

Team Reference Document

Heltion

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1 Contest

1.1 Makefile

```
1 %: %.cpp
2 g++ $< -o $$@ -std=gnu++20 -O2 \
3 -Wall -Wextra -Wconversion \
4 -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC
```

1.2 debug.h

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 template <class T, size_t size = tuple_size<T>::value>
4 string to_debug(T, string s = "")
5     requires(not ranges::range<T>);
6 string to_debug(auto x)
7     requires requires(ostream& os) { os << x; }
8 {
9     return static_cast<ostringstream>(ostringstream() << x).str();
10 }
11 string to_debug(ranges::range auto x, string s = "")
12     requires(not is_same_v<decltype(x), string>)
13 {
14     for (auto xi : x) { s += ", " + to_debug(xi); }
15     return "[" + s.substr(s.empty() ? 0 : 2) + "]";
16 }
17 template <class T, size_t size>
18 string to_debug(T x, string s)
19     requires(not ranges::range<T>)
20 {
21     [&<size_t... I>(index_sequence<I...>) {
22         ((s += ", " + to_debug(get<I>(x))), ...);
23     }(make_index_sequence<size>());
24     return "(" + s.substr(s.empty() ? 0 : 2) + ")";
25 }
26 #define debug(...) \
27     cerr << __FILE__ ":" << __LINE__ \
28     << " : " #__VA_ARGS__ " " << to_debug(tuple(__VA_ARGS__)) << "\n"
```

1.3 template.cpp

```
1 #include <bits/extc++.h>
2 using namespace std;
3 using namespace __gnu_pbds;
4 #ifndef ONLINE_JUDGE
5 #include "debug.h"
6 #else
7 #define debug(...) void(0)
```

```
8 #endif
9 template <class T>
10 using RBTtree = tree<T, null_type, less<T>, rb_tree_tag,
11     tree_order_statistics_node_update>;
12 using i64 = int64_t;
13 int main() {
14     cin.tie(nullptr)->sync_with_stdio(false);
15     cout << fixed << setprecision(20);
16 }
```

1.4 .clang-format

```
1 BasedOnStyle: Google
2 IndentWidth: 2
3 ColumnLimit: 200
```

2 Data Structure

2.1 Segment Tree

```
1 template <class T, auto f, auto e>
2     requires is_constructible_v<function<T(T, T)>, decltype(f)> and
3         is_constructible_v<function<T()>, decltype(e)>
4 struct SegmentTree {
5     int n;
6     vector<T> s;
7     SegmentTree(int n) : n(n), s(n * 2, e()) {}
8     void set(int i, T v) {
9         for (s[i += n] = v; i /= 2;) s[i] = f(s[i * 2], s[i * 2 + 1]);
10 }
11 // Returns the product of elements in [l, r).
12 T product(int l, int r) {
13     T rl = e(), rr = e();
14     for (l += n, r += n; l != r; l /= 2, r /= 2) {
15         if (l % 2) rl = f(rl, s[l++]);
16         if (r % 2) rr = f(s[r - 1], rr);
17     }
18     return f(rl, rr);
19 };
```

2.2 Disjoint Set Union

```
1 struct DisjointSetUnion {
2     vector<int> dsu;
3     DisjointSetUnion(int n) : dsu(n, -1) {}
4     int find(int u) { return dsu[u] < 0 ? u : dsu[u] = find(dsu[u]); }
```

```

5 void merge(int u, int v) {
6     u = find(u);
7     v = find(v);
8     if (u != v) {
9         if (dsu[u] > dsu[v]) {
10             swap(u, v);
11         }
12         dsu[u] += dsu[v];
13         dsu[v] = u;
14     }
15 }
16 };
17 struct RollbackDisjointSetUnion {
18     vector<pair<int, int>> stack;
19     vector<int> dsu;
20     RollbackDisjointSetUnion(int n) : dsu(n, -1) {}
21     int find(int u) { return dsu[u] < 0 ? u : find(dsu[u]); }
22     int time() { return ssize(stack); }
23     bool merge(int u, int v) {
24         if ((u = find(u)) == (v = find(v))) {
25             return false;
26         }
27         if (dsu[u] < dsu[v]) {
28             swap(u, v);
29         }
30         stack.emplace_back(u, dsu[u]);
31         dsu[v] += dsu[u];
32         dsu[u] = v;
33         return true;
34     }
35     void rollback(int t) {
36         while (ssize(stack) > t) {
37             auto [u, dsu_u] = stack.back();
38             stack.pop_back();
39             dsu[dsu[u]] -= dsu_u;
40             dsu[u] = dsu_u;
41         }
42     }
43 };

```

2.3 Sparse Table

```

1 struct SparseTable {
2     vector<vector<int>> table;
3     SparseTable() {}
4     SparseTable(const vector<int>& a) {
5         int n = a.size(), h = bit_width(a.size());
6         table.resize(h);
7         table[0] = a;
8         for (int i = 1; i < h; i += 1) {
9             table[i].resize(n - (1 << i) + 1);
10             for (int j = 0; j + (1 << i) <= n; j += 1) {

```

```

11         table[i][j] = min(table[i - 1][j], table[i - 1][j + (1 << (i - 1))]);
12     }
13 }
14 }
15 int query(int l, int r) {
16     int h = bit_width(unsigned(r - l)) - 1;
17     return min(table[h][l], table[h][r - (1 << h)]);
18 }
19 };
20 struct DisjointSparseTable {
21     vector<vector<int>> table;
22     DisjointSparseTable(const vector<int>& a) {
23         int h = bit_width(a.size() - 1), n = a.size();
24         table.resize(h, a);
25         for (int i = 0; i < h; i += 1) {
26             for (int j = 0; j + (1 << i) < n; j += (2 << i)) {
27                 for (int k = j + (1 << i) - 2; k >= j; k -= 1) {
28                     table[i][k] = min(table[i][k], table[i][k + 1]);
29                 }
30                 for (int k = j + (1 << i) + 1; k < j + (2 << i) and k < n; k += 1) {
31                     table[i][k] = min(table[i][k], table[i][k - 1]);
32                 }
33             }
34         }
35     }
36     int query(int l, int r) {
37         if (l + 1 == r) {
38             return table[0][l];
39         }
40         int i = bit_width(unsigned(l ^ (r - 1))) - 1;
41         return min(table[i][l], table[i][r - 1]);
42     }
43 };

```

2.4 Treap

```

1 struct Node {
2     static constexpr bool persistent = true;
3     static mt19937_64 mt;
4     Node *l, *r;
5     u64 priority;
6     int size, v;
7     i64 sum;
8     Node(const Node& other) { memcpy(this, &other, sizeof(Node)); }
9     Node(int v) : v(v), sum(v), priority(mt()), size(1) { l = r = nullptr; }
10    Node* update(Node* l, Node* r) {
11        Node* p = persistent ? new Node(*this) : this;
12        p->l = l;
13        p->r = r;
14        p->size = (l ? l->size : 0) + 1 + (r ? r->size : 0);
15        p->sum = (l ? l->sum : 0) + v + (r ? r->sum : 0);
16        return p;

```

