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REVISION 2

2016

MATHEMATICS METHODS

Units 1 & 2

Semester 2

SOLUTIONS

SECTION 1 – Calculator-free

Question 1 **(6 marks)**

Complete the following

$$f(x) = x^2 - 4x$$

$$f(x+h) = (x+h)^2 - 4(x+h)$$

$$= x^2 + 2x^2h + \underline{h^2 - 4x - 4h} \quad \checkmark$$

By definition

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{(x+h) - x}\right)$$

and

$$f(x+h) - f(x) = \underline{\cancel{x^2} + 2x^2h + h^2 \cancel{-4x} - 4h - \left(\cancel{x^2} - \cancel{4x}\right)} \quad \checkmark$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2x^2h + h^2 - 4h}{h} \quad \checkmark$$

$$= \underline{2x^2 + h - 4} \quad \checkmark$$

Therefore

$$f'(x) = \lim_{h \to 0}\left(2x^2 + h - 4\right) \quad \checkmark$$

$$\therefore \quad f'(x) = \underline{2x^2 - 4} \quad \checkmark$$

Question 2 **(7 marks)**

(a) $(x-1)(x^2+x+1)=x^3-1$

$(x-1)=0 \;\; or \;\; (x^2+x+1)=0$ ✓

$x=1 \qquad x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

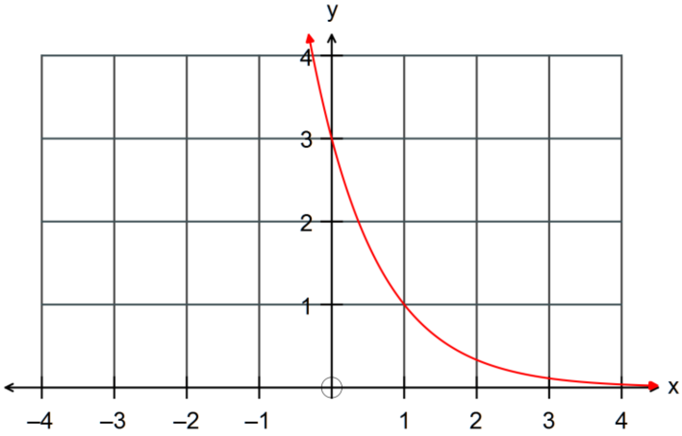
$b^2-4ac=1-4\times1\times1$

$b^2-4ac=-3$

$b^2-4ac<0$ ✓

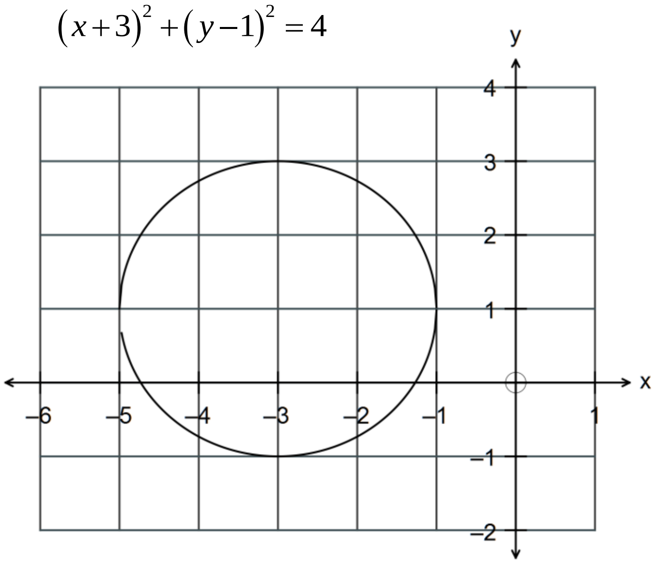
Therefore the only solution is $x=1$

(b) (i) $y=3^{1-x}$



✓✓✓

(ii) $(x+3)^2+(y-1)^2=4$



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Question 3 **(9 marks)**

(a) $\left(\dfrac{8^{-1}x^3}{y^{-6}}\right)^{-\frac{1}{3}} = \left(\dfrac{x^3y^6}{8}\right)^{-\frac{1}{3}} = \left(\dfrac{8}{x^3y^6}\right)^{\frac{1}{3}} = \dfrac{2}{xy^2}$
✓ ✓ ✓

