Neap

Trial Examination 2020

# VCE Specialist Mathematics Units 3&4

## Written Examination 2

## Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

**Structure of booklet**

| *Section* | *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|---|
| A | 20 | 20 | 20 |
| B | 5 | 5 | 60 |
| | | | Total 80 |

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**

Question and answer booklet of 20 pages

Formula sheet

Answer sheet for multiple-choice questions

**Instructions**

Write your **name** and your **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

**At the end of the examination**

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

## SECTION A – MULTIPLE-CHOICE QUESTIONS

**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g$ ms$^{-2}$, where $g = 9.8$.

### Question 1

The graph with equation $y = \dfrac{1}{x^2 + x - 2}$ has asymptotes given by

**A.** $x = -2$, $x = 1$ and $y = -\dfrac{1}{2}$

**B.** $x = -2$ and $x = 1$ only

**C.** $x = 2$, $x = -1$ and $y = 0$

**D.** $x = -2$, $x = 1$ and $y = 0$

**E.** $x = 2$ and $x = -1$ only

### Question 2

The maximal domain and the range of the function $f(x) = \arccos(4x - 1) + \dfrac{\pi}{3}$ are respectively

**A.** $\left[-\dfrac{1}{2}, 0\right]$ and $[0, \pi]$

**B.** $\left[-\dfrac{1}{2}, 0\right]$ and $\left[\dfrac{\pi}{3}, \dfrac{4\pi}{3}\right]$

**C.** $\left[0, \dfrac{1}{2}\right]$ and $[0, \pi]$

**D.** $\left[0, \dfrac{1}{2}\right]$ and $\left[\dfrac{\pi}{3}, \dfrac{4\pi}{3}\right]$

**E.** $[0, \pi]$ and $\left[0, \dfrac{1}{2}\right]$


SMU34EX2_QA_2020.FM

**Question 3**

If $\sin(x) = -\frac{1}{3}$ and $\pi \le x \le \frac{3\pi}{2}$, then $\cot(x)$ is equal to

**A.** $-2\sqrt{2}$

**B.** $2\sqrt{2}$

**C.** $-\frac{1}{2\sqrt{2}}$

**D.** $\frac{1}{2\sqrt{2}}$

**E.** $-\frac{3}{2\sqrt{2}}$

**Question 4**

The algebraic fraction $\frac{4x^2}{(x-1)(x-2)^2}$ could be expressed in partial fraction form as

**A.** $\frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-2}$

**B.** $\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-2}$

**C.** $\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

**D.** $\frac{A}{x-1} + \frac{B}{x-2} + \frac{Cx+D}{(x-2)^2}$, $C \ne 0$

**E.** $\frac{A}{x-1} + \frac{B}{x-2}$

**Question 5**

In an Argand diagram, points $U$ and $V$ represent the complex numbers $u = 2 + 3i$ and $v = iu$, respectively.

The area of triangle $OUV$, where $O$ is the origin, is equal to

**A.** $\frac{\sqrt{13}}{2}$

**B.** $\frac{25}{2}$

**C.** $\frac{5}{2}$

**D.** $\sqrt{13}$

**E.** $\frac{13}{2}$

**Question 6**

Consider the complex numbers $w = u + vi$ and $z = x + yi$ where $u, v, x, y \in R$.

If $\text{Re}\left(\frac{w}{z}\right) = \frac{\text{Re}(w)}{\text{Re}(z)}$, where $\text{Re}(z) \neq 0$, which one of the following is correct?

**A.** $\text{Im}(z) = 0$ only

**B.** $\text{Im}\left(\frac{w}{z}\right) = 0$ only

**C.** $\text{Im}(z) = 0$ or $\text{Im}\left(\frac{w}{z}\right) = 0$

**D.** $\text{Im}(z) = 0$ or $\text{Im}\left(\frac{w}{z}\right) = \frac{vx}{u}$

**E.** $\text{Im}(z) = 0$ or $\text{Im}\left(\frac{w}{z}\right) = \frac{u}{vx}$

**Question 7**

If $P(z) = z^3 - 3z^2 + 9z - 27$, $z \in C$, then a linear factor of $P(z)$ is

**A.** $z^2 + 9$

**B.** $z + 3i$

**C.** $z + 3$

**D.** $3i$

**E.** $3$

**Question 8**

The set of points in the complex plane defined by $\text{Arg}\left(\frac{z-4}{i}\right) = \text{Arg}(1 + i)$ corresponds to

