

VCE Mathematical Methods Units 3&4

Written Examination 2

Suggested Solutions

SECTION A – MULTIPLE-CHOICE QUESTIONS

| | | | | | |
|----|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|
| 1 | <input type="checkbox"/> A | <input checked="" type="checkbox"/> B | <input type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 2 | <input type="checkbox"/> A | <input type="checkbox"/> B | <input checked="" type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 3 | <input type="checkbox"/> A | <input type="checkbox"/> B | <input type="checkbox"/> C | <input type="checkbox"/> D | <input checked="" type="checkbox"/> E |
| 4 | <input type="checkbox"/> A | <input type="checkbox"/> B | <input checked="" type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 5 | <input type="checkbox"/> A | <input type="checkbox"/> B | <input checked="" type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 6 | <input checked="" type="checkbox"/> A | <input type="checkbox"/> B | <input type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 7 | <input type="checkbox"/> A | <input checked="" type="checkbox"/> B | <input type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 8 | <input checked="" type="checkbox"/> A | <input type="checkbox"/> B | <input type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 9 | <input type="checkbox"/> A | <input type="checkbox"/> B | <input checked="" type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 10 | <input type="checkbox"/> A | <input type="checkbox"/> B | <input type="checkbox"/> C | <input checked="" type="checkbox"/> D | <input type="checkbox"/> E |
| 11 | <input type="checkbox"/> A | <input checked="" type="checkbox"/> B | <input type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 12 | <input type="checkbox"/> A | <input type="checkbox"/> B | <input checked="" type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 13 | <input checked="" type="checkbox"/> A | <input type="checkbox"/> B | <input type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 14 | <input type="checkbox"/> A | <input type="checkbox"/> B | <input type="checkbox"/> C | <input type="checkbox"/> D | <input checked="" type="checkbox"/> E |
| 15 | <input type="checkbox"/> A | <input type="checkbox"/> B | <input type="checkbox"/> C | <input type="checkbox"/> D | <input checked="" type="checkbox"/> E |
| 16 | <input checked="" type="checkbox"/> A | <input type="checkbox"/> B | <input type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 17 | <input type="checkbox"/> A | <input type="checkbox"/> B | <input checked="" type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 18 | <input type="checkbox"/> A | <input checked="" type="checkbox"/> B | <input type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 19 | <input type="checkbox"/> A | <input checked="" type="checkbox"/> B | <input type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |
| 20 | <input type="checkbox"/> A | <input type="checkbox"/> B | <input checked="" type="checkbox"/> C | <input type="checkbox"/> D | <input type="checkbox"/> E |

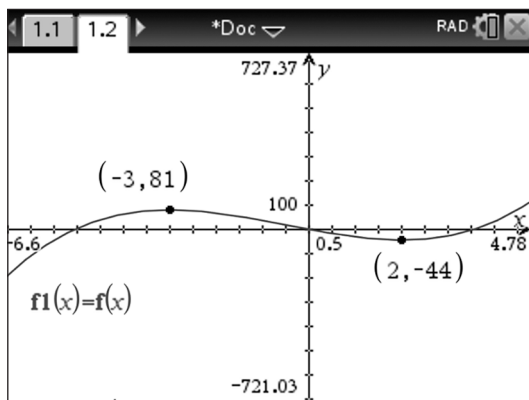
Question 1 B

$$\text{period} = \frac{2\pi}{\pi} = 2$$

$$\text{range} = [2 - 3, 2 + 3] = [-1, 5]$$

Question 2 C

Using a CAS calculator gives:



C is not a true statement and is therefore the required response. The graph does have a point of inflection.

A, B and **D** are true statements and are therefore not the required response.

E is a true statement and is therefore not the required response. The function does not have an inverse since it is not monotonic.

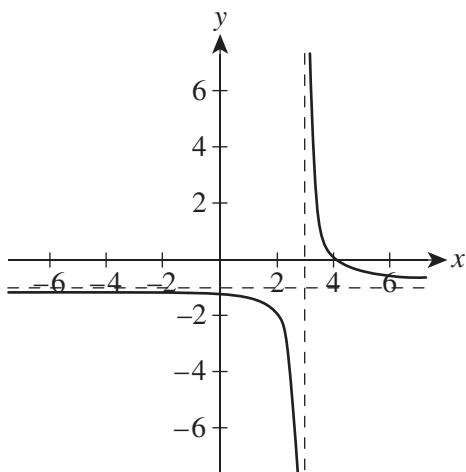
Question 3 E

Using a CAS calculator to consider three cases where the second ball is a different colour to the first ball gives:

$$\frac{4}{10} \cdot \frac{6}{9} + \frac{5}{10} \cdot \frac{5}{9} + \frac{1}{10} \cdot \frac{9}{9} = \frac{29}{45}$$

Question 4 C

C is correct. The graph of $f(x) = -1 + \frac{1}{x-3}$ is as follows.