(b) $2\left(81^{-\frac{1}{4}}\right) - \dfrac{1}{16^{-0.5}} + 17^0 = 2\left(\dfrac{1}{3}\right) - \sqrt{16} + 1 = \dfrac{2}{3} - 4 + 1 = -2\dfrac{1}{3}$
✓ ✓ ✓

(c) $16^x - 3\left(4^x\right) + 2 = 0$

Let $y = 4^x$

$y^2 - 3y + 2 = 0$ ✓

$(y-1)(x-2) = 0$

$y = 1\ or\ \ y = 2$

$4^x = 1\ or\ \ 4^x = 2$

$x = 0\ \ or\ \ 2^{2x} = 2^1$
✓

$x = \dfrac{1}{2}$
✓

Question 4 **(13 marks)**

(a) (i) $AP\ \ a = 10,\ d = -2$ ✓

$T_n = a + (n-1)d$

$T_n = 10 + (n-1)(-2)$

$T_n = 12 - 2n$ ✓

(ii) $T_{n+1} = T_n - 2, \quad T_1 = 10$ ✓✓

(b) 12, 6, 3, …..

$T_n = ar^{n-1}$

$T_n = 12 \times \left(\dfrac{1}{2}\right)^{n-1}$ ✓

$T_n = 24 \times \left(\dfrac{1}{2}\right)^{n}$ ✓ $= 24 \times 2^{-n} = 3 \times 2^{3-n}$

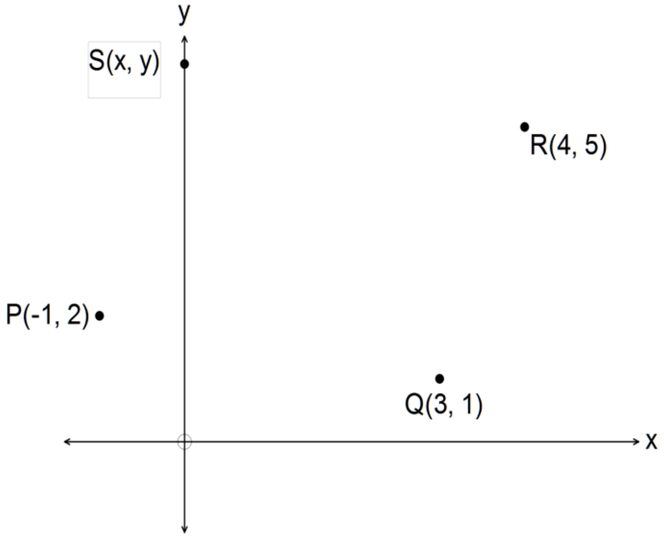
(c) $2, 1, \dfrac{1}{2}, \ldots.\ \ S_\infty = \dfrac{a}{1-r} = \dfrac{2}{1-\dfrac{1}{2}} = 4$ ✓✓

The tree will grow to 4 metres. ✓

(d) (i) $^{8}C_5 = \frac{8!}{5!3!} = \frac{8\times7\times6}{3\times2\times1} = 56$ ✓✓

(ii) $^{6}C_5 = 6$ ✓✓

Question 5 **(8 marks)**.



