Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

MATHEMATICS

3C/3D

Calculator-free

WACE Examination 2014

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

Section One: Calculator-free (50 marks)

Question 1 (9 marks)

Evaluate the following:

(a) $\int_0^3 \left(6x^2 + 2x + 1\right) dx$ (3 marks)

Solution
$\int_0^3 \left(6x^2 + 2x + 1\right) dx$ $= \left[2x^3 + x^2 + x\right]_0^3$ $= (54 + 9 + 3) - (0)$ $= 66$
Specific behaviours
✓ integrates correctly ✓ substitutes two limits for x ✓ calculates the correct value for the integral

(b) $\int_1^2 \frac{d}{dx}\left(\frac{x^5}{x^2+1}\right) dx$ (3 marks)

Solution
$\int_1^2 \frac{d}{dx}\left(\frac{x^5}{x^2+1}\right) dx$ $= \left[\frac{x^5}{x^2+1}\right]_1^2$ $= \left(\frac{32}{5}\right) - \left(\frac{1}{2}\right)$ $= \frac{59}{10}$
Specific behaviours
✓ determines the correct expression for the integral ✓ substitutes correct values for the two limits for x ✓ calculates the correct value for the integral

(c) $\dfrac{d}{dx}\displaystyle\int_4^{x^2} \frac{2}{3t^3-1}dt.$ (3 marks)

Solution
$\dfrac{d}{dx}\displaystyle\int_4^{x^2} \frac{2}{3t^3-1}dt$ $=\dfrac{2}{3\left(x^2\right)^3-1}2x$ $=\dfrac{4x}{3x^6-1}$
Specific behaviours
✓ uses the Fundamental Theorem of Calculus   ✓ applies the chain rule   ✓ determines the simplified expression for the derivative

Question 2 **7 marks)**

(a) Simplify the expression $2-\dfrac{1}{2-\dfrac{1}{x}}$. (3 marks)

Solution
$2-\dfrac{1}{2-\dfrac{1}{x}}$ $=2-\dfrac{1}{\dfrac{2x-1}{x}}$ $=2-\dfrac{x}{2x-1}$ $=\dfrac{4x-2-x}{2x-1}$ $=\dfrac{3x-2}{2x-1}$
Specific behaviours
✓ simplifies the denominator of the given fraction   ✓ subtracts the fraction correctly   ✓ determines the simplified expression

(b) Solve the inequality $\dfrac{2x^3}{(x-2)(x+4)} > 0$. (4 marks)

Solution				
expression	$x < -4$	$-4 \le x < 0$	$0 \le x < 2$	$x \ge 2$
$2x^3$	–ve	–ve	+ve	+ve
$x-2$	–ve	–ve	–ve	+ve
$x+4$	–ve	+ve	+ve	+ve
result	–ve	+ve	–ve	+ve

$-4 < x < 0,\ x > 2$

Specific behaviours

✓ determines the critical points –4, 0, 2
✓ correctly determines signs of at least two terms over **R**
✓ states at least one correct interval
✓ states the correct two intervals

Question 3 **(4 marks)**

When two fair six-sided dice are rolled, event A occurs when the sum of the uppermost faces is odd. Event B occurs when the sum of the uppermost faces is two, three, eight or nine.

Explain whether events A and B are mutually exclusive, independent or neither. Justify your answer.

Solution

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(A)=\frac{18}{36}=\frac{1}{2}$ $\quad$ $P(B)=\frac{12}{36}=\frac{1}{3}$ $\quad$ $P(A\cap B)=\frac{6}{36}=\frac{1}{6}$

