Neap

Trial Examination 2023

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (4 marks)

a. $f(x) = (4x-2)^{-1}$

$f'(x) = -(4x-2)^{-2} \times 4$ **OR** $\dfrac{-4}{(4x-2)^2}$ **OR** $\dfrac{-1}{(2x-1)^2}$ A1

b. **i.** $\displaystyle\int \frac{1}{4x-2}dx = \frac{1}{4}\log_e(4x-2)+c$ **OR** $\dfrac{1}{4}\log_e(2x-1)+c$ A1

Note: Responses do not require c in order to obtain full marks.

ii. $\displaystyle\int_1^5 f(x)dx = \frac{1}{4}\left[\log_e(4x-2)\right]_1^5 = \frac{1}{4}\left(\log_e(18)-\log_e(2)\right)$ M1

$= \dfrac{1}{4}\log_e(9)$

$= \log_e\left(\sqrt{3}\right)$ A1

Question 2 (2 marks)

$f(x) = \displaystyle\int 3\sin(2x)dx$

$= -\dfrac{3}{2}\cos(2x)+c$ M1

$f\left(\dfrac{\pi}{3}\right) = 1 \Rightarrow -\dfrac{3}{2}\cos\left(\dfrac{2\pi}{3}\right)+c = 1$

$\left(-\dfrac{3}{2}\right)\times\left(-\dfrac{1}{2}\right)+c = 1$

$c = \dfrac{1}{4}$

$f(x) = -\dfrac{3}{2}\cos(2x)+\dfrac{1}{4}$ A1

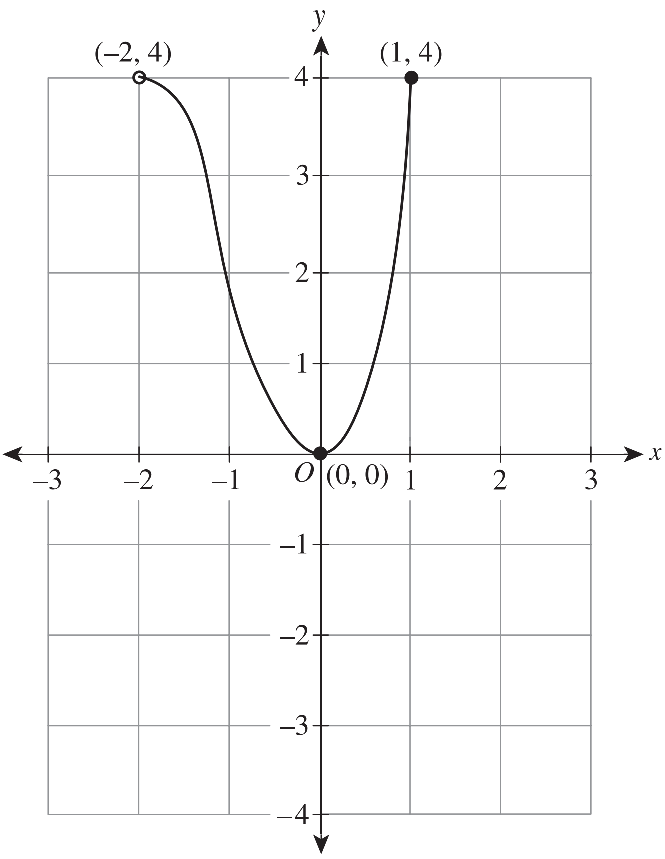
Question 3 (4 marks)

a. $f'(x) = 3x^2+6x$ M1

$f'(x) = 0 \Rightarrow 3x(x+2) = 0 \Rightarrow x = 0$ or $x = -2 \notin D_f$

$f(0) = 0 \Rightarrow (0,0)$ A1

b.



correct shape with an inflection point A1

correct endpoints and stationary point with (–2, 4) excluded A1

Question 4 (3 marks)

a.

	B	B'	
A	k^2	0.2	
A'	0.1		$1.6k$

$\Pr(A' \cap B') = 1 - \left(k^2 + 0.2 + 0.1\right) = 0.7 - k^2$ **OR** $\Pr(A' \cap B') = 1.6k - 0.1$ A1

Note: Responses do not require a table to obtain full marks.

b. $\Pr(A') = 1.6k = 0.1 + 0.7 - k^2$ M1

$k^2 + 1.6k - 0.8 = 0$

$5k^2 + 8k - 4 = 0$

$k = -2$ or $k = \dfrac{2}{5}$

$k = \dfrac{2}{5}$ A1

Question 5 (3 marks)

$\cos^2(3x) = \frac{1}{4}$ M1

$3x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\left[\begin{array}{l} \cos(3x) = \frac{1}{2} \\ \cos(3x) = -\frac{1}{2} \end{array}\right. \Rightarrow \left[\begin{array}{l} 3x = \frac{\pi}{3} \\ 3x = -\frac{\pi}{3} \end{array}\right.$ M1

$x = -\frac{\pi}{9}$ or $x = \frac{\pi}{9}$ A1

Question 6 (2 marks)

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x_new = (x - c)/b
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A1

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y_new = ay + d
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A1

Question 7 (4 marks)

a. Three numbers are obtained.

