

**SPECIALIST MATHS**

**TRIAL** **EXAMINATION 2**

**SOLUTIONS**

**2019**

## Section A – Multiple-choice answers

**1.** C **6.** E **11.** D **16.** E

**2.** E **7.** B **12.** A **17.** C

**3.** B **8.** A **13.** A **18.** C

**4.** D **9.** A **14.** B **19.** D

**5.** B **10.** C **15.** A **20.** A

## Section A - Multiple-choice solutions

### Question 1

For ****.

The graph of  has been dilated by a factor of ‘*a*’ from the *y*-axis. This does not affect the range.

The answer is C.

**Question 2**

****

The answer is E.

**Question 3**









The answer is B.

**Question 4**

Do a quick sketch.



The complex number *z* has been rotated 90° in a clockwise direction to become the complex number .

Hence the line segment joining *O* and  lies at an angle of 90° to the line segment joining *O* and .

The quadrilateral is therefore a rectangle.



The answer is D.

**Question 5**







Since  is a solution to the equation ,

then 

For option A, if  so option A can be true.

For option B, we have .



but  so option B cannot be true.

The answer is B.

**Question 6**

.



The closest answer is 402.7.

The answer is E.

**Question 7**





The answer is B.

**Question 8**



Using the formula on the formula sheet, and noting that ,



The closest answer is 9.5.

The answer is A.

**Question 9**

Given that  then the graph of *f* has a negative gradient and is concave down over its entire domain.

An example of such a graph is shown.



Both the function and the gradient of *f* will be strictly decreasing. There can be no non-stationary points of inflection because the graph does not change its concavity i.e.  is negative over the entire domain.

The answer is A.

**Question 10**

Along the *x*-axis, the gradient is zero.

For option A,  except at the origin so reject option A.

For option B, the same applies so reject option B.

For option C, , this satisfies the *x*-axis markings where  for all values of *x*. When , the gradient is approximately 1 as is the case when .

For other values of *y* the differential equation also holds.

The answer is C.

**Question 11**

For option A, a rhombus has all four sides of equal length so .

For option B, .

Since , which is the angle between , is greater than 90°,



For option C, .

For option E, the scalar product of the diagonal vectors  is zero because these two vectors are perpendicular in a rhombus.

For option D, .

The answer is D.

**Question 12**

vector resolute of  perpendicular to 



The answer is A.

**Question 13**



Solve  for *t*.



so period of time is closest to 0.25 hour.

The answer is A.

**Question 14**





Since the path is circular then .



The direction of motion at .

The answer is B.

**Question 15**



Initial momentum is 



The answer is A.

**Question 16**

Mark in the forces.



Resolving horizontally:



Resolving vertically:



The answer is E.

**Question 17**





The answer is C.

**Question 18**

Let *X* represent the weights of the population of the sea birds.



For the sample taken, .



The closest answer is 0.0929.

The answer is C.

**Question 19**





The answer is D.

**Question 20**

Let *A* represent the annual rainfall in Alban.

Let *B* represent the annual rainfall in Beachtown.





The closest answer is 0.7339.

The answer is A.

**SECTION B**

# Question 1 (10 marks)

1. 



**(1 mark)**

So .

**(1 mark)**



**(1 mark)** – correct shape

**(1 mark)** – correct stationary point

**(1 mark)** – correct endpoints

1. **i.**  using part **a.**

**(1 mark)**

1. 



**(1 mark)**

1.  **(1 mark)**



**(1 mark)**

1. 

 cubic units (correct to 2 decimal places)

**(1 mark)**

**Question 2** (9 marks)

1. 



**(1 mark)**

1. 





**(1 mark)**

Using CAS, solve  simultaneously.

The two points of intersection are .

**(1 mark)**



**(1 mark)** for *S*

**(1 mark)** for 

1. Using the graph from part **c.**, the minimum value of  for which the ray intersects **both** graphs is zero and the maximum value is .

So .

**(1 mark)** – left endpoint

**(1 mark)** – right endpoint

1. The area required is shaded in the diagram below.

Note that the centre of the circle *S* lies at .





**(1 mark)**

**(1 mark)**

**Question 3** (9 marks)

1. 