A is incorrect. This option has the asymptote $x = 3$ only.

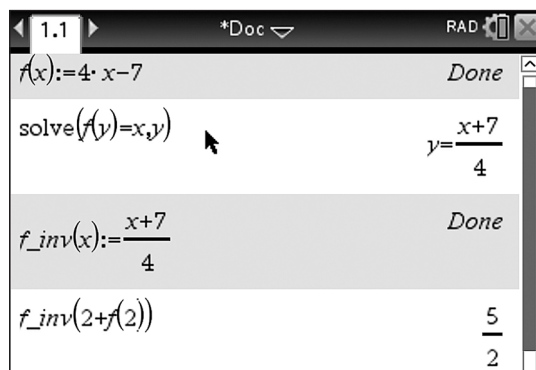
B is incorrect. This option has the asymptote $y = -1$ only.

D is incorrect. This option has the asymptotes $x = -1$ and $y = 3$.

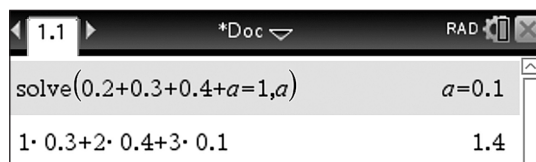
E is incorrect. This option has the asymptotes $x = 3$ and $y = 0$.

Question 5 C

Using a CAS calculator gives:

**Question 6 A**

Using a CAS calculator gives:



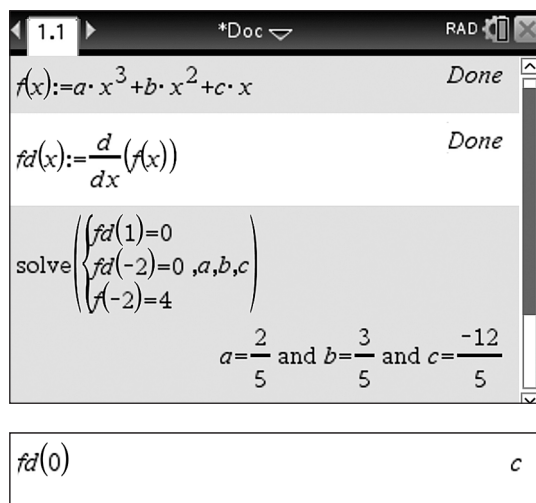
Question 7 B

Determining the average value gives:

$$\begin{aligned}\frac{1}{4-0} \int_0^4 f(x) dx &= \frac{1}{4} \text{ of area under } f(x) \\ &= \frac{1}{4} \left(\frac{3}{2}(4+1) + \frac{1}{2}(1+3) \right) \\ &= \frac{19}{8}\end{aligned}$$

Question 8 A

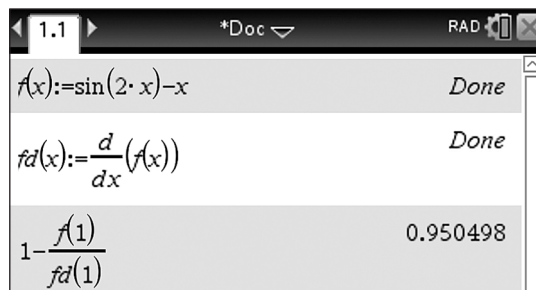
Using a CAS calculator gives:

**Question 9 C**

The algorithm returns an angle multiplied by $\frac{180}{\pi}$ or $\frac{\pi}{180}$ depending on the initial unit. This is used to convert a given angle between degrees and radians.

