



MATHEMATICS

3A/3B

Calculator-free

WACE Examination 2014

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

Section One: Calculator-free

(50 Marks)

Question 1

(4 marks)

- (a) Determine $\frac{dy}{dx}$, given $y = \frac{7x^4 - 5x}{x}$. (2 marks)

Solution
$y = \frac{7x^4 - 5x}{x} = 7x^3 - 5$ $\frac{dy}{dx} = 21x^2$
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly simplifies equation ✓ correctly determines derivative

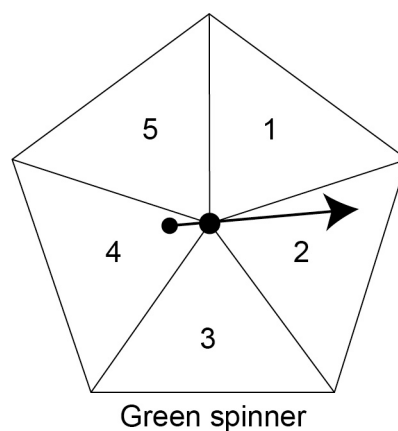
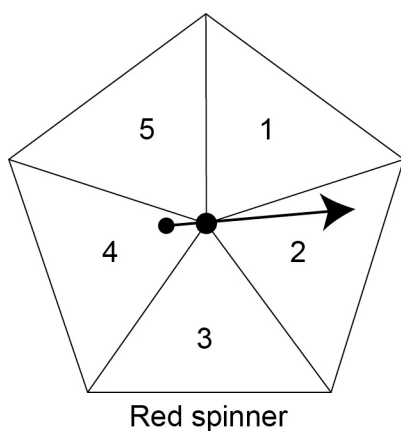
- (b) Determine $\frac{dy}{dx}$ using the product rule, given $y = (3x^2 + 2)(5x - x^3)$.
(Do not simplify your answer.) (2 marks)

Solution
$\frac{dy}{dx} = (5x - x^3)(6x) + (3x^2 + 2)(5 - 3x^2)$
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly uses product rule ✓ correctly differentiates each term

Question 2

(7 marks)

Two spinners, one red and one green, are spun. Some of the outcomes are shown in the table below.



(a) Complete the table.

(1 mark)

Solution						
Red spinner						
Green spinner		1	2	3	4	5
	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)
Specific behaviours						
✓ completes table correctly						

(b) What is the probability that both spinners show the same number?

(1 mark)

Solution	
$\frac{5}{25} = \frac{1}{5}$	
Specific behaviours	
✓ calculates correct probability	

(c) What is the probability that the number on the red spinner is higher than the number on the green spinner?

(1 mark)

Solution	
$\frac{10}{25} = \frac{2}{5}$	
Specific behaviours	
✓ calculates correct probability	

- (d) What is the probability that the green spinner shows a prime number? (1 mark)

Solution
$\frac{15}{25} = \frac{3}{5}$
Specific behaviours
✓ calculates correct probability

- (e) What is the probability of spinning a four on one spinner and a number greater than three on the other spinner? (1 mark)

Solution
$\frac{3}{25}$
Specific behaviours
✓ calculates correct probability

- (f) What is the probability that the total of the two spinners is even, given that the green spinner shows a number larger than the red spinner? (2 marks)

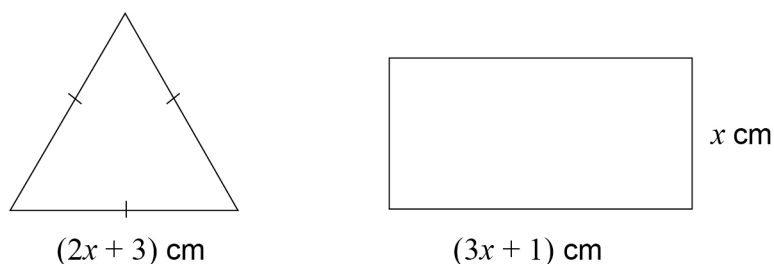
Solution
$\frac{4}{10} = \frac{2}{5}$
Specific behaviours
✓ calculates correct numerator
✓ calculates correct denominator

Question 3 (6 marks)

- (a) Solve the inequality $3 - 2x \leq x + 4$. (2 marks)

Solution
$3 - 2x \leq x + 4$ $-3x \leq 1$ $x \geq -\frac{1}{3}$
Specific behaviours
✓ correctly collects like terms
✓ correctly simplifies the inequality to determine $x \geq -\frac{1}{3}$

- (b) In the diagram below, the perimeter of the equilateral triangle is less than the perimeter of the rectangle.



