SHENTON COLLEGE

Examination Semester One 2019
Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 3

Section Two
(Calculator-assumed)

Your name ______________________________

Time allowed for this section

Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this section

To be provided by the supervisor
Question/answer booklet for Section Two.
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this examination

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Student Score
Section One: Calculator-free	8	8	50	52	
Section Two: Calculator-assumed	13	13	100	98	
			Total	150	

Section Two: Calculator-assumed **(98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(7 marks)**

(a) Determine the values of the real constant a and the real constant b given that $z-4+2i$ is a factor of z^3+az+b.

(4 marks)

(b) Clearly show that $2+i$ is a root of the equation $z^3-7z^2+17z-15=0$. (2 marks)

(c) State all three solutions of $z^3-7z^2+17z-15=0$. (1 mark)

Question 10 **(10 marks)**

(a) Consider the system of simultaneous equations… (6 marks)

$$\begin{gathered} 2x - 4y + 2z = 8 \\ -x + 5y + z = -9 \\ x + y + (p^2 + 2)z = p \end{gathered}$$

(i) Express the system as an augmented matrix and use row reduction techniques so that the coefficients of x and y in the third equation above are both zero

Hence determine value/s for *p* so that the system has:

(ii) No solutions

(iii) An infinite number of solutions

(iv) A unique solution

(b) Determine whether the following system of equations has a unique solution, an infinite number of solutions, or no solutions. Provide a brief geometric interpretation of your findings. (3 marks)

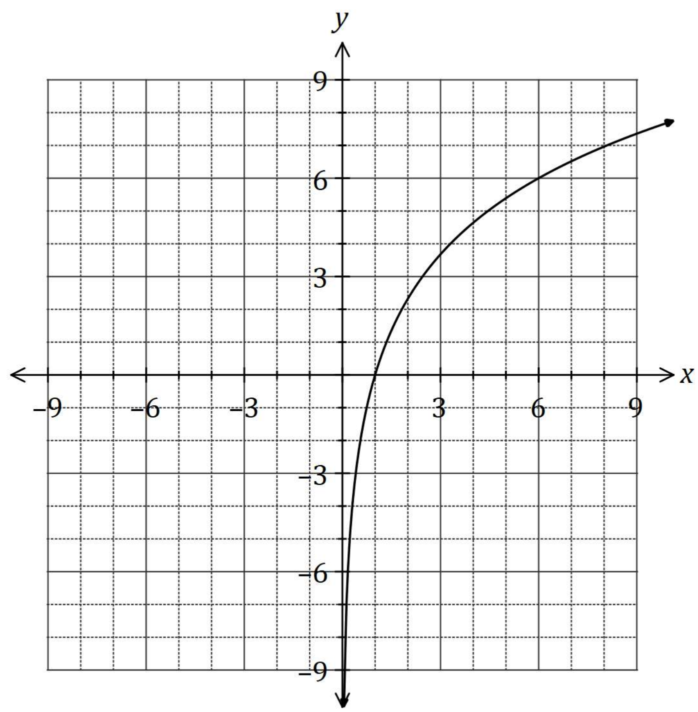
$$\begin{aligned} x + 2y + z &= 3 \\ 2x + 4y + 2z &= 7 \\ 2x + y + 2z &= 4 \end{aligned}$$

(c) Describe the geometric interpretation of a system that has an infinite number of solutions. (1 marks)

Question 11 **(6 marks)**

(a) Explain why the function $f(x) = \cos x$, where $x \in \mathbb{R}$, is not one-to-one. (1 mark)

(b) The graph of $y = g(x)$ is shown below. Sketch the graph of $y = g^{-1}(x)$ on the same axes. (2 marks)



(c) The inverse function of h is defined as $h^{-1}(x) = x^2 - 8x + 17$ for $x \leq 4$. Determine the defining rule for $h(x)$ and state its domain. (3 marks)

