3.7 Fibonacci number

3.7.1 Fibonacci number using iteration

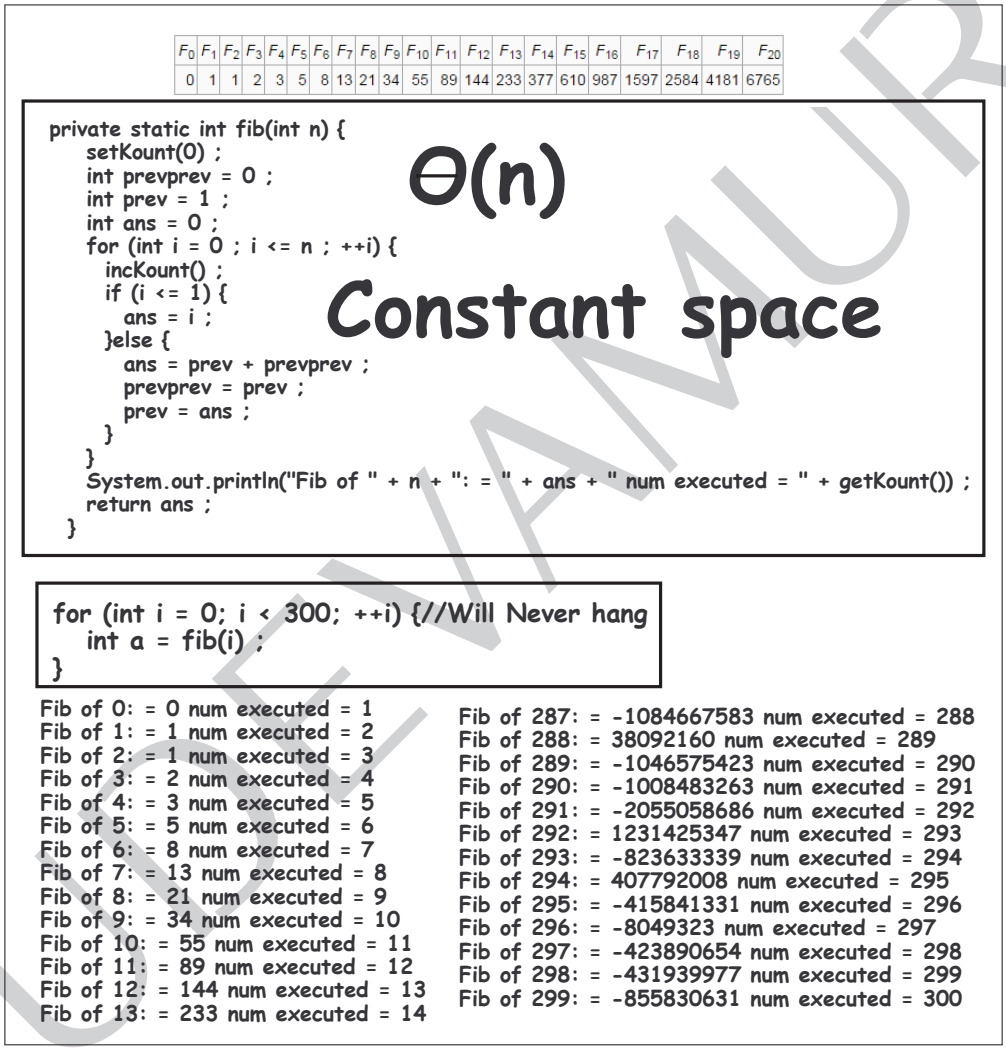

Figure 3.11: Fibonacci number using iteration

3.7.2 Fibonacci number using recursion

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}	F_{17}	F_{18}	F_{19}	F_{20}
0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	987	1597	2584	4181	6765

```
private static int fib_r1(int n) {
    incKount() ;
    if (n <= 1) {
      return n ;
    }
    return (fib_r1(n-2) + fib_r1(n-1)) ;
 }
```

