PERTH MODERN SCHOOL

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INDEPENDENT PUBLIC SCHOOL

**WAEP Semester One Examination, 2018**

**Question/Answer booklet**

# MATHEMATICS SPECIALIST UNIT 1

## Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures | | | | | | | | |

In words _____

Your name _____

## Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet
Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 8 | 8 | 50 | 53 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 97 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**See next page**

**Section Two: Calculator-assumed** **65% (97 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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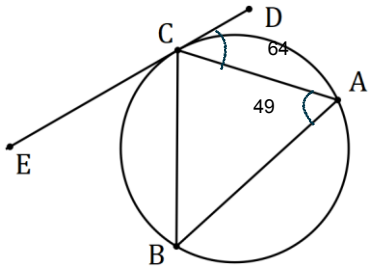
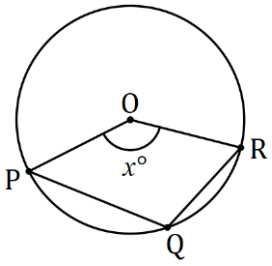
**Question 9** **(7 marks)**

(a) In the diagram below, points $A$ and $B$ lie on a circle, $DE$ is a tangent to the circle at $C$, $\angle ACD = 64°$ and $\angle BAC = 49°$. Determine the sizes of $\angle ABC, \angle BCE$ and $\angle BCA$. (3 marks)




(b) In the next diagram, $P, Q$ and $R$ lie on a circle with centre $O$ and $\angle POR = x°$. Determine, with reasons, the size of $\angle PQR$ in terms of $x$. (4 marks)



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**Question 10** **(7 marks)**

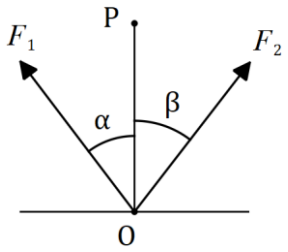
(a) A body travels with a velocity $12\mathbf{i} - 5\mathbf{j}$ ms$^{-1}$. Determine its speed and the bearing on which it is moving, assuming the positive $y$-axis to be due north. (3 marks)

(b) Given that $\lambda(5\mathbf{i} - 2\mathbf{j}) + \mu(-7\mathbf{i} + 4\mathbf{j}) = 25\mathbf{i} - 13\mathbf{j}$, determine the values of $\lambda$ and $\mu$. (4 marks)

**See next page**

**Question 11** **(8 marks)**

Two forces, $F_1 = 230$ N and $F_2 = 272$ N, act on a body at $O$, and make angles of $\alpha = 15°$, and $\beta = 20°$ respectively with the vertical $OP$, as shown in the diagram below.




(a) Determine the magnitude of the resultant force and the angle it makes with the vertical. (5 marks)

(b) The magnitude of $F_2$ is to be adjusted so that the direction of the resultant is vertical. Determine the required magnitude of $F_2$. (3 marks)



**See next page**

**Question 12** **(6 marks)**

The largest Australian family recently met with the largest English family. Between them, these two families had 37 children.

(a) Three of the children were chosen at random to feature in a TV documentary about the two families. Determine the number of different selections of three children that were possible. (1 mark)

(b) Prove that at least four of the children were born in the same month of the year. (2 marks)

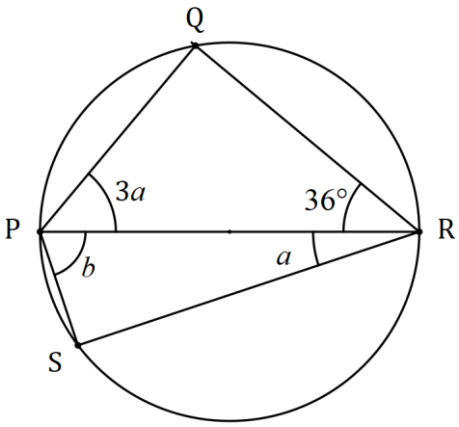
There were more children in the English family than the Australian family and the English children all had blue, brown, hazel or grey coloured eyes.

(c) Show that at least five English children had the same eye colour. (3 marks)

**See next page**

**Question 13** **(8 marks)**

(a) Determine the size of angles $a$ and $b$ in the diagram below, where $Q$ and $S$ lie on the circumference of the circle with diameter $PR$. (3 marks)




(b) Triangle $ABC$ has sides of length $AB = 5$ cm, $BC = 7$ cm and $AC = 9$ cm. Prove, using the method of contradiction, that if $AC$ is a diameter of a circle then $B$ does not lie on the circumference of the circle. (5 marks)



**See next page**

**Question 14** **(8 marks)**

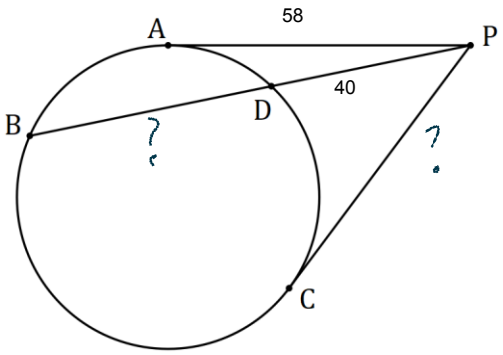
(a) Simplify $(4\mathbf{a} - 2\mathbf{b}) \cdot (\mathbf{a} - 3\mathbf{b})$ given that $|\mathbf{a}| = 5, |\mathbf{b}| = 3$ and vector $\mathbf{a}$ is parallel and in the opposite direction to vector $\mathbf{b}$. (4 marks)

