



Semester Two Examination, 2017

Question/Answer booklet

**MATHEMATICS  
METHODS  
UNITS 1 AND 2**  
Section One:  
Calculator-free

# SOLUTIONS

Student Number: In figures

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In words

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Your name

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**Time allowed for this section**

Reading time before commencing work: five minutes

Working time: fifty minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					100

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free**

**35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

**Question 1**

**(5 marks)**

(a) Determine  $f'(x)$  if

(i)  $f(x) = 5x^4 + x.$

**(1 mark)**

Solution
$f'(x) = 20x^3 + 1$
Specific behaviours
✓ differentiates

(ii)  $f(x) = (2x + 3)^2.$

**(2 marks)**

Solution
$f(x) = 4x^2 + 12x + 9$
$f'(x) = 8x + 12$
Specific behaviours
✓ expands
✓ differentiates

(b) The area of an oil slick, at time  $t$  hours, is given by  $A(t) = 0.5t^3 - 2t^2 + 7$  square meters. Determine the instantaneous rate of change of the area of the slick when  $t = 10$  hours.

**(2 marks)**

Solution
$A'(t) = 1.5t^2 - 4t$
$A'(10) = 150 - 40 = 110 \text{ m}^2/\text{h}$
Specific behaviours
✓ differentiates correctly
✓ substitutes and simplifies

**Question 2****(7 marks)**

- (a) The tenth term of an arithmetic sequence is 98 and the sixteenth term is 80.

Determine the sum of the first 20 terms of the sequence.

**(4 marks)**

<b>Solution</b>
$d = \frac{80 - 98}{16 - 10} = -\frac{18}{6} = -3$ $98 = a + (10 - 1)(-3)$ $a = 125$ $S_{20} = \frac{20}{2}(2(125) + (20 - 1)(-3))$ $S_{20} = 10(250 - 57)$ $= 1930$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ determines common difference</li> <li>✓ determines first term</li> <li>✓ substitutes correctly into sum formula</li> <li>✓ evaluates sum</li> </ul>

- (b) The limiting sum of the series  $x + x^2 + x^3 + x^4 + \dots$  is 10.

Determine the value of  $x$ .

**(3 marks)**

<b>Solution</b>
<p>GP: <math>a = x, r = x</math></p> $10 = \frac{x}{1 - x}$ $10 - 10x = x$ $x = \frac{10}{11}$
<b>Specific behaviours</b>
<ul style="list-style-type: none"> <li>✓ indicates sequence is geometric</li> <li>✓ substitutes into sum to infinity formula</li> <li>✓ determines value of <math>x</math></li> </ul>

**Question 3**

**(7 marks)**

- (a) Evaluate  $x^{2a} \cdot x^b$  when  $x = 64$ ,  $a = 2$  and  $b = -4.5$ .

**(3 marks)**

Solution
$x^{2a+b} = 64^{-\frac{1}{2}}$ $= \frac{1}{\sqrt{64}}$ $= \frac{1}{8}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ eliminates negative indices</li> <li>✓ eliminates fractional indices</li> <li>✓ states value</li> </ul>

- (b) The first two terms of a geometric sequence are  $3 \times 10^{-4}$  and  $6 \times 10^{-6}$ . Calculate the fifth term of the sequence, giving your answer in scientific notation.

**(4 marks)**

Solution
$r = \frac{6 \times 10^{-6}}{3 \times 10^{-4}} = 2 \times 10^{-2}$ $T_4 = (3 \times 10^{-4})(2 \times 10^{-2})^4$ $= 3 \times 10^{-4} \times 2^4 \times 10^{-8}$ $= 48 \times 10^{-12}$ $= 4.8 \times 10^{-11}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ evaluates ratio</li> <li>✓ indicates expression for 4th term</li> <li>✓ simplifies</li> <li>✓ expresses term in scientific notation</li> </ul>

**Question 4****(8 marks)**

The graph of  $y = ax^3 + bx + c$  has a stationary point at  $(2, 29)$  and a gradient of 18 when  $x = 1$ .

(a) Determine the values of the constants  $a$ ,  $b$  and  $c$ .