T(n) = 1 for n = 0/1
= T(n-2)+T(n-2) +C

$\Theta(2^n)$

```
 private static int fib_r(int n) {
   setKount(0) ;
   int a = fib_r1(n) ;
   System.out.println("Fib of " + n + ": = " + a + " num executed = " + getKount()) ;
   return a ;
 }
```

```
for (int i = 0; i < 40; ++i) { //Run with 300 to see hangs at 50
    int a = fib_r(i) ;
 }
```

```
Fib of 0: = 0 num executed = 1
Fib of 1: = 1 num executed = 1
Fib of 2: = 1 num executed = 3
Fib of 3: = 2 num executed = 5
Fib of 4: = 3 num executed = 9
Fib of 5: = 5 num executed = 15
Fib of 6: = 8 num executed = 25
Fib of 7: = 13 num executed = 41
Fib of 8: = 21 num executed = 67
Fib of 9: = 34 num executed = 109
Fib of 10: = 55 num executed = 177
Fib of 11: = 89 num executed = 287
Fib of 12: = 144 num executed = 465
Fib of 13: = 233 num executed = 753
Fib of 14: = 377 num executed = 1219
Fib of 15: = 610 num executed = 1973
Fib of 16: = 987 num executed = 3193
Fib of 17: = 1597 num executed = 5167
Fib of 18: = 2584 num executed = 8361
Fib of 19: = 4181 num executed = 13529
Fib of 20: = 6765 num executed = 21891
Fib of 21: = 10946 num executed = 35421
Fib of 22: = 17711 num executed = 57313
Fib of 23: = 28657 num executed = 92735
Fib of 24: = 46368 num executed = 150049
Fib of 25: = 75025 num executed = 242785
Fib of 26: = 121393 num executed = 392835
Fib of 27: = 196418 num executed = 635621
Fib of 28: = 317811 num executed = 1028457
Fib of 29: = 514229 num executed = 1664079
Fib of 30: = 832040 num executed = 2692537
Fib of 31: = 1346269 num executed = 4356617
Fib of 32: = 2178309 num executed = 7049155
Fib of 33: = 3524578 num executed = 11405773
Fib of 34: = 5702887 num executed = 18454929
Fib of 35: = 9227465 num executed = 29860703
Fib of 36: = 14930352 num executed = 48315633
Fib of 37: = 24157817 num executed = 78176337
Fib of 38: = 39088169 num executed = 126491971
Fib of 39: = 63245986 num executed = 204668309
```