**A.** the ray $y = 4 - x, x < 4$

**B.** the ray $y = x - 4, x < 4$

**C.** the line $y = 4 - x$

**D.** the line $y = x - 4$

**E.** the ray $y = -x, x < 0$

**Question 9**

With a suitable substitution, $\int \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$ can be expressed as

**A.** $2\int \sin(u)du$

**B.** $\frac{1}{2}\int \sin(u)du$

**C.** $\int \sin(u)du$

**D.** $2\int \sin(\sqrt{u})du$

**E.** $\frac{1}{2}\int \sin(\sqrt{u})du$

**Question 10**

To solve the differential equation $2\frac{dy}{dx} + \arctan(e^x) = \sin(x)$, with the initial condition $y = 1$ when $x = 0$, Euler's method is used with a step size of 0.2.

When $x = 0.4$, the approximation for $y$ is given by

**A.** $\left(1 - \frac{\pi}{40}\right) + 0.2(\sin(0.2) - \arctan(e^{0.2}))$

**B.** $1 + 0.1(\sin(0.2) - \arctan(e^{0.2}))$

**C.** $\left(1 - \frac{\pi}{20}\right) + 0.1(\sin(0.2) - \arctan(e^{0.2}))$

**D.** $1 + 0.2(\sin(0.2) - \arctan(e^{0.2}))$

**E.** $\left(1 - \frac{\pi}{40}\right) + 0.1(\sin(0.2) - \arctan(e^{0.2}))$

**Question 11**

A curve $C$ has equation $4\cos(y) = 3 - 2\sin(x)$, where $x, y \in R$.

The equation of the tangent to $C$ at the point $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ intersects the $x$-axis at

**A.** $(0, 0)$

**B.** $\left(-\frac{\pi}{2}, 0\right)$

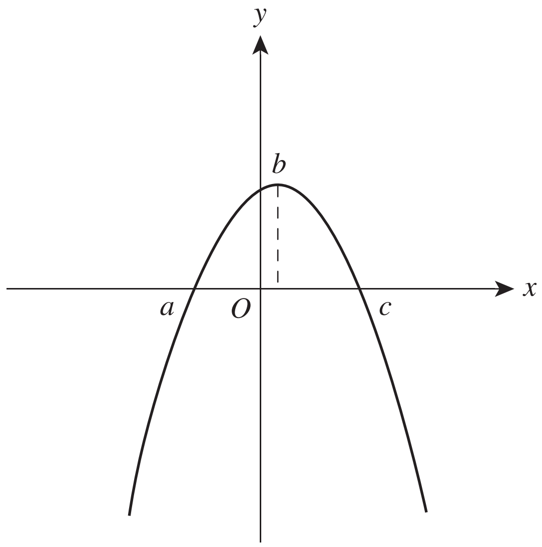
**C.** $\left(-\frac{5\pi}{6}, 0\right)$

**D.** $\left(\frac{\pi}{2}, 0\right)$

**E.** $\left(\frac{5\pi}{6}, 0\right)$

**Question 12**

The graph of $y = f'(x)$ is shown below, where $a < b < c$.




Which one of the following statements about the graph of $y = f(x)$ is **incorrect**?

**A.** The gradient is positive for $a < x < c$.

**B.** There is a stationary point of inflection at $x = b$.

**C.** The gradient is decreasing for $x > b$.

**D.** There is a local minimum at $x = a$.

**E.** There is a local maximum at $x = c$.

**Question 13**

The second derivative of a function $f$ is given by $f''(x) = x^2 \sin(x) - 2$.

For $-8 \le x \le 8$, the graph of $f$ has

**A.** 1 point of inflection.

**B.** 2 points of inflection.

**C.** 4 points of inflection.

**D.** 5 points of inflection.

**E.** 7 points of inflection.

**Question 14**

Which one of the following differential equations is satisfied by $y = e^{-x}\sin(x)$?

