

Student Name……………………………………

### MATHEMATICAL METHODS UNITS 3 & 4

### TRIAL EXAMINATION 2

**2020**

**(Adjusted Study Design)**

#### Reading Time: 15 minutes

Writing time: 2 hours

###### Instructions to students

This exam consists of Section A and Section B.

Section A consists of 20 multiple-choice questions, which should be answered on the detachable answer sheet which can be found on page 25 of this exam.

Section B consists of 5 extended-answer questions.

Section A begins on page 2 of this exam and is worth 20 marks.

Section B begins on page 10 of this exam and is worth 60 marks.

There is a total of 80 marks available.

All questions in Section A and Section B should be answered.

In Section B, where more than one mark is allocated to a question, appropriate working must be shown.

Where a numerical answer is required, an exact value must be given unless otherwise directed.

Diagrams in this exam are not to scale except where otherwise stated.

Students may bring one bound reference into the exam.

Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.

A formula sheet can be found on pages 23 and 24 of this exam.

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**SECTION A – Multiple-choice questions**

**Question 1**

Let ****.

The period and range of *f* are respectively

1. 
2. 
3. 
4. 
5. 

**Question 2**

Part of the graph of a polynomial function is shown below.



A possible rule for this function is

1. 
2. 
3. 
4. 
5. 

**Question 3**

Given that  is a factor of , then *k* is equal to

**A.** – 4

**B.** – 2

**C.** – 1

**D.** 2

**E.** 4

**Question 4**

The graph of the function *f* has adjacent asymptotes at .

A possible rule for *f* is

1. 
2. 
3. 
4. 
5. 

**Question 5**

Let , where *a* is a positive real number.

The average rate of change of *f* between  is

1. 
2. 
3. 
4. 
5. 

**Question 6**

The equation , where *p* is a constant, has two real solutions for

1. 
2. 
3. 
4. 
5. 

**Question 7**

Let .

The gradient of the tangent to the graph of *f* at the point where , is .

The value of *a* is equal to

1.  only
2. – 1 only
3. 3 only
4. – 4 or – 1
5. – 1 or 3

**Question 8**

The simultaneous linear equations  have no solutions for

1. 
2. 
3. 
4. 
5. 

**Question 9**

Part of the graph of  is shown below.



The shaded rectangle shown has corner points located at the origin, ,  and .

The maximum area of the rectangle is closest to

**A.** 0.2329

**B.** 0.3975

**C.** 0.4372

**D.** 0.5326

**E.** 0.5628

**Question 10**

The random variable *X* has a binomial distribution where .

 is equal to

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 11**

Let .

If *y* is a real number, then which one of the following equations is **not** true?

1. 
2. 
3. 
4. 
5. 

**Question 12**

The transformation  maps the graph of  onto the graph of .

The rule for *T* is given by

1. 
2. 
3. 
4. 
5. 

**Question 13**

The function with rule  has a maximal domain and .

The graph of *f* is, in order, dilated by a factor of 2 from the *x*-axis, dilated by a factor of *a* from the *y*-axis, translated 1 unit downwards and translated  units left.

One of the *x*-intercepts of the transformed graph is located at

1. 
2. 
3. 
4. 
5. 

**Question 14**

Let , where *a* is a positive constant.

The graph of *f* and its inverse , intersect at their right endpoints.

The value of *a* is

1. 
2. 1
3. 2
4. 4
5. 6

**Question 15**

Ella takes a bus to get to school. The probability that she misses the early bus to school is 0.1. Whether she misses the early bus to school one day is independent of whether she misses it on another day.

Given that over the last thirty occasions Ella attended school she missed the early bus less than five times, then the probability that she missed it exactly twice is equal to

**A.** 0.2267

**B.** 0.2456

**C.** 0.2761

**D.** 0.2939

**E.** 0.8245

**Question 16**

The function *f* is a cubic function. The graph of the derivative function  has *x*-intercepts at

 and has a maximum value of 4.

The graph of *f* has a local minimum at

**A.** 

**B.** 

**C.** 

**D.** 

**E.** 

**Question 17**

Let *f* be a function with an average value of 1 over the interval . The graph of *f* over this interval could be



**Question 18**

The area under the graph of , between , is 10 units.

Given that , the area under the graph of , is

1. 34 
2. 38 
3. 40 
4. 42 
5. 46 

**Question 19**

The function  is a differentiable function.

The derivative of  with respect to *x* is equal to

1. 
2. 
3. 
4. 
5. 

**Question 20**

Let .

Consider the inequality 

If this inequality is true, then

1. 
2. 
3. 
4. 
5. 

**SECTION B**

Answer all questions in this section.

**Question 1** (10 marks)

Let .

1. Find the coordinates of the stationary point of the graph of *f*. 2 marks
2. Find the values of *x* for which *f* is strictly decreasing. 1 mark
3. The average value of *f* between  can be expressed in the form  where *a*, *b* and *c* are positive integers. Find *a*, *b* and *c*. 2 marks
4. Let the function *t* represent the tangent line to the graph of *f* at the point where .
   1. Find the rule for *t*. 1 mark
   2. Find the values of *x* for which , expressing interval endpoints correct to four decimal places. 1 mark
5. Find the values of  has exactly one positive solution. 1 mark
6. Find the values of  has two solutions. 2 marks

**Question 2** (12 marks)

An airline logo is based around a particular shape.

The lower and upper boundaries of this shape are modelled respectively by the functions



The side boundaries of the shape are vertical lines.

