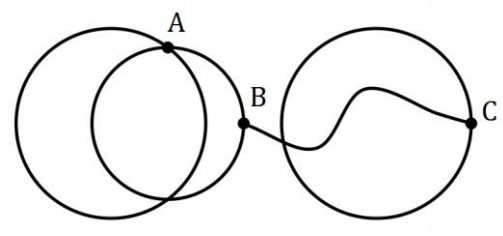
Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

Consider the following graph G.



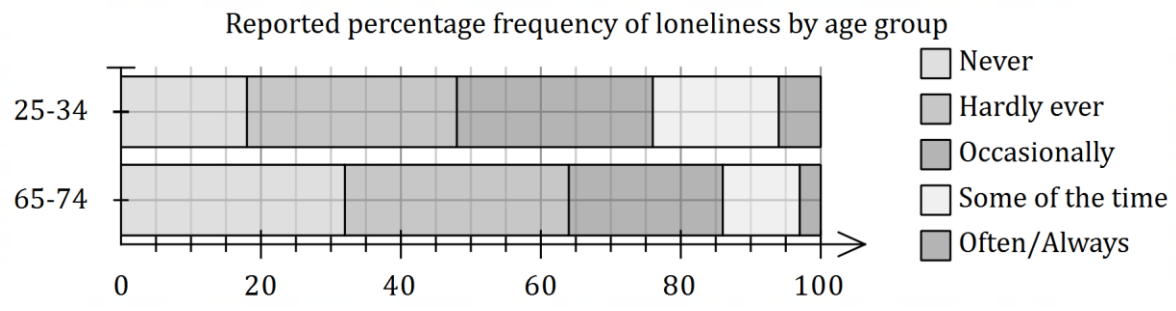
(a) Draw G in the plane, to clearly show that it is planar. (2 marks)

(b) State the degree of each vertex and the degree sum for G. (2 marks)

(c) Explain how to recognise a bridge in a connected graph and state, with justification, whether G contains a bridge. (2 marks)

Question 2 **(7 marks)**

A recent survey asked people their age and the question "How often do you feel lonely?". The responses for the age groups 25-34 years and 65-74 years are shown below. The categories in the key are shown from left to right in the stacked percentage frequency graph.



(a) State the largest category for the 25-34 age group. (1 mark)

(b) State the percentage of those

(i) in the 65-74 age group who responded with 'Often/Always'. (1 mark)

(ii) in the 25-34 age group who responded with 'Occasionally' or 'Some of the time'. (1 mark)

(c) Use the graph to explain why the data suggests an association exists between feeling lonely and age. (2 marks)

(d) Briefly discuss non-causal explanations for the observed association. (2 marks)

Question 3 **(6 marks)**

(a) Connected planar graph G_1 has 3 vertices and 4 edges.

(i) Use Euler's formula to determine the number of faces in G_1. (2 marks)

(ii) Sketch a possible graph G_1. (2 marks)

(b) Graph G_2 has 3 vertices and is Eulerian. The length of the Euler cycle is 5. Sketch a possible graph G_2. (2 marks)

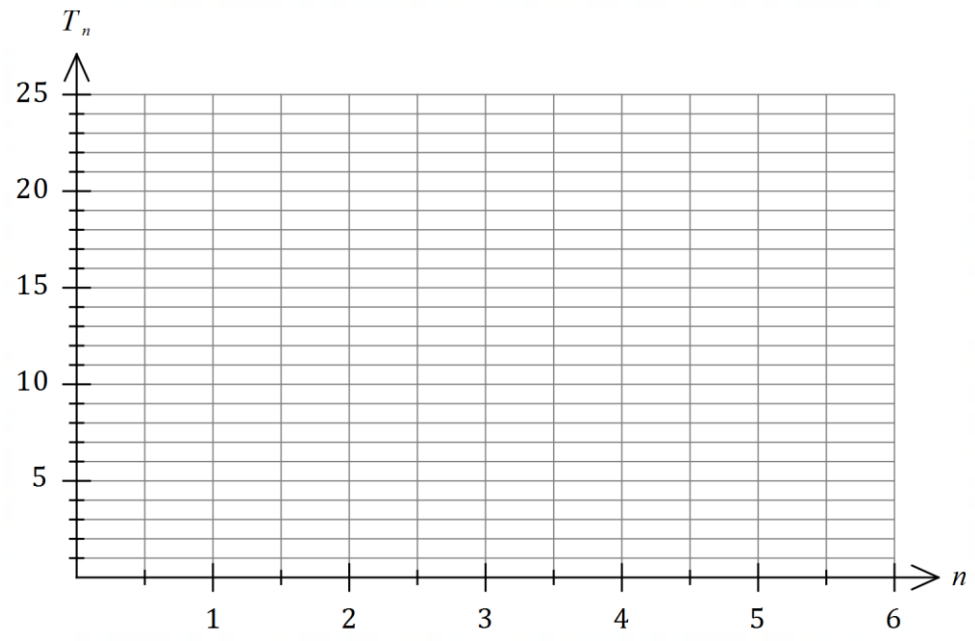
Question 4 **(6 marks)**

A recursive rule for a sequence is $T_{n+1} = T_n - 3.5,\ T_3 = 13.5$.

