GREENWOOD COLLEGE

LEARN GROW CHANGE

Semester One Examination, 2018

Question/Answer booklet

MATHEMATICS METHODS UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

Student number: In figures

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

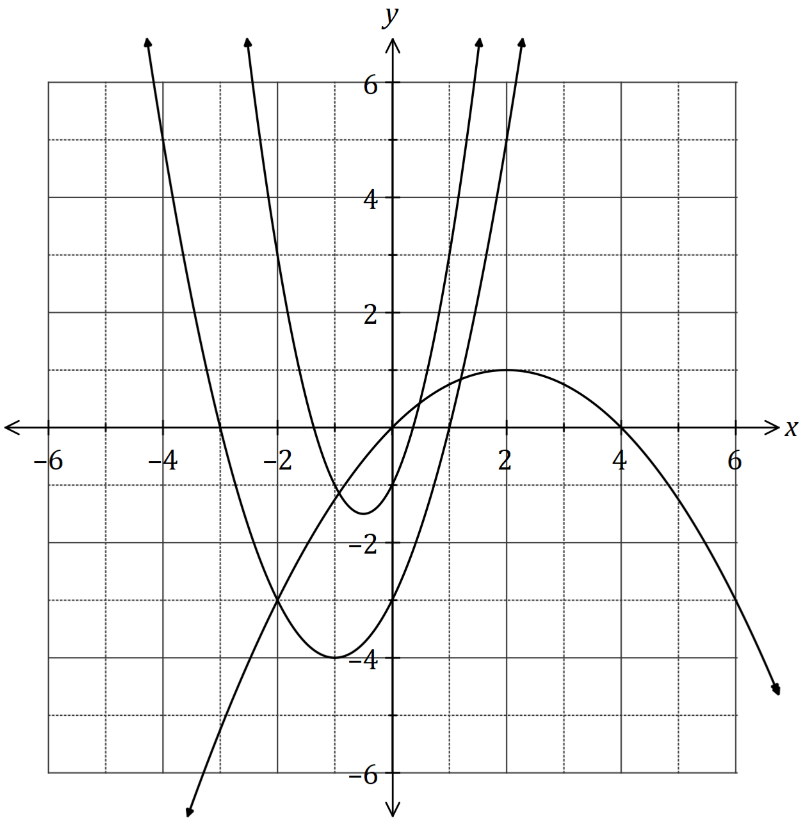
Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(4 marks)**

The graphs of $y = 2x^2 + 2x + c$, $y = a(x-2)^2 + 1$ and $y = (x+b)(x+3)$ are shown below.

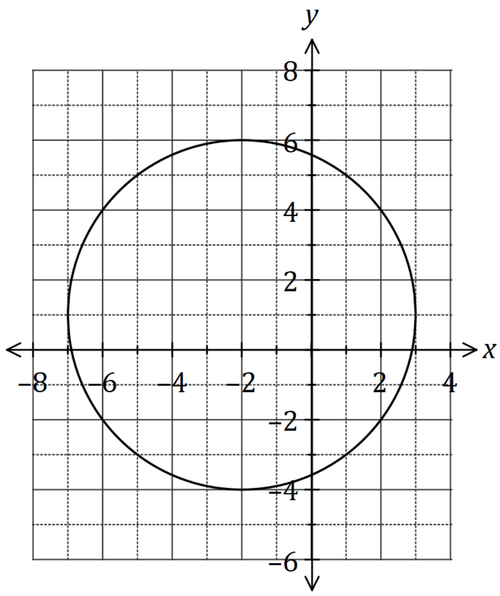


Determine the values of the constants a, b and c.

Solution
$x = 4 \Rightarrow 0 = a(4-2)^2 + 1$ $a = -\frac{1}{4}$ $b = -1$ (Other root at -3) $c = -1$ (y-intercept)
Specific behaviours
✓ uses point on inverted parabola ✓ value of a ✓ value of b ✓ value of c

Question 10 **(7 marks)**

(a) The graph of a relationship is circular, as shown below.



