Semester Two Examination, 2016

Question/Answer Booklet

MATHEMATICS SPECIALIST UNITS 1 AND 2

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work:	ten minutes
Working time for section:	one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	149	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Booklet.

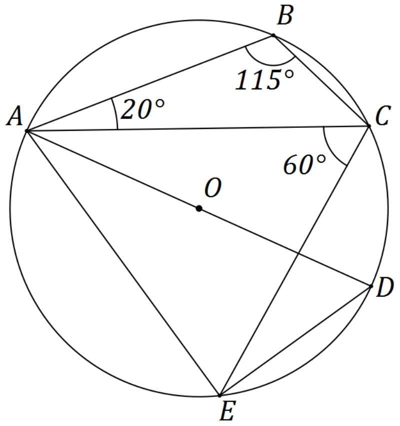
Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(5 marks)**

Points B, C and E lie on the circle with diameter AOD as shown below. $\angle ABC = 115°$, $\angle BAC = 20°$ and $\angle ACE = 60°$.



Determine the size of the following angles.

(a) $\angle ADE$. (1 mark)

Solution
$60°$
Specific behaviours
✓ states angle

(b) $\angle EAD$. (1 mark)

Solution
$30°$
Specific behaviours
✓ states angle

(c) $\angle AEC$. (1 mark)

Solution
$65°$
Specific behaviours
✓ states angle

(d) $\angle CAD$. (1 mark)

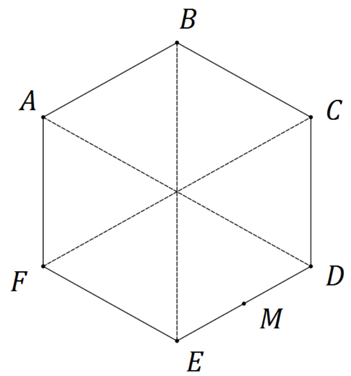
Solution
$25°$
Specific behaviours
✓ states angle

(e) $\angle CED$. (1 mark)

Solution
$25°$
Specific behaviours
✓ states angle

Question 9 **(7 marks)**

(a) $ABCDEF$ is a regular hexagon. The midpoint of side DE is M.



Let $\mathbf{a} = \overrightarrow{AB}$ and $\mathbf{b} = \overrightarrow{AF}$. Express each of the following in terms of $\mathbf{a}$ and $\mathbf{b}$.

(i) $\overrightarrow{BC}$. (1 mark)

Solution
$\overrightarrow{BC} = \mathbf{a} + \mathbf{b}$
Specific behaviours
✓ states correct expression

(ii) $\overrightarrow{AE}$. (1 mark)

Solution
$\overrightarrow{AE} = \overrightarrow{AF} + \overrightarrow{FE} = \mathbf{b} + (\mathbf{a} + \mathbf{b}) = \mathbf{a} + 2\mathbf{b}$
Specific behaviours
✓ states correct expression

(iii) $\overrightarrow{MB}$. (1 mark)

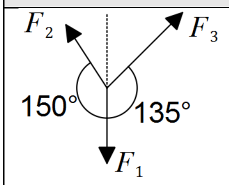
Solution
$\overrightarrow{MB} = \overrightarrow{ME} + \overrightarrow{BE} = -0.5\mathbf{a} - 2\mathbf{b}$
Specific behaviours
✓ states correct expression

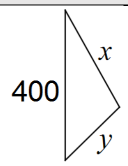
(b) Three forces, F_1, F_2 and F_3 act on a body that remains in equilibrium.

F_1 has a magnitude of 400 N. The angle between the directions of F_1 and F_2 is 150°, between F_1 and F_3 is 135° and between F_2 and F_3 is 75°.

Determine the magnitudes of F_2 and F_3, rounding your answers to the nearest whole number. (4 marks)

Solution





For equilibrium, $F_1 + F_2 + F_3 = 0$ - nose to tail vectors in triangle.

