WA Exams Practice Paper E, 2015

Question/Answer Booklet

MATHEMATICS METHODS UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures

In words __________________________________

Your name __________________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 9 **(6 marks)**

(a) A and B are **acute** angles such that $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$.

(i) Determine the exact values of $\cos A$ and $\sin B$. (2 marks)

$$\cos A = \frac{4}{5}$$

$$\sin B = \frac{5}{13}$$

(ii) Show that the exact value of $\cos(A+B)$ is $\frac{33}{65}$. (2 marks)

$$\begin{aligned}\cos(A+B) &= \cos A \cos B - \sin A \sin B \\ &= \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13} \\ &= \frac{48-15}{65} \\ &= \frac{33}{65}\end{aligned}$$

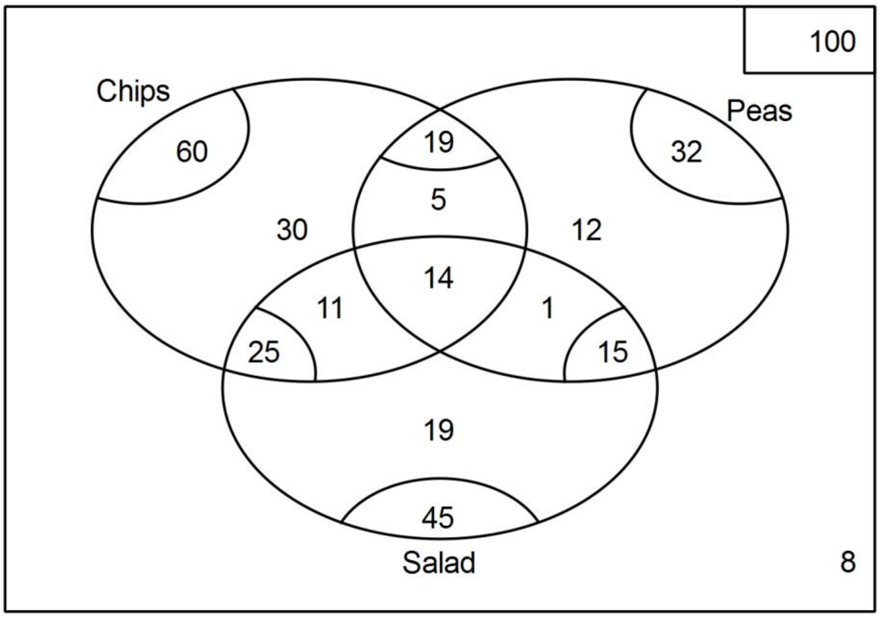
(b) A and B are **obtuse** angles such that $\sin A = \frac{3}{5}$ and $\cos B = -\frac{12}{13}$. Determine the exact value of $\cos(A-B)$. (2 marks)

$$\begin{aligned}\cos(A-B) &= \cos A \cos B + \sin A \sin B \\ &= -\frac{4}{5} \times -\frac{12}{13} + \frac{3}{5} \times \frac{5}{13} \\ &= \frac{48+15}{65} \\ &= \frac{63}{65}\end{aligned}$$

Question 10 **(8 marks)**

In a cafe, customers may choose any combination of chips, peas or salad as a side to their main course. Records show that the percentage of customers who choose salad only is 19%, chips only is 30%, peas and chips 19%, salad and peas 15%, salad and chips 25%, salad and chips and peas 14% and 8% choose none.

(a) Display this information in the Venn diagram below, indicating clearly on your diagram the percentages assigned to each region. (4 marks)



Determine the probability the next randomly chosen customer

(b) chooses chips. (1 mark)

$$\boxed{\frac{60}{100}}$$

(c) chooses salad or peas. (1 mark)

$$\boxed{\frac{62}{100}}$$

(d) chooses salad, given they choose chips or peas. (1 mark)

$$\boxed{\frac{26}{73}}$$

(e) chooses just one side, given they choose at least one side. (1 mark)

$$\boxed{\frac{61}{92}}$$

Question 11 **(9 marks)**

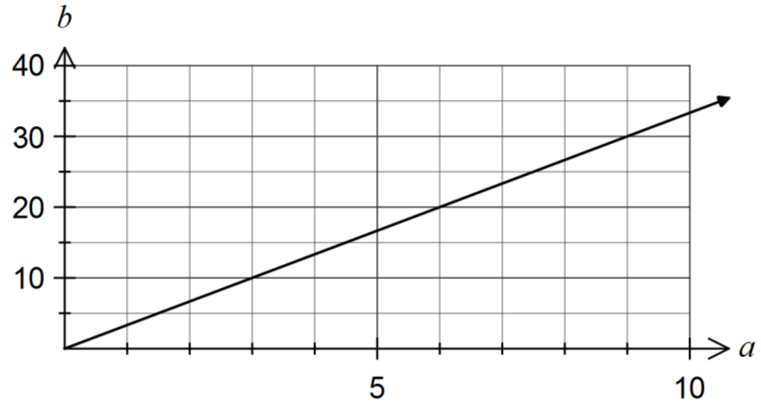
(a) The variable a is directly proportional to the variable b, such that when $a = 9$, $b = 30$.

