



Semester Two Examination, 2017

Question/Answer booklet

**MATHEMATICS
APPLICATIONS
UNITS 1 AND 2**

**Section One:
Calculator-free**

SOLUTIONS

Student Number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free

35% (52 Marks)

This section has **six (6)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(7 marks)

A vehicle accelerates along a road. The distance d of the vehicle from its starting point at time t is given by the formula $d = \frac{1}{2}at^2$, where a is the acceleration.

(a) Calculate d when

(i) $a = 2.5$ and $t = 4$.

(1 mark)

Solution
$d = \frac{1}{2} \times 2.5 \times 4 \times 4 = 20$
Specific behaviours
✓ calculates value

(ii) $a = 16$ and $t = \frac{1}{2}$.

(1 mark)

Solution
$d = \frac{1}{2} \times 16 \times \frac{1}{2} \times \frac{1}{2} = 2$
Specific behaviours
✓ calculates value

(b) Calculate the distance travelled by the body between $t = 1$ and $t = 3$ when $a = 4$.

(3 marks)

Solution
$d_1 = \frac{1}{2} \times 4 \times 1 \times 1 = 2$
$d_2 = \frac{1}{2} \times 4 \times 3 \times 3 = 18$
$d = 18 - 2 = 16$
Specific behaviours
✓ calculates d_1
✓ calculates d_2
✓ calculates difference

(c) Determine a when $d = 250$ and $t = 10$.

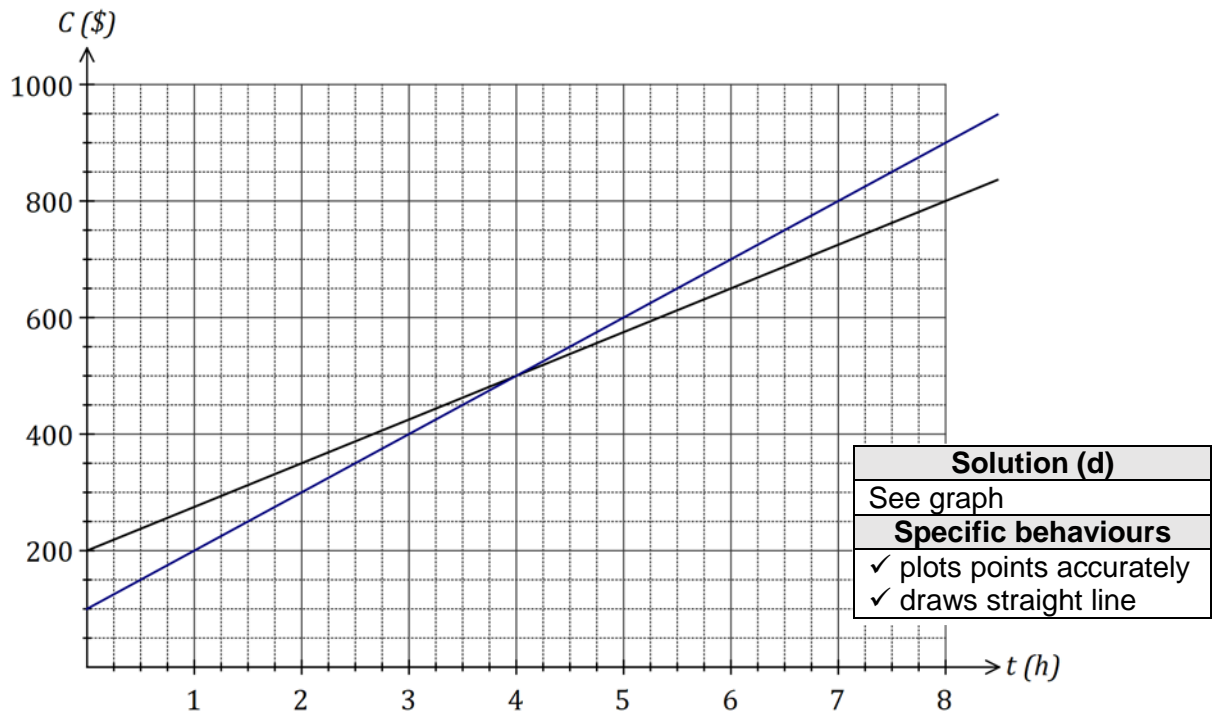
(2 marks)

Solution
$250 = \frac{1}{2} \times a \times 10 \times 10$
$250 = 50a$
$a = 5$
Specific behaviours
✓ substitutes and simplifies
✓ states value of a

Question 2

(10 marks)

The cost of using tree contractor *A* to remove fallen trees is shown below, where the cost is the sum of a fixed callout fee and a variable cost, depending on how many hours the job takes.