```

17     }
18 };
19 mt19937_64 Node::mt;
20 pair<Node*, Node*> split_by_v(Node* p, int v) {
21     if (not p) {
22         return {};
23     }
24     if (p->v < v) {
25         auto [l, r] = split_by_v(p->r, v);
26         return {p->update(p->l, l), r};
27     }
28     auto [l, r] = split_by_v(p->l, v);
29     return {l, p->update(r, p->r)};
30 }
31 pair<Node*, Node*> split_by_size(Node* p, int size) {
32     if (not p) {
33         return {};
34     }
35     int l_size = p->l ? p->l->size : 0;
36     if (l_size < size) {
37         auto [l, r] = split_by_size(p->r, size - l_size - 1);
38         return {p->update(p->l, l), r};
39     }
40     auto [l, r] = split_by_size(p->l, size);
41     return {l, p->update(r, p->r)};
42 }
43 Node* merge(Node* l, Node* r) {
44     if (not l or not r) {
45         return l ? r;
46     }
47     if (l->priority < r->priority) {
48         return r->update(merge(l, r->l), r->r);
49     }
50     return l->update(l->l, merge(l->r, r));
51 }

```

2.5 Lines Maximum

```

1 struct Line {
2     mutable i64 k, b, p;
3     bool operator<(const Line& rhs) const { return k < rhs.k; }
4     bool operator<(const i64& x) const { return p < x; }
5 };
6 struct Lines : multiset<Line, less<>> {
7     static constexpr i64 inf = numeric_limits<i64>::max();
8     static i64 div(i64 a, i64 b) { return a / b - ((a ^ b) < 0 and a % b); }
9     bool isect(iterator x, iterator y) {
10         if (y == end()) {
11             return x->p == inf, false;
12         }
13         if (x->k == y->k) {
14             x->p = x->b > y->b ? inf : -inf;

```

```

15     } else {
16         x->p = div(y->b - x->b, x->k - y->k);
17     }
18     return x->p >= y->p;
19 }
20 void add(i64 k, i64 b) {
21     auto z = insert({k, b, 0}), y = z++, x = y;
22     while (isect(y, z)) {
23         z = erase(z);
24     }
25     if (x != begin() and isect(--x, y)) {
26         isect(x, y = erase(y));
27     }
28     while ((y = x) != begin() and (--x)->p >= y->p) {
29         isect(x, erase(y));
30     }
31 }
32 optional<i64> get(i64 x) {
33     if (empty()) {
34         return {};
35     }
36     auto it = lower_bound(x);
37     return it->k * x + it->b;
38 }
39 };

```

2.6 Segments Maximum

```

1 struct Segment {
2     i64 k, b;
3     i64 get(i64 x) { return k * x + b; }
4 };
5 struct Segments {
6     struct Node {
7         optional<Segment> s;
8         Node *l, *r;
9     };
10    i64 tl, tr;
11    Node* root;
12    Segments(i64 tl, i64 tr) : tl(tl), tr(tr), root(nullptr) {}
13    void add(i64 l, i64 r, i64 k, i64 b) {
14        function<void(Node*&, i64, i64, Segment)> rec = [&](Node*& p, i64 tl, i64
15            tr, Segment s) {
16            if (p == nullptr) {
17                p = new Node();
18            }
19            i64 tm = midpoint(tl, tr);
20            if (tl >= l and tr <= r) {
21                if (not p->s) {
22                    p->s = s;
23                    return;

```

```

24     auto t = p->s.value();
25     if (t.get(tl) >= s.get(tl)) {
26         if (t.get(tr) >= s.get(tr)) {
27             return;
28         }
29         if (t.get(tm) >= s.get(tm)) {
30             return rec(p->r, tm + 1, tr, s);
31         }
32         p->s = s;
33         return rec(p->l, tl, tm, t);
34     }
35     if (t.get(tr) <= s.get(tr)) {
36         p->s = s;
37         return;
38     }
39     if (t.get(tm) <= s.get(tm)) {
40         p->s = s;
41         return rec(p->r, tm + 1, tr, t);
42     }
43     return rec(p->l, tl, tm, s);
44 }
45 if (l <= tm) {
46     rec(p->l, tl, tm, s);
47 }
48 if (r > tm) {
49     rec(p->r, tm + 1, tr, s);
50 }
51 };
52 rec(root, tl, tr, {k, b});
53 }
54 optional<i64> get(i64 x) {
55     optional<i64> res = {};
56     function<void(Node*, i64, i64)> rec = [&](Node* p, i64 tl, i64 tr) {
57         if (p == nullptr) {
58             return;
59         }
60         i64 tm = midpoint(tl, tr);
61         if (p->s) {
62             i64 y = p->s.value().get(x);
63             if (not res or res.value() < y) {
64                 res = y;
65             }
66         }
67         if (x <= tm) {
68             rec(p->l, tl, tm);
69         } else {
70             rec(p->r, tm + 1, tr);
71         }
72     };
73     rec(root, tl, tr);
74     return res;
75 }
76 };

```

2.7 Segment Beats

```

1 struct Mv {
2     static constexpr i64 inf = numeric_limits<i64>::max() / 2;
3     i64 mv, smv, cmv, tmv;
4     bool less;
5     i64 def() { return less ? inf : -inf; }
6     i64 mmv(i64 x, i64 y) { return less ? min(x, y) : max(x, y); }
7     Mv(i64 x, bool less) : less(less) {
8         mv = x;
9         smv = tmv = def();
10        cmv = 1;
11    }
12    void up(const Mv& ls, const Mv& rs) {
13        mv = mmv(ls.mv, rs.mv);
14        smv = mmv(ls.mv == mv ? ls.smv : ls.mv, rs.mv == mv ? rs.smv : rs.mv);
15        cmv = (ls.mv == mv ? ls.cmv : 0) + (rs.mv == mv ? rs.cmv : 0);
16    }
17    void add(i64 x) {
18        mv += x;
19        if (smv != def()) {
20            smv += x;
21        }
22        if (tmv != def()) {
23            tmv += x;
24        }
25    }
26 };
27 struct Node {
28     Mv mn, mx;
29     i64 sum, tsum;
30     Node *ls, *rs;
31     Node(i64 x = 0) : sum(x), tsum(0), mn(x, true), mx(x, false) { ls = rs = nullptr; }
32     void up() {
33         sum = ls->sum + rs->sum;
34         mx.up(ls->mx, rs->mx);
35         mn.up(ls->mn, rs->mn);
36     }
37     void down(int tl, int tr) {
38         if (tsum) {
39             int tm = midpoint(tl, tr);
40             ls->add(tl, tm, tsum);
41             rs->add(tm, tr, tsum);
42             tsum = 0;
43         }
44         if (mn.tmv != mn.def()) {
45             ls->ch(mn.tmv, true);
46             rs->ch(mn.tmv, true);
47             mn.tmv = mn.def();
48         }
49         if (mx.tmv != mx.def()) {
50             ls->ch(mx.tmv, false);

```

