

**MATHS METHODS 3 & 4**

**TRIAL** **EXAMINATION 2**

**SOLUTIONS**

**2020**

**(Adjusted Study Design)**

# SECTION A – Multiple-choice answers

1. **A 9. A 17. E**
2. **C 10. E 18. A**
3. **E 11. D 19. E**
4. **D 12. B 20. D**
5. **D 13. B**
6. **D 14. E**
7. **E 15. C**
8. **C 16. B**

# SECTION A – Multiple-choice solutions

# Question 1



The amplitude is 2 and there is a translation 1 unit down. So range.

The answer is A.

**Question 2**

The graph touches the *x*-axis at , so the rule has a repeated factor of .

The graph cuts the *x*-axis at . The graph is an inverted quartic.



The answer is C.

**Question 3**

.

Since  is a factor, then .

So 



The answer is E.

**Question 4**

The period of the graph of .



If the function *f* has adjacent asymptotes at

,

then the period is .



A possible rule is .

The answer is D.

**Question 5**



The answer is D.

**Question 6**

The equation  is a quadratic in the variable *x*.



For two real solutions, .

Solve  for *p*.

So, 

In interval notation, this is given by .

The answer is D.

**Question 7**



Solve  for *a*



The graph of *f* is a hyperbola with two branches and two asymptotes. One asymptote has equation  and the other .

The vertical asymptote with equation  can be to the left or right of the *y*-axis, or on the *y*-axis depending on the value of *a* as shown below.



So both values of *a* are possible since .

The answer is E.

**Question 8**

The simultaneous equations can be expressed as a matrix equation.



If , then there are no solutions or infinitely many solutions i.e.





We can now reject options B and E.



For no solutions, .

(i.e. if  there are infinitely many solutions and for all other values of *k* there are no solutions.)

Reject option A.



For no solutions .

Reject option D.

So for no solutions, the only possible option given is .

The answer is C.

**Question 9**



Solve  for *x* using CAS.





The closest answer is 0.2329.

The answer is A.

**Question 10**

Since *X* has a binomial distribution, .





The answer is E.

**Question 11**

****

****

If time, check option E.



The answer is D.

# Question 12

To transform  we need to

* translate 1 unit left



* translate 2 units down



* reflect in the *x*-axis



* dilate by a factor of 3 from the *y*-axis



The transformation *T* is .

Option B gives this.

**Question 12** (cont’d)

If you have time you can do a confirmation.

Let  be the image point. Using the matrix equation given in option B,



Image equation is  as required.

The answer is B.

**Question 13**

We start with .

After a dilation by a factor of 2 from the *x*-axis, we have



After a dilation by a factor of *a* from the *y*-axis, we have



After a translation of 1 unit downwards, we have



After a translation of  units left, we have



The *x*-intercept occurs when ,





and so on.

Only  is offered.

The answer is B.

**Question 14**

The graphs of *f* and  intersect at a point along the line .



So .

The answer is E.

**Question 15**

Let *X* represent the number of times that Ella has missed the early bus over the last thirty occasions that she attended school.







Note that the numerator in the fraction is obtained using binomPdf(30,0.1,2).

The denominator in the fraction is obtained using binomCdf(30,0.1,0,4).

The closest answer is 0.2761.

The answer is C.

**Question 16**



Because the graph of *f* is a cubic function, then the

graph of must be a quadratic function.

Because this graph of  has a maximum value, then

the graph of *f*  has a negative gradient for , a

positive gradient for  and a negative gradient

for .

So *f* must be anegative cubic (ie comes down

from the left or alternatively goes down to the right).

A possible graph of *f*  is shown. Note that we don’t

know exactly what the graph of *f* looks like because

we don’t have the rule for *f.* We do know however that

it will have a local minimum at  (and a local

maximum at ).

The answer is B.

**Question 17**

Draw a horizontal line through the



average value, i.e. .

The area above and below **this line**,

between the boundaries 

and the function, must be equal.

The only function where you can achieve

this is option E.

The answer is E.

**Question 18**





The answer is A.

**Question 19**

 is a composite function.

We use the chain rule to differentiate it.



The answer is E.

**Question 20**

The graphs below show the graph of  for values of *k* which are less than 1, equal to 1 and greater than 1.



For option A in the question, . If ie if the area enclosed by the graph of , the *x*-axis and the lines  is above the *x*-axis, then we would need . Option A offers just  so reject option A.

Similarly for option B which offers . Reject option B.

Consider option C which has .

When , we have . (a property of the definite integral)

Reject option C.

Options D and E offer .

Consider .

 (a property of the definite integral)



Looking at the graphs below, we see that the area enclosed by the graph of  and the *x*-axis is above the *x-*axis for  if .

So when  the inequality will be true.



Option D gives  and is therefore correct.

The answer is D.

**SECTION B**

**Question 1 (10 marks)**

1. Stationary points occur when .

  **(1 mark)**

Stationary point is . **(1 mark)**

1. Sketch the graph of *f* .



Using part **a**., we see that

the function is strictly

decreasing for .

**(1 mark)**

1.  **(1 mark)**



So  **(1 mark)**

1. **i.** Using CAS, . **(1 mark)**
2. Solve  for *x,* using CAS.



The tangent intersects the graph at 

The tangent touches the graph at  (point of tangency). Sketch the graphs.



 where 0.4065 is correct to four decimal places. **(1 mark)**

1. The graph of *f* passes through the origin and has a horizontal asymptote at .

If we translate the graph of *f* to the right, we will have one positive solution.