(i) Determine an equation for the relationship between a and b. (2 marks)

$a = kb$: $9 = k \times 30 \Rightarrow k = 0.3$

$a = 0.3b$

(ii) Sketch a graph of the relationship between a and b. (2 marks)



(b) The pressure, P, in an air bubble varies inversely with the volume, V, of the bubble. It is known that $P = 2.4$ kPa when $V = 5$ cm^3.

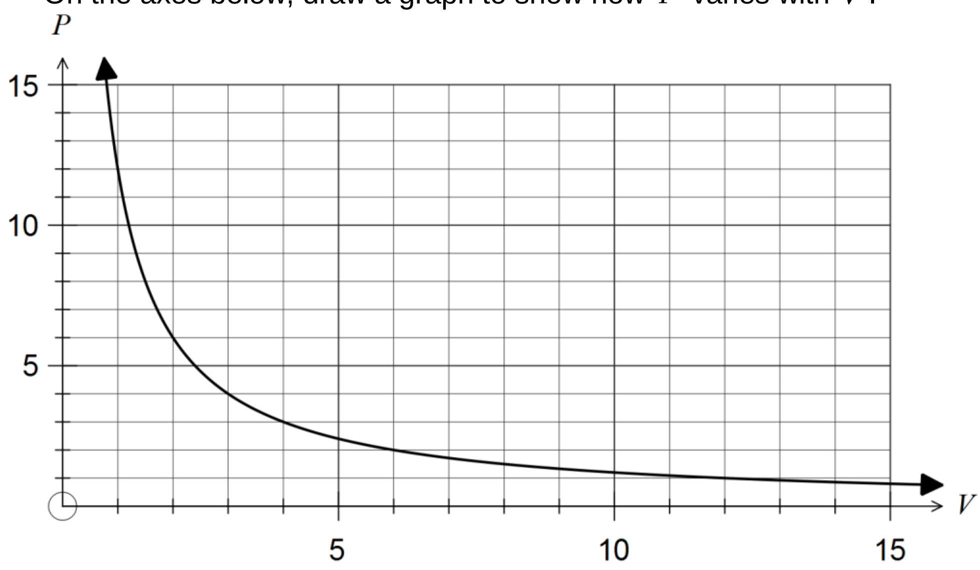
(i) Find the value of the constant k in the equation $P = \frac{k}{V}$. (1 mark)

$2.4 = \frac{k}{5} \Rightarrow k = 12$

(ii) Determine the value of V when $P = 10$ kPa. (1 mark)

$10 = \frac{12}{V} \Rightarrow V = 1.2$ cm^3

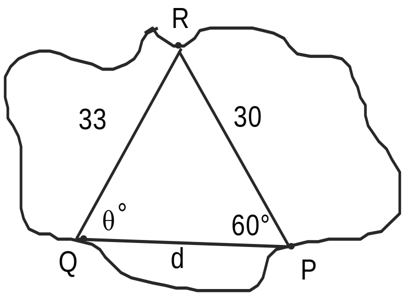
(iii) On the axes below, draw a graph to show how P varies with V. (3 marks)



Question 12 **(6 marks)**

P, Q and R are three campsites on the shore of a lake. The distance across the lake from R to P is 30 km, from R to Q is 33 km and $\angle RPQ$ is 60°.

(a) Show this information on the diagram below. (1 mark)



(b) If the distance from P to Q is d, use the cosine rule to show that $d^2 - 30d - 189 = 0$. (2 marks)

$$33^2 = 30^2 + d^2 - 2 \times 30 \times d \times \cos 60$$

$$1089 = 900 + d^2 - 30d$$

$$d^2 - 30d - 189 = 0$$

(c) Hence calculate the distance between the campsites at P and Q. (1 mark)

$d = 35.3$ km

(d) Determine an expression for θ, the size of $\angle PQR$, but do not calculate θ. (2 marks)

$$\frac{\sin\theta}{30} = \frac{\sin 60}{33}$$

$$\theta = \sin^{-1}\left(\frac{10 \sin 60}{11}\right)$$

Question 13 **(9 marks)**

(a) A regular six-sided dice marked with numbers one to six is thrown. Let three events be defined as follows:

Event A occurs when the dice shows a prime number.
Event B occurs when the dice shows an even number.
Event C occurs when the dice shows a factor of five.