If magnitudes of $F_2 = x$ and $F_3 = y$ then $\frac{x}{\sin 45} = \frac{y}{\sin 30} = \frac{400}{\sin 105}$

Solving gives $x = F_2 = 293$ N and $y = F_3 = 207$ N.

Specific behaviours

✓ sketch diagram
✓ triangle or equation indicates force vectors must sum to zero
✓ solves for x
✓ solves for y

Question 10 **(7 marks)**

(a) A number is to be formed by randomly selecting three **different** digits from those in the number 93265. Determine how many different numbers

(i) start with an odd digit. (1 mark)

Solution
$3 \times 4 \times 3 = 36$ numbers
Specific behaviours
✓ calculates correct number

(ii) end with an even digit. (1 mark)

Solution
$2 \times 4 \times 3 = 24$ numbers
Specific behaviours
✓ calculates correct number

(iii) start with an odd digit or end in an even digit. (2 marks)

Solution
$3 \times 2 \times 3 = 18$ numbers start with an odd digit and end in an even digit $36 + 24 - 18 = 42$ numbers
Specific behaviours
✓ calculates set intersection ✓ calculates correct number for set union

(b) A computer user has forgotten their six character, case-sensitive password, but know that they always use a permutation of F, F, 1, 9, 9, and 9 - their initials and the year they were born. Determine how many passwords are possible if

(i) the F's must both be uppercase. (2 marks)

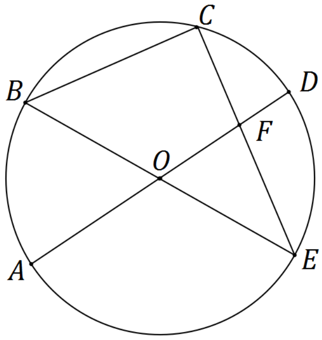
Solution
$\frac{6!}{2! \times 3!} = \frac{6 \times 5 \times 4}{2} = 60$ passwords
Specific behaviours
✓ shows correct method ✓ calculates correct number

(ii) either F can be lowercase or uppercase. (1 mark)

Solution
FF can be replaced with Ff, fF or ff, so $4 \times 60 = 240$ passwords
Specific behaviours
✓ calculates correct number

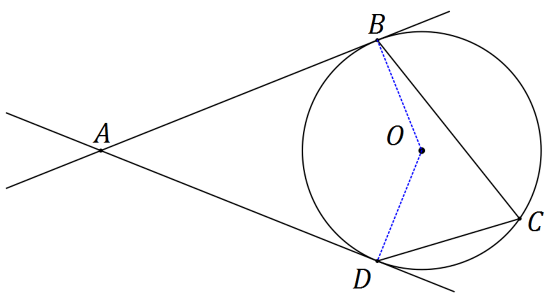
Question 11 **(8 marks)**

(a) Triangle BCE is such that B, C and E lie on a circle with centre O and radius 29 cm. Diameter AD and chord CE intersect at F, so that $DF = 8.5$ cm and $EF = 25.5$ cm. Determine the lengths OF, CF and BC. (5 marks)



Solution
$OF = 29 - 8.5 = 20.5$ cm $CF \times EF = DF \times AF \Rightarrow CF = 8.5 \times \frac{29+20.5}{25.5} = 16.5$ cm ΔBCE is right-angled $\Rightarrow BC = \sqrt{(29+29)^2 - (16.5+25.5)^2} = 40$ cm
Specific behaviours
✓ calculates OF ✓ uses intersecting chord theorem   ✓ calculates CF ✓ uses angle in semicircle   ✓ calculates BC

(b) In the diagram below, points B, C and D lie on a circle with centre O. The tangents to the circle at B and D intersect at point A. If $\angle BAD = x$, prove that $\angle BCD = 90° - \frac{x}{2}$. (3 marks)