Question 12 **(6 marks)**

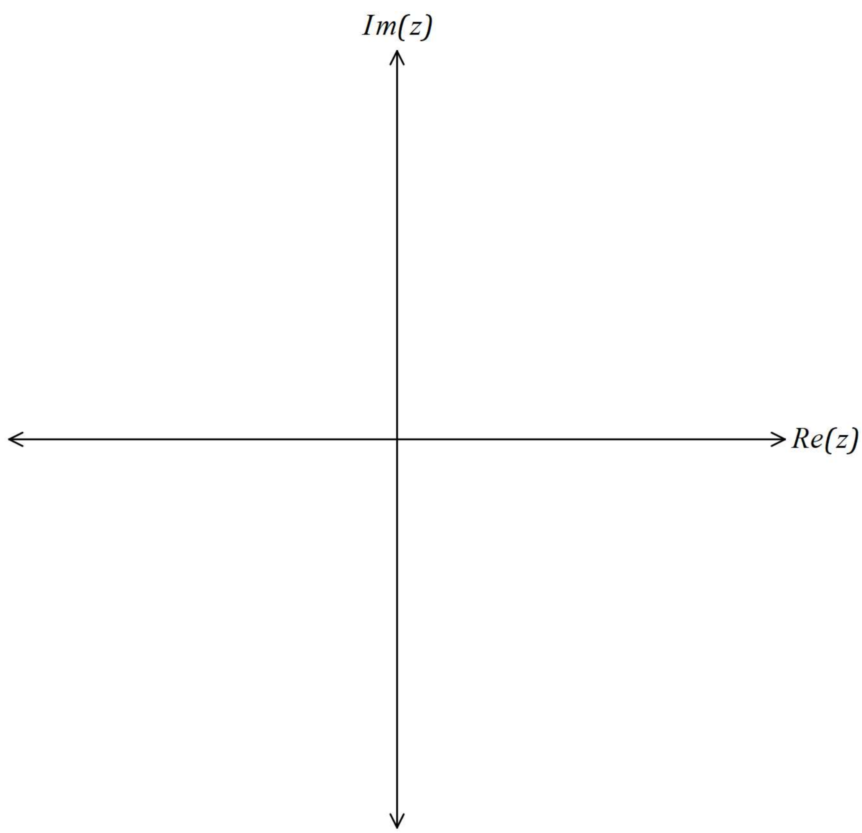
The line **L** passes through the points **a** and **b,** given by the position vectors $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ respectively. A third point **p** is located at $\mathbf{i} + \mathbf{j} + \mathbf{k}$**.** Determine the position of the point on **L** that is closest to **p** and calculate this minimum distance. (6 marks)

Question 13 **(9 marks)**

Let $w = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$.

(a) Express w, w^2, w^3 and w^4 in the form $r \operatorname{cis} \theta, -\pi < \theta \leq \pi$. (2 marks)

(b) Sketch w, w^2, w^3 and w^4 on the Argand diagram below. (2 marks)



(c) Describe the transformation in the complex plane of any point z when it is multiplied by w. (2 marks)

(d) Simplify

(i) $w^0 + w^1 + w^2$. (1 mark)

(ii) $w^0 + w^1 + w^2 + \cdots + w^{2018} + w^{2019}$. (2 marks)

Question 14 **(7 marks)**

(a) Solve the equation $z^5 + 32 = 0$, writing your solutions in polar form $r \operatorname{cis} \theta$. (4 marks)

(b) Use your answers from (a) to show that $\cos\left(\frac{\pi}{5}\right) + \cos\left(\frac{3\pi}{5}\right) = \frac{1}{2}$. (3 marks)

Question 15 **(8 marks)**

The position vectors of two particles at time t are given below, where a is a constant.

$$\mathbf{r}_\mathrm{A} = 8\mathbf{i} - 5\mathbf{j} - \mathbf{k} + t(\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \text{ and } \mathbf{r}_\mathrm{B} = 3\mathbf{i} + a\mathbf{j} + \mathbf{k} + t(\mathbf{3i} - \mathbf{j} - 2\mathbf{k})$$

The paths of the particles cross at P but the particles do not meet.

(a) Determine the value of the constant a and the position vector of P. (5 marks)

(b) Show that the point $(1, -5, 4)$ lies in the plane containing the two lines. (3 marks)

Question 16 **(7 marks)**

(a) Point A has coordinates $(8, -3, 3)$ and plane Π has equation $2x - y + 2z = 16$. Determine

(i) a vector equation for the straight line through A perpendicular to Π.

