

**SPECIALIST MATHS**

**TRIAL** **EXAMINATION 2**

**SOLUTIONS**

**2020**

**(Adjusted Study Design)**

## Section A – Multiple-choice answers

**1.** E **6.** A **11.** B **16.** A

**2.** C **7.** E **12.** E **17.** B

**3.** D **8.** B **13.** D **18.** D

**4.** E **9.** A **14.** C **19.** D

**5.** C **10.** E **15.** B **20.** E

## Section A - Multiple-choice solutions

### Question 1



, for *x*.

Method 1 – using CAS



or .

The answer is E.

Method 2 – by hand

We require 

i.e. 

If 





If ,



So 

So 

The answer is E.

**Question 2**



Do a quick sketch of .



The answer is C.

**Question 3**

The graph of  is created by translating the graph of  to the right.

The graph of  therefore has asymptotes located at .

The graph of  has asymptotes located at .

Reject option A.

The graphs of  have asymptotes located at .

Reject options B and C.

The graph of  has asymptotes at  so option D is correct.

Note that for , the asymptotes occur at .

The answer is D.

**Question 4**



The answer is E.

**Question 5**

Using CAS to solve, we see that there are 5 solutions to this equation.

They are .

The answer is C.

**Question 6**

The solutions to  lie on a circle with its centre at the origin and with a radius of



Only Option A gives a circle with centre at the origin and with a radius of 5.

The answer is A.

**Question 7**

For option A, since  (from the diagram)

then , so option A is true.

For option B, since a straight line that passes through the origin can be drawn from , then option B is true.

For option C, since the complex number  has been rotated anticlockwise by an angle greater than  to produce , so option C is true.

For option D, since  has been rotated by  to produce , then .

Since  so option D is true.

Option E cannot be true since .

The answer is E.

**Question 8**





 will be real when 



The answer is B.

**Question 9**





The answer is A.

**Question 10**

Since 





The answer is E.

**Question 11**

The differential equation  would only have a direction field for . Reject option A.

Similarly for , the direction field would only exist for . Reject option E.

For , the direction field would only exist for  and  and and . Reject option D.

The gradient .





So option B is correct.

To double check, for option C, at say (1, 2),



The answer is B.

**Question 12**





The closest answer is 5.02.

The answer is E.

**Question 13**

****

**** (separation of variables)

The answer is D.

**Question 14**



scalar resolute of  in the direction of 



Solve  for *m*

, but *m* is a positive constant so .

The answer is C.

**Question 15**

The particles will collide when



The particles will collide if .

The answer is B.

**Question 16**

If  are linearly dependent then .



Equating the  components:



Equating the  components:



Equating the  components:



(2) gives 



In (4) 



The answer is A.

**Question 17**





Method 1 Method 2



The answer is B.



When ,



The answer is B.

**Question 18**

Because the particle is in equilibrium we can use Lami’s theorem.



For option A, involving *P* and *R*, we have



Option A is true.

For option B, involving *Q* and *R*, we have



Option B is true.

For option C, involving *P* and *Q*, we have



Option C is true.

For option D, involving *Q* and *P*, we have



Option D is **not** true.

If you have time, check option E just to confirm that D is the correct answer.



Option E is true.

The answer is D.

**Question 19**

Resolving forces horizontally:





Using 





The closest answer is 192.

The answer is D.

**Question 20**

Draw in the forces.



Around the  particle:



 \_\_\_\_\_\_\_\_\_\_\_(\*)

Around the  particle:



Using (\*) we have,





The answer is E.

**SECTION B**

# Question 1 (12 marks)

1. 

The centre is located at . The radius is 1.

**(1 mark)**



**(1 mark)** – graph of *L*

1. **i.** 

**** **(1 mark)**

1. *a* is the radius of circle *L.* From the diagram, the distance from the point  (Pythagoras).

So . **(1 mark)**

**d.** **i.**  



**(1 mark)**

**ii.** Equation of *L* is 



Points of intersection are  **(1 mark)** & **(1 mark)**

**e.**



**(1 mark)**

The required area is shaded. 

 **(1 mark)**



**f.**

We see from the diagram that the radius of circle *L* that has it’s endpoint at the origin (shown as a dotted line), is perpendicular to the ray with equation .

Since a tangent to a circle is perpendicular to the radius drawn from the point of contact (circle properties), then this ray must be tangential to circle *L* at the origin.