A and B are not mutually exclusive since $P(A\cap B)\neq 0$

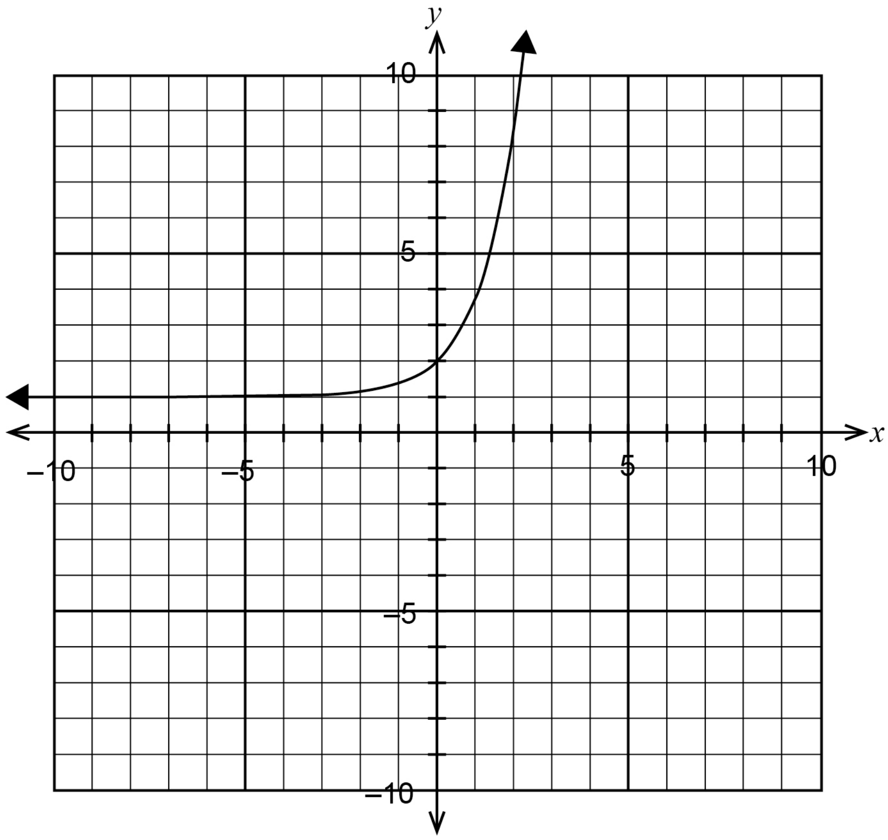
A and B are independent since $P(A\cap B)=P(A)\times P(B)$

Specific behaviours

- ✓ constructs a sample space for the events
- ✓ determines $P(A)$ and $P(B)$
- ✓ determines $P(A\cap B)$ and states that A and B are not mutually exclusive
- ✓ reasons correctly that A and B are independent

