



MATHEMATICS

3A/3B

Calculator-assumed

WACE Examination 2014

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

Section Two: Calculator-assumed

(100 Marks)

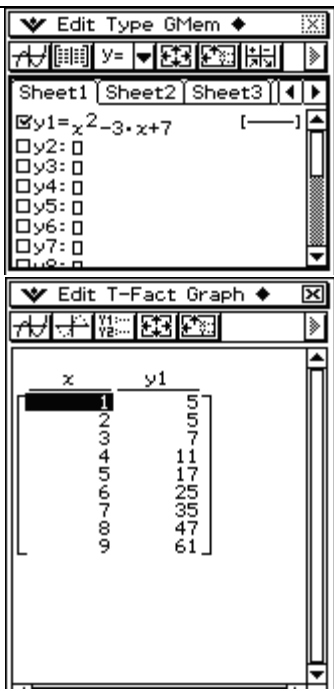
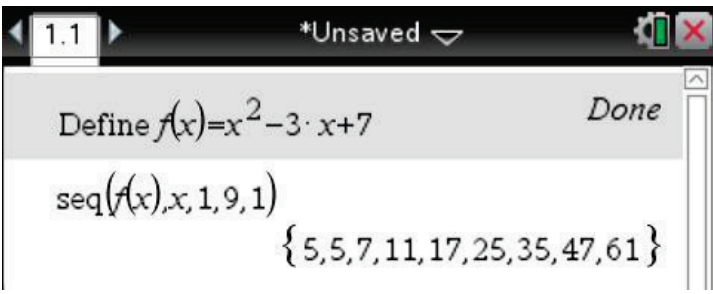
Question 8

(5 marks)

- (a) Adam made the conjecture that the function $f(x) = x^2 - 3x + 7$ generated prime numbers for $1 \leq x \leq 9$, where x is an integer.

Was Adam's conjecture true? If not, state a value of x for which the conjecture is false.

(2 marks)

Solution	
	 <p>Adam's conjecture was false. Either $x = 6$ or $x = 7$ will make the conjecture false</p>
Specific behaviours	
<ul style="list-style-type: none"> ✓ states the conjecture is false ✓ states a value of x for which the conjecture is false 	

- (b) Prove that $(3n+1)^2 - (3n-1)^2$ is a multiple of six for all positive integer values of n .

(3 marks)

Solution
$(3n+1)^2 - (3n-1)^2 = 9n^2 + 6n + 1 - (9n^2 - 6n + 1) = 12n = 6(2n)$, which has a factor of six therefore must be a multiple of six.
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly expands expression ✓ correctly simplifies expression ✓ correctly identifies expression has a factor 6

Question 9

(16 marks)

A biologist was conducting trials of a new fertiliser. She obtained ten almost identical plants and applied different amounts of fertiliser to test its effectiveness. The heights of the plants were measured after two months. The results are shown in the table below.

Plant	1	2	3	4	5	6	7	8	9	10
Fertiliser (f) (grams)	8	9	11	12	13	15	17	20	21	23
Height (h) (centimetres)	24	26	29	28	32	33	35	35	34	33

- (a) Determine the correlation coefficient r_{fh} . (1 mark)

Solution
$r = 0.85$
Specific behaviours
✓ correctly calculates the correlation coefficient

- (b) Describe, in words, the relationship between f and h . (2 marks)

Solution
Strong, positive relationship
Specific behaviours
✓ states that the relationship is strong
✓ states that the relationship is positive