Question 10 D

Using a CAS calculator gives:

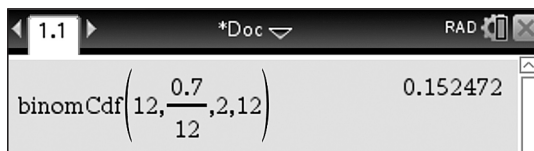


Question 11 B

$$\begin{aligned}
 \Pr(17.2 \leq X \leq 18.4) &= \Pr\left(\frac{17.2-18}{0.4} \leq Z \leq \frac{18.4-18}{0.4}\right) \\
 &= \Pr(-2 \leq Z \leq 1) \\
 &= \Pr(-1 \leq Z \leq 2)
 \end{aligned}$$

Question 12 C

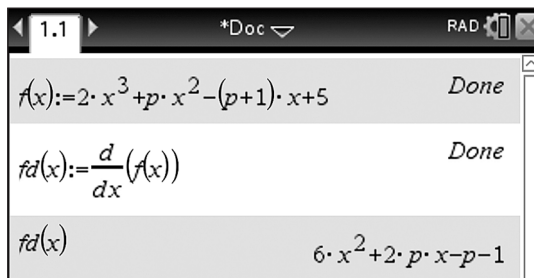
Using a CAS calculator gives:



A screenshot of a CAS calculator interface. The input is $\text{binomCdf}\left(12, \frac{0.7}{12}, 2, 12\right)$ and the output is 0.152472.

Question 13 A

Using a CAS calculator gives:



A screenshot of a CAS calculator interface showing the differentiation of a function. The input is $f(x) := 2 \cdot x^3 + p \cdot x^2 - (p+1) \cdot x + 5$. The derivative is calculated as $f'(x) := \frac{d}{dx}(f(x))$, resulting in $6 \cdot x^2 + 2 \cdot p \cdot x - p - 1$.

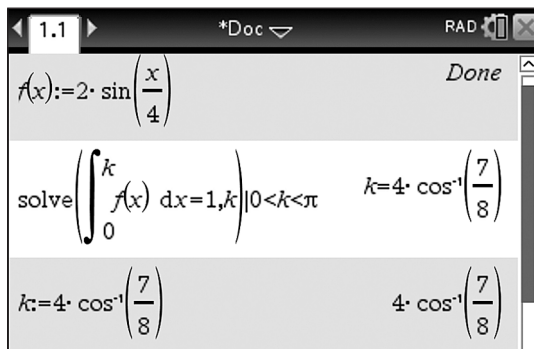
If a function has a turning point, then it is not monotonic; hence, it has no inverse.

So, looking for a positive discriminant gives:

$$\begin{aligned}
 (2p)^2 - 4 \times 6 \times (-p-1) &> 0 \\
 4p^2 + 24p + 24 &> 0 \\
 p^2 + 6p + 6 &> 0
 \end{aligned}$$

Question 14 E

Using a CAS calculator gives:



A screenshot of a CAS calculator interface showing the solution of an integral equation. The input is $f(x) := 2 \cdot \sin\left(\frac{x}{4}\right)$. The problem is to solve $\int_0^k f(x) dx = 1, k$ for $0 < k < \pi$. The solution is $k = 4 \cdot \cos^{-1}\left(\frac{7}{8}\right)$.



A screenshot of a CAS calculator interface showing the numerical value of the integral. The input is $\int_1^k f(x) dx$ and the output is 0.751299.

Question 15 E

The three numbers can be expressed as x , $2x$ and $100 - 3x$.

Using a CAS calculator gives:

| | | |
|--|----------------------------------|-----|
| 1.1 | *Doc | RAD |
| $f(x) := x \cdot 2 \cdot x \cdot (100 - 3 \cdot x)$ | Done | |
| $\text{solve}\left(\frac{d}{dx}(f(x)) = 0, x\right)$ | $x = 0. \text{ or } x = 22.2222$ | |
| $f(22)$ | 32912 | |
| $f(23)$ | 32798 | |

Hence, the maximum product is 32 912.

Question 16 A

The answer can be obtained by trial and error using a CAS calculator.

| | | |
|--|----------|-----|
| 1.1 | *Doc | RAD |
| $f(n) := \text{binomCdf}(n, 0.24, 3, n)$ | Done | |
| $f(17)$ | 0.812347 | |
| $f(18)$ | 0.842994 | |
| $f(16)$ | 0.776767 | |

Hence, the smallest possible value of n is 17.

Question 17 C

Reflection in the y -axis maps $y = \sin(2x)$ to $y = \sin(-2x)$.