```

51     rs->ch(mx.tmv, false);
52     mx.tmv = mx.def();
53 }
54 }
55 bool cmp(i64 x, i64 y, bool less) { return less ? x < y : x > y; }
56 void add(int tl, int tr, i64 x) {
57     sum += (tr - tl) * x;
58     tsum += x;
59     mx.add(x);
60     mn.add(x);
61 }
62 void ch(i64 x, bool less) {
63     auto &lhs = less ? mn : mx, &rhs = less ? mx : mn;
64     if (not cmp(x, rhs.mv, less)) {
65         return;
66     }
67     sum += (x - rhs.mv) * rhs.cmv;
68     if (lhs.smv == rhs.mv) {
69         lhs.smv = x;
70     }
71     if (lhs.mv == rhs.mv) {
72         lhs.mv = x;
73     }
74     if (cmp(x, rhs.tmv, less)) {
75         rhs.tmv = x;
76     }
77     rhs.mv = lhs.tmv = x;
78 }
79 void add(int tl, int tr, int l, int r, i64 x) {
80     if (tl >= l and tr <= r) {
81         return add(tl, tr, x);
82     }
83     down(tl, tr);
84     int tm = midpoint(tl, tr);
85     if (l < tm) {
86         ls->add(tl, tm, l, r, x);
87     }
88     if (r > tm) {
89         rs->add(tm, tr, l, r, x);
90     }
91     up();
92 }
93 void ch(int tl, int tr, int l, int r, i64 x, bool less) {
94     auto &lhs = less ? mn : mx, &rhs = less ? mx : mn;
95     if (not cmp(x, rhs.mv, less)) {
96         return;
97     }
98     if (tl >= l and tr <= r and cmp(rhs.smv, x, less)) {
99         return ch(x, less);
100     }
101     down(tl, tr);
102     int tm = midpoint(tl, tr);
103     if (l < tm) {

```

```

104         ls->ch(tl, tm, l, r, x, less);
105     }
106     if (r > tm) {
107         rs->ch(tm, tr, l, r, x, less);
108     }
109     up();
110 }
111 i64 get(int tl, int tr, int l, int r) {
112     if (tl >= l and tr <= r) {
113         return sum;
114     }
115     down(tl, tr);
116     i64 res = 0;
117     int tm = midpoint(tl, tr);
118     if (l < tm) {
119         res += ls->get(tl, tm, l, r);
120     }
121     if (r > tm) {
122         res += rs->get(tm, tr, l, r);
123     }
124     return res;
125 }
126 };

```

2.8 Tree

2.8.1 Least Common Ancestor

```

1 struct LeastCommonAncestor {
2     SparseTable st;
3     vector<int> p, time, a, par;
4     LeastCommonAncestor(int root, const vector<vector<int>>& g) {
5         int n = g.size();
6         time.resize(n, -1);
7         par.resize(n, -1);
8         function<void(int)> dfs = [&](int u) {
9             time[u] = p.size();
10            p.push_back(u);
11            for (int v : g[u]) {
12                if (time[v] == -1) {
13                    par[v] = u;
14                    dfs(v);
15                }
16            }
17        };
18        dfs(root);
19        a.resize(n);
20        for (int i = 1; i < n; i += 1) {
21            a[i] = time[par[p[i]]];
22        }
23        st = SparseTable(a);
24    }

```

```

25 int query(int u, int v) {
26     if (u == v) {
27         return u;
28     }
29     if (time[u] > time[v]) {
30         swap(u, v);
31     }
32     return p[st.query(time[u] + 1, time[v] + 1)];
33 }
34 };

```

2.8.2 Link Cut Tree

```

1 template <class T, class E, class REV, class OP>
2 struct Node {
3     T t, st;
4     bool reversed;
5     Node* par;
6     array<Node*, 2> ch;
7     Node(T t = E()) : t(t), st(t), reversed(false), par(nullptr) { ch.fill(
8         nullptr); }
9     int get_s() {
10         if (par == nullptr) {
11             return -1;
12         }
13         if (par->ch[0] == this) {
14             return 0;
15         }
16         if (par->ch[1] == this) {
17             return 1;
18         }
19         return -1;
20     }
21     void push_up() { st = OP()(ch[0] ? ch[0]->st : E(), OP()(t, ch[1] ? ch
22         [1]->st : E())); }
23     void reverse() {
24         reversed ^= 1;
25         st = REV()(st);
26     }
27     void push_down() {
28         if (reversed) {
29             swap(ch[0], ch[1]);
30             if (ch[0]) {
31                 ch[0]->reverse();
32             }
33             if (ch[1]) {
34                 ch[1]->reverse();
35             }
36             reversed = false;
37         }
38     }
39     void attach(int s, Node* u) {

```

```

38     if ((ch[s] = u)) {
39         u->par = this;
40     }
41     push_up();
42 }
43 void rotate() {
44     auto p = par;
45     auto pp = p->par;
46     int s = get_s();
47     int ps = p->get_s();
48     p->attach(s, ch[s ^ 1]);
49     attach(s ^ 1, p);
50     if (~ps) {
51         pp->attach(ps, this);
52     }
53     par = pp;
54 }
55 void splay() {
56     push_down();
57     while (~get_s() and ~par->get_s()) {
58         par->par->push_down();
59         par->push_down();
60         push_down();
61         (get_s() == par->get_s() ? par : this)->rotate();
62         rotate();
63     }
64     if (~get_s()) {
65         par->push_down();
66         push_down();
67         rotate();
68     }
69 }
70 void access() {
71     splay();
72     attach(1, nullptr);
73     while (par != nullptr) {
74         auto p = par;
75         p->splay();
76         p->attach(1, this);
77         rotate();
78     }
79 }
80 void make_root() {
81     access();
82     reverse();
83     push_down();
84 }
85 void link(Node* u) {
86     u->make_root();
87     access();
88     attach(1, u);
89 }
90 void cut(Node* u) {

```



```

91     u->make_root();
92     access();
93     if (ch[0] == u) {
94         ch[0] = u->par = nullptr;
95         push_up();
96     }
97 }
98 void set(T t) {
99     access();
100    this->t = t;
101    push_up();
102 }
103 T query(Node* u) {
104     u->make_root();
105     access();
106     return st;
107 }
108 };

```

3 Graph

3.1 Connected Components

3.1.1 Strongly Connected Components

Returns strongly connected components in topologically order.

