



# Mercedes College

WA Exams Practice Paper A, 2016

Question/Answer Booklet

## MATHEMATICS APPLICATIONS UNITS 3 AND 4

Section One:  
Calculator-free

## SOLUTIONS

Student Number: In figures

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In words \_\_\_\_\_

Your name \_\_\_\_\_

### Time allowed for this section

Reading time before commencing work: five minutes  
Working time for section: fifty minutes

### Materials required/recommended for this section

*To be provided by the supervisor*

This Question/Answer Booklet  
Formula Sheet

*To be provided by the candidate*

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

### Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

### Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	51	35
Section Two: Calculator-assumed	12	12	100	100	65
<b>Total</b>				151	100

### Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer Booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
  - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
  - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

**Section One: Calculator-free****35% (51 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

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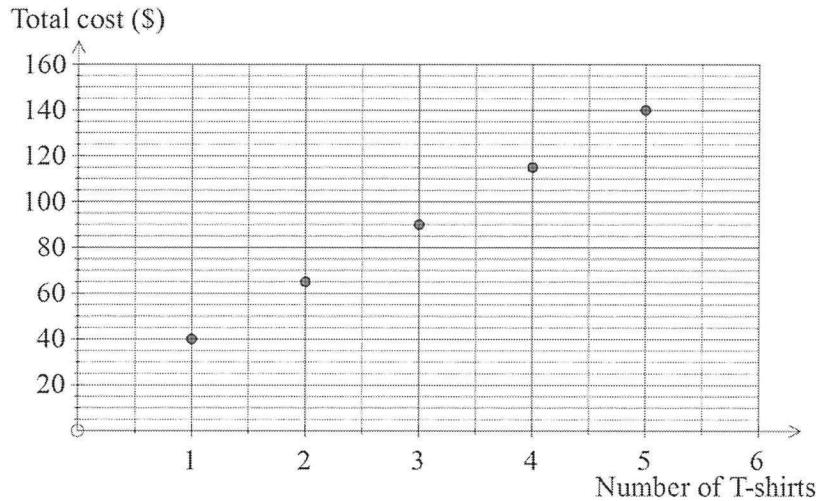
**Question 1****(7 marks)**

An online company sell T-shirts for \$25 each plus a single \$15 shipping fee.

- (a) Complete the table below to show the total cost of purchasing up to five T-shirts. (2 marks)

Number of T-shirts	1	2	3	4	5
Total cost (\$)	40	65	90	115	140

- (b) Display the information in the table in graphical form on the axes below. (2 marks)



- (c) State a rule for the total cost  $C$  of purchasing  $n$  T-shirts. (1 mark)

$$C = 25n + 15$$

- (d) Determine how many T-shirts can be purchased for \$265. (2 marks)

$$\begin{aligned} 25n + 15 &= 265 \\ 25n &= 250 \Rightarrow n = 10 \end{aligned}$$

**Question 2**

(7 marks)

The distances, in metres, along direct tracks between six campsites, A, B, C, D, E and F, are shown in the table below. Not all of the campsites have direct tracks between them.

	A	B	C	D	E	F
A	50					
B	50		90	155	80	
C		90		75		
D		155	75		60	115
E		80		60		180
F				115	180	

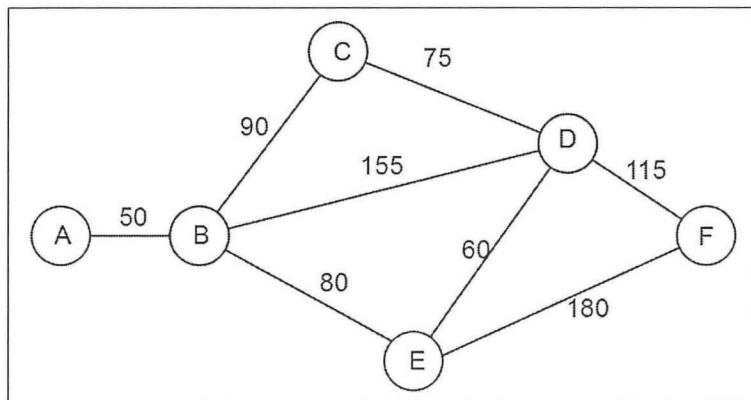
- (a) Use the fact that there are eight edges between the six vertices to determine the number of faces that a planar graph of the above information will have. (2 marks)

$$v + f = e + 2$$

$$6 + f = 8 + 2 \Rightarrow f = 4$$

Four faces.

- (b) Draw a weighted planar graph to represent the information in the table. (3 marks)



- (c) Is the graph above Eulerian, semi-Eulerian or neither? Explain your answer. (2 marks)

Semi-Eulerian.

A trail exists from A to E (or vice-versa) that includes every edge once and once only.

**Question 3**

(7 marks)

The seasonal indices for the first three quarters of a year for the turnover in a restaurant are shown in the table below.

Quarter	One	Two	Three	Four
Seasonal index	0.85	1.02	1.17	-

- (a) In the first and second quarters of this year, the restaurant had turnovers of \$248 700 and \$249 600. In which of these first two quarters will the deseasonalised value of the turnover be greatest? Explain your answer. (3 marks)

Quarter One.