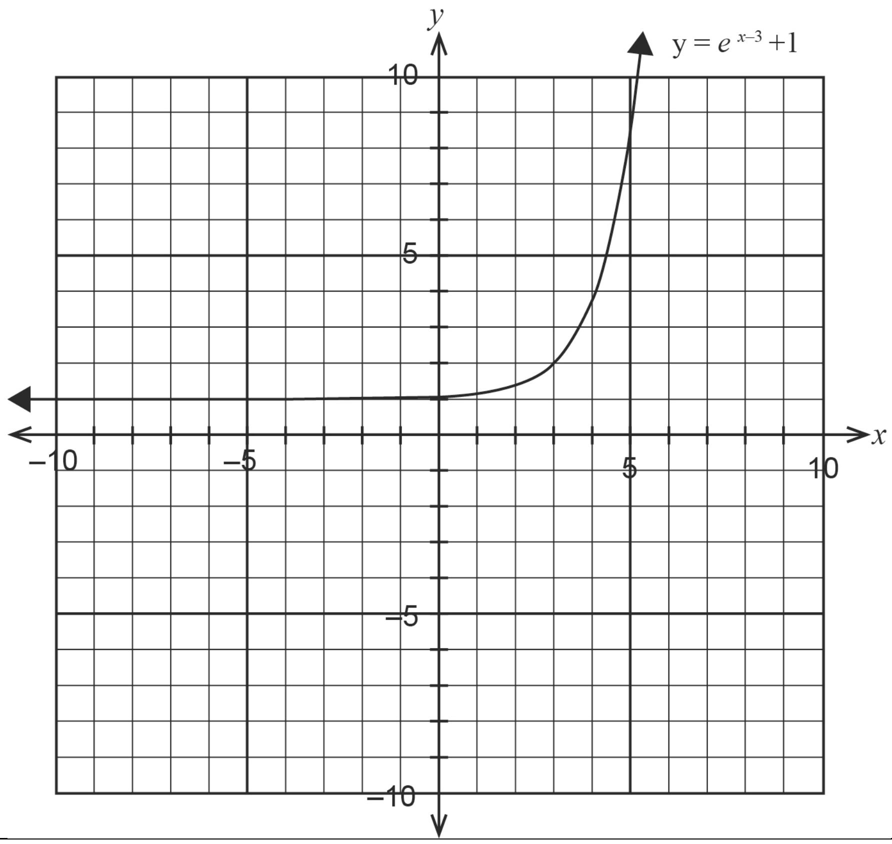
Question 4 **(8 marks)**

The function $f(x) = e^x + 1$ is graphed on the axes below.



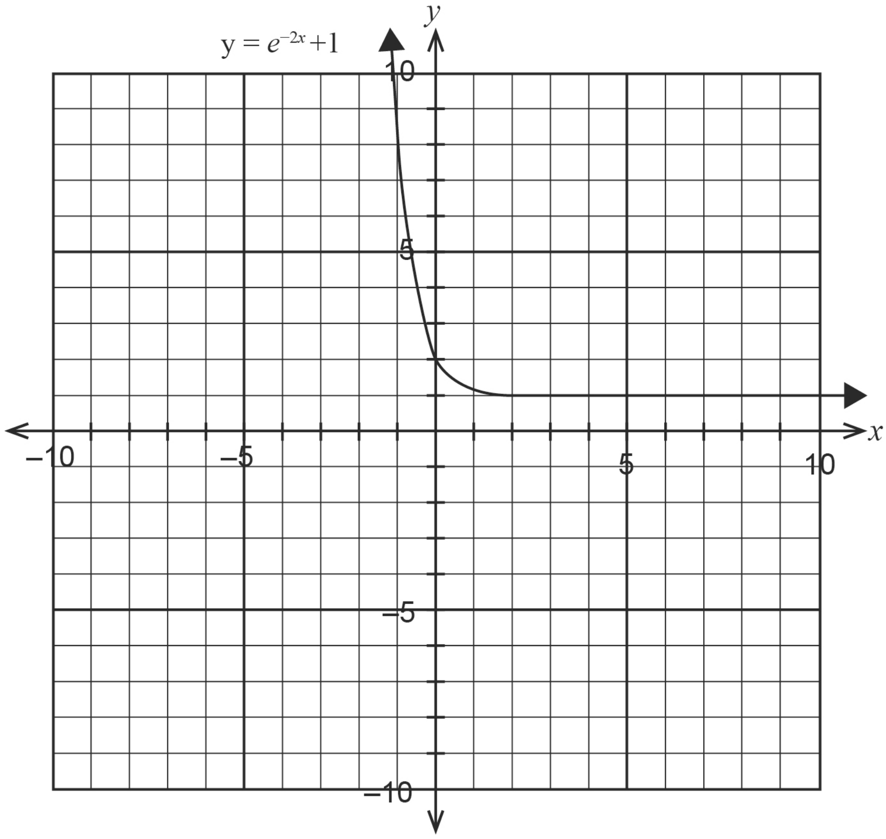
(a) On the same axes, sketch the following functions, showing all relevant features.
Label each graph.