**(1 mark)**

1. 

 (separation of variables) **(1 mark)**



When 



**(1 mark)**

**(1 mark)**

1. The solution given is .

So 

The differential equation given is



So the solution given satisfies the d.e. given. **(1 mark)**

Check the initial conditions:



So the solution given satisfies the initial conditions given.

**(1 mark)**

1. Initially there is 400 litres of sugar solution in the tank.

Each minute, 4 litres is lost. So it takes  minutes.

**(1 mark)**

1. Concentration of sugar in the sugar solution is . **(1 mark)**



so the concentration becomes



which is constant.

**(1 mark)**

**Question 4** (11 marks)

1. **i.** Solve 

 seconds

**(1 mark)**

1. To hit the tiny object, the stone must be 6 m horizontally from the release point and 2.5 m vertically above the release point at the same time.

Solve  for *t.*



Neither of these values of *t* equal 0.8 hence the stone cannot be at the point (6, 2.5) and hence cannot hit the tiny object.

**(1 mark)**

1. From part **i.,** the stone is 6 metres from the release point at  seconds. At this time its height above the release point is given by



 **(1 mark)**



The stone passes 4.76 m (correct to two decimal places) vertically above the tiny object.

**(1 mark)**

1. 

 **(1 mark)**

The stone is vertically above the tiny object when  (from part **a.**).



**(1 mark)**

1. The position of the tiny object relative to the release point is (6, 2.5).

Solve  simultaneously for *t* and .**(1 mark)**

Some solutions must be rejected because  and .



So the two possible angles of projection are 30.7° and 81.9° (both correct to one decimal place).

**(1 mark)** for 30.7°

**(1 mark)** for 81.9°

1. Method 1 – using a diagram

From part **b.** .

Draw a diagram.



**(1 mark)**

If the distance of the stone on the ground from the base of the wall on the opposite side of the wall from where it was thrown is 16.4228 metres, then the angle of projection must have been 30.7°, (correct to 1 decimal place).

**(1 mark)**

Method 2 **–** using result to part **b.**

From part **b.** 

If  then solve



Horizontal distance from projection



**(1 mark)**

So distance from base of opposite side of wall is .

The angle of projection must have been 30.7° (correct to 1 decimal place). **(1 mark)**

If you opted to test  for *t*.



Horizontal distance from point of projection



So distance from base of opposite side of wall is  so angle of projection can’t have been 81.9° and must have been 30.7°.

**Question 5** (11 marks)



**(1 mark)** for forces around the *m* kg mass

**(1 mark)** for forces around the 4 kg mass

1. Resolving:

Around the 4 kg mass: Around the *m* kg mass

 



**(1 mark)**

1. The mass accelerates downwards so the equation of motion is





**(1 mark)**

1.  from part **c**.



**(1 mark)**

Method 1 – using CAS

Method 2 – by hand



When ,



 **(1 mark)**

Using the boundary conditions

,



**(1 mark)**

1.  (from part **d.**)

****

as **.**

The limiting (terminal velocity) is 2g ms-1.

**(1 mark)**

1.  (from part **e.**)



**(1 mark)**

The liquid is 28.3 metres deep (correct to 1 decimal place).

**(1 mark)**

1.  (from part **c.**)



**(1 mark)**

The 4 kg mass is 7.6 metres below the surface (correct to 1 decimal place) when .

**(1 mark)**

**Question 6** (10 marks)

1. Let *S* represent the distribution of weights of small muffins.

Let *L* represent the distribution of weights of large muffins.





**(1 mark)** – mean

**(1 mark)** - sd

1. Let *B* represent the distribution of weights of boxes of 25 small muffins.



Mean weight of a box is 2700 grams. **(1 mark)**

Standard deviation of the weight of a box is .

**(1 mark)**

1. 

 (correct to 3 decimal places)

**(1 mark)**

1. 



**(1 mark)**

1.  **(1 mark)**



**(1 mark)**

1. Since  from part **e.**) we should reject . **(1 mark)**



**g.** Find *c* such that 





**(1 mark)**

Note that the largest number (correct to 1 decimal place) that is less than 443.9938… is 443.9. Rounding 443.9938… to one decimal place, does not answer the question.