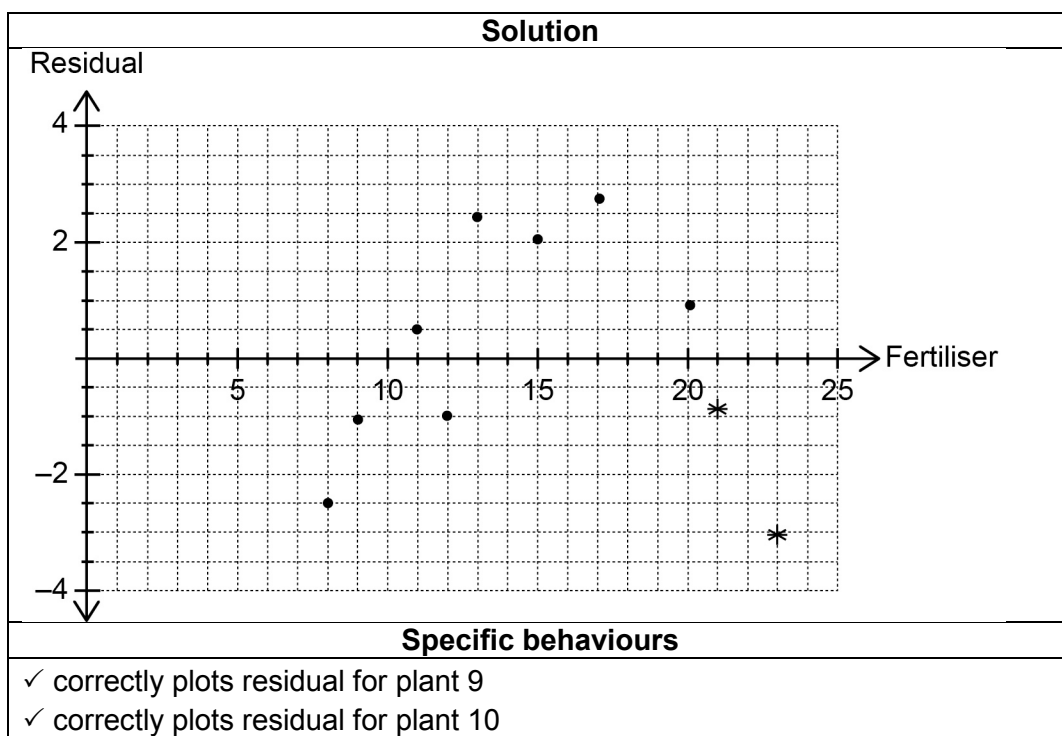
- (c) Determine the regression line of h on f . (1 mark)

Solution
$h = 0.642f + 21.34$
Specific behaviours
✓ correctly determines the regression line

- (d) The difference between the actual plant height and the predicted plant height using the regression line for each different amount of fertiliser is calculated. This difference is called the residual.
- (i) Calculate the residuals for plants 9 and 10 and complete the table below. The eight residuals are recorded in the table correct to two decimal places. (2 marks)

Solution										
Plant	1	2	3	4	5	6	7	8	9	10
Residual	-2.47	-1.11	0.60	-1.04	2.32	2.04	2.75	0.83	-0.82	-3.10
Specific behaviours										
✓ calculates residual for plant 9										
✓ calculates residual for plant 10										

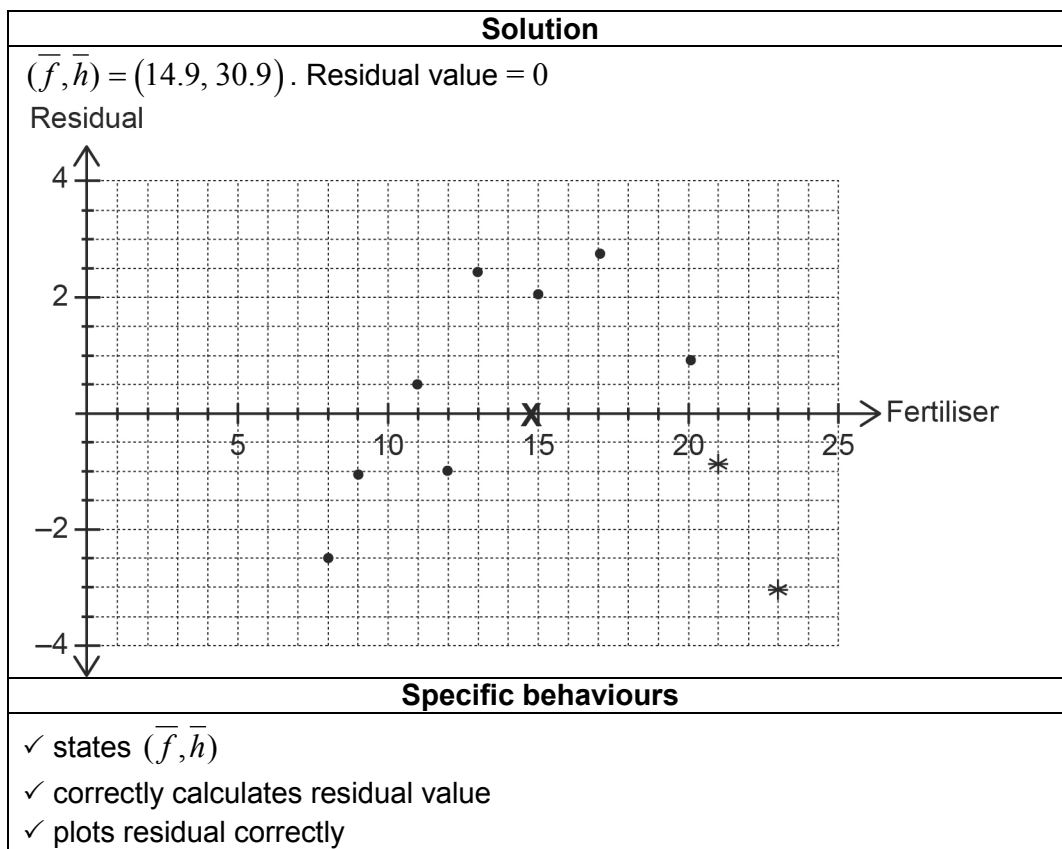
- (ii) The residuals for the first eight plants have been plotted below. Complete the residual plot using the results for plants 9 and 10. (2 marks)



