MATHEMATICS APPLICATIONS UNITS 1 AND 2
Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	13	13	100	99	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Booklet.

Section Two: Calculator-assumed   **65% (99 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8   **(6 marks)**

A worker in a chemical factory earns \$42.70 per hour on a weekday, time and a half at weekends, and a fixed allowance of \$39.15 per day.

(a)   Calculate the workers' weekly pay when she works from 9 am until 1 pm and then 1.30 pm until 4.30 pm from Wednesday to Sunday inclusive.   (4 marks)

Solution
Daily hours: $4 + 3 = 7$ hours per day
Weekdays: $42.70 \times 7 + 39.15 = 338.05$ per day
Weekends: $42.70 \times 1.5 \times 7 + 39.15 = 487.50$ per day
Total: $3 \times 338.05 + 2 \times 487.50 = 1014.15 + 975.00 = \$1989.15$ per week
Specific behaviours
✓ calculates daily hours
✓ calculates weekday pay
✓ calculates weekend pay
✓ calculates total pay

(b)   Calculate the workers' total holiday pay when she takes four weeks leave, if the company pays her for 35 hours per week at her base rate, does not pay the fixed allowance but does add leave loading of 17.5%.   (2 marks)

Solution
Weekly: $4 \times 42.70 \times 35 \times 1.175 = 5978 \times 1.175 = \$7024.15$
Specific behaviours
✓ calculates 4 wks at base rate
✓ increases by 17.5%

Question 9   **(8 marks)**

Sam and Tom recorded their daily exercise time over a two-week period. Their times are shown below to the nearest minute.

Sam	29	27	30	28	27	28	31	31	25	32	26	27	32	26
Tom	27	24	25	26	27	23	28	26	29	25	27	29	26	29

(a) The left-hand side of the stem plot below displays the exercise times for Sam. Complete the right-hand side for the times that Tom exercised.   (2 marks)

	Stem	
	23.	0
	24.	0
0	25.	0 0
0 0	26.	0 0 0
0 0 0	27.	0 0 0
0 0	28.	0
0	29.	0 0 0
0	30.	
0 0	31.	
0 0	32.	

Exercise times for Sam (LHS) — Exercise times for Tom (RHS)

Solution
See stem plot
Specific behaviours
✓ plots 14 zeroes
✓ aligns zeroes in vertical columns

(b) Calculate the mean of the times spent exercising for each person and explain how their difference relates to a feature of the stem plot.   (3 marks)

Solution
$\bar{x}_{SAM} = 28.5$ and $\bar{x}_{TOM} = 26.5$ minutes
The leaves for Sam (LHS) are clearly further down the scale than the leaves for Tom (RHS).
Specific behaviours
✓ calculates first mean
✓ calculates second mean
✓ explains difference using feature of stem plot

(c) Calculate the standard deviations of the times spent exercising for each person and explain how their difference relates to a feature of the stem plot.   (3 marks)

Solution
$sd_{SAM} = 2.26$ and $sd_{TOM} = 1.80$ minutes
The leaves for Sam (LHS) are more spread out than for Tom (RHS)
Specific behaviours
✓ states first standard deviation
✓ states second standard deviation
✓ explains difference using feature of stem plot

Question 10 **(6 marks)**

A semi-circular grassed area has a radius of 45 m.

(a) A fence is to be installed around the edge of the area at a cost of $85 per metre. Calculate the cost of fencing to the nearest hundred dollars. (3 marks)

Solution
Curved length: $2 \times \pi \times 45 \div 2 = 141.4$ m
Total: $2 \times 45 + 141.4 = 231.4$ m
Cost: $231.4 \times 85 = 19669 \approx \$19\,700$
Specific behaviours
✓ calculates curved side
✓ calculates total length
✓ calculates cost, rounding to nearest $100

