

Chapter 2

Analysis of Algorithms


2.1 Introduction

2.1.1 Algorithm

Algorithm

Algorithm is an effective method for solving a problem expressed as a finite sequence of instructions

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A stamp issued September 6, 1963 in the Soviet Union, commemorating al-Khwarizmi's (approximate) 1200th birthday.

Born	c. 780
Died	c. 850
Ethnicity	Persian
Known for	Contributions to mathematics

Algorithm LargestNumber

Input: A List L that has N numbers and $N \geq 1$
Output: The largest number in the list L.

```
largest = L[0]
for (i = 1 to N-1) {
    if (L[i] > largest)
        largest = item L[i]
}
return largest
```

1. Precise
2. Unambiguous
3. Mechanical
4. Efficient
5. Correct

1. Is it correct?
2. How much time does it take, in terms of n
3. And can we do better?

Figure 2.1: Algorithm

2.1.2 Finding a number in an unsorted array



Figure 2.2: Finding a number in an unsorted array

2.1.3 Finding a fake coin

2.1. INTRODUCTION



You are given 70 gold coins, one of which is a fake coin (the fake coin has lesser weight). You have a scale, shown in the picture. What is the smallest number of weighing you can do in order to find the fake coin?

Figure 2.3: Finding fake coin

2.2 Mathematics

2.2.1 Logarithm

Logarithm

How many time
I should multiply
1 by 2 to get 8?

How many time
I should divide
8 by 2 to get 1?

$1 * 2 * 2 * 2$
 $2^? = 8$
 exponent $((8/2)/2)/2$
 $\log_2(8) = ?$
 base

$5^? = 625$
 $\log_5(625) = ?$
 $5 * 5 * 5 * 5 = 625$
 $\log_5(625) = 4$

$2^{10} = 1024$
 $\log_2(1024) = 10$
 $2^0 = 1$
 $\log_2 1 = 0$
 $2^1 = 2$
 $\log_2 2 = 1$
 what is $\log_2 64$
 $2^8 = 64$
 $\log_2 64 = 8$

$\log_{10} 100 == \log_{10} 100$ **Engineers**
 $\log_e 7.389 == \ln(7.389) \approx 2$ **Math**
 $e = \text{Euler Number} = 2.71828$
 $\log_2 16 == \lg(16) = 4$ **CS**

The logarithm $\log_b(x)$ can be written

$$\log_b(x) = \frac{\log_k(x)}{\log_k(b)}$$

Java: $(\text{Math.log}(n) = \log_e n)$ **TO base e**
 $\log_2 n = \frac{\log_e n}{\log_e 2}$
 $\log_2 n = (\text{Math.log}(n) / \text{Math.log}(2))$

$\log_b(xy) = \log_b(x) + \log_b(y)$
 $\log_b(b^x) = x \log_b(b) = x$
 $b^{\log_b(y)} = y$

Figure 2.4: Logarithm

Logarithm Table

log 0 is NOT defined

Table of base 10, base 2 and base e (ln) logarithms:

x	$\log_{10} x$	$\log_2 x$	$\log_e x$
1	0.000000	0.000000	0.000000
2	0.301030	1.000000	0.693147
3	0.477121	1.584963	1.098612
4	0.602060	2.000000	1.386294
5	0.698970	2.321928	1.609438
6	0.778151	2.584963	1.791759
7	0.845098	2.807355	1.945910
8	0.903090	3.000000	2.079442
9	0.954243	3.169925	2.197225
10	1.000000	3.321928	2.302585
20	1.301030	4.321928	2.995732
30	1.477121	4.906891	3.401197
40	1.602060	5.321928	3.688879
50	1.698970	5.643856	3.912023
60	1.778151	5.906991	4.094345
70	1.845098	6.129283	4.248495
80	1.903090	6.321928	4.382027
90	1.954243	6.491853	4.499810
100	2.000000	6.643856	4.605170
200	2.301030	7.643856	5.298317
300	2.477121	8.228819	5.703782
400	2.602060	8.643856	5.991465
500	2.698970	8.965784	6.214608
600	2.778151	9.228819	6.396930
700	2.845098	9.451211	6.551080
800	2.903090	9.643856	6.684612
900	2.954243	9.813781	6.802395
1000	3.000000	9.965784	6.907755
10000	4.000000	13.287712	9.210340