The actual turnovers are very close to each other, but when the figures are deseasonalised, the Q1 figure will increase considerably (by over 15%) and the Q2 figure will decrease slightly (by close to 2%). Thus Q1 will become greater than Q2.

- (b) Determine the seasonal index for the fourth quarter. (1 mark)

$$4 - (0.85 + 1.02 + 1.17) = 4 - 3.04 = 0.96$$

- (c) The equation of the trend line fitted to the deseasonalised turnover  $T$  for each quarter  $Q$  is

$$T = 285\ 000 - 9\ 500 \times Q.$$

- (i) Explain, in the context of the question, the meaning of the figure 9 500 in this equation. (1 mark)

The underlying turnover of the restaurant is decreasing at an average rate of \$9 500 per quarter.

- (ii) Use the equation to write down an expression to predict the actual turnover in the fourth quarter of the first year, but **do not** evaluate it. (2 marks)

$$(285000 - 9500 \times 4) \times 0.96$$

**Question 4**

(6 marks)

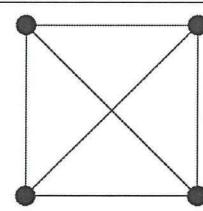
A complete graph with  $n$  vertices will have  $\frac{n(n-1)}{2}$  edges.

- (a) Determine the number of edges a complete graph with 10 vertices will have. (1 mark)

$$\frac{10 \times 9}{2} = 45 \text{ edges.}$$

- (b) A complete graph has 4 vertices.

- (i) Draw the graph, and explain why it is planar. (2 marks)



Planar since

- satisfies Euler's formula:  $v+f=e+2$ , or  $4+4=6+2$ ; or
- can be drawn in plane with no edges that cross.

- (ii) Represent the graph with an adjacency matrix. (1 mark)

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

- (c) Are all complete graphs also planar? Justify your answer. (2 marks)

No. All complete graphs with 5 or more vertices cannot be drawn in the plane without at least one pair of edges crossing.

**Question 5****(4 marks)**

- (a) A couple are saving the same amount of money each week towards buying a home. State which of the following three investment options would grow their savings at the fastest rate, and explain why. (2 marks)

A: An account offering 5% interest per annum, compounded monthly.

B: An account offering 5% simple interest per annum.

C: An account offering 5% interest per annum, compounded annually.

Option A.

Since all options have 5% pa interest, the most frequently compounding option will be best - monthly.

- (b) A student wishes to have the smallest possible monthly repayment on a loan to purchase a car costing \$20 000. The loan attracts interest of 0.75% per month. State which of the following four options would achieve this aim, and explain why. (2 marks)

P: Take a loan for 80% of the cost over the shortest possible period of time.

Q: Take a loan for 90% of the cost over the shortest possible period of time.

R: Take a loan for 80% of the cost over the longest possible period of time.

S: Take a loan for 90% of the cost over the longest possible period of time.

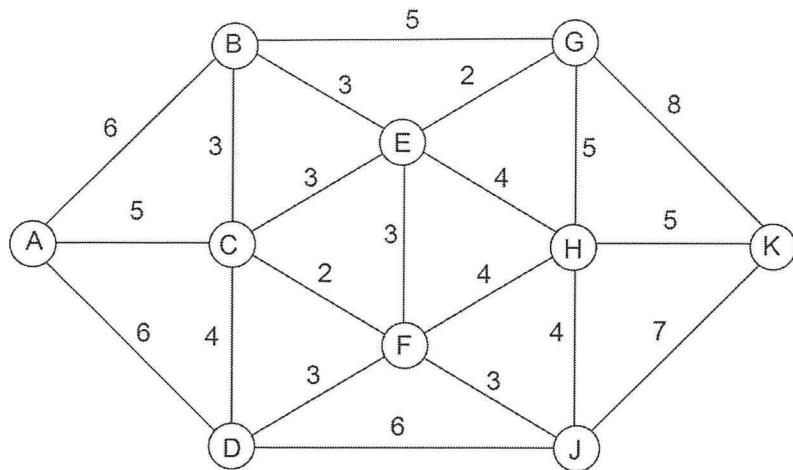
Option R.

In this situation, the student must borrow the least amount of money for the longest possible time to minimise repayments.

**Question 6**

(7 marks)

The time taken, in minutes, to walk on footpaths between various buildings on a school campus is shown on the weighted graph below.



- (a) Determine the minimum time needed to walk from

- (i) B to K.

(1 mark)

12 minutes

- (ii) G to D.

(1 mark)

8 minutes

- (b) Determine the minimum time needed to walk from A to K, stating the route and the time.  
(3 marks)

A C F H K

16 minutes

- (c) A student wanted to walk along every footpath on the campus, without using the same footpath more than once. If this is possible, explain where they should start. If it is not possible, explain why not.  
(2 marks)

Eulerian or semi-Eulerian trail not possible, as there are more than two odd vertices (A, C, E, F, H and K).

