Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS

3A/3B

Calculator-free

WACE Examination 2013

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

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Section One: Calculator-free (50 Marks)

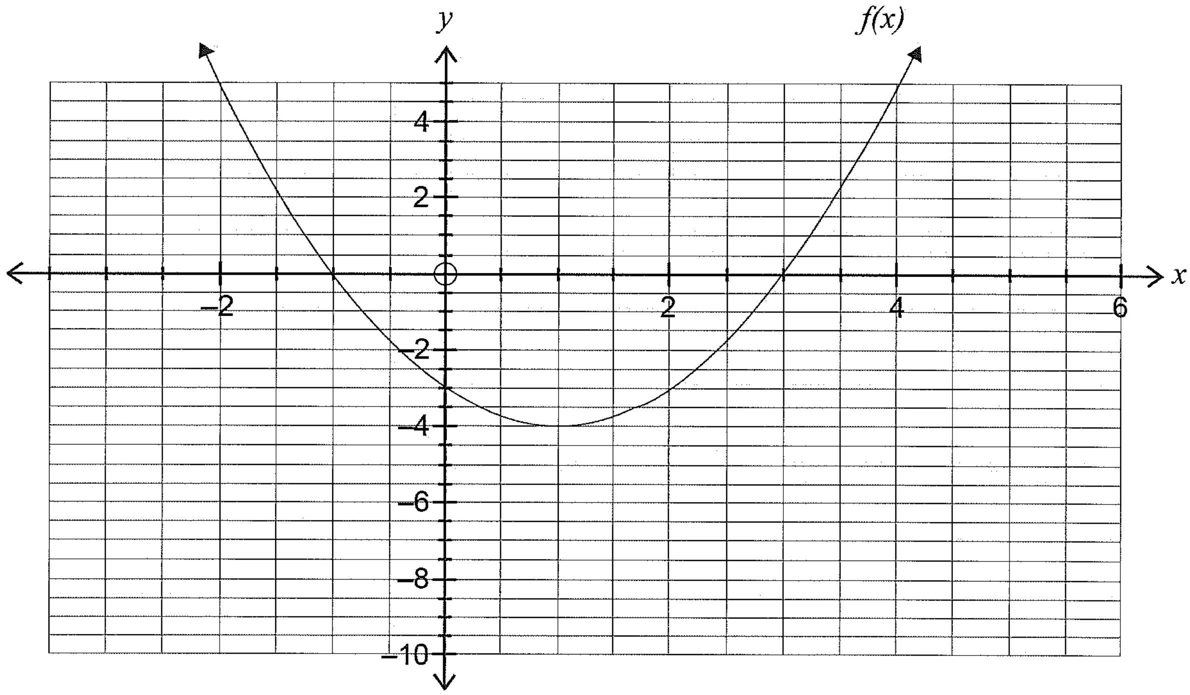
Question 1 (4 marks)

A recursive sequence is defined by $u_n = pu_{n-1} + q$. Given that $u_1 = -8, u_2 = 8$ and $u_3 = 4$, write down **two** equations and solve simultaneously to determine the values of p and q.

Solution	
$8 = -8p + q$	
$4 = 8p + q$	
adding gives $2q = 12 \quad \therefore\ q = 6$	
using equation one $8p = 4 - 6 \quad \therefore\ p = -\dfrac{1}{4}$	
Specific behaviours	
✓	correctly formulates equation one
✓	correctly formulates equation two
✓	correctly calculates q
✓	correctly substitutes and calculates p

Question 2 (9 marks)

The function $y = f(x)$ shown below is transformed to produce $g(x) = -f(x+1)$.