**A.** $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2y = 0$

**B.** $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = 0$

**C.** $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - 2y = 0$

**D.** $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - 2y = 0$

**E.** $\dfrac{d^2y}{dx^2} + 2y = 0$

**Question 15**

At time $t$ seconds, the side length of a cube is $x$ cm, the surface area of the cube is $S$ cm$^2$ and the volume of the cube is $V$ cm$^3$. The surface area of the cube is increasing at a constant rate of 8 cm$^2$ s$^{-1}$.

The volume of the cube, $V$, at time $t$ seconds, satisfies the differential equation

**A.** $\dfrac{dV}{dt} = \dfrac{1}{2}V^{-\frac{1}{3}}$

**B.** $\dfrac{dV}{dt} = 2V^{-\frac{1}{3}}$

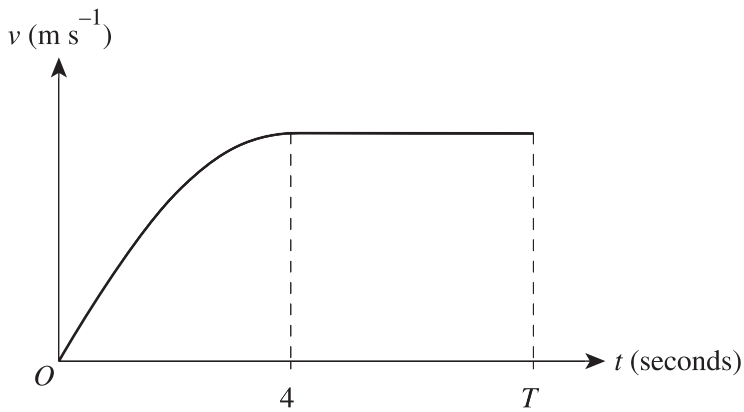
**C.** $\dfrac{dV}{dt} = \dfrac{9}{2}V^{\frac{1}{3}}$

**D.** $\dfrac{dV}{dt} = \dfrac{1}{2}V^{\frac{1}{3}}$

**E.** $\dfrac{dV}{dt} = 2V^{\frac{1}{3}}$

**Question 16**

At time $t$ seconds, where $0 \le t \le T$, a particle moves with velocity, $v$ m s$^{-1}$, as shown in the velocity–time graph below.




For $0 \le t \le 4$, the velocity of the particle is given by $v = 3t - \dfrac{3}{8}t^2$.

Which one of the following statements about the particle's motion is **incorrect**?

**A.** The particle's maximum velocity is 6 m s$^{-1}$.

**B.** For $4 \le t \le T$, the particle's acceleration is zero.

**C.** At $t = 2$, the particle's acceleration is 1.5 m s$^{-2}$.

**D.** For $4 \le t \le T$, the particle travels a distance of $6T$ metres.

**E.** The particle travels 16 metres in the first 4 seconds of its motion.

**Question 17**

Consider the vectors $\underset{\sim}{a} = \underset{\sim}{i} + \underset{\sim}{k}$, $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{c} = p\underset{\sim}{i} + q\underset{\sim}{j}$, where $p$ and $q$ are non-zero constants.

The value of $\frac{p}{q}$ such that $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ form a linearly dependent set of vectors is

**A.** $-2$

**B.** $-\frac{1}{2}$

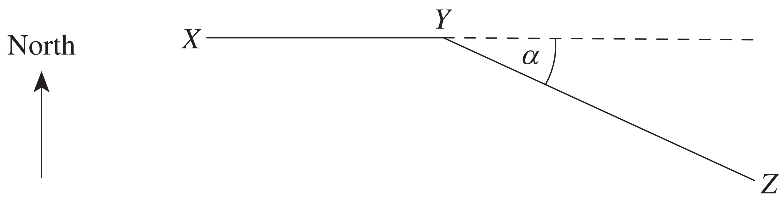
**C.** $\frac{1}{2}$

**D.** 1

**E.** 2

**Question 18**

An aeroplane is flying at a speed of 600 km h$^{-1}$ while maintaining a constant altitude. Its flight from $X$ to $Y$ takes 30 minutes, and its flight from $Y$ to $Z$ takes 60 minutes. The aeroplane's path is shown in the diagram below.