## Question 7

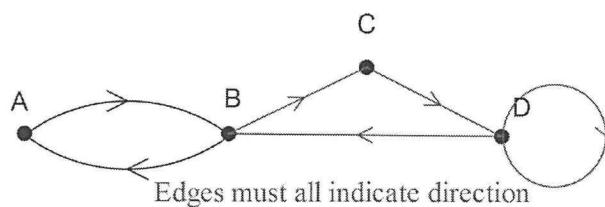
(7 marks)

The adjacency matrix for a digraph with four vertices is

$$\begin{array}{l} A \quad B \quad C \quad D \\ \hline A \left[ \begin{matrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{matrix} \right] \\ B \\ C \\ D \end{array}$$

- (a) Complete the digraph below.

(2 marks)



- (b) List, in order visited, the vertices on a Hamilton path for the digraph starting at vertex C. (1 mark)

C, D, B, A

- (c) State the vertices, if any, from which a Hamilton path for the digraph cannot start. Explain your answer. (2 marks)

Cannot start from B or D.

If these were starting vertices, then not possible to visit all vertices without passing through another vertex twice.

- (d) A Hamilton cycle will exist if one directed edge is added to the digraph. State a suitable edge to add and list, in order, the vertices that will lie on the cycle. (2 marks)

Add an edge from A to C (or from D to A).

Vertices on cycle will be\* A, C, D, B, A (or A, B, C, D, A).

\* Can start and finish at any vertex on cycle

**Question 8**

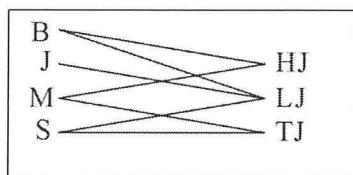
(6 marks)

- (a) The athletics events that four students are proficient in are shown in the table below.

Student	Proficient in these events
Branden	High Jump, Long Jump
Joyce	Long Jump
Marti	High Jump, Triple Jump
Sol	Long Jump, Triple Jump

Represent this information using a bipartite graph.

(2 marks)



- (b) Another three students plan to enter as a team for a triathlon. Their best times (in minutes) for each leg of the triathlon are shown in the table below.

	Swim	Cycle	Run
Gene	26	35	29
Rhea	34	22	35
Tandra	21	19	25

Each student must complete just one of the three legs. Show use of the Hungarian algorithm to determine which student should be allocated to each leg in order to minimise their total time for the triathlon, and state what this time is. (4 marks)

$$\begin{bmatrix} 26 & 35 & 29 \\ 34 & 22 & 35 \\ 21 & 19 & 25 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 9 & 3 \\ 12 & 0 & 13 \\ 2 & 0 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 9 & 0 \\ 12 & 0 & 10 \\ 2 & 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 11 & 0 \\ 10 & 0 & 8 \\ 0 & 0 & 1 \end{bmatrix}$$

Reduce rows

Reduce cols

Only 2 lines cover 0's so subtract/add 2

3 lines cover 0's so done:  $21 + 22 + 29 = 72$  minutes

Gene runs, Rhea cycles and Tandra swims for total of 72 minutes.



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WA Exams Practice Paper A, 2016

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## MATHEMATICS APPLICATIONS UNITS 3 AND 4

Section Two:  
Calculator-assumed

## SOLUTIONS

Student Number: In figures

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In words \_\_\_\_\_

Your name \_\_\_\_\_

### Time allowed for this section

Reading time before commencing work: ten minutes

Working time for section: one hundred minutes

### Materials required/recommended for this section

#### *To be provided by the supervisor*

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Formula Sheet (retained from Section One)

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Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

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**Section Two: Calculator-assumed****65% (100 Marks)**

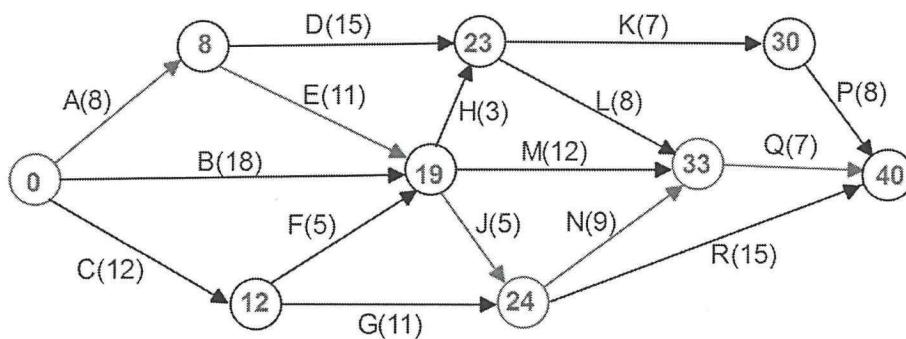
This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

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**Question 9****(7 marks)**

The project network below consists of 16 tasks, with completion times shown in days.