Determine the equation of this circle in the form $x^2 + y^2 = a + bx + cy$, where a, b and c are constants. (4 marks)

Solution
Centre at $(-2, 1)$and $r = 5$ $(x+2)^2 + (y-1)^2 = 5^2$ $x^2 + y^2 = 20 - 4x + 2y$
Specific behaviours
✓ indicates centre   ✓ indicates radius   ✓ factored form   ✓ re-arranges as required

(b) The line $x + y + 1 = 0$ intersects the circle at the points A and B. Show that the line passes through the centre of the circle, and hence determine the distance AB. (3 marks)

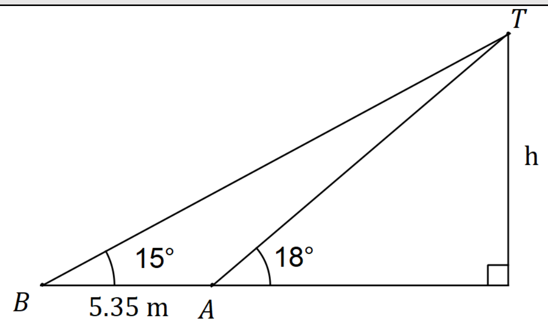
Solution
Sub centre $(-2,1)$: $-2+1+1=0$ Hence line passes through centre AB is a diameter $AB = 2 \times 5 = 10$
Specific behaviours
✓ correct substitution   ✓ indicates AB is diameter   ✓ correct distance

Question 11 **(6 marks)**

A thin pole stands vertically in the middle of a level playing ground. From point A on the ground, the angle of elevation to the top of the pole, T, is 18º.

From point B, also on the ground but 5.35 metres further from the foot of the pole than A, the angle of elevation to the top of the pole is 15º.

(a) Draw a sketch to represent this information. (1 mark)

Solution

Specific behaviours
✓ sketch with right-angle, two given angles and distance AB

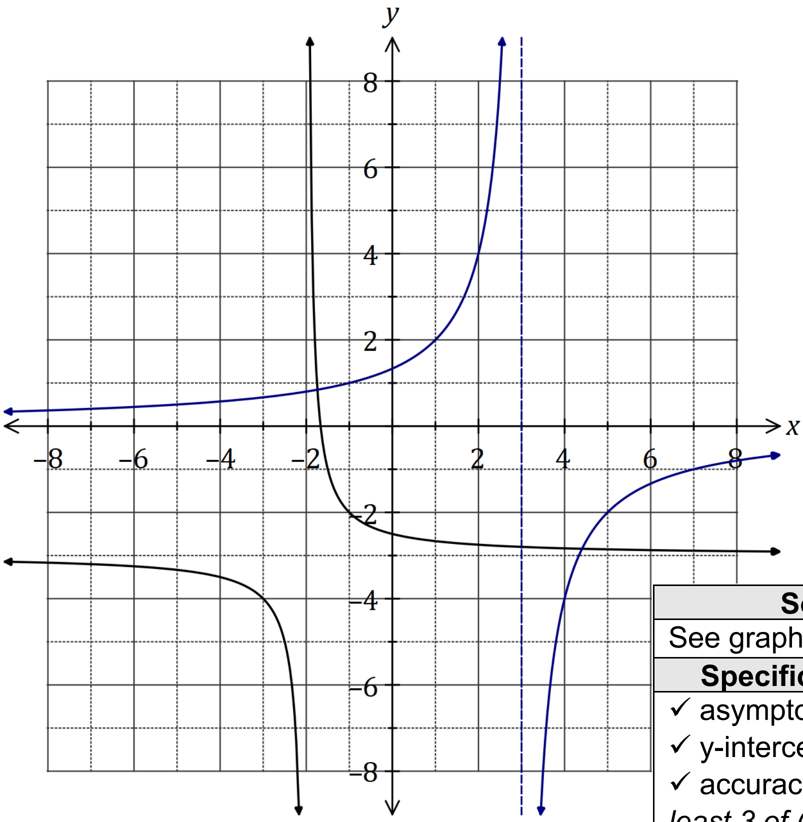
(b) Showing use of trigonometry, determine the height of the post. (5 marks)

Solution
$\dfrac{AT}{\sin 15} = \dfrac{5.35}{\sin 3} \Rightarrow AT = 26.46$ $h = 26.46 \times \sin 18 = 8.18$
Specific behaviours
✓ angle BTA ✓ equation using sine rule ✓ solves for AT ✓ use of trig in right triangle ✓ determines h

Question 12 **(7 marks)**

Let $f(x) = \dfrac{4}{3-x}$ and $g(x) = \dfrac{1}{x+p} + q$, where p and q are constants.

