

## **Trial Examination 2021**

# VCE Specialist Mathematics Units 3&4

# Written Examination 1

## **Question and Answer Booklet**

Reading time: 15 minutes Writing time: 1 hour

Student's Name:		
Teacher's Name:		

#### Structure of booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

#### **Materials supplied**

Question and answer booklet of 12 pages

Formula sheet

Working space is provided throughout the booklet.

#### **Instructions**

Write your name and your teacher's name in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

## At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2021 VCE Specialist Mathematics Units 3&4 Written Examination 1.

## **Instructions**

Answer all questions in the spaces provided.

Unless otherwise specified, an exact answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms<sup>-2</sup>, where g = 9.8.

## **Question 1** (4 marks)

Relative to a fixed origin O, the point A has position vector  $\mathbf{a} = \mathbf{i} + \mathbf{j}$  and the point B has position vector  $\mathbf{b} = -5\mathbf{k}$ . The point C has position vector  $\mathbf{c}$  where  $\mathbf{c} = \mathbf{a} + \mathbf{b}$ .

1 ma
3 ma

<b>Question 2</b> (3 marks) Find the set of real values of x for which $ x + 2  > 2 x - 2 $ .		
That the set of real values of x for which $ x  + 2 x  = 2 x $ .		

<b>Question 3</b> (3 marks) A particle of mass 3 kg moves under a force $\mathbf{F}$ newtons so that its position vector $\mathbf{r}$ at time $t$ seconds is given by $\mathbf{r} = \cos(\pi t)\mathbf{i} + \sin(2\pi t)\mathbf{j}$ , $t \ge 0$ . Find the particle's change in momentum from $t = \frac{1}{2}$ to $t = 1$ .

Question 4 (4 marks)
Consider two independent random variables $X$ and $Y$ . The mean of $X$ and the mean of $Y$ are both $-2$ . The variance of $X$ is 1 and the variance of $Y$ is 2.
If the mean and the variance of $aX + bY$ are 2 and 9 respectively, find the value of $a$ and of $b$ , where $a, b \in Z$ .

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A curve C has the equation  $ye^{-x} = x + y^2$ .

**a.** Find  $\frac{dy}{dx}$  in terms of x and y.

2 marks

**b.** Point P lies on curve C and has coordinates (0, 1).

Find the equation of the normal to curve C at point P.

2 marks

**Question 6** (4 marks)

a. Show that the equation  $\csc(x) + \cot(x) = 2\sin(x)$  can be expressed as  $2\cos^2(x) + \cos(x) - 1 = 0$ .

**b.** Hence, solve the equation  $\csc(x) + \cot(x) = 2\sin(x)$  for  $0 < x < \pi$ .

2 marks

**Question 7** (4 marks)

**a.** Express  $\frac{3}{(x+1)(x-2)}$  in partial fractions.

2 marks

**b.** Hence, by solving the differential equation  $\frac{dx}{dt} = t^2(x+1)(x-2)$ , where x > 2,

2 marks

show that  $\frac{x-2}{x+1} = ke^{t^3}$ , where k is a constant.

Question 8 (3 marks)
A curve C has equation $y = \frac{1}{6} \left( e^{3x} + e^{-3x} \right), 0 \le x \le \log_e(k)$ , where $k > 1$ .
Show that the length of curve <i>C</i> is $\frac{1}{6} \left( k^3 - \frac{1}{k^3} \right)$ .

Question 9 (5 marks)	
The functions $f$ and $g$ are defined by	
	$f(x) = \arctan\left(\frac{x+1}{1-x}\right)$
	$g(x) = \arctan(x)$

W

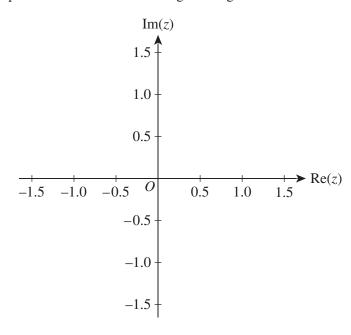
	$e x < 1.$ Show that $G(u) = \sigma(u)$		2
By using an appropriate trigonometric identity, verify the result obtained in part a. 2 m	Show that $f'(x) = g'(x)$ .		3 r
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## **Question 10** (6 marks)

The roots of the equation  $z^3 - 1 = 0$ , where  $z \in C$ , are denoted by 1, w and  $w^2$ .

**a.** Plot the points that represent these roots on the Argand diagram below.

1 mark



**b.** Show that  $1 + w + w^2 = 0$ .

2 marks

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Find a cubic equation with integer coefficients that has roots 1, $1+w$ and $1+w^2$ .	3 ma

END OF QUESTION AND ANSWER BOOKLET