(1 mark)

(ii) the perpendicular distance of A from Π. (3 marks)

(b) Prove that the perpendicular distance from the origin to the plane $\mathbf{r} \cdot \mathbf{n} = k$ is $\dfrac{k}{|\mathbf{n}|}$.

(3 marks)

Question 17 **(8 marks)**

Sphere S has diameter PQ, where P and Q have coordinates $(2, -3, 1)$ and $(-4, 7, 5)$ respectively.

(a) Determine the vector equation of the sphere. (3 marks)

(b) Show that the point $(1, -1, 2)$ lies inside the sphere. (2 marks)

(c) Show that the line with equation $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is tangential to the sphere.

(3 marks)

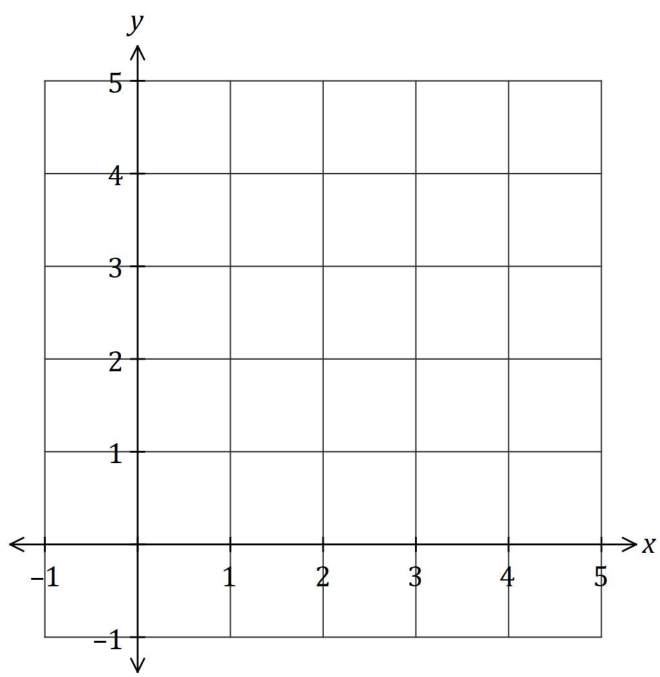
Question 18 **(8 marks)**

Let $f(x) = \sqrt{x-1}$, $g(x) = \dfrac{3}{x}$ and $h(x) = f \circ g(x)$.

(a) Determine an expression for $h(x)$ and show that the domain of $h(x)$ is $0 < x \leq 3$. (3 marks)

(b) Determine an expression for $h^{-1}(x)$, the inverse of $h(x)$. (1 mark)

(c) Sketch the graphs of $y = h(x)$ and $y = h^{-1}(x)$ on the axes below. (3 marks)



(d) Solve $h(x) = h^{-1}(x)$, correct to 0.01 where necessary. (1 mark)

Question 19 **(8 marks)**

If $f(x) = \frac{1}{x+2}$ and $g(x) = x^2 - 5$,

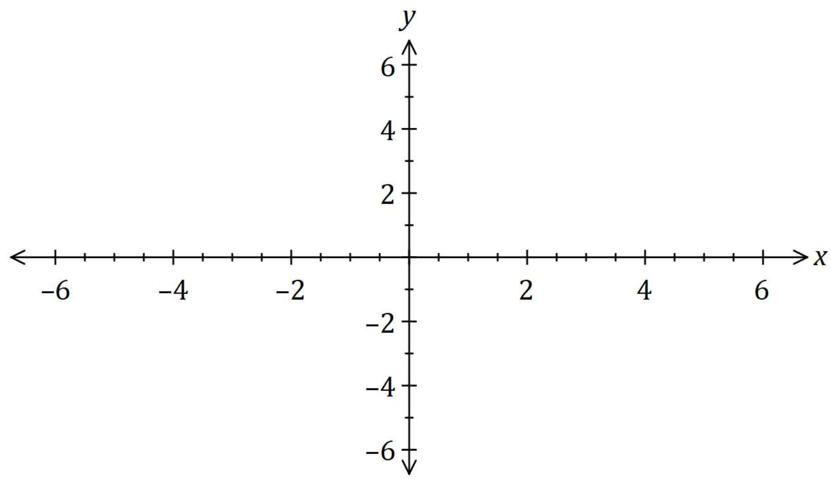
(a) Determine the domain and range of the composition $f(g(x))$. (4 marks)