The graph of $y = g(x)$ is shown below.



Solution
See graph
Specific behaviours
✓ asymptotes ✓ y-intercept ✓ accuracy *[i.e. thu' at least 3 of (-1,1), (1,2), (2,4), (4,-4) or (7,-1)]*

(a) Sketch the graph of $y = f(x)$ on the axes above. (3 marks)

(b) Determine the values of p and q. (2 marks)

Solution
$p = 2, \qquad q = -3$
Specific behaviours
✓ value of p ✓ value of q

(c) Solve the equation $f(x) = g(x)$, giving your solution(s) to one decimal place. (2 marks)

Solution
$x = -1.7, \qquad x = 4.4$
Specific behaviours
✓ one solution ✓ second solution *(Rounding for guidance only but penalise answers given as coordinates)*

Question 13 **(6 marks)**

(a) Determine the equation of the axis of symmetry for the graph of $y = 3x^2 + 12x + 40$. (2 marks)

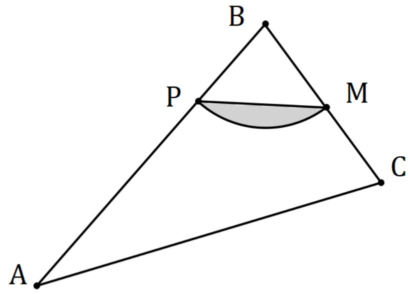
Solution
$x = -\frac{b}{2a} = -\frac{12}{2 \times 3} = -2$ $x = -2$
Specific behaviours
✓ indicates use of formula   ✓ correct equation

(b) The graph of $y = ax^2 + bx + 13$ passes through the points $(-3, -23)$ and $(4, 5)$. Determine the values of the constants a and b. (4 marks)

Solution
$-23 = (-3)^2 a - 3b + 13$ $-23 = 9a - 3b + 13$ $5 = 4^2 a + 4b + 13$ $5 = 16a + 4b + 13$ Solve simultaneously using CAS $a = -2, \quad b = 6$
Specific behaviours
✓ substitutes first point   ✓ substitutes second point   ✓ solves for a ✓ solves for b

Question 14 **(10 marks)**

A logo with triangular outline ABC contains a shaded segment bounded by the straight line PM and the circular arc PM with centre B and radius $BM = 18$ cm, as shown below.



Given that $\angle ABC = \frac{5\pi}{12}$, $\angle BCA = 2\angle BAC$ and M is the midpoint of BC, determine

(a) the size of $\angle ABC$ in degrees. (1 mark)

Solution
$\frac{5\pi}{12} \times \frac{180}{\pi} = 75°$
Specific behaviours
✓ converts angle

(b) the area of the shaded segment. (2 marks)

Solution
$A = \frac{1}{2}(18)^2\left(\frac{5\pi}{12} - \sin\left(\frac{5\pi}{12}\right)\right) \approx 55.6 \text{ cm}^2$
Specific behaviours
✓ indicates substitution into segment area formula ✓ evaluates area

(c) the perimeter of the shaded segment. (3 marks)

Solution
$PM_{arc} = 18 \times \frac{5\pi}{12} = \frac{15\pi}{2} \approx 23.56$ $b = \sqrt{18^2 + 18^2 - 2(18)(18)\cos 75} \approx 21.92$ $\text{Perimeter} = 23.56 + 21.92 \approx 45.5 \text{ cm}$
Specific behaviours
✓ calculates arc length ✓ indicates use of cosine rule to find PM ✓ evaluates PM and states perimeter