- (iii) Comment on the information the residual plot revealed to the biologist. (2 marks)

Solution
There is a pattern, therefore the linear regression model is not appropriate.
Specific behaviours
✓ states there is a pattern in the residual plot
✓ concludes that linear regression model is not appropriate

- (iv) State the coordinates (\bar{f}, \bar{h}) , determine the corresponding residual for this point and plot it on the residual plot in part (ii). (3 marks)



- (e) (i) Use the regression line to predict the height of a plant after two months when 22 grams of fertiliser are applied. (1 mark)

Solution
Height is 35.5 cm
Specific behaviours
✓ correctly calculates height

- (ii) Comment on the reliability of the prediction in part (e) (i). (2 marks)

Solution
The prediction is not reliable even though it is interpolation, as the linear regression model is not appropriate (from (d) (iii)).
Specific behaviours
✓ correctly concludes it is not reliable
✓ correctly gives an adequate reason for the unreliability

Question 10

(8 marks)

On 2 March, Michael is assigned to read a particular novel for his Literature class in preparation for a test on 22 March. On the first night, he reads 80 pages before falling asleep. On the next night, he reads 60 pages and on the third night, 45 pages. He continues to read 25% fewer pages each consecutive night.

- (a) The situation can be represented by a recursive rule in the form $a_{n+1} = pa_n$, where $a_1 = q$.

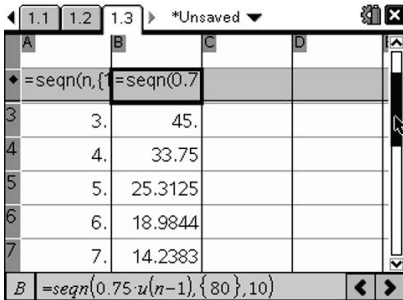
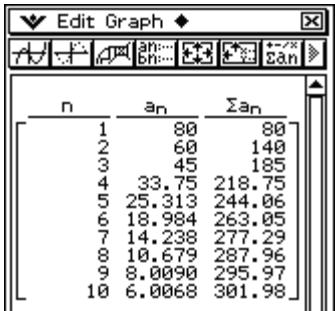
Determine the values of p and q .

(2 marks)

Solution
$a_{n+1} = 0.75a_n$, where $a_1 = 80 \Rightarrow p = 0.75$ and $q = 80$
Specific behaviours
✓ correctly determines the value of p
✓ correctly determines the value of q

- (b) How many pages, to the nearest whole number, will Michael read on the fifth night?

(2 marks)

Solution
$a_5 = 0.75(33.75) = 25.3125 = 25$ (to the nearest whole number) OR using CAS opposite
 
Specific behaviours
✓ correctly determines the value of the fifth term
✓ correctly rounds down to the nearest whole number

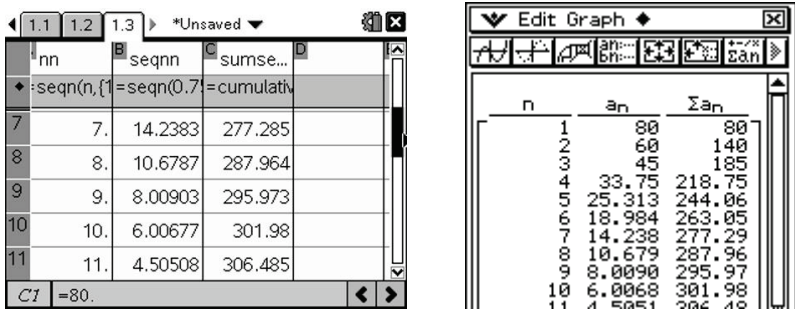
- (c) How long will it take Michael to read at least 300 pages?

(1 mark)

Solution

10 nights or more

using CAS opposite



The CAS interface shows a sequence table with columns n, a_n, and Σa_n. The table contains values for n from 7 to 11. The edit graph shows the sequence values and their cumulative sum.

n	a _n	Σa _n
7	14.2383	277.285
8	10.6787	287.964
9	8.00903	295.973
10	6.00677	301.98
11	4.50508	306.485

Specific behaviours

✓ correctly determines the number of nights using either CAS or otherwise

- (d) If the book has 320 pages, will Michael complete the book by the test date? Justify your answer.

