Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

MATHEMATICS

3A/3B

Calculator-assumed

WACE Examination 2012

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

When examiners design an examination, they develop provisional marking keys that can be reviewed at a marking key ratification meeting and modified as necessary in the light of candidate responses.

Section Two: Calculator-assumed **(100 Marks)**

Question 8 **(7 marks)**

A ball is dropped from a height of 60 cm onto a horizontal surface. The height reached by the ball after each bounce is two-thirds of the height of the previous bounce.

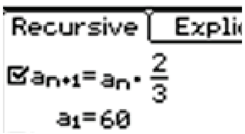
(a) Write a recursive rule to show the distance travelled by each successive downward motion of the ball. (2 marks)

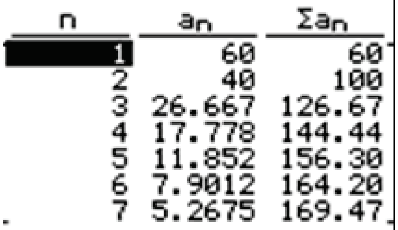
Solution
$T_{n+1} = \frac{2}{3} T_n \quad , T_1 = 60$
Specific behaviours
✓ correctly writes a recursive rule ✓ correctly states first term

(b) What is the maximum height reached by the ball after the third bounce, correct to **two (2)** decimal places? (2 marks)

Solution

T_4 = 17.78 cm





n	a_n	Σa_n
1	60	60
2	40	100
3	26.667	126.67
4	17.778	144.44
5	11.852	156.30
6	7.9012	164.20
7	5.2675	169.47

Specific behaviours

✓ states the correct term
✓ states answer correct to two decimal places

(c) What is the total distance travelled by the ball just as it hits the ground for the seventh time? (3 marks)

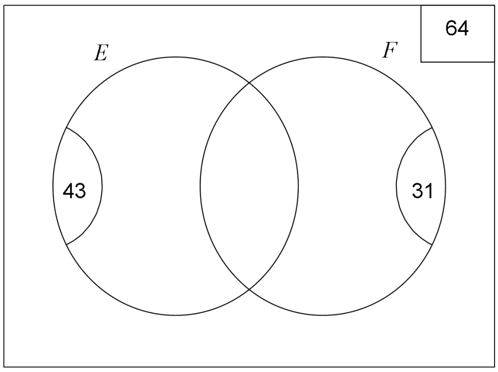
Solution
Sum of the first seven downward movements (169.47) Plus the sum of the last six upward movements (169.47 – 60 = 109.47) Total distance travelled is 278.94 cm (278.93 cm if using full capacity)
Specific behaviours
✓ correctly calculates the total of the downward movements ✓ correctly calculates the total of the upward movements ✓ correctly states total distance travelled

Question 9 **(5 marks)**

E and F are two sets for which $n(E) = 43$, $n(F) = 31$ and $n(U) = 64$.

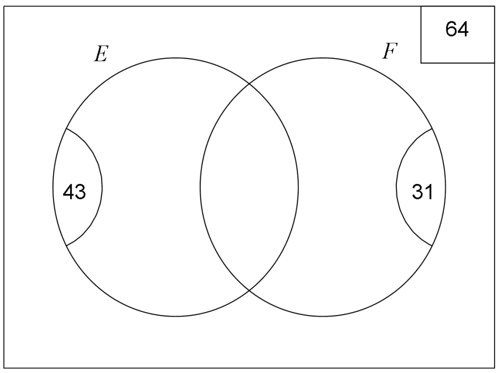
Using the Venn diagram or otherwise, determine

(a) $n(E \cap F)$ if $n(\overline{E \cup F}) = 0$. (2 marks)



Solution
$n(E \cap F) = (43 + 31) - 64 = 10$
Specific behaviours
✓ uses $n(\overline{E \cup F}) = 0$ ✓ correctly calculates $n(E \cap F)$

