WAEP WA Exams Practice Paper E, 2016

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section Two: Calculator-assumed

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

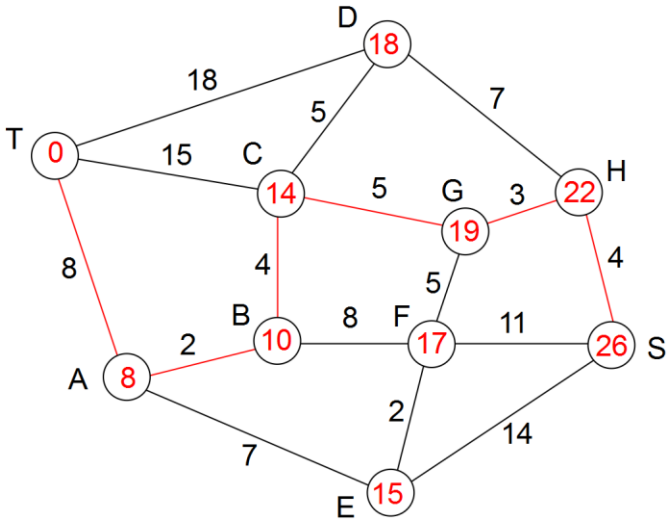
Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(5 marks)**

The numbers on edges of the weighted graph below show the time, in minutes, to travel between stops on a tram network.

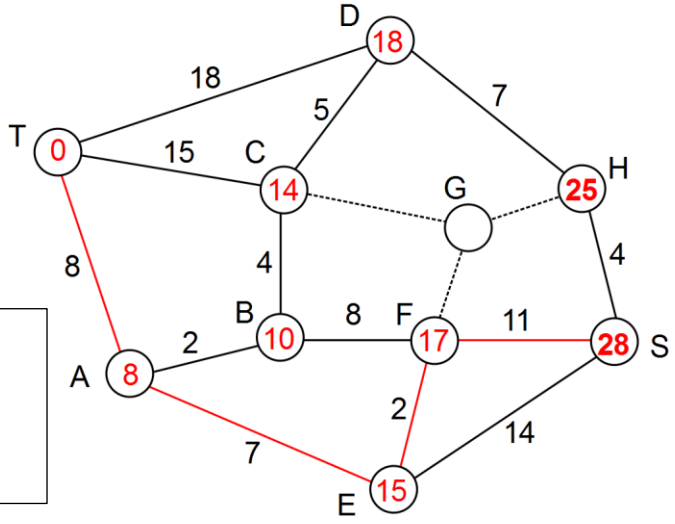


(a) Determine the fastest path from the town centre T to the train station S, stating the minimum time. (3 marks)

Path: T A B C G H S

Time: 26 minutes

(b) An accident at stop G made it impossible for trams to travel via this stop. What effect, if any, will this have on your answer to (a)? Justify your answer. (2 marks)
A copy of the graph has been supplied for your use, if required.