(a) (i) $m_{PQ} = -\frac{1}{4}$ $m_{QR} = 4$ ✓

$m_{PQ} \times m_{QR} = -\frac{1}{4}\times 4 = -1$ ✓

$\therefore\ PQ \perp QR$

(ii) $PQ = \sqrt{(-4)^2 + 1^2} = \sqrt{17}$ ✓ $QR = \sqrt{1^2 + 4^2} = \sqrt{17}$ ✓

Therefore PQ = PR

(b) $m_{PQ} = -\frac{1}{4}$ $m_{QR} = 4$

Therefore equation of RS: $y = -\frac{x}{4} + c$ $(4,5) \to 5 = -1 + c$ $y = -\frac{x}{4} + 6$

Therefore equation of PS: $y = 4x + c$ $(-1,2) \to 2 = -4 + c$ $y = 4x + 6$ ✓

Find the intersection of PQ and QR:

$-\frac{x}{4} + 6 = 4x + 6 \quad \Rightarrow \quad x = 0$

If $x = 0$, then $y = 6$

$S(0,6)$ ✓

(c) Midpoint of PR is $x = \frac{-1+4}{2} = 1.5,$ $y = \frac{2+5}{2} = 3.5 \Rightarrow (1.5, 3.5)$ ✓

Midpoint of SQ is $x = \frac{0+3}{2} = 1.5,$ $y = \frac{6+1}{2} = 3.5 \Rightarrow (1.5, 3.5)$ ✓

The midpoints are the same.

Therefore the diagonals of the parallelogram bisect each other.

Question 6 **(6 marks)**

(a) (i) $10^6 \times 2^4 = 1.6 \times 10^7$ ✓✓

(ii) $10^6 \div 2^3 = 0.125 \times 10^6 = 1.25 \times 10^5$ ✓✓

(iii) 10.20 a.m. + 10 minutes, so 10.30 a.m. ✓✓

Question 7 **(12 marks)**

(a) $\int (10 - 3x^2 + 12x)\,dx = 10x - x^3 + 6x^2 + c$ ✓

(b) $\int \left(\frac{x^2 + x - 12}{x-3} \right) dx = \int \left(\frac{(x+4)(x-3)}{x-3} \right) dx = \int (x+4)\,dx = \frac{x^2}{2} + 4x + c$ ✓