(b) $P(E \mid \overline{F})$ if $n(\overline{E \cup F}) = 10$. (3 marks)



Solution
$n(E \cup F) = 54$, hence $n(E \cap \overline{F}) = 23$ and $n(\overline{F}) = 33$ Therefore $P(E \mid \overline{F}) = \dfrac{23}{33}$
Specific behaviours
✓ recognises that $n(\overline{F}) = 33$ ✓ recognises that $n(E \cap \overline{F}) = 23$ ✓ states correct probability

Question 10 **(12 marks)**

Researchers believe that a lake is inhabited by a native species of fish thought to be endangered. At a point approximately in the middle of the lake, 50 of these fish were captured, tagged and released. The following day, at the same point, another 50 fish were caught and, before they were released, it was noted that 10 of these were tagged.

(a) Show how the researchers could use this information to estimate that the total population (P) of these fish in the lake was 250. (2 marks)

Solution
$\frac{10}{50}=\frac{50}{P}$ $P=250$
Specific behaviours
✓ states correct ratio ✓ solves for P correctly

In general, the number of tagged fish caught (t) allowed researchers to estimate P according to the relationship in this table.

t	5	10	20	25	50
P	**500**	250	**125**	100	**50**

(b) (i) Complete the table above for population estimates (P) for different numbers of tagged fish caught (t) in lots of 50. (3 marks)

Solution
See above table
Specific behaviours
✓✓✓ one mark for each correct entry

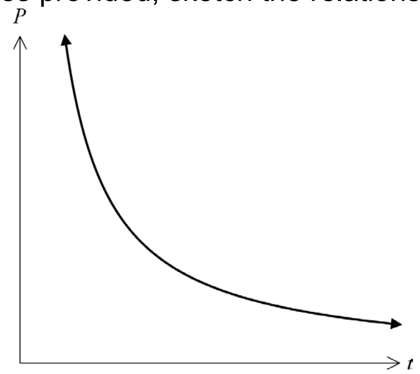
(ii) Describe the relationship between t and P. (1 mark)

Solution
Relationship is inverse proportion (OR reciprocal function)
Specific behaviours
✓ states correct relationship

(iii) Write an equation for P in terms of t. (2 marks)

Solution
$P=\frac{2500}{t}$
Specific behaviours
✓ determines correct numerator ✓ determines correct denominator

(iv) On the axes provided, sketch the relationship found in Part (b) (iii). (2 marks)



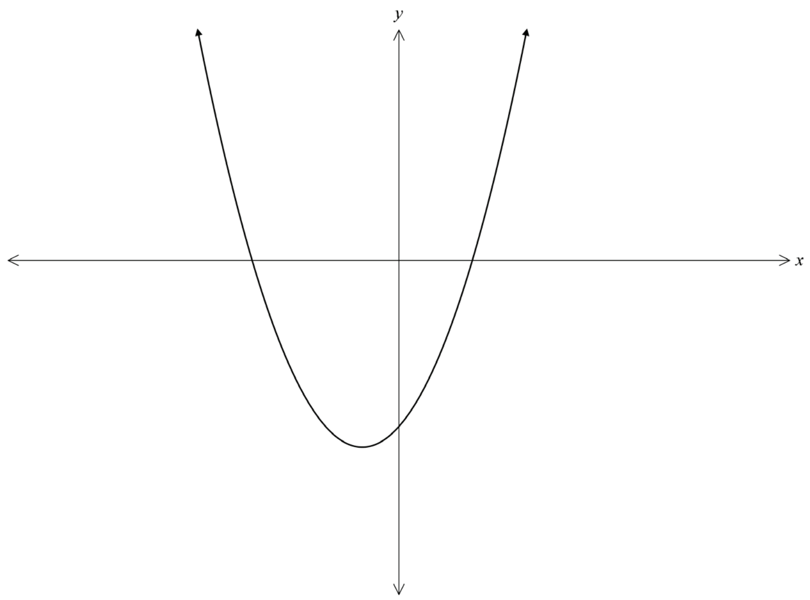
Solution
See above graph
Specific behaviours
✓ sketches graph with correct shape ✓ labels axes correctly

