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Student Name……………………………………

### MATHEMATICAL METHODS UNITS 3 & 4

### TRIAL EXAMINATION 2

**2019**

#### Reading Time: 15 minutes

Writing time: 2 hours

###### Instructions to students

This exam consists of Section A and Section B.

Section A consists of 20 multiple-choice questions, which should be answered on the detachable answer sheet which can be found on page 25 of this exam.

Section B consists of 5 extended-answer questions.

Section A begins on page 2 of this exam and is worth 20 marks.

Section B begins on page 9 of this exam and is worth 60 marks.

There is a total of 80 marks available.

All questions in Section A and Section B should be answered.

In Section B, where more than one mark is allocated to a question, appropriate working must be shown.

Where a numerical answer is required, an exact value must be given unless otherwise directed.

Diagrams in this exam are not to scale except where otherwise stated.

Students may bring one bound reference into the exam.

Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.

A formula sheet can be found on pages 23 and 24 of this exam.

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**SECTION A – Multiple-choice questions**

**Question 1**

Let ****.

The period of this function is

1. 2
2. 3
3. 4
4. 6
5. 12

**Question 2**

The range of the function , is

1. 
2. 
3. 
4. 
5. *R*

**Question 3**

If , then  over the interval

1. 
2. 
3. 
4. 
5. 

**Question 4**

The graph of *f* undergoes a transformation to become the graph of *h* where .

The point  lies on the graph of *f* and is mapped to the point *Q* which lies on the graph of *h*.

*Q* has the coordinates

1. 
2. 
3. 
4. 
5. 

**Question 5**

Sian randomly selects one ball from a bucket containing six green and three yellow tennis balls and a second ball from a bucket containing two green and three yellow tennis balls.

If Sian selects two green tennis balls she serves first in a tennis match. If she doesn’t, her friend gets to serve first.

The probability that Sian’s friend serves first in the tennis match is

1. 
2. 
3. 
4. 
5. 

**Question 6**

The discrete random variable *X* has a binomial distribution. The mean of *X* is 1.8 and the variance of *X* is 1.44. The number of independent trials *n*, and the probability of success in each trial *p*, are given by

1. 
2. 
3. 
4. 
5. 

**Question 7**

Let .

The function  is

1. 
2. 
3. 
4. 
5. 

**Question 8**

For the two mutually exclusive events *A* and *B*, 

 is equal to

1. 0.05
2. 0.1
3. 0.2
4. 0.25
5. 0.6

**Question 9**

Given that  is equal to

1. –6
2. –2
3. 2
4. 4
5. 6

**Question 10**

Let .

A tangent to the graph of *g* has a gradient of – 2.

The *y*-intercept of this tangent is

1. 2
2. 
3. 
4. 
5. 4

**Question 11**

The function *f* has the property  for all real numbers *x*.

The rule for *f* could be

1. 
2. 
3. 
4. 
5. 

**Question 12**

Over the interval for which , the graph of , has exactly one asymptote.

A possible value of *a* is

1. 
2. 
3. 
4. 
5. 

**Question 13**

In a particular city, the unemployment rate for 17 – 20 year olds is 18%. If a random sample of 12 young people in this age bracket is taken, the probability that more than half of them will be unemployed is

1. 0.0018
2. 0.0021
3. 0.0116
4. 0.4771
5. 0.5658

**Question 14**

Let , where *p* is a positive constant.

The graph of *f* will have two stationary points when

1. 
2. 
3. 
4. 
5. 

**Question 15**

The shaded rectangle in the diagram below is formed using part of the coordinate axes and the point , which lies on the curve .



The maximum area of the rectangle is closest to

1. 1.3
2. 2.8
3. 3.6
4. 5.0
5. 6.4

**Question 16**

Let .

The graphs of *f* and *g* are shown in the diagram below.



An expression that gives the total area of the shaded region is

1. 
2. 
3. 
4. 
5. 

**Question 17**

Let 

The function *f* has an inverse function .

The instantaneous rate of change of  at the point where  is .

The value of *a* is

1. 
2. 
3. 1
4. 2
5. 4

**Question 18**

The graphs of *f* and *g* are shown below. The graphs have the same scale.