```

1 vector<vector<int>> strongly_connected_components(const vector<vector<int>>&
  g) {
2     int n = g.size();
3     vector<bool> done(n);
4     vector<int> pos(n, -1), stack;
5     vector<vector<int>> res;
6     function<int(int)> dfs = [&](int u) {
7         int low = pos[u] = stack.size();
8         stack.push_back(u);
9         for (int v : g[u]) {
10             if (not done[v]) {
11                 low = min(low, ~pos[v] ? pos[v] : dfs(v));
12             }
13         }
14         if (low == pos[u]) {
15             res.emplace_back(stack.begin() + low, stack.end());
16             for (int v : res.back()) {
17                 done[v] = true;
18             }
19             stack.resize(low);
20         }
21         return low;
22     };
23     for (int i = 0; i < n; i += 1) {
24         if (not done[i]) {

```

```

25         dfs(i);
26     }
27 }
28 ranges::reverse(res);
29 return res;
30 }

```

3.1.2 Two-vertex-connected Components

```

1 vector<vector<int>> two_vertex_connected_components(const vector<vector<int
  >>& g) {
2     int n = g.size();
3     vector<int> pos(n, -1), stack;
4     vector<vector<int>> res;
5     function<int(int, int)> dfs = [&](int u, int p) {
6         int low = pos[u] = stack.size(), son = 0;
7         stack.push_back(u);
8         for (int v : g[u]) {
9             if (v != p) {
10                 if (~pos[v]) {
11                     low = min(low, pos[v]);
12                 } else {
13                     int end = stack.size(), lowv = dfs(v, u);
14                     low = min(low, lowv);
15                     if (lowv >= pos[u] and (~p or son++)) {
16                         res.emplace_back(stack.begin() + end, stack.end());
17                         res.back().push_back(u);
18                         stack.resize(end);
19                     }
20                 }
21             }
22         }
23         return low;
24     };
25     for (int i = 0; i < n; i += 1) {
26         if (pos[i] == -1) {
27             dfs(i, -1);
28             res.emplace_back(move(stack));
29         }
30     }
31     return res;
32 }

```

3.1.3 Two-edge-connected Components

```

1 vector<vector<int>> bcc(const vector<vector<int>>& g) {
2     int n = g.size();
3     vector<int> pos(n, -1), stack;
4     vector<vector<int>> res;
5     function<int(int, int)> dfs = [&](int u, int p) {

```

```

6   int low = pos[u] = stack.size(), pc = 0;
7   stack.push_back(u);
8   for (int v : g[u]) {
9       if (~pos[v]) {
10          if (v != p or pc++) {
11              low = min(low, pos[v]);
12          }
13      } else {
14          low = min(low, dfs(v, u));
15      }
16  }
17  if (low == pos[u]) {
18      res.emplace_back(stack.begin() + low, stack.end());
19      stack.resize(low);
20  }
21  return low;
22 };
23 for (int i = 0; i < n; i += 1) {
24     if (pos[i] == -1) {
25         dfs(i, -1);
26     }
27 }
28 return res;
29 }

```

3.1.4 Three-edge-connected Components

```

1  vector<vector<int>> three_edge_connected_components(const vector<vector<int
   >>& g) {
2      int n = g.size(), dft = -1;
3      vector<int> pre(n, -1), post(n), path(n, -1), low(n), deg(n);
4      DisjointSetUnion dsu(n);
5      function<void(int, int)> dfs = [&](int u, int p) {
6          int pc = 0;
7          low[u] = pre[u] = dft += 1;
8          for (int v : g[u]) {
9              if (v != u and (v != p or pc++)) {
10                 if (pre[v] != -1) {
11                     if (pre[v] < pre[u]) {
12                         deg[u] += 1;
13                         low[u] = min(low[u], pre[v]);
14                     } else {
15                         deg[u] -= 1;
16                         for (int& p = path[u]; p != -1 and pre[p] <= pre[v] and pre[v] <=
                             post[p];) {
17                             dsu.merge(u, p);
18                             deg[u] += deg[p];
19                             p = path[p];
20                         }
21                     }
22                 } else {
23                     dfs(v, u);

```

```

24         if (path[v] == -1 and deg[v] <= 1) {
25             low[u] = min(low[u], low[v]);
26             deg[u] += deg[v];
27         } else {
28             if (deg[v] == 0) {
29                 v = path[v];
30             }
31             if (low[u] > low[v]) {
32                 low[u] = min(low[u], low[v]);
33                 swap(v, path[u]);
34             }
35             for (; v != -1; v = path[v]) {
36                 dsu.merge(u, v);
37                 deg[u] += deg[v];
38             }
39         }
40     }
41 }
42 post[u] = dft;
43 };
44 for (int i = 0; i < n; i += 1) {
45     if (pre[i] == -1) {
46         dfs(i, -1);
47     }
48 }
49 vector<vector<int>> _res(n);
50 for (int i = 0; i < n; i += 1) {
51     _res[dsu.find(i)].push_back(i);
52 }
53 vector<vector<int>> res;
54 for (auto& res_i : _res) {
55     if (not res_i.empty()) {
56         res.emplace_back(move(res_i));
57     }
58 }
59 return res;
60 }
61 }

```

3.2 Euler Walks

```

1  optional<vector<vector<pair<int, bool>>>> undirected_walks(int n, const
   vector<pair<int, int>>& edges) {
2      int m = ssize(edges);
3      vector<vector<pair<int, bool>>> res;
4      if (not m) {
5          return res;
6      }
7      vector<vector<pair<int, bool>>> g(n);
8      for (int i = 0; i < m; i += 1) {
9          auto [u, v] = edges[i];
10         g[u].emplace_back(i, true);

```

```

11     g[v].emplace_back(i, false);
12 }
13 for (int i = 0; i < n; i += 1) {
14     if (g[i].size() % 2) {
15         return {};
16     }
17 }
18 vector<pair<int, bool>> walk;
19 vector<bool> visited(m);
20 vector<int> cur(n);
21 function<void(int)> dfs = [&](int u) {
22     for (int& i = cur[u]; i < ssize(g[u]);) {
23         auto [j, d] = g[u][i];
24         if (not visited[j]) {
25             visited[j] = true;
26             dfs(d ? edges[j].second : edges[j].first);
27             walk.emplace_back(j, d);
28         } else {
29             i += 1;
30         }
31     }
32 };
33 for (int i = 0; i < n; i += 1) {
34     dfs(i);
35     if (not walk.empty()) {
36         ranges::reverse(walk);
37         res.emplace_back(move(walk));
38     }
39 }
40 return res;
41 }
42 optional<vector<vector<int>>> directed_walks(int n, const vector<pair<int,
43     int>>& edges) {
44     int m = ssize(edges);
45     vector<vector<int>> res;
46     if (not m) {
47         return res;
48     }
49     vector<int> d(n);
50     vector<vector<int>> g(n);
51     for (int i = 0; i < m; i += 1) {
52         auto [u, v] = edges[i];
53         g[u].push_back(i);
54         d[v] += 1;
55     }
56     for (int i = 0; i < n; i += 1) {
57         if (ssize(g[i]) != d[i]) {
58             return {};
59         }
60     }
61     vector<int> walk;
62     vector<int> cur(n);
63     vector<bool> visited(m);

```

```

63     function<void(int)> dfs = [&](int u) {
64         for (int& i = cur[u]; i < ssize(g[u]);) {
65             int j = g[u][i];
66             if (not visited[j]) {
67                 visited[j] = true;
68                 dfs(edges[j].second);
69                 walk.push_back(j);
70             } else {
71                 i += 1;
72             }
73         }
74     };
75     for (int i = 0; i < n; i += 1) {
76         dfs(i);
77         if (not walk.empty()) {
78             ranges::reverse(walk);
79             res.emplace_back(move(walk));
80         }
81     }
82     return res;
83 }

```

3.3 Dominator Tree