(v) Comment on the rate of change of P as t nears zero and how this might have consequences on the stability of this model for making predictions with very small values of t. (2 marks)

Solution
As t nears zero, P increases rapidly. As a consequence, it would be difficult to make predictions for small values of t
Specific behaviours
✓ states that P increases for small values of t ✓ states a valid reason for the stability of the model for small values of t

Question 11 **(10 marks)**

The quadratic function $y = f(\mathrm{x})$ is shown below. The turning point has coordinates (a, b) and y-intercept has coordinates $(0, c)$.



(a) Determine the equation of the line of symmetry for $y = f(x-1)$. (2 marks)

Solution
$f(x-1)$ is a horizontal translation of one unit to the right of $f(x)$, therefore the equation of the line of symmetry is $x = a + 1$
Specific behaviours
✓ identifies the horizontal translation ✓ states the correct equation of the line of symmetry

(b) Determine the coordinates of the turning point of $y = f(x+4)+5$. (2 marks)

Solution
$y = f(x+4)+5$ is a horizontal translation of four units to the left and a vertical translation of five units up from $f(x)$, therefore the coordinates of the turning point of $y = f(x+4)+5$ is $(a-4,\ b+5)$
Specific behaviours
✓ identifies the horizontal and vertical translation ✓ states the correct coordinates of the turning point

(c) Determine the coordinates of the y-intercept of $y = -f(x) - 2$. (3 marks)

Solution
The coordinates of the y-intercept of $f(x)$ is $(0,\ c)$, therefore the coordinates of the y-intercept of $-f(x)$ is $(0, -c)$ (reflection in the x-axis). After a vertical translation of two units down the y-intercept becomes $(0, -c-2)$
Specific behaviours
✓ recognises a reflection in the x-axis ✓ recognises a vertical translation ✓ states the correct coordinates of the y-intercept

Another quadratic function $y = g(x)$ has a turning point at $(1, 8)$ and intersects the x-axis at $(3, 0)$.

(d) Determine the coordinates of the x-intercepts of $y = g(2x)$. (3 marks)

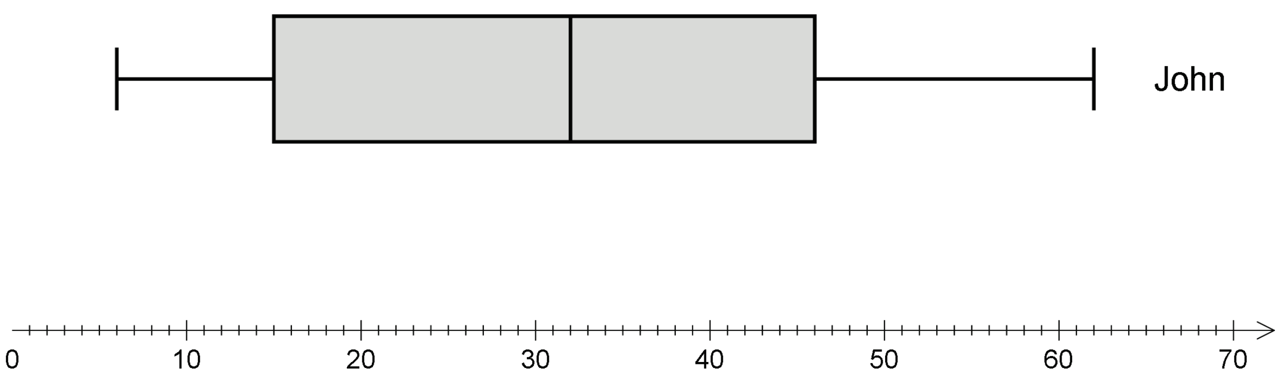
Solution
The line of symmetry of $y = g(2x)$ is $x = \frac{1}{2}$ due to the dilation factor of a half in the x direction. Therefore, intercept of $(3, 0)$ now becomes $(\frac{3}{2}, 0)$ and by symmetry, the coordinates of the other x-intercept is $(-\frac{1}{2}, 0)$. $\therefore$ solutions are $(-\frac{1}{2}, 0)$ and $(\frac{3}{2}, 0)$
Specific behaviours
✓ uses the dilation factor to determine new line of symmetry ✓ determines the new x-intercept ✓ reflects about the line of symmetry to determine the correct coordinates of the other x-intercept