Solution
In quadrilateral $ABOD$, $\angle ABO = \angle ADO = 90°$ (tangent-radius angle) $\angle BOD = 360 - 90 - 90 - x = 180 - x$ (angle sum in quadrilateral) $\angle BCD = \frac{1}{2} \times \angle BOD = \frac{1}{2} \times 180 - x = 90° - \frac{x}{2}$ (angle at centre twice that on circ.)
Specific behaviours
✓ adds radii to diagram noting right-angles   ✓ determines $\angle BOD$ with reason   ✓ determines $\angle BCD$ with reason

Question 12 **(9 marks)**

Transformation A is an anti-clockwise rotation about the origin of 90° and matrix $B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

(a) Represent transformation A as a 2×2 matrix. (2 marks)

Solution
$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
Specific behaviours
✓ writes matrix for 90° rotation ✓ matrix is for anticlockwise rotation

(b) Describe the transformation represented by matrix B. (2 marks)

Solution
A dilation parallel to x-axis of scale factor 2 and dilation parallel to y-axis of scale factor 3.
Specific behaviours
✓ two dilations ✓ fully qualifies both dilations with directions and scale factors

(c) Determine the coordinates of the point $P(-15, -11)$ following transformation A and then transformation B. (2 marks)

Solution
$P' = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \times \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} -15 \\ -11 \end{bmatrix}$ $= \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \times \begin{bmatrix} 11 \\ -15 \end{bmatrix} = \begin{bmatrix} 22 \\ -45 \end{bmatrix}$
Specific behaviours
✓ calculates first image using A ✓ calculates P'

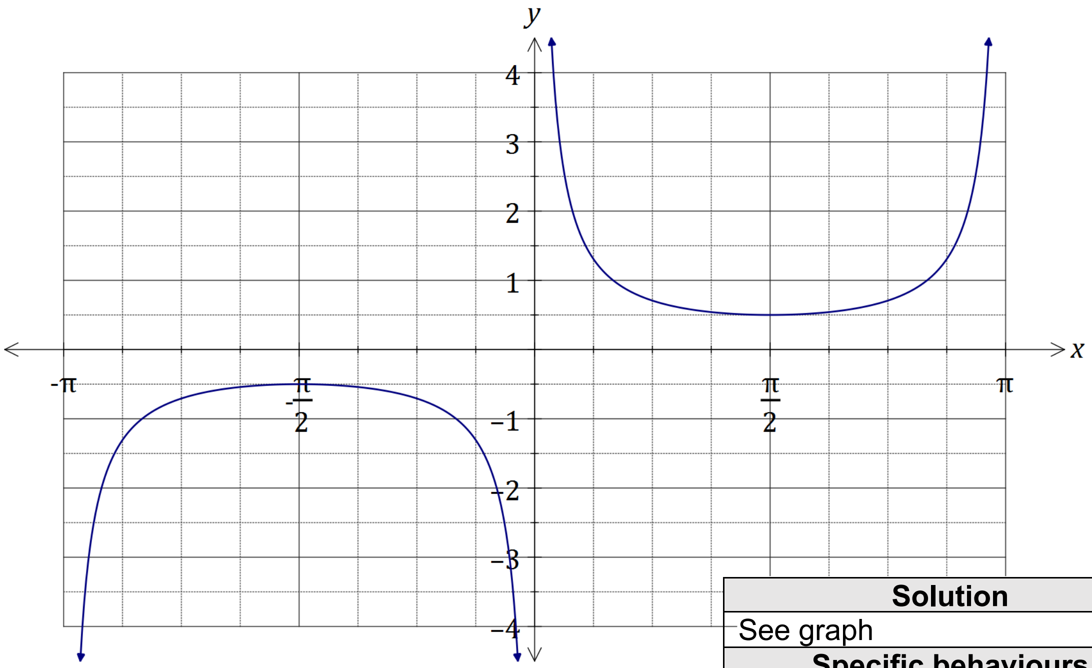
(d) Following transformation B and then transformation A, point Q is transformed to point $Q'(12, 7)$.

