \angle ORQ = \angle OQR = 50°$ (base angles isos Δ) $\angle POQ = 50° + 50° = 100°$ (exterior angle $\Delta\ OQR$) In $\Delta\ OPQ$ $OP = OQ$ (equal radii) $\therefore \quad \angle OPQ = \angle OQP$ (base angles isos Δ) $2\theta + 100 = 180$ (angle sum Δ) $2\theta = 80$ $\theta = 40$ (if done using circle geometry properties, it becomes easier) $\angle Q = 90°$ (angle in semicircle) $\theta + 50 + 90 = 180°$ (angle sum Δ) $\theta = 180 - 140 = 40°$ 	B
47.	 $\tan \angle BAD = \frac{36}{15}$ $\angle BAD = \tan^{-1}\left(\frac{36}{15}\right)$ $= 67.380$ $= 67°$ (nearest degree) $\angle BCA = 180° - (90 + 67)$ (Angle sum $\Delta\ ABC$) $= 23°$	A
48.	$V = \frac{1}{2} \times \frac{4}{3} \pi r^3$ $= \frac{2}{3} \times \pi \times 8.4^3$ $= 1241.356$ $= 1240\ m^3$ ($3\ sf$)	C

49.	Slant height $= l$ $l^2 = 200^2 + 45^2$ $= 42025$ $l = \sqrt{42025} = 205$ $SA = \frac{1}{2} \times 90 \times 205 \times 4 + 90^2$ $= 45\,000 \text{ cm}^2$	D
50.	$18x^2 - 32 = 2(9x^2 - 16)$ $= 2(3x - 4)(3x + 4)$	B
51.	$\frac{1}{x-4} + \frac{2}{x-3} = \frac{x-3+2(x-4)}{(x-4)(x-3)}$ $= \frac{x-3+2x-8}{(x^2-3x-4x+12)}$ $= \frac{3x-11}{x^2-7x+12}$	D
52.	$x - 2y = 5$ $y = 3x - 5$ $x - 2(3x - 5) = 5$ $x - 6x + 10 = 5$ $-5x = -5$ $x = 1$	C
53.	$3x^2 - 3x - 2 = 0$ $a = 3, b = -3, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{3 \pm \sqrt{9 - 4 \times 3 \times (-2)}}{2 \times 3}$ $x = \frac{3 \pm \sqrt{33}}{6}$	B
54.	Fastest change is the steepest section of the graph which is between 00:00 and 02:00 Change is 10 m in 2 hours which is 5 m/h.	A

55.	Since concave down, it must be either B or D. When $y = 0, x = \pm 2$, so test these. Sub $x = 2$ into $y = 8 - x^2$ $y = 8 - 2^2 = 8 - 4 = 4$ Sub $x = 2$ into $y = 8 - 2x^2$ $y = 8 - 2 \times 2^2 = 8 - 2 \times 4 = 0$ Similarly for $x = -2$ So correct graph is D	D
56.	 Those which meet the requirement A or B but not both are marked *. $P(A \text{ or } B \text{ but not both}) = \frac{8}{12} = \frac{2}{3}$	B
57.	Archery and Golf had 6 who played both. There were 60 players altogether. $P(A \text{ and } G) = \frac{6}{60} = \frac{1}{10}$  	C
58.	74 is the lower quartile and 82 is the median, so a quarter of all people lie between these values. $\frac{1}{4}$ of $240 = \frac{240}{4} = 60$	A

59.	From calculator Standard deviation $\sigma_n = 1.26$	C
60.	As all scores increase by 2, the mean will increase by 2, and since all increase by the same amount, the spread will be the same, so the standard deviation will not change. New mean = 88.2 + 2 = 90.2 New SD = Old SD = 4.3	D

School Name

Year 10 Half Yearly Examination

Advanced Mathematics Course 2016

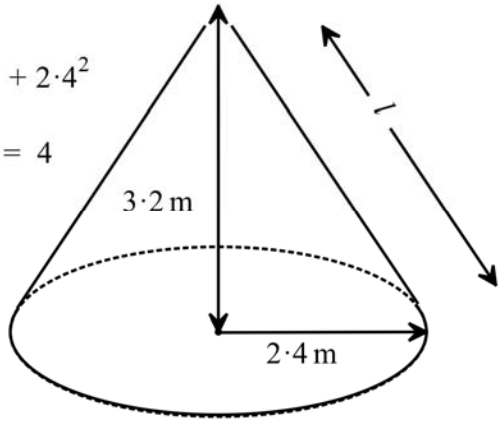
Multiple Choice Section Answer Sheet

Name _____ Teacher _____

Completely fill the response oval representing the most correct answer.
Use a black or blue pen or 2B pencil.