Question 12 **(7 marks)**

John and Jane work at different phone shops. They recorded the number of phones they sold each month over a period of twelve months. Jane's sales for the twelve months were:

34 47 1 15 57 24 20 11 19 50 28 37

John's data are displayed on the following box and whisker plot.

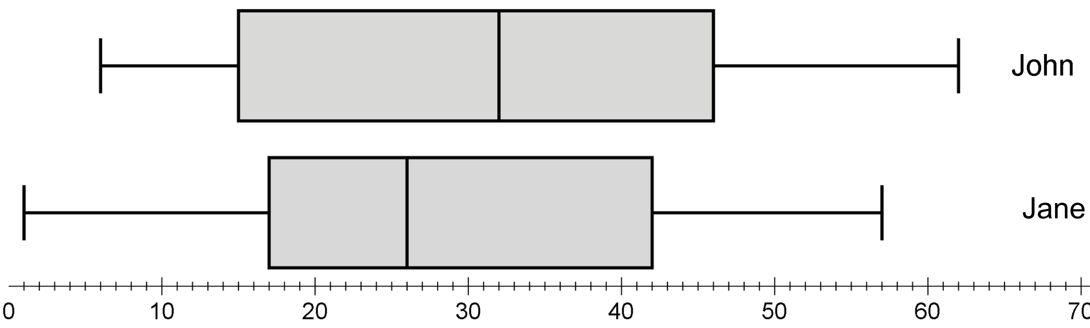


(a) Calculate the median and interquartile range of phone sales for Jane.

(2 marks)

Solution
Median = 26 IQR = 25
Specific behaviours
✓ calculates median correctly ✓ calculates IQR correctly

(b) Construct a box and whisker plot for Jane's sales on the above diagram. (Draw your plot directly underneath John's plot). (3 marks)



Solution
See diagram above
Specific behaviours
✓ correctly plots end points ✓ correctly plots median ✓ correctly plots quartiles

(c) Who had the better sales performance? Justify your answer. (2 marks)

Solution
John's highest and lowest sales are both higher than Jane's corresponding sales. John's median sales figure is higher than Jane's. These results suggest that John consistently sells more phones than Jane.
Specific behaviours
✓ states that John had the better sales performance ✓ justifies choice based on higher extreme and/or higher median

Question 13 **(3 marks)**

Joanne, a Year 10 student, achieved a final Semester Two mark of 79% in mathematics. The table below gives the weightings and her mean score for the given categories.

	Weighting (%)	Mean score (%)
Homework	10	78
Tests	25	x
Investigations	20	75
Examinations	45	80

Calculate her mean score for tests. Give your answer to the nearest whole percentage.

Solution
$0.1\times 78+0.25x+0.2\times 75+0.45\times 80=79$ $x=80.8\%$ So $x=81\%$ to nearest whole percentage
Specific behaviours
✓ states correct equation ✓ solves correctly for x ✓ states correct mean score to the nearest whole percentage

Question 14 **(4 marks)**

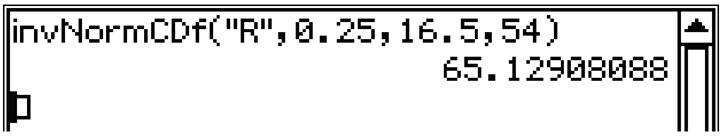
The marks for a mathematics examination at a school are normally distributed with a mean of 54% and a standard deviation of 16.5%.