Dilation by a factor of 2 from the y -axis maps $y = \sin(-2x)$ to $y = \sin(-x)$.

Translation of $\frac{\pi}{2}$ units in the positive direction of the x -axis maps $y = \sin(-x)$ to:

$$\begin{aligned}
 y &= \sin\left(-\left(x - \frac{\pi}{2}\right)\right) \\
 &= \sin\left(\frac{\pi}{2} - x\right) \\
 &= \cos(x)
 \end{aligned}$$

Question 18 B

Using a CAS calculator gives:

| | |
|---|-------------|
| $f(x) := e^{2 \cdot x} + 1$ | Done |
| $g(x) := \text{tangentLine}(f(x), x=a)$ | Done |
| $\triangle \text{solve}(g(2)=0, a)$ | $a=2.50335$ |
| $a := 2.50335$ | 2.50335 |
| $g(0)$ | -597.645 |

Question 19 B

Using a CAS calculator gives:

| | |
|---|---|
| $\text{domain}\left(5 \cdot \tan\left(\frac{x}{3}\right) - 2, x\right)$ | $x \neq \frac{3 \cdot (2 \cdot n1 - 1) \cdot \pi}{2}$ |
|---|---|

The answer is not immediately identifiable. Further manipulation gives:

$$\begin{aligned} \frac{3\pi}{2}(2k-1) &= \frac{3\pi}{2}(2k-1) + 2\pi m \quad (\text{where } k \text{ and } m \text{ are any integers}) \\ &= \frac{3\pi}{2}(2k-1) + \frac{3\pi}{2} \left(\frac{2}{3\pi} \times 2\pi m \right) \\ &= \frac{3\pi}{2} \left(2k-1 + \frac{4m}{3} \right) \end{aligned}$$

Choosing $m = 3$, $\frac{3\pi}{2}(2k+3)$.

Question 20 C

Stationary points exist when $\frac{d}{dx}f(g(x)) = 0$.

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \times g'(x)$$

$$f'(g(x)) \times g'(x) = 0 \Rightarrow \begin{cases} g'(x) = 0 \\ f'(g(x)) = 0 \end{cases}$$

Observing the graphs:

$$\begin{cases} g'(x) = 0 \\ f'(g(x)) = 0 \end{cases} \Rightarrow \begin{cases} 1 \text{ solution: } x = 3 \\ g(x) \approx 0.5 \Rightarrow 2 \text{ solutions} \\ g(x) \approx 3.5 \Rightarrow 1 \text{ solution} \\ g(x) \approx 6.5 \Rightarrow \text{no solution} \end{cases}$$

Note that the question asks for solutions for $0 \leq x \leq 5$. Therefore, there are 4 stationary points.

SECTION B**Question 1** (9 marks)

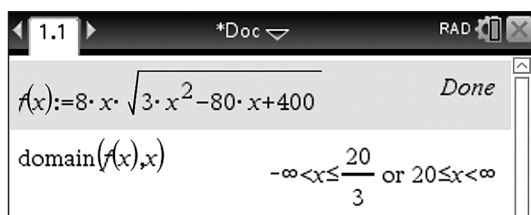
a. $KM = \frac{80 - 6x - 2x}{2} = 40 - 4x$ M1

b. $KO = \sqrt{KM^2 - MO^2}$
 $= \sqrt{(40 - 4x)^2 - (2x)^2}$ M1
 $= 2\sqrt{3x^2 - 80x + 400}$ A1

$A = \frac{1}{2} \times KO \times (KL + MN)$
 $= \frac{1}{2} \times (2\sqrt{3x^2 - 80x + 400}) \times 8x$ M1
 $= 8x\sqrt{3x^2 - 80x + 400}$

Note: Consequential on answer to Question 1a.

c. Using a CAS calculator gives:



The valid interval for x such that the trapezium exists needs to be selected.