```

1 vector<int> dominator(const vector<vector<int>>& g, int s) {
2     int n = g.size();
3     vector<int> pos(n, -1), p, label(n), dom(n), sdom(n), dsu(n), par(n);
4     vector<vector<int>> rg(n), bucket(n);
5     function<void(int)> dfs = [&](int u) {
6         int t = p.size();
7         p.push_back(u);
8         label[t] = sdom[t] = dsu[t] = pos[u] = t;
9         for (int v : g[u]) {
10             if (pos[v] == -1) {
11                 dfs(v);
12                 par[pos[v]] = t;
13             }
14             rg[pos[v]].push_back(t);
15         }
16     };
17     function<int(int, int)> find = [&](int u, int x) {
18         if (u == dsu[u]) {
19             return x ? -1 : u;
20         }
21         int v = find(dsu[u], x + 1);
22         if (v < 0) {
23             return u;
24         }
25         if (sdom[label[dsu[u]]] < sdom[label[u]]) {
26             label[u] = label[dsu[u]];
27         }
28         dsu[u] = v;

```

```

29     return x ? v : label[u];
30 };
31 dfs(s);
32 iota(dom.begin(), dom.end(), 0);
33 for (int i = ssize(p) - 1; i >= 0; i -= 1) {
34     for (int j : rg[i]) {
35         sdom[i] = min(sdom[i], sdom[find(j, 0)]);
36     }
37     if (i) {
38         bucket[sdom[i]].push_back(i);
39     }
40     for (int k : bucket[i]) {
41         int j = find(k, 0);
42         dom[k] = sdom[j] == sdom[k] ? sdom[j] : j;
43     }
44     if (i > 1) {
45         dsu[i] = par[i];
46     }
47 }
48 for (int i = 1; i < ssize(p); i += 1) {
49     if (dom[i] != sdom[i]) {
50         dom[i] = dom[dom[i]];
51     }
52 }
53 vector<int> res(n, -1);
54 res[s] = s;
55 for (int i = 1; i < ssize(p); i += 1) {
56     res[p[i]] = p[dom[i]];
57 }
58 return res;
59 }

```

3.4 Directed Minimum Spanning Tree

```

1 struct Node {
2     Edge e;
3     int d;
4     Node *l, *r;
5     Node(Edge e) : e(e), d(0) { l = r = nullptr; }
6     void add(int v) {
7         e.w += v;
8         d += v;
9     }
10    void push() {
11        if (l) {
12            l->add(d);
13        }
14        if (r) {
15            r->add(d);
16        }
17        d = 0;
18    }

```

```

19 };
20 Node* merge(Node* u, Node* v) {
21     if (not u or not v) {
22         return u ? v;
23     }
24     if (u->e.w > v->e.w) {
25         swap(u, v);
26     }
27     u->push();
28     u->r = merge(u->r, v);
29     swap(u->l, u->r);
30     return u;
31 }
32 void pop(Node*& u) {
33     u->push();
34     u = merge(u->l, u->r);
35 }
36 pair<i64, vector<int>> directed_minimum_spanning_tree(int n, const vector<
37     Edge>& edges, int s) {
38     i64 ans = 0;
39     vector<Node*> heap(n), edge(n);
40     RollbackDisjointSetUnion dsu(n), rbdsu(n);
41     vector<pair<Node*, int>> cycles;
42     for (auto e : edges) {
43         heap[e.v] = merge(heap[e.v], new Node(e));
44     }
45     for (int i = 0; i < n; i += 1) {
46         if (i == s) {
47             continue;
48         }
49         for (int u = i;;) {
50             if (not heap[u]) {
51                 return {};
52             }
53             ans += (edge[u] = heap[u])->e.w;
54             edge[u]->add(-edge[u]->e.w);
55             int v = rbdsu.find(edge[u]->e.u);
56             if (dsu.merge(u, v)) {
57                 break;
58             }
59             int t = rbdsu.time();
60             while (rbdsu.merge(u, v)) {
61                 heap[rbdsu.find(u)] = merge(heap[u], heap[v]);
62                 u = rbdsu.find(u);
63                 v = rbdsu.find(edge[v]->e.u);
64             }
65             cycles.emplace_back(edge[u], t);
66             while (heap[u] and rbdsu.find(heap[u]->e.u) == rbdsu.find(u)) {
67                 pop(heap[u]);
68             }
69         }
70     }
71     for (auto [p, t] : cycles | views::reverse) {

```

```

71     int u = rbdsu.find(p->e.v);
72     rbdsu.rollback(t);
73     int v = rbdsu.find(edge[u]->e.v);
74     edge[v] = exchange(edge[u], p);
75 }
76 vector<int> res(n, -1);
77 for (int i = 0; i < n; i += 1) {
78     res[i] = i == s ? i : edge[i]->e.u;
79 }
80 return {ans, res};
81 }

```

3.5 K Shortest Paths

```

1 struct Node {
2     int v, h;
3     i64 w;
4     Node *l, *r;
5     Node(int v, i64 w) : v(v), w(w), h(1) { l = r = nullptr; }
6 };
7 Node* merge(Node* u, Node* v) {
8     if (not u or not v) {
9         return u ? v;
10    }
11    if (u->w > v->w) {
12        swap(u, v);
13    }
14    Node* p = new Node(*u);
15    p->r = merge(u->r, v);
16    if (p->r and (not p->l or p->l->h < p->r->h)) {
17        swap(p->l, p->r);
18    }
19    p->h = (p->r ? p->r->h : 0) + 1;
20    return p;
21 }
22 struct Edge {
23     int u, v, w;
24 };
25 template <typename T>
26 using minimum_heap = priority_queue<T, vector<T>, greater<T>>;
27 vector<i64> k_shortest_paths(int n, const vector<Edge>& edges, int s, int t,
28     int k) {
29     vector<vector<int>> g(n);
30     for (int i = 0; i < ssize(edges); i += 1) {
31         g[edges[i].u].push_back(i);
32     }
33     vector<int> par(n, -1), p;
34     vector<i64> d(n, -1);
35     minimum_heap<pair<i64, int>> pq;
36     pq.push({d[s] = 0, s});
37     while (not pq.empty()) {
38         auto [du, u] = pq.top();

```

