MATHEMATICS APPLICATIONS UNITS 3 AND 4
Section One:
Calculator-free

SOLUTIONS

Student Number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction
fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that
you do not have any unauthorised notes or other items of a non-personal nature in the
examination room. If you have any unauthorised material with you, hand it to the supervisor
before reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	51	35
Section Two: Calculator-assumed	12	12	100	100	65
			Total	151	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section One: Calculator-free 35% (51 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 (7 marks)
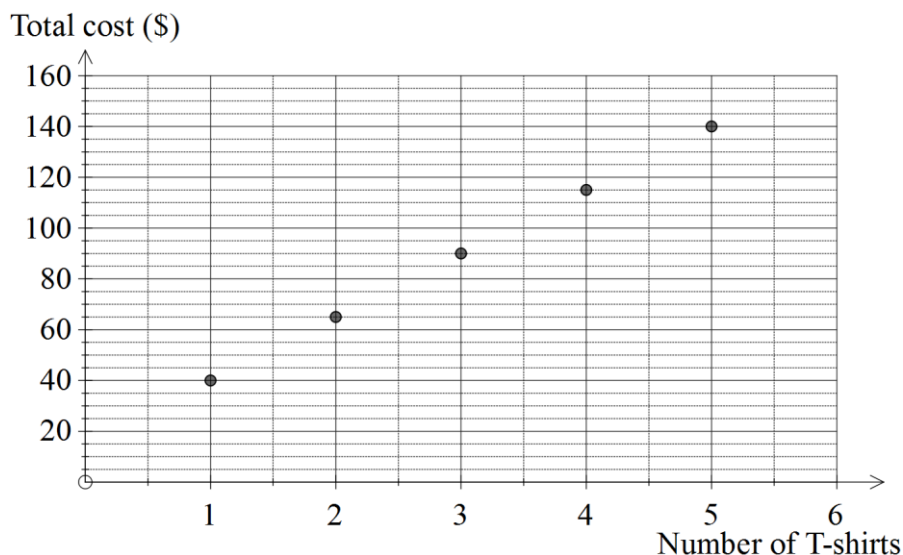
An online company sell T-shirts for $25 each plus a single $15 shipping fee.

(a) Complete the table below to show the total cost of purchasing up to five T-shirts. (2 marks)

Number of T-shirts	1	2	3	4	5
Total cost ($)	40	**65**	**90**	**115**	**140**

(b) Display the information in the table in graphical form on the axes below. (2 marks)

(c) State a rule for the total cost C of purchasing n T-shirts. (1 mark)

$$C = 25n + 15$$

(d) Determine how many T-shirts can be purchased for $265. (2 marks)

$$25n + 15 = 265$$
$$25n = 250 \Rightarrow n = 10$$

See next page

Question 2 **(7 marks)**

The distances, in metres, along direct tracks between six campsites, A, B, C, D, E and F, are shown in the table below. Not all of the campsites have direct tracks between them.

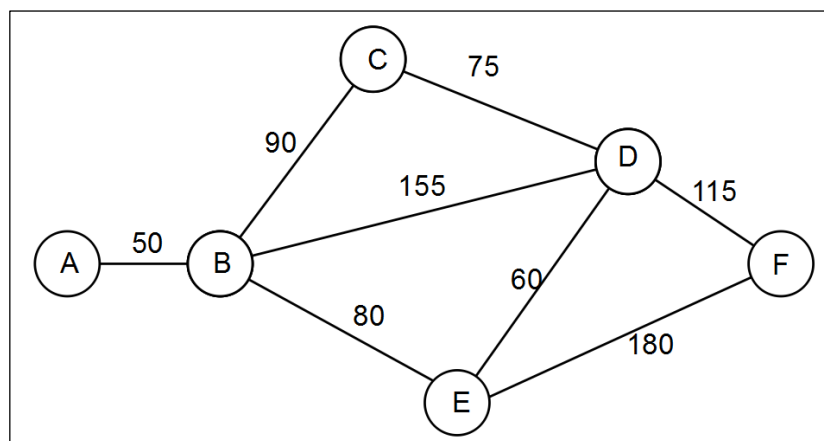|	A	B	C	D	E	F
A		50				
B	50		90	155	80	
C		90		75		
D		155	75		60	115
E		80		60		180
F				115	180	

(a) Use the fact that there are eight edges between the six vertices to determine the number of faces that a planar graph of the above information will have. (2 marks)

$$v + f = e + 2$$

$$6 + f = 8 + 2 \implies f = 4$$

Four faces.

(b) Draw a weighted planar graph to represent the information in the table. (3 marks)

(c) Is the graph above Eulerian, semi-Eulerian or neither? Explain your answer. (2 marks)

Semi-Eulerian.

A trail exists from A to E (or vice-versa) that includes every edge once and once only.

See next page

Question 3 **(7 marks)**

The seasonal indices for the first three quarters of a year for the turnover in a restaurant are shown in the table below.