$0 < x < \frac{20}{3}$ A1

Note: Responses must use strict inequalities.

d. Using a CAS calculator gives:

CAS calculator interface showing:
 $f(x) := 8 \cdot x \cdot \sqrt{3 \cdot x^2 - 80 \cdot x + 400}$
 $\text{solve}\left(\frac{d}{dx}(f(x)) = 0, x\right)$
 $x = \frac{-10 \cdot (\sqrt{3} - 3)}{3} \text{ or } x = \frac{10 \cdot (\sqrt{3} + 3)}{3}$

CAS calculator interface showing:
 $x = 4.2265 \text{ or } x = 15.7735$
 $f\left(\frac{-10 \cdot (\sqrt{3} - 3)}{3}\right) = 363.333$

$$\frac{dA}{dx} = 0 \Rightarrow x = 4.2 \dots \text{ or } x = 15.7 \dots$$

M1

$$\text{Since } 15.7 \dots \notin \left(0, \frac{20}{3}\right), x = 4.2 \dots$$

$$A(4.2 \dots) = 363.3$$

A1

e. Using a CAS calculator gives:

CAS calculator interface showing:
 $f(x) := 8 \cdot x \cdot \sqrt{3 \cdot x^2 - 80 \cdot x + 400}$
 $\frac{1}{\frac{20}{3} - 0} \int_0^{\frac{20}{3}} f(x) \, dx = 255.631$

$$\frac{1}{\frac{20}{3} - 0} \int_0^{\frac{20}{3}} A(x) \, dx = 255.6$$

M1 A1

Question 2 (11 marks)

a. Solving for k using a CAS calculator gives:

A screenshot of a CAS calculator interface. The top bar shows '1.1', '*Doc', and 'RAD'. The main display area shows three lines of code: $g(x) := \frac{-1}{16} \cdot x^2 - \frac{1}{4} \cdot x + \frac{221}{16}$, $h(x) := \frac{1}{20} \cdot x^2 - \frac{41}{20} \cdot x + k$, and $g(8)$. The results for the first two lines are 'Done'. The result for $g(8)$ is $\frac{125}{16}$.

A screenshot of a CAS calculator interface. The main display area shows two lines of code: $h(8)$ and $\text{solve}(g(8)=h(8),k)$. The result for $h(8)$ is $k - \frac{66}{5}$. The result for $\text{solve}(g(8)=h(8),k)$ is $k = \frac{1681}{80}$.

$$g(8) = \frac{125}{16}$$

M1

$$h(8) = k - \frac{66}{5}$$

$$g(8) = h(8) \Rightarrow k = \frac{1681}{80}$$

M1

b. Using a CAS calculator gives:

A CAS calculator interface showing the differentiation of two functions. The first row shows $gd(x) := \frac{d}{dx}(g(x))$ with a 'Done' button. The second row shows $hd(x) := \frac{d}{dx}(h(x))$ with a 'Done' button. The third row shows the result of $gd(8)$ as $-\frac{5}{4}$.

A CAS calculator interface showing the differentiation of two functions. The first row shows $hd(8)$ as $-\frac{5}{4}$. The second row shows $gd(x)$ as $-\frac{x}{8} - \frac{1}{4}$. The third row shows $hd(x)$ as $\frac{x}{10} - \frac{41}{20}$.

$$g'(x) = -\frac{1}{8}x - \frac{1}{4}$$

$$h'(x) = \frac{1}{10}x - \frac{41}{20}$$

$$g'(8) = -\frac{5}{4}$$

$$h'(8) = -\frac{5}{4}$$

$g'(x)$ and $h'(x)$ M1

$g'(8)$ and $h'(8)$ A1

$$\begin{cases} g(8) = h(8) \\ g'(8) = h'(8) \end{cases} \Rightarrow f(x) \text{ is differentiable at } x = 8$$

M1

c. Using a CAS calculator gives:

A CAS calculator interface showing the evaluation of $g(0)$ as $\frac{221}{16}$ and solving the equation $h(x) = 0$ for x , resulting in $x = \frac{41}{2}$.

height: $g(0) = \frac{221}{16}$ m

A1

length: $h(x) = 0 \Rightarrow x = \frac{41}{2}$ m

A1

- d. i. The curve is concave down for the given interval, so the trapezium will always be under the curve. Hence, the approximation will be less than the actual area.