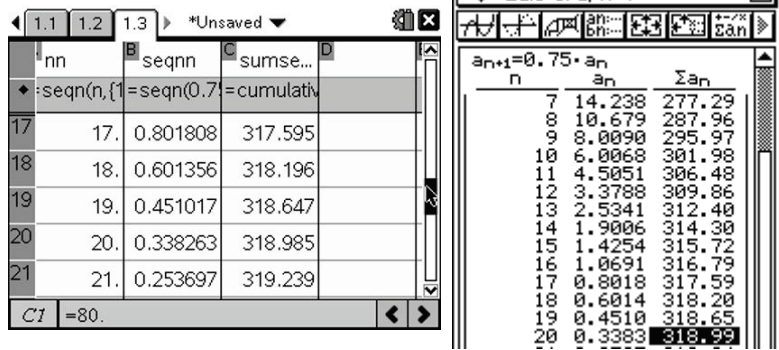
(3 marks)

Solution

2 March to 22 March is 20 nights

After 20 nights he has only read 319 pages (to nearest whole number)

Therefore Michael will not complete the book by the test date.



The CAS interface shows a sequence table with columns n, a_n, and Σa_n. The table contains values for n from 17 to 21. The edit graph shows the sequence values and their cumulative sum.

n	a _n	Σa _n
17	0.801808	317.595
18	0.601356	318.196
19	0.451017	318.647
20	0.338263	318.985
21	0.253697	319.239

Specific behaviours

✓ correctly determines the number of nights

✓ correctly states the number of pages read for the 20 nights

✓ correctly concludes that he will not complete the book by the test date

Question 11

(6 marks)

Consider the following data written in ascending order and consisting of seven integer values with the middle three values missing. The mean is 11.14 (correct to two decimal places) and the median is 11.

3, 4, __, __, __, 17, 26

- (a) If the smallest two scores from the data are both increased by one and the largest two scores are both reduced by one, state the effect on

- (i) the standard deviation. (1 mark)

Solution
The standard deviation will decrease.
Specific behaviours
✓ correctly states the effect on the standard deviation

- (ii) the mean. (1 mark)

Solution
The mean will stay the same.
Specific behaviours
✓ correctly states the effect on the mean

- (iii) the median. (1 mark)

Solution
The median will stay the same.
Specific behaviours
✓ correctly states the effect on the median

- (b) Determine an eighth data point which, when included with the original data would produce the boxplots shown below.



Solution
eighth value is 2
Specific behaviours
✓ correctly identifies the eighth value

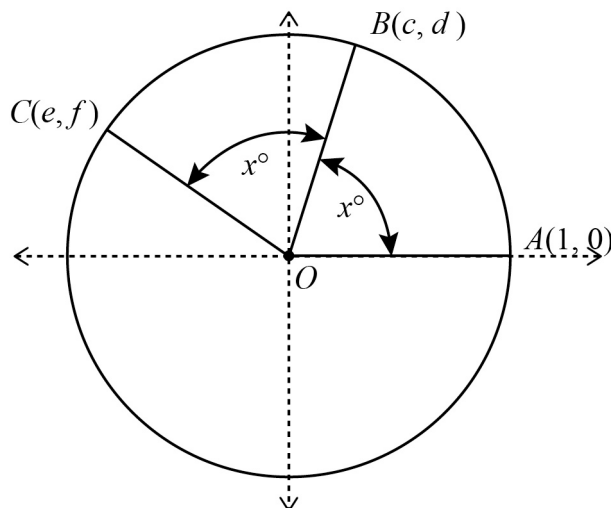


Solution
If the original median was 11 and there was an odd number of data points, 11 must have been in the data set. Therefore to get a median of 10.5 when an eighth value is included, the eighth value must be 10.
Specific behaviours
✓ identifies that 10.5 is an average of two integers
✓ correctly deduces that 10 must be the missing value

Question 12

(7 marks)

The unit circle below, with centre O at the origin, has the points $A(1, 0)$, $B(c, d)$ and $C(e, f)$ on the circumference and $\angle COB = \angle BOA = x^\circ$, $45^\circ < x^\circ < 90^\circ$.