- (a) List, in order, the tasks on the critical path and determine the minimum completion time for this project. (3 marks)

A E J N Q

MCT is 40 days

- (b) The time to complete task F increases by four days. Determine the effect, if any, on your answers to part (a). (2 marks)

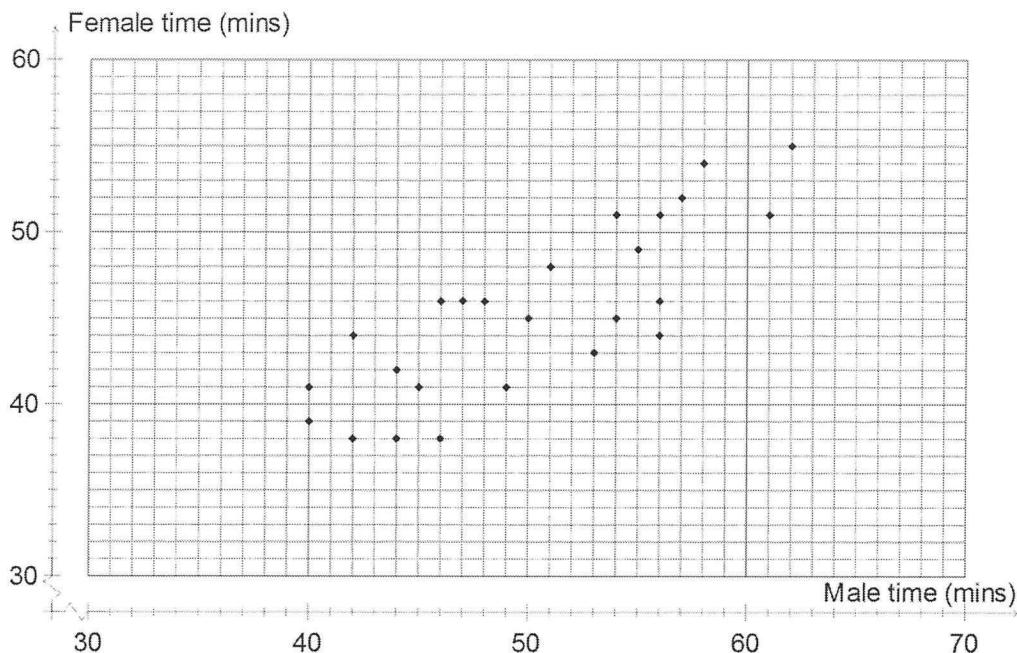
F has 2 days slack, so MCT will increase by 2 days to 42 days and critical path will change to C F J N Q

- (c) An additional task that is not a predecessor for any other task, is to be added to the network and can commence once task D is complete. How many days can this new task be allocated without increasing the minimum completion time? (2 marks)

New task can start on day 23 and must finish by day 40, so must take no more than 17 days.

**Question 10****(9 marks)**

A cycling event was open to teams consisting of one male and one female competitor. The scatterplot below shows the times, rounded to the nearest minute, taken by each pair of competitors in the event.



- (a) The male competitor in one team took 50 minutes. How long did his female team mate take? (1 mark)

**45 minutes**

- (b) How many teams had a total time of 110 minutes or more? (1 mark)

**3 teams**

- (c) Describe a feature of the scatterplot that suggests the presence of an association between the two variables. (1 mark)

**As the male times increase so the female times tend to increase.**

The correlation coefficient between the pairs of times is 0.86 and the equation of the least-squares line is  $f = 0.635m + 13.55$ , where  $f$  is the female time and  $m$  is the male time.

- (d) State which is the explanatory variable for this relationship. (1 mark)

**$m$ , the male times.**

- (e) What percentage of the variation in the female times can be explained by the variation in the male times? (1 mark)

$r^2 = 0.7396$ , and so approximately 74% of the variation can be explained in this way.

- (f) Predict the time taken, to the nearest minute, by the female team mate of a male who took

- (i) 52 minutes (1 mark)

$$f = 0.635(52) + 13.55 = 46.57 \approx 47 \text{ minutes}$$

- (ii) 65 minutes (1 mark)

$$f = 0.635(65) + 13.55 = 54.825 \approx 55 \text{ minutes}$$

- (g) Which prediction in (f) do you consider to be the most reliable? Explain your answer.

(2 marks)

The first prediction of 52 minutes, as it involves interpolation whereas the second involves extrapolation and so is unreliable.

**Question 11**

(7 marks)

- (a) Determine the exact value of  $T_7$  for the following two sequences, one of which is arithmetic and the other is geometric.

(i) 14, 17, 20, 23, 26, ...

(1 mark)

32

(ii) 500, 400, 320, 256, 204.8, ...

(1 mark)

131.072

- (b) A sequence is given by  $T_{n+1} = 5 - 2T_n$      $T_1 = 3$ .

(i) Calculate  $T_2$ .

(1 mark)

5 - 2(3) = -1

(ii) What is the smallest value of  $n$  for which  $T_n \geq 300$ .

(1 mark)

n = 9 ( $T_9 = 343$ )

- (c) After clean-up operations began on a small oil slick, it was observed that the area of the slick decreased by 8% every hour. The initial area of the oil slick was  $550\text{m}^2$ .