A1

- ii. Using the CAS calculator gives:

The screenshot shows a CAS calculator interface. The top bar has tabs for 1.1, 1.2, and 1.3, with 1.3 selected. Below the tabs, the expression $\int_0^8 g(x) dx$ is entered, and the result $\frac{551}{6}$ is displayed. Below this, the expression $\frac{1}{2} \cdot (g(0) + 2 \cdot (g(1) + g(2) + g(3) + g(4) + g(5) + g(6)) + g(8))$ is entered, and the result $\frac{367}{4}$ is displayed. At the bottom, the expression $\frac{551}{6} - \frac{367}{4}$ is entered, and the result 0.083333 is displayed.

$$\text{actual area} = \int_0^8 g(x) dx = \frac{551}{6}$$

A1

$$\text{approximate area} = \frac{1}{2} \left(g(0) + 2 \left(g(1) + g(2) + g(3) + g(4) + g(5) + g(6) + g(7) \right) + g(8) \right) = \frac{367}{4}$$

A1

$$\text{difference} = \frac{551}{6} - \frac{367}{4} = 0.083$$

A1

Question 3 (14 marks)

a. $\int_8^{12} \frac{1}{40}(t-8)dt = \frac{[(t-8)^2]_8^{12}}{80} = \frac{16-0}{80} = \frac{16}{80}$ A1

$$\int_{12}^{15} \frac{1}{40}(20-t)dt = \frac{[(20-t)^2]_{12}^{15}}{-80} = \frac{25-64}{-80} = \frac{39}{80}$$
 A1

$$\begin{aligned} \Pr(T \leq 15) &= \frac{16}{80} + \frac{39}{80} \\ &= \frac{55}{80} \\ &= \frac{11}{16} \end{aligned}$$
 M1

b. Using a CAS calculator gives:

The screenshot shows a CAS interface with the following content:

- Line 1.1: $f(t) := \begin{cases} \frac{1}{40} \cdot (t-8), & 8 \leq t \leq 12 \\ \frac{1}{40} \cdot (20-t), & 12 \leq t \leq 20 \end{cases}$ (labeled "Done")
- Line 1.2: $\int_8^{12} f(t) dt$ (result: $\frac{16}{55}$)
- Line 1.3: $\frac{\frac{11}{16}}{\frac{16}{55}}$

$$\frac{\int_8^{12} f(t) dt}{\frac{11}{16}} = \frac{16}{55}$$

M1 A1

c.

The screenshot shows a CAS interface with the following content:

- Line 1.1: $\int_8^{20} (t \cdot f(t)) dt$ (result: 13.8667)

$$\int_8^{20} t \times f(t) dt = 13.9 \text{ minutes}$$

M1 A1

d.

Calculator screen showing:
 $p := \int_8^{12} f(t) dt = \frac{1}{5}$
 $\text{binomCdf}(10, p, 6, 10) = 0.006369$

 X = number of trips completed in less than 12 minutes

$$X \sim \text{Bi}(10, p)$$

$$p = \int_8^{12} f(t) dt = \frac{1}{5}$$

M1

$$\Pr(X \geq 6) = 0.0064$$

A1

e.

Calculator screen showing:
 $a := \text{invNorm}(0.65, 0, 1) = 0.38532$
 $\text{solve}\left(\frac{13-12}{s} = a, s\right) = s = 2.59524$

$$\Pr(U < 13) = \Pr(Z < a)$$

$$a = 0.3853 \dots$$

M1

$$\frac{13-12}{\sigma} = a$$

$$\sigma = 2.5952$$

A1

f.

Calculator screen showing:
 $p := \text{normCdf}(-\infty, 3, 3.4, 0.8) = 0.308538$
 $\text{binomCdf}(30, p, 15, 24) = 0.021959$

 Y = number of times a trip is interrupted by red light

$$Y \sim N(3.4, 0.8^2)$$

M1

 X = number of trips with less than three red light interruptions

$$X \sim \text{Bi}(30, p)$$

$$p = \Pr(Y < 3) = 0.3085 \dots$$

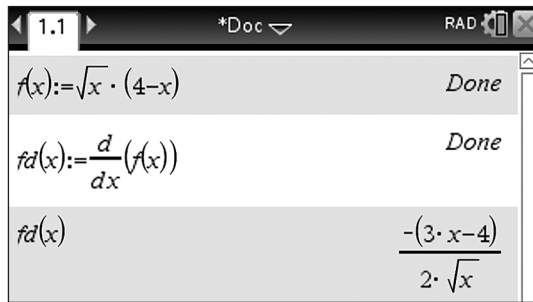
A1

$$\hat{P} = \frac{X}{30} \Rightarrow \Pr(0.5 \leq \hat{P} \leq 0.8) = \Pr(15 \leq X \leq 24) = 0.0220$$