(b) The restriction $x \leq 0$ is applied to the composite function $f(g(x))$ so that its inverse exists. Determine the equation of this inverse and state the domain and range. (4 marks)

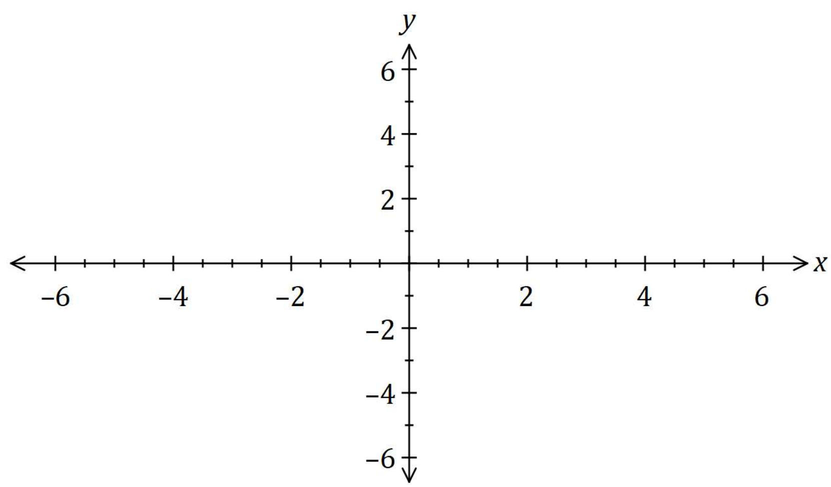
Question 20 **(8 marks)**

Let $f(x) = 3 - |2x - 6|$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (2 marks)



(b) Sketch the graph of $y = f(|x|)$ and hence solve $f(|x|) - 3 = 0$. (3 marks)

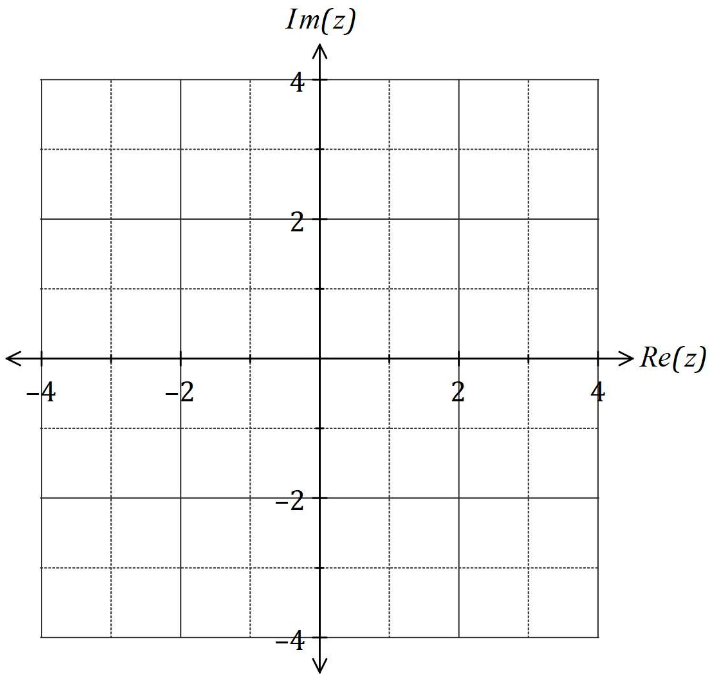


(c) The equation $f(x) = a|x + b| + c$ is true only for $0 \leq x \leq 3$. Determine the value of each of the constants a, b and c. (3 marks)

Question 21 **(6 marks)**

Sketch the locus of the complex number z given by

(a) $|z+3| \leq |z-1+2i|$. (3 marks)



(b) $|z+2| = |z| - 2$. (3 marks)