(a) Briefly explain which feature of the recursive rule indicates that the sequence is arithmetic. (1 mark)

(b) Determine T_5 and T_1. (2 marks)

(c) Graph the first six terms of the sequence on the axes below. (2 marks)



(d) What feature of the graph indicates that the sequence is arithmetic? (1 mark)

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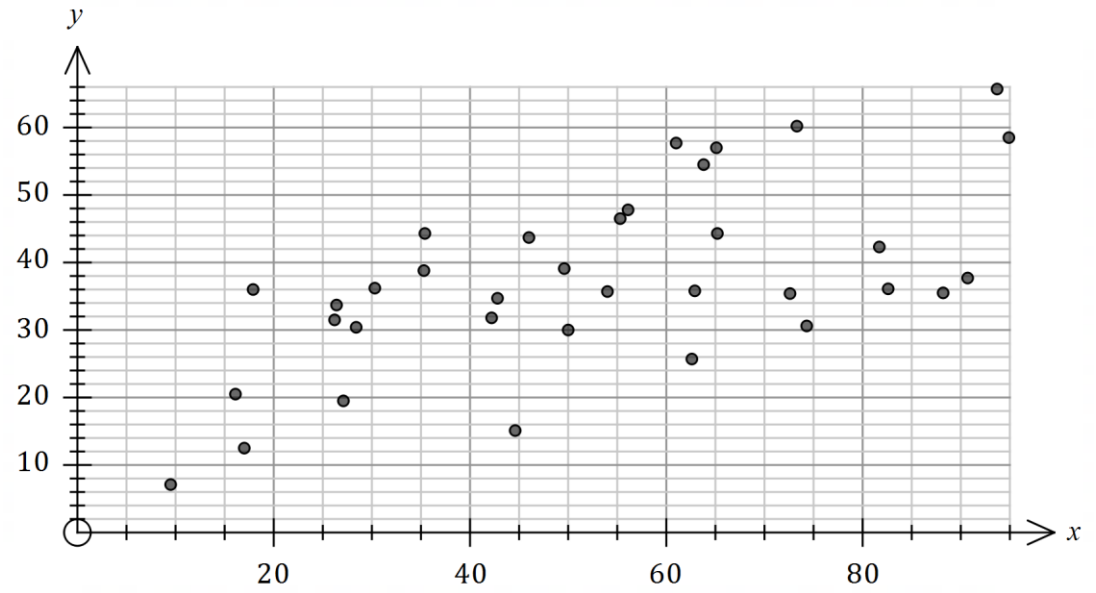
Question 5 **(7 marks)**

The scatterplot below shows the number of personal computers (PC's) per 100 people on the x-axis and the growth domestic product per employee (GDP/employee), in thousands of dollars, on the y-axis for a selection of countries in 2017.



(a) Describe the strength and direction of the association between the variables. (2 marks)

(b) The equation of the least-squares line for the data is $y = 19.1 + 0.35x$. Interpret the intercept and the slope of this line. (3 marks)

(c) A newspaper article used the graph to claim that increasing the number of PC's per person in a country caused the GDP/employee to rise. Comment on this claim. (2 marks)

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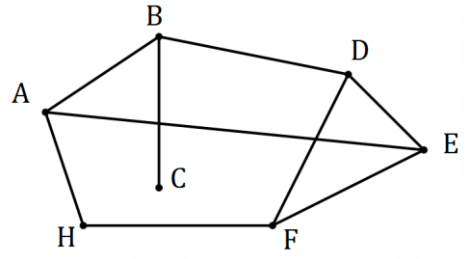
Question 6 **(6 marks)**



Graph G is shown. G_1 and G_2 are subgraphs of G, so that each subgraph has 7 vertices but one less edge than G.

(a) G_1 does not satisfy Euler's formula. State which edge must be removed from G, and show that G_1 does not satisfy Euler's formula. (3 marks)

(b) G_2 is bipartite. State which edge must be removed from G, and draw G_2 to clearly show the partite sets. (3 marks)

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Question 7 **(7 marks)**

In a set of 5 pages on a website, there are hyperlinks from page 1 to page 3, from page 2 to pages 4 and 5, from page 3 to page 2, from page 4 to page 1, and from page 5 to page 4.

(a) Construct digraph D to show the above information, where pages are represented by vertices and hyperlinks by directed edges. (2 marks)

(b) Construct an adjacency matrix for D. (2 marks)

(c) List, starting at page 3 and in the order visited, vertices in D that form a

(i) walk of length 2. (1 mark)

(ii) cycle of length 4. (1 mark)

(iii) trail of length 5. (1 mark)

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Question 8 **(7 marks)**

Every day, 10% of the water in a tank is drained for crop irrigation and then the tank is topped up with 40 kL of water. The tank has a maximum capacity of 350 kL.

At the start of Day 1, before water is drained for the crops, the tank contains 50 kL.

(a) Determine the amount of water in the tank at the start of Day 2. (2 marks)

(b) Determine a recursive rule for the amount of water, A_n, in the tank at the start of Day n. (2 marks)

(c) Explain why the tank will never empty. (1 mark)

(d) State, with justification, whether the tank will overflow. (2 marks

End of questions