**(6 marks)**

Solution
$\frac{dy}{dx} = 3ax^2 + b$ $0 = 3a(2)^2 + b \Rightarrow 12a + b = 0$ $18 = 3a(1)^2 + b \Rightarrow 3a + b = 18$ $9a = -18 \Rightarrow a = -2$ $b = -12(-2) = 24$ $29 = -2(2)^3 + 24(2) + c \Rightarrow c = -3$ $a = -2, \quad b = 24, \quad c = -3$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ differentiates equation</li> <li>✓ equation using <math>y' = 0</math> when <math>x = 2</math></li> <li>✓ equation using <math>y' = 18</math> when <math>x = 1</math></li> <li>✓ determines value of <math>a</math></li> <li>✓ determines value of <math>b</math></li> <li>✓ uses <math>(2, 29)</math> to determine <math>c</math></li> </ul>

(b) Determine the coordinates of any other stationary points.

**(2 marks)**

Solution
$\frac{dy}{dx} = -6x^2 + 24$ $= 0 \text{ when } x = \pm 2$ $-2(-2)^3 + 24(-2) - 3 = -35$ <p>Other SP at <math>(-2, -35)</math></p>
Specific behaviours
<ul style="list-style-type: none"> <li>✓ determines other <math>x</math>-coordinate</li> <li>✓ states coordinates</li> </ul>

**Question 5**

(7 marks)

Solve the following equations for  $x$ :

(a)  $2 \cos x = 1, 0 \leq x \leq 360^\circ.$

(2 marks)

Solution
$\cos x = \frac{1}{2} \Rightarrow x = 60^\circ, 300^\circ$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ determines one solution</li> <li>✓ determines both solutions</li> </ul>

(b)  $\frac{x-3}{x+2} = \frac{2}{3}.$

(2 marks)

Solution
$3(x-3) = 2(x+2)$ $3x-9 = 2x+4$ $x = 13$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ cross multiplies and expands</li> <li>✓ correct solution</li> </ul>

(c)  $(2x-3)^2 - 36 = 0.$

(3 marks)

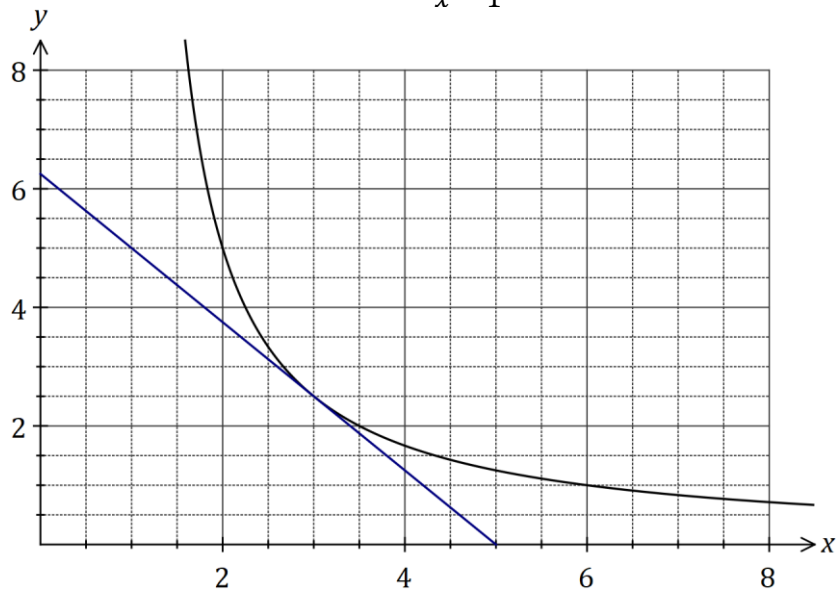
Solution
$(2x-3)^2 = 36$ $2x-3 = \pm 6$ $2x = 9, -3$ $x = \frac{9}{2}, x = -\frac{3}{2}$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ square roots both sides, with <math>\pm</math> one side</li> <li>✓ solves for <math>2x</math></li> <li>✓ correct solutions</li> </ul>

## Question 6

(6 marks)

The graph of the function  $y = f(x)$  is shown below, where

$$f(x) = \frac{5}{x-1}.$$



- (a) Draw the tangent to the graph at  $x = 3$  so that it cuts both axes, and use the tangent to estimate the value of  $f'(3)$ . (3 marks)

Solution
See graph. Slope of tangent $\approx -\frac{6}{5} \approx -1.2$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ tangent drawn as straight line</li> <li>✓ tangent cuts <math>y</math>-axis between 5.5 and 7</li> <li>✓ estimate of gradient of tangent</li> </ul>

- (b) Calculate the average rate of change of the function as  $x$  increases from 3 to 3.5. (3 marks)

Solution
$f(3) = 2.5, \quad f(3.5) = 2$
$\text{ARoC} = \frac{2 - 2.5}{3.5 - 3} = \frac{-0.5}{0.5} = -1$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ calculates change in <math>x</math>-coordinates</li> <li>✓ calculates change in <math>y</math>-coordinates</li> <li>✓ states correct gradient</li> </ul>



**Question 7**

**(7 marks)**

- (a) Determine the coefficient of the  $a^3$  term in the expansion of  $(2a - 3)^4$ .