Determine the single matrix that will transform Q' back to Q and hence determine the coordinates of point Q. (3 marks)

Solution
$Q' = ABQ \Rightarrow Q = B^{-1}A^{-1}Q'$ $B^{-1}A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{1}{3} & 0 \end{bmatrix}$ $Q = \begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{1}{3} & 0 \end{bmatrix} \times \begin{bmatrix} 12 \\ 7 \end{bmatrix} = \begin{bmatrix} 3.5 \\ -4 \end{bmatrix}$
Specific behaviours
✓ determines inverse of B and inverse of A ✓ determines single matrix   ✓ determines coordinates of Q

Question 13 **(8 marks)**

(a) On the axes below sketch the graph of $y = \frac{1}{2}\sec\left(x - \frac{\pi}{2}\right)$. (3 marks)



Solution
See graph
Specific behaviours
✓ max/min correct locations ✓ vertical asymptotes ✓ smooth curves

(b) Consider the function $f(t) = 2\sin t - 5\cos t,\ t \geq 0$.

(i) $f(t)$ can be expressed in the form $r\sin(t - \alpha)$, where $r > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$. Determine the values of r and α, rounded to 2 decimal places. (3 marks)

Solution
$2\sin t - 5\cos t = r\sin(t - \alpha)$ $= r\sin t\cos\alpha - r\cos t\sin\alpha$ $r\cos\alpha = 2$ and $r\sin\alpha = 5 \Rightarrow r = \sqrt{29} \approx 5.39,\ \alpha = \tan^{-1}\frac{5}{2} \approx 1.19$
Specific behaviours
✓ uses difference identity ✓ determines r ✓ determines α

(ii) Hence or otherwise determine the minimum value of $f(t)$ and the smallest value of t for this minimum to occur. (2 marks)

Solution
Minimum value is $-\sqrt{29}$ Occurs when $t - \tan^{-1}\frac{5}{2} = \frac{3\pi}{2} \Rightarrow t \approx 5.90$
Specific behaviours
✓ states minimum value ✓ determines first time, $t \geq 0$

Question 14 **(8 marks)**

(a) Consider the vectors $\mathbf{p} = (24, -143)$ and $\mathbf{q} = (20, -21)$. Determine

(i) the angle between the directions of vectors $\mathbf{p}$ and $\mathbf{q}$. (1 mark)

Solution
Using CAS, angle is 34.1°
Specific behaviours
✓ states correct angle.

(ii) two vectors that are perpendicular to $\mathbf{q}$ and have the same magnitude as $\mathbf{p}$. (3 marks)

Solution
Magnitude of required vectors are $\lvert\mathbf{p}\rvert = 145$ Unit vectors $\perp$ to $\mathbf{q}$ are $\pm\frac{1}{29}(21, 20)$ Required vectors are $\pm\frac{145}{29} \times (21, 20) = (105, 100)$ and $(-105, -100)$
Specific behaviours
✓ calculates magnitudes of $\mathbf{p}$ and $\mathbf{q}$ ✓ determines at least one perpendicular vector ✓ states both required vectors

(b) If $\overrightarrow{AB} = (3, 4)$ and $\overrightarrow{AC} = (-2, 1)$, determine

(i) the component of $\overrightarrow{AB}$ parallel to $\overrightarrow{AC}$. (2 marks)

Solution
Vector projection of $\overrightarrow{AB}$ on $\overrightarrow{AC}$: $\frac{(3, 4) \cdot (-2, 1)}{(-2, 1) \cdot (-2, 1)} \times (-2, 1) = (0.8, -0.4)$
Specific behaviours
✓ substitutes correctly into vector projection formula ✓ evaluates component

(ii) the component of $\overrightarrow{AB}$ perpendicular to $\overrightarrow{AC}$. (2 marks)