- (a) State the cost of using tree contractor *A* for a 3-hour job.

(1 mark)

Solution
$C = \$425$
Specific behaviours
✓ states exact cost

- (b) For tree contractor *A*, determine

- (i) the fixed call out fee.

(1 mark)

Solution
Fee = \$200
Specific behaviours
✓ states fee

- (ii) the hourly rate charged.

(1 mark)

Solution
Rate = \$75 per hour
Specific behaviours
✓ states rate

- (c) State an equation that can be used to calculate the cost, C , of using contractor *A* for a job lasting t hours.

(2 marks)

Solution
$C = 75t + 200$
Specific behaviours
✓ uses coefficients from (b)
✓ correct formula

- (d) Tree contractor *B* charges \$300 for a 2-hour job and \$700 for a 6-hour job. Given that the relationship between the job time and cost is linear, use these two points to add a line representing the cost of using tree contractor *B* to the graph on the previous page. (2 marks)

- (e) State the call-out fee for contractor *B*. (1 mark)

Solution
Fee = \$100
Specific behaviours
✓ states fee

- (f) For what length of job

- (i) do both contractors charge the same amount? (1 mark)

Solution
4 hours
Specific behaviours
✓ states time

- (ii) is contractor *A* \$50 cheaper than contractor *B*? (1 mark)

Solution
6 hours
Specific behaviours
✓ states time

Question 3

(8 marks)

The cost per metre of a fabric, usually \$3.50, is reduced when larger quantities are purchased, as shown in this table.

Length of fabric purchased (x m)	$0 \leq x < 5$	$5 \leq x < 10$	$10 \leq x < 25$	$x \geq 25$
Cost per metre (C \$)	3.50	3.00	2.50	2.00

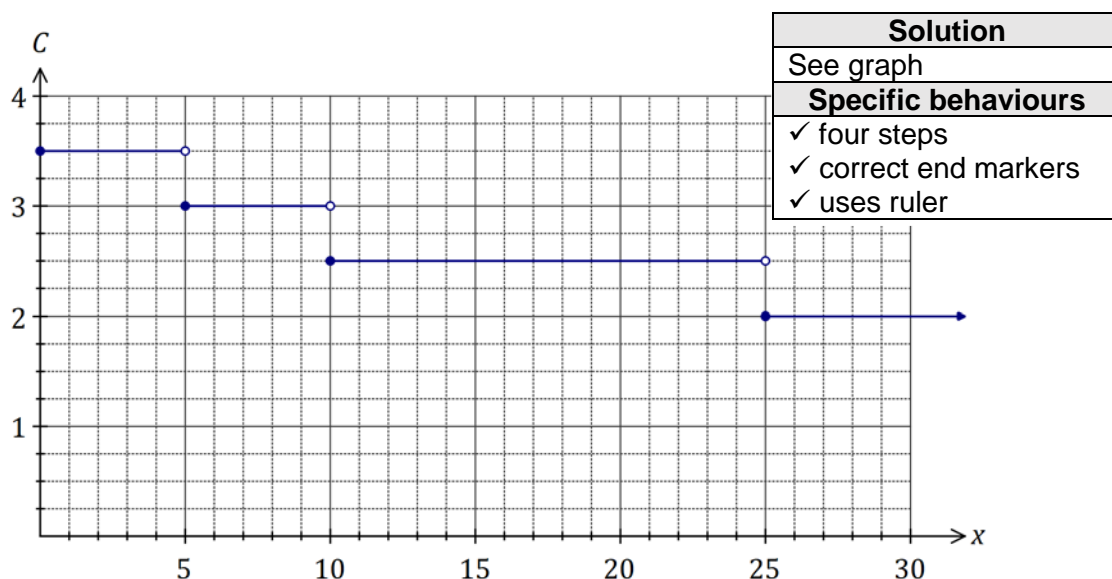
- (a) Calculate the cost of buying 20 m of the fabric.

(2 marks)

Solution
$C = 2.50$ per m $C = 2.50 \times 20 = \$50$
Specific behaviours
<ul style="list-style-type: none"> ✓ uses correct cost per metre ✓ states cost

- (b) Sketch a piece-wise graph of cost per metre against length purchased on the axes below.

(3 marks)



- (c) A manufacturer has an order for 2 shirts per week that use this fabric, with each shirt requiring 1.5 metres of the material. Calculate the total saving made over a period of ten weeks if the manufacturer buys all the material at once, instead of weekly.