Figure 2.5: Logarithm table

2.2. MATHEMATICS

2.2.2 Arithmetic series

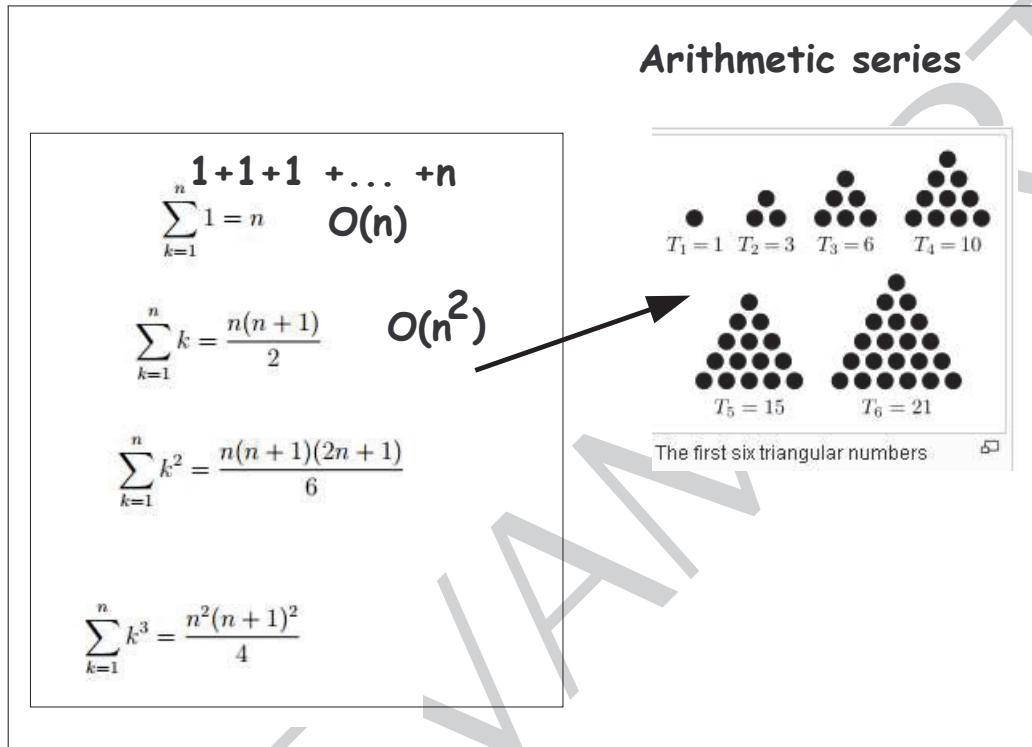


Figure 2.6: Arithmetic series

2.2.3 Harmonic series