A1

Question 4 (16 marks)

a. Using a CAS calculator gives:



$$f'(x) = \frac{4-3x}{2\sqrt{x}}$$

M1

$$f'(x) = 0 \Rightarrow x = \frac{4}{3}$$

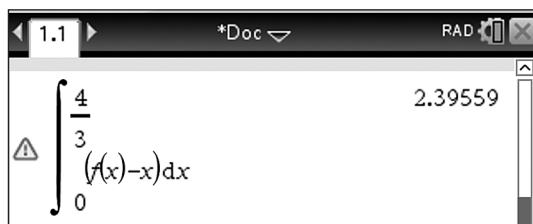
A1

If $D_f = [0, k]$ does not contain any turning points, then f has an inverse. Therefore,

$$0 < k \leq \frac{4}{3}.$$

M1

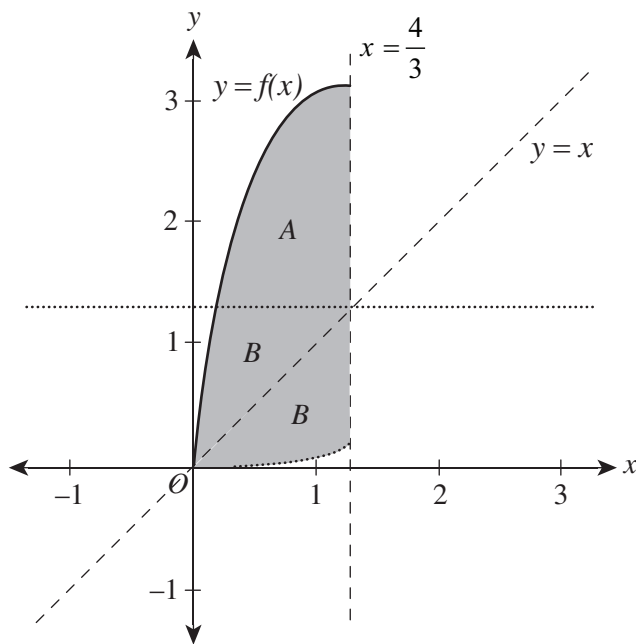
b.



$$\int_0^{\frac{4}{3}} (f(x) - x) dx = 2.40$$

M1 A1

- c. Rather than trying to find a rule for f^{-1} , the area can be found using symmetrical regions.



Using the graph above, the total area can be represented by $A + 2B$.

1.1 1.2 *Doc RAD

solve $\left(f(x)=\frac{4}{3}x\right)$ $x=0.117967$ or $x=3.26173$

$a := \int_{0.117967}^{\frac{4}{3}} \left(f(x) - \frac{4}{3}\right) dx$ 1.55786

$b := \int_0^{\frac{4}{3}} (f(x) - x) dx - a$ 0.837734

$a + 2 \cdot b$ 3.23333

$$f(x) = \frac{4}{3} \Rightarrow x = 0.1179 \dots$$

M1

$$A = \int_{0.1179 \dots}^{\frac{4}{3}} \left(f(x) - \frac{4}{3}\right) dx = 1.5578 \dots$$

M1

$$B = \int_0^{\frac{4}{3}} (f(x) - x) dx - A = 0.8377$$

$$A + 2B = 3.23$$

A1

d.

TI-84 Plus calculator screen showing the definition of $g(x)$ and the integral $s(a)$.

Line 1: $g(x) := a \cdot \sqrt{x} \cdot (4 - a \cdot x)$ Done

Line 2: $\int_0^{\frac{4}{3 \cdot a}} (g(x) - x) dx$ $\frac{256 \cdot \sqrt{\frac{3}{a}}}{135} - \frac{8}{9 \cdot a^2}$

Line 3: $s(a) := \frac{256 \cdot \sqrt{\frac{3}{a}}}{135} - \frac{8}{9 \cdot a^2}$ Done

TI-84 Plus calculator screen showing the solution for a and the value of $s(1.05429)$.

Line 1: $\text{solve}\left(\frac{d}{da}(s(a))=0, a\right)$ $a=1.05429$

Line 2: $s(1.05429)$ 2.3991

Let the area be $S(a)$:

$$S(a) = \int_0^{\frac{4}{3a}} (g(x) - x) dx$$

$$= \frac{256}{135} \sqrt{\frac{3}{a}} - \frac{8}{9a^2}$$

M1

$$S'(a) = 0 \Rightarrow a = 1.05 \dots$$

A1

$$S(1.05 \dots) = 2.40$$

A1

e. i.