```

38     pq.pop();
39     if (du > d[u]) {
40         continue;
41     }
42     p.push_back(u);
43     for (int i : g[u]) {
44         auto [_, v, w] = edges[i];
45         if (d[v] == -1 or d[v] > d[u] + w) {
46             par[v] = i;
47             pq.push({d[v] = d[u] + w, v});
48         }
49     }
50 }
51 if (d[t] == -1) {
52     return vector<i64>(k, -1);
53 }
54 vector<Node*> heap(n);
55 for (int i = 0; i < ssize(edges); i += 1) {
56     auto [u, v, w] = edges[i];
57     if (~d[u] and ~d[v] and par[v] != i) {
58         heap[v] = merge(heap[v], new Node(u, d[u] + w - d[v]));
59     }
60 }
61 for (int u : p) {
62     if (u != s) {
63         heap[u] = merge(heap[u], heap[edges[par[u]].u]);
64     }
65 }
66 minimum_heap<pair<i64, Node*>> q;
67 if (heap[t]) {
68     q.push({d[t] + heap[t]->w, heap[t]});
69 }
70 vector<i64> res = {d[t]};
71 for (int i = 1; i < k and not q.empty(); i += 1) {
72     auto [w, p] = q.top();
73     q.pop();
74     res.push_back(w);
75     if (heap[p->v]) {
76         q.push({w + heap[p->v]->w, heap[p->v]});
77     }
78     for (auto c : {p->l, p->r}) {
79         if (c) {
80             q.push({w + c->w - p->w, c});
81         }
82     }
83 }
84 res.resize(k, -1);
85 return res;
86 }

```

3.6 Global Minimum Cut

```

1 i64 global_minimum_cut(vector<vector<i64>>& w) {
2     int n = w.size();
3     if (n == 2) {
4         return w[0][1];
5     }
6     vector<bool> in(n);
7     vector<int> add;
8     vector<i64> s(n);
9     i64 st = 0;
10    for (int i = 0; i < n; i += 1) {
11        int k = -1;
12        for (int j = 0; j < n; j += 1) {
13            if (not in[j]) {
14                if (k == -1 or s[j] > s[k]) {
15                    k = j;
16                }
17            }
18        }
19        add.push_back(k);
20        st = s[k];
21        in[k] = true;
22        for (int j = 0; j < n; j += 1) {
23            s[j] += w[j][k];
24        }
25    }
26    for (int i = 0; i < n; i += 1) {
27    }
28    int x = add.rbegin()[1], y = add.back();
29    if (x == n - 1) {
30        swap(x, y);
31    }
32    for (int i = 0; i < n; i += 1) {
33        swap(w[y][i], w[n - 1][i]);
34        swap(w[i][y], w[i][n - 1]);
35    }
36    for (int i = 0; i + 1 < n; i += 1) {
37        w[i][x] += w[i][n - 1];
38        w[x][i] += w[n - 1][i];
39    }
40    w.pop_back();
41    return min(st, stoer_wagner(w));
42 }

```

3.7 Minimum Perfect Matching on Bipartite Graph

```

1 minimum_perfect_matching_on_bipartite_graph(const vector<vector<i64>>& w) {
2     i64 n = w.size();
3     vector<int> rm(n, -1), cm(n, -1);
4     vector<i64> pi(n);
5     auto resid = [&](int r, int c) { return w[r][c] - pi[c]; };
6     for (int c = 0; c < n; c += 1) {

```

```

7         int r = ranges::min(views::iota(0, n), {}, [&](int r) { return w[r][c];
8             });
9         pi[c] = w[r][c];
10        if (rm[r] == -1) {
11            rm[r] = c;
12            cm[c] = r;
13        }
14    }
15    vector<int> cols(n);
16    iota(cols.begin(), cols.end(), 0);
17    for (int r = 0; r < n; r += 1) {
18        if (rm[r] != -1) {
19            continue;
20        }
21        vector<i64> d(n);
22        for (int c = 0; c < n; c += 1) {
23            d[c] = resid(r, c);
24        }
25        vector<int> pre(n, r);
26        int scan = 0, label = 0, last = 0, col = -1;
27        [&]() {
28            while (true) {
29                if (scan == label) {
30                    last = scan;
31                    i64 min = d[cols[scan]];
32                    for (int j = scan; j < n; j += 1) {
33                        int c = cols[j];
34                        if (d[c] <= min) {
35                            if (d[c] < min) {
36                                min = d[c];
37                                label = scan;
38                            }
39                            swap(cols[j], cols[label++]);
40                        }
41                    }
42                    for (int j = scan; j < label; j += 1) {
43                        if (int c = cols[j]; cm[c] == -1) {
44                            col = c;
45                            return;
46                        }
47                    }
48                }
49                int c1 = cols[scan++], r1 = cm[c1];
50                for (int j = label; j < n; j += 1) {
51                    int c2 = cols[j];
52                    i64 len = resid(r1, c2) - resid(r1, c1);
53                    if (d[c2] > d[c1] + len) {
54                        d[c2] = d[c1] + len;
55                        pre[c2] = r1;
56                        if (len == 0) {
57                            if (cm[c2] == -1) {
58                                col = c2;
                                return;

```

```

59         }
60         swap(cols[j], cols[label++]);
61     }
62 }
63 }
64 }
65 }();
66 for (int i = 0; i < last; i += 1) {
67     int c = cols[i];
68     pi[c] += d[c] - d[col];
69 }
70 for (int t = col; t != -1;) {
71     col = t;
72     int r = pre[col];
73     cm[col] = r;
74     swap(rm[r], t);
75 }
76 }
77 i64 res = 0;
78 for (int i = 0; i < n; i += 1) {
79     res += w[i][rm[i]];
80 }
81 return {res, rm};
82 }

```

3.8 Matching on General Graph

```

1 vector<int> matching(const vector<vector<int>>& g) {
2     int n = g.size();
3     int mark = 0;
4     vector<int> matched(n, -1), par(n, -1), book(n);
5     auto match = [&](int s) {
6         vector<int> c(n), type(n, -1);
7         iota(c.begin(), c.end(), 0);
8         queue<int> q;
9         q.push(s);
10        type[s] = 0;
11        while (not q.empty()) {
12            int u = q.front();
13            q.pop();
14            for (int v : g[u])
15                if (type[v] == -1) {
16                    par[v] = u;
17                    type[v] = 1;
18                    int w = matched[v];
19                    if (w == -1) {
20                        [&](int u) {
21                            while (u != -1) {
22                                int v = matched[par[u]];
23                                matched[matched[u] = par[u]] = u;
24                                u = v;
25                            }

```

```

26                }(v);
27                return;
28            }
29            q.push(w);
30            type[w] = 0;
31        } else if (not type[v] and c[u] != c[v]) {
32            int w = [&](int u, int v) {
33                mark += 1;
34                while (true) {
35                    if (u != -1) {
36                        if (book[u] == mark) {
37                            return u;
38                        }
39                        book[u] = mark;
40                        u = c[par[matched[u]]];
41                    }
42                    swap(u, v);
43                }
44            }(u, v);
45            auto up = [&](int u, int v, int w) {
46                while (c[u] != w) {
47                    par[u] = v;
48                    v = matched[u];
49                    if (type[v] == 1) {
50                        q.push(v);
51                        type[v] == 0;
52                    }
53                    if (c[u] == u) {
54                        c[u] = w;
55                    }
56                    if (c[v] == v) {
57                        c[v] = w;
58                    }
59                    u = par[v];
60                }
61            };
62            up(u, v, w);
63            up(v, u, w);
64            for (int i = 0; i < n; i += 1) {
65                c[i] = c[c[i]];
66            }
67        }
68    };
69 };
70 for (int i = 0; i < n; i += 1) {
71     if (matched[i] == -1) {
72         match(i);
73     }
74 }
75 return matched;
76 }

```

3.9 Maximum Flow