(a) State the median examination score. (1 mark)

Solution
The median score is 54% by symmetry
Specific behaviours
✓ correctly states the median score

(b) Determine the interquartile range of the examination scores. (3 marks)

Solution
$P(X>Q_3)=0.25$ $\therefore\ Q_3 = 65.1\%$ Therefore $Q_1=42.9\%$ by symmetry IQR = $65.1-42.9=22.2\%$ {note: if using full capacity, IQR $=22.26\%=22.3\%$}
Specific behaviours
✓ states probability that $P(X>Q_3)=0.25$ ✓ determines lower quartile ✓ calculates the IQR



Question 15 **(13 marks)**

A theatre company performed for three weeks at a large venue capable of seating 4200 people. The attendances, in hundreds, at the evening performances are shown in the following table.

Day		Time (t)	Attendance (in hundreds)	Three-point moving average	Residual
First Week	Wednesday	1	20		
	Friday	2	16	20	−4
	Saturday	3	24	22.7	1.3
Second Week	Wednesday	4	28	26	2
	Friday	5	26	29.3	−3.3
	Saturday	6	34	30.7	3.3
Third Week	Wednesday	7	32	31.7	0.3
	Friday	8	A	34	B
	Saturday	9	41		

(a) Calculate the value of the missing entries marked by A and B.

(3 marks)

Solution
$\dfrac{32+41+A}{3}=34 \quad \Rightarrow A=29$ $B=29-34=-5$
Specific behaviours
✓ uses three-point moving average to determine an equation for A ✓ solves for A ✓ correctly calculates the resulting value of B

(b) Calculate the seasonal component for the Wednesday performances.

(2 marks)

Solution
$\dfrac{2+0.3}{2}=1.15$
Specific behaviours
✓ uses Wednesday residuals   ✓ correctly averages residuals to calculate seasonal components

(c) A regression line was fitted to the three-point moving averages given above. Determine the equation of this regression line correct to **two (2)** decimal places. (2 marks)

Solution
$\hat{y} = 16.22 + 2.31t$
Specific behaviours
✓ correctly states gradient of line of regression ✓ correctly states y-intercept of line of regression

(d) This equation was used to represent the trend of the time series. Use this equation, together with the seasonal component for Wednesday, to predict the attendance for Wednesday in the fourth week to the nearest hundred. (4 marks)

Solution
$t = 10 \Rightarrow \hat{y} = 39.32$ (accept 39.33 if students use full capacity) $\therefore$ prediction $= 39.32 + 1.15 = 40.47 \approx 4000$ (accept $39.33 + 1.15 = 40.48$)
Specific behaviours
✓ determines $t = 10$ ✓ calculates $\hat{y}$ correctly ✓ adds the seasonal component ✓ rounds correctly to the nearest hundred

(e) Should performances be extended for a fourth week? Give a reason for your answer. (2 marks)

Solution
Yes, as the attendance is increasing (increasing trend)
Specific behaviours
✓ correctly answers 'yes' ✓ states valid reason with reference to increasing trend

Question 16 **(12 marks)**

A laboratory that distils natural oils from plant materials produces two types of oil concentrate: an olive oil product for general lubricating use and a refined eucalyptus oil product for use as a cosmetic base and in natural medicines.