TI-84 Plus calculator screen showing the definition of $g(x)$ and the inequality solve command.

Line 1: $g(x)$ $-a \cdot \sqrt{x} \cdot (a \cdot x - 4)$

Line 2: $\text{solve}\left(g\left(\frac{4}{3 \cdot a}\right) < \frac{4}{3 \cdot a}, a\right)$ $0 < a < 0.572357$

$$g\left(\frac{4}{3a}\right) < \frac{4}{3a}$$

M1

$$0 < a < 0.57$$

A1

ii.

$a := \frac{1}{2}$
 $\text{solve}(g(x)=x, x) \quad x=0 \text{ or } x=-8 \cdot (\sqrt{3}-2)$
 $\int_0^{-8 \cdot (\sqrt{3}-2)} (g(x)-x) dx + \int_{-8 \cdot (\sqrt{3}-2)}^{\frac{8}{3 \cdot a}} (x-g(x)) dx = 1.33927$

$$g(x) = x \Rightarrow x = 16 - 8\sqrt{3}$$

M1

$$\int_0^{16-8\sqrt{3}} (g(x)-x) dx + \int_{16-8\sqrt{3}}^{\frac{8}{3}} (x-g(x)) dx = 1.34$$

M1 A1

Note: Award the second M1 for one correct definite integral.

Question 5 (10 marks)

a.

$f(x) := 2 \cdot \sin(2 \cdot x) + \cos(x)$
 $f'(x) := \frac{d}{dx}(f(x))$
 $f'(x) \quad 4 \cdot \cos(2 \cdot x) - \sin(x)$

Hence, $f'(x) = 4\cos(2x) - \sin(x)$.

A1

b.

$x1 := 1 - \frac{f(1)}{f'(1)} \quad 1.94128$
 $x2 := x1 - \frac{f(x1)}{f'(x1)} \quad 1.50041$
 $\text{solve}(f(x)=0, x) | 0 < x < 2 \quad x = \frac{\pi}{2}$
 $\text{solve}(f(x)=0, x) | 0 < x < 2 \quad x = 1.5708$

$$x_1 = 1 - \frac{f(1)}{f'(1)} = 1.9412 \dots$$

M1

$$x_2 = 1 - \frac{f(x_1)}{f'(x_1)} = 1.5004$$

A1

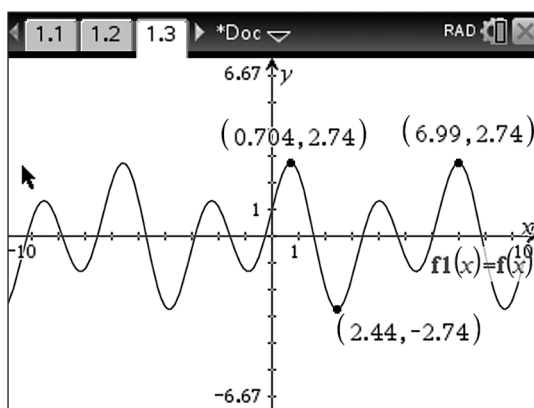
$$f(x) = 0 \Rightarrow \text{closest root is } x = \frac{\pi}{2} \approx 1.5708 > x_2$$

A1

- c. i. period = $LCM(2\pi, \pi) = 2\pi$

A1

ii.



$$-2.74 \leq y \leq 2.74$$

A1

- d. i. Observation and trial and error give:

$$a = -1$$

A1

$$b = -\frac{1}{2}$$

A1

ii. $f\left(\frac{x}{2} - \pi\right) = 2\sin\left(2\left(\frac{x}{2} - \pi\right)\right) + \cos\left(\frac{x}{2} - \pi\right)$

$$= 2\sin(x - 2\pi) + \cos\left(\pi - \frac{x}{2}\right)$$

M1

$$= 2\sin(x) - \cos\left(\frac{x}{2}\right)$$

$$= -\left(-2\sin(x) + \cos\left(\frac{x}{2}\right)\right)$$

$$= -\left(2\sin(-x) + \cos\left(-\frac{x}{2}\right)\right)$$

M1

$$= -f\left(-\frac{x}{2}\right)$$