(b) A rotunda is to be built in the middle of the grassed area with a hemispherical glass roof. The cost of the rotunda is $17 800 plus $235 per square metre for the glass used in the hemispherical roof. If the radius of the hemisphere is 4.5 m, calculate the cost of building the rotunda to the nearest hundred dollars. (3 marks)

Solution
Area of glass: $4 \times \pi \times 4.5^2 \div 2 = 127.2$ m^2
Cost: $17800 + 127.2 \times 235 = 17800 + 29892 = 47692 \approx \$47\,700$
Specific behaviours
✓ uses sphere area formula
✓ adjusts for hemisphere
✓ calculates total cost, rounding to nearest $100

See next page

Question 11   **(7 marks)**

A young couple purchased a house for $419 000 and sold it one year later for $432 000.

(a)   Calculate the percentage profit they made.   (2 marks)

Solution
$Profit = 432000 - 419000 = 13000$
$\frac{13000}{419000} \times 100 = 3.1\%$ profit
Specific behaviours
✓ calculates profit
✓ calculates percentage profit

(b)   The couple financed their purchase of the house with an interest only loan of $336 000 at a rate of 4.85% pa. Calculate the simple interest payable for the first month.   (2 marks)

Solution
$I = 336000 \times 0.0485 \times \frac{1}{12} = 16296 \times \frac{1}{12} = \$1358$
Specific behaviours
✓ calculates interest for one year
✓ calculates interest for one month

(c)   The couple used the services of a real estate agent to sell the house. The agent charged 2.5% of the selling price on the first $100 000 and 0.75% on the remainder. Calculate the agent's fee for selling the house.   (2 marks)

Solution
$0.025 \times 100000 + 0.0075 \times 332000 = 2500 + 2490 = \$4990$
Specific behaviours
✓ splits selling price correctly
✓ works out percentages and adds

(d)   The agent bought advertising through an internet company for $423.50, including GST at 10%. How much of the advertising cost was GST?   (1 mark)

Solution
$GST = \frac{423.50}{11} = \$38.50$
Specific behaviours
✓ calculates GST

Question 12 **(7 marks)**

Stalls at a pop-up market cost $60 per day. A student, who makes pairs of earrings that cost $5 each and sell for $11 each, hires a stall for a day.

(a) If the student sells x pairs of earrings at the market one day, explain why

(i) the total cost for the day is given by $5x + 60$. (1 mark)

Solution
Cost is sum of making earrings ($5x$) and stall hire (60)
Specific behaviours
✓ identifies parts of expression

(ii) the total revenue for the day is given by $11x$. (1 mark)

Solution
Revenue is number sold (x) multiplied by price (11)
Specific behaviours
✓ reasonable explanation

(b) Complete the table below to show the cost and revenue for the given values of x.

(2 marks)

x	0	6	12	18
Total cost, $	60	90	120	150
Total revenue, $	0	66	132	198

Solution
See table
Specific behaviours
✓ all costs
✓ all revenues

(c) Graph the two straight lines that represent the daily cost and revenue when the student sells x pairs of earrings at the market on the axes below. (2 marks)