The vector $\overrightarrow{XZ}$ is given by

**A.** $300(1 + 2\sin(\alpha))\underset{\sim}{i} - 600\cos(\alpha)\underset{\sim}{j}$

**B.** $300(1 + 2\cos(\alpha))\underset{\sim}{i} + 600\sin(\alpha)\underset{\sim}{j}$

**C.** $300(1 + 2\cos(\alpha))\underset{\sim}{i} - 600\sin(\alpha)\underset{\sim}{j}$

**D.** $300(1 + 2\sin(\alpha))\underset{\sim}{i} + 600\cos(\alpha)\underset{\sim}{j}$

**E.** $600(1 + \cos(\alpha))\underset{\sim}{i} - 600\sin(\alpha)\underset{\sim}{j}$

**Question 19**

A body of mass $m$ kg is travelling in a straight line. Its velocity decreases from $U$ m s$^{-1}$ to $V$ m s$^{-1}$, where $U > V > 0$, in a time of $t$ seconds.

The change of momentum of the body in kg m s$^{-1}$, in the direction of its motion, is given by

**A.** $\frac{m(V-U)}{t}$

**B.** $V - U$

**C.** $m(V - U)$

**D.** $\frac{m(U-V)}{t}$

**E.** $m(U - V)$

**Question 20**

A particle of mass $m$ kg is acted on by two forces, $\underset{\sim}{F}_1 = p\underset{\sim}{i} + q\underset{\sim}{j}$ and $\underset{\sim}{F}_2 = r\underset{\sim}{i} + s\underset{\sim}{j}$, where $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$ are measured in newtons and $p, q, r, s \in R$.

The magnitude of the particle's acceleration in m s$^{-2}$ is

**A.** $\dfrac{\sqrt{(p-r)^2 + (q-s)^2}}{m}$

**B.** $\dfrac{\sqrt{(p+q)^2 + (r+s)^2}}{m}$

**C.** $\sqrt{(p+r)^2 + (q+s)^2}$

**D.** $\dfrac{(p+r) + (q+s)}{m}$

**E.** $\dfrac{\sqrt{(p+r)^2 + (q+s)^2}}{m}$

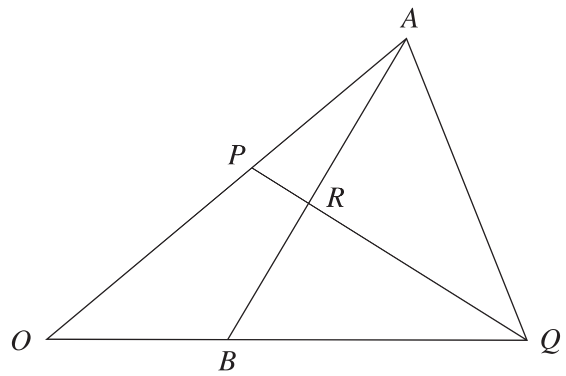
## SECTION B

**Instructions for Section B**

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude $g$ ms$^{-2}$, where $g = 9.8$.

**Question 1** (8 marks)

The diagram below shows triangle $OAQ$. Point $P$ lies on $OA$ such that $OP : OA = 3 : 5$. Point $B$ lies on $OQ$ such that $OB : BQ = 1 : 2$.




Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.

**a.** Given that $\overrightarrow{AR} = m\overrightarrow{AB}$, where $0 < m < 1$, show that $\overrightarrow{OR} = (1 - m)\underset{\sim}{a} + m\underset{\sim}{b}$. 2 marks

**b.** Given that $\overrightarrow{PR} = n\overrightarrow{PQ}$, where $0 < n < 1$, show that $\overrightarrow{OR} = \frac{3}{5}(1 - n)\underset{\sim}{a} + 3n\underset{\sim}{b}$. 2 marks

