



## Semester One Examination, 2021

### Question/Answer booklet

# MATHEMATICS APPLICATIONS UNIT 3

## Section One: Calculator-free

Your name \_\_\_\_\_

Teacher's name \_\_\_\_\_

### Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Number of additional  
answer booklets used  
(if applicable):

### Materials required/recommended for this section

#### *To be provided by the supervisor*

This Question/Answer booklet

Formula sheet

#### *To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					100

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free**

**35% (52 Marks)**

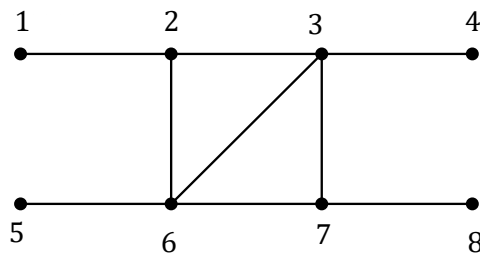
This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

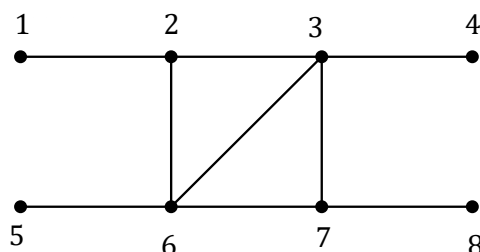
**Question 1**

**(6 marks)**

Graph  $G$  is shown:



- (a) State the number of edges and the number of faces in  $G$  and hence show that the graph satisfies Euler's formula. (2 marks)
- (b) State the length of the longest trail in  $G$  and highlight the edges in a trail of this length on the graph above. (2 marks)
- (c) State the length of the longest path in  $G$  and highlight the edges in a path of this length on the copy of  $G$  below. (2 marks)



**Question 2****(7 marks)**

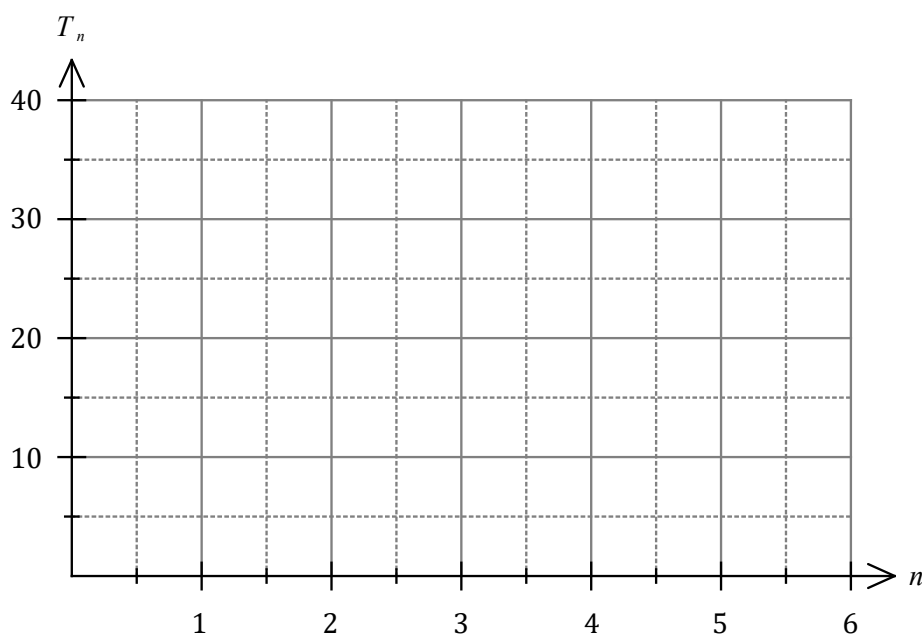
A sequence is defined by  $T_{n+1} = T_n - 6$ ,  $T_1 = 35$ .

- (a) Write the first six terms of the sequence in the following table.

**(2 marks)**

$n$	1	2	3	4	5	6
$T_n$						

- (b) Graph the first six terms of the sequence on the axes below.