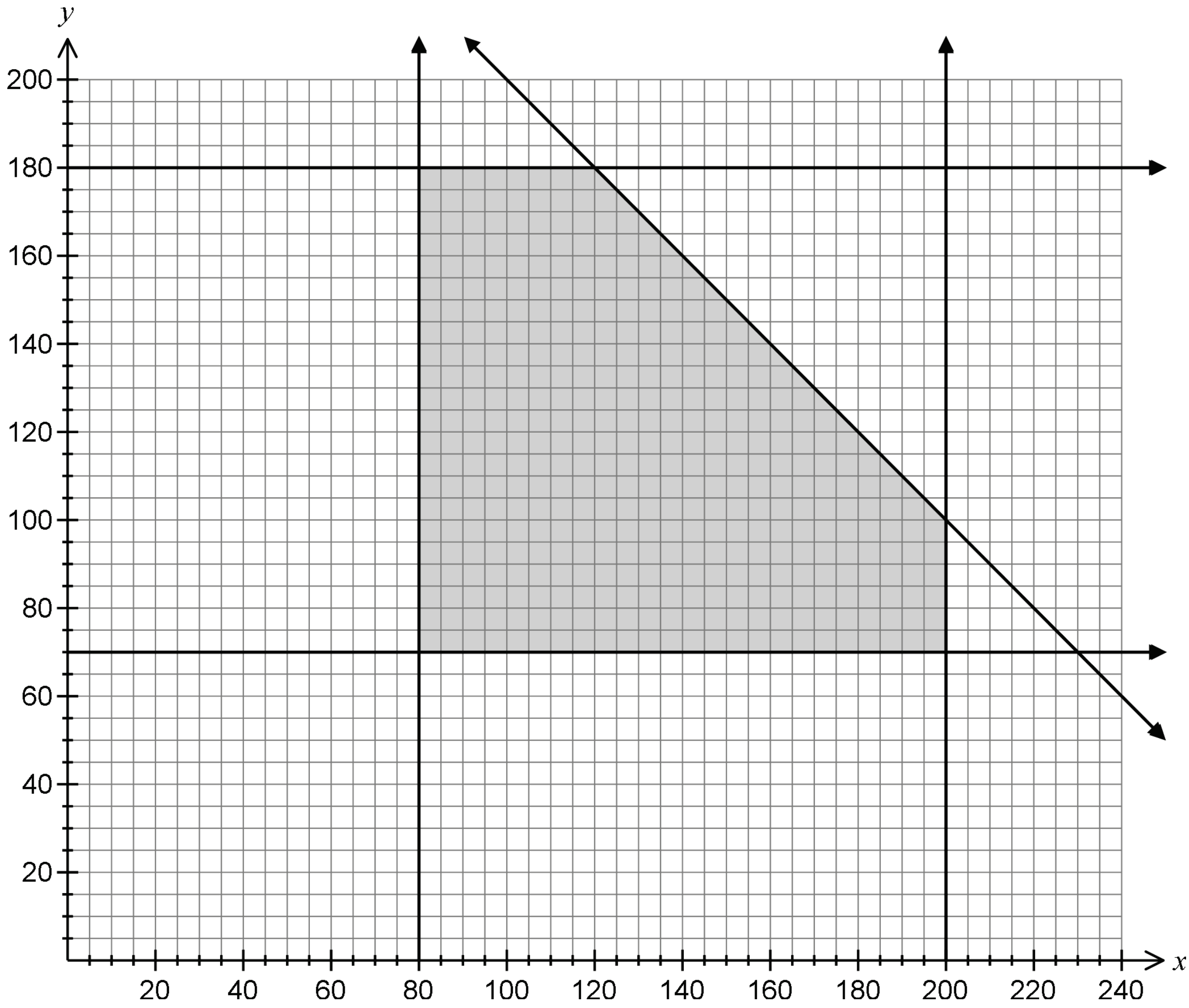
Fixed demands require the production of at least 80 litres of the olive oil and at least 70 litres of eucalyptus oil each week. The laboratory has the capacity to produce up to 300 litres of distilled oil each week, of which a maximum of 200 litres can be olive oil or a maximum of 180 litres can be eucalyptus oil.

Let x be the number of litres of olive oil produced and y be the number of litres of eucalyptus oil produced.