```

1 struct HighestLabelPreflowPush {
2     int n;
3     vector<vector<int>> g;
4     vector<Edge> edges;
5     HighestLabelPreflowPush(int n) : n(n), g(n) {}
6     int add(int u, int v, i64 f) {
7         if (u == v) {
8             return -1;
9         }
10        int i = ssize(edges);
11        edges.push_back({u, v, f});
12        g[u].push_back(i);
13        edges.push_back({v, u, 0});
14        g[v].push_back(i + 1);
15        return i;
16    }
17    i64 max_flow(int s, int t) {
18        vector<i64> p(n);
19        vector<int> h(n), cur(n), count(n * 2);
20        vector<vector<int>> pq(n * 2);
21        auto push = [&](int i, i64 f) {
22            auto [u, v, _] = edges[i];
23            if (not p[v] and f) {
24                pq[h[v]].push_back(v);
25            }
26            edges[i].f -= f;
27            edges[i ^ 1].f += f;
28            p[u] -= f;
29            p[v] += f;
30        };
31        h[s] = n;
32        count[0] = n - 1;
33        p[t] = 1;
34        for (int i : g[s]) {
35            push(i, edges[i].f);
36        }
37        for (int hi = 0;;) {
38            while (pq[hi].empty()) {
39                if (not hi--) {
40                    return -p[s];
41                }
42            }
43            int u = pq[hi].back();
44            pq[hi].pop_back();
45            while (p[u] > 0) {
46                if (cur[u] == ssize(g[u])) {
47                    h[u] = n * 2 + 1;
48                    for (int i = 0; i < ssize(g[u]); i += 1) {
49                        auto [_, v, f] = edges[g[u][i]];
50                        if (f and h[u] > h[v] + 1) {
51                            h[u] = h[v] + 1;

```

```

52                cur[u] = i;
53            }
54        }
55        count[h[u]] += 1;
56        if (not(count[hi] -= 1) and hi < n) {
57            for (int i = 0; i < n; i += 1) {
58                if (h[i] > hi and h[i] < n) {
59                    count[h[i]] -= 1;
60                    h[i] = n + 1;
61                }
62            }
63        }
64        hi = h[u];
65    } else {
66        int i = g[u][cur[u]];
67        auto [_, v, f] = edges[i];
68        if (f and h[u] == h[v] + 1) {
69            push(i, min(p[u], f));
70        } else {
71            cur[u] += 1;
72        }
73    }
74    }
75    }
76    return i64(0);
77 }
78 };
79
80 struct Dinic {
81     int n;
82     vector<vector<int>> g;
83     vector<Edge> edges;
84     vector<int> level;
85     Dinic(int n) : n(n), g(n) {}
86     int add(int u, int v, i64 f) {
87         if (u == v) {
88             return -1;
89         }
90        int i = ssize(edges);
91        edges.push_back({u, v, f});
92        g[u].push_back(i);
93        edges.push_back({v, u, 0});
94        g[v].push_back(i + 1);
95        return i;
96    }
97    i64 max_flow(int s, int t) {
98        i64 flow = 0;
99        queue<int> q;
100        vector<int> cur;
101        auto bfs = [&]() {
102            level.assign(n, -1);
103            level[s] = 0;
104            q.push(s);

```



```

105 while (not q.empty()) {
106     int u = q.front();
107     q.pop();
108     for (int i : g[u]) {
109         auto [_, v, c] = edges[i];
110         if (c and level[v] == -1) {
111             level[v] = level[u] + 1;
112             q.push(v);
113         }
114     }
115 }
116 return ~level[t];
117 };
118 auto dfs = [&](auto& dfs, int u, i64 limit) -> i64 {
119     if (u == t) {
120         return limit;
121     }
122     i64 res = 0;
123     for (int& i = cur[u]; i < ssize(g[u]) and limit; i += 1) {
124         int j = g[u][i];
125         auto [_, v, f] = edges[j];
126         if (level[v] == level[u] + 1 and f) {
127             if (i64 d = dfs(dfs, v, min(f, limit)); d) {
128                 limit -= d;
129                 res += d;
130                 edges[j].f -= d;
131                 edges[j ^ 1].f += d;
132             }
133         }
134     }
135     return res;
136 };
137 while (bfs()) {
138     cur.assign(n, 0);
139     while (i64 f = dfs(dfs, s, numeric_limits<i64>::max())) {
140         flow += f;
141     }
142 }
143 return flow;
144 }
145 };

```

3.10 Minimum Cost Maximum Flow

Constraints: there is no edge with negative cost.

```

1 struct MinimumCostMaximumFlow {
2     template <typename T>
3     using minimum_heap = priority_queue<T, vector<T>, greater<T>>;
4     int n;
5     vector<Edge> edges;
6     vector<vector<int>>> g;
7     MinimumCostMaximumFlow(int n) : n(n), g(n) {}

```

```

8     int add_edge(int u, int v, i64 f, i64 c) {
9         int i = edges.size();
10        edges.push_back({u, v, f, c});
11        edges.push_back({v, u, 0, -c});
12        g[u].push_back(i);
13        g[v].push_back(i + 1);
14        return i;
15    }
16    pair<i64, i64> flow(int s, int t) {
17        constexpr i64 inf = numeric_limits<i64>::max();
18        vector<i64> d, h(n);
19        vector<int> p;
20        auto dijkstra = [&]() {
21            d.assign(n, inf);
22            p.assign(n, -1);
23            minimum_heap<pair<i64, int>>> q;
24            q.emplace(d[s] = 0, s);
25            while (not q.empty()) {
26                auto [du, u] = q.top();
27                q.pop();
28                if (du > d[u]) {
29                    continue;
30                }
31                for (int i : g[u]) {
32                    auto [_, v, f, c] = edges[i];
33                    if (f and d[v] > d[u] + h[u] - h[v] + c) {
34                        p[v] = i;
35                        q.emplace(d[v] = d[u] + h[u] - h[v] + c, v);
36                    }
37                }
38            }
39            return ~p[t];
40        };
41        i64 f = 0, c = 0;
42        while (dijkstra()) {
43            for (int i = 0; i < n; i += 1) {
44                h[i] += d[i];
45            }
46            vector<int> path;
47            for (int u = t; u != s; u = edges[p[u]].u) {
48                path.push_back(p[u]);
49            }
50            i64 mf = edges[ranges::min(path, {}, [&](int i) { return edges[i].f; })
51                ].f;
52            f += mf;
53            c += mf * h[t];
54            for (int i : path) {
55                edges[i].f -= mf;
56                edges[i ^ 1].f += mf;
57            }
58        }
59        return {f, c};
60    }

```