**(2 marks)**

- (c) The rule for the  $n^{\text{th}}$  term of the sequence is  $T_n = an + b$ . State the value of the constant  $a$  and the value of the constant  $b$ . **(3 marks)**

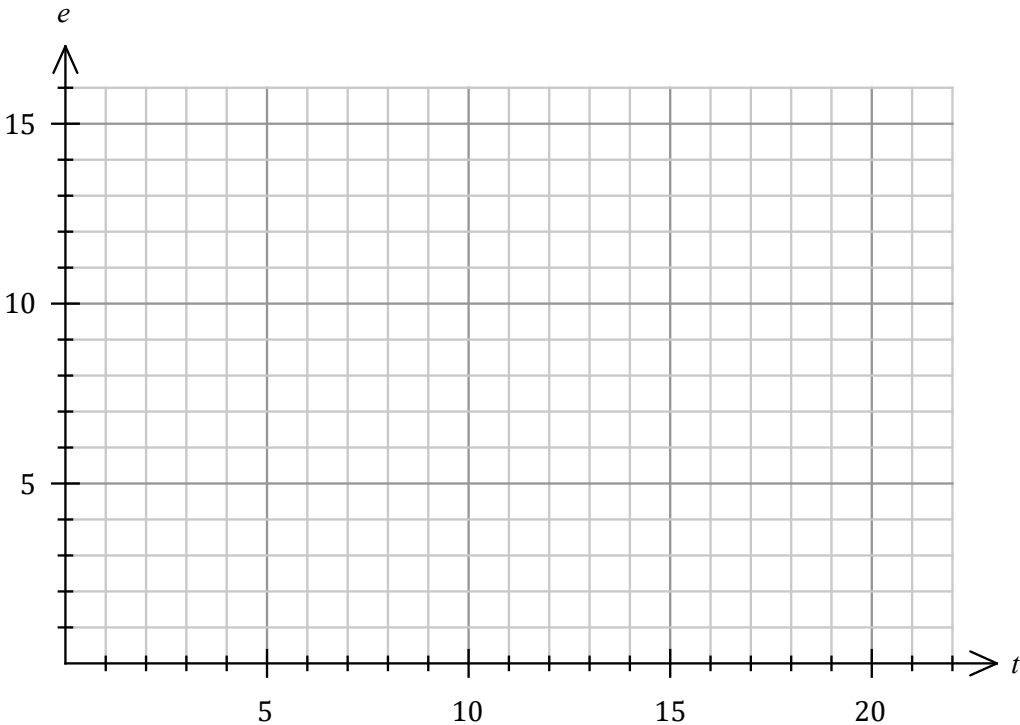
Question 3

(6 marks)

A student recorded the time taken and the number of errors made when completing nine multiple choice tests, each with 40 different questions, in the table below.

Time, $t$ minutes	15	22	20	18	11	15	13	6	8
Number of errors, $e$	8	4	7	10	15	13	9	16	12

- (a)
- Construct a scatterplot of this data on the axes below.
- (2 marks)



- (b)
- Describe the strength and direction of the association between the two variables.
- (2 marks)

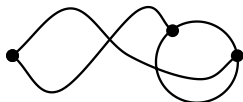
- (c)
- The student used the data to conclude that taking more time to answer multiple choice tests caused them to answer more questions correctly. Explain whether this conclusion is justified.
- (2 marks)

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**Question 4****(7 marks)**

- (a) Graph  $G$  is shown below. Redraw  $G$  in the plane, to clearly show that it is planar.

(1 mark)



- (b) Let  $K_n$  be the complete graph with  $n$  vertices. Draw, and state the number of edges in,

(i)  $K_4$ .

(2 marks)

(ii)  $K_5$ .

(2 marks)

- (c) State, with reasoning, whether  $K_4$  is a planar graph.