(i) $f(x-3)$ (2 marks)

Solution

Specific behaviours
✓ sketches curve with the horizontal asymptote unchanged ✓ moves curve three units to the right

(ii) $f(-2x)$ (3 marks)

Solution



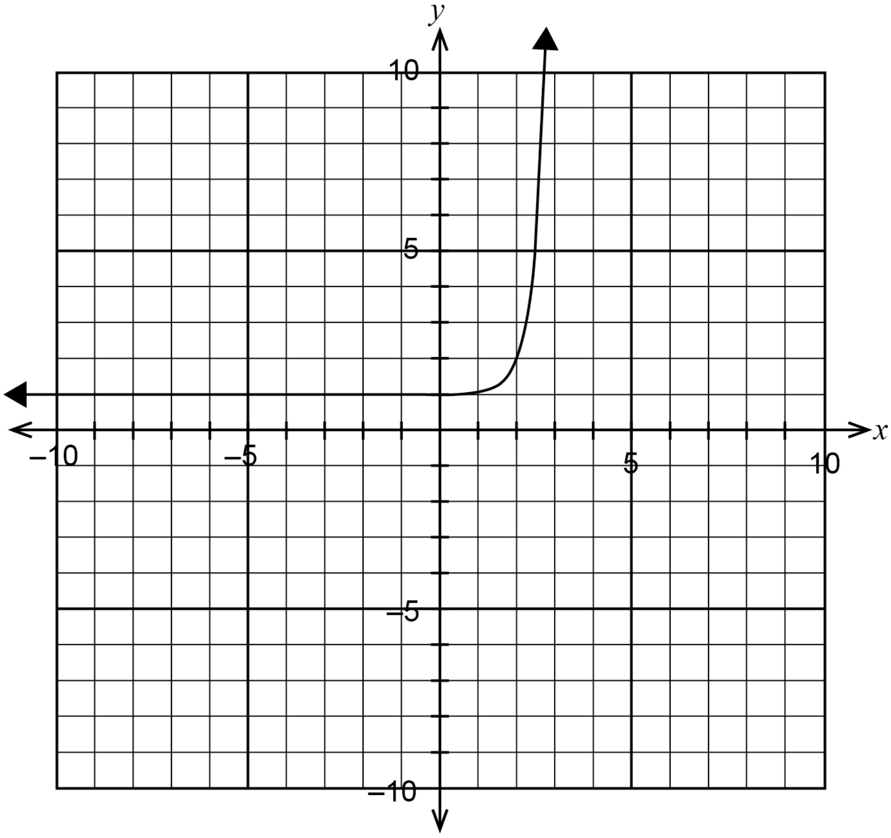
Specific behaviours

✓ reflects curve in the y-axis

✓ dilates graph by a factor of $\frac{1}{2}$ parallel to x-axis

✓ indicates correct horizontal asymptote

(b) The graph $y = g(x)$ is drawn below. Given that $g(x) = f(ax - 6)$ where a is a constant, determine the value of a. (3 marks)



Solution
y-intercept $(0,2)$ moved firstly to $(6,2)$ then dilated to $(2,2)$ by $\frac{1}{a}$ dilation of factor one-third, therefore $a = 3$
Specific behaviours
✓ uses initial translation of 6 units to right ✓ uses dilation of $\frac{1}{a}$ parallel to x-axis ✓ determines $a = 3$

OR

Alternative solution
Substitute (2,2) into $f(ax-6)$ $2 = e^{2a-6} + 1$ $1 = \frac{e^{2a}}{e^6}$ $2a = 6$ $a = 3$
Specific behaviours
✓ substitutes a known point into $g(x)$ ✓ simplifies an expression involving a ✓ determines $a = 3$

Question 5 **(4 marks)**

Given that $y = x^{\frac{1}{3}}$, use $x = 1000$ and the increments formula $\delta y \approx \frac{dy}{dx}\delta x$ to determine an approximate value for $\sqrt[3]{1006}$.

Solution
$\frac{dy}{dx} = \frac{1}{3}x^{\frac{-2}{3}}$ $\delta y \approx \frac{1}{3}x^{\frac{-2}{3}} \times 6$ When $x = 1000$, $\delta y \approx 2 \times \frac{1}{\left(\sqrt[3]{1000}\right)^2}$ $\approx \frac{2}{100}$ $\therefore \sqrt[3]{1006} \approx 10.02$
Specific behaviours
✓ substitutes for x correctly ✓ determines $\frac{dy}{dx}$ ✓ uses $\frac{\delta y}{\delta x}$ correctly ✓ determines approximate value

Question 6 **(6 marks)**

Let $f(x)=x-7$ and $g(x)=\frac{1}{x}$.