The first number can be any number; hence, the probability is $\frac{6}{6}$.

The second number must be the same as the first; hence, the probability is $\frac{1}{6}$.

The third number must be the same as the first; hence, the probability is $\frac{1}{6}$.

Multiplying all the probabilities gives:

$\frac{6}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ A1

b. The first number can be any number; hence, the probability is $\frac{6}{6}$.

The second number must be the same as the first; hence, the probability is $\frac{1}{6}$.

The third number must be different to the first; hence, the probability is $\frac{5}{6}$.

The order of the numbers can be arranged in three ways.

Multiplying all the probabilities by the number of possible ways gives:

$3 \times \frac{6}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{15}{36}$ **OR** $\frac{5}{12}$ A1

c. A: all numbers are greater than 3

B: exactly two numbers are the same

Determining $A \cap B$:

The first number must be greater than 3; hence, the probability is $\frac{3}{6}$.

The second number must be the same as the first; hence, the probability is $\frac{1}{6}$.

The third number must be greater than 3 but not the same as the previous number; hence, the probability is $\frac{2}{6}$.

The order of the numbers can be arranged in three ways.

Multiplying all the probabilities by the number of possible ways gives:

$$3 \times \frac{3}{6} \times \frac{1}{6} \times \frac{2}{6}$$

Determining B:

The answer from **part b.** $\left(\frac{15}{36}\right)$ is used.

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)} \qquad \text{M1}$$

$$= \frac{3 \times \frac{3}{6} \times \frac{1}{6} \times \frac{2}{6}}{\frac{15}{36}}$$

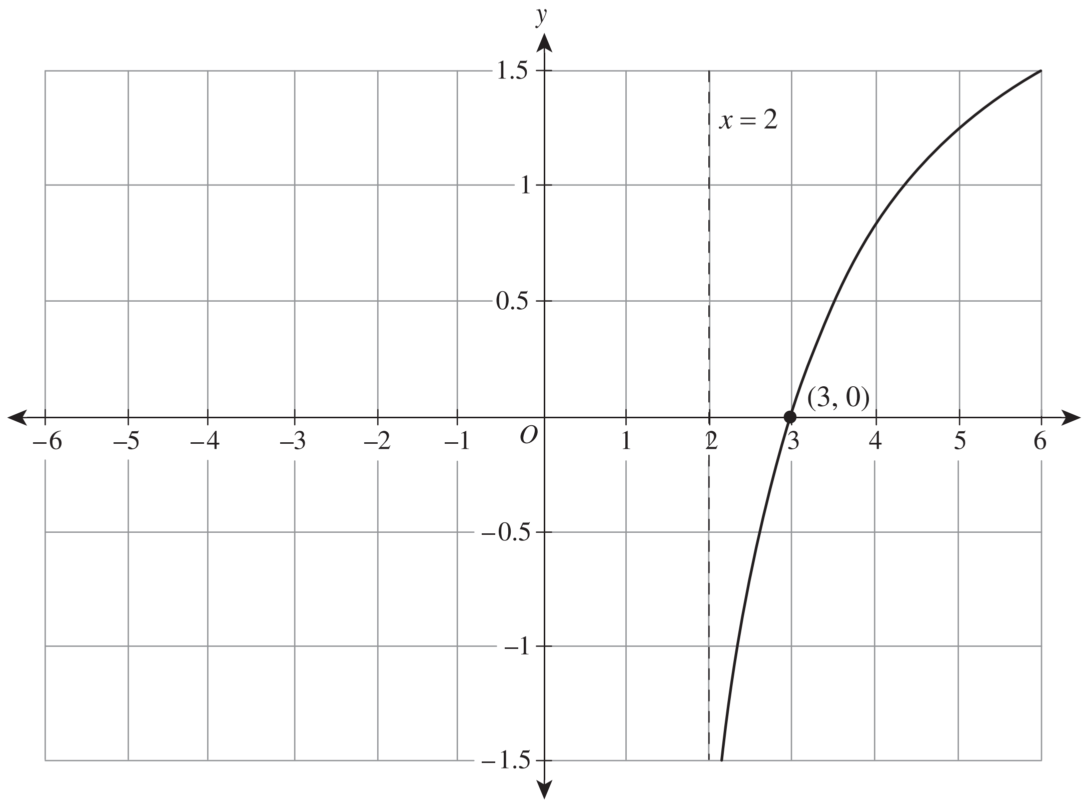
$$= \frac{1}{5} \qquad \text{A1}$$

Note: For M1, a correct numerator or denominator is sufficient to obtain the mark.