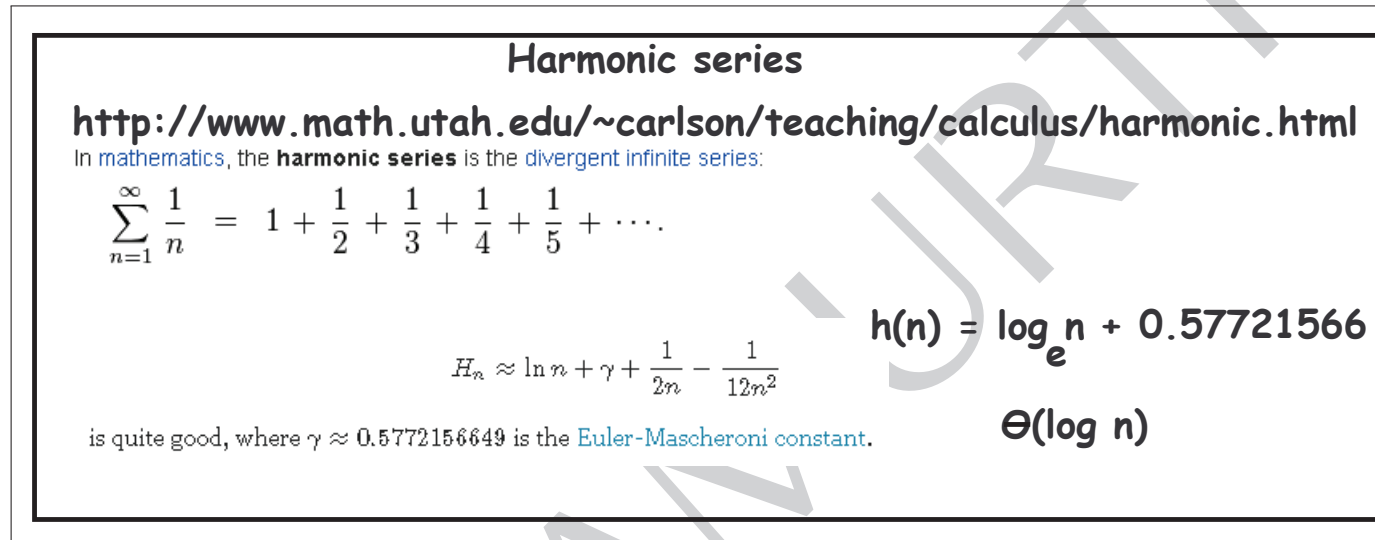


Figure 2.7: Harmonic series

2.2. MATHEMATICS

2.2.4 Geometric series

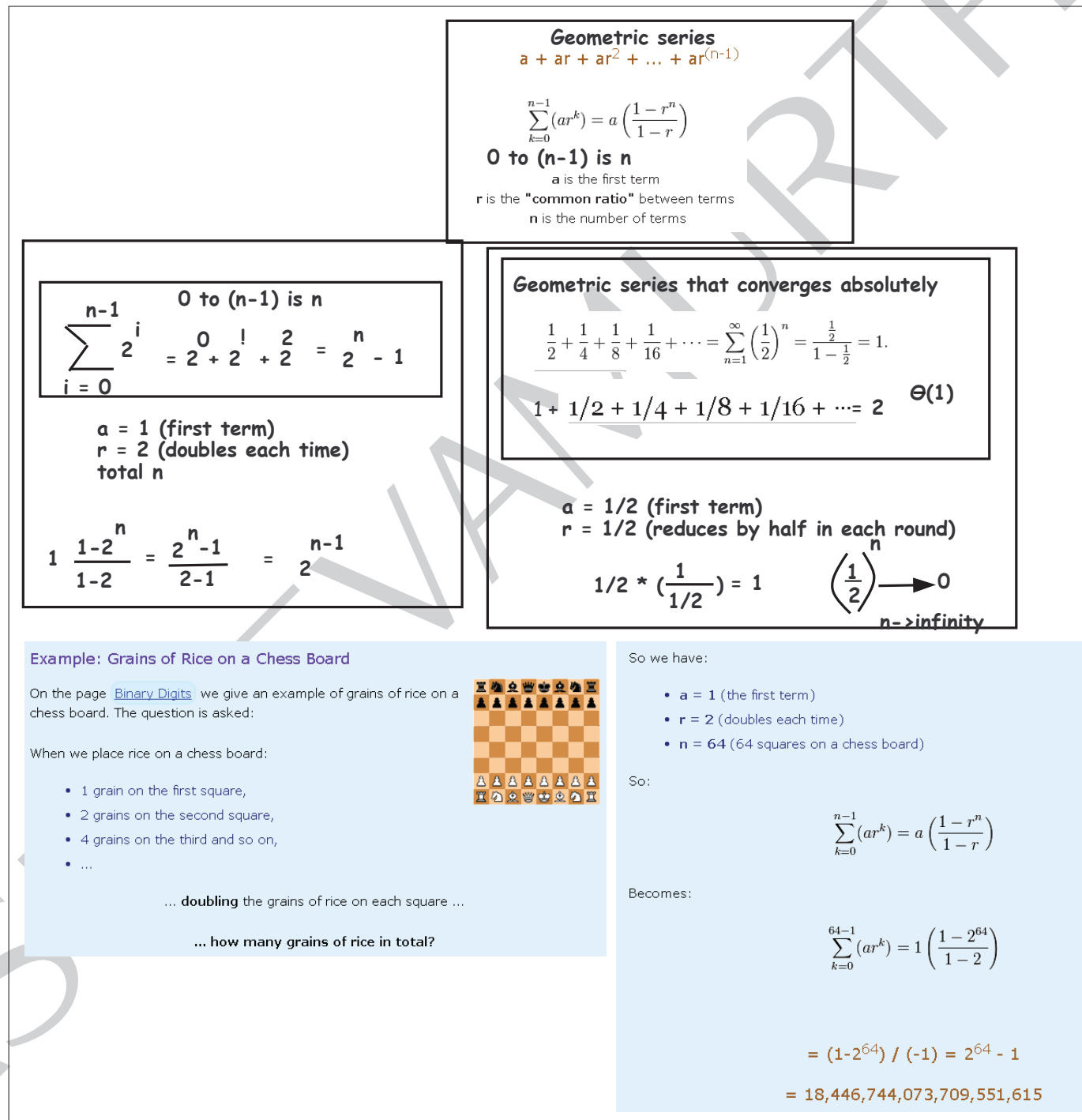


Figure 2.8: Geometric series