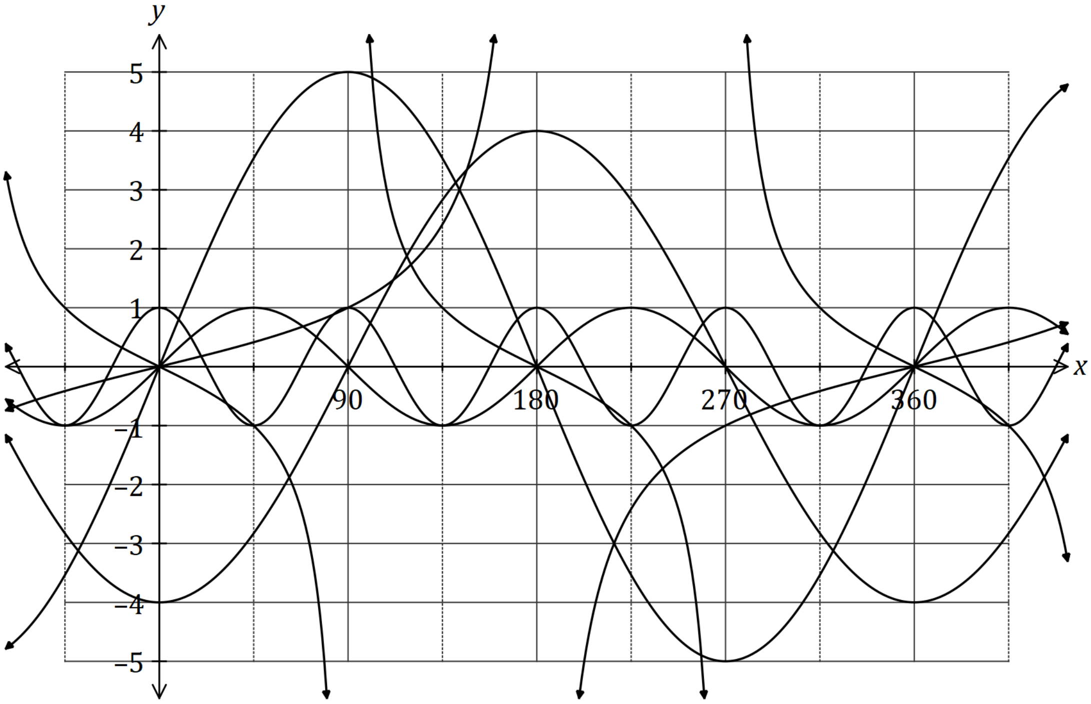
(d) the area of triangle ABC. (4 marks)

Solution
$\angle A + \angle C = 180 - 75$ $\angle A + 2\angle A = 105 \Rightarrow \angle A = 35$ $\frac{AC}{\sin 75} = \frac{2 \times 18}{\sin 35}$ $AC = 60.63$ $\text{Area} = \frac{1}{2}(36)(60.63)\sin(2 \times 35) \approx 1025 \text{ cm}^2$
Specific behaviours
✓ indicates use of equation to find second angle ✓ evaluates second angle and indicates use in sin rule ✓ evaluates second side ✓ evaluates triangle area

Question 15 **(9 marks)**

(a) The graphs of the following, where a, b, c, d, e and f are constants, are shown below.

$y = \sin(ax)$ $y = b\cos(x)$ $y = \tan(cx)$ $y = d\sin(x)$ $y = \cos(ex)$ $y = f\tan(x)$



State the values of a, b, c, d, e and f. (6 marks)

Constant	Value
a	2
b	-4
c	0.5
d	5
e	4
f	-1

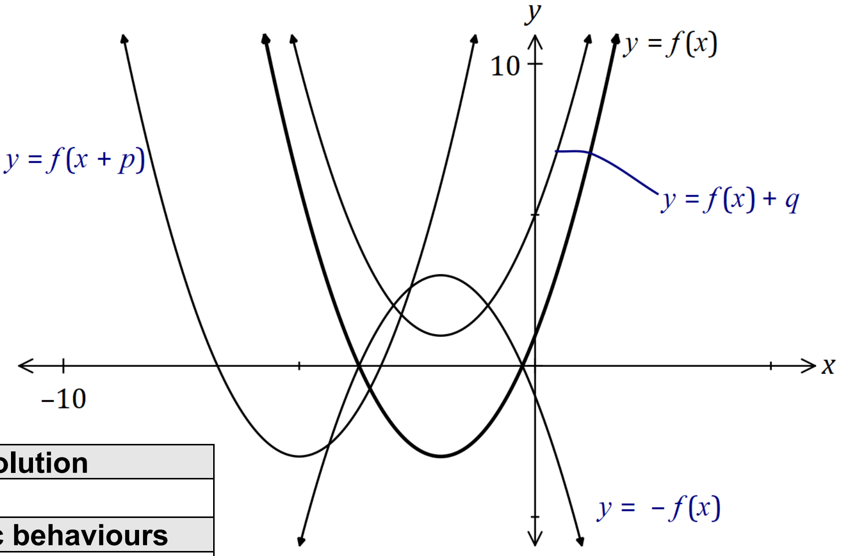
Solution
See table
Specific behaviours
✓ each value