21.	A	○	B	○	C	●	D	○	41.	A	○	B	○	C	●	D	○
22.	A	○	B	●	C	○	D	○	42.	A	○	B	●	C	○	D	○
23.	A	○	B	○	C	○	D	●	43.	A	●	B	○	C	○	D	○
24.	A	○	B	○	C	●	D	○	44.	A	●	B	○	C	○	D	○
25.	A	●	B	○	C	○	D	○	45.	A	○	B	○	C	○	D	●
26.	A	○	B	○	C	●	D	○	46.	A	○	B	●	C	○	D	○
27.	A	○	B	●	C	○	D	○	47.	A	●	B	○	C	○	D	○
28.	A	○	B	○	C	○	D	●	48.	A	○	B	○	C	●	D	○
29.	A	●	B	○	C	○	D	○	49.	A	○	B	○	C	○	D	●
30.	A	●	B	○	C	○	D	○	50.	A	○	B	●	C	○	D	○
31.	A	○	B	○	C	●	D	○	51.	A	○	B	○	C	○	D	●
32.	A	○	B	●	C	○	D	○	52.	A	○	B	○	C	●	D	○
33.	A	○	B	○	C	○	D	●	53.	A	○	B	●	C	○	D	○
34.	A	○	B	●	C	○	D	○	54.	A	●	B	○	C	○	D	○
35.	A	○	B	●	C	○	D	○	55.	A	○	B	○	C	○	D	●
36.	A	●	B	○	C	○	D	○	56.	A	○	B	●	C	○	D	○
37.	A	●	B	○	C	○	D	○	57.	A	○	B	○	C	●	D	○
38.	A	○	B	○	C	○	D	●	58.	A	●	B	○	C	○	D	○
39.	A	○	B	○	C	○	D	●	59.	A	○	B	○	C	●	D	○
40.	A	○	B	○	C	●	D	○	60.	A	○	B	○	C	○	D	●

Year 10	*WME Solutions Advanced Mathematics Half Yearly*	Calculator Allowed
Section 2 Part B	Longer Answer Section	
ANSWERS		

		Marks
61.	a) $l^2 = 3 \cdot 2^2 + 2 \cdot 4^2$ $= 16$ $l = \sqrt{16} = 4$ 	1 mark for correct answer
	b) $SA = \pi r^2 + \pi rl$ $= \pi \times 2.4^2 + \pi \times 2.4 \times 4$ $= 48.254863$ $= 48.3\ m^2$ (3 sig fig)	1 mark for correct answer
62.	a) $\tan 50° = \frac{TP}{22}$ $TP = 22 \tan 50°$ $= 26.218579$ $= 26.2$ (3 sig fig)	1 mark for correct answer

	b) $\tan \angle TBP = \dfrac{26.2}{45}$ $\angle TBP = \tan^{-1}\left(\dfrac{26.2}{45}\right)$ $= 30.22658$ $= 30°$ (nearest degree)	1 mark for correct answer
63.	a) $2x(2x-3)(3x-2) = 2x(6x^2-4x-9x+6)$ $= 2x(6x^2-13x+6)$ $= 12x^3-26x^2+12x$	1 mark for correct answer
	b) $3x^2-5x+2 = 3x^2-3x-2x+2$ $= 3x(x-1)-2(x-1)$ $= (3x-2)(x-1)$	1 mark for correct answer
	c) $\dfrac{12x^3-26x^2+12x}{14x^2-21x} \times \dfrac{7x+7}{3x^2-5x+2}$ $= \dfrac{2x(2x-3)(3x-2)}{7x(2x-3)} \times \dfrac{7(x+1)}{(x-1)(3x-2)}$ $= \dfrac{2\cancel{x}\,\cancel{(2x-3)}\cancel{(3x-2)}}{\cancel{7}\,\cancel{x}\,\cancel{(2x-3)}} \times \dfrac{\cancel{7}(x+1)}{(x-1)\cancel{(3x-2)}}$ $= \dfrac{2(x+1)}{x-1}$ $= \dfrac{2x+2}{x-1}$	1 mark for correct answer in either factorised or expanded form

64.	$S(-3, -4)$ and $T(-8, 8)$ $ST = \sqrt{(-3 - -8)^2 + (-4 - 8)^2}$ $= \sqrt{(5)^2 + (12)^2}$ $= \sqrt{25 + 144}$ $= \sqrt{169}$ $= 13$ So ST = TU = 13 units, Hence the triangle is isosceles. $T(-8, 8)$ and $U(5, 8)$ TU is a vertical line. $TU\ 5 - -8 = 13$	2 marks for any correct proof, which includes finding at least two distances and showing them equal. A diagram alone will not suffice unless calculation has been done using Pythagoras to find distances and show equal. 1 mark for worked solution with a minor error, or which is not complete e.g. finding one distances only
65.	a) In $2x^2 + 9x + 8 = 0$ $a = 2, b = 9$ and $c = 8$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-9 \pm \sqrt{9^2 - 4 \times 2 \times 8}}{2 \times 2}$ $x = \frac{-9 \pm \sqrt{81 - 64}}{4}$ $x = \frac{-9 \pm \sqrt{17}}{4}$ $= -1.21922359$ or -3.280776 $= -1.2$ or -3.3 (correct to 1 decimal $p\lambda ce$)	2 marks for 2 correct answers. 1 mark for one correct answer or for working with a minor error that leads to incorrect answers.

	b) $s = ut + \frac{1}{2}at^2$ $s - ut = \frac{1}{2}at^2$ $2(s - ut) = at^2$ $a = \frac{2(s - ut)}{t^2}$ * $a = \frac{2s - 2ut}{t^2}$ * Either answer (*) is okay	

66.

