

Section Two: Calculator-assumed**65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9**(8 marks)**

A random sample of 46 drivers was taken at a test centre. Each pair of letters shown below represents one driver. The first letter shows the driving test outcome (Pass, P or Fail, F) for the person and the second letter shows if they were taking the test for the first time (Yes, Y or N, No).

PN PY PN PY FY PY PY PY FN PN FY PY FN PY PN FN PY FY PN PY FN PY PY
PN FN PY PN FN FY PN PY PY PN PN FY PN PY PY PY PY PY FN PN PN FN FN

- (a) Two categorical variables have been recorded for each driver. Name one of the variables and explain why it is categorical. (2 marks)

- (b) Summarise the data by completing the two-way frequency table below. (2 marks)

	Yes	No
Pass		
Fail		

- (c) Convert the two-way frequency table to show column percentages (correct to the nearest percentage). (2 marks)

%	Yes	No
Pass		
Fail		

- (d) Discuss whether this sample data suggests the presence of an association between passing the driving test and taking the test for the first time. (2 marks)

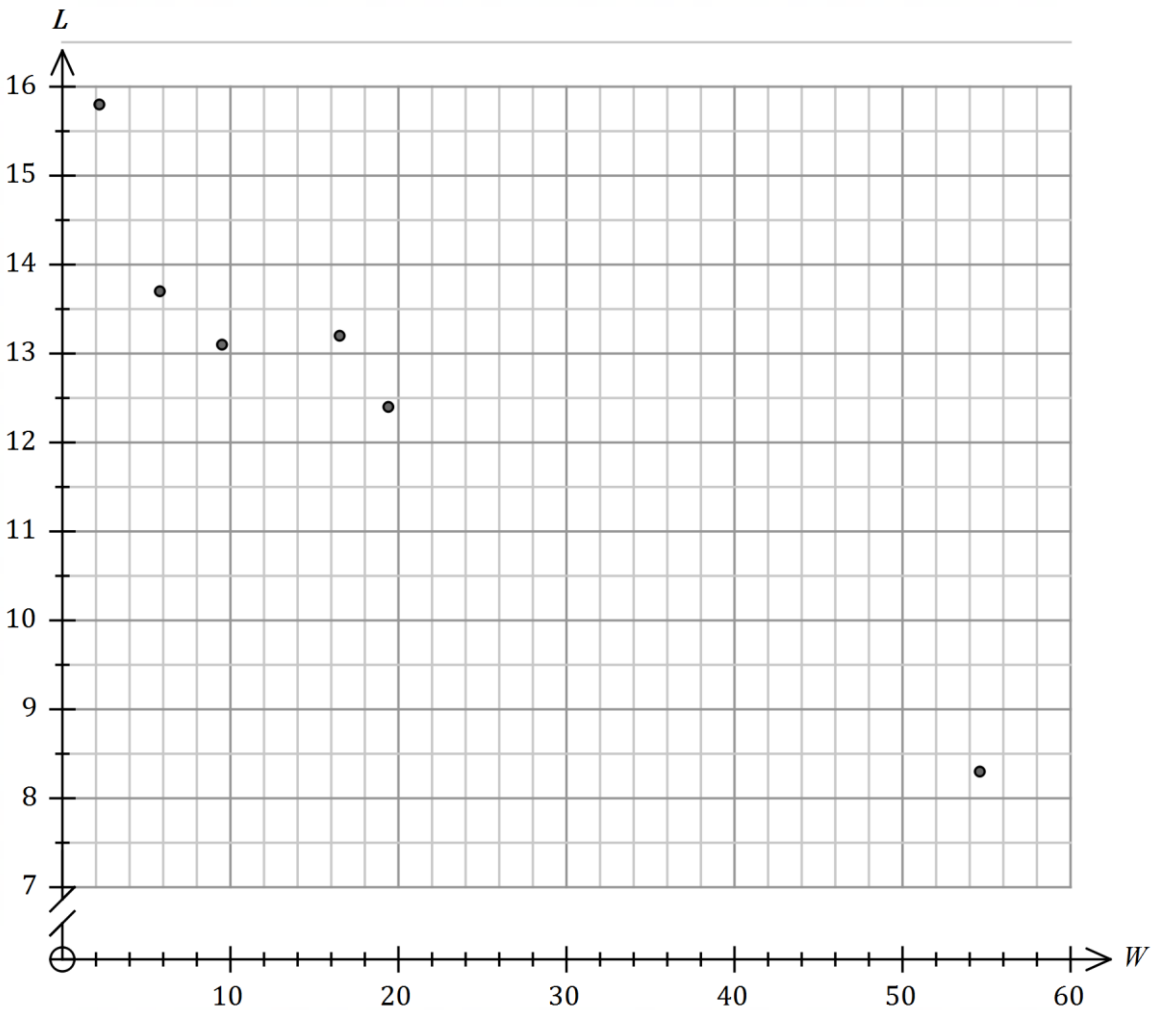
Question 10

(10 marks)

The table below shows the average lifespan L years and the average adult weight W kg of male dogs for a variety of breeds.

