**Year 2018**

**VCE**

**Mathematical Methods**

**Trial Examination 2**

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Victorian Certificate of Education

**2018**

###### STUDENT NUMBER

Letter

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| Figures  Words |  |  |  |  |  |  |  |  |  |  |  |
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###### MATHEMATICAL METHODS

**Trial Written Examination 2**

Reading time: 15 minutes

Total writing time: 2 hours

**QUESTION AND ANSWER BOOK**

**Structure of book**

|  |
| --- |
| *Section Number of Number of questions Number of*  *questions to be answered marks* |
| A 20 20 20  B 4 4 60  Total 80 |

* Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, their full functionality may be used.
* Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

* Question and answer booklet of 33 pages.
* Detachable sheet of miscellaneous formulas at the end of this booklet.
* Answer sheet for multiple-choice questions.

**Instructions**

* Detach the formula sheet from the end of this book during reading time.
* Write your **student number** in the space provided above on this page.
  + Write your **name** and **student number** on your answer sheet for multiple-choice questions, and sign your name in the space provided to verify this.
  + Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
* All written responses must be in English.

**At the end of the examination**

* Place the answer sheet for multiple-choice questions inside the front cover of this book.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

**SECTION A**

**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1 mark, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No mark will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

# Question 1

Consider the function  where . If the range is 

then the domain is

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 2**

Let , where  is a polynomial. When 

is divided by  the remainder is 6, when  is divided by  the remainder is

12. When  is divided by  the remainder is

**A.** 

**B.** 12

**C.** 

**D.** 

**E.** 

**Question 3**

Which of the following is **not** a continuous function?

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 4**

Several students were investigating the system of linear simultaneous equations

 and  where *k* is a real constant.

Alan stated that there is no solution when  and an infinite number of solutions when 

Ben stated that there is no solution when  and an infinite number of solutions when 

Colin stated that there is a unique solution when 

David stated that there is a unique solution when  or 

Edward stated that there is no solution when  and an infinite number of solutions when , and a unique solution when .

Then

**A.** Alan and Colin are correct.

**B.** Ben and Colin are correct.

**C.** Alan and David are correct.

**D.** Ben and David are correct.

**E.** Edward is correct.

**Question 5**

If *f* and *g* are two continuous and differentiable functions with the following properties



If  then

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 6**

The same students where investigating the function

 where 

Alan stated the range is *R*.

Ben stated the average value of the function is  and

the average rate of change of the function is *b*.

Colin stated the average value of the function is  and

the average rate of change of the function is.

David stated that the function is always increasing and that the gradient at  does not exist.

Edward stated that the function is always increasing and that  .

Then

**A.** Alan and Colin are correct.

**B.** Colin and David are correct.

**C.** Alan and David are correct.

**D.** Ben and David are correct.

**E.** Ben and Edward are correct.

**Question 7**

A box contains six black and five red marbles. Three marbles are drawn from the box without replacement. The probability that there are at least one of each colour is

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 8**

The graph of  is shown below, then



**A.**  and 

**B.**  and 

**C.**  and 

**D.**  and 

**E.**  and 

**Question 9**

If  and then  is equal to

**A.** 7

**B.** 6

**C.** 4

**D.** 

**E.** 

**Question 10**

The graphs of the functions  and  have a common tangent

at , as shown.



Then which of the following is **false**?

**A. **

**B.** 

**C.** 

**D.** 

**E.** 

**Question 11**

The graphs of  and  do not intersect when

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 12**

Which of the following is the inverse function of 

where *a* is a positive real number.

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 13**

If  is a non-zero even function, and is a non-zero odd function, then if 

 is equal to

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 14**

A certain curve has its gradient given by . If the curve crosses the *x*-axis at

 then the equation of the curve is

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 15**

Let . Which of the following statements about *f* is **not** true?

**A.**  for 

**B.**  for 

**C.**  for 

**D.** 

**E.**  for 

**Question 16**

The graph of  is transformed into the graph of  by

**A.** A dilation by a scale factor of  parallel to the *y*-axis, a reflection in the *x*-axis,

and a translation of  to the right parallel to the *x*-axis.

**B.** A dilation by a scale factor of  away from the *y*-axis, a reflection in the *x*-axis,

and a translation of  to the left parallel to the *x*-axis.

**C.** A dilation by a scale factor of  parallel to the *y*-axis, a reflection in the *y*-axis,

and a translation of  to the right parallel to the *y*-axis.

**D.** A dilation by a scale factor of  away from the *y*-axis, a reflection in the *y*-axis,

and a translation of  to the left parallel to the *x*-axis.