So for , we require *d* to be a negative number ie .

**(1 mark)**

1. From part **a.** we know that the stationary point, which is a maximum turning point, is located at the point .



The graph currently has one *x*-intercept and hence the equation  has one solution when .

As , the graph approaches the *x*-axis from above; that is, there is a horizontal asymptote at .

If , then the graph will be translated *h* units down.

When , only the turning point will be touching the *x*-axis and hence  will have one solution.

For two solutions, we require .

**(1 mark)** – left endpoint

**(1 mark)** – right endpoint

**Question 2** (12 marks)

1. 



**(1 mark)**

1. Find the point of intersection using CAS.

*P* is the point .

**(1 mark)**

1. The rules of the functions *l* and *u* are given respectively by .

In the transformation of *l* to *u*, we see that *x* is replaced by  so there is a translation of 2 to the left.

We see that *y* is replaced by  so there is a translation of  upwards.

So .

**(1 mark)**

There are no dilations and no reflections.

So .

**(1 mark)**

1. The distance *D* is a function of *x*, i.e. .

Solve  for *x*.

**(1 mark)**



So min value occurs at .

Since , this is the point where the graph of *u* and of *l* touch, i.e. at the point

*P**.*

**(1 mark)**

1. From part **d.** we saw that the only value of *x* for which  occurred at .

From the graph, the maximum vertical distance between the two graphs could occur at  or at  so check both.



**(1 mark)**

So the maximum value of *D* is  and this occurs at .

**(1 mark)**

1.  **(1 mark)**



**(1 mark)**

1. The shaded area to the left in the diagram below, is represented by .

Note that from part **a.**, the period of function *l* is 6 and so its maximum value occurs at .

The shaded area to the right in the diagram below, is represented by .

From part **c.**, because there are no dilations, the shaded areas to the left and right of point *P* are equal.

The minimum value of function *u* is located at .



So because of symmetry, .

**(1 mark)** for *m*

**(1 mark)** for *k.*

**Question 3** (14 marks)

1. Define *f* on your CAS.

**(1 mark)**



Length of footpath is .

**(1 mark)**

1. Solve  for *x*. **(1 mark)**



The furthest distance north is 25.16 metres and this occurs when . **(1 mark)**

1. **i.** To model this change we replace *x* with  in the rule for *f.*



 **(1 mark)**

Alternatively, using CAS,

1.  (using part **a.**)

 (where endpoints are correct to 1 decimal place)

**(1 mark)**

1. Since the point  lies on the graph of , then



**(1 mark)**

1. ****

**(1 mark)**

1. The equation of the line representing the shunting track is



The gradient of the line representing the shunting track is . **(1 mark)**

The gradient of the existing curved track at point .

At  these gradients are equal.

Solve  for *p* using CAS. **(1 mark)**

.





**(1 mark)**

1. Since the shunting track is tangential to the curved line represented by  at the point , then



**(1 mark)**

1.   **(1 mark)**



**(1 mark)**

**Question 4** (11 marks)

**a.** 



 **(1 mark)**

**b.** 

**(1 mark)**



 **(1 mark)**

**c.**  **(1 mark)**



**(1 mark)**

Note that if less than two of the workouts are completed in less than 25 minutes then we want the probability of zero or one of the workouts being completed in less than 25 minutes.

**d.**  **(1 mark)**



 **(1 mark)**

**e.** Solve . **(1 mark)**





The maximum value of *W* is 0.6695 which occurs when *p* equals 0.2899 where both values are expressed correct to 4 decimal places. **(1 mark)**

**f.** , where 

When , we have  **(1 mark)**

So  using invNorm(0.28989…, 30,2)

So  **(1 mark)**

**Question 5** (13 marks)

1. If  exists, then *f* must be a  function.

If *a* is to be the least possible value, then .

**(1 mark)**

1.  from part **a.**



  **(1 mark)**



Swap *x* and *y* for inverse.



**(1 mark)**

1. Do a quick sketch of the graphs of .



The graph of  is shown as a dotted

line.

The point of intersection of the graphs

occurs on the line .

Solve  for *x*, to find this

point of intersection.

It occurs at .



**(1 mark)**

**(1 mark)**

1. **i.**  so let 

* after a dilation by a factor of 3 from the *x*-axis we have 



* after a dilation by a factor of  from the *y*-axis we have 
* after a reflection in the *y*-axis we have 



 as required. **(1 mark)**

**ii.** We know that .

After the two dilations, the domain of the resulting function is still .

After the reflection in the *y*-axis, .

**(1 mark)**

**e.**



**(1 mark)** for  including correct endpoint at  and correct asymptote of 

**(1 mark)** for  including correct endpoint at  and correct asymptote of 

**f.** 

Since ,



**(1 mark)**

**(1 mark)**

. Therefore we choose the negative branch when taking the square root.



**g.** The shaded region to the left of the

*y*-axis represents the area given by

. Note that .

The shaded region to the right of the

*y*-axis represents the area given by

.

Note that these two shaded regions

are equal in area.

So the two regions are equal when

. **(1 mark)**



If , then the area of the

region to the right of the *y*-axis will

be less than that to the left of the

*y*-axis, as shown in the diagram.

So, possible values of *k* are

.

**(1 mark)**

**(1 mark)** – left endpoint

**(1 mark)** – right endpoint