(b) Calculate the acute angle in degrees between the lines $y = x + 5$ and $y = 3x - 1$, rounding your answer to one decimal place. (3 marks)

Solution
$\alpha = \tan^{-1}(1) = 45°$ $\beta = \tan^{-1}(3) = 71.565°$ $\beta - \alpha = 26.6°\ \ (1\text{ dp})$
Specific behaviours
✓ angle of inclination of first line ✓ angle of inclination of second line ✓ acute angle, to one decimal place

Question 16 **(6 marks)**

(a) The graph of $y = f(x)$ is shown in bold below. The graphs of $y = -f(x)$, $y = f(x+p)$ and $y = f(x) + q$ are also shown, where p and q are constants.



Solution
See graph
Specific behaviours
✓✓✓ each correct label

Clearly label the remaining graphs with $y = -f(x), y = f(x+p)$ or $y = f(x) + q$. (3 marks)

(b) The one-to-one relation $y = 7 - 3x$ has domain and range given by $\{x: x = -2, 3, a\}$ and $\{y: y = -8, -2, b\}$ respectively. Determine the values of constants a and b. (3 marks)

Solution
$x = 3, y = -2$ $x = -2, y = 13 = b$ $x = a, y = 7 - 3a = -8 \Rightarrow a = 5$
Specific behaviours
✓ value of b ✓ indicates a is mapped onto -8 ✓ solves for value of a

Question 17 **(9 marks)**

The wind speed at a weather station, v metres per second, t hours after recording began, can be modelled by the function

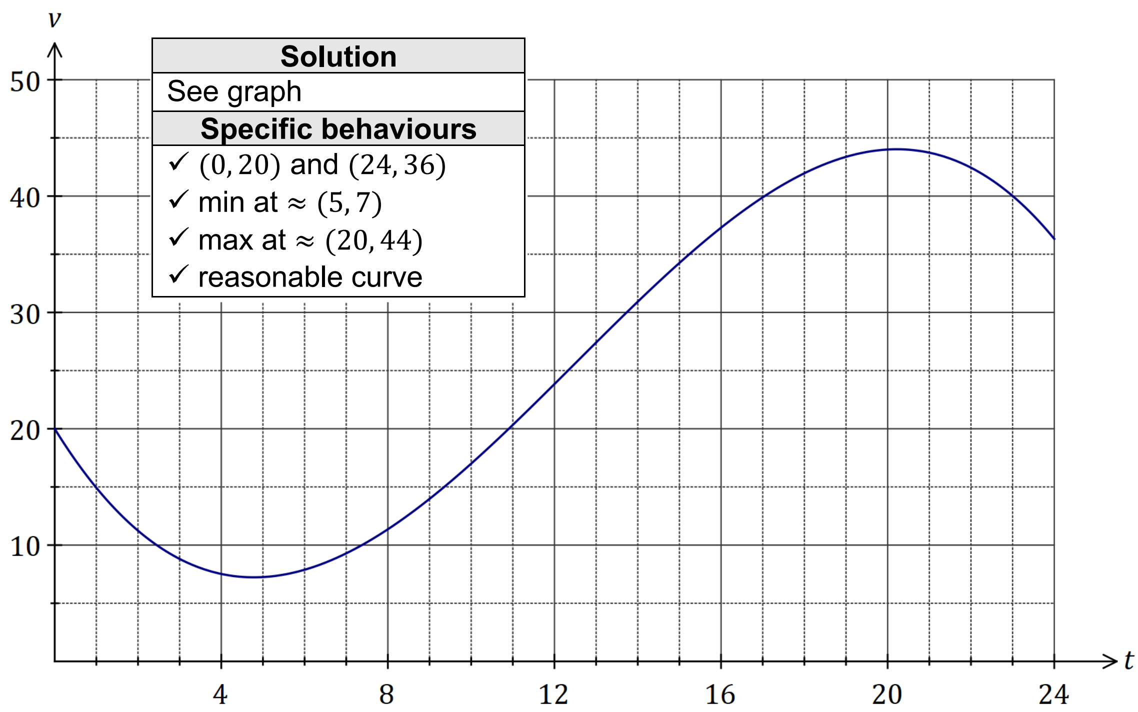
$$v = 20 - 5.8t + 0.75t^2 - 0.02t^3, 0 \leq t \leq 24$$