**E.** A dilation by a scale factor of  away from the *x*-axis, a reflection in the *x*-axis,

and a translation of  to the left parallel to the *y*-axis.

**Question 17**

If  ,  ,  and , then

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 18**

It is known that 55% of the population have brown eyes. For a random sample of 20 students,

 is the random variable of the proportion of students who have brown eyes. Then

, correct to four decimal places is

**A.** 0.1771

**B.** 0.1593

**C.** 0.2493

**D.** 0.5914

**E.** 0.7507

**Question 19**

The random variable *Z*  has the standard normal distribution, with ,

and  , where , then is equal to

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 20**

A discrete random variable *X* has the following probability distribution.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |

Given that  then

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**END OF SECTION A**

**SECTION B**

**Instructions for Section B**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

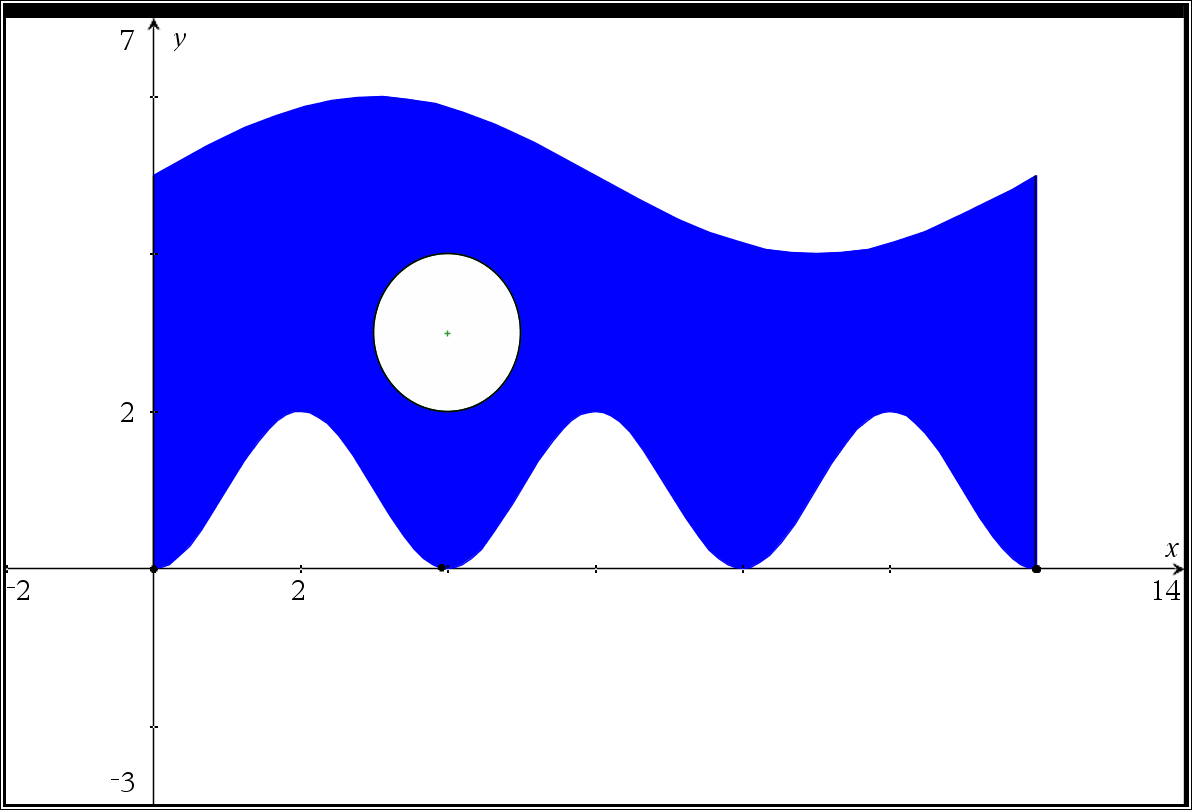
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1** (12 marks)

The diagram shows a plane cross-sectional view of a swimming pool at a hotel resort.

The *x* and *y* axes are shown, and are in the directions of east-west and north-south respectively. Distance are measured in metres and the length of the pool is 12 metres.



N

W

E

S

The pool is bounded by the northern most curve which is the function



and the southern most curve which is the function

.

The pool also contains a circular island, which has the equation .

To get to the island, a bridge is to be built from the centre of the circular island to the northern

most point on the graph of *f*.

**a.** Find the length of this bridge in metres.

2 marks

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Another bridge is to be built as a tangent line, from the southern curve at the point where.