**(3 marks)**

Solution
$(2a - 3)^4 = \dots + \binom{4}{1} (2a)^3 (-3)^1 + \dots$ <p>Coefficient is <math>4 \times 2^3 \times (-3) = -96</math></p>
Specific behaviours
<ul style="list-style-type: none"> <li>✓ identifies correct term</li> <li>✓ uses <math>\binom{4}{1}</math> in expansion</li> <li>✓ correct coefficient</li> </ul>

- (b) Consider the equation  $x^3 - 4x^2 - 11x + 30 = 0$ .

- (i) Show that  $x = 2$  is a solution of the equation.

**(1 mark)**

Solution
$8 - 16 - 22 + 30 = 38 - 38 = 0$
Specific behaviours
<ul style="list-style-type: none"> <li>✓ substitutes and expands</li> </ul>

- (ii) Determine all other solutions.

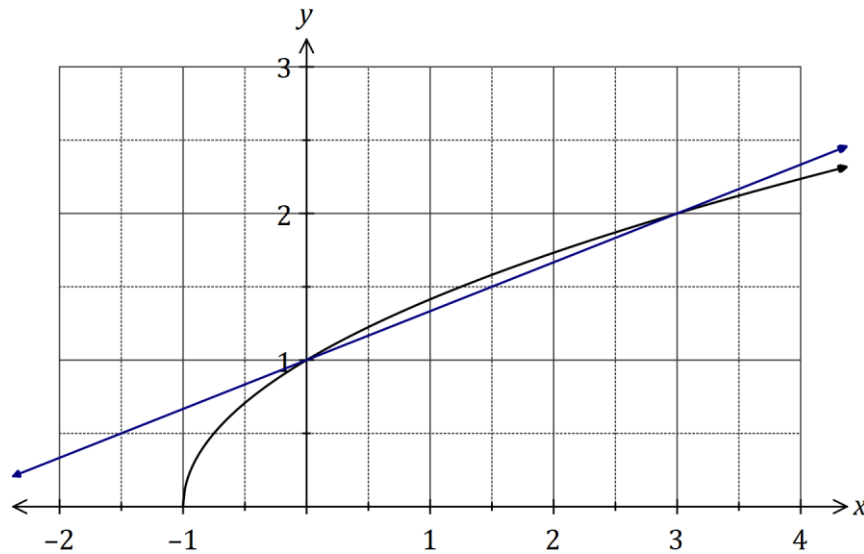
**(3 marks)**

Solution
$x^3 - 4x^2 - 11x + 30 = (x - 2)(x^2 - 2x - 15)$ $= (x - 2)(x + 3)(x - 5)$ <p>Other solutions: <math>x = -3, x = 5</math></p>
Specific behaviours
<ul style="list-style-type: none"> <li>✓ determines quadratic factor</li> <li>✓ factorises cubic</li> <li>✓ states other two solutions</li> </ul>

## Question 8

(5 marks)

The graph of  $y = f(x)$  is shown below, where  $f(x) = \sqrt{x+1}$ .



The difference quotient is shown here:

$$\frac{f(x+h) - f(x)}{h}$$

- (a) Add to the graph a secant whose slope represents the difference quotient when  $x = 0$  and  $h = 3$ , and state the value of this slope. (2 marks)

Solution
See graph - slope is $\frac{1}{3}$
Specific behaviours
✓ secant through (0, 1) and (3, 2)
✓ correct slope

- (b) Evaluate the difference quotient as  $h \rightarrow 0$  to determine the slope of  $f(x)$  when  $x = 0$ . (3 marks)

Solution
$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{\sqrt{0+h+1} - \sqrt{0+1}}{h} \\ &= \frac{\sqrt{h+1} - 1}{h} \times \frac{\sqrt{h+1} + 1}{\sqrt{h+1} + 1} \\ &= \frac{h}{h(\sqrt{h+1} + 1)} \\ &= \frac{1}{\sqrt{h+1} + 1} \Big _{h=0} \\ &= \frac{1}{2} \end{aligned}$
Specific behaviours
✓ substitutes $f(x)$ , $x = 0$ and simplifies ✓ removes surd from numerator ✓ substitutes $h = 0$ and simplifies

Additional working space

Question number: \_\_\_\_\_