(a) Calculate the wind speed when $t = 11$. (1 mark)

Solution
$v(11) = 20.33 \text{ m/s}$
Specific behaviours
✓ value

(b) Sketch the graph of wind speed against time on the axes below. (4 marks)

Solution
See graph
Specific behaviours
✓ $(0, 20)$ and $(24, 36)$
✓ min at $\approx (5, 7)$
✓ max at $\approx (20, 44)$
✓ reasonable curve



(c) During the 24-hour period, determine

(i) the time at which the wind speed was greatest. (1 mark)

Solution
$t = 20.2 \text{ h}$
Specific behaviours
✓ value *(at least 1dp)*

(ii) the minimum wind speed. (1 mark)

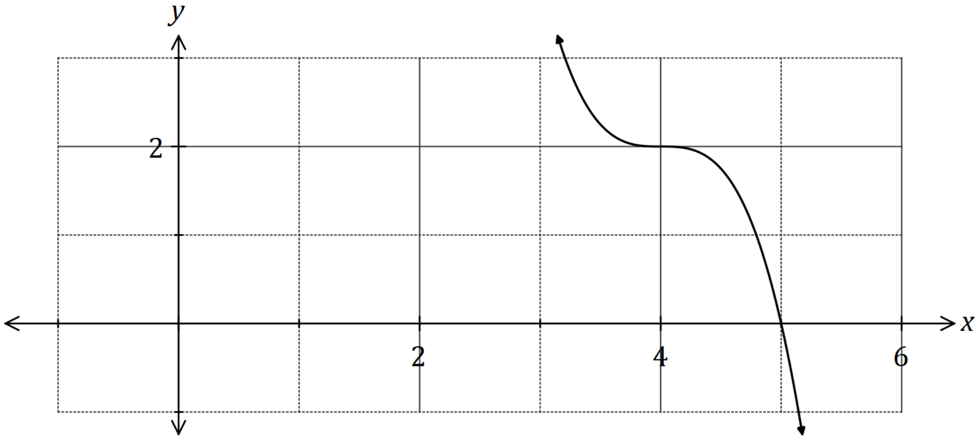
Solution
$v_{MIN} \approx 7.23 \text{ m/s}$
Specific behaviours
✓ value *(at least 1 dp)*

(iii) the length of time, in hours and minutes, that the wind speed was increasing. (2 marks)

Solution
$20.219 - 4.781 = 15.438$ $= 15 \text{ h } 26 \text{ min}$
Specific behaviours
✓ interval in hours
✓ interval in hours and minutes

Question 18 **(6 marks)**

(a) Part of the graph of $y = f(x)$ is shown below, where $f(x) = -2(x - b)^3 + c$, and b and c are constants.



(i) State the degree of $f(x)$. (1 mark)

Solution
3
Specific behaviours
✓ correct degree

(ii) Determine the value of b. (1 mark)

Solution
$b = 4$
Specific behaviours
✓ correct value

(iii) Determine $f(0)$. (2 marks)

Solution
$f(x) = -2(x - 4)^3 + 2$ $f(0) = -2(-4)^3 + 2 = 130$
Specific behaviours
✓ indicates value of c ✓ evaluates

(b) Another function is given by $g(x) = f(x + 8)$.

Describe how to obtain the graph of $y = g(x)$ from the graph of $y = f(x)$. (2 marks)

Solution
Translate graph 8 units to the left.
Specific behaviours
✓ uses translation   ✓ indicates distance and direction