**c.** Find the values of $m$ and $n$. 3 marks

**d.** Find $PR : PQ$. 1 mark

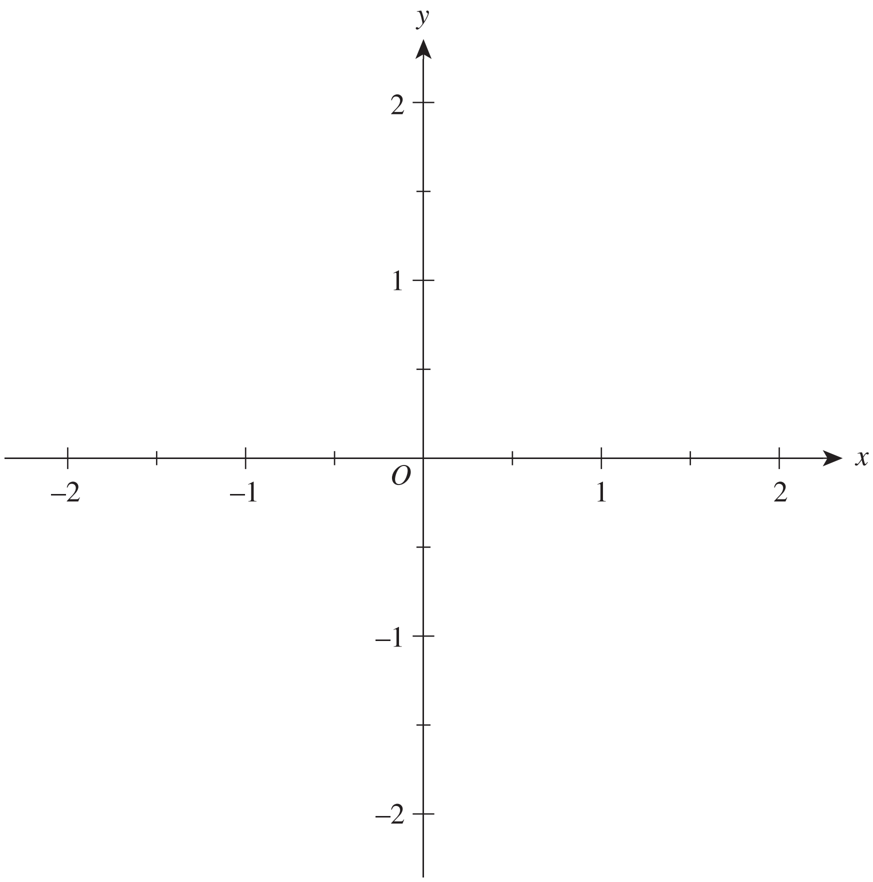
 

**Question 2** (13 marks)

A curve $C$ is defined parametrically by $x = \cos(t),\ y = \frac{1}{2}\sin(2t)$, where $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$.

**a.** Show that $C$ can be represented by the cartesian equation $y^2 = x^2(1 - x^2)$. 2 marks

**b.** Sketch $C$ on the axes below, labelling the coordinates of any points of intersection with the axes. 3 marks





SMU34EX2_QA_2020.FM

**c.** The region bounded by $C$ is rotated through $\pi$ radians about the $x$-axis.

Find the volume of the solid formed. Give your answer in the form $\frac{a\pi}{b}$ where $a, b \in Z^{+}$. 2 marks

**d.** **i.** Find the equation of the normal to $C$ at point $P$ where $t = \frac{2\pi}{3}$. 2 marks

**ii.** The normal to $C$ at point $P$ intersects $C$ again at point $Q$.

Find the coordinates of $Q$. Give your answer correct to three decimal places. 4 marks

**Question 3** (11 marks)

Consider $z = r\text{cis}(\theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$.