Given Pythagoras's Theorem: 'In a right triangle PQR where p, q are the short sides and r is the hypotenuse, $r^2 = p^2 + q^2$,' use the pronumerals c, d, e and f to:

- (a) state a relationship between c and d not involving x . (2 marks)

Solution
$c^2 + d^2 = 1$
Specific behaviours
✓ uses Pythagoras's Theorem
✓ correctly states the relationship between c and d

- (b) determine an equation for $\sin x^\circ$. (1 mark)

Solution
$\sin x^\circ = d$
Specific behaviours
✓ correctly states the value of $\sin x^\circ$

- (c) evaluate $\cos 2x^\circ$. (2 marks)

Solution
$\cos 2x^\circ = e$
Specific behaviours
<ul style="list-style-type: none"> ✓ identifies that $2x^\circ$ is the angle AOC ✓ correctly states the value of $\cos 2x^\circ$

- (d) determine the area of triangle OBC . (2 marks)

Solution
$A = \frac{1}{2}(1)(1)\sin x^\circ = \frac{d}{2}$
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly uses area of triangle formula ✓ correctly determines the area

Question 13 (4 marks)

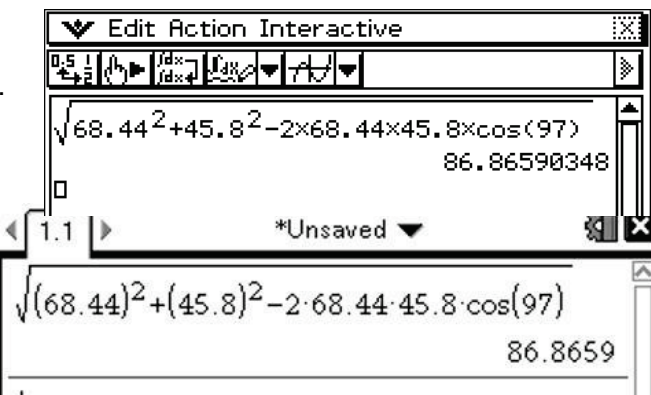
A farmer wishes to fence a triangular paddock ABC . He first needs to calculate the perimeter of the paddock. He determines that point B is 45.8 m from point A and on a bearing of 055° and point C is 68.44 m from point A and on a bearing of 318° .

- (a) Draw a diagram to represent this information. (2 marks)

Solution
Specific behaviours
<ul style="list-style-type: none"> ✓ draws an appropriate diagram ✓ correctly labels sides and angles

- (b) Calculate the perimeter of the paddock.

(2 marks)

Solution	
<p>Using CAS $BC = 86.87$ m Therefore the perimeter is 201.11 m.</p>	 <p>The screenshot shows a CAS calculator window titled 'Edit Action Interactive'. The input field contains the formula $\sqrt{68.44^2 + 45.8^2 - 2 \times 68.44 \times 45.8 \times \cos(97)}$, and the result is 86.86590348. Below this, the formula is repeated with parentheses: $\sqrt{(68.44)^2 + (45.8)^2 - 2 \cdot 68.44 \cdot 45.8 \cdot \cos(97)}$, resulting in 86.8659. The window also shows a zoom level of 1.1 and an unsaved status.</p>
Specific behaviours	
<ul style="list-style-type: none"> ✓ correctly calculates the length of BC ✓ correctly calculates the perimeter of the paddock 	

Question 14

(12 marks)

The table below shows the cost of gas for a Perth household. Accounts are sent at the end of each time period. Accounts for 2010 are unavailable, but some calculated statistics are provided.

Time (<i>t</i>)	Months/Year	Cost (\$)	Four-point centred moving averages (<i>m</i>)	Residual
1	November 2009–January 2010	<i>D</i>	-	-
2	February 2010–April 2010	-	-	-
3	May 2010–July 2010	-	-	-
4	August 2010–October 2010	<i>A</i>	-	-
5	November 2010–January 2011	121.52	138.84	-17.32
6	February 2011–April 2011	140.14	139.07	1.08
7	May 2011–July 2011	153.26	140.13	<i>C</i>
8	August 2011–October 2011	143.11	141.73	1.38
9	November 2011–January 2012	126.47	144.68	-18.21
10	February 2012–April 2012	148.02	148.07	-0.05
11	May 2012–July 2012	169.00	151.15	17.85
12	August 2012–October 2012	154.45	152.33	2.12
13	November 2012–January 2013	139.78	152.58	-12.80
14	February 2013–April 2013	144.17	154.30	-10.13
15	May 2013–July 2013	174.80	155.32	19.48
16	August 2013–October 2013	162.43	155.80	6.63
17	November 2013–January 2014	139.96	156.73	-16.77
18	February 2014–April 2014	147.88	157.77	-9.89
19	May 2014–July 2014	178.50	-	-
20	August 2014–October 2014	167.05	-	-

- (a) What is the time period of the household gas accounts? (1 mark)