(a) Give the equation of $f(x)$ in the form $y = (x-p)^2 + d$. (2 marks)

Solution	
$y = (x-1)^2 - 4$	
Specific behaviours	
✓	correctly indicates the value of p
✓	correctly indicates the value of d

(b) (i) Describe the transformations required to produce $g(x)$ from $f(x)$. (2 marks)

Solution
Translation of one unit in the negative x direction followed by a reflection in the x-axis. (order does not matter)
Specific behaviours
✓ correctly states first transformation
✓ correctly states second transformation

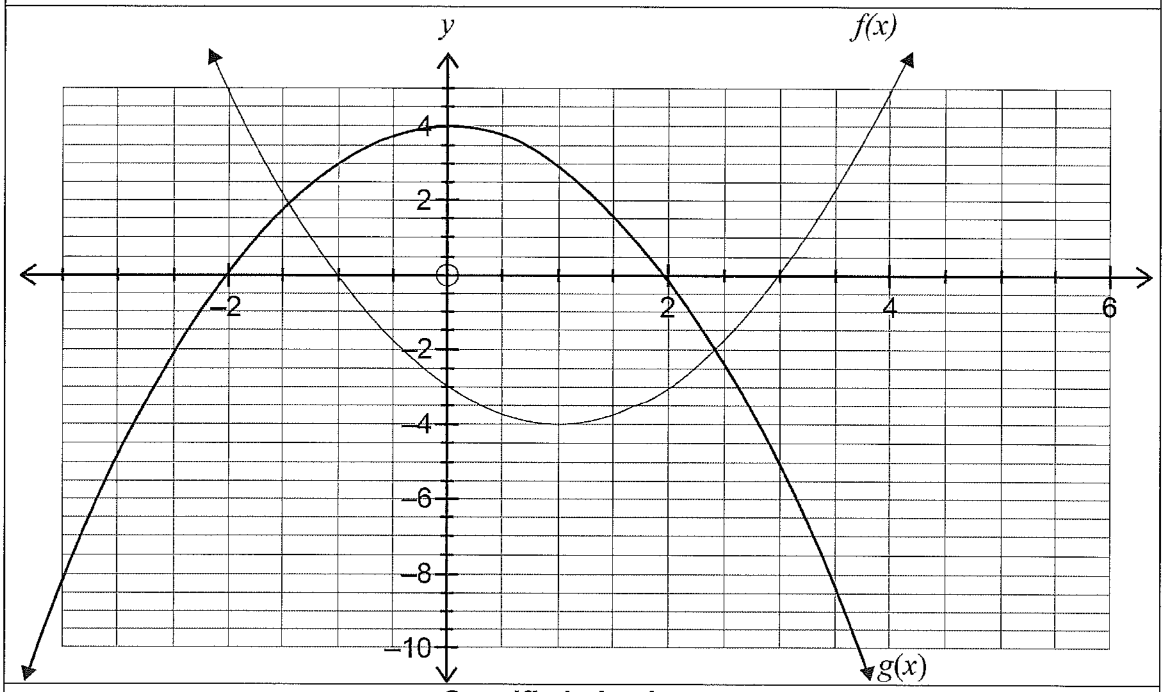
(ii) State the coordinates of the turning point of $g(x)$. (1 mark)

Solution
$(0,4)$
Specific behaviours
✓ correctly states coordinates

(c) On the grid above, draw the function $y = g(x)$, showing the x and y intercepts.

(2 marks)

Solution



Specific behaviours

✓ draws inverted parabola passing through the turning point from part (b) (ii)

✓ correctly shows x-intercepts and y-intercept

(d) State the domain and range of $y = g(x)$. (2 marks)

Solution
Domain: all reals Range: $y \le 4$
Specific behaviours
✓ states correct domain ✓ states correct range

Question 3 **(5 marks)**

(a) Give a reason why the following statement is false for real numbers.

$$(-4)^{\frac{3}{2}} \times (-4)^{\frac{3}{2}} = (-4)^3 = -64$$ (1 mark)

Solution
$(-4)^{\frac{3}{2}} = \sqrt{(-4)^3} = \sqrt{-64}$ which is undefined, therefore the statement is false
Specific behaviours
✓ correctly recognises the negative square root is undefined

(b) In the following, b and c are positive integers. If the statement is correct, write **true** next to the statement. If the statement is false, rewrite the right-hand side of the equation to make the statement true.