(a) Given that $x \geq 80$ and $y \leq 180$, state **three (3)** further inequalities involving x and y, other than $x \geq 0$ and $y \geq 0$. (3 marks)

Solution
$y \geq 70,\ x \leq 200$ $x + y \leq 300$
Specific behaviours
✓ states restriction on production of eucalyptus oil (i.e. $y \geq 70$) ✓ states restriction on capacity of olive oil (i.e. $x \leq 200$) ✓ states restriction on maximum capacity (i.e. $x + y \leq 300$)

(b) Graph these inequalities on the grid below and shade the feasible region. (4 marks)



Solution
See above graph
Specific behaviours
✓ correctly graphs remaining horizontal constraint ✓ correctly graphs remaining vertical constraint ✓ correctly graphs oblique constraint ✓ shades correct feasible region

(c) If the profit on olive oil is \$10 per litre and the profit on eucalyptus oil is \$20 per litre, calculate the maximum profit the distillery can make each week. (3 marks)

Solution

(x, y)	$P = 10x + 20y$
(80, 70)	$P = 800 + 1400 = \$2200$
(80, 180)	$P = 800 + 3600 = \$4400$
(120, 180)	$P = 1200 + 3600 = \$4800$
(200, 100)	$P = 2000 + 2000 = \$4000$
(200, 75)	$P = 2000 + 1500 = \$3500$

Feasible point (120, 180) gives a profit of \$4800
Feasible point (200, 100) gives a profit of \$4000
Therefore the maximum profit is \$4800

Specific behaviours

✓ states the correct objective function
✓ calculates the profit at the feasible points
✓ states the correct maximum profit

(d) Specific orders require the distillery to produce exactly 160 litres of olive oil in one week. What is the maximum profit it can make in that week? (2 marks)

Solution

(160, 140) is the point maximising profit
$160 \times 10 + 140 \times 20 = \$4400$

Specific behaviours

✓ correctly determines coordinates for maximum profit
✓ correctly calculates maximum profit

Question 17 **(12 marks)**

Simone and Lucy are driving down a highway at constant speeds of 95 km/h and 85 km/h respectively. Lucy drives 15 kilometres further than Simone.

Let the distance Lucy travels be x km.

(a) Write an expression in terms of x for the time taken, in hours, for both Simone and Lucy to complete their respective journeys. (2 marks)

Solution
Lucy's time = $\frac{x}{85}$ hours Simone's time = $\frac{x-15}{95}$ hours
Specific behaviours
✓ correctly states Lucy's time ✓ correctly states Simone's time

(b) Simone completes her journey in 15 minutes less than Lucy.

Explain why $\frac{x}{85}-\frac{x-15}{95}=\frac{1}{4}$. (2 marks)

Solution
The difference in the time taken between the two women is 15 minutes, and that is one-quarter of an hour.
Specific behaviours
✓ correctly states that the equation represents the difference between the times taken by the two women ✓ states that 15 minutes is equal to $\frac{1}{4}$ of an hour

(c) Calculate x and hence state the times taken by Simone and Lucy to complete their respective journeys. Give your answers correct to the nearest minute. (4 marks)

Solution
By CAS, $x = 74.375$ Hence Lucy's time is $\frac{x}{85} = \frac{74.375}{85} = 0.875$ hours ≈ 53 minutes.   And Simone's time is $\frac{x-15}{95} = \frac{74.375-15}{95} = 0.625$ hours ≈ 38 minutes.
Specific behaviours
✓ correctly calculates x ✓ correctly calculates Lucy's time   ✓ correctly calculates Simone's time   ✓ rounds times correct to the nearest minute

(d) Assuming Simone maintains her constant speed of 95 km/h, at what speed would Lucy need to be travelling to finish her journey in the same time as Simone? Give this speed in metres per second. (4 marks)

Solution
Lucy's speed $= \frac{74.375}{0.625}$ $= 119$ km/h $= 119 \times 1000 \div 3600$ m/s $= 33.0\dot{5}$ m/s
Specific behaviours
✓ uses correct distance   ✓ uses correct time   ✓ converts kilometres to metres correctly   ✓ converts from hours to seconds correctly

Question 18 **(9 marks)**

Consider the function $f(x) = x^4 - 8x^3 - 270x^2$.