- (i) Write an inequality and solve it to determine the possible values of x . (3 marks)

Solution
$3(2x + 3) < 2x + 2(3x + 1)$ $6x + 9 < 8x + 2$ $7 < 2x \Rightarrow x > 3.5$
Specific behaviours
<ul style="list-style-type: none"> ✓ writes correct inequality ✓ correctly expands brackets and simplifies ✓ correctly solves for x

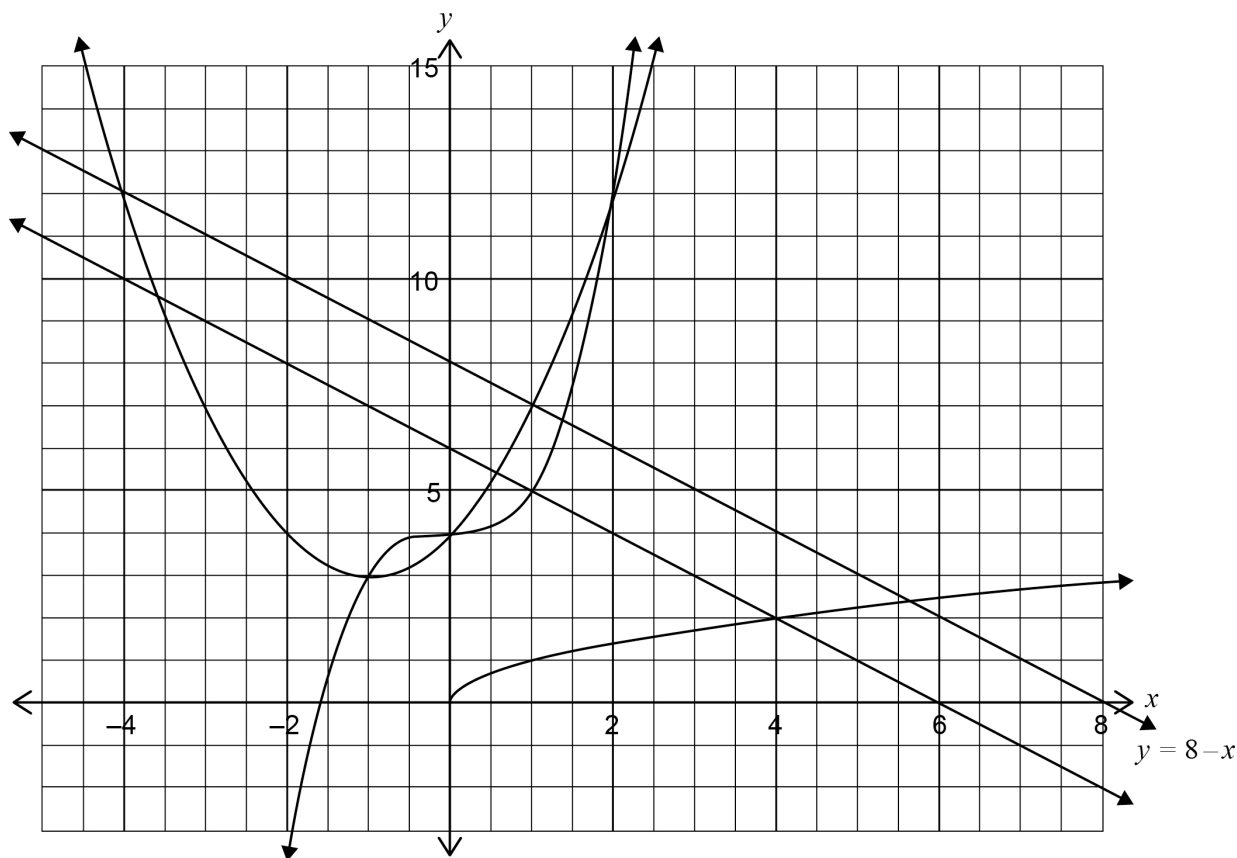
- (ii) What is the smallest possible integer value of x ? (1 mark)

Solution
$x = 4$
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly identifies the smallest integer value of x

Question 4

(9 marks)

The functions $y = x^3 + 4$, $y = \sqrt{x}$, $x + y = 6$ and $y = (x+1)^2 + 3$ are graphed below.



Use the graph to solve the following equations.