Solution			
(i)	$c^2 \times c^{-2} = b^0$	(true)	(1 mark)
(ii)	$(3bc)^2 = 6b^2c^2$	$\left(9b^2c^2\right)$	(1 mark)
(iii)	$c^2 \div 3bc = \dfrac{2c^2}{b}$	$\left(\dfrac{c}{3b}\right)$	(1 mark)
(iv)	$2b^{-1} = -2b$	$\left(\dfrac{2}{b}\right)$	(1 mark)
Specific behaviours			
(i) ✓	recognises that the equation is true		
(ii) ✓	correctly rewrites the expression as $9b^2c^2$		
(iii) ✓	correctly rewrites the expression as $\left(\dfrac{c}{3b}\right)$		
(iv) ✓	correctly rewrites the expression as $\left(\dfrac{2}{b}\right)$		

Question 4 **(5 marks)**

Determine the gradient of $y = x^2 - 5x - 24$ at the point(s) where it crosses the x-axis.

Solution	
$x^2 - 5x - 24 = 0$ $(x-8)(x+3) = 0 \Rightarrow$ roots are $x = 8$ and $x = -3$ $\frac{dy}{dx} = 2x - 5$ $\left.\frac{dy}{dx}\right\|_{x=8} = 11$ and $\left.\frac{dy}{dx}\right\|_{x=-3} = -11$	
Specific behaviours	
✓	correctly factorises quadratic
✓	correctly determines the roots
✓	correctly differentiates y
✓	correctly determines the gradient at $x = 8$
✓	correctly determines the gradient at $x = -3$

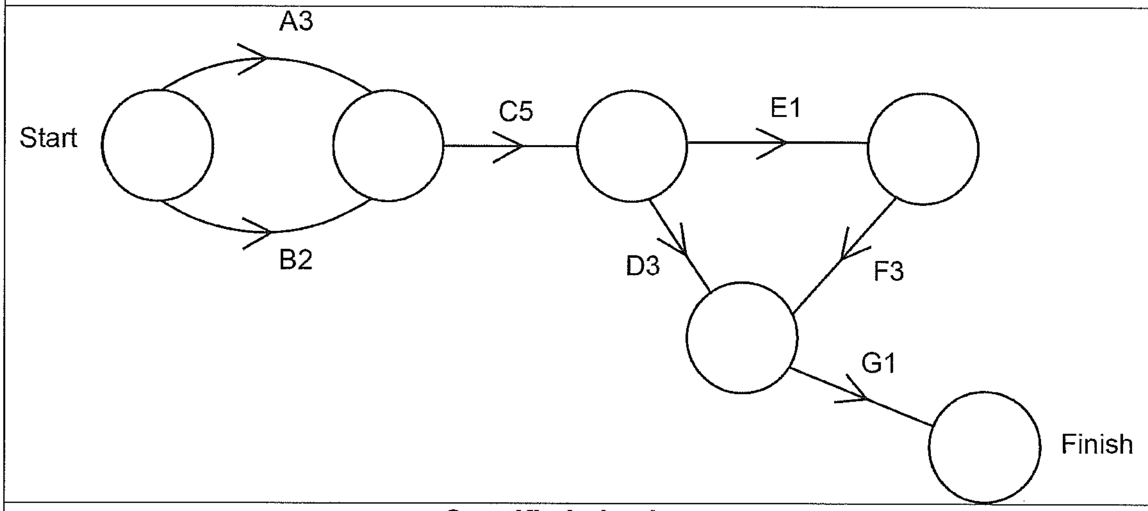
Question 5 **(3 marks)**

The activities A to G, their immediate predecessors and the time taken to complete each activity, are shown in the table below.

Activity	Immediate predecessors	Time (days)
A	-	3
B	-	2
C	A,B	5
D	C	3
E	C	1
F	E	3
G	D, F	1

Construct a project network for this information.