(a) Using calculus techniques, show that the function has two stationary points for the domain $-2 \le x \le 20$. (3 marks)

Solution
$f(x) = x^4 - 8x^3 - 270x^2$ $f'(x) = 4x^3 - 24x^2 - 540x$ $= 0 \text{ when } x = 0, -9, 15$ Since -9 is outside the domain, there are only two stationary points within the given domain
Specific behaviours
✓ correctly determines the first derivative ✓ equates equal to zero and solves for x ✓ correctly concludes there are only two turning points within the given domain

(b) Determine the coordinates of the stationary points identified in Part (a) and state their nature. (3 marks)

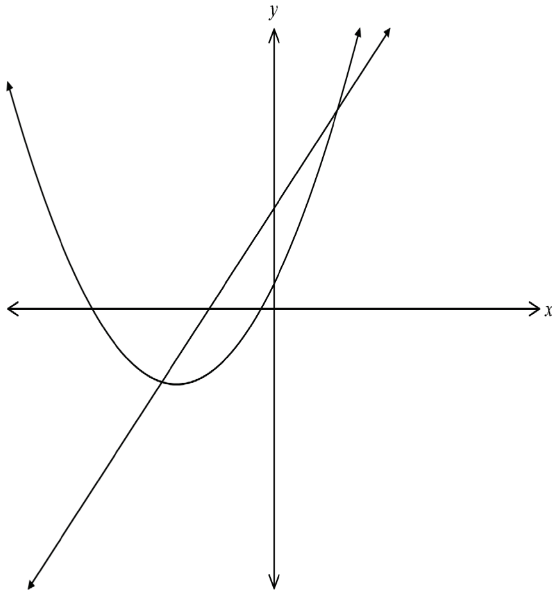
Solution
$(0,0)$ and $(15, -37\ 125)$ maximum and minimum respectively
Specific behaviours
✓ correctly determines the coordinates of each stationary point ✓ correctly states that (0,0) is maximum turning point ✓ correctly states that $(15, -37\ 125)$ is a minimum turning point

(c) For the domain $-k \le x \le k$, k an integer, the function has a global maximum of 0 and a global minimum of –25 000. Determine the value of k. (3 marks)

Solution
Solving $f(x) = 0$ gives $x = -12.9,\ 0,\ 20.9$ correct to 1d.p. Solving $f(x) = -25\ 000$ gives $x = 10,\ 18.6$, correct to 1d.p. Since k is an integer and cannot be zero, k =10
Specific behaviours
✓ correctly solves $f(x) = 0$ ✓ correctly solves $f(x) = -25\ 000$ ✓ correctly states the value of k

Question 19 **(6 marks)**

The functions $f(x)=x^2+4x+1$ and $g(x)=3x+d$, where d is a constant, are shown below.



For what value(s) of d does the equation $f(x)=g(x)$ produce:

(a) one solution? (5 marks)

Solution
For one solution, $g(x)$ is a tangent to $f(x)$ $f'(x)=2x+4$ $\quad =3$ when $x=-0.5$ $\therefore f(-\frac{1}{2})=\frac{1}{4}-2+1=-\frac{3}{4}$ $\Rightarrow -\frac{3}{4}=3(-\frac{1}{2})+d \therefore d=\frac{3}{4}$
Specific behaviours
✓ recognises that one solution occurs when $g(x)$ is a tangent to $f(x)$ ✓ correctly solves $f'(x)=3$ ✓ correctly substitutes $x=-\frac{1}{2}$ in $f(x)$ ✓ correctly writes an equation in d ✓ solves this equation to determine the correct value of d

(b) two solutions? (1 mark)

Solution
Two solutions occur when $d>\frac{3}{4}$
Specific behaviours
✓ correctly states solution