Breed	Weight W (kg)	Lifespan L (years)
English Setter	33.1	11.2
Jack Russell	5.8	13.7
Saint Bernard	43.5	8.9
Chihuahua	2.2	15.8
Beagle	9.5	13.1
Bullmastiff	54.6	8.3
Golden Retriever	31.7	11.6
Border Collie	16.5	13.2
German Shepherd	34.3	10.5
Spaniel	19.4	12.4

- (a) Complete the scatterplot below by plotting the four missing breeds bolded in the table above (English Setter, Saint Bernard, Golden Retriever and German Shepherd). (2 marks)



(b) Determine

(i) the correlation coefficient between W and L . (1 mark)

(ii) the equation of the least-squares line that can be used to predict L from W . (2 marks)

(c) Add the least-squares line to the scatterplot. (2 marks)

(d) A breed of dog has an average male weight of 52.5 kg.

(i) Predict the average lifespan of males of this breed. (1 mark)

(ii) Briefly discuss two factors that support the validity of your prediction. (2 marks)

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Question 11**(7 marks)**

The recursive rule $A_{n+1} = 1.0036A_n - 320$, $A_0 = 7\,500$ can be used to model the repayment of a loan, where A_n is the amount owing in dollars after n weekly repayments of \$320.

(a) Determine

(i) the initial amount of the loan. (1 mark)

(ii) the amount owing after 8 repayments to the nearest cent. (1 mark)

(iii) the minimum number of repayments required to reduce the amount owing to no more than \$1 500. (1 mark)

(b) After 8 repayments, changes to the financial circumstances of the borrower meant that the weekly repayment was reduced by \$140. Determine the change in the minimum number of repayments required to reduce the amount owing to no more than \$1 500. (4 marks)

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Question 12

(5 marks)

Six students were asked to create a short presentation to explain the meaning of some chemistry terms. The following table shows which terms each student offered to present.

Student	Terms
Qi	Acid, Cation
Ruby	Dipole, Enzyme
Sid	Base
Tess	Base, Fusion
Vic	Dipole, Fusion
Will	Base, Cation

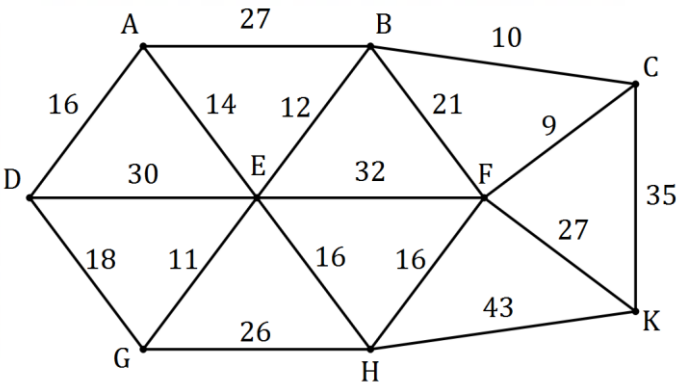
- (a) Draw a bipartite graph to represent this information. (3 marks)

- (b) Determine how many more edges must be added to the bipartite graph in (a) so that it would be a complete bipartite graph. (2 marks)

Question 13

(7 marks)

In the graph below, the vertices represent towns and the weights on each edge represent the distance, in kilometres, between pairs of towns. A parcel delivery service is based at town D.



- (a) Complete the table below to show the shortest distance d km from town D to each of the other towns. (4 marks)

Town	A	B	C	E	F	G	H	K
d , km								

- (b) State the route that gives the minimum distance between towns D and K. (1 mark)

