WACE
Western Australian Certificate of Education

MATHEMATICS

3A/3B

Calculator-free

WACE Examination 2015

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

Section One: Calculator-free   **$33\frac{1}{3}$% (50 Marks)**

Question 1   **(8 marks)**

(a)   Determine $\dfrac{dy}{dx}$ for each of the following:

      (i) $y = \dfrac{2}{3}ax^3 + a^2$ (*a* is a constant)   (2 marks)

Solution
$\dfrac{dy}{dx} = (3)\dfrac{2}{3}ax^2$ $\quad = 2ax^2$
Specific behaviours
✓ correctly differentiates first term   ✓ correctly differentiates constant term

      (ii) $y = \dfrac{6x - 3x^3}{2x}$.   (2 marks)

Solution
$y = 3 - \dfrac{3x^2}{2} \Rightarrow \dfrac{dy}{dx} = -3x$
Specific behaviours
✓ correctly simplifies   ✓ correctly differentiates

(b)   Determine the equation of the tangent to the curve $y = x - x^3 - 2 + 2x^2$ at the point where $x = 1$.   (4 marks)

Solution
$\dfrac{dy}{dx} = 1 - 3x^2 + 4x$ at $x = 1$ $\dfrac{dy}{dx} = 2$ $\therefore y = 2x + c$ when $x = 1, y = 0 \Rightarrow c = -2 \therefore y = 2x - 2$
Specific behaviours
✓ correctly differentiates   ✓ correctly calculates gradient   ✓ correctly calculates the value of y when $x = 1$ ✓ correctly calculates the value of c and hence the equation of the tangent

Question 2   **(11 marks)**

(a)   (i)   Express, as a power of a, $a^x \times a^y \div a^{x-y}$.   (2 marks)

Solution
$a^{x+y} \div a^{x-y}$ $= a^{2y}$
Specific behaviours
✓ simplifies product   ✓ simplifies quotient

(ii)   Evaluate $\left(\dfrac{16}{9}\right)^{\frac{3}{2}}$.   (2 marks)

Solution
$\left(\dfrac{16}{9}\right)^{\frac{3}{2}} = \left(\dfrac{4}{3}\right)^3 = \dfrac{64}{27}$ Note: order is not important
Specific behaviours
✓ square roots expression   ✓ cubes expression

(b)   (i)   Solve the equation $\sqrt{3^{2x+4}} = 27^x$ for x.   (3 marks)

Solution
$3^{x+2} = 3^{3x}$ $x + 2 = 3x$ $\quad x = 1$
Specific behaviours
✓ simplifies to common base   ✓ equates indices   ✓ correctly solves for x

(ii) Solve the following simultaneous equations for a and b.   (4 marks)

$$2^{a+b} = 4$$

$$3^{5a-2b} = 27$$

Solution
$2^{a+b} = 4 \Rightarrow 2^{a+b} = 2^2$
$3^{5a-2b} = 27 \Rightarrow 3^{5a-2b} = 3^3$
$a + b = 2 \text{ -----} (1)$
$5a - 2b = 3 \text{ ----} (2)$
$(2) + 2 \times (1) \Rightarrow 7a = 7 \therefore a = 1 \text{ and } b = 1$
Specific behaviours
✓ simplifies to common base
✓ equates indices
✓ correctly solves for a
✓ substitutes and solves for b

Question 3   **(7 marks)**

A game is played using three coloured marbles (red, green and blue) and a four-sided die. The marble is drawn at random from a bag (and replaced), and then the die is rolled.

(a) Complete the table below.   (1 mark)

		Die			
		1	**2**	**3**	**4**
Marble	Red	(R, 1)	**(R, 2)**	**(R, 3)**	(R,4)
	Green	**(G, 1)**	(G, 2)	**(G, 3)**	**(G, 4)**
	Blue	**(B, 1)**	**(B, 2)**	(B, 3)	**(B, 4)**

Solution
See table above
Specific behaviours
✓ correctly completes table

(b) What is the probability of choosing a green marble and rolling a five?   (1 mark)

Solution
$\Pr(G,5) = \dfrac{0}{12}$
Specific behaviours
✓ calculates correct probability

(c)   What is the probability of choosing a red marble or rolling a number greater than one?

(2 marks)

Solution
$\dfrac{4+9-3}{12} = \dfrac{10}{12}$ or $= \dfrac{5}{6}$
Specific behaviours
✓ correctly calculates the numerator
✓ correctly calculates probability

(d)   What is the probability of choosing a blue marble, given that the score on the die was even? (2 marks)

Solution
$\dfrac{2}{6}$ or $= \dfrac{1}{3}$
Specific behaviours
✓ correctly identifies reduced sample space
✓ correctly calculates outcomes within the sample space

(e)   What is the probability of choosing a non-red marble and a prime number on the die?