Solution
Every three months, or quarterly.
Specific behaviours
✓ correctly states the period of the household gas accounts

- (b) Calculate the missing entries marked by *A* and *C*. (2 marks)

Solution
$139.07 = \frac{\frac{A}{2} + 121.52 + 140.14 + 153.26 + \frac{143.11}{2}}{4} \Rightarrow A = 139.61$
$C = 153.26 - 140.13 = 13.13$
Specific behaviours
✓ correctly evaluates <i>A</i>
✓ correctly evaluates <i>C</i>

- (c) Calculate the seasonal component for the November–January period. (2 marks)

Solution
$\frac{-17.32 - 18.21 - 12.8 - 16.77}{4} = -16.28$
Specific behaviours
<ul style="list-style-type: none"> ✓ uses correct residuals ✓ correctly calculates seasonal component

- (d) The regression equation for the time (t) against the four-point centred moving average, m , is $m = 1.534t + 131.72$. Predict the cost for the November 2014–January 2015 account. (3 marks)

Solution
$t = 21 \Rightarrow m = 1.534(21) + 131.72 = 163.93$ Future prediction = $163.93 - 16.28 = \$147.65$
Specific behaviours
<ul style="list-style-type: none"> ✓ states correct t value ✓ correctly calculates the value of m ✓ correctly adds the seasonal component to calculate the predicted amount for 2015

- (e) Comment on the reliability of your prediction from part (d). (2 marks)

Solution
The prediction is fairly reliable as it is within one cycle of the data.
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly states that the prediction is fairly reliable ✓ correctly gives a reason for this statement

- (f) The seasonally adjusted figure for November 2009–January 2010 is \$133.79. Determine the value of the missing entry marked by D . (2 marks)

Solution
$D - (-16.28) = 133.79 \Rightarrow D = 117.51$
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly subtracts the seasonal component from D ✓ correctly solves the equation for D

Question 15

(12 marks)

- (a) Given $f(x) = 7 - 3x$ with domain $\left\{-2, 1, \frac{1}{3}, 4\right\}$, determine the range of $f(x)$. (2 marks)

Solution
Range: $\{13, 4, 6, -5\}$
Specific behaviours
✓ correctly calculates two of the four range values
✓ correctly calculates all four of the range values

- (b) (i) The function $g(x) = \sqrt{2(x+1)}$ can be formed by transforming $k(x) = \sqrt{x}$. Determine the transformations required to obtain $g(x)$ from $k(x)$ and state the transformations in the correct order. (2 marks)

Solution
1. dilation in the x -direction, factor $\frac{1}{2}$
2. translation of 1 unit in the negative x direction (1 unit to the left)
Specific behaviours
✓ correctly states first transformation
✓ correctly states second transformation

- (ii) Determine the domain of $g(x)$. (2 marks)

Solution
$2(x+1) \geq 0$ $x+1 \geq 0$ $x \geq -1$
Specific behaviours
✓ correctly writes an inequality in x
✓ correctly solves the inequality for the domain of $g(x)$

- (c) (i) Determine the equation of the function $h(x)$ shown on the axes below. (3 marks)

Solution
$h(x) = k(x-1)^3$ <p>Using the point $(2, 2) \Rightarrow 2 = k(1)^3 \therefore k = 2$</p> $\therefore h(x) = 2(x-1)^3$
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly identifies a cubic relationship ✓ correctly identifies a translation of 1 unit to the right ✓ correctly uses the point $(2, 2)$ to calculate the value of dilation factor in the x-direction

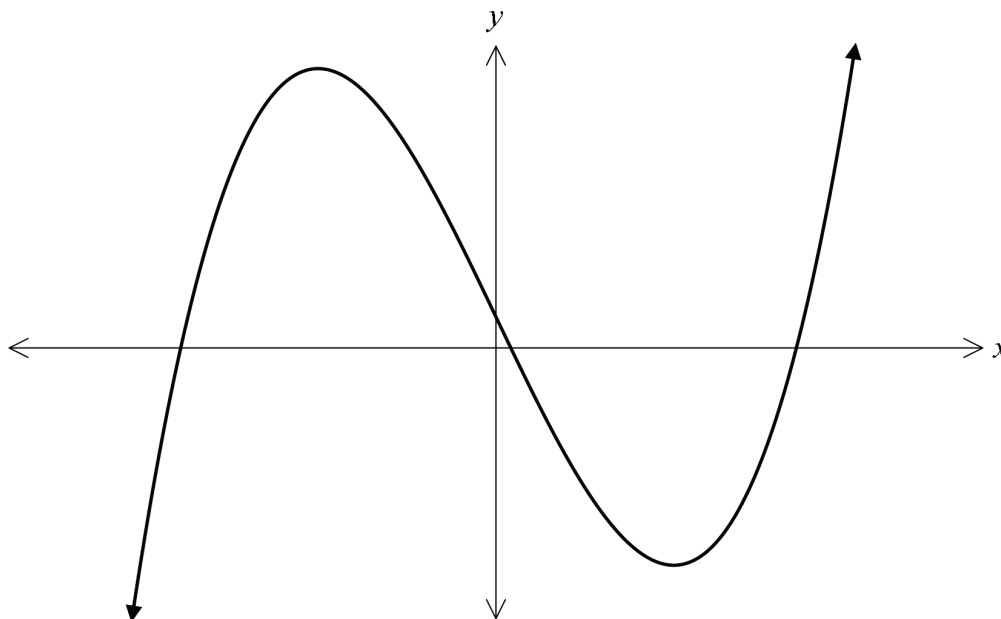
- (ii) Use the axes above to sketch the function $y = h(-x) + 2$. (3 marks)