Figure 3.12: Fibonacci number using recursion

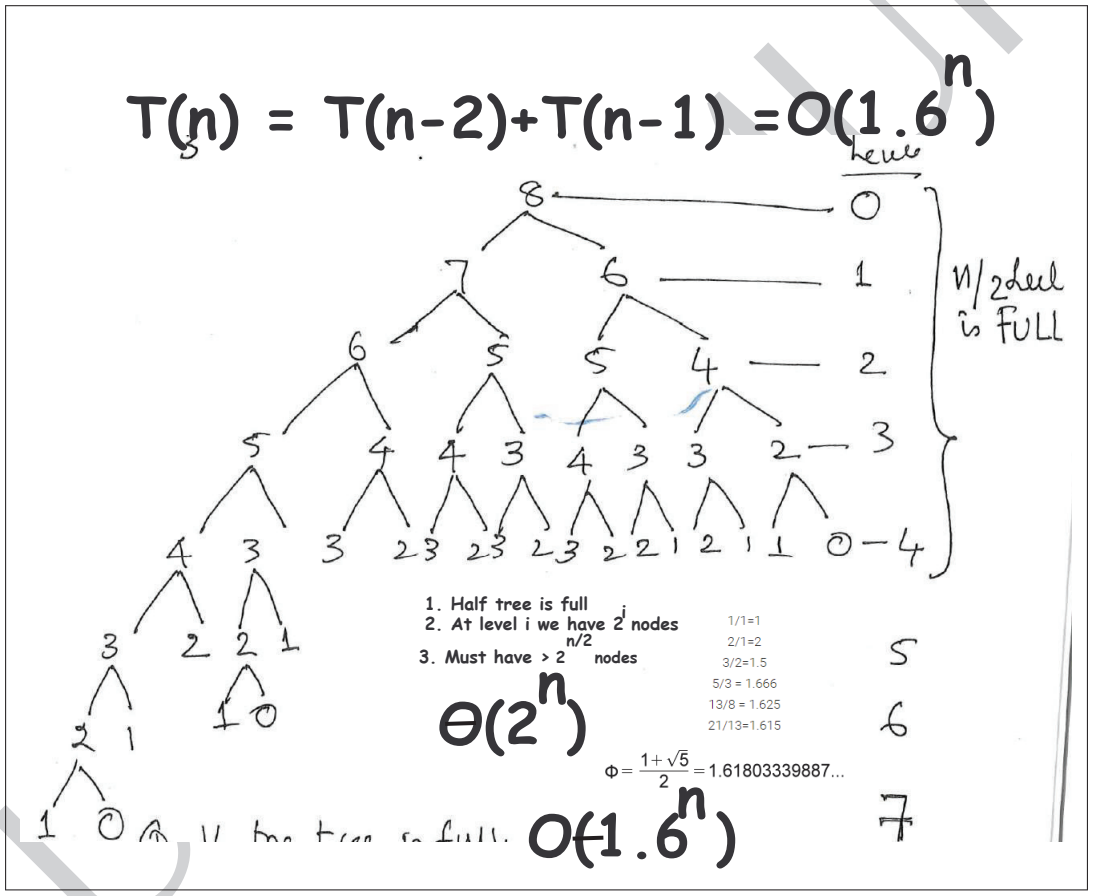

Figure 3.13: Complexity of computing Fibonacci number using recursion

3.7.3 Fibonacci number using tail recursion

```
  private static int fibTR(int n, int prevprev, int prev) {
      incKount() ;
      if (n == 0) {
        return prevprev ;
      }
      if (n == 1) {
        return prev ;
      }
      return fibTR(n-1,prev,prev+prevprev);
    }
```

```
private static int fibWithTailR(int n) {
    setKount() ;
    int a = fibTR(n,0,1);
    System.out.println("fibWithTailR of " + n + ": = " + a + " num executed = " + getKount()) ;
    return a ;
  }
```

```
for (int i = 0; i < 300; ++i) { //Will Never hang
   int a = fibWithTailR(i) ;
 }
```

```
fibWithTailR of 0: = 0 num executed = 1
fibWithTailR of 1: = 1 num executed = 1
fibWithTailR of 2: = 1 num executed = 2
fibWithTailR of 3: = 2 num executed = 3
fibWithTailR of 4: = 3 num executed = 4
fibWithTailR of 5: = 5 num executed = 5
fibWithTailR of 6: = 8 num executed = 6
fibWithTailR of 7: = 13 num executed = 7
fibWithTailR of 8: = 21 num executed = 8
fibWithTailR of 9: = 34 num executed = 9
fibWithTailR of 10: = 55 num executed = 10
fibWithTailR of 11: = 89 num executed = 11
fibWithTailR of 12: = 144 num executed = 12
fibWithTailR of 13: = 233 num executed = 13
fibWithTailR of 14: = 377 num executed = 14
```

```
fibWithTailR of 290: = -1008483263 num executed = 290
fibWithTailR of 291: = -2055058686 num executed = 291
fibWithTailR of 292: = 1231425347 num executed = 292
fibWithTailR of 293: = -823633339 num executed = 293
fibWithTailR of 294: = 407792008 num executed = 294
fibWithTailR of 295: = -415841331 num executed = 295
fibWithTailR of 296: = -8049323 num executed = 296
fibWithTailR of 297: = -423890654 num executed = 297
fibWithTailR of 298: = -431939977 num executed = 298
fibWithTailR of 299: = -855830631 num executed = 299
```

Figure 3.14: Fibonacci number using tail recursion

3.7.4 Fibonacci number using stack

3.8 Tower of Hanoi

3.8.1 Tower of Hanoi using recursion

