MATHEMATICS SPECIALIST

SAMPLE FORMULA SHEET
2016

This document is valid for teaching and examining until 31 December 2016.

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Vectors

Magnitude: $|(a_1, a_2, a_3)| = \sqrt{a_1^{\,2} + a_2^{\,2} + a_3^{\,2}}$

Dot product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta = a_1 b_1 + a_2 b_2 + a_3 b_3$

Triangle inequality: $|\mathbf{a} + \mathbf{b}| \le |\mathbf{a}| + |\mathbf{b}|$

Vector equation of a line in space:
one point and the slope: $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$
two points A and B: $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$

Cartesian equations of a line in space: $\dfrac{x - a_1}{b_1} = \dfrac{y - a_2}{b_2} = \dfrac{z - a_3}{b_3}$

Parametric form of vector equation of a line in space:

$$x = a_1 + \lambda b_1 \text{......}(1)$$
$$y = a_2 + \lambda b_2 \text{......}(2)$$
$$z = a_3 + \lambda b_3 \text{......}(3)$$

Vector equation of a plane in space: $\mathbf{r} \bullet \mathbf{n} = c$ or $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$

Cartesian equation of a plane: $ax + by + cz = d$

Trigonometry

In any triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$A = \frac{1}{2} ab \sin C$$

In a circle of radius r, for an arc subtending angle θ (radians) at the centre:

Length of arc $= r\theta$

Area of segment $= \frac{1}{2} r^2 (\theta - \sin\theta)$ Area of sector $= \frac{1}{2} r^2 \theta$

Identities:

$$\cos^2\theta + \sin^2\theta = 1$$

$$\cos(\theta \pm \varphi) = \cos\theta \cos\varphi \mp \sin\theta \sin\varphi$$

$$\sin(\theta \pm \varphi) = \sin\theta \cos\varphi \pm \cos\theta \sin\varphi$$

$$\tan(\theta \pm \varphi) = \frac{\tan\theta \pm \tan\varphi}{1 \mp \tan\theta \tan\varphi}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

Simple Harmonic Motion: If $\dfrac{d^2 x}{dt^2} = -k^2 x$ then $x = A\sin(kt + \alpha)$ or $x = A\cos(kt + \beta)$ and

$v^2 = k^2 (A^2 - x^2)$, where A is the amplitude of the motion, α and β are phase angles, v is the velocity and x is the displacement.

Functions

Differentiation:

If $f(x) = y$ then $f'(x) = \dfrac{dy}{dx}$

If $f(x) = x^n$ then $f'(x) = nx^{n-1}$

If $f(x) = e^x$ then $f'(x) = e^x$

If $f(x) = \ln x$ then $f'(x) = \dfrac{1}{x}$

If $f(x) = \sin x$ then $f'(x) = \cos x$

If $f(x) = \cos x$ then $f'(x) = -\sin x$

If $f(x) = \tan x$ then $f'(x) = \sec^2 x = \dfrac{1}{\cos^2 x}$

Product rule:

If $y = f(x)\,g(x)$

then $y' = f'(x)\,g(x) + f(x)\,g'(x)$

or

If $y = uv$

then $\dfrac{dy}{dx} = \dfrac{du}{dx}v + u\dfrac{dv}{dx}$

Quotient rule:

If $y = \dfrac{f(x)}{g(x)}$

then $y' = \dfrac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$

or

If $y = \dfrac{u}{v}$

then $\dfrac{dy}{dx} = \dfrac{\dfrac{du}{dx}v - u\dfrac{dv}{dx}}{v^2}$

Incremental formula: $\delta y \simeq \dfrac{dy}{dx}\delta x$ or $f(x+h) - f(x) \simeq f'(x)h$

Chain rule:

If $y = f(g(x))$

then $y' = f'(g(x))\,g'(x)$

or

If $y = f(u)$ and $u = g(x)$

then $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

Integration:

Powers: $\int x^n dx = \dfrac{x^{n+1}}{n+1} + c,\ n \neq -1$

Exponentials: $\int e^x dx = e^x + c$

Logarithms: $\int \dfrac{1}{x}\,dx = \ln|x| + c$

Trigonometric:

$\int \sin x\,dx = -\cos x + c$

$\int \cos x\,dx = \sin x + c$

$\int \sec^2 x\,dx = \tan x + c$

Fundamental Theorem of Calculus:

$\dfrac{d}{dx}\int_a^x f(t)\,dt = f(x)$ and $\int_a^b f'(x)\,dx = f(b) - f(a)$

Functions

Quadratic function:

If $y = ax^2 + bx + c$ and $y = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $x \in C$

Absolute value function: $|x| = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$

Complex numbers

For $z = a + ib$, where $i^2 = -1$

Argument: $\arg z = \theta$, where $\tan\theta = \dfrac{b}{a}$ and $-\pi < \theta \leq \pi$

Modulus: $\text{mod } z = |z| = |a + ib| = \sqrt{a^2 + b^2} = r$

Product: $|z_1 z_2| = |z_1|\,|z_2|$ $\qquad \arg(z_1 z_2) = \arg z_1 + \arg z_2$

Quotient: $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$ $\qquad \arg\dfrac{z_1}{z_2} = \arg z_1 - \arg z_2$

Polar form:
For $z = r\,\text{cis}\,\theta$, where $r = |z|$ and $\theta = \arg z$:

$\text{cis}(\theta + \varphi) = \text{cis}\,\theta\,\text{cis}\,\varphi$ $\qquad \text{cis}\,\theta = \cos\theta + i\sin\theta$

$\text{cis}(-\theta) = \dfrac{1}{\text{cis}\,\theta}$ $\qquad \text{cis}(0) = 1$

$z_1 z_2 = r_1 r_2\,\text{cis}(\theta + \varphi)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta - \varphi)$

For complex conjugates:

$z = a + bi$ $\qquad \bar{z} = a - bi$

$z = r\,\text{cis}\,\theta$ $\qquad \bar{z} = r\,\text{cis}(-\theta)$

$z\bar{z} = |z|^2$ $\qquad z^{-1} = \dfrac{1}{z} = \dfrac{\bar{z}}{|z|^2}$

$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$ $\qquad \overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$

Exponentials and logarithms

For a, $b > 0$ and m, n real:

$$a^m a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n}$$

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn} \qquad (ab)^m = a^m b^m$$

For m an integer and n a positive integer:

$$a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

For a, b, y, m and n positive real and k real:

$$1 = a^0 \Leftrightarrow \log_a 1 = 0 \qquad y = a^x \Leftrightarrow \log_a y = x$$

$$\log_a mn = \log_a m + \log_a n \qquad a = a^1 \Leftrightarrow \log_a a = 1$$

$$\log_a m = \frac{\log_b m}{\log_b a} \quad \text{(change of base)} \qquad \log_a(m^k) = k \log_a m$$

If $\frac{dP}{dt} = kP$, then $P = P_0 e^{kt}$

Mathematical reasoning

De Moivre's theorem:

$$(\text{cis } \theta)^n = (\cos \theta + i\sin \theta)^n$$

$$(\text{cis } \theta)^n = \cos n\theta + i\sin n\theta$$

$$z^n = |z|^n \text{cis } (n\theta)$$

$$z^{\frac{1}{q}} = |z|^{\frac{1}{q}}\left(\cos\left(\frac{\theta + 2\pi k}{q}\right) + i\sin\left(\frac{\theta + 2\pi k}{q}\right)\right) \text{ for } k \text{ an integer}$$

Measurement

Circle:	$C = 2\pi r = \pi D$, where C is the circumference, r is the radius and D is the diameter $A = \pi r^2$, where A is the area
Triangle:	$A = \frac{1}{2}bh$, where b is the base and h is the perpendicular height
Parallelogram:	$A = bh$
Trapezium:	$A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides
Prism:	$V = Ah$, where V is the volume and A is the area of the base
Pyramid:	$V = \frac{1}{3}Ah$
Cylinder:	$S = 2\pi rh + 2\pi r^2$, where S is the total surface area $V = \pi r^2 h$
Cone:	$S = \pi rs + \pi r^2$, where s is the slant height $V = \frac{1}{3}\pi r^2 h$
Sphere:	$S = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

See next page

Chance and Data

A confidence interval for the mean of a population is:

$$\overline{X} - z\frac{\sigma}{\sqrt{n}} \le \mu \le \overline{X} + z\frac{\sigma}{\sqrt{n}}$$

where μ is the population mean,

σ is the population standard deviation,

$\overline{X}$ is the sample mean,

n is the sample size,

z is the cut off value on the standard normal distribution corresponding to the confidence level.

Sample size: $n = \left(\frac{z \times \sigma}{d}\right)^2$ where d is the required value of the difference from the mean.

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.