**a.** **i.** Show that $r\text{cis}\left(\frac{\theta}{2}\right)\left(\text{cis}\left(\frac{\theta}{2}\right) - \text{cis}\left(-\frac{\theta}{2}\right)\right) = r\text{cis}(\theta) - r$. 2 marks

**ii.** Hence, show that $\frac{z}{z - r} = \frac{1}{2} - \frac{1}{2}i\cot\left(\frac{\theta}{2}\right)$. 3 marks

**b.** Solve the equation $z^2 - 2z + 4 = 0$, $z \in C$. Express your answers in the form $z = r\text{cis}(\theta)$, where $-\pi < \theta \leq \pi$. 1 mark

**c.** Find a suitable transformation from $z$ to $w$ such that the equation $z^2 - 2z + 4 = 0$, where $z \in C$, can be expressed as $\dfrac{4w^2}{(w-1)^2} - \dfrac{4w}{w-1} + 4 = 0$, where $w \in C$. 2 marks

**d.** Using the results from **part a.ii.**, **part b.** and **part c.**, or otherwise, find the roots of the equation $\dfrac{4w^2}{(w-1)^2} - \dfrac{4w}{w-1} + 4 = 0$. Give your answers in the form $\dfrac{1}{2} - \dfrac{1}{2}i\cot(\alpha)$. 3 marks

**Question 4** (15 marks)

A ball of mass 0.1 kg is projected vertically upwards from ground level with an initial speed of $5g$ m s$^{-1}$. While in flight, the forces acting on the ball are its weight ($W$ newtons) and air resistance ($R$ newtons) where $R = 0.02v$ and $v$ m s$^{-1}$ is the velocity of the ball. The time after projection is denoted by $t$ seconds.

**a.** Draw a diagram showing all the forces acting on the ball during its upward motion. 1 mark

**b.** For the ball's upward motion, show that $\frac{dv}{dt} = -0.2(5g + v)$. 1 mark

**c.** Use integration to solve the differential equation in **part b.** and hence show that $v = 5g(2e^{-0.2t} - 1)$. 4 marks

**d.** The ball reaches a maximum height of $H$ metres.

Find the value of $H$. Give your answer in the form $ag(b - \log_e(c))$, where $a$, $b$ and $c$ are positive integers. 4 marks

**e.** Explain why the ball's downward motion can also be described by the differential equation in **part b.** 1 mark

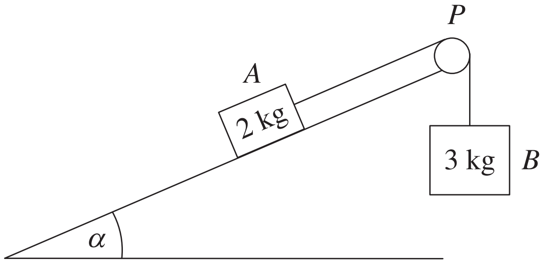
**f.** The ball reaches the ground again when $t = T$.

**i.** Show that $\frac{T}{10} + e^{-0.2T} = 1$. 3 marks

**ii.** Find the value of $T$. Give your answer correct to two decimal places. 1 mark

**Question 5** (13 marks)

Two masses, $A$ and $B$, of 2 kg and 3 kg respectively, are attached by a light inextensible string that passes over a smooth pulley, $P$. Mass $A$ is at rest on a rough plane inclined at an angle $\alpha$ to the horizontal, where $\tan(\alpha) = \frac{3}{4}$. Mass $B$ hangs freely at the end of the inclined plane vertically below $P$, as shown in the diagram below.




The two masses, the pulley and the string lie in a vertical plane parallel to the line of greatest slope of the inclined plane. At $t = 0$, the two masses are held at rest with the string taut. The two masses are then released. Mass $A$ begins to move up the inclined plane, where the constant frictional force between mass $A$ and the plane has magnitude 10 N.

**a.** Show that the acceleration, in m s$^{-2}$, of the two masses immediately after they are released is given by $a = \frac{9g}{25} - 2$. 2 marks

**b.** Show that the tension in the string, in newtons, is given by $T = \frac{48g}{25} + 6$. 1 mark

Let $F$ represent the force exerted by the string on $P$.

**c.** Find the direction and magnitude of $F$. Give your answers correct to one decimal place. 4 marks

**d.** Two seconds after release, the string breaks while both masses are moving.

Assuming that mass $A$ does not reach the pulley, find the **total** distance travelled by mass $A$ moving up the plane from the instant the masses were released. Give your answer correct to two decimal places. 6 marks

**END OF QUESTION AND ANSWER BOOKLET**