The graph of *f* has been transformed onto the graph of *g* according to the transformation *T*.

*T* could be

1. 
2. 
3. 
4. 
5. 

**Question 19**

A probability density function *f* is given by





A possible value of *k* is

1. 2
2. 
3. 
4. 
5. 

**Question 20**

Consider the functions  where *p* is a positive integer and .

The average value of *f* is always less than the average value of *g* over the interval

1. 
2. 
3. 
4. 
5. 

**SECTION B**

Answer all questions in this section.

**Question 1** (12 marks)

Waste material in a container is slowly poured into an empty machine to await processing. After passing through the processing area of the machine, the processed waste material passes out of the machine as shown in the diagram below.



The height, , in metres, of waste material awaiting processing in the machine, can be modelled by the function



where *t* is the time, in minutes, after the pouring begins.

1. **i.** For how many minutes is there waste material awaiting processing in the machine,

after the container starts being poured? 1 mark

1. What is the average height of waste material awaiting processing in the machine during this time? Give your answer in metres correct to two decimal places. 2 marks

Suppose that two minutes after this first container of waste material starts being poured into the machine, a second, identical container of waste starts being poured into the machine.

The height, in metres, of waste material awaiting processing in the machine contributed individually by these two containers is shown in the graph below.



1. **i.** On the graph above, sketch the total height of waste material awaiting processing in the machine for . 2 marks
2. Find the maximum height of waste material awaiting processing in the machine for  and the time(s) when this maximum occurs. 2 marks

Suppose instead, that *a* minutes after the first container of waste material starts being poured into the machine, a second, identical container of waste starts being poured into the machine.

The height, in metres, of waste material from this second container awaiting processing can be modelled by the function



where *t* is the time, in minutes, after the pouring of the first container begins.

The graphs of  are shown below.



1. Describe the transformation that maps the graph of . 1 mark

The second container of waste material starts being poured into the machine

* after the first container starts being poured but
* whilst there is still waste material from the first container awaiting processing in the machine.

1. Find the possible values of *a*. 1 mark

**e.** Find the values of *t* for which waste material from both containers remains in the machine

waiting to be processed. 1 mark

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**f.** If the maximum height of waste material in the machine waiting to be processed is limited

to  metres, find the possible values of *a*. 2 marks

**Question 2** (11 marks)

Two stage curtains, which are symmetrical in shape, are being designed for a theatre.

The rectangular stage surround has a width of ten metres and a height of six metres.

The curved edge of the right curtain can be modelled by the function



where the *x*-axis represents the stage floor, *x* represents the distance from the middle of the stage, and *y* represents the height above the stage floor.

All units are measured in metres.

The curved edge of the left curtain can be modelled by the function .

The graphs of are shown in the diagram below.



1. Write down the domain of function *l*. 1 mark
2. Explain why the rule for *l* is given by  1 mark
3. Find the total area covered by the two curtains for this design, correct to the nearest square metre. 2 marks

A second design is drawn up.

A model of the edge of the right curtain in this second design is obtained by applying a transformation to .

This transformation involves

* a dilation by a factor of 2 from the *y*-axis
* a translation of 1 unit to the left

A graph of this second design is shown below.



The edge of the right curtain for this second design is modelled by the function .

1. Define the function . 3 marks

A third and final design is drawn up. In this design, the left and right curtains are still symmetrical. The overlap of the left and right curtains, which forms a double layer of curtain, is indicated by the darker shading in the diagram below.



The curved edge of the left curtain is modelled by the function



where *a* is a constant.

1. Find the value of *a* correct to four decimal places. 1 mark
2. Find to the nearest square metre, the area that is covered by a single layer of curtain as indicated by the lighter shading in the diagram above. 3 marks

**Question 3** (11 marks)

Consider the quartic function .

Part of the graph of  is shown below.



Point *P* lies on the graph and has an *x*-coordinate of 1.

1. Find the coordinates of the stationary points of the graph. 2 marks

**b.** Find the equation of the tangent to *f* at point *P*. 1 mark

**c.** Show that the tangent found in part **b.** intersects with *f* at only two points. 1 mark

1. Find the area of the region enclosed by the tangent found in part **b.** and . 2 marks

Consider the gradient function 

The graph of  is shown below.