Question 19 **(12 marks)**

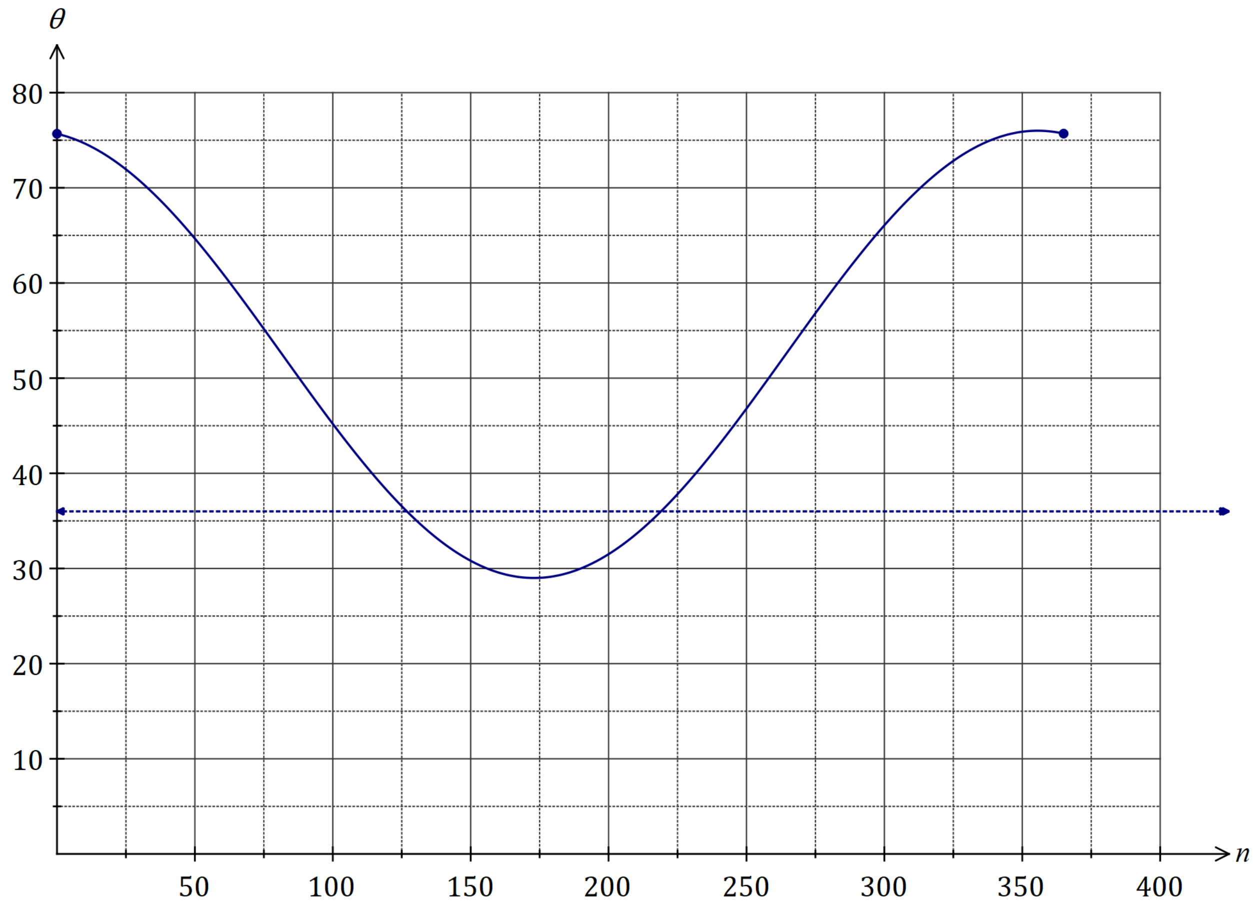
During 2018, the altitude of the sun, θ degrees, at noon in Melbourne on the n^{th} day of the year can be modelled by the equation

$$\theta = 23.5\sin\left(\frac{8\pi(101+n)}{1461}\right) + 52.2$$

(a) On the 26th of January, the altitude was $71.4°$. Calculate the altitude ten days earlier. (2 marks)

Solution
$n = 16, \qquad \theta = 73.4°$
Specific behaviours
✓ indicates n ✓ correct angle

(b) Graph the altitude on the axes below for $0 \le n \le 365$. (4 marks)



Solution
See graph
Specific behaviours
✓ endpoints, $\theta \approx 75°$ ✓ minimum close to $(173, 29°)$ ✓ maximum close to right endpoint ✓ smooth curve

(c) State the minimum altitude of the sun at noon in Melbourne and on which day of the year this occurred. (2 marks)

Solution
$\theta_{MIN} = 28.7°$ on day 173
Specific behaviours
✓ altitude, correct to 1dp ✓ day of year, rounded to whole number

(d) Solar panels on the roof of a Melbourne business are designed to meet its entire power needs on cloudless days when the altitude of the sun is at least $36°$ at noon.