Solution

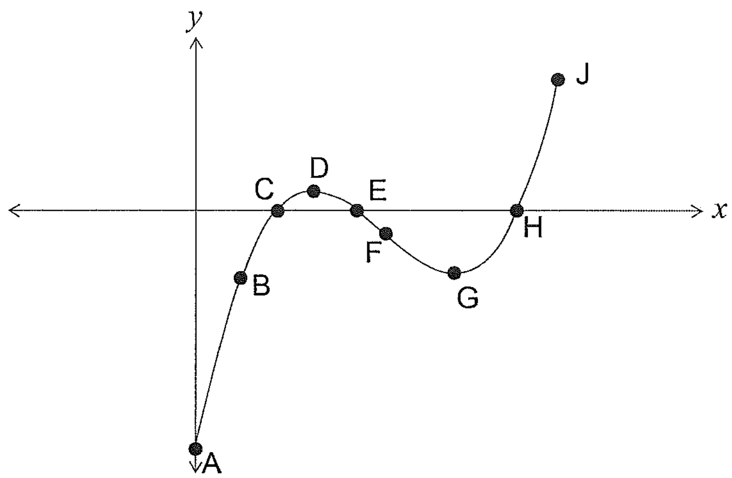


Specific behaviours

- ✓ places activities in correct order
- ✓ correctly indicates times on arcs
- ✓ places arrows on arcs in correct direction

Question 6 **(9 marks)**

The function $y=(x-1)(x-2)(x-4)$, shown below, has been graphed for the domain $0 \le x \le 4.5$. The function has turning points at D and G and a point of inflection at F.



(a) Determine the coordinates of the y-intercept. (2 marks)

Solution	
$x=0 \Rightarrow y=-1\times-2\times-4=-8 \therefore$ y-intercept is $(0,-8)$	
Specific behaviours	
✓	correctly substitutes zero into the function
✓	correctly states the coordinates of the y-intercept

(b) Which of the points on the graph labelled A to J shows the

(i) global maximum? (1 mark)

Solution
J
Specific behaviours
✓ states correct point

(ii) local minimum? (1 mark)

Solution
G
Specific behaviours
✓ states correct point

(c) Calculate the global maximum for the function. (3 marks)

Solution
$x = 4.5 \Rightarrow y = 3.5 \times 2.5 \times 0.5$ $= \frac{7}{2} \times \frac{5}{2} \times \frac{1}{2} = \frac{35}{8}$
Specific behaviours
✓ identifies that $x = 4.5$ gives the global maximum   ✓ correctly substitutes $x = 4.5$ into the equation   ✓ correctly evaluates product

(d) Between which two points for the given domain is the function concave up? (2 marks)

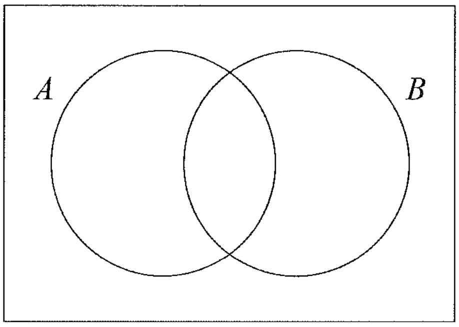
Solution
Between F and J
Specific behaviours
✓ correctly identifies F as the point of inflection   ✓ identifies correct interval

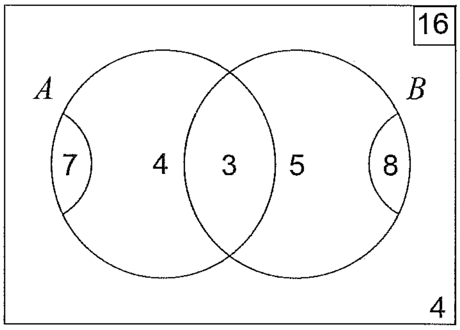
Question 7 **(6 marks)**

In a Year 12 mathematics class, seven students used a Brand 'A' calculator and eight students used a Brand 'B' calculator. Three students used both brands of calculator and four students used neither brand of calculator.

Let A represent the set of students who used a Brand 'A' calculator and B represent the set of students who used a Brand 'B' calculator.