(1 mark)

Solution
$\dfrac{4}{12}$ or $= \dfrac{1}{3}$
Specific behaviours
✓ correctly calculates the numerator and hence the correct probablilty

Question 4 **(4 marks)**

A unit circle is given below.
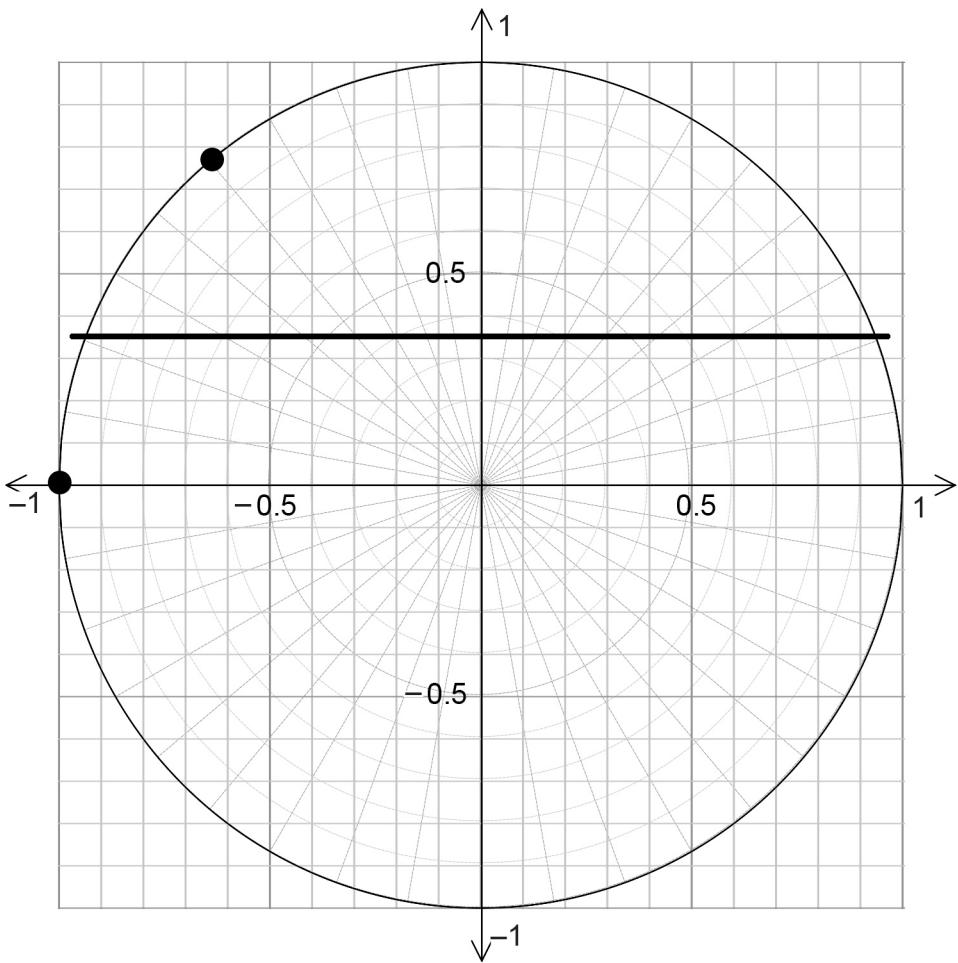
Use the unit circle to estimate each of the following:

(a) $\sin 130°$ (1 mark)

Solution
≈ 0.77
Specific behaviours
✓ estimates correct value

(b) $\cos 180°$ (1 mark)

Solution
-1
Specific behaviours
✓ estimates correct value

(c) θ given that $sin\,\theta = 0.35$ for $0° \leq \theta \leq 180°$.   (2 marks)

Solution
$\approx 20°$ or $160°$
Specific behaviours
✓ correctly estimates $20°$
✓ correctly estimates $160°$