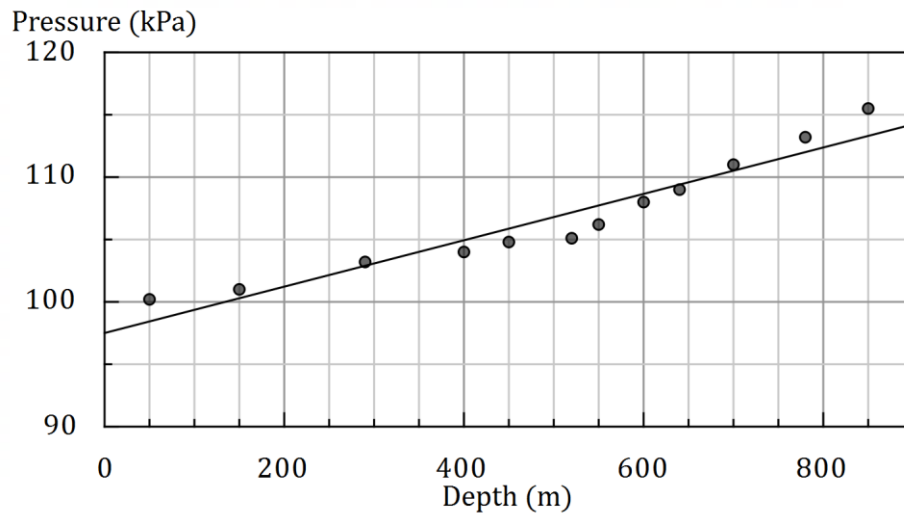
- (c) One day the delivery service has two parcels to deliver, one at A and the other at K. Determine the shortest path from D to K that passes through A and state the length of this path. (2 marks)

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Question 14

(7 marks)

The graph below shows pressure and depth readings collected from a variety of mines in a country, together with the least-squares line for the linear association between the variables.



- (a) The correlation coefficient r for the linear association is one of the values shown in the list below. Circle this value and justify your choice. (3 marks)

$\{-0.96, -0.83, -0.41, -0.09, 0.41, 0.83, 0.96\}$

- (b) Determine the coefficient of determination for the linear association and interpret its value. (2 marks)

- (c) State, with reasons, whether the nature of the relationship between the variables is linear or non-linear. (2 marks)

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Question 15

(7 marks)

The scores of a sample of students who sat two tests are shown in the table below.

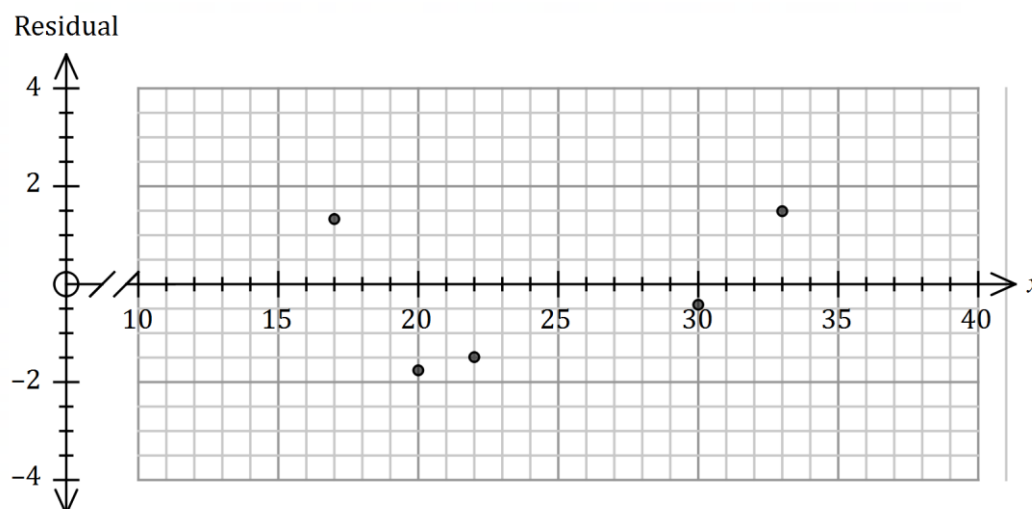
Student	1	2	3	4	5	6	7	8
Test A	33	22	30	24	17	20	32	31
Test B	14	18	14	21	24	19	15	10
Residual	1.49	-1.49	-0.42	p	1.33	-1.76	1.85	-3.78

Two students missed Test B and their teacher planned to predict their marks for this test using their scores from Test A and the linear relationship modelled by the least-squares line between the response (y) and explanatory (x) variables.

The equation is $y = -0.635x + 33.46$ and the correlation coefficient is -0.867 . This equation was used to determine the residuals shown in the table above.

- (a) Determine the value of p in the table above. (2 marks)

- (b) Complete the residual plot for the data on the axes below by plotting student 4, student 7 and student 8. (2 marks)



- (c) Using the residual plot and other relevant factors, comment on the teacher's plan. (3 marks)

Question 16

(8 marks)

A photocopier was purchased for \$5 750. Its value depreciates at a rate of 6.5 cents per copy. Let V_n be the value of the photocopier in dollars after n copies have been made, where $V_n = a + bn$.