(a) Using this information, complete the Venn diagram. (2 marks)



Solution


	Specific behaviours
✓	transfers given information onto the Venn diagram
✓	completes the remaining regions of the Venn diagram (i.e. 4 and 5)

(b) Determine

(i) $P(A \cup B)$. (1 mark)

Solution
$P(A \cup B) = \frac{12}{16}$

	Specific Behaviours
✓	correctly states the probability of $A \cup B$

(ii) $P\left(B \cap \overline{A}\right)$. (1 mark)

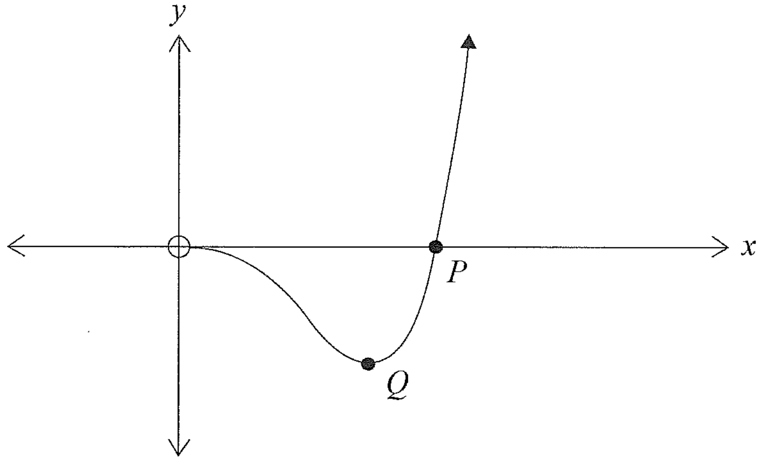
Solution
$P\left(B \cap \overline{A}\right) = \frac{5}{16}$
Specific behaviours
✓ correctly states the probability of $B \cap \overline{A}$

(iii) the proportion of students who used a Brand 'B' calculator, given that they did not use a Brand 'A' calculator. (2 marks)

Solution
$P\left(B \mid \overline{A}\right) = \frac{5}{9}$ $\therefore$ the proportion is $\frac{5}{9}$
Specific behaviours
✓ identifies the reduced sample size (i.e. 9) ✓ states correct proportion

Question 8 **(5 marks)**

The function $y = 2x^3(x-k)$, where k is a positive constant, has been graphed below for $x > 0$.



(a) Given that the point P has coordinates $(2, 0)$, determine the value of k. (1 mark)

Solution	
$k = 2$	
Specific behaviours	
✓	correctly determines the value of k

(b) Determine the x-coordinate of the local minimum point Q. (4 marks)

Solution

$y = 2x^3(x-2) = 2x^4 - 4x^3$

$y' = 8x^3 - 12x^2$

$= 4x^2(2x-3)$

$= 0$ when $x = 0$ or $x = \dfrac{3}{2}$

$\therefore$ the x-coordinate is $x = \dfrac{3}{2}$ since the function is only defined for $x > 0$

Specific behaviours	
✓	correctly differentiates y
✓	correctly factorises the derivative
✓	correctly equates the derivative to zero and solves for x
✓	correctly states the x-coordinate of the point Q

Question 9 **(4 marks)**

The following set of 14 integers is arranged in ascending order and has a mean of 10.

2, 2, 2, p, 5, 6, 9, 11, 11, 13, 14, q, 21, 24

(a) Determine all possible values for p and q. (2 marks)

Solution
$p+q=140-120=20$ Therefore (p,q) can be $(2,18)$, $(3,17)$, $(4,16)$ or $(5,15)$
Specific Behaviours
✓ correctly calculates the sum of p and q ✓ correctly gives the four possible pairs for p and q

(b) Determine the smallest possible value for the interquartile range. (2 marks)

Solution
$\text{IQR} = 14-5=9$
Specific Behaviours
✓ correctly identifies the lower quartile (5) and upper quartile (14) ✓ correctly calculates interquartile range

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