Question 8 (12 marks)

a. $x > 2$ **OR** $(2, \infty)$ A1

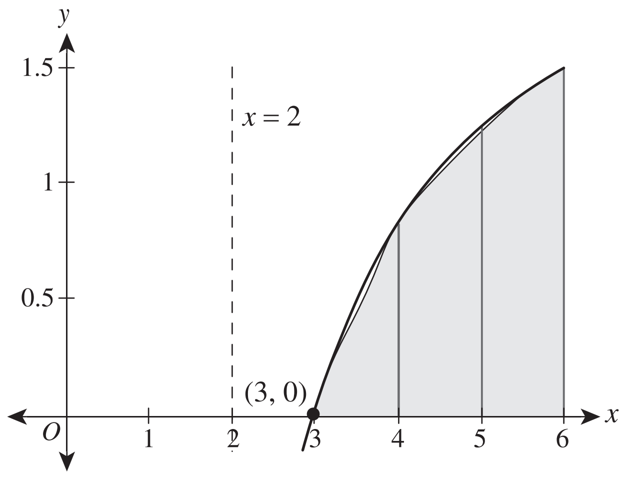
b.



correct shape A1

correct x-intercept and vertical asymptote A1

c.



$$\frac{1}{2}\left(f(3)+2f(4)+2f(5)+f(6)\right)=\frac{1}{2}\left(0+2\log_e(2)+2\log_e(3)+\log_e(4)\right)$$ M1

$$=\frac{1}{2}\left(\log_e(4)+\log_e(9)+\log_e(4)\right)$$ M1

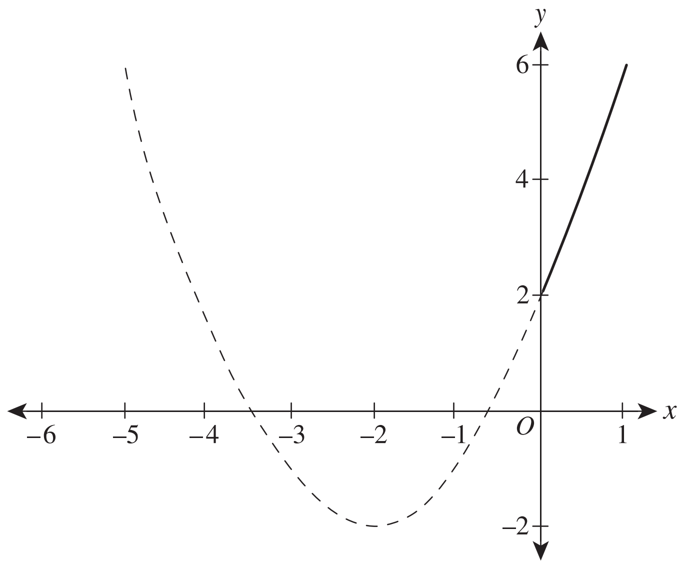
$$=\frac{1}{2}\log_e(144)$$

$$=\log_e(12)$$ A1

Note: Responses do not require a graphic to obtain full marks.

d. $R_g \subseteq D_f = (2, \infty)$ M1

The restricted graph of $g(x)$ from a to ∞ such that its range is contained in $(2, \infty)$ is as follows.



M1

Note: Accept any equivalent graphical or non-graphical method.

Hence, $a = 0$. A1

e. $h(x) = \log_e\left(x^2 + 4x\right)$ A1

f. $D_h = D_g = (0, \infty)$ A1

Note: Accept the response from ***part d.*** *for this mark.*

For $x > 0$, $x^2 + 4x \in R^+ \Rightarrow \log_e\left(x^2 + 4x\right) \in R$.

$R_h = R$ A1

Question 9 (6 marks)

a. $f'(x) = 4 - 2x$

$m = f'(a) = 4 - 2a$ M1

$y - f(a) = m(x - a)$

$y - 4a + a^2 = (4 - 2a)(x - a)$

$y = (4 - 2a)x - 4a + 2a^2 + 4a - a^2$ M1

$y = (4 - 2a)x + a^2$

b. $S(a)=\int_0^2\big((4-2a)x+a^2-f(x)\big)dx$ M1

$=\left[(2-a)x^2+a^2x-2x^2+\dfrac{x^3}{3}\right]_0^2$ M1

$=4(2-a)+2a^2-8+\dfrac{8}{3}$

$=2a^2-4a+\dfrac{8}{3}$ A1

c. $S'(a)=4a-4$

$S'(a)=0\Rightarrow a=1$ gives the minimum area.

The maximum area occurs for $a=0$ or $a=2$. A1