(i) State which two of the above events are mutually exclusive and explain why. (2 marks)

B and C

They have no numbers in common:
B = { 2, 4, 6 } and C = { 1, 5 }

(ii) State which two of the above events are independent and explain why. (2 marks)

A and C

A = { 2, 3, 5 } and so $P(A) = P(A \mid C) = \frac{1}{2}$.

(b) A variety box of confectionary contains 17 different chocolates.

(i) Determine the number of different selections of four chocolates that can be made from the 17. (2 marks)

$^{17}C_4 = 2380$

(ii) Two people independently write down their selections of four chocolates. What is the probability that they both write down the same four chocolates? (1 mark)

$\frac{1}{2380}$

(iii) When n chocolates are chosen, the largest possible number of selections from the 17 occurs. Determine the value(s) of n and state the number of different selections possible when this number of chocolates are chosen. (2 marks)

n can be either 8 or 9, when 24310 selections are possible.

Question 14 **(9 marks)**

The table below shows the results of a scientific study which involved 2649 patients with a certain disease. The study was designed to evaluate the effectiveness of four different drugs in treating the disease.

	Clinical improvement		Individuals where clinical improvement was seen by extent of improvement		
Drug received	Seen	Not seen	Small improvement	Large improvement	Full recovery
A	254	264	123	86	45
B	276	355	135	90	51
C	318	401	85	176	57
D	408	373	103	230	75
Total people	1256	1393	446	582	228

Table notes:

- all individuals were administered only one of the four drugs
- where clinical improvement was seen, all individuals were allocated to the one most relevant category for extent of improvement.

(a) One patient is selected at random from those in the study, determine the probability that

(i) clinical improvement was not seen in the patient. (1 mark)

$$\boxed{\frac{1393}{2649}}$$

(ii) the patient was given drug B and experienced a large clinical improvement. (1 mark)

$$\boxed{\frac{90}{2649}}$$

(iii) the patient was given drug A or experienced a full clinical recovery. (1 mark)

$$\boxed{\frac{254+264+51+57+75}{2649}=\frac{701}{2649}}$$

(b) Given that a clinical improvement was seen in a randomly selected patient, what is the probability that the patient was administered drug D and did not experience a full recovery? (2 marks)

$$\boxed{\frac{103+230}{1256}=\frac{333}{1256}}$$

(c) Determine the probability that a patient randomly selected from those where clinical improvement was seen, experienced a full clinical recovery given they used drug A. (1 mark)

$$P(F \mid A) = \frac{45}{254} \approx 0.177$$

(d) For those individuals where clinical improvement was seen, do you consider that making a full clinical recovery is independent of the drug received? Justify your answer. (3 marks)

$$P(F) = \frac{228}{1256} \approx 0.182$$

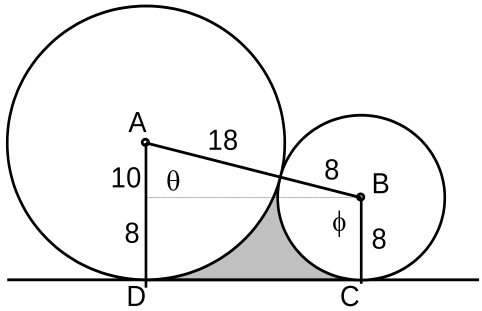
$$P(F \mid A) \approx 0.177$$
$$P(F \mid B) \approx 0.185$$
$$P(F \mid C) \approx 0.179$$
$$P(F \mid D) \approx 0.184$$

Yes, independent.

The probability of such an individual making a full recovery is 0.182, and the conditional probabilities are all very close to this figure.

Question 15 **(9 marks)**

Two circles, one of radius 8 cm and the other of radius 18 cm, with a common tangent, touch each other as shown in the diagram.