Solution
Let component be $\mathbf{r}$. $\mathbf{r} + (0.8, -0.4) = \overrightarrow{AB}$ $\mathbf{r} = (2.2, 4.4)$
Specific behaviours
✓ shows use of vector addition ✓ evaluates component

Question 15 **(8 marks)**

(a) Express the recurring decimal $1.1\overline{58}$ as a rational number. (2 marks)

Solution
Let $x = 1.158585858585\ldots$ then $100x = 115.858585858\ldots$ So $100x - x = 99x = 114.7 \Rightarrow x = \frac{1147}{990}$
Specific behaviours
✓ multiplies by 100 ✓ expresses as rational number in lowest terms

(b) Use a counterexample to explain why the statement $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(2xy = 24)$ is false. (2 marks)

Solution
If $x = 24$, then $y = \frac{1}{2}$ and so statement is false, as an integer value for y clearly does not exist for all values of x.
Specific behaviours
✓ counterexample ✓ explanation

(c) Prove, by contradiction, that $\sqrt{6}$ is irrational. (4 marks)

Solution
Assume that $\sqrt{6}$ is rational and can be expressed in the form $\frac{a}{b}$ where a and b are both integers with no common factors. Then 6= $\frac{a^2}{b^2} \Rightarrow a^2 = 6b^2 = 2(3b^2)$, so that a^2 and hence a must be even. If $a = 2k$ (k an integer) then $(2k)^2 = 6b^2 \Rightarrow 3b^2 = 2k^2$, so that $3b^2$ and hence b^2 and b must also be even. But if both a and b are multiples of 2, this contradicts the original assumption, which means it is false and so $\sqrt{6}$ is not rational, and so must be irrational.
Specific behaviours
✓ clearly makes rational assumption ✓ deduces that a must be even ✓ deduces that b must be even ✓ explains contradiction

Question 16 **(7 marks)**

(a) Let the angle $\theta = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$.

(i) Use your calculator to write down an exact value for $\sin\left(\frac{\pi}{12}\right)$. (1 mark)

Solution
$\frac{\sqrt{2}(\sqrt{3}-1)}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$
Specific behaviours
✓ writes an exact value

(ii) Use an angle sum or difference identity to show how to obtain the above exact value for $\sin\left(\frac{\pi}{12}\right)$. (3 marks)

Solution
$\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$ $= \frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2} - \frac{1}{2}\times\frac{\sqrt{2}}{2}$ $= \frac{\sqrt{6}-\sqrt{2}}{4}$
Specific behaviours
✓ uses difference identity   ✓ shows substitution of four correct values   ✓ shows simplification

(b) Prove the identity $\sin x + \sin 2x + \sin 3x = (1 + 2\cos x)\sin 2x$. (3 marks)

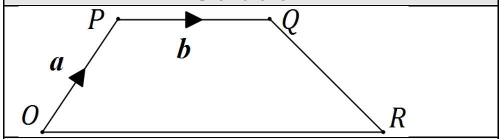
Solution
$RHS = (1 + 2\cos x)\sin 2x$ $= \sin 2x + 2\sin 2x\cos x$ $= \sin 2x + \sin(2x + x) + \sin(2x - x)$ $= \sin x + \sin 2x + \sin 3x$ $= LHS$
Specific behaviours
✓ expands RHS   ✓ uses product as sum and difference   ✓ completes proof in logical steps

Question 17 **(9 marks)**

Trapezium $OPQR$ has parallel sides PQ and OR such that $|\overrightarrow{OR}| = k|\overrightarrow{PQ}|$. Let $\overrightarrow{OP} = \mathbf{a}$ and $\overrightarrow{PQ} = \mathbf{b}$.