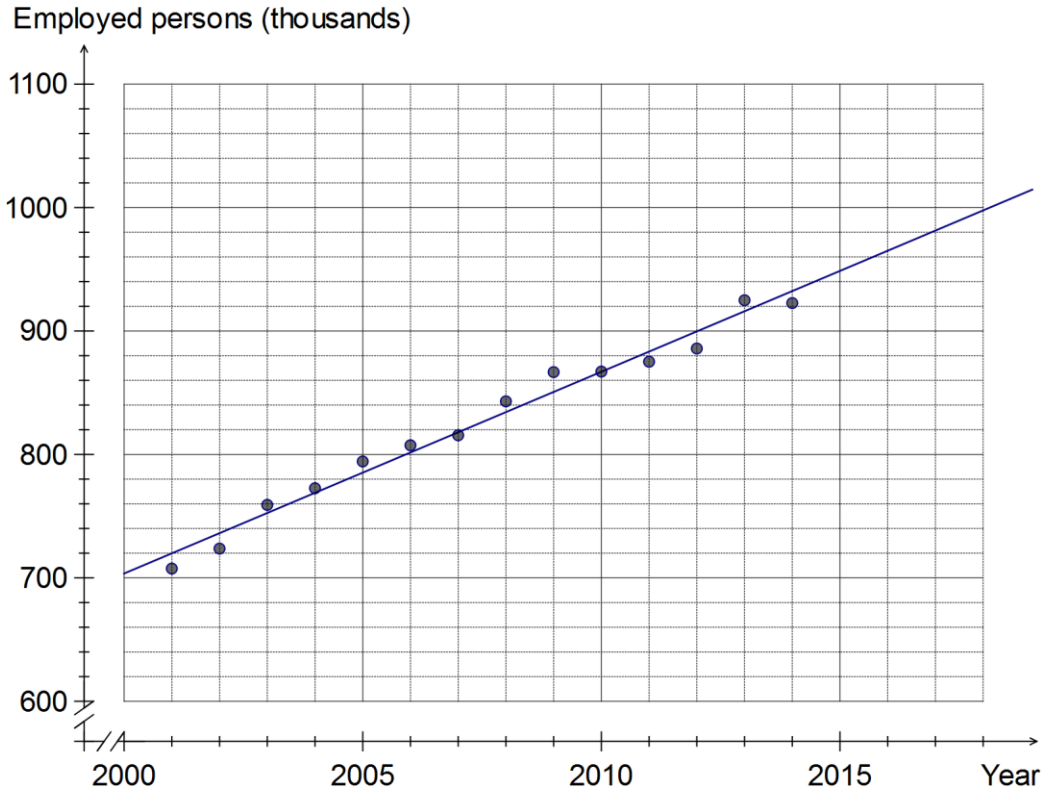
Path will change and time will increase.

New route: T A E F S
New time: 28 minutes

See next page

Question 9 **(10 marks)**

The scatterplot below shows the number of employed young persons (aged 15 to 24 years) in the Australian workforce, measured in June of each year.



(a) How many employed young persons were there in June 2003? (1 mark)

760 000

(b) There were 922 600 employed young persons in June 2014. Plot this additional data point on the axes above. (1 mark)

The equation of the least-squares line is $y = 16.35x - 32000$, where x is the year and y is the number of employed young persons.

(c) Draw the least-squares line on the scatterplot. (2 marks)

Hint: Use two points from (e).

(d) State, including units, the average rate of increase in the number of employed young persons from the year 2005 until 2014. (2 marks)

16350 persons per year.

(e) Estimate, to the nearest thousand, the number of employed young persons in

(i) June 2000. (1 mark)

$16.35 \times 2000 - 32000 = 700$ thousand persons

(ii) June 2018. (1 mark)

$16.35 \times 2018 - 32000 = 994$ thousand persons

(f) Which of the estimates in (e) do you consider the most reliable? Explain your answer. (2 marks)

June 2000, as it involves the least extrapolation.

See next page

Question 10 **(8 marks)**

A research company received 582 responses to the question '*Which of the following best describes how you feel about Christmas?*' and were interested in whether attitude to Christmas is associated with age.The responses, A to E, were split into age groups as shown in the table below.

Age group	18 - 54	55+
A I enjoy Christmas a lot	120	105
B I enjoy some things about Christmas	112	140
C I don't enjoy Christmas at all	15	22
D I don't really care one way or the other about Christmas	15	48
E Don't know	2	3

(a) State the explanatory variable for the investigation. (1 mark)

Age.

(b) Of those who don't really care one way or the other about Christmas, what percentage were aged 18 - 54? (1 mark)

$$\frac{15}{15+48} = \frac{15}{63} \approx 23.8\%$$

(c) Of those aged 55+, what percentage enjoyed Christmas a lot? (1 mark)

$$\frac{105}{105+140+22+48+3} = \frac{105}{318} \approx 33.0\%$$

(d) Construct a table of percentages that can be used to help identify whether attitude to Christmas is associated with age. (3 marks)

Response	18 - 54	55+
A	45.5%	33.0%
B	42.4%	44.0%
C	5.7%	6.9%
D	5.7%	15.1%
E	0.8%	0.9%
Total	100%	100%