(a) Calculate the perimeter of the shaded region. (5 marks)

$CD = \sqrt{26^2 - 10^2} = 24$ cm

$\theta = \cos^{-1}\dfrac{10}{26} = 1.176^r$

$\phi = \pi - 1.176 = 1.966^r$

Long arc $= 18 \times 1.176 = 21.17$

Short arc $= 8 \times 1.966 = 15.72$

Perimeter $= 24 + 21.17 + 15.72 = 60.89$ cm

(b) Calculate the area of the shaded region. (4 marks)

Trapezium ABCD $= \dfrac{18+8}{2} \times 24 = 312$

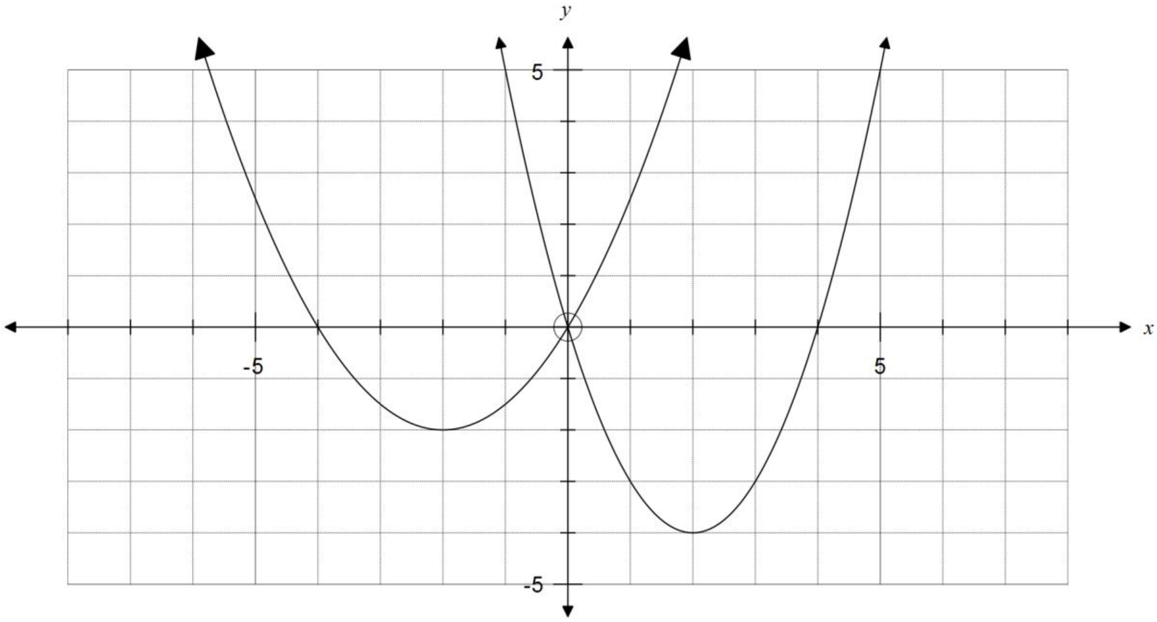
Large sector $= \dfrac{1}{2} \times 18^2 \times 1.176 = 190.51$

Small sector $= \dfrac{1}{2} \times 8^2 \times 1.966 = 62.90$

Total area $= 312 - 190.51 - 62.90 = 58.59$ cm^2

Question 16 **(8 marks)**

The function $y = f(x)$ is shown below.



(a) State the equation of $f(x)$ in the form $y = a(x+p)^2 + q$. (2 marks)

$$\boxed{y = 0.5(x+2)^2 - 2}$$

(b) State the domain and range of $f(x)$. (2 marks)

$$\boxed{x \in \mathbb{R},\ y \geq -2}$$

Another function is given by $g(x) = 2f(x-4)$.

(c) Describe the transformations required to produce $g(x)$ from $f(x)$. (2 marks)

In either order:

- translation of 4 units in the positive x direction

- vertical dilation of scale factor 2

(d) Draw the graph of $y = g(x)$ on the axes above. (2 marks)

Question 17 **(8 marks)**

At the end of a technology course, all students sat a practical and a theory examination, with 20% achieving a distinction in the practical examination, 3% of students achieving distinctions in both examinations and 76% achieving no distinction in either examination.

(a) A student is chosen at random. Determine the probability that the student did not achieve a distinction in the theory examination given that they did achieve a distinction in the practical examination. (3 marks)

$$P(T \mid P) = \frac{0.03}{0.20} = 0.15$$

$$P(\overline{T} \mid P) = 1 - 0.15$$
$$= 0.85$$

(Tree diagram useful to visualise events)

(b) What is the probability that a student chosen at random from the course achieved a distinction in the theory examination or the practical examination? (2 marks)

$$P(\overline{P} \cap T) = 1 - (0.2 + 0.76) = 0.04$$

$$P(P \cup T) = 0.20 + 0.04$$
$$= 0.24$$

(c) Are the events 'achieving a distinction in the practical examination' and 'achieving a distinction in the theory examination' independent? Explain your answer. (3 marks)

No

$$P(T) = 0.03 + 0.04$$
$$= 0.07$$
$$\neq P(T \mid P)$$

(See above)

Question 18 **(7 marks)**

(a) State the equation of the axis of symmetry for the graph $y = 2x^2 - 8x + 7$. (1 mark)

$$x = -\frac{-8}{2(2)}$$

$$x = 2$$

(b) Determine the discriminant of the quadratic equation $4x^2 - 20x + 25 = 0$ and hence state how many solutions the equation has. (2 marks)

$$(-20)^2 - 4(4)(25) = 400 - 400$$
$$= 0$$

Hence one solution.