**b.i.** Find the equation of this tangent line. 2 marks

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**ii.** Find the coordinates where this bridge intersects the northern curve.

Give your answers correct to three decimal places.

2 marks

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**iii.** Determine the length of this bridge, in metres, giving your answer correct to three

decimal places.

1 mark

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**c.i.** Write down a definite integral which gives the total cross-sectional area in square metres of the water in the swimming pool . 1 mark

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**ii.** The pool is filled to a constant depth of 1.5 metres. Find the volume in cubic metres of water in the swimming pool.

1 mark

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**d.** Find the maximum and minimum widths, in metres measured north-south of the swimming pool. Give your answers correct to three decimal places.

3 marks

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**Question 2** (14 marks)

Consider the function , where 

For different values of *c* and *d*, the graph has different properties.

**a.i.** Find a range of values of *c*, for which the graph of the function *f* has two stationary points.

1 mark

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**ii.** For the case when , find a range of values of *d*, when the graph of the function *f*, crosses the *x*-axis at three distinct points.

3 marks

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**iii.** If the graph of the function *f* has one stationary point at , find the values of *c*

and *d*, in this particular case and determine the coordinates of the other stationary point.

1 mark

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**b.i.** Find the value of *c*, for which the graph of the function *f* has exactly one stationary point.

1 mark

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**ii.** If the graph of the function *f* is translated *p* units to the left away from the *y*-axis, it becomes the graph of . Find the values of *c*, *d* and *p* in this particular case.

2 marks

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**c.** If the graph of the function *f* crosses the *y*-axis at , and the tangent to the function *f*

at , is , find the values of *a*, *c*, and *d* in this particular case.

2 marks

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**d.i.** Let *A* be the area bounded by the graph of the function *f*, the coordinate axes, and the

line , and that  for . If this area is approximated by four equally

spaced left rectangles, the area is . If this area is approximated by four equally spaced right rectangles, the area is , find the values of *c* and *d* in this particular case, and the true area *A*.

3 marks

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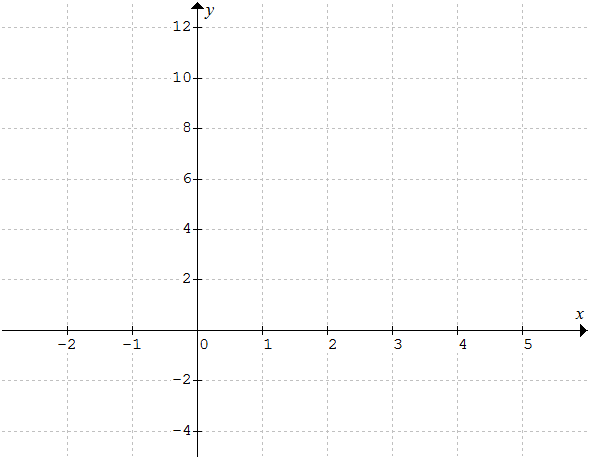
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**ii.** Sketch the graph of  on the axes below, for this case, shading the required area.

1 mark



**Question 3** (16 marks)

**a.** The length of time for which a certain type of candle burns before going out is normally distributed with a mean of one hour and a standard deviation of ten minutes.

**i.** Find the probability that a randomly selected candle of this type burns for between 45 and 55 minutes, giving your answer correct to four decimal places.

1 mark

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**ii.** Find the probability that a candle of this type will burn for less than 45 minutes, giving your answer correct to four decimal places. 1 mark

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**iii.** A birthday cake has 10 such candles, assuming that all candles burn independently,

find the probability that at least one candle burns for less than 45 minutes, giving your answer correct to four decimal places.

1 mark

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**b.** The manufacturers of the candles decide to take a random sample of 80 of these types of candles. For samples of size 80 from the population which burn for a mean of one hour

and a standard deviation of ten minutes, let  be the random variable of the distribution

of sample proportions of these types of candles which burn for less than 45 minutes.

**i.** Find the probability . Give your answer to four decimal places.

Do not use a normal approximation.

2 marks

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**ii.** The manufacturers find that in a particular sample of 80 of these types of candles, 16 of them burn for less than 45 minutes. Determine a 90% confidence interval for the manufacturers estimate of the proportion of these candles which burn for less than 45 minutes. Give your values correct to three decimal places.

1 mark

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**c.** For another type of long life candles, the times that these candles burn for is found to be normally distributed. It is found that 25% of these candles burn for more than 252 minutes, while 9% of these candles burn for less than 246 minutes. Find the mean and standard deviation of the times that these candles burn for. Give your answers to the nearest minute.