Note that the origin is not included in the ray so  does not intersect with *L.* Similarly,  does not intersect with *L*.

Note also that  (formula sheet).

So required values of *n* are . **(1 mark)** correct brackets

**(1 mark)** endpoints in left interval **(1 mark)** endpoints in right interval

**Question 2** (12 marks)

1. 



Vertical asymptote at . **(1 mark)**

Curved asymptote at . **(1 mark)**

1. **i.** Solve , for *x*



Stationary point occurs at , correct to 2 decimal places.

**(1 mark)**

1. solve  for *x*



When , so the point of inflection is at .

**(1 mark)**



**(1 mark)** – correct asymptotes

**(1 mark)** – correct shapes of branches

**(1 mark)** *–* correct labelling of coordinates

**d.** 

Using working from part **a.**, asymptotes are located at .

The point of intersection of these two asymptotes occurs at . **(1 mark)**

**e. i.** The equation  will have two solutions when the graph of *f* has its stationary point located on the *x*-axis. From part **d.** we see that the location of the curved asymptote is dependent on the value of *k.*

The stationary point of the the graph of *f* is located at the point where   

Note that the *x*-coordinate of the stationary point is not affected by the value of *k.*

The *y*-coordinate **is** however affected by the value of *k.*

 **(1 mark)**

Note that in part **b.**, we saw that the stationary point of the graph of *f* when  was located at .For the stationary point to be located on the *x*-axis, we require that





**(1 mark)**

**ii.** Using part **i.** and the graph above, the values of *k* for which  has three solutions are  where the right endpoint of the interval is expressed correct to 2 decimal places. **(1 mark)**

**f.** The point of inflection occurs when .

We require the *y* - coordinate of the point of inflection to be zero.



**(1 mark)**

**Question 3** (11 marks)

1. **i.** 



**(1 mark)**

* 1. ****

**(1 mark)**

1. 



**(1 mark)**

1. **i.**  (chain rule) \_\_\_\_\_\_\_\_\_\_(\*) **(1 mark)** for chain rule

Start by finding .

Using working from part **a.i**., and recognizing that the variable *y* is representing the height of the water for , we have



When .

Alternatively, use  from part **a. ii.**



**(1 mark)**

**(1 mark)**

**ii.** 



The rate at which the height of the water is decreasing is 

**(1 mark)**

1.  (from part **c.**)



**(1 mark)**

**(1 mark)**

1. 



**(1 mark)**

**(1 mark)**

**Question 4** (13 marks)

1. 

 **(1 mark)**



**(1 mark)**





**(1 mark)** – endpoints in correct positions

**(1 mark)** – correct shape

**(1 mark)** – correct arrow indicating direction of motion

1. 



**(1 mark)**

**(1 mark)**

1. distance travelled



**(1 mark)**

**(1 mark)**

1. The direction of motion of the boat at time *t* is given by the velocity vector.



The direction of motion of the woman at time *t* is constant.



Direction will be the same if  **(1 mark)**



and the motion of the boat is defined only for  so the woman and the boat are never travelling in the same direction during this period.

**(1 mark)**

**f.** Method 1

From part **e.**, we saw that, relative to the Cartesian axes, the gradient of the path of the boat was .

When . **(1mark)**

Let  be the angle that the path of the boat makes with the positive branch of the *x*-axis when.

Let  be the angle that the path of the woman makes with the positive branch of the *x*-axis. Note that this is constant for .



The acute angle between the path of the woman

and that of the boat at  is equal to .



**(1 mark)**

Method 2

 (from part **c**.)

 and  **(1 mark)**

Let  be the angle between the path of the woman and that of the boat at .



**(1 mark)**

**Question 5** (12 marks)



**(1 mark)**

1. Around the container Around the block





**(1 mark)**



Around the container Around the block

(motion down the plane) (motion upwards)



**(1 mark)**



**(1 mark)**

Acceleration is constant.





**(1 mark)**

1. 

**(1 mark)**

1. Around the container Around the block

(motion down the plane) (motion upwards)

****

**(1 mark)**

****

**(1 mark)**

1.  from part **e.**, and maximum velocity occurs when .

So ,. **(1 mark)**

Method 1



**(1 mark)**

Method 2



When 



**(1 mark)**

**g.**  from part **f.**

 **(1 mark)**



The time between the first and second attempts being made is therefore . **(1 mark)**