(a) State $g\circ f(x)$ with its domain and range. (3 marks)

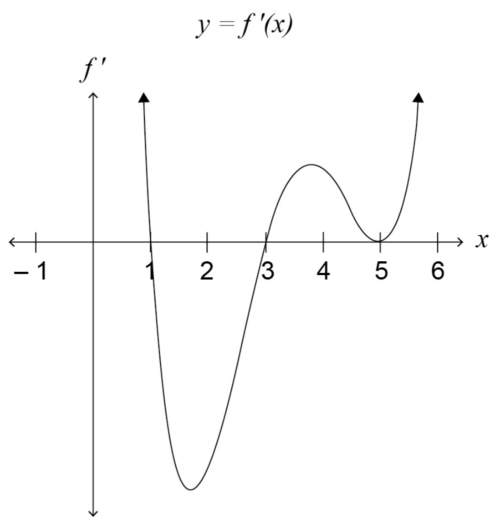
Solution
$g\circ f(x)=\frac{1}{x-7}$ Domain: $x\neq 7$ Range: $y\neq 0$
Specific behaviours
✓ states correct rule for $g\circ f(x)$ ✓ states correct domain ✓ states correct range

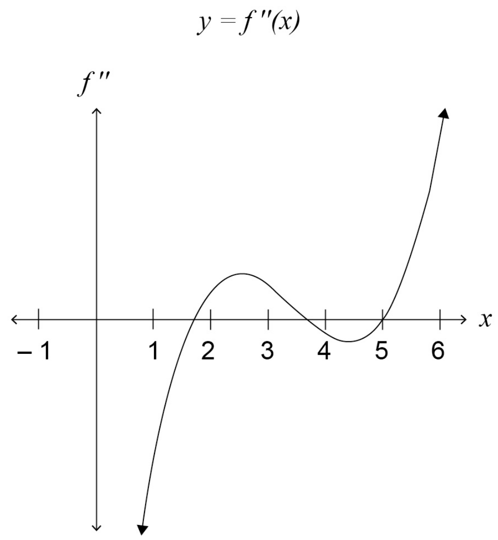
(b) Determine $h(x)$ if $h\circ f(x)=10x-49$. (3 marks)

Solution
let $f(x)=y$ then $y=x-7$ $x=y+7$ $h(y)=10(y+7)-49$ $=10y+70-49$ $=10y+21$ $\therefore h(x)=10x+21$
Specific behaviours
✓ substitutes $f(x)$ into $h\circ f(x)$ ✓ manipulates expression ✓ states $h(x)=10x+21$

Question 7 **(4 marks)**

The graphs of $y = f'(x)$ and $y = f''(x)$ of a function $y = f(x)$ are shown below. The function $y = f(x)$ passes through points $(1,4),(3,-2)$ and $(5,1)$.





(a) Determine the coordinates of the maximum and minimum points of $y = f(x)$. (2 marks)