✓

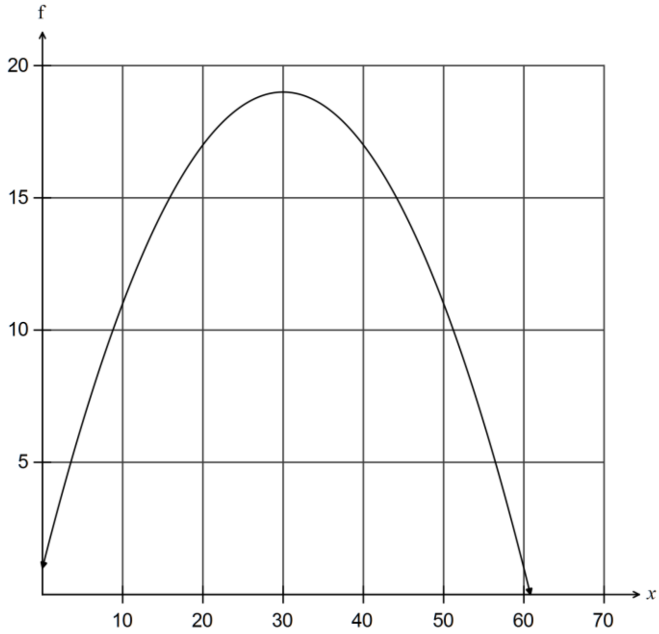
SECTION 2 – Calculator-assumed

Question 8 **(5 marks)**

(a) (i) 60.82 m ✓✓

(ii) 19 m ✓

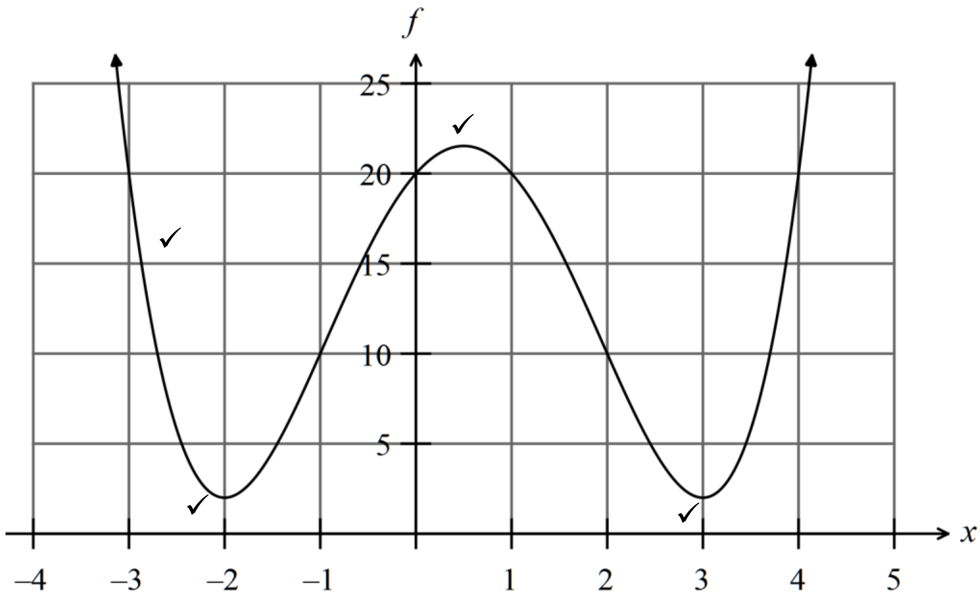
(iii)



✓✓

Question 9 **(10 marks)**

(a)



(b) (i) A(-3, 0) ✓✓

(ii) B is a maximum turning point, a local maximum and a double root where x = 0. ✓✓

(iii) $f(x) = x^4 - 9x^2$

$f'(x) = 4x^3 - 18x = 2x\left(2x^2 - 9\right)$ ✓

$If \quad f'(x) = 0, \quad x = 0 \quad or \quad x^2 = \dfrac{9}{2}$ The x ordinate for C is $\dfrac{3}{\sqrt{2}}$. ✓

Largest x is $x = +\sqrt{\dfrac{9}{2}} = \dfrac{3}{\sqrt{2}}$

Question 10 **(14 marks)**

(a) $v = 6 - 6t\, m/s$

$x = \int\left(6 - 6t\right)dt$ ✓

$x = 6t - 3t^2 + c$

$At \quad t = 0, \quad x = 0 \quad so \; c = 0$ ✓

$\therefore \quad x = 6t - 3t^2$ ✓

(b) Changes direction when $v = 0$ i.e. at $t = 1$. ✓✓

$x = 6t - 3t^2$

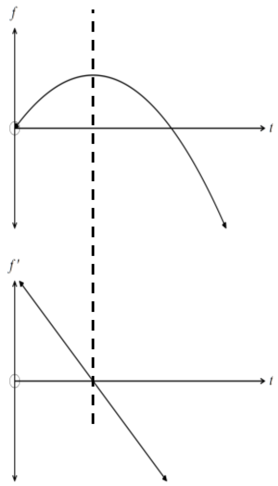
$At \quad t = 1, x = 3m$ ✓✓

(c) $v = 6 - 6t\, m/s$

$At \quad t = 2, \quad v = 6 - 6(2) = -6$ ✓

Speed is $6m/s$ ✓

(d)



✓✓

✓✓

(e) The turning point on the displacement graph has a gradient of 0.
This corresponds to the x intercept on the velocity graph. Where the displacement is increasing, the gradient on the velocity graph is positive and where the displacement is decreasing, this corresponds to where the velocity graph is negative. ✓✓✓

Question 11 **(6 marks)**

(a) $y = x^2 - 4$

$\frac{dy}{dx} = 2x$ ✓

Slope of 3?