Quarter	One	Two	Three	Four
Seasonal index	0.85	1.02	1.17	-

(a) In the first and second quarters of this year, the restaurant had turnovers of $248 700 and $249 600. In which of these first two quarters will the deseasonalised value of the turnover be greatest? Explain your answer. (3 marks)

> Quarter One.
>
> The actual turnovers are very close to each other, but when the figures are deseasonalised, the Q1 figure will increase considerably (by over 15%) and the Q2 figure will decrease slightly (by close to 2%). Thus Q1 will become greater than Q2.

(b) Determine the seasonal index for the fourth quarter. (1 mark)

> $4 - (0.85 + 1.02 + 1.17) = 4 - 3.04 = 0.96$

(c) The equation of the trend line fitted to the deseasonalised turnover T for each quarter Q is

$T = 285\,000 - 9\,500 \times Q$.

(i) Explain, in the context of the question, the meaning of the figure 9 500 in this equation. (1 mark)

> The underlying turnover of the restaurant is decreasing at an average rate of $9 500 per quarter.

(ii) Use the equation to write down an expression to predict the actual turnover in the fourth quarter of the first year, but **do not** evaluate it. (2 marks)

> $(285000 - 9500 \times 4) \times 0.96$
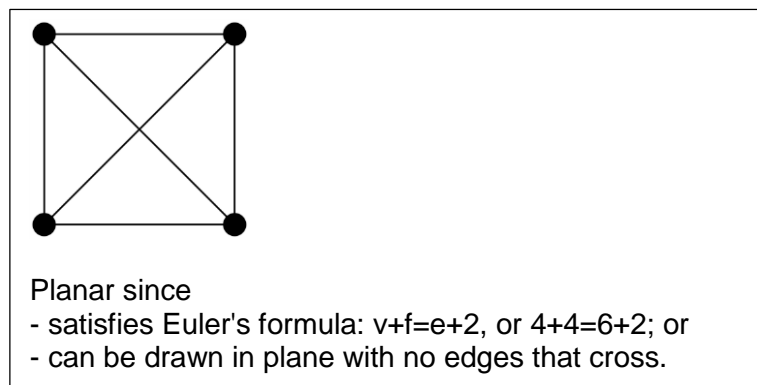
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Question 4 **(6 marks)**

A complete graph with n vertices will have $\dfrac{n(n-1)}{2}$ edges.

(a) Determine the number of edges a complete graph with 10 vertices will have. (1 mark)

$$\frac{10 \times 9}{2} = 45 \text{ edges.}$$

(b) A complete graph has 4 vertices.

(i) Draw the graph, and explain why it is planar. (2 marks)

Planar since
- satisfies Euler's formula: v+f=e+2, or 4+4=6+2; or
- can be drawn in plane with no edges that cross.

(ii) Represent the graph with an adjacency matrix. (1 mark)

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(c) Are all complete graphs also planar? Justify your answer. (2 marks)

No. All complete graphs with 5 or more vertices cannot be drawn in the plane without at least one pair of edges crossing.

See next page

Question 5 **(4 marks)**

(a) A couple are saving the same amount of money each week towards buying a home. State which of the following three investment options would grow their savings at the fastest rate, and explain why. (2 marks)

A: An account offering 5% interest per annum, compounded monthly.

B: An account offering 5% simple interest per annum.

C: An account offering 5% interest per annum, compounded annually.

> Option A.
>
> Since all options have 5% pa interest, the most frequently compounding option will be best - monthly.

(b) A student wishes to have the smallest possible monthly repayment on a loan to purchase a car costing $20 000. The loan attracts interest of 0.75% per month. State which of the following four options would achieve this aim, and explain why. (2 marks)

P: Take a loan for 80% of the cost over the shortest possible period of time.

Q: Take a loan for 90% of the cost over the shortest possible period of time.

R: Take a loan for 80% of the cost over the longest possible period of time.

S: Take a loan for 90% of the cost over the longest possible period of time.

> Option R.
>
> In this situation, the student must borrow the least amount of money for the longest possible time to minimise repayments.

See next page

Question 6   **(7 marks)**

The time taken, in minutes, to walk on footpaths between various buildings on a school campus is shown on the weighted graph below.
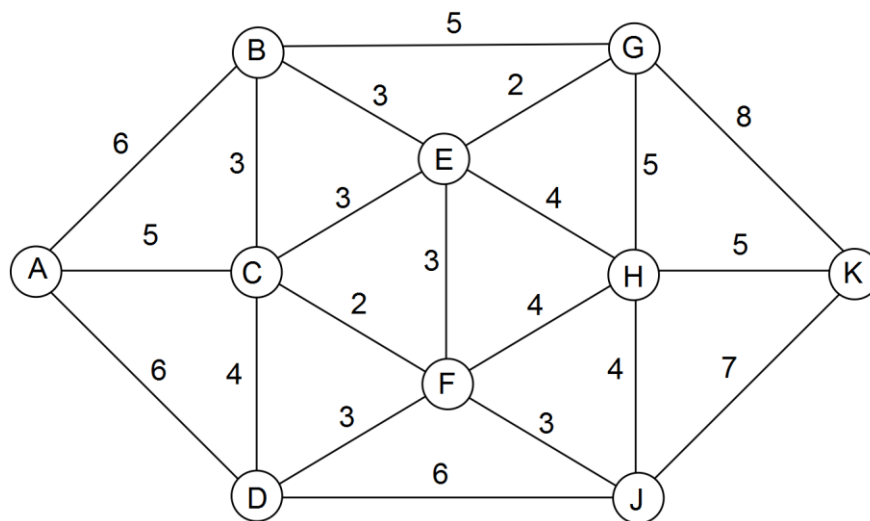
(a)   Determine the minimum time needed to walk from