Solution
See graph
Specific behaviours
✓ straight line for cost
✓ straight line for revenue
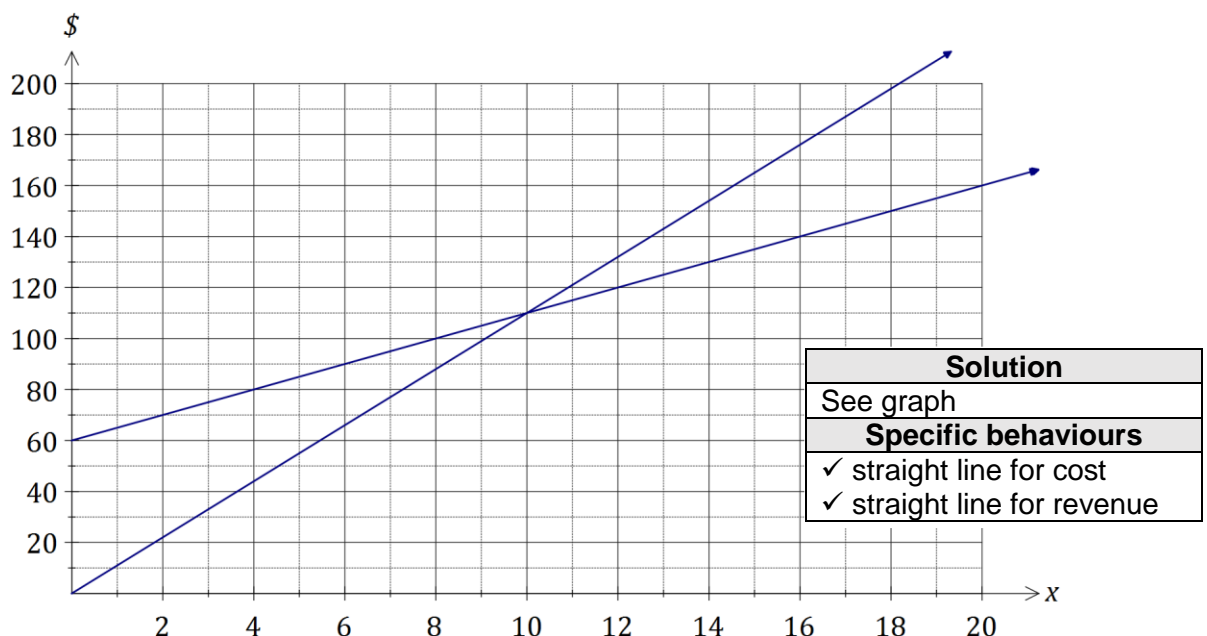
(d) Using your graph, or otherwise, determine the number of pairs of earrings that the student must sell per day in order to break-even at the market. (1 mark)

Solution
10 pairs of earrings
Specific behaviours
✓ states x value that reflects intersection of lines

See next page

Question 13   **(8 marks)**

(a)   A young person working in a grocery store has to label shelves with the price per 100 mL of the following bottles of detergent made by the same company:

- a small 450 mL bottle priced at $1.69
- a medium 720 mL bottle priced at $2.99
- a large 1.25 L bottle priced at $4.95

(i)   Determine the unit cost for each bottle as required by the store and hence state which size represents the best value.   (3 marks)

Solution
Small: $169 \times \frac{100}{450} = 37.6$ c/100mL
Medium: $299 \times \frac{100}{720} = 41.5$ c/100mL
Large: $495 \times \frac{100}{1250} = 39.6$ c/100mL
Small is the best value.

Specific behaviours
✓ calculates 100 mL costs for small and medium
✓ calculates 100 mL cost for large
✓ selects best value

(ii)   The company that made the detergent had just increased the wholesale price of a box of 24 small bottles from $26.40 to $27.30, claiming that the increase was below the current 2.4% annual rate of inflation. Was this claim correct? Justify your answer.   (2 marks)

Solution
Increase is $\frac{0.9}{26.40} \times 100 = 3.4\%$
Claim incorrect, as increase of 3.4% was above inflation rate of 2.4%.

Specific behaviours
✓ states claim incorrect, with reason
✓ justifies mathematically

(b)   A sum of $5 500 was invested for four years in an account paying 3.75% interest per annum compounded annually.

(i)   Calculate the amount in the account at the end of the four years.   (2 marks)

Solution
$FV = 5500 \times 1.0375^4 = \$6372.58$

Specific behaviours
✓ substitutes into compound interest formula
✓ calculates future value

(ii)   Determine the total interest accumulated over the four years.   (1 mark)

Solution
Interest: $6372.58 - 5500.00 = \$872.58$

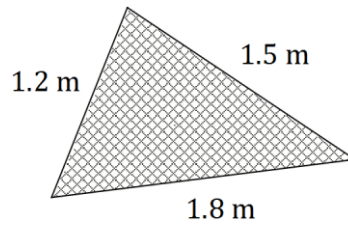| Specific behaviours |
|---|
| ✓ calculates interest |

See next page

Question 14 **(7 marks)**

A triangular sign has sides of lengths 1.2 m, 1.5 m and 1.8 m.