(i) What was the area of the slick 1 hour after clean-up began? (1 mark)

550 × 0.92 = 506  $\text{m}^2$ 

- (ii) Write a rule to determine  $A_n$ , the area of the slick  $n$  hours after clean-up operations began. (2 marks)

 $A_n = 550(0.92)^n$

**Question 12**

(8 marks)

A lump sum of money is invested in a savings fund that compounds interest annually.

The amount of money in the savings fund,  $T_n$ , is given by the recursive rule  
 $T_{n+1} = 1.05T_n$ ,  $T_0 = 2250$ , where  $n$  is the number of years after the initial deposit.

- (a) State the initial amount of the lump sum.

(1 mark)

\$2 250

- (b) State the annual interest rate offered by the savings fund.

(1 mark)

5% pa

- (c) Determine

- (i) the amount of money, to the nearest cent, in the savings fund after four years.  
(2 marks)

$2250(1.05)^4 = \$2\ 734.89$

- (ii) the amount of interest accumulated during the first four years.  
(1 mark)

$2734.89 - 2250 = \$484.89$

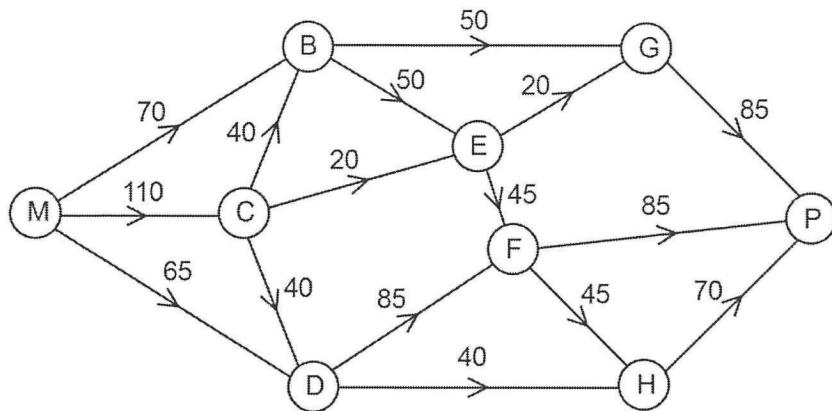
- (d) Another savings fund was offering an interest rate of 4.9% pa compounded monthly.  
Would this fund be a better choice for savers investing the same amount for four years?  
Justify your answer.  
(3 marks)

$$2250 \left(1 + \frac{4.9}{12 \times 100}\right)^{12 \times 4} = \$2\ 736.09$$

Yes, but only just. This fund returns an extra \$1.20 in interest over four years.

**Question 13****(8 marks)**

The directed graph below shows the available conveyor belts to transport raw product from mine M to port P. The number on each edge is the maximum weight of raw product that can be moved along it, in hundreds of tonnes per hour.

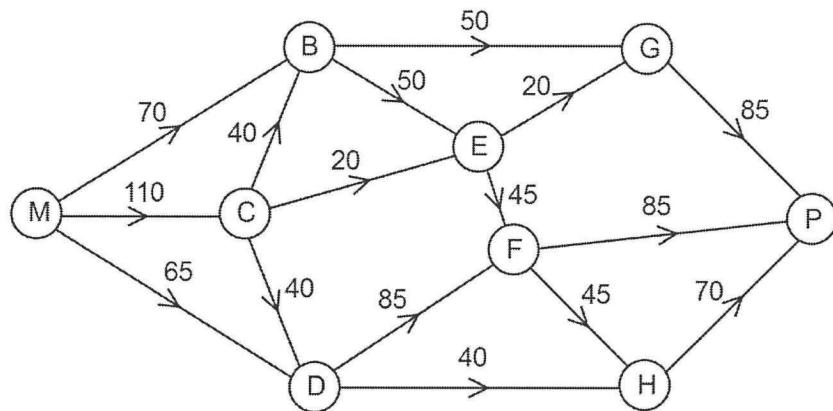


- (a) Determine the maximum weight of raw product that can be transported in one hour from the mine M to the port P. Show systematic working. (5 marks)

MBGP	50
MBEGP	20
MCEFP	30
MCEFP	15
MCDFP	40
MDFHP	45
MDHP	20
Total	220
Transport 22 000 tonnes of raw product per hour.	

- (b) The mine transport manager has funds to increase the capacity of CB or CD by up to 3000 tonnes per hour. Determine which, if either, would be the best to choose and calculate the new maximum weight of raw product that can be transported in one hour from the mine M to the port P. Justify your answer. (3 marks)

*A copy of the original directed graph is provided below for your use, if required.*



CB already has spare capacity, so choose CD.

Along path MCDHP, note that MC, DH and HP have spare capacity of 25, 20 and 15 respectively.

So can increase CD by 15 to reduce spare capacity in HP to nothing.

This will increase tonnage by 1 500, so the new maximum will be 23 500 tonnes per hour.

**Question 14****(8 marks)**

The table below shows the responses of 120 students when asked which one of three school facilities they used most often during term time.