   (i)   B to K.   (1 mark)

   ┌──────────────────┐
   │ 12 minutes   │
   └──────────────────┘

   (ii)   G to D.   (1 mark)

   ┌──────────────────┐
   │ 8 minutes   │
   └──────────────────┘

(b)   Determine the minimum time needed to walk from A to K, stating the route and the time.
                                                                                 (3 marks)

   ┌──────────────────────┐
   │ A C F H K   │
   │   │
   │ 16 minutes   │
   └──────────────────────┘

(c)   A student wanted to walk along every footpath on the campus, without using the same footpath more than once. If this is possible, explain where they should start. If it is not possible, explain why not.   (2 marks)

   ┌──┐
   │ Eulerian or semi-Eulerian trail not possible, as there │
   │ are more than two odd vertices (A, C, E, F, H and K). │
   └──┘

Question 7 **(7 marks)**

The adjacency matrix for a digraph with four vertices is

$$\begin{array}{c c c c c} & A & B & C & D \\ A & \begin{bmatrix} 0 & 1 & 0 & 0 \\ B & 1 & 0 & 1 & 0 \\ C & 0 & 0 & 0 & 1 \\ D & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}.$$

(a) Complete the digraph below. (2 marks)

Edges must all indicate direction
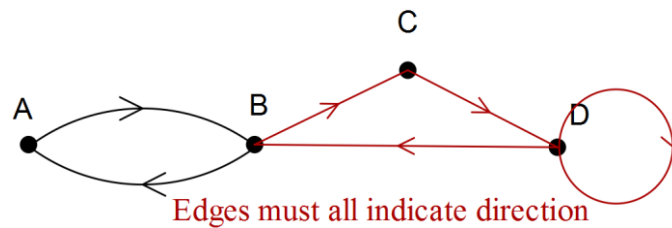
(b) List, in order visited, the vertices on a Hamilton path for the digraph starting at vertex C. (1 mark)

C, D, B, A

(c) State the vertices, if any, from which a Hamilton path for the digraph cannot start. Explain your answer. (2 marks)

Cannot start from B or D.

If these were starting vertices, then not possible to visit all vertices without passing through another vertex twice.

(d) A Hamilton cycle will exist if one directed edge is added to the digraph. State a suitable edge to add and list, in order, the vertices that will lie on the cycle. (2 marks)

Add an edge from A to C (or from D to A).

Vertices on cycle will be* A, C, D, B, A (or A, B, C, D, A).

* Can start and finish at any vertex on cycle

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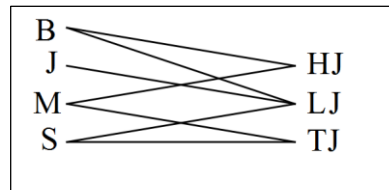
Question 8 **(6 marks)**

(a) The athletics events that four students are proficient in are shown in the table below.

Student	Proficient in these events
Branden	High Jump, Long Jump
Joyce	Long Jump
Marti	High Jump, Triple Jump
Sol	Long Jump, Triple Jump

Represent this information using a bipartite graph. (2 marks)

(b) Another three students plan to enter as a team for a triathlon. Their best times (in minutes) for each leg of the triathlon are shown in the table below.

	Swim	Cycle	Run
Gene	26	35	29
Rhea	34	22	35
Tandra	21	19	25

Each student must complete just one of the three legs. Show use of the Hungarian algorithm to determine which student should be allocated to each leg in order to minimise their total time for the triathlon, and state what this time is. (4 marks)

$$\begin{bmatrix} \underline{26} & 35 & 29 \\ 34 & \underline{22} & 35 \\ 21 & \underline{19} & 25 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 9 & \underline{3} \\ 12 & 0 & 13 \\ 2 & 0 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 9 & 0 \\ 12 & 0 & 10 \\ \underline{2} & 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 11 & \underline{0} \\ 10 & \underline{0} & 8 \\ \underline{0} & 0 & 1 \end{bmatrix}$$

Reduce rows
Reduce cols
Only 2 lines cover 0's so subtract/add 2

3 lines cover 0's so done: $21 + 22 + 29 = 72$ minutes
Gene runs, Rhea cycles and Tandra swims for total of 72 minutes.

End of questions

Additional working space

Question number: _____