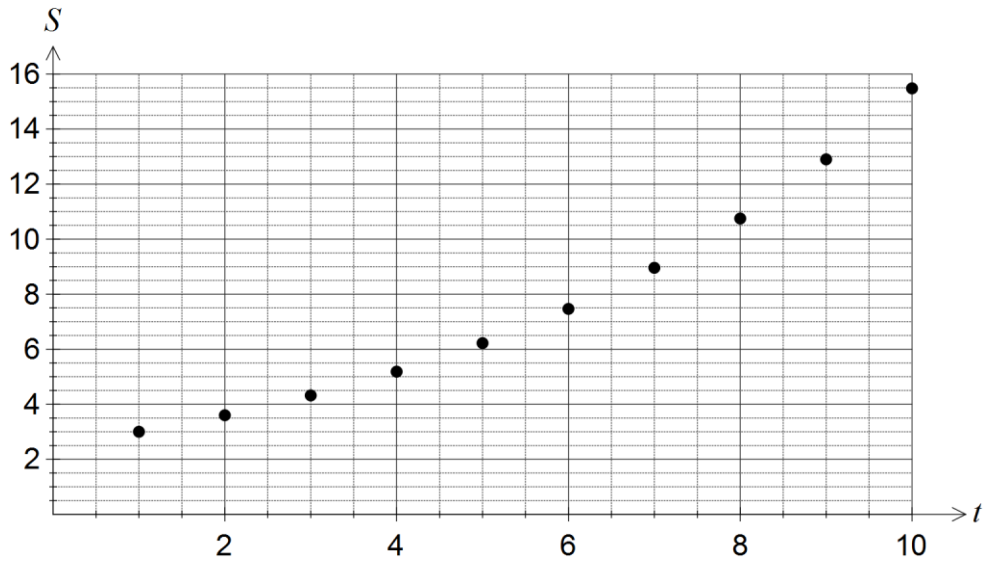
(e) Comment on whether this data provides any evidence that attitude to Christmas is associated with age. (2 marks)

Yes, as although similar percentages of both age groups responded with B, C and E, a significantly different percentage of both responded with A and D.

See next page

Question 11 **(8 marks)**

The total number of sales, S in thousands, of a new phone app were recorded weekly and are shown in the graph below, where t is the week number.



(a) Use the graph to determine

(i) the sales of the app in the first week. (1 mark)

3 000

(ii) the week when sales first exceeded 5 000. (1 mark)

Week 4

(b) The sales of the app, in thousands, can be modelled by the rule $S = 2.5 \times 1.2^t$. Complete the two calculations missing in the table below. (2 marks)

Week (t)	Calculation	Sales (S) in thousands
1	2.5×1.2	3
2	$2.5 \times 1.2 \times 1.2$	3.6
3	$2.5 \times 1.2 \times 1.2 \times 1.2$	4.32
4	$2.5 \times 1.2 \times 1.2 \times 1.2 \times 1.2$	5.184

(c) State the percentage rate of growth of sales of the app. (2 marks)

20% per week. *(must include per week)*

(d) Comment, with reasons, on the use of the model $S = 2.5 \times 1.2^t$ in the long-term. (2 marks)

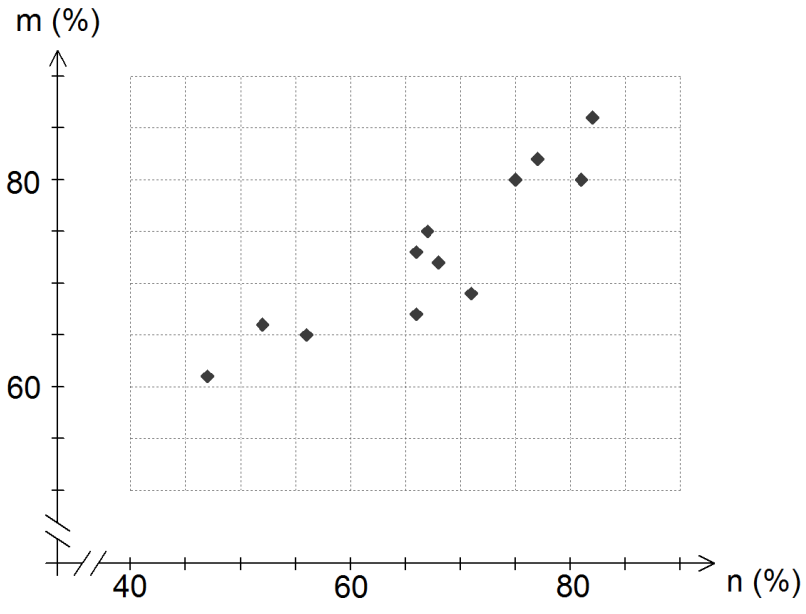
Unrealistic, as sales increase exponentially and will eventually exceed the number of people on the planet.