3 marks

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**d.** Every year on her birthday Lilly has either a chocolate cake or an ice-cream cake. If she has an ice-cream cake one year, the probability that she has an ice-cream cake the following year is 0.65, while if she has a chocolate cake one year, the probability she has a chocolate cake the following year is 0.55. Suppose that on her 13th birthday she had an ice-cream cake, determine the probability that from her 13th to 16th birthday inclusive she has exactly three ice-cream cakes. Give your answer to four decimal places.

2 marks

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**e.** When Lilly has a party, the time *t*, in hours that the guests stay, for has found to satisfy a continuous probability density function defined by



**i.** Find the probability that guests stay for more than two hours, if it is known that they stayed for at least one hour.

2 marks

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**ii.**  Find the mean and variance of the times in hours that guests stay at the party.

2 marks

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**iii.** Find the median time in hours that guests stay at the party.

Give your answer correct to three decimal places.

1 mark

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**Question 4**  (18 marks)

Consider the function 

**a.** The function *f* can be obtained from the graph of  under a transformation,

 , 

Find the values of *a*, *b*, *h* and *k*.

2 marks

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**b.** State the maximal domain *B* of the function *f*.

1 mark

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**c.** Find  , the inverse function *f*.

2 marks

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**d.** Find all values of *u*, such that .

1 mark

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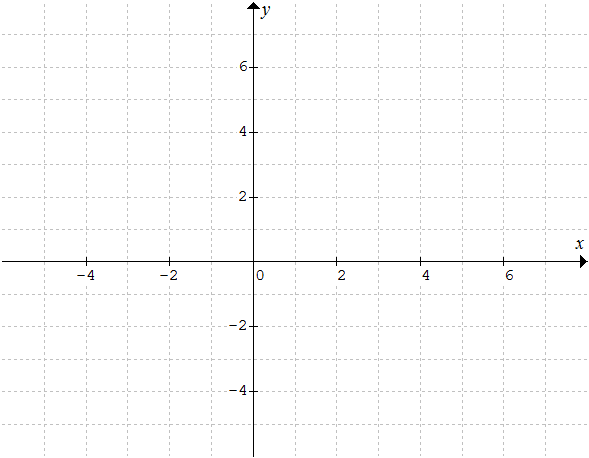
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**e.** Sketch the graphs of and the graph of  on the diagram below.

2 marks



**f.i.** Find the area enclosed between the graphs of  and .

1 mark

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**ii.** Find the area enclosed between the graphs of  and .

1 mark

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**g.** Find the angle between the tangents to both curves  and  at 

Give your answer in degrees correct to one decimal place.

2 marks

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Consider now the function  where 

**h.** Describe the transformations that map the graph of  onto the graph of *g*.

1 mark

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**i.** Find  the inverse function *g*, stating the maximal domain *D* of *g*.

1 mark

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**j.** The point with co-ordinates  lies on the graphs of  and  Express *c* in terms of *k*.

1 mark

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**k.** Find values of *k*, for when the graphs of  and  do not intersect.

1 mark

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**l.** Determine the value of *k*, when the tangents to both curves  and  intersect at  at 

2 marks

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**END OF EXAMINATION**

**EXTRA WORKING SPACE**

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**MATHEMATICAL METHODS**

**Written examination 2**

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Mathematical Methods formulas

## Mensuration

|  |  |  |  |
| --- | --- | --- | --- |
| area of a trapezium |  | volume of a pyramid |  |
| curved surface area  of a cylinder |  | volume of a sphere |  |
| volume of a cylinder |  | area of triangle |  |
| volume of a cone |  |  |  |

## Calculus

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
| product rule |  | quotient rule |  |
| chain rule |  |  | |

## Probability

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
|  | |  | |
| mean |  | variance |  |

|  |  |  |  |
| --- | --- | --- | --- |
| **Probability distribution** | | **Mean** | **Variance** |
| discrete |  |  |  |
| continuous |  |  |  |

## 

## Sample proportions

|  |  |  |  |
| --- | --- | --- | --- |
|  | | mean |  |
| standard  deviation |  | approximate  confidence  interval |  |

**END OF FORMULA SHEET**

## 

**ANSWER SHEET**

###### STUDENT NUMBER

Letter

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Figures  Words |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |

###### SIGNATURE \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**SECTION A**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |
| **1** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **2** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **3** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **4** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **5** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **6** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **7** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **8** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **9** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **10** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **11** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **12** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **13** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **14** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **15** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **16** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **17** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **18** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **19** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
| **20** |  | **A** |  | **B** |  | **C** |  | **D** |  | **E** |  |
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