Solution
<p>The graph shows a coordinate plane with x and y axes ranging from -6 to 6. A solid cubic curve $h(x)$ is plotted, passing through the point $(2, 2)$. A dashed cubic curve $y = h(-x) + 2$ is also plotted, passing through the points $(-1, 2)$, $(-2, 4)$, and $(0, 0)$.</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly identifies reflection in y-axis ✓ correctly identifies vertical translation ✓ correctly plots the function passing through at least two of the following points: $(-1, 2)$, $(-2, 4)$, $(0, 0)$

Question 16

(9 marks)

The cubic function $y = x^3 - 12x + 2$ is graphed below.



- (a) By differentiating the function, determine the coordinates of the turning points. (4 marks)

Solution
$\frac{dy}{dx} = 3x^2 - 12 = 0$ when $3x^2 - 12 = 0 \Rightarrow x = \pm 2$ when $x = 2, y = -14$ and when $x = -2, y = 18$ \therefore coordinates are $(2, -14)$ and $(-2, 18)$
Specific behaviours
✓ correctly differentiates function ✓ correctly equates derivative to zero and solves for x ✓ determines coordinate $(2, -14)$ ✓ determines coordinate $(-2, 18)$

- (b) For what values of x is the function decreasing? (2 marks)

Solution
$-2 < x < 2$
Specific behaviours
✓ states correct values of x ✓ does not include end points

- (c) Determine the equation of the tangent to the curve at the point $(-1, 13)$. (2 marks)

Solution	
$\left. \frac{dy}{dx} \right _{x=-1} = -9 \therefore$ equation of tangent is $y = -9x + c$	
using $(-1, 13) \Rightarrow c = 4 \therefore y = -9x + 4$	
Specific behaviours	
✓ correctly calculates gradient of tangent	
✓ correctly calculates c	

- (d) Verify that this tangent passes through the minimum turning point. (1 mark)

Solution	
When $x = 2, y = -9(2) + 4 = -14 \Rightarrow$ tangent passes through the minimum turning point	
Specific behaviours	
✓ correctly substitutes $x = 2$ into equation of tangent	

Question 17

(10 marks)

A retailer determines that he can sell 60 headbands per week if he charges \$10 per headband. The retailer estimates that for each 50 cent reduction in price he can sell 15 extra head bands per week.

- (a) Complete the table below. (2 marks)

Solution		
Number of 50 cent reductions	Price(\$)	Number of head bands sold
0	10	60
1	9.50	75
2	9.00	90
3	8.50	105
x	$10 - 0.5x$	$60 + 15x$
Specific behaviours		
✓ correctly determines price/number of headbands sold for one, two and three 50c reductions		
✓ correctly writes an expression for the number of headbands sold in the last cell of column 3		

- (b) Given that Revenue(R) = Sale Price (P) \times Number Sold (N), and ignoring any other overhead costs, show that $R = 600 + 120x - 7.5x^2$. (2 marks)

Solution	
$R = (10 - 0.5x)(60 + 15x) = 600 + 120x - 7.5x^2$	
Specific behaviours	
✓ expresses R as the product of two factors	
✓ expands and simplifies brackets correctly	

- (c) Using calculus techniques, calculate the amount that should be charged per headband to maximise the revenue. (3 marks)

Solution
$\frac{dR}{dx} = 120 - 15x = 0$ when $x = 8 \therefore \text{Price} = 10 - 0.5(8) = 6$ Therefore he should charge \$6 per headband to maximise his revenue.
Specific behaviours
<ul style="list-style-type: none"> ✓ differentiates correctly ✓ equates the derivative equal to zero and correctly solves for x ✓ correctly calculates the price of the headband to maximise revenue