(a) Use Heron's rule to determine the area of the sign. (3 marks)

Solution
$s = \frac{1.2+1.5+1.8}{2} = 2.25$
$A = \sqrt{2.25(2.25-1.2)(2.25-1.5)(2.25-1.8)} \approx 0.893$ sq m
Specific behaviours
✓ calculates semi-perimeter
✓ substitutes into Heron's rule
✓ calculates area, rounding sensibly

(b) Show use of trigonometry to determine the size of the angle opposite the side of length 1.8 m. (2 marks)

Solution
$1.8^2 = 1.2^2 + 1.5^2 - 2 \times 1.2 \times 1.5 \times \cos A$
$A = 82.8°$
Specific behaviours
✓ substitutes correctly into cosine rule
✓ calculates angle

(c) Use trigonometry to determine the size of the smallest angle in the triangle. (2 marks)

Solution
$\frac{1.2}{\sin A} = \frac{1.8}{\sin 82.8}$
$A = 41.4°$
Specific behaviours
✓ shows use of trig
✓ calculates smallest angle

See next page

Question 15   **(8 marks)**

Nasim won $12 500 in a lottery and decided to invest $7 000 in the stock market and spend the rest on a holiday.

(a)   Nasim booked a holiday in Egypt, converting $750 Australian dollars (AUD) spending money into Egyptian pounds (EGP) before he left. The exchange rates advertised by his local bank for 1 AUD to EGP were buy: 7.5768 and sell: 5.7723.

     (i)   Calculate how many Egyptian pounds Nasim took with him, giving your answer to the nearest 10 pounds.   (3 marks)

Solution
$750 \times 5.7723 = 4329.23 \approx 4330$ Egyptian pounds
Specific behaviours
✓ uses sell rate
✓ multiplies to convert
✓ rounds to nearest 10 pounds

     (ii)   Nasim returned from holiday with 950 Egyptian pounds. How many Australian dollars would his local bank give him for these pounds?   (2 marks)

Solution
$950 \div 7.5768 = \$125.38$
Specific behaviours
✓ uses buy rate
✓ divides to convert

(b)   The cost of the shares Nasim bought at $11.55 each in a company, together with his brokerage fee of $70, came to exactly $7 000. The company forecast it would pay a full year dividend of 35 cents per share.

     (i)   Determine how many shares Nasim bought in the company.   (2 marks)

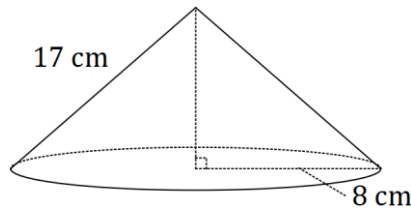| Solution |
|---|
| $7000 - 70 = 6930$ |
| $6930 \div 11.55 = 600$ shares |
| **Specific behaviours** |
| ✓ calculates amount spent on shares |
| ✓ calculates number of shares |

     (ii)   What is his expected full year dividend payment?   (1 mark)

Solution
$600 \times 0.35 = \$210$
Specific behaviours
✓ multiplies number of shares by dividend, showing units

See next page

Question 16   **(8 marks)**

A solid cone has a base radius of 8 cm and a slant height of 17 cm.