$y = x^2 + 4x + 6$

x intercepts solve $x^2 + 4x + 6 = 0$

$b^2 - 4ac = 16 - 4 \times 1 \times 6 = 16 - 24 = -8$

so no solutions, so no x intercepts.

axis of symmetry $x = -\dfrac{b}{2a} = -\dfrac{4}{2 \times 1} = -2$

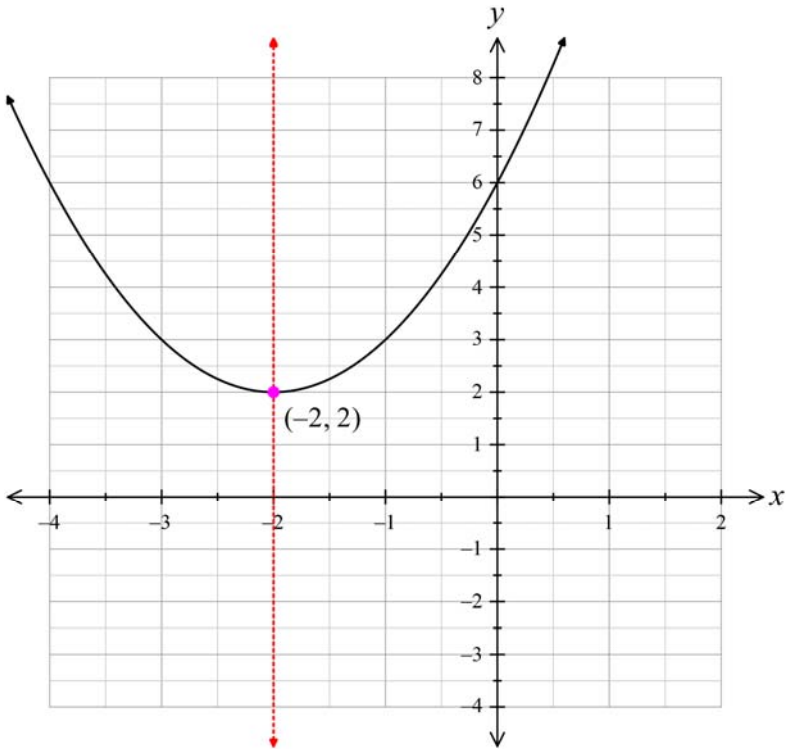
Vertex y value $y = (-2)^2 + 4 \times (-2) + 6$

$= 4 - 8 + 6$

$= 2$

Vertex $(-2, 2)$

y intercept $y = (0)^2 + 4 \times (0) + 6 = 6$



3 marks for a graph including
- correct shape (concave up parabola),
- correct axis of symmetry and vertex and
- correct y intercept (and no x intercepts).

2 marks for correct working with a minor error and graph drawn correctly from this,

or 2 marks for correct graph with axis, vertex or intercept missing.

1 mark for some working towards obtaining the graph, or graph with more than one error or missing feature.

67.	a) $\left(1-2\sqrt{3}\right)\left(5+2\sqrt{3}\right) = 1\times 5+1\times 2\sqrt{3} - 2\sqrt{3}\times 5-2\sqrt{3}\times 2\sqrt{3}$ $= 5+2\sqrt{3}-10\sqrt{3}-4\sqrt{9}$ $= 5-8\sqrt{3}-12$ $= -7-8\sqrt{3}$	1 mark for correct answer
	b) $\dfrac{1-2\sqrt{7}}{5-\sqrt{7}} = \dfrac{1-2\sqrt{7}}{5-\sqrt{7}}\times\dfrac{5+\sqrt{7}}{5+\sqrt{7}}$ $= \dfrac{5+\sqrt{7}-10\sqrt{7}-14}{25-7}$ $= \dfrac{-9-9\sqrt{7}}{18}$ $= \dfrac{9(-1-\sqrt{7})}{18}$ $= \dfrac{-1-\sqrt{7}}{2}$	2 marks for correct answer, in simplified form 1 mark for worked solution with a minor error, or which is not complete but has substantial correct working
68.	a) $\sigma_n = 14.27$ which is bimodal with few values around the centre of the distribution.	1 mark for the correct standard deviation
	b) The shape of the distribution is **positively skewed.**	1 mark for the correct term