(3 marks)

Solution
<p>Weekly purchase: $2 \times 1.5 = 3$ m. $3 \times 3.5 = 10.50$ per week, so total of \$105.</p> <p>Bulk buy: $2 \times 1.5 \times 10 = 30$ m. Total = $30 \times 2 = \\$60$</p> <p>Saving: \$45</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ calculates weekly cost ✓ calculates bulk buy cost ✓ states saving

Question 4

(8 marks)

Six matrices are given by

$$P = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}, \quad Q = \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}, \quad R = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad S = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad T = \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \quad U = \begin{bmatrix} 8 & 7 \\ 5 & 9 \end{bmatrix}.$$

(a) Calculate, where possible, the following. If not possible, give a reason why.

(i) $U - Q$.

(1 mark)

Solution
$\begin{bmatrix} 8 & 7 \\ 5 & 9 \end{bmatrix} - \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 2 & 7 \end{bmatrix}$
Specific behaviours
✓ states difference

(ii) SP .

(1 mark)

Solution
$\begin{bmatrix} 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \end{bmatrix}$
Specific behaviours
✓ states product

(iii) $2T - 3R$.

(2 marks)

Solution
$2 \begin{bmatrix} 2 \\ -3 \end{bmatrix} - 3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \end{bmatrix}$
Specific behaviours
✓ calculates multiples
✓ states difference

(iv) QP .

(2 marks)

Solution
$\begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 11 \\ 3 & 4 \end{bmatrix}$
Specific behaviours
✓ at least 2 elements correct
✓ states product

(b) If V is the square of one of the above matrices, and $v_{11} = 22$, determine v_{22} .

(2 marks)

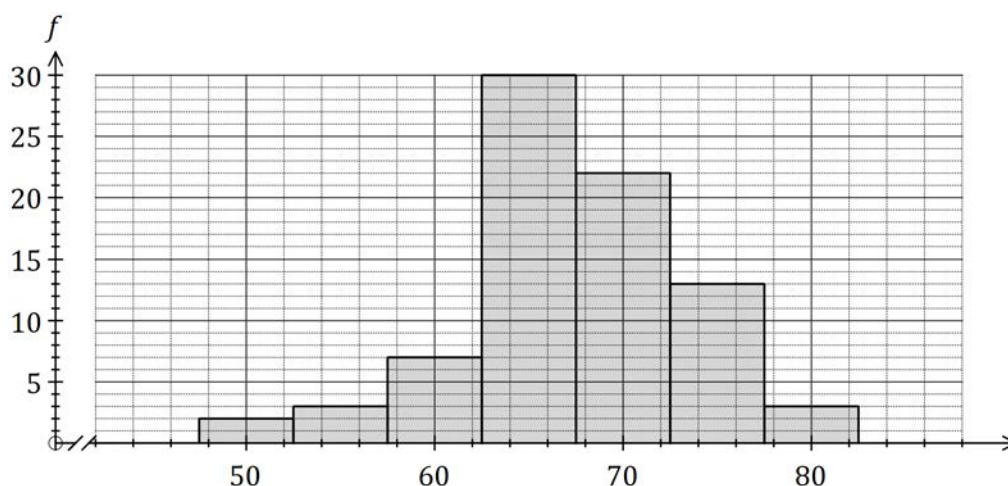
Solution
$V = Q^2 = \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 22 & \dots \\ \dots & 1 \end{bmatrix}$
$v_{22} = 1$
Specific behaviours
✓ identifies matrix
✓ states element

Question 5

(12 marks)

The marks of 80 students in an exam are listed below in ascending order and summarised in the histogram.

51, 52, 55, 56, 57, 58, 58, 59, 60, 61, 61, 61, 63, 63, 63, 64, 64, 64, 64, 64,
 64, 65, 65, 65, 65, 65, 65, 66, 66, 66, 66, 67, 67, 67, 67, 67, 67, 67, 67, 67,
 67, 67, 68, 68, 68, 68, 69, 69, 69, 69, 69, 70, 70, 70, 70, 71, 71, 71, 72, 72,
 72, 72, 72, 72, 73, 73, 73, 73, 74, 74, 74, 75, 75, 76, 76, 76, 77, 78, 78, 80.



- (a) Describe the shape of the distribution of marks in terms of modality and shape. (2 marks)

Solution
The scores are unimodal, with just one peak, and most of them are bunched around the mid-sixties. The spread of scores is not very symmetrical, and a tail to the left indicates negative skew.
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates unimodal ✓ indicates negative skew

- (b) State the

- (i) median mark.

- (ii) modal mark.

- (iii) range of marks.

- (iv) inter-quartile range of marks.