(a)   Determine the total surface area of the cone.   (2 marks)
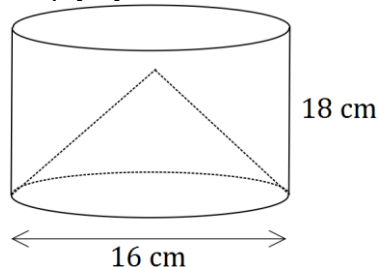
Solution
$TSA = \pi \times 8^2 + \pi \times 8 \times 17$
$TSA = 628.3$ cm^2

Specific behaviours
✓ substitutes into formula
✓ evaluates TSA

(b)   Calculate the perpendicular height of the cone.   (2 marks)

Solution
$h^2 + 8^2 = 17^2$
$h = 15$

Specific behaviours
✓ uses Pythagoras
✓ calculates height

The solid cone is placed inside an empty cylindrical vase of diameter 16 cm and height 18 cm.

(c)   Water is then poured into the vase until it reaches the top. Assuming that the cone does not float, how much water will be required to fill the vase?   (4 marks)

Solution
$V_{cone} = \frac{1}{3} \times \pi \times 8^2 \times 15 = 1005$
Cylinder radius $= \frac{16}{2} = 8$
$V_{cyl} = \pi \times 8^2 \times 18 = 3619$
$V = 3619 - 1005 = 2614$ cm^3

Specific behaviours
✓ calculates volume of cone
✓ uses correct cylinder radius
✓ calculates volume of cylinder
✓ subtracts to find volume of water

See next page

Question 17   **(11 marks)**

An energy saving globe costs $12 to buy and $0.005 per hour to run. A regular globe that delivers the same amount of light as the energy saving globe costs $2.50 to buy and $0.03 per hour to run.

(a)   Assuming that neither globe will need to be replaced,

     (i)   explain why the cost, in dollars, of buying and running one energy saving globe for x hours is given by the expression $12 + 0.005x$.   (1 mark)

Solution
12 = cost of globe, plus 0.005 multiplied by number of hours run
Specific behaviours
✓ reasonable explanation of both parts of expression

     (ii)   write an expression for the cost, in dollars, of buying and running one regular globe for x hours.   (1 mark)

Solution
$2.5 + 0.03x$
Specific behaviours
✓ correct expression

     (iii)   determine the number of hours for which the cost of buying and running the globes is the same.   (2 marks)

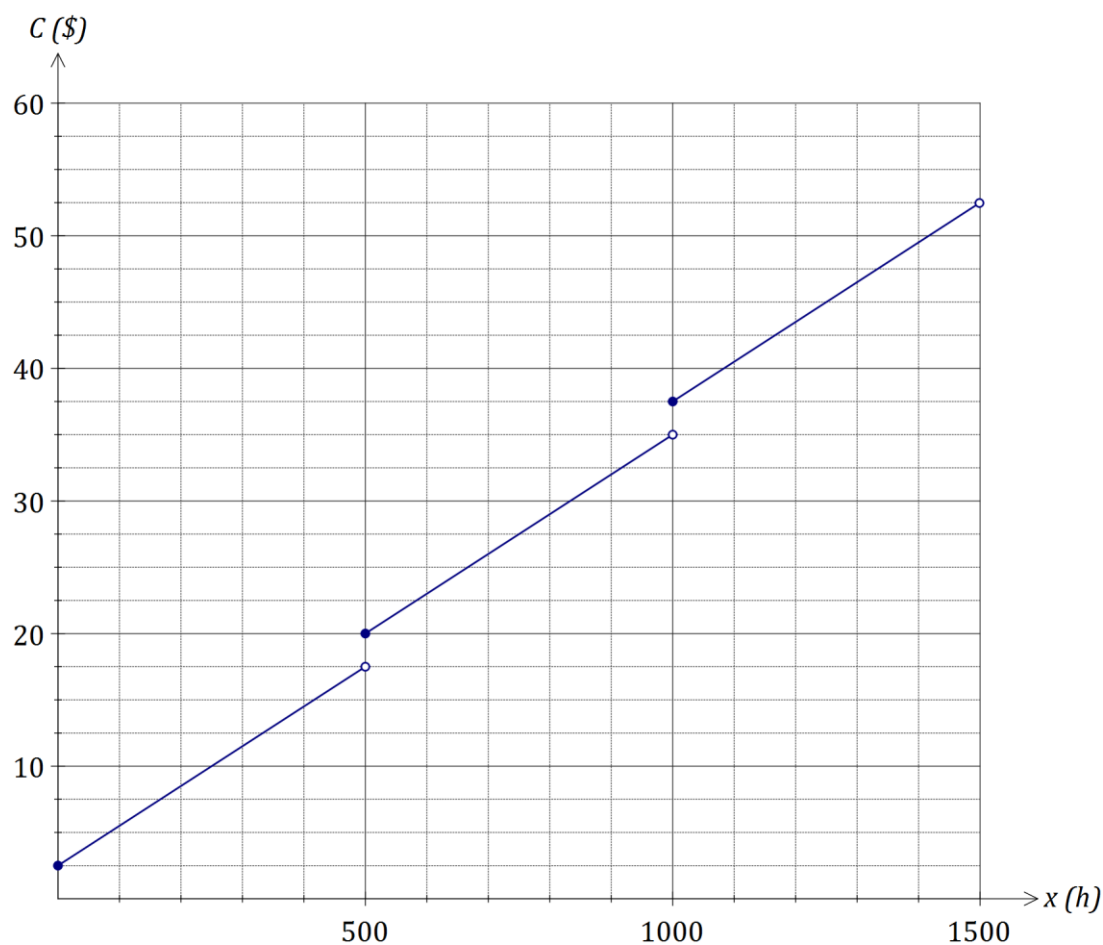| **Solution** |
| --- |
| Solve $12 + 0.005x = 2.5 + 0.03x$ to get $x = 380$ hours |
| **Specific behaviours** |
| ✓ writes equation
✓ solves equation |