(b) Using $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$, demonstrate a vector method to show that if the diagonals $\overrightarrow{OB}$ and $\overrightarrow{AC}$ of parallelogram $OABC$ are perpendicular, then the parallelogram is a rhombus. (4 marks)

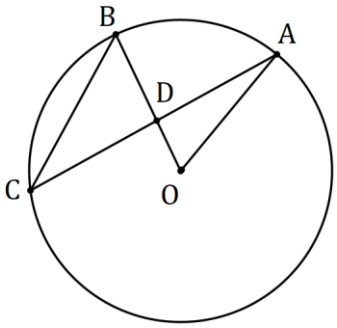
**See next page**

**Question 15** **(9 marks)**

(a) In the diagram below, $PA$ and $PC$ are tangents to the circle, with $PA = 58$ cm. Secant $PB$ cuts the circle at $D$, so that $PD = 40$ cm. Determine the lengths of $PC$ and $BD$. (4 marks)




(b) In the diagram below, $A, B$ and $C$ lie on the circumference of the circle with centre $O$, with $AC$ intersecting $OB$ at $D$. Prove that $\angle DAO = \angle DBC - \angle DCB$. (5 marks)



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**Question 16** **(9 marks)**

(a) Determine the number of integers between $1$ and $370$ that are divisible by $4$ or $7$. (4 marks)

(b) A pigeon fancier has $5$ Florentine, $6$ King and $8$ Maltese pigeons and must choose three of them to enter in a local show. Determine the number of different ways the three pigeons can be chosen if

(i) there are no restrictions. (1 mark)

(ii) the fancier decides to take one of each breed. (2 marks)

(iii) the fancier decides to take at least two Maltese pigeons. (2 marks)

**Question 17** **(6 marks)**

Three vectors are $\mathbf{u} = a\mathbf{i} + b\mathbf{j}, \mathbf{v} = -3\mathbf{i} + 5\mathbf{j}$ and $\mathbf{w} = -\mathbf{i} + 4\mathbf{j}$.

(a) Determine the vector projection of $\mathbf{v}$ on $\mathbf{w}$ in exact form. (2 marks)

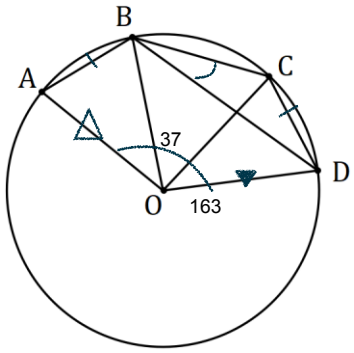
(b) If $\mathbf{u}$ has the same magnitude as $\mathbf{v}$ and is perpendicular to $\mathbf{w}$, determine the exact values of the coefficients $a$ and $b$. (4 marks)



**See next page**

**Question 18** **(7 marks)**

(a) In the diagram below, points $B$ and $C$ lie on the minor arc $AD$ of the circle with centre $O$. The lengths of chords $AB$ and $CD$ are congruent, $\angle BOC = 37°$ and $\angle AOD = 163°$. Determine the size of $\angle CBD$. (3 marks)




(b) Line segment $AC$ intersects line segment $BD$ at $N$. Given that $AC$ and $BD$ are non-parallel and the lengths $AN, AC, BN$ and $BD$ are 6, 41, 21 and 31 cm respectively, explain whether the points $A, B, C$ and $D$ are concyclic. (4 marks)

**Question 19** **(8 marks)**

(a) Triangle $ABC$ has vertices with position vectors $A(2,-6), B(-3,14)$ and $C(6,8)$. Point $P$ lies on side $BC$ so that $2\overrightarrow{BP} = \overrightarrow{PC}$. Determine the vector $\overrightarrow{AP}$. (4 marks)

(b) $OPQR$ is a parallelogram. Point $M$ is the midpoint of side $PQ$ and point $N$ is on side $QR$ so that $QN = \frac{3}{5}QR$. If $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$, determine $\overrightarrow{MN}$ in terms of $\mathbf{p}$ and $\mathbf{r}$. (4 marks)



**See next page**

**Question 20** **(7 marks)**

A small boat leaves jetty $A$ to travel to jetty $B$, $276$ m away on a bearing of $240°$. A steady current of $2.5$ ms$^{-1}$ runs in the river between the jetties on a bearing $130°$. If the small boat travels at a constant speed of $6.5$ ms$^{-1}$, determine the bearing it should steer to reach jetty $B$ and how long the journey will take.

**See next page**

**Question 21** **(7 marks)**

A child is playing with thirteen coloured cubes, all the same size. There are six pink cubes, three navy and one each of red, blue, orange and green.

(a) If the child stacks cubes one on top of another to make a column, determine the number of different coloured columns that can be made using

(i) all the red, blue and green cubes. (1 mark)

(ii) all the pink, red and orange cubes. (2 marks)

(iii) all the cubes. (2 marks)

(b) If all but one of the cubes are used to make a column, determine the number of different coloured columns that can now be made. Justify your answer. (2 marks)



**End of questions**

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

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Question number: _____