(c) The parabola $y = ax^2 + bx - 10$ passes through the points (4.5, 8) and (-2.5, 15). Determine the values of a and b. (4 marks)

$$8 = 4.5^2 a + 4.5b - 10$$
$$15 = (-2.5)^2 a - 2.5b - 10$$

Solve simultaneously to get

$a = 2,\ b = -5$

Question 19 **(8 marks)**

(a) In a class of 17 students, 11 are female. If two students are chosen at random for an activity, determine the probability that they are both of the same gender. (3 marks)

$$\begin{aligned} P(FF) &= \frac{11}{17}\times\frac{10}{16}=\frac{110}{272} \\ P(MM) &= \frac{6}{17}\times\frac{5}{16}=\frac{30}{272} \\ P(FF)+P(MM) &= \frac{110}{272}+\frac{30}{272} \\ &= \frac{140}{272}=\frac{35}{68} \end{aligned}$$

(b) The probability that a door to door salesman convinces a customer to buy is 0.4.

Assuming that sales are independent, find the probability that the salesman makes at least one sale before reaching the fourth house. (2 marks)

$$\begin{aligned} &1-(0.6)^3 \\ &=0.784 \end{aligned}$$

(c) A and B are two independent events such that $P(A)=0.2$ and $P(B)=0.15$.

Evaluate

(i) $P(A|B)$ (1 mark)

$$0.2$$

(ii) $P(A\cap B)$ (1 mark)

$$0.2\times 0.15=0.03$$

(iii) $P(A\cup B)$ (1 mark)

$$0.2+0.15-0.03=0.32$$

Question 20 **(5 marks)**

(a) Simplify $\sin(A+B)-\sin(A-B)$. (1 mark)

$$\boxed{2\cos(A)\cdot\sin(B)}$$

(b) Solve

(i) $2\sin^2 x-1=0$, $0\le x\le 2\pi$. (2 marks)

$$\sin x=\pm\frac{1}{\sqrt{2}}$$
$$x=\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}$$

(ii) $2\cos^2 3x+\cos 3x=0$, $-90<x<90$. (2 marks)

$$\cos 3x(2\cos 3x-1)=0$$
$$x=-80°,\ -40°,\ -30°,\ 30°,\ 40°,\ 80°$$

Question 21 **(6 marks)**

(a) Use the angle sum and difference identities to show that

(i) $\cos(2A) = \cos^2 A - \sin^2 A$. (1 mark)

$$\begin{aligned}\cos 2A &= \cos(A + A)\\ &= \cos A\cos A - \sin A\sin A\\ &= \cos^2 A - \sin^2 A\end{aligned}$$

(ii) $\sin A = \cos(90^\circ - A)$. (1 mark)

$$\begin{aligned}\cos(90 - A) &= \cos 90\cos A + \sin 90\sin A\\ &= 0 \times \cos A + 1 \times \sin A\\ &= \sin A\end{aligned}$$

(b) The exact values of the sine and cosine of 36° are $\dfrac{\sqrt{10 - 2\sqrt{5}}}{4}$ and $\dfrac{1+\sqrt{5}}{4}$ respectively.

Use both identities from (a) to show that the exact value of the sine of 18° is $\dfrac{\sqrt{5}-1}{4}$.

(4 marks)

$$\begin{aligned}\sin 18 &= \cos(90 - 18)\\ &= \cos 72\\ &= \cos(2 \times 36)\\ &= \cos^2 36 - \sin^2 36\\ &= \left(\frac{1+\sqrt{5}}{4}\right)^2 - \frac{10 - 2\sqrt{5}}{16}\\ &= \frac{1 + 2\sqrt{5} + 5 - 10 + 2\sqrt{5}}{16}\\ &= \frac{\sqrt{5} - 1}{4}\end{aligned}$$

End of questions

Additional working space

Question number: _________

Additional working space

Question number: _________

Additional working space

Question number: _________