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MTF072 Computational Fluid Dynamics of Turbulent Flow

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Task K1

The task should be carried out (using MATLAB) in groups of one or two students. Below the computational domain, boundary conditions, etc. are given for 40 cases. Your group number defines the case you should pick. For example, group 20 should resolve case 20, group 45 should pick case 5, and group 90 should resolve case number 10.

We are going to study the diffusion equation for temperature T (i.e. heat conduction equation), which can be written

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+b=0 \tag{1}$$

Discretise this equation according to the textbook (Eq. 4.57)

$$a_P T_P = a_E T_E + a_W T_W + a_N T_N + a_S T_S + b\Delta x\Delta y. \tag{2}$$

The algebraic equation system should be solved using:

- Gauss-Seidel

How to proceed

- Read carefully the coding standard `CodingStandard.pdf`. It will be beneficial for your code and for you.
- Open and read the matlab script `GuideTaskK1.m`. You can find hints to structure your code.
- Create your grid (you can use functions as `linspace` and `meshgrid`). Start with a 10x10 equidispaced grid. Once everything is working, you can refine your mesh adding more cells.
- Start to write your program to solve Eq. 1. Resolve your temperature field using Gauss-Seidel method. You only need `for` and `while` loops. Do not use command like *"backslash"* but resolve your equation cell by cell.

- Implement the residual criteria as in Eqs. 3 and 4.
- Use different meshes to solve the problem (i.e. 10×10, 20×20 and 40×40). After, refine your largest grid in regions where you expect large gradients using non equidistant spacing. The domain can be rectangular, thus you can have different spacing and different number of cells in x and y direction.
- You can use a log law or a geometrical law of your choice to make the non equidistant refinement. When you refine keep in mind that the decrease or increase of the cell dimension (from one cell to the neighbouring one) should not be larger than 15% to avoid numerical errors.

Convergence

It is very important to verify that a converged solution has been obtained. At each iteration compute the residual as

$$\varepsilon = \frac{1}{F}\left(\sum_{\text{allcells}} |a_E T_{i+1,j} + a_W T_{i-1,j} + a_N T_{i,j+1} + a_S T_{i,j-1} + S_u - a_P T_{i,j}|\right) \tag{3}$$

where F is a temperature flux used to normalize the residual. The temperature flux F should be representative of the total flux in the domain. In the present work it is suitable to take F as the boundaries temperature flux.

$$F = \sum_{boundaryCells} \left| k_i A \left(\frac{\partial T}{\partial x_i}\right)\right| \tag{4}$$

where T is the temperature of each cell. A is the exchange area between two neighbouring cells (in this case is the face length since the problem is 2D). The solution is considered as converged when $\varepsilon < 0.001$.

Presentation of the work

The work should be presented both orally and in a form of a short report (max 10 pages. **This is a strict limit**). The code **must** be included as appendix at the end of the report. The oral presentation should be approximately **5 minutes** (use some slides and aim for 4 minutes). Try to discuss the results from a physical and numerical point of view. Present the results for example as contour plots of the temperature. The presentation and the report must include the following parts:

1. Show and compare the results on the meshes you have build. Both equidispaced and non equispaced.
2. How sensitive is the solution to the coefficient of conductivity k? Increase and reduce k by a factor 100. Explain why the solution is changed or not!

3. What happens if you change the boundary conditions? If you have Neumann b.c. somewhere: change Neumann to Dirichlet (given T) along one side. If you don't have Neumann b.c. anywhere: change Dirichlet to Neumann along one side. Discuss how/why the temperature field is changed.

4. In order to illustrate the heat flow, plot the heat flux vector $\dot{q}_x, \dot{q}_y$ (you can use the `quiver` function in MATLAB)

$$\dot{q}_x = -k\frac{\partial T}{\partial x},\ \dot{q}_y = -k\frac{\partial T}{\partial y} \tag{5}$$

as a vector plot. Discuss and investigate the relation between the heat flow and the temperature contours.

The general geometry

The geometry and boundaries should be presented as below. L, H and boundary conditions vary according to your specific case.

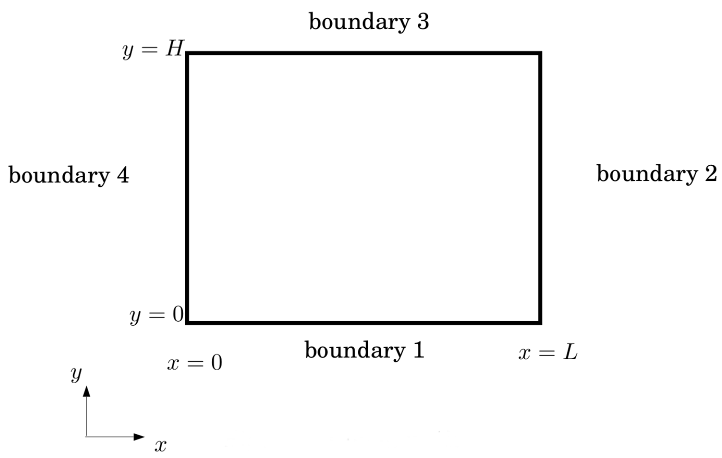

Figure 1: *Configuration.*

Cases from 1 to 40

Case	T_1	T_2	T_3
1	10	$10 + 20\sin(\pi y/H)$	10
2	15	$10 + 5(1 - y/H) + 15\sin(\pi y/H)$	10
3	15	$15\cos(2\pi y/H)$	15
4	10	$10 + 5y/H + 10\sin(\pi y/H)$	15
5	15	$-5y/H + 15\cos(2\pi y/H)$	10

Table 1: Definition of case 1 - 5. $L = 1$, $H = 0.5$, $dT/dx = 0$ *at boundary 4. Constant source term (per area) b=-1.5. Coefficient of conductivity* $k = 5(1 + 100x/L)$.