$3 = 2x \rightarrow (1.5, -1.75)$

✓ ✓

(b) $y = 3x + c$

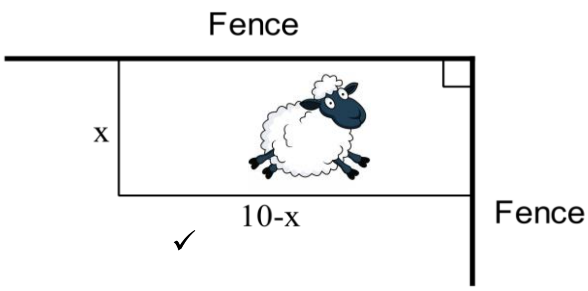
$(1.5, -1.75) \rightarrow -1.75 = 1.5 \times 3 + c$

$c = -6.25$ ✓

$y = 3x - 6.25$ ✓

(c) $-\frac{1}{3}$ ✓

Question 12 **(6 marks)**



✓

$A = x(10 - x)$ ✓

$A = 10x - x^2$

For maximum area $\frac{dA}{dx} = 0$

$\frac{dA}{dx} = 10 - 2x$ ✓

$If\ \frac{dA}{dx} = 0.\ 0 = 10 - 2x$

$x = 5$ ✓

Test for maximum

x	0	5	6
$\frac{dA}{dx}$	+	0	−

✓

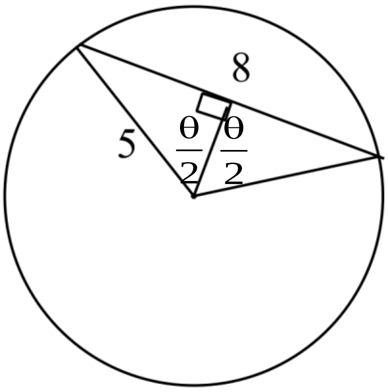
Therefore maximum

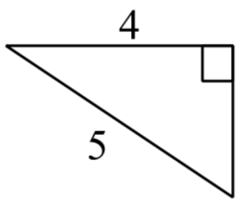
$If\ \ x = 5,\ 10 - 5 = 5$

Therefore the dimensions of the maximum sized pen are 5m x 5m. ✓

Question 13 **(12 marks)**

(a)





$$sin\left(\frac{\theta}{2}\right)=\frac{4}{5}$$

$$\theta=1.8546 \quad \checkmark$$

$$A_{segment}=\frac{1}{2}r^2\left(\theta-sin(\theta)\right)$$

$$A_{segment}=\frac{1}{2}\times 5^2\left(1.8546-sin(1.8546)\right)$$

$$A_{segment}\approx 11.18\ cm^2 \quad \checkmark$$

(b) (i) $sin\left(\frac{x}{2}\right)=\frac{1}{\sqrt{2}}$ for $-\frac{\pi}{2}\le x\le \pi$

$$\frac{x}{2}=\frac{\pi}{4}+n(2\pi) \quad or \quad \frac{x}{2}=\frac{3\pi}{4}+n(2\pi)$$

$$x=\frac{\pi}{2}+4n\pi \qquad x=\frac{3\pi}{2}+4n\pi \quad \checkmark$$

$$x=\frac{\pi}{2} \quad \checkmark$$

(ii) $tan\left(2x+\frac{\pi}{3}\right)=-\sqrt{3}$ for $0\le x\le \frac{\pi}{2}$

$$2x+\frac{\pi}{3}=\frac{2\pi}{3}\pm n\pi \quad \checkmark$$

$$2x=\frac{\pi}{3}\pm n\pi$$

$$x=\frac{\pi}{6}\pm\frac{n\pi}{2}$$

$$x=\frac{\pi}{6} \quad \checkmark$$

(c) $cos\left(\frac{7\pi}{12}\right) = cos\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$$= cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$= cos\left(\frac{\pi}{4}\right)cos\left(\frac{\pi}{3}\right) - sin\left(\frac{\pi}{4}\right)sin\left(\frac{\pi}{3}\right) \quad \checkmark$$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{2\sqrt{2}}\left(1 - \sqrt{3}\right) \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$cos\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2}}{4}\left(1 - \sqrt{3}\right) \quad \checkmark$$