Question 5   **(11 marks)**

In the triangle ABC below (not drawn to scale), the side lengths are $(2x-1)$ cm, $(7-2x)$ cm and x cm.
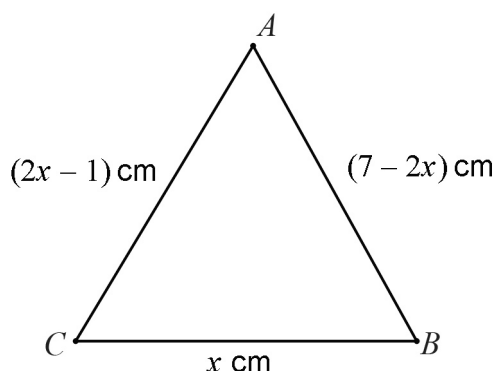
(a)   If $AB = AC$ then triangle ABC would be isosceles. There are two other possibilities for which the triangle is isosceles. Using this information, write down **three** separate equations in terms of x for which triangle ABC is isosceles.   (3 marks)

Solution
$2x-1 = 7-2x$
$2x-1 = x$
$7-2x = x$
Specific behaviours
✓✓✓ correctly writes each equation

(b)   Solve each equation found in part (a) for x.   (3 marks)

Solution
$2x-1 = 7-2x \Rightarrow 4x = 8 \therefore x = 2$
$2x-1 = x \Rightarrow x = 1$
$7-2x = x \Rightarrow 3x = 7 \therefore x = \dfrac{7}{3}$
Specific behaviours
✓✓✓ correctly solves each equation for x

(c)   Write down the side lengths of each triangle using the values of x obtained in part (b).

(3 marks)

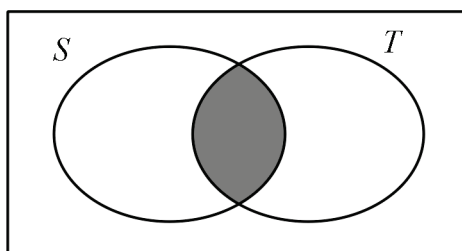| Solution |
|---|
| $x = 2$ gives side lengths of : $3, 3$ and 2 |
| $x = \dfrac{7}{3}$ gives side lengths of : $\dfrac{7}{3}, \dfrac{7}{3}$ and $\dfrac{11}{3}$ |
| $x = 1$ gives side lengths of : $1, 1$ and 5 |
| **Specific behaviours** |
| ✓✓✓ correctly states the side lengths of each possible triangle |

(d)   One of these triangles is not possible. State, with reasons, the triangle that is not possible.

(2 marks)

Solution
$1, 1, 5$ since $1 + 1 = 2 < 5 \therefore$ the triangle is impossible
Specific behaviours
✓ identifies correct triangle
✓ states correct reason

Question 6   **(9 marks)**

(a)   For the sets S and T, shade the region $S \cap \left(\bar{S} \cup T \right)$.   (1 mark)

Solution
see Venn diagram above
Specific behaviours
✓ shades correct region

(b)   Use set notation to describe the shaded region below.   (1 mark)
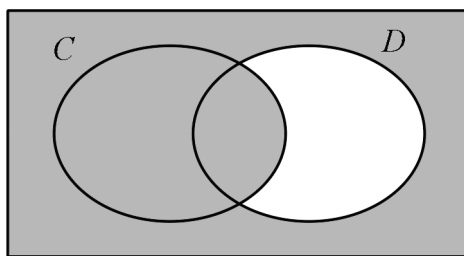
Solution
$C \cup \bar{D}$
Specific behaviours
✓ correctly describes region

(c)   (i)   If $n\left(\overline{A \cup B}\right) = x$ and $n\left(B \cap \overline{A}\right) = 7$, complete the Venn diagram below.   (2 marks)
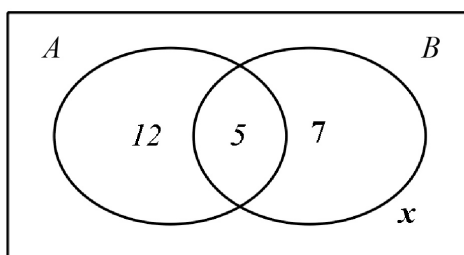
Solution
see Venn diagram above
Specific behaviours
✓✓ correctly completes Venn diagram

(ii)   Determine $n\left(\overline{A}\right)$.   (1 mark)

Solution
$7 + x$
Specific behaviours
✓ states correct value

(iii)   Determine $n\left(\overline{A} \cup B\right)$.   (1 mark)

Solution
$7 + x + 5 = 12 + x$
Specific behaviours
✓ states correct value

(iv)   If $P\left(\overline{A}\mid\overline{B}\right)=\dfrac{1}{7}$, calculate the value of x.   (3 marks)

Solution
$\dfrac{x}{12+x}=\dfrac{1}{7}\Rightarrow 7x=12+x\Rightarrow 6x=12 \therefore x=2$
Specific behaviours
✓ states correct numerator i.e. x ✓ states correct denominator i.e. $12 + x$ ✓ correctly solves equation for x