See next page

Question 12 **(8 marks)**

The table and scatterplot show the percentage scores for a group of twelve candidates who took both numerical reasoning (n) and mechanical aptitude (m) tests as part of a job selection process.

n (%)	m (%)
47	61
82	86
67	75
66	67
71	69
56	65
68	72
66	73
52	66
81	80
77	82
75	80



The equation of the least-squares line is $m = 0.638n + 30$ and the correlation coefficient between the two variables is 0.91.

(a) Describe two features of the scatterplot that are reflected in the value of the correlation coefficient. (2 marks)

As n increases, so m tends to increase, reflected in the positive value of r.

The points lie fairly close to a straight line, reflected in the closeness of r to 1.

(b) A new candidate, who saw the above scatterplot and who had a low numerical reasoning score, decided not to take the mechanical aptitude test because he said his low reasoning score would cause his aptitude score to also be low. Comment on the candidates reasoning. (2 marks)

Reasoning is flawed.

There is no evidence that a low reasoning score *causes* a low aptitude score, just that the two variables are associated.

(c) Calculate the coefficient of determination between the two variables. (1 mark)

$0.91^2 = 0.828$

(d) What percentage of the variation in the mechanical aptitude scores can be explained by the variation in numerical reasoning scores? (1 mark)

Approximately 83%.

(e) Another candidate for the job scored 61 on the numerical reasoning test but due to illness was unable to complete the mechanical aptitude test. Predict the mechanical aptitude score for this person and explain, with reasons, how valid your prediction is. (2 marks)

$0.638 \times 61 + 30 = 68.918$

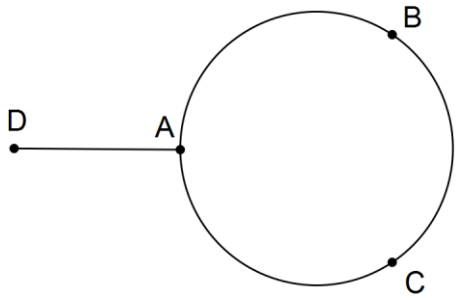
Predict a score of 69%.

This prediction is valid as a strong relationship exists and the prediction involves interpolation.

See next page

Question 13 **(7 marks)**

A simple connected graph is shown below.



(a) The graph contains a semi-Eulerian trail. State the length of this trail. (1 mark)

4 edges.

(b) List the vertices of a **closed** walk of length two on the graph that is **not** a trail and explain why it is not a trail. (2 marks)

A B A *(etc, must start and end at same vertex)* Not a trail because an edge is used more than once.

(c) List the vertices of an **open** walk of length three on the graph that is **not** a path and explain why it is not a path. (2 marks)

A C A D *(etc, must **NOT** start and end at same vertex)* Not a path because edge AC and vertex A both visited more than once.

(d) The graph contains a semi-Hamiltonian trail. State which vertex this trail cannot start at, and explain your answer. (2 marks)

A. Would have to pass through A twice in order to reach D and all other vertices as part of trail (eg ABCAD). To be semi-Hamiltonian, must only visit vertices once.

Question 14 **(8 marks)**

A company purchases equipment at a cost of \$44 000 and expect the equipment to be used in the business for eight years. At the end of this time they expect to sell the equipment for \$7 000.

(a) Calculate the total loss in value of the equipment. (1 mark)

$44000 - 7000 = \$37\ 000$

Under the straight line depreciation method, the loss in value is spread equally over the eight years.

(b) Calculate the annual loss in value of the equipment. (1 mark)

$37000 \div 8 = \$4\ 625$ per year

(c) State a recursive rule for the value, V_n, of the equipment at the end of year *n*. (2 marks)

$V_{n+1} = V_n - 4625,\ V_0 = 44000$

(d) Determine the value of the equipment at the end of the fourth year. (1 mark)