(b)   It is expected that the regular globe will require replacing every 500 hours.

     (i)   Complete the table below to show the total cost of buying a regular globe and replacements, and running them for the hours shown.   (3 marks)

Hours of running time (x)	0	250	500	750	1000
Cost of globe, in dollars	2.50	-	2.50	-	2.50
Running cost, in dollars	0	7.50	15.00	22.50	30.00
Total cost in dollars (C)	2.50	10.00	20.00	27.50	37.50

Solution
See table
Specific behaviours
✓ running costs ✓ total costs

(ii)   Plot the total cost of buying a regular globe and replacements, and running them
        for up to 1 500 hours, on the axes below.   (4 marks)

Solution
See graph
Specific behaviours
✓ plots five points from table in (b)(i)
✓ attempt at piecewise graph
✓ all three pieces correct
✓ correct markers at end of each piece

Question 18   **(8 marks)**

(a)   The speeds of cars travelling along a stretch of road with a 60 km/h limit were observed to be normally distributed with mean and standard deviation of 58.5 km/h and 7.5 km/h respectively.

     (i)   Determine the probability that a randomly selected car travels along the road below the speed limit.   (1 mark)

Solution
$P(X < 60) = 0.579$
Specific behaviours
✓ calculates probability

     (ii)   Determine the probability that a randomly selected car travels along the road within 10 km/h of the speed limit.   (2 marks)

Solution
$P(50 < X < 70) = 0.809$
Specific behaviours
✓ calculates upper and lower limits
✓ calculates probability

     (iii)   Two percent of cars exceed a speed of s on the road. Determine the value of s, giving your answer to the nearest km/h.   (2 marks)

Solution
$P(X > s) = 0.02 \Rightarrow s = 73.9 \approx 74$ km/h
Specific behaviours
✓ shows use of right tail
✓ determines s

(b)   A student scored 63% in a geometry test, in which the mean and standard deviation for the class was 55% and 10% respectively.