(d) $y = 1 - cos\left(\frac{x}{2}\right)$ ✓✓

(e)

$$cos\,B = \frac{a^2 + c^2 - b^2}{2ac}$$

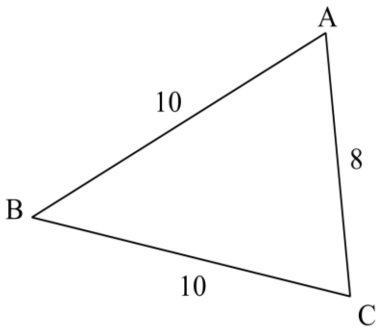
$$= \frac{10^2 + 10^2 - 8^2}{2(10)(10)}$$

$$\angle ABC = 47.156^o \quad \checkmark$$

$$Area_\Delta = \frac{1}{2}ac\,sin(B)$$

$$= \frac{1}{2} \times 10 \times 10 \times sin(47.156^o)$$

$$= 36.66\,unit^2 \quad \checkmark$$



Question 14 **(19 marks)**

(a) (i) $P(t) = 5(4)^t$ ✓

(ii) $P(2) = 5(4)^2$ ✓

$P(2) = 80$ ✓

(iii) $25 = 5(4)^t$

$x = 1.1609$ ✓

In about 14 months ✓

(b) $1\frac{1}{4}+2\frac{1}{8}+3\frac{1}{16}+\ldots.=\left(1+2+3+\ldots..\right)+\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots..\right)$

AP $a=1,\ d=1, n=10$ ✓ GP $a=\frac{1}{4},\ r=\frac{1}{2}, n=10$ ✓

$S_n=\frac{n}{2}\left(2a+\left(n-1\right)d\right)$ $S_n=\frac{a\left(1-r^n\right)}{1-r}$

$S_{10}=\frac{10}{2}\left(2+\left(9\right)1\right)$ $S_n=\frac{\frac{1}{4}\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}}$

$S_{10}=55$ ✓ $S_{10}=0.49951$ ✓

$\therefore\ S_{10}=55.49951$ ✓

Question 15 **(4 marks)**

(a) (i) $y=3-4x^3+6x$

$\frac{dy}{dx}=-12x^2+6$ ✓✓

(ii) $f(x)=-4\left(2-x^3\right)=-8+4x^3$ ✓

$f\,'(x)=12x^2$ ✓

Question 16 **(9 marks)**

(a) (i) $\frac{f\left(1.1\right)-f\left(1\right)}{0.1}=\frac{2.79-3}{0.1}=-2.1$ ✓ ✓ ✓

(ii) You have found the gradient of the interval from x = 1 to x = 1,1 ✓✓
which is an approximation to the slope at x = 1.

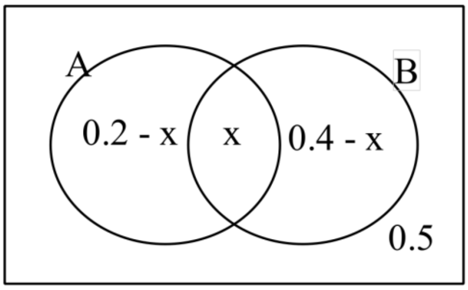
(iii) The average rate of change is the gradient of an interval. ✓
The instantaneous rate of change is the slope at a point which is the same as the slope of the tangent at that point. ✓

(b) The area is effectively the product of velocity and time and gives the displacement.