(2 marks)

**Question 5**

**(7 marks)**

Bivariate data analysis of the eye diameter  $D$  mm, length  $L$  mm and width  $W$  mm of a large number of cardinal fish yielded the following correlation coefficients and least-squares lines:

$$r_{WL} = 0.9, \quad L = 7W - 4, \quad r_{DL} = 0.8, \quad L = 16D + 5.$$

- (a) Determine the percentage of the variation in the lengths of these fish that can be explained by the variation in their eye diameters. (2 marks)
- (b) One of the least-squares lines would be better than the other as a predictor for the lengths of these fish. Write the equation of the line below and explain your choice. (2 marks)
- (c) Use the equation from part (b) to predict the length of a fish that has a width of 15 mm and an eye diameter of 6 mm. (1 mark)
- (d) Explain why it is difficult to comment on the validity of the prediction made in part (c). (2 marks)

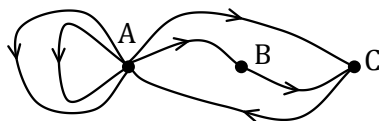
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## Question 6

(7 marks)

- (a) Digraph  $G_1$  is shown. Complete the adjacency matrix  $M$  for  $G_1$ .

(2 marks)



$M$	A	B	C
A			
B			
C			

- (b) The adjacency matrix  $A$  for the non-directed graph  $G_2$  with 4 vertices is shown below.

$$A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 3 & 0 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

- (i)  $G_3$  is a subgraph of  $G_2$ , and has 4 vertices. State, with reasoning, the minimum number of edges that must be removed from  $G_2$  so that  $G_3$  is a simple graph. (3 marks)

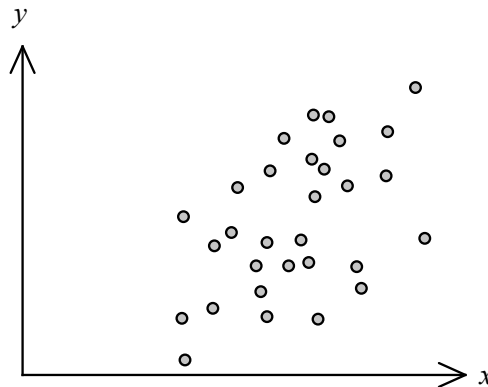
- (ii) In the matrix  $A^4$ , the entry  $a_{3,2} = 68$ . Use precise terminology associated with graphs to fully explain the meaning of this entry in  $A^4$ . (2 marks)



Question 7

(6 marks)

The scatterplot below shows data from 30 samples drawn from different suburbs in a city. The variables are the percentage of people in each sample who have grey hair ( $x$ ) and who have heart disease ( $y$ ).

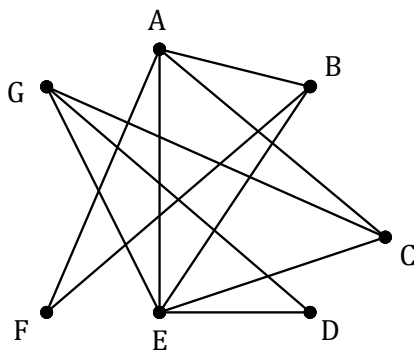


- (a) The correlation coefficient  $r_{xy}$  for this data is one of 0.8, 0.5, 0.2,  $-0.2$ ,  $-0.5$ ,  $-0.8$ . State  $r_{xy}$  and explain your choice. (2 marks)
- (b) The least-squares line for the data is  $y = ax + b$ , where  $a$  and  $b$  are constants.
- (i) State the name of the response variable for this least-squares line. (1 mark)
- (ii) Explain whether the variable  $a$  would be a positive or negative number. (1 mark)
- (c) Identify and explain a possible non-causal explanation for the observed association between the variables in this data. (2 marks)

**Question 8****(6 marks)**

- (a) Let  $K_{m,n}$  be the complete bipartite graph with  $m$  vertices in one set and  $n$  vertices in the second set. Draw  $K_{3,3}$  and explain whether  $K_{3,3}$  is Hamiltonian, semi-Hamiltonian or neither. (3 marks)

- (b) The vertices and edges in the graph below represent students and friendships, respectively. Determine whether it is possible for the students to sit in a circle so that every student is sitting between two friends. If it is possible, draw a possible seating plan. If it is not possible, explain why. (3 marks)



Supplementary page

Question number: \_\_\_\_\_

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MARKER	Markers use only		
	Question	Maximum	Mark
TSEN	1	6	
	2	7	
	3	6	
	4	7	
	5	7	
	6	7	
	7	6	
	8	6	
	S1 Total	52	
	S2 Total	98	
	TOTAL	150	
	Percent	100%	