Facility	Age group	
	11 - 13 years	14 - 16 years
Gym	8	38
Library	11	22
Canteen	21	20

- (a) How many 14 - 16 year old responded? (1 mark)

$$38 + 22 + 20 = 80$$

- (b) What percentage of 11 - 13 year olds used the gym most often? (1 mark)

$$\frac{8}{8+11+21} \times 100 = \frac{8}{40} \times 100 = 20\%$$

- (c) Construct an appropriately percentaged two-way table that can be used to help identify whether the use of a facility is associated with age group. (3 marks)

%	Age group	
Facility	11 - 13 years	14 - 16 years
Gym	20	47.5
Library	27.5	27.5
Canteen	52.5	25
Total	100%	100%

- (d) Does the use of a facility appear to be associated with age group? Justify your answer. (3 marks)

Yes - the gym and the canteen.

Just 20% of 11-13 yr olds frequently used the gym compared to 47.5% of 14-16 yr olds.

52.5% of 11-13 yr olds frequently used the canteen compared to just 25% of 14-16 yr olds.

**Question 15**

(7 marks)

- (a) The size of a population of feral cats can be represented by the recurrence relation  
 $p_{n+1} = 0.8p_n + 50$ ,  $p_0 = 76$ , where  $p_n$  is the size of the population after  $n$  months.

- (i) Determine the size of the population after 1 month. (1 mark)

$$110.8 \approx 111 \text{ cats}$$

- (ii) Determine the size of the population after 12 months. (1 mark)

$$238.04 \approx 238 \text{ cats}$$

- (iii) Comment on the size of the cat population in the long term. (2 marks)

Increases until it reaches a steady state of 250 cats (after approximately 27 months).

- (b) Another population of 24 feral cats was observed to grow to 44 after one year and increase to 74 after two years. The size of this population can be represented by  
 $p_{n+1} = r \times p_n + k$ ,  $p_0 = a$ , where  $p_n$  is the size of the population after  $n$  years.

- Determine the values of  $r$ ,  $k$  and  $a$ . (3 marks)

$$a = 24$$

$$44 = r \times 24 + k$$

$$74 = r \times 44 + k$$

Solve simultaneously to get  $r = 1.5$  and  $k = 8$ .

**Question 16**

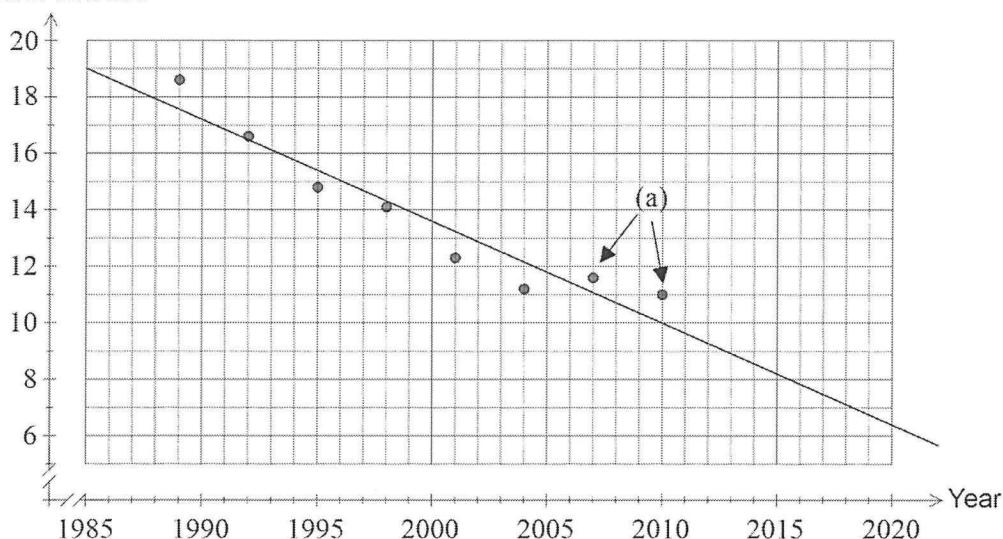
(8 marks)

The average number of cigarettes smoked per week by Australian adults aged between 18 and 24 years is shown in the table below for a number of years.

Year ( $t$ )	1989	1992	1995	1998	2001	2004	2007	2010
Number smoked ( $n$ )	18.6	16.6	14.8	14.1	12.3	11.2	11.6	11.0

- (a) Complete the scatterplot below. (1 mark)

Number smoked



- (b) Determine the correlation coefficient between  $t$  and  $n$ . (1 mark)

$$r = -0.956$$

- (c) Describe the association between the variables in terms of direction and strength. (2 marks)

The association is strong (correlation is close to -1) and negative (as time increase, number smoked tends to decrease).

- (d) Determine the equation of the least-squares line in the form  $n = at + b$ . (1 mark)

$$n = -0.3603t + 734.23$$

- (e) Interpret, in the context of this question, the value of the gradient of the least-squares line in (d). (1 mark)

The average number of cigarettes smoked per week by people in this age group is decreasing by an average of 0.36 per year.