The upper and lower boundaries touch at point *P*.

The shape is indicated by the shaded region in the diagram below. All measurements are in centimetres.



1. State the period of function *l*. 1 mark
2. Find the coordinates of point *P*. 1 mark
3. A transformation that maps the graph of  is given by



Find the values of *a*, *b*, *c* and *d*. 2 marks

Let *D* be the vertical distance between the upper and lower boundaries of the shape.

1. Show that the minimum value of *D* occurs at *P*. 2 marks
2. Find the maximum value of *D* and the value(s) of *x* for which this occurs. 2 marks
3. Find the area of the airline logo in square centimetres. 2 marks

**g.** Given that , find the value of the constant *m* and of the

constant *k*. 2 marks

**Question 3** (14 marks)

A miniature train runs along a horizontal track in a children’s park.

The track is modelled by the function



where *x* is the distance east, in metres, from the origin at *O* and *y* is the distance north, in metres, from *O* as shown on the graph below.



The train starts at point  and finishes at point .

That part of the *x*-axis for which  represents a footpath.

1. Find the length, in metres, of the footpath, correct to two decimal places. 2 marks
   1. Find the furthest distance north of the footpath that the track goes and the value(s) of *x* for which this occurs. 2 marks
   2. The original plan for the laying of the track involved laying it in the same shape but five metres to the east of the present track.

For this original plan,

1. state a rule, in terms of *x*, that would model the position of the track. 1 mark
2. state a domain for the rule found in part **i.**, with endpoint values correct to one decimal place. 1 mark

In order to run a second miniature train on the track, a straight shunting track is built.

It starts at the origin and meets the existing curved track at the point .

It is indicated by the dotted line on the graph below.



Regardless of whether a train approaches point *C* from the origin along the shunting track or from point *A* along the existing curved track, it would be heading in exactly the same direction at point *C*. That is, there is a smooth join where the shunting track meets the existing curved track.

1. Write down an expression for *q* in terms of *p*. 1 mark
2. Find . 1 mark
3. Find the length of the shunting track correct to two decimal places. 3 marks
4. Find the angle that the tangent to the graph of *f* at the point , makes with the positive branch of the *x*-axis correct to the nearest degree. 1 mark
5. Find the area, in square metres, enclosed by the shunting track, the existing curved track and the *x*-axis correct to four decimal places. 2 marks

**Question 4** (11 marks)

Bhavani exercises by using a particular workout written by a trainer at her gym.

The time *T*, in minutes, that Bhavani takes to complete her workout is normally distributed with a mean of 30 minutes and a standard deviation of 2 minutes.

1. Find the probability that Bhavani takes less than 25 minutes to complete her workout, correct to four decimal places. 1 mark
2. Find the probability that Bhavani takes less than 25 minutes to complete her workout given that she takes less than 30 minutes, correct to four decimal places. 2 marks

Bhavani completes five workouts each week.

1. What is the probability that less than two of the five workouts are completed in less than 25 minutes, correct to four decimal places? 2 marks

Bhavani continues to complete five workouts each week.

Her trainer wants to monitor the time that her workouts are taking. He wants to know the time, *m* minutes, that Bhavani takes to complete a workout, so that the probability that she completes one or two workouts a week in less than *m* minutes is maximized.

Let *W* represent the probability that Bhavani completes one or two of her five weekly workouts in less than *m* minutes.

1. Given that , show that . 2 marks

1. Find the maximum value of *W* and the value of *p* for which this occurs, each correct to fourdecimal places. 2 marks

1. Find the value of *m* for which the maximum value of *W* occurs, correct to twodecimal places. 2 marks

**Question 5** (13 marks)

Part of the graph of  is shown below.



Let , where *a* is the least possible value such that , the inverse of *f* , exists.

**a.** Find the value of *a*. 1 mark

**b.** Find the rule and domain of . 2 marks

**c.** Find the area, in square units, bounded by the graphs of *f ,* and the *x* and *y* axes, correct

to four decimal places. 2 marks

The graph of *f* is

* + dilated by a factor of 3 from the *x*-axis
  + dilated by a factor of  from the *y*-axis
  + reflected in the *y*-axis

to become the graph of *h*.

**d. i.** Show that the rule for *h* is given by . 1 mark

**ii.** Find the domain of *h.* 1 mark

**e.** On the set of axes below, sketch the graphs of *h* and its inverse function .

Indicate clearly the coordinates of any endpoints and the equation of any asymptotes. 2 marks



**f.** Using the fact that , find the rule for . 2 marks

**g.** Given that , find the possible values of *k*. 2 marks

**Mathematical Methods formulas**

## Mensuration

|  |  |  |  |
| --- | --- | --- | --- |
| area of a trapezium |  | volume of a pyramid |  |
| curved surface area of a cylinder |  | volume of a sphere |  |
| volume of a cylinder |  | area of a triangle |  |
| volume of a cone |  |  | |

## Calculus

|  |  |  |  |
| --- | --- | --- | --- |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
|  | |  | |
| product rule |  | quotient rule |  |
| chain rule |  |  | |

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**Probability**

|  |  |
| --- | --- |
|  |  |
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|  |  |  |  |
| --- | --- | --- | --- |
| **Probability distribution** | | **Mean** | **Variance** |
| Bernoulli |  |  |  |
| binomial |  |  |  |
| normal |  |  |  |

**END OF FORMULA SHEET**