(a) State the value of the constant a and the value of the constant b . (2 marks)

(b) Determine V_{2000} . (1 mark)

(c) Determine n when $V_n = 4671$. (2 marks)

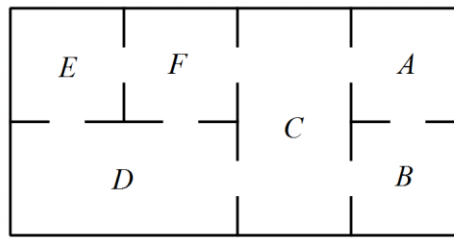
(d) Calculate the value of the photocopier after 65 000 copies have been made. (1 mark)

(e) The photocopier will be replaced as soon as its value falls below \$300. Determine the number of copies the photocopier will make before it is replaced. (2 marks)

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Question 17**(8 marks)**

A warehouse has dividing walls that split its interior into six areas, as shown in the plan below. The gaps in the dividing walls are doorways that allow people to move from one area to another.



- (a) Construct a graph to represent the warehouse areas and doorways, with each area being a vertex and each doorway an edge. (2 marks)
- (b) An inspector started in one area and followed a route that went through all doorways exactly once before stopping in another area. State where their route started and stopped and explain how the Eulerian properties of the graph in (a) help to identify these locations. (3 marks)
- (c) Another inspector wishes to start in an area, follow a route that visits all the other areas exactly once and end up back where they started. Comment on whether this is possible, referring to the Hamiltonian properties of the graph in (a) to justify your response. (3 marks)

Question 18

(9 marks)

The value T_n , in dollars, of a property n years after it was bought can be represented by the rule $T_{n+1} = 1.079T_n$, $T_0 = 450\,000$.

- (a) State the value of the property when it was bought and the annual percentage increase in its value. (2 marks)
- (b) Determine the value of the property after 2 years. (1 mark)
- (c) Determine how many years it will take for the value of the property to at least double. (2 marks)
- (d) If the annual percentage increase in value of the property changed to 4.4% after 3 years, determine the value (to the nearest \$100) of the property 8 years after it was bought. (4 marks)

Question 19**(7 marks)**

The average mid-year commuting times for full-time workers in Melbourne (m minutes) and in Perth (p minutes) between the years 2004 ($t = 4$) and 2011 ($t = 11$) are shown in the table below.

Year, t	4	5	6	7	8	9	10	11
Melbourne, m	29.5	29.9	31.2	32.3	32.8	32.1	33.4	33.5
Perth, p	24.2	25.8	27.2	26.1	27.7	27.1	30.2	29.5

The least-squares line to model the linear relationship between t and p is $p = 0.719t + 21.8$ and $r_{tp} = 0.902$.

- (a) Determine the least-squares line to model the linear relationship between t and m and state the correlation coefficient for this association. (2 marks)
- (b) Predict the average commuting times in Melbourne and Perth in the year 2022 and state, with justification, which prediction you are most confident in. (3 marks)
- (c) Predict the year in which the average commuting time will be the same in both cities and comment on how confident you are of this prediction. (2 marks)

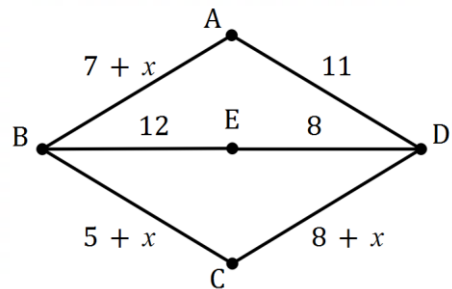
Question 20

(7 marks)

The edges in the graph (not to scale) represent roads and the weight on each edge is the time, in minutes, that it takes to drive along that road. The times to drive along AB , BC and DC vary throughout the day.

The variable x can only take whole number values.

An inspector wishes to drive along each road at least once, starting and finishing at B , in the minimum possible time.



- (a) Briefly explain why the edges on a path between B and D will have to be repeated. (1 mark)
- (b) List all possible paths between B and D , and state how long each would take, in terms of x where appropriate. (2 marks)
- (c) Determine all possible values of x so that BC would be one of the repeated edges. (2 marks)
- (d) For the case when $x = 6$, determine the time required for the inspectors' drive. (2 marks)

Question 21**(8 marks)**

A nail is hammered into a piece of wood. The distances moved by the tip of the nail during the first, second and third hits are 20, 14 and 9.8 mm respectively.

- (a) Show that the distances can be modelled by a geometric sequence. (2 marks)
- (b) Write a rule for the distance moved by the tip of the nail D_n during the n^{th} hit of the hammer in the form $D_n = a(r)^{n-1}$. (1 mark)
- (c) Determine which hit first moves the tip of the nail less than 2 mm, and state the distance moved during this hit, rounded to one decimal place. (2 marks)
- (d) The piece of wood is 65 mm thick. State, with justification, whether the tip of the nail will pass all the way through the piece of timber, stating any assumptions that you make. (3 marks)

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