- (f) Predict the average number of cigarettes that will be smoked per week by Australian adults aged between 18 and 24 years in 2016, explaining any dangers associated with your prediction. (2 marks)

$$y = -0.3603(2016) + 734.23 = 7.9$$

From least-squares line, expect close to 8 cigarettes per week, but prediction involves extrapolation and so should be treated with caution.

**Question 17**

(7 marks)

- (a) A way to repay a loan of \$5 900 using monthly repayments of \$1 000 is shown in the table below. At the end of each month, interest of 1.5% of the amount owing at the start of the month is added to the loan and a repayment of \$1 000 is then subtracted. The final loan repayment, of less than \$1 000, brings the balance of the loan to zero.

Month	Balance at start of month	Interest	Repayment	Balance carried forward to start of next month
1	5 900.00	88.50	1 000.00	4 988.50
2	4 988.50	74.83	1 000.00	4 063.33
3	4 063.33	60.95	1 000.00	3 124.28
4	3 124.28	46.86	1 000.00	2 171.14
5	2 171.14	32.57	1 000.00	
6	A	B	1 000.00	C
7			D	0.00

Determine the values of A, B, C and D in the table above.

(4 marks)

$$A = 2171.14 + 32.57 - 1000 = \$1\ 203.71$$

$$B = 1203.71 \times 1.5\% = 18.06$$

$$C = 1203.71 + 18.06 - 1000 = 221.77$$

$$D = 221.77 + 221.77 \times 1.5\% = 221.77 + 3.33 = \$225.10$$

- (b) A loan of \$5 250, with interest of 7.2% per annum compounded monthly, is to be repaid with equal monthly repayments over two and a half years.

- (i) How many repayments are required?

(1 mark)

$$2.5 \times 12 = 30 \text{ repayments}$$

- (ii) Determine the amount of each repayment, rounding your answer to the nearest dollar. (2 marks)

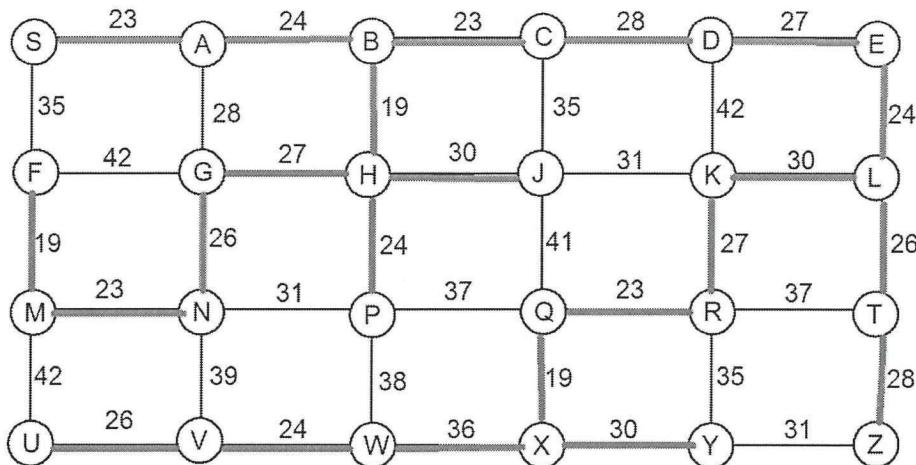
Compound Interest	
N	30
I%	7.2
PV	5250
PMT	-191.743312
FV	0
P/Y	12
C/Y	12

Using a financial calculator, amount is \$191.75 which rounds to \$192.

**Question 18**

(7 marks)

The cost, in thousands of dollars, of installing underground power lines between various locations and a substation located at S is shown in the weighted graph below.



- (a) Identify the minimum spanning tree on the above graph. (3 marks)
- (b) What is the cost of installing underground power lines to all the locations, based on the minimum spanning tree? (2 marks)

$$23 + 24 + 23 + 28 + 27 + 19 + 24 + 27 + 30 + 30 + 19 + 23 \\ + 26 + 24 + 23 + 27 + 26 + 26 + 24 + 36 + 19 + 30 + 28 = 586$$

Cost is \$586 000

- (c) If, for security reasons, power lines must be installed between S and F, F and M and M and U, determine what effect this has on the cost of installing underground power to all the locations. (2 marks)

Include SF and MU (increase of  $35+42=77$ )

Remove GH and WX (decrease of  $27+36=63$ ).

Net increase of \$14 000.