- (d) If the retailer initially purchased 200 headbands at \$4 each, calculate his expected profit given that the price is set to maximise the revenue. (3 marks)

Solution
Costs = $200 \times 4 = \$800$ Revenue = $600 + 120(8) - 7.5(8)^2 = \1080 Profit = $1080 - 800 = \$280$
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly calculates costs ✓ correctly calculates revenue ✓ correctly calculates the expected profit

Question 18

(3 marks)

The heights of a small population of people living on a Pacific island were all measured for a regional census. It was found that 99.7% of the population were between 124 cm and 190 cm tall and 68% of the population were between 146 cm and 168 cm tall. If the heights are normally distributed, use this information to estimate the mean and standard deviation of the population heights.

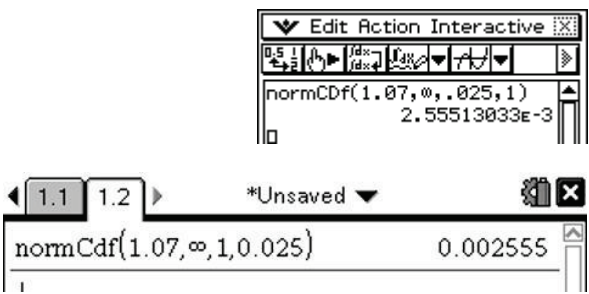
Solution
68% of the heights were within one standard deviation of the mean 99.7% of the heights were within three standard deviations of the mean Therefore $190 - 168 = 22$ cm would be two standard deviations This gives a standard deviation of 11 cm Therefore the mean is $168 - 11 = 157$ cm
Specific behaviours
✓ uses the percentage rules to determine the standard deviations from the mean ✓ calculates the standard deviation ✓ uses symmetry to calculate the mean

Question 19

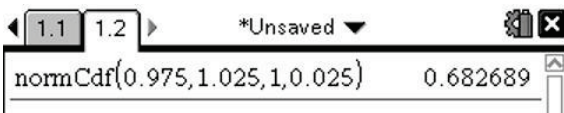

(8 marks)

A jeweller purchased cultured pearls for the production of necklaces. The diameters of the pearls were found to be normally distributed, with a mean of 1 cm and a standard deviation of 0.025 cm.


- (a) What proportion of the pearls will have a diameter exceeding 1.07 cm? Give your answer correct to three significant figures. (2 marks)

Solution
Using CAS with mean = 1 cm and standard deviation = 0.025 cm, the probability is 0.002 555. <div style="text-align: center;">  </div>
Therefore the proportion of pearls with a diameter exceeding 1.07 cm is 0.002 56 correct to three significant figures.
Specific behaviours
✓ calculates correct probability ✓ correctly rounds proportion to three significant figures

- (b) What is the probability that a pearl will have a diameter between 0.975 cm and 1.025 cm? (1 mark)

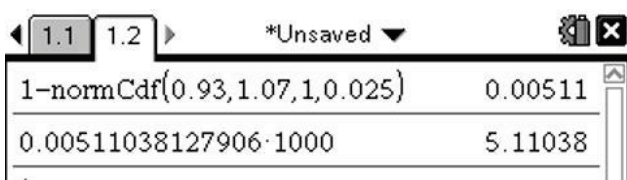
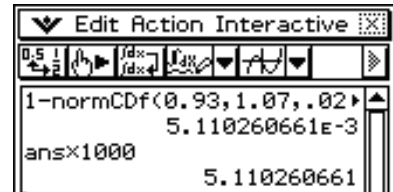
Solution	
<p>Using CAS Probability is 0.6827 (4 d.p.)</p> 	
Specific behaviours	
✓ correctly calculates probability	

- (c) Below what size will the diameter of 10% of the pearls fall? (2 marks)

Solution	
<p>Using CAS left tail = 10% gives a diameter of 0.97 cm (2 d.p.)</p> 	
Specific behaviours	
✓ correctly uses left tail of 10% ✓ calculates correct diameter	

Industry standards dictate that for the consistency of the necklace, pearls with a diameter of less than 0.93 cm or greater than 1.07 cm cannot be used.

- (d) For every 1000 pearls, determine the number of pearls that will **not** be suitable for use in the necklaces. (3 marks)

Solution	
<p>Using CAS Approximately five pearls will not be suitable.</p> <p>The probability could have been obtained by doubling the value found in part (a).</p> 	
Specific behaviours	
✓ correctly calculates probability ✓ uses probability to calculate the number of pearls unsuitable from the sample of 1000 pearls ✓ rounds correctly to five pearls	

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