- (a) $\sqrt{x} = 6 - x$, for $-5 \leq x \leq 8$. (1 mark)

Solution
$x = 4$
Specific behaviours
✓ correctly identifies the solution

- (b) $(x+1)^2 + 3 = x^3 + 4$, for $x > 0$. (2 marks)

Solution
$x = 2$
Specific behaviours
✓ uses the domain correctly
✓ correctly identifies the solution

- (c) $(x+1)^2 + 3 = 4$, for $-5 \leq x \leq 8$. (2 marks)

Solution
$x = 0$ or $x = -2$
Specific behaviours
✓✓ correctly identifies each solution

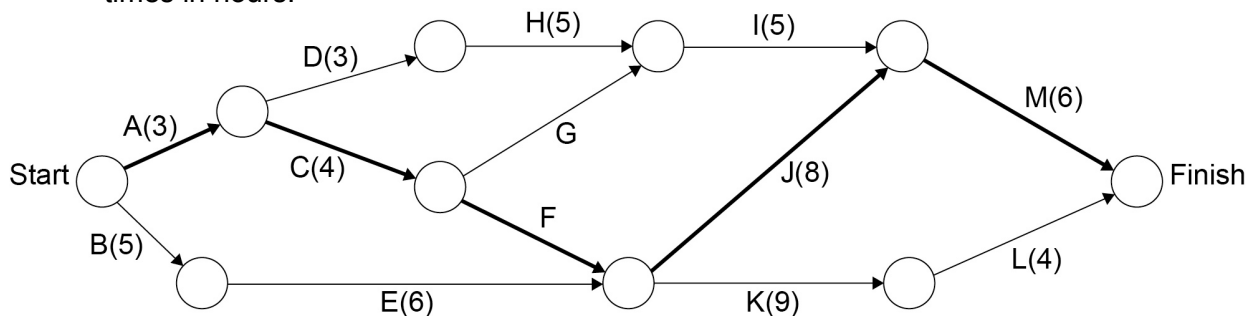
- (d) $x + (x+1)^2 + 3 = 8$, for $-5 \leq x \leq 8$, by drawing a suitable straight line on the graph. (4 marks)

Solution
$x + (x+1)^2 + 3 = 8$ $(x+1)^2 + 3 = 8 - x$ Draw the line $y = 8 - x$ on the graph $x = -4$ or $x = 1$
Specific behaviours
✓ correctly rearranges equation ✓ correctly identifies $y = 8 - x$ as the straight line ✓ correctly draws line on graph ✓ correctly identifies solutions

Question 5

(9 marks)

- (a) The project network below consists of 13 tasks, from task A to task M, with completion times in hours.



The minimum completion time for this project is 26 hours and task F is on the critical path.

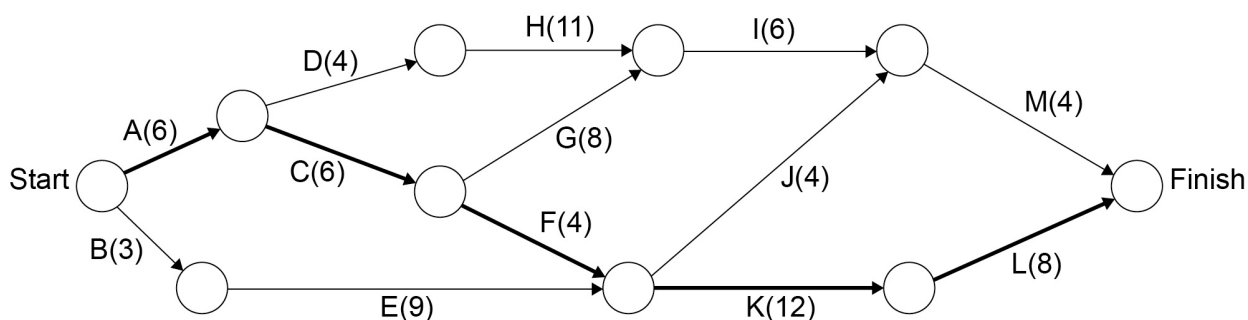
- (i) Determine the completion time for task F. (2 marks)

Solution
<p>Let f denote time taken to complete task F.</p> $7 + f + 8 + 6 = 26$ $f = 5$ <p>Therefore task F takes 5 hours to complete.</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly identifies critical path ✓ determines correct completion time for task F

- (ii) Determine the possible completion times for task G. (2 marks)

Solution
<p>Let g denote time taken to complete task time for task G.</p> $7 + g + 5 + 6 < 26$ $g < 8$ <p>Therefore task G must take less than 8 hours to complete.</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly writes an inequality involving time for task G ✓ determines possible completion times for task G

- (b) The project network below has different completion times, in hours, to that of part (a).