1. **i.** Find the value of *a*, where *a* is a positive constant, for which  has exactly two solutions. 2 marks
   1. Find the values of *x* for which this occurs. 1 mark

One of the two tangents to *f* which have a gradient of *a*, found in part **e.** **i**., is translated *m* units in the positive direction of the *x*-axis. It then coincides with the second tangent.

1. Using your answer to part **b.**, find the value of *m*. 2 marks

**Question 4** (16 marks)

A car manufacturer discovers that 15% of the cars it has produced were fitted with faulty airbags.

1. A random sample is taken of 20 cars produced by this manufacturer. Find, correct to four decimal places, the probability that exactly five of these cars have faulty airbags. 2 marks
2. Another random sample of 20 cars produced by this manufacturer is taken. The proportion of cars in this sample with faulty airbags is 0.15.

Determine an approximate 90% confidence interval for the proportion of cars with faulty airbags, correct to four decimal places. 2 marks

1. Further random samples of 20 cars produced by this manufacturer are taken. Let  be the random variable that represents the proportion of these cars with a faulty airbag.

Find  correct to four decimal places. 3 marks

Car dealerships must organize the recall of cars made by this manufacturer so that the airbags can be replaced.

The time taken for car owners to respond to this recall is normally distributed with a mean of seven weeks and a standard deviation of one and a half weeks.

1. Find the probability that a randomly selected car owner whose car has been recalled, responds to the recall within nine weeks. Give your answer correct to four decimal places. 1 mark
2. One randomly selected car owner had not responded to the recall within seven weeks. Find, correct to four decimal places, the probability that they respond within 12 weeks. 2 marks
3. Eighty percent of car owners whose cars had been recalled, respond to the recall within *w* weeks. Find the value of *w* correct to four decimal places. 1 mark

For a randomly selected car that has been recalled, the time taken to replace its airbags has the probability density function given by



where *t* is measured in minutes.

1. Find the median time, in minutes, for a randomly selected car that has been recalled, to have its airbags replaced. Give your answer correct to one decimal place. 2 marks
2. Find the probability that a randomly selected car owner whose car is recalled, will have their airbags replaced in less than an hour. Give your answer correct to four decimal places. 1 mark

Of all the car owners who had their car recalled, 42% were female and the rest were male.

Of these female car owners, two thirds had their airbags replaced in less than an hour.

1. Find the probability that a randomly selected male car owner who had his car recalled, had the airbags replaced in less than an hour. Give your answer correct to three decimal places. 2 marks

**Question 5** (10 marks)

Consider the function  where *a* is a positive real number.

1. Find the coordinates of the local maximum and local minimum of *f* in terms of *a*. 2 marks

The line  where *k* is a constant, passes through the local maximum and local minimum points found in part **a**.

1. Show that the average rate of change of *f* between  and  equals *k*. 1 mark

The function *h* has the same rule as *f* but is strictly decreasing over its entire domain.

1. Given that *h* has a maximal domain, find the rule and domain of *h*. 2 marks
2. The function *h*  has an inverse function .

Find the area bounded by the graph of  and the *x* and *y* axes. 2 marks

1. Find the area enclosed by the graphs of . 3 marks

**Mathematical Methods formulas**

## Mensuration

|  |  |  |  |
| --- | --- | --- | --- |
| area of a trapezium |  | volume of a pyramid |  |
| curved surface area of a cylinder |  | volume of a sphere |  |
| volume of a cylinder |  | area of a triangle |  |
| volume of a cone |  |  | |

## Calculus

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
| product rule |  | quotient rule |  |
| chain rule |  |  | |

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**Probability**

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
|  | |  | |
| mean |  | variance |  |

|  |  |  |  |
| --- | --- | --- | --- |
| **Probability distribution** | | **Mean** | **Variance** |
| discrete | = |  |  |
| continuous |  |  |  |

## Sample proportions

|  |  |  |  |
| --- | --- | --- | --- |
|  | | mean |  |
| standard  deviation |  | approximate  confidence  interval |  |