(a) Sketch the trapezium. (1 mark)

Solution



Specific behaviours

✓ draws labelled trapezium

(b) Determine vectors for $\overrightarrow{OQ}$ and $\overrightarrow{PR}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$. (2 marks)

Solution
$\overrightarrow{OQ} = \mathbf{a} + \mathbf{b}$ and $\overrightarrow{PR} = k\mathbf{b} - \mathbf{a}$.
Specific behaviours
✓ states first vector ✓ states second vector

(c) Show that the scalar product of $\overrightarrow{OQ}$ and $\overrightarrow{PR}$ is $k|\mathbf{b}|^2 - |\mathbf{a}|^2 + (k-1)\mathbf{a} \cdot \mathbf{b}$. (2 marks)

Solution
$(\mathbf{a}+\mathbf{b}) \cdot (k\mathbf{b} - \mathbf{a}) = k\mathbf{a} \cdot \mathbf{b} - \mathbf{a}.\mathbf{a} + k\mathbf{b}.\mathbf{b} - \mathbf{a}.\mathbf{b}$ $= k|\mathbf{b}|^2 - |\mathbf{a}|^2 + (k-1)\mathbf{a} \cdot \mathbf{b}$
Specific behaviours
✓ expands scalar product ✓ simplifies scalar product

(d) Simplify your result from (c) if $k = 1$, $\mathbf{a} = \mathbf{i} + 4\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} - 2\sqrt{2}\mathbf{j}$. (2 marks)

Solution
$k\lvert\mathbf{b}\rvert^2 - \lvert\mathbf{a}\rvert^2 + (k-1)\mathbf{a}\cdot\mathbf{b} = \lvert\mathbf{b}\rvert^2 - \lvert\mathbf{a}\rvert^2 + (1-1)\mathbf{a}\cdot\mathbf{b}$ $= (9+8) - (1+16) + 0$ $= 0$
Specific behaviours
✓ substitutes $k = 1$ to eliminate $\mathbf{a}\cdot\mathbf{b}$ ✓ determines magnitudes and simplifies expression to zero

(e) Explain the geometric significance of your result from (d). (2 marks)

Solution
The values of k, $\mathbf{a}$ and $\mathbf{b}$ have turned the trapezium into a rhombus and as the scalar product is zero, the diagonals must intersect at right angles.
Specific behaviours
✓ identifies significance of values   ✓ uses scalar product to conclude that diagonals intersect at right angles

Question 18 **(7 marks)**

(a) The work done, in joules, by a force $\mathbf{F}$ Newtons in changing the displacement of an object $\mathbf{s}$ metres is given by the scalar product of $\mathbf{F}$ and $\mathbf{s}$. Calculate the work done when a force of 750 N moves an object a distance of 85 cm at an angle of 5° to the force.

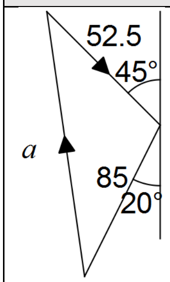
(2 marks)

Solution
$750 \times 0.85 \times \cos 5° = 635.1$ N
Specific behaviours
✓ substitutes correct values into scalar product ✓evaluates work done

(b) A drone flies with a constant velocity and height above level ground, over which a wind blows from the north west at 3.5 metres per second. After 15 seconds, the drone reaches a point 85 metres on a bearing of 020° from where it was launched. Determine the velocity of the drone, giving its magnitude to two decimal places and bearing to the nearest degree.

(5 marks)

Solution



Wind component is $15 \times 3.5 = 52.5$ m

$a^2 = 52.5^2 + 85^2 - 2(52.5)(85)\cos 115 \Rightarrow a = 117.2737$

Speed of drone is $117.2737 \div 15 = 7.82$ m/s (2dp).

$\frac{52.5}{\sin x} = \frac{106.4211}{\sin 115} \Rightarrow x = 24°$ and so bearing is $020 - 24 = 356°$.