$V_4 = \$25\ 500$

(e) After an accounting review, it was found that the equipment was actually to be used for ten years, after which time it could be sold for \$5 000. Use this information to revise your answer to (d), assuming the straight line depreciation method is still used and showing all working. (3 marks)

$$\frac{44000 - 5000}{10} = 3900$$

$V_{n+1} = V_n - 3900,\ V_0 = 44000$

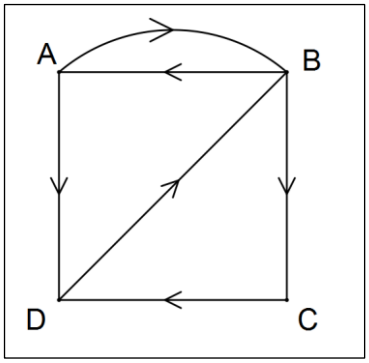
$V_4 = \$28\ 400$

See next page

Question 15 **(8 marks)**

Adele, Bob, Chen and Daisy all have work in the same office and have their own email addresses. Adele and Bob both know each other's email addresses and Adele and Chen both know Daisy's email address. Bob also knows Chen's email address and Daisy knows Bob's email address.

(a) Construct a digraph to show the above information, ignoring the fact that everyone knows their own email address. (3 marks)



(b) Construct an adjacency matrix, A, for the digraph in (a) so that the entry A_{RC} is the number of directed edges from vertex R to vertex C. (2 marks)

		To			
		A	B	C	D
From	A	0	1	0	1
	B	1	0	1	0
	C	0	0	0	1
	D	0	1	0	0

(c) Calculate the matrix B, where $B = A^2 + A$. (2 marks)

$$B = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(d) Use matrix B to identify which of the workers, if any, are unable to email the other three workers either directly or by asking one other worker to forward their email. (1 mark)

Chen cannot email Adele in this way as B_{31} is a zero.

See next page

Question 16 **(8 marks)**

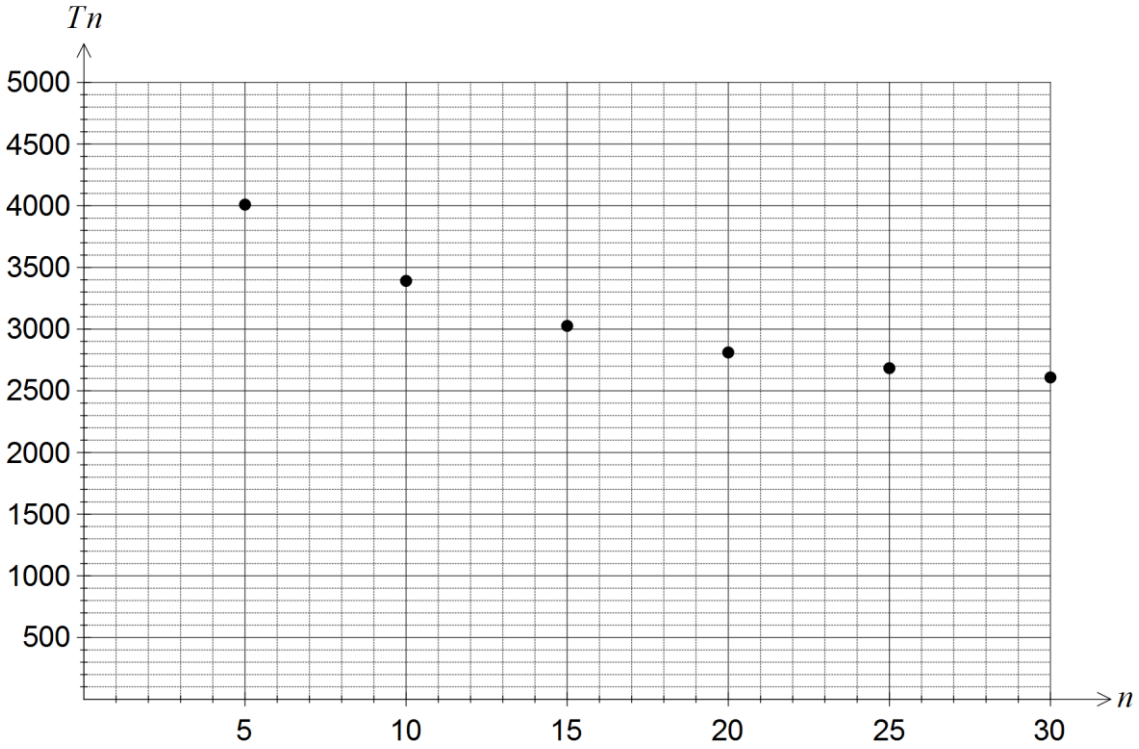
A plantation has 4 800 trees. The plantation manager is interested in modelling what would happen if each year, 10% of the existing trees were cut down for timber and another 250 new trees planted.