Solution
Median: 67
Mode: 67
Range: 29
IQR: $72 - 64 = 8$
Specific behaviours
<ul style="list-style-type: none"> ✓ median ✓ mode ✓ range ✓ identifies Q_1 and Q_3 ✓ IQR

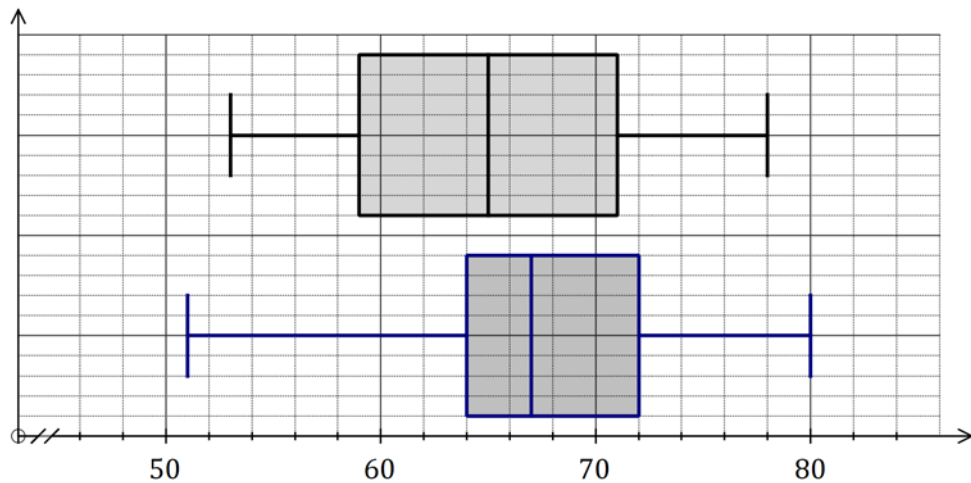
(1 mark)

(1 mark)

(1 mark)

(2 marks)

The marks of another group of students, group B, who sat the same exam are shown in the box plot below.



- (c) Add a box plot on the axes above for the data given on the previous page for group A.

(3 marks)

Solution
See graph
Specific behaviours
✓ indicates neat box and whiskers
✓ correct min, max and median
✓ correct Q_1 and Q_3

- (d) Use information from the box plots to compare the performance of group A and group B in the exam.

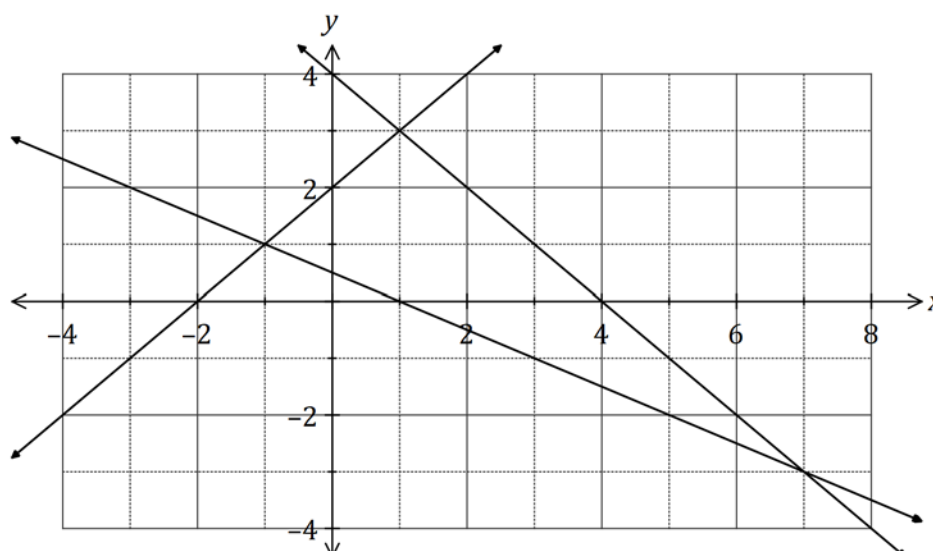
(2 marks)

Solution
Group A performed better, as their median was higher than that of group B.
Group A had a smaller IQR, indicating that their marks were less spread and more consistent than those of group B.
Specific behaviours
✓ statement using median
✓ statement using IQR

Question 6

(7 marks)

(a) The graphs of three straight lines are shown below.



Using the graph, or otherwise, solve the simultaneous equations

(i) $y = 4 - x$ and $y = 2 + x$.

Solution
$x = 1, \quad y = 3$
Specific behaviours
✓ x value
✓ y value

(2 marks)

(ii) $x + 2y = 1$ and $x + y = 4$.

Solution
$x = 7, \quad y = -3$
Specific behaviours
✓ x value
✓ y value

(2 marks)

(b) Solve the simultaneous equations $2a - b = 13$ and $3a + b = 12$.

(3 marks)

Solution
$2a - b = 13$ $3a + b = 12$ $5a = 25$ $a = 5$ $2(5) - b = 13$ $b = -3$
Specific behaviours
✓ sum of equations ✓ states value of a ✓ states value of b

Additional working space

Question number: _____