Specific behaviours

✓ sketch displacement vector diagram
✓ uses cosine rule to determine drone displacement
✓ calculates drone speed
✓ uses sine rule to determine angle
✓ determines bearing

Question 19 **(8 marks)**

(a) A high school has 5 male and 9 female volunteers from which to choose a debating team of 5 students. Determine the number of different teams that can be formed if

(i) there are no special requirements. (1 mark)

Solution
$\binom{14}{5} = 2002$ teams
Specific behaviours
✓ evaluates correct number

(ii) there must be a captain and a vice-captain. (2 marks)

Solution
$\binom{14}{1} \times \binom{13}{1} \times \binom{12}{3} = 40040$ teams
Specific behaviours
✓ shows a suitable method ✓ evaluates correct number

(iii) there must be more females than males, but at least one male. (2 marks)

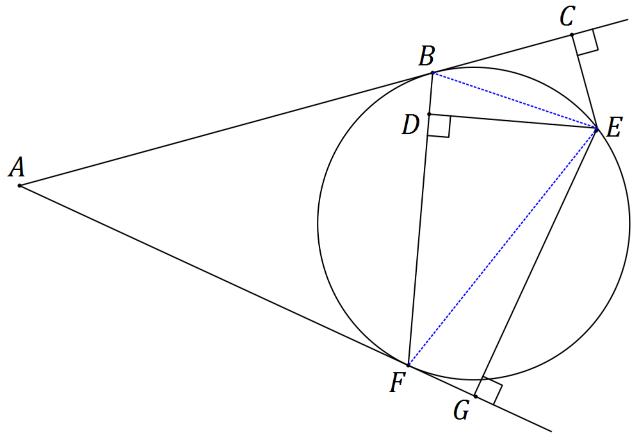
Solution
$\binom{5}{1} \times \binom{9}{4} + \binom{5}{2} \times \binom{9}{3} = 1470$ teams
Specific behaviours
✓ shows a suitable method ✓ evaluates correct number

(b) Determine how many **different** numbers must be selected from the first 25 positive integers to be certain that at least one of them will be twice the other. (3 marks)

Solution
Partition integers into pigeonholes, where double included if possible: {1, 2}, {3, 6}, {4, 8}, {5, 10}, {7, 14}, {9, 18}, {11, 22}, {12, 24}, {13}, {15}, {16}, {17}, {19}, {20}, {21}, {23}, {25}. *NB Other partitions possible.* There are 17 partitions (pigeonholes) and so 18 numbers (pigeons) are required to ensure that at least one will be twice the other.
Specific behaviours
✓ partitions most integers ✓ systematically partitions all integers ✓ applies pigeonhole principle to get correct number

Question 20 **(7 marks)**

In the diagram below, the tangents from point A touch the circle at B and F. Point E lies on the major arc BF and D lies on BF so that $DE \perp BF$. Points C and G lie on AB and AF extended respectively such that $EC \perp AC$ and $EG \perp AG$.



(a) Show that ΔBCE and ΔFDE are similar. (3 marks)

Solution
$\angle CBE = \angle DFE$ (alternate segment theorem) $\angle BCE = \angle FDE$ (both right) Hence $\Delta BCE \sim \Delta FDE$ (AA)
Specific behaviours
✓ shows one pair of angles equal with reason ✓ shows second pair of angles equal with reason ✓ makes conclusion that similar

(b) Show that $DE^2 = CE \times GE$. (4 marks)

Solution
$\angle GFE = \angle DBE$ (alternate segment theorem) $\angle BDE = \angle FGE$ (both right) Hence $\Delta BDE \sim \Delta FGE$ (AA) So ratio of sides is $\frac{DE}{GE} = \frac{BE}{FE}$ But ratio of sides from (a) is $\frac{CE}{DE} = \frac{BE}{FE} \Rightarrow \frac{CE}{DE} = \frac{DE}{GE} \Rightarrow DE^2 = CE \times GE$.
Specific behaviours
✓ realises second pair of similar triangles required ✓ uses same reasoning from (a) to show $\Delta BDE \sim \Delta FGE$ ✓ states ratio of corresponding sides for both pairs of triangles ✓ uses common ratio to obtain result

End of questions

Additional working space

Question number: _________