The number of trees, T_n, at the start of year n can be modelled by $T_{n+1} = 0.9T_n + 250,\ T_1 = 4800$.

(a) Use the recurrence relation to complete the missing values in the following table. (2 marks)

n	5	10	15	20	25	30
T_n	4009	**3391**	**3026**	**2811**	**2684**	2608

(b) Plot the values from the table on the axes below. (2 marks)



(c) Comment on how the number of trees in the plantation is changing. (2 marks)

The number of trees is decreasing, but at a slower and slower rate.

(d) Does the model predict that eventually there will be no trees left in the planation? Justify your answer. (2 marks)

No - the number will never fall below 2 500, the long term steady-state solution of the sequence.

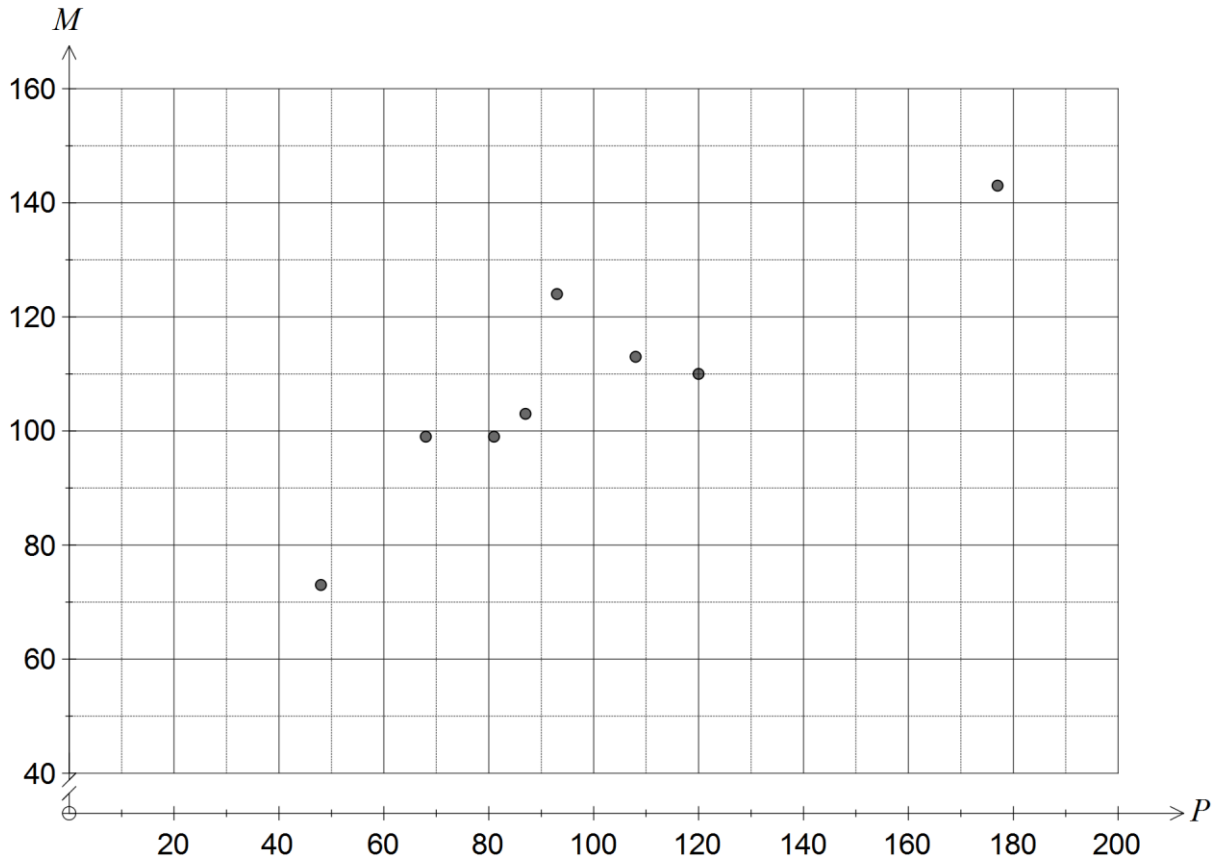
See next page

Question 17 **(10 marks)**

The table below shows the total population (P, in millions) and the number of mobile telephone subscriptions (M, in millions) in eight countries at the end of 2014.