**Question 19**

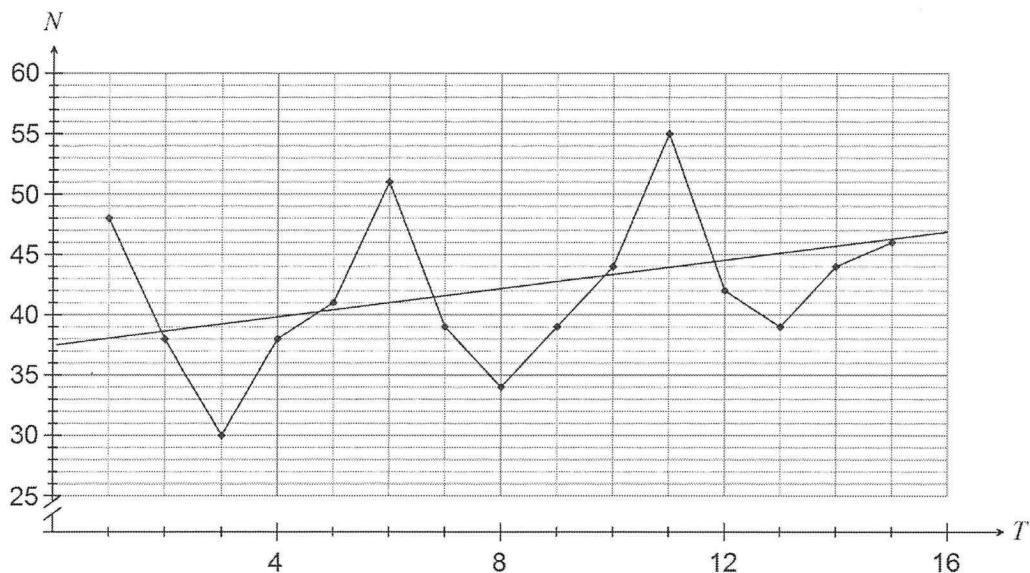
(16 marks)

The table below shows  $N$ , the number of calls per weekday to an enquiry centre, over a three-week period.

Day	Week 1					Week 2					Week 3				
	M	T	W	T	F	M	T	W	T	F	M	T	W	T	F
$T$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$N$	48	38	30	38	41	51	39	34	39	44	55	42	39	44	46

- (a) Construct a time series plot of this data on the axes below.

(3 marks)



- (b) Comment on features of the above graph, including trend and seasonality. (2 marks)

The underlying trend is upwards, so that the number of calls is increasing over the long term.

A weekly cycle with five seasons is evident, reflected in decreasing calls Mon to Weds and then increasing again to the next Mon.

- (c) What is noticeable about the number of calls to the enquiry line on Wednesdays? (1 mark)

The number of calls on Wednesdays is always the lowest for each week.

- (d) Calculate the five-point moving average for Tuesday of Week 2. (1 mark)

$$\frac{41+51+39+34+39}{5} = 40.8$$

The seasonal index for each weekday is shown in the table below.

Day	Mon	Tue	Wed	Thu	Fri
Seasonal index	1.23	0.95	0.82	0.96	1.04

This table shows the deseasonalised values of the number of calls per weekday,  $D$ .

Day	Week 1					Week 2					Week 3				
	M	T	W	T	F	M	T	W	T	F	M	T	W	T	F
$T$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$D$	39.1	40.1	36.7	39.4	39.3	41.6	41.1	41.6	40.5	42.1	44.8	44.3	47.7	A	B

- (e) Calculate the values of A and B in the table above. (2 marks)

$$A = 44 \div 0.96 = 45.8$$

$$B = 46 \div 1.04 = 44.2$$

- (f) Determine the equation of the least-squares regression line that can be used to predict  $D$  from time  $T$ . (2 marks)

$$\text{Using table above, } D = 0.5752T + 37.3$$

$$(NB \text{ Using original data } D = 0.5705T + 37.3)$$

- (g) Draw the line of regression calculated in (f) on the graph in (a). (2 marks)

- (h) Forecast the expected number of calls to the enquiry centre on Monday of Week 4. (3 marks)

$$T = 16$$

$$D = 0.574 \times 16 + 37.3 = 46.484$$

$$46.484 \times 1.23 = 57.17$$

Expect 57 calls.

$$(NB \text{ Using TimeS program, } 46.435 \times 1.2265 = 56.953)$$

**Question 20**

(8 marks)

A lottery winner received a prize of \$760 000.

- (a) The winner used one-half of their winnings to set up an annuity so that they received a monthly payment of \$8 600. The annuity attracted interest of 8.4% per annum, compounded monthly.

- (i) Show that after the first payment, the annuity is worth \$374 060. (3 marks)

$$760000 \div 2 = 380000$$

$$380000 \times 8.4 \div 12 \div 100 = 2660$$

$$380000 + 2660 - 8600 = \$374\ 060$$

- (ii) What is the value of the annuity after the second payment has been made? (1 mark)

Compound Interest	
N	2
I%	8.4
PV	-380000
PMT	8600
FV	374060
P/Y	12
C/Y	12

Using FC, \$368 078.42

- (iii) To the nearest month, how long will the annuity last and how much will the winner have received in total from the annuity after the final payment of \$8 600? (2 marks)

Compound Interest	
N	58.04958560
I%	8.4
PV	-380000
PMT	8600
FV	0
P/Y	12
C/Y	12

Using FC, 53 months.

Total:  $53 \times 8600 = \$455\ 800$ .

- (b) If the winner had chosen to invest all their prize money in an annuity paying 0.7% per month, and did not want the value of the annuity to fall below \$760 000, determine the largest possible monthly payment. (2 marks)

$$Q = \frac{760000 \times 0.7}{100} \\ = \$5320$$