     (i)   Calculate the students standard score in the geometry test.   (1 mark)

Solution
$z = \dfrac{63-55}{10} = 0.8$
Specific behaviours
✓ calculates z-score

     (ii)   The mean and standard deviation for the same class in a statistics test was 66% and 20%. If the student achieved the same standard score in both tests, calculate their score in the statistics test.   (2 marks)
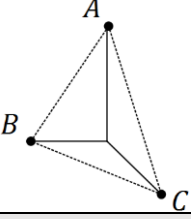
Solution
$0.8 = \dfrac{x-66}{20} \Rightarrow x = 82\%$
Specific behaviours
✓ substitutes into standard score equation
✓ solves for score

See next page

Question 19   **(8 marks)**

Student A is 60 m due North of a pole standing on a level playing field. Student B is 45 m due West of the pole. Student C is at a distance of 40 m and on a bearing of 130° from the pole.

(a)   Draw a sketch of the positions of the pole and the three students.   (1 mark)

Solution
Specific behaviours
✓ sketch with students labelled and in roughly correct positions *(No need for dimensions to be shown)*

(b)   Calculate the bearing of student A from student B.   (2 marks)

Solution
$\tan^{-1}\frac{60}{45} \approx 53.1$ Bearing is $90 - 53.1 \approx 036.9°$
Specific behaviours
✓ calculates angle in right-triangle ✓ states bearing

(c)   Calculate the distance of student A from student C.   (2 marks)

Solution
$AC^2 = 60^2 + 40^2 - 2 \times 60 \times 40 \times \cos 130$ $AC = 91.0$
Specific behaviours
✓ uses cosine rule ✓ calculates distance

(d)   Calculate the bearing of student C from student A.   (3 marks)

Solution
$\frac{40}{\sin x} = \frac{91.0}{\sin 130} \Rightarrow x = 19.7$ Bearing is $180 - 19.7 = 160.3°$
Specific behaviours
✓ uses sin rule ✓ calculates angle in triangle ✓ determines bearing

See next page

**Question 20**   **(7 marks)**

A machine makes spherical balls of ice. A setting on the machine can change the diameter of the spheres from 1 cm up to 5 cm.

(a)   Assuming that water does not change its volume as it turns into ice, determine how many 3 cm diameter spheres of ice the machine could make from 15 litres of water.   (3 marks)
       Hint: One litre of water takes up volume of 1 000 cm³.

Solution
$15 \times 1000 = 15000$ cm³ of water
$V = \frac{4}{3} \times \pi \times 1.5^3 = 14.14$
$15000 \div 14.14 \approx 1061$ ice spheres
Specific behaviours
✓ calculates volume of water
✓ calculates volume of one sphere
✓ calculates number of spheres

(b)   The machine made 500 spheres of ice with a diameter of 4 cm and a worker put them all in a rectangular tray on a level table. The tray, in the shape of a rectangular prism, had a length of 62.5 cm, a width of 40 cm and a height of 12.5 cm.
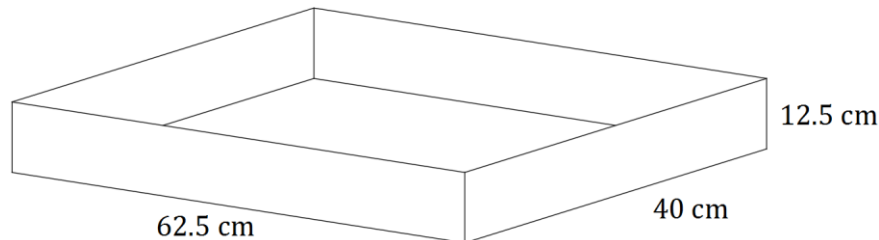
After the cubes had melted back to water, determine the depth of water in the tray.
                                                                                (4 marks)

Solution
Volume of one sphere: $V = \frac{4}{3} \times \pi \times 1.5^3 = 33.51$
Volume of all spheres: $33.51 \times 500 = 16755$
Volume of prism: $16755 = 62.5 \times 40 \times h$
Height: $h = 6.7$ cm
Specific behaviours
✓ calculates volume of one sphere
✓ calculates volume of all spheres
✓ uses formula for rectangular prism
✓ calculates height

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____