Country	A	B	C	D	E	F	G	H
Population	48	68	81	87	120	108	93	177
Mobile Subscriptions	73	99	99	103	110	113	124	143

(a) Plot this data on the axes below. (2 marks)



A global communications company was investigating whether the number of mobile subscriptions in a country could be determined from its population.

(b) Name the response variable for their investigation. (1 mark)

Mobile subscriptions

(c) Describe the direction and strength of the linear association between P and M that is apparent from the scatterplot. (1 mark)

Positive and moderate linear association.

(d) Calculate the correlation coefficient, r, for the linear association between P and M, giving your answer to three decimal places. (1 mark)

$r = 0.897$

(e) The communications company concluded that the number of mobile subscriptions in any country in the world could reliably be determined from its population.

(i) Give one reason that supports their conclusion. (1 mark)

Correlation is close to 1, indicating a moderate to strong relationship exists.

(ii) Give one reason that does not support their conclusion. (1 mark)

Investigation is based on only eight countries and these may not be representative of all countries.

(f) State the equation of the least-squares line that models the relationship between the population and the number of mobile subscriptions for the eight countries. (2 marks)

$M = 0.4697P + 62.08$

(g) Another country had a population of 150 million people. Predict the number of mobile telephone subscriptions in this country. (1 mark)

$M = 0.4697(150) + 62.08 = 132.5$ million.

See next page

Question 18 **(9 marks)**

(a) A sequence is defined by $T_{n+1} = 0.75T_n,\ T_1 = 160$.

(i) Calculate T_4. (1 mark)

$T_4 = 67.5$

(ii) Determine how many terms of the sequence are larger than 1. (1 mark)

18 terms

(iii) State whether the sequence contains at least one negative number, explaining your answer. (2 marks)

No. All terms will be positive and getting closer and closer to zero.

(b) The first two terms, in order, of a geometric sequence are $\frac{1}{2}$ and $\frac{2}{3}$.

(i) Calculate the next term of this sequence, leaving your answer as a fraction in simplest form. (2 marks)

$$r = \frac{2}{3} \div \frac{1}{2} = \frac{4}{3}$$

$$T_3 = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

(ii) State a rule for the n^{th} term of this sequence. (2 marks)

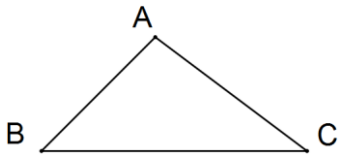
$$T_n = \frac{1}{2} \times \left(\frac{4}{3}\right)^{n-1}$$

(iii) Determine the minimum number of terms of this sequence that are required to have a sum of at least 10. (1 mark)

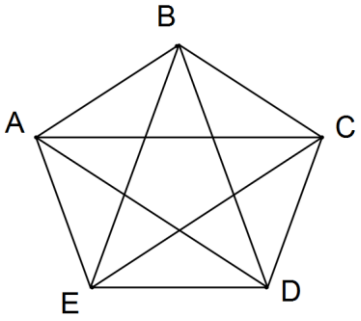
8 terms

Question 19 **(9 marks)**

Shown below is a complete graph with three vertices.



(a) Construct a complete graph with five vertices. (3 marks)



Five vertices all connected to every other vertex.

(b) State the number of edges in a complete graph with five vertices. (1 mark)

10 edges.

(c) Complete the table below for the number of edges, T_n, in a complete graph with n vertices. (2 marks)

n	1	2	3	4	5	6
T_n	0	**1**	**3**	**6**	**10**	15

(d) Use an example from the table to show that the number of edges, T_n, in a complete graph with n vertices is given by the recurrence relation $T_{n+1} = T_n + n, \quad T_1 = 0$. (2 marks)

$T_3 = T_2 + 2 \Rightarrow 3 = 1 + 2 = 3$ Hence true.

(Or other example)

(e) Determine the number of edges in a complete graph with 12 vertices. (1 mark)

Using CAS, $T_{12} = 66$

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____