- (i) Determine the critical path and the minimum completion time. (2 marks)

Solution
Critical path: ACFKL Minimum completion time is 36 hours
Specific behaviours
✓ correctly identifies critical path ✓ correctly states minimum completion time

- (ii) Describe the effect of delaying task D by 6 hours on the minimum completion time. (1 mark)

Solution
It would increase the minimum completion time by one hour
Specific behaviours
✓ correctly identifies the increase in completion time

- (iii) A new task, Q, with a completion time of 24 hours is added to the network. It has an immediate predecessor of task B. Task L has immediate predecessors of tasks K and Q. Draw task Q on the network and state what effect it would have on the minimum completion time. (2 marks)

Solution
<pre> graph LR Start((Start)) -- A(6) --> Node1(()) Start -- B(3) --> Node2(()) Node1 -- C(6) --> Node3(()) Node1 -- D(4) --> Node4(()) Node2 -- E(9) --> Node5(()) Node2 -- Q(24) --> Node6(()) Node3 -- F(4) --> Node5 Node3 -- G(8) --> Node4 Node4 -- H(11) --> Node7(()) Node5 -- K(12) --> Node6 Node7 -- I(6) --> Node8(()) Node7 -- J(4) --> Node6 Node8 -- M(4) --> Finish((Finish)) Node6 -- L(8) --> Finish </pre> <p>This new task has no effect on the minimum completion time.</p>
Specific behaviours
✓ draws and labels task Q on the network ✓ states that the new task has no effect on the minimum completion time

Question 6

(10 marks)

Solve the following equations for x .

(a) $x^2 + x - 72 = 0$.

(2 marks)

Solution
$x^2 + x - 72 = 0$ $(x + 9)(x - 8) = 0 \Rightarrow x = -9 \text{ or } x = 8$
Specific behaviours
✓ correctly factorises trinomial ✓ correctly solves for x

(b) $-2x(x + 2)(1 - 3x) = 0$.

(2 marks)

Solution
$-2x(x + 2)(1 - 3x) = 0$ $x = 0 \text{ or } x = -2 \text{ or } x = \frac{1}{3}$
Specific behaviours
✓ states two solutions for x ✓ states all three solutions for x

(c) $2x^{\frac{1}{3}} + 1 = 5$.

(3 marks)

Solution
$2x^{\frac{1}{3}} + 1 = 5$ $2x^{\frac{1}{3}} = 4$ $x^{\frac{1}{3}} = 2$ $\therefore x = 8$
Specific behaviours
✓ collects like terms ✓ divides both sides by 2 ✓ solves for x

(d) $2^{x+1} = 4^{1-2x}$.

(3 marks)

Solution
$2^{x+1} = 4^{1-2x} \Rightarrow 2^{x+1} = 2^{2-4x} \Rightarrow x+1 = 2-4x \Rightarrow 5x = 1 \therefore x = \frac{1}{5}$
Specific behaviours
✓ changes to common base ✓ equates indices ✓ solves for x

Question 7

(5 marks)

For the function $f(x) = ax^2 - 2bx^3$, where a and b are positive constants, $f(1) = 2$ and $f'(2) = -24$

- (a) Establish the simultaneous equations $a - 2b = 2$ and $a - 6b = -6$.

(3 marks)

Solution
$f(1) = a \times 1^2 - 2b \times 1^3 = a - 2b = 2$Equation 1 $f'(x) = 2ax - 6bx^2$ $f'(2) = 2a \times 2 - 6b \times 2^2 = 4a - 24b = -24$ and dividing by 4 gives $a - 6b = -6$Equation 2
Specific behaviours
✓ correctly substitutes and establishes first equation ✓ correctly differentiates f ✓ correctly substitutes and establishes second equation

- (b) Solve the simultaneous equations from part (a) to determine the values of a and b .

(2 marks)

Solution
Subtracting Equation 2 from Equation 1 gives $4b = 8 \Rightarrow b = 2$ Substituting $b = 2$ into Equation 1 gives $a = 6$
Specific behaviours
✓ correctly solves for b ✓ correctly substitutes and solves for a

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