Case	T_2	T_4
6	$10 + 20\sin(\pi y/H)$	10
7	$10 + 20\sin(\pi y/H)$	30
8	$5(y/H - 1) + 15\cos(\pi y/H)$	15
9	$5(y/H - 1) + 15\cos(\pi y/H)$	30
10	$10 + 5\sin(\pi y/H)$	10

Table 2: Definition of case 6 - 10. $L = 1.5$, $H = 0.5$, $T_1 = 10$, $dT/dy = 0$ *at boundary 3. Coefficient of conductivity* $k = 0.01$ *in the region* $0.7 < x < 1.1$, $0.3 < y < 0.4$, *and in the remaining of the computational domain* $k = 20$. *The source* $b = 0$.

Case	c_1	c_2	T_3
11	20	0.2	$20x/L$
12	25	0.1	$10(1 + 2x/L)$
13	25	0.3	$15 + 5x/L$
14	20	0.4	$5 + 15x/L$
15	25	0.25	$5 + 3(1 + 5x/L)$

Table 3: Definition of case 11 - 15. $L = 1$, $H = 1$, $T_1 = 10$, $T_2 = 20$, $dT/dx = 0$ *at boundary 4. Coefficient of conductivity* $k = 2(1 + 20T/T_1)$. *Heat source over the whole computational domain* $b = 15(c_1 - c_2T^2)$.

Case	c_1	c_2	T_3
16	20	0.2	$20x/L$
17	25	0.1	$10(1 + 2x/L)$
18	25	0.3	$15 + 5x/L$
19	20	0.4	$5 + 15x/L$
20	25	0.25	$5 + 3(1 + 5x/L)$

Table 4: Definition of case 16 - 20. $L = 1$, $H = 7$, $T_1 = 10$, $T_2 = 20$, $dT/dx = 0$ *at boundary 4. Coefficient of conductivity* $k = 16(y/H + 30T/T_1)$. *Heat source over the whole computational domain* $b = 15(c_1 - c_2T^2)$.

Case	$\mathbf{T_1}$	$\mathbf{T_4}$	$\mathbf{T_3}$
21	10	$10 + 20\sin(\pi y/H)$	10
22	15	$10 + 5(1 - y/H) + 15\sin(\pi y/H)$	10
23	15	$15\cos(2\pi y/H)$	15
24	10	$10 + 5y/H + 10\sin(\pi y/H)$	15
25	15	$-5y/H + 15\cos(2\pi y/H)$	10

Table 5: Definition of case 21 - 25. $L = 1$, $H = 0.5$, $dT/dx = 0$ *at boundary 2. Constant source term (per area) b=-1.5. Coefficient of conductivity* $k = 5(1 + 100x/L)$.

Case	$\mathbf{T_1}$	$\mathbf{T_3}$
26	$10 + 20\sin(\pi x/L)$	10
27	$10 + 20\sin(\pi x/L)$	30
28	$5(y/H - 1) + 15\cos(\pi x/L)$	15
29	$5(y/H - 1) + 15\cos(\pi x/L)$	30
30	$10 + 5\sin(\pi x/L)$	10

Table 6: Definition of case 26 - 30. $L = 1.5$, $H = 0.5$, $T_1 = 10$, $dT/dx = 0$ *at boundary 4. Coefficient of conductivity* $k = 0.01$ *in the region* $0.7 < x < 1.1$, $0.3 < y < 0.4$, *and in the remaining of the computational domain* $k = 20$. *The source* $b = 0$.

Case	$\mathbf{c_1}$	$\mathbf{c_2}$	$\mathbf{T_4}$
31	20	0.2	$20y/H$
32	25	0.1	$10(1 + 2y/H)$
33	25	0.3	$15 + 5y/H$
34	20	0.4	$5 + 15y/H$
35	25	0.25	$5 + 3(1 + 5y/H)$

Table 7: Definition of case 31 - 35. $L = 1$, $H = 1$, $T_1 = 10$, $T_2 = 20$, $dT/dy = 0$ *at boundary 3. Coefficient of conductivity* $k = 2(1 + 20T/T_1)$. *Heat source over the whole computational domain* $b = 15(c_1 - c_2T^2)$.

Case	$\mathbf{c_1}$	$\mathbf{c_2}$	$\mathbf{T_4}$
36	20	0.2	$20y/H$
37	25	0.1	$10(1 + 2y/H)$
38	25	0.3	$15 + 5y/H$
39	20	0.4	$5 + 15y/H$
40	25	0.25	$5 + 3(1 + 5y/H)$

Table 8: Definition of case 36 - 40. $L = 1$, $H = 7$, $T_1 = 10$, $T_2 = 20$, $dT/dy = 0$ *at boundary 3. Coefficient of conductivity* $k = 16(y/H + 30T/T_1)$. *Heat source over the whole computational domain* $b = 15(c_1 - c_2T^2)$.