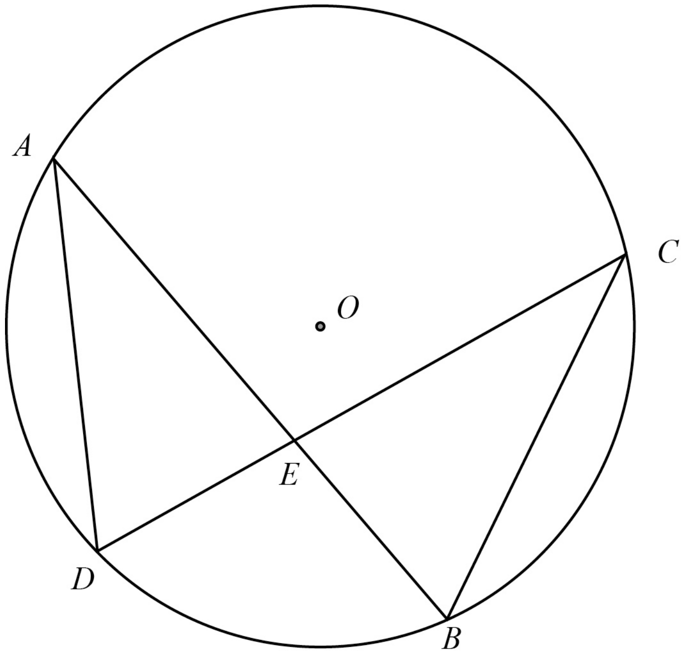
Solution
Stationary points at $x = 1, x = 3$ and $x = 5$ as $f'(x) = 0$ Maximum turning point at (1, 4) as $f''(x) < 0$ and minimum turning point at (3, –2) as $f''(x) > 0$.
Specific behaviours
✓ determines x values of turning points ✓ labels the turning points as maximum or minimum and states coordinates

(b) Determine whether there exists a horizontal point of inflection. Give reasons. (2 marks)

Solution
There exists a horizontal point of inflection at $x = 5$ as gradient is positive on either side of the stationary point at $x = 5$ **OR** There exists a horizontal point of inflection at $x = 5$ as both $f'(x) = 0 = f''(x)$.
Specific behaviours
✓ identifies $x = 5$ as an inflection point ✓ explains why the inflection point is horizontal

Question 8 **(5 marks)**

Points A, B, C and D lie on the circle with centre O, as shown below, with $\overline{AB}, \overline{CD}, \overline{AD}$ and $\overline{CB}$ chords of the same circle. Point E is the point of intersection of chords $\overline{AB}$ and $\overline{CD}$.



Prove $\dfrac{AE}{AD} = \dfrac{CE}{CB}$.

Solution
Angles $\angle DAB$ and $\angle DCB$ are on the same arc and are therefore congruent. i.e. $\angle A = \angle C$ Likewise $\angle ADC$ and $\angle ABC$ are congruent. i.e. $\angle D = \angle B$ Due to the AA test, ΔDAE is similar to ΔBCE with A corresponding to C and D corresponding to B. Therefore $\dfrac{AE}{AD} = \dfrac{CE}{CB}$.
Specific behaviours
✓ recognises that triangles ΔDAE and ΔBCE are similar ✓ identifies one pair congruent angles in triangles with reason ✓ identifies two pairs of congruent angles in triangles with reason ✓ states corresponding sides or angles in triangles ✓ states required equation

Question 9 **(3 marks)**

The derivatives of the sequence $1, x, x^2, x^3, \ldots, x^{n-1}$ are $0, 1, 2x, 3x^2, \ldots, (n-1)x^{n-2}$. If the sum of the power series $1 + x + x^2 + x^3 + \ldots + x^{n-1} = \dfrac{1-x^n}{1-x}$, show that the sum of the series of derivatives $1 + 2x + 3x^2 + \ldots + (n-1)x^{n-2} = \dfrac{x^n(n-1) - nx^{n-1} + 1}{(1-x)^2}$

Solution
As, $\dfrac{d}{dx}\left(1+x+x^2+x^3+\ldots+x^{n-1}\right) = 1+2x+3x^2+\ldots+(n-1)x^{n-2}$, differentiating RHS, gives the sum of the second series (Use quotient rule) $\dfrac{d}{dx}\left(\dfrac{1-x^n}{1-x}\right) = \dfrac{(1-x)(-nx^{n-1})+(1-x^n)}{(1-x)^2}$ $\dfrac{d}{dx}\left(\dfrac{1-x^n}{1-x}\right) = \dfrac{-nx^{n-1}+nx^n+1-x^n}{(1-x)^2}$ $\dfrac{d}{dx}\left(\dfrac{1-x^n}{1-x}\right) = \dfrac{x^n(n-1)-nx^{n-1}+1}{(1-x)^2}$
Specific behaviours
✓ applies the quotient rule ✓ expands the numerator ✓ simplifies the numerator to show result

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