(i) Draw a straight line on your graph to represent this requirement. (1 mark)

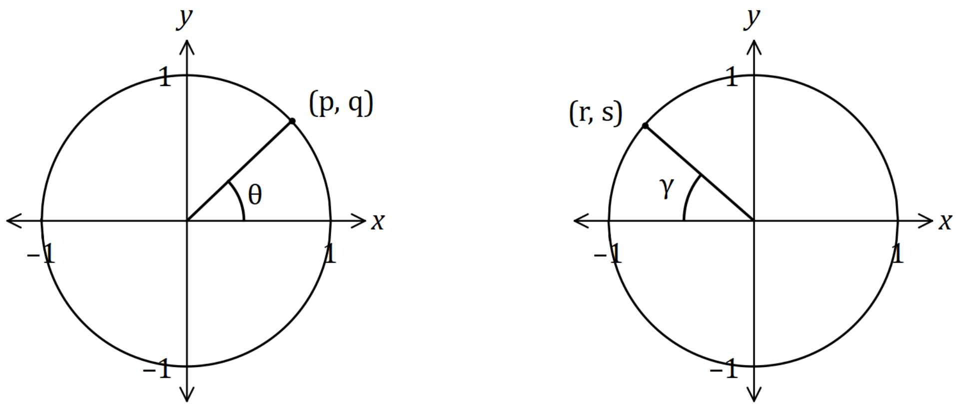
Solution
See graph - horizontal line $\theta = 36°$
Specific behaviours
✓ neat, straight line

(ii) Determine the number of days the panels are expected to achieve this aim during 2018, ignoring the possibility of cloud cover. (3 marks)

Solution
$\theta > 36° \Rightarrow 126 \le n \le 220$ $220 - 126 = 94$ $365 - 94 = 271$ days
Specific behaviours
✓ one value of n ✓ second value of n ✓ maximum number of days

Question 20 **(7 marks)**

Consider the points with coordinates (p, q) and (r, s) that lie in the first and second quadrants respectively of the unit circles shown below, where θ and γ are acute angles.



Determine the following in terms of p, q, r and s, simplifying your answers where possible.

(a) $\tan\theta$. (1 mark)

(b) $\sin(180° - \theta)$. (1 mark)

(c) $\cos\ \gamma$. (1 mark)

(d) $\sin(\pi + \gamma)$. (1 mark)

Solutions
(i) $\frac{q}{p}$
(ii) q
(iii) $-r$
(iv) $-s$
Specific behaviours
✓ each correct response

(e) $\cos(\gamma - \theta)$. (3 marks)

Solution
$\cos(\gamma - \theta) = \cos\gamma\cos\theta + \sin\gamma\sin\theta$ $= (-r)(p) + (s)(q)$ $= qs - pr$
Specific behaviours
✓ uses identity ✓ at least two correct trig values ✓ correct expression

Question 21 **(9 marks)**

(a) Use your calculator to

(i) determine the exact value of $\cos 36°$. (1 mark)

(ii) determine the exact value of $\sin 105°$. (1 mark)

Solution
(i) $\dfrac{\sqrt{5}+1}{4}$ (ii) $\dfrac{\sqrt{2}(\sqrt{3}+1)}{4}$
Specific behaviours
✓ each exact value

(iii) solve $\cos(x+60°) = \sin x$ for $-270° \le x \le 270°$. (2 marks)

Solution
$x = -165°, \qquad x = 15°, \qquad x = 195°$
Specific behaviours
✓ two correct solutions   ✓ all three correct   *NB Graphical or numerical solve quickest - use of exact solve slow*

(b) Using suitable exact values of acute angles and an angle sum and difference identity, justify your above value of $\sin 105°$. (5 marks)

Solution
$\sin(105) = \sin(45+60)$ $= \sin 45 \cos 60 + \cos 45 \sin 60$ $= \dfrac{\sqrt{2}}{2} \times \dfrac{1}{2} + \dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{3}}{2}$ $= \dfrac{\sqrt{2}(1+\sqrt{3})}{4}$
Specific behaviours
✓ chooses suitable pair of angles   ✓ chooses correct identity   ✓ first pair of exact values   ✓ second pair of exact values   ✓ simplifies to match previous answer

End of questions

Supplementary page

Question number: _________

Supplementary page

Question number: _________