✓✓

Question 17 **(11 marks)**

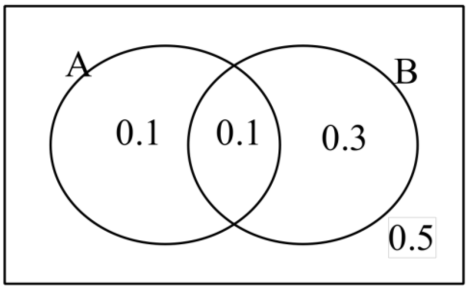
(a) (i) $P(A)=0.2,\ P(B)=0.4,\ P(\overline{A\cup B})=0.5$



$0.2-x+x+0.4-x+0.5=1$

$-x+1.1=1$

$x=0.1$ ✓



If events A and B are independent, then $P(A\cap B)=P(A)\times P(B)$ ✓

$P(A)\times P(B)=0.2\times 0.4=0.08$

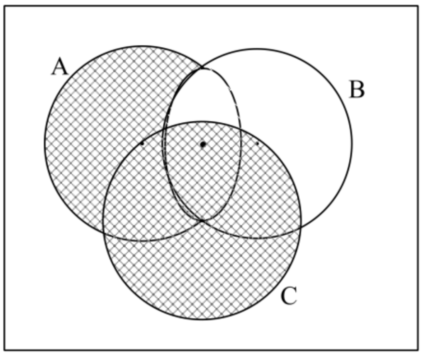
$P(A\cap B)=0.1$ ✓

Therefore events A and B are NOT independent.

(ii) If events A and B are mutually exclusive, then $P(A\cap B)=0$.

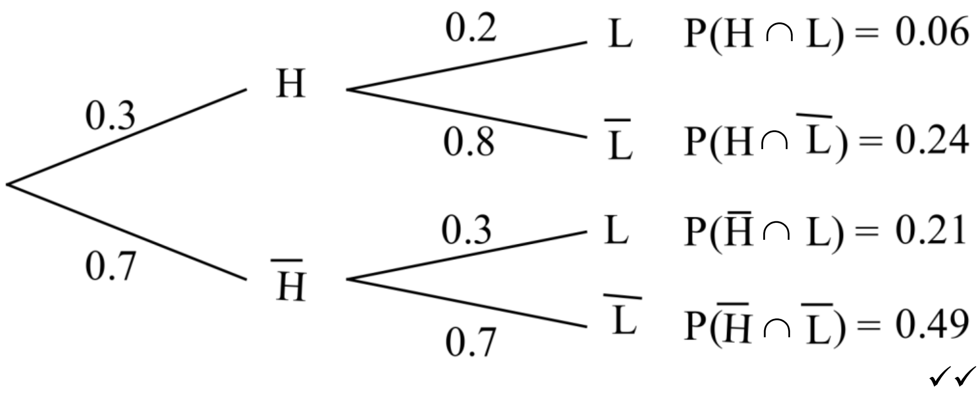
But $P(A\cap B)=0.1$ so events A and B are NOT mutually exclusive. ✓

(b) $(A\cap\overline{B})\cup C$



✓✓

(c) (i)



(ii) $P(L) = P(H \cap L) + P(\overline{H} \cap L)$

$= 0.06 + 0.21$

$= 0.27$ ✓

(iii) $P(H \mid \overline{L}) = \dfrac{P(H \cap \overline{L})}{P(\overline{L})} = \dfrac{0.24}{0.24 + 0.49} = \dfrac{0.24}{0.73} = \dfrac{24}{73}$

✓ ✓

Question 18 **(11 marks)**

(a) 5, 7 ,9 AP a = 5, d = 2 ✓

$S_n = \dfrac{n}{2}\left(2a + (n-1)d\right)$ ✓

$200 = \dfrac{n}{2}\left(10 + (n-1)2\right)$

$n \approx 8.2$ ✓✓

“At least” implies Ruth would need to go out and drive 9 times. ✓

(b) $S_n = \dfrac{n}{2}\left(2a + (n-1)d\right)$

$S_{10} = 5\left(10 + 4 \times 2\right)$ ✓

$S_{10} = 140 \text{ km}$ ✓✓

(c) $S_n = 480 \text{ km} = ?$

$480 = \dfrac{n}{2}\left(10 + (n-1)2\right)$ ✓✓

$n = 20$ ✓

Ruth took 20 lessons!

End of solutions