```
60 };
```

4 String

4.1 Z

```
1 vector<int> fz(const string& s) {
2     int n = s.size();
3     vector<int> z(n);
4     for (int i = 1, j = 0; i < n; i += 1) {
5         z[i] = max(min(z[i - j], j + z[j] - i), 0);
6         while (i + z[i] < n and s[i + z[i]] == s[z[i]]) {
7             z[i] += 1;
8         }
9         if (i + z[i] > j + z[j]) {
10            j = i;
11        }
12    }
13    return z;
14 }
```

4.2 Lyndon Factorization

```
1 vector<int> lyndon_factorization(string const& s) {
2     vector<int> res = {0};
3     for (int i = 0, n = s.size(); i < n; i++) {
4         int j = i + 1, k = i;
5         for (; j < n and s[k] <= s[j]; j += 1) {
6             k = s[k] < s[j] ? i : k + 1;
7         }
8         while (i <= k) {
9             res.push_back(i += j - k);
10        }
11    }
12    return res;
13 }
```

4.3 Border

```
1 vector<int> fborder(const string& s) {
2     int n = s.size();
3     vector<int> res(n);
4     for (int i = 1; i < n; i += 1) {
5         int& j = res[i] = res[i - 1];
6         while (j and s[i] != s[j]) {
7             j = res[j - 1];
8         }
9     }
```

```
9     j += s[i] == s[j];
10 }
11 return res;
12 }
```

4.4 Manacher

```
1 vector<int> manacher(const string& s) {
2     int n = s.size();
3     vector<int> p(n);
4     for (int i = 0, j = 0; i < n; i += 1) {
5         if (j + p[j] > i) {
6             p[i] = min(p[j * 2 - i], j + p[j] - i);
7         }
8         while (i >= p[i] and i + p[i] < n and s[i - p[i]] == s[i + p[i]]) {
9             p[i] += 1;
10        }
11        if (i + p[i] > j + p[j]) {
12            j = i;
13        }
14    }
15    return p;
16 }
```

4.5 Suffix Array

```
1 pair<vector<int>, vector<int>> binary_lifting(const string& s) {
2     int n = s.size(), k = 0;
3     vector<int> p(n), rank(n), q, count;
4     iota(p.begin(), p.end(), 0);
5     ranges::sort(p, {}, [&](int i) { return s[i]; });
6     for (int i = 0; i < n; i += 1) {
7         rank[p[i]] = i and s[p[i]] == s[p[i - 1]] ? rank[p[i - 1]] : k++;
8     }
9     for (int m = 1; m < n; m *= 2) {
10        q.resize(m);
11        iota(q.begin(), q.end(), n - m);
12        for (int i : p) {
13            if (i >= m) {
14                q.push_back(i - m);
15            }
16        }
17        count.assign(k, 0);
18        for (int i : rank) {
19            count[i] += 1;
20        }
21        partial_sum(count.begin(), count.end(), count.begin());
22        for (int i = n - 1; i >= 0; i -= 1) {
23            p[count[rank[q[i]]] - 1] = q[i];
24        }
25    }
```

```

25     auto previous = rank;
26     previous.resize(2 * n, -1);
27     k = 0;
28     for (int i = 0; i < n; i += 1) {
29         rank[p[i]] = i and previous[p[i]] == previous[p[i - 1]] and previous[p[
30             i] + m] == previous[p[i - 1] + m] ? rank[p[i - 1]] : k++;
31     }
32     vector<int> lcp(n);
33     k = 0;
34     for (int i = 0; i < n; i += 1) {
35         if (rank[i]) {
36             k = max(k - 1, 0);
37             int j = p[rank[i] - 1];
38             while (i + k < n and j + k < n and s[i + k] == s[j + k]) {
39                 k += 1;
40             }
41             lcp[rank[i]] = k;
42         }
43     }
44     return {p, lcp};
45 }

```

4.6 Aho-Corasick Automaton

```

1  constexpr int sigma = 26;
2  struct Node {
3      int link;
4      array<int, sigma> next;
5      Node() : link(0) { next.fill(0); }
6  };
7  struct AhoCorasick : vector<Node> {
8      AhoCorasick() : vector<Node>(1) {}
9      int add(const string& s, char first = 'a') {
10         int p = 0;
11         for (char si : s) {
12             int c = si - first;
13             if (not at(p).next[c]) {
14                 at(p).next[c] = size();
15                 emplace_back();
16             }
17             p = at(p).next[c];
18         }
19         return p;
20     }
21     void init() {
22         queue<int> q;
23         for (int i = 0; i < sigma; i += 1) {
24             if (at(0).next[i]) {
25                 q.push(at(0).next[i]);
26             }
27         }

```

```

28     while (not q.empty()) {
29         int u = q.front();
30         q.pop();
31         for (int i = 0; i < sigma; i += 1) {
32             if (at(u).next[i]) {
33                 at(at(u).next[i]).link = at(at(u).link).next[i];
34                 q.push(at(u).next[i]);
35             } else {
36                 at(u).next[i] = at(at(u).link).next[i];
37             }
38         }
39     }
40 }
41 };

```

4.7 Suffix Automaton

```

1  struct Node {
2      int link, len;
3      array<int, sigma> next;
4      Node() : link(-1), len(0) { next.fill(-1); }
5  };
6  struct SuffixAutomaton : vector<Node> {
7      SuffixAutomaton() : vector<Node>(1) {}
8      int extend(int p, int c) {
9         if (~at(p).next[c]) {
10             // For online multiple strings.
11             int q = at(p).next[c];
12             if (at(p).len + 1 == at(q).len) {
13                 return q;
14             }
15             int clone = size();
16             push_back(at(q));
17             back().len = at(p).len + 1;
18             while (~p and at(p).next[c] == q) {
19                 at(p).next[c] = clone;
20                 p = at(p).link;
21             }
22             at(q).link = clone;
23             return clone;
24         }
25         int cur = size();
26         emplace_back();
27         back().len = at(p).len + 1;
28         while (~p and at(p).next[c] == -1) {
29             at(p).next[c] = cur;
30             p = at(p).link;
31         }
32         if (~p) {
33             int q = at(p).next[c];
34             if (at(p).len + 1 == at(q).len) {
35                 back().link = q;

```

```

36     } else {
37         int clone = size();
38         push_back(at(q));
39         back().len = at(p).len + 1;
40         while (~p and at(p).next[c] == q) {
41             at(p).next[c] = clone;
42             p = at(p).link;
43         }
44         at(q).link = at(cur).link = clone;
45     }
46 } else {
47     back().link = 0;
48 }
49 return cur;
50 }
51 };

```

4.8 Palindromic Tree

```

1 struct Node {
2     int sum, len, link;
3     array<int, sigma> next;
4     Node(int len) : len(len) {
5         sum = link = 0;
6         next.fill(0);
7     }
8 };
9 struct PalindromicTree : vector<Node> {
10     int last;
11     vector<int> s;
12     PalindromicTree() : last(0) {
13         emplace_back(0);
14         emplace_back(-1);
15         at(0).link = 1;
16     }
17     int get_link(int u, int i) {
18         while (i < at(u).len + 1 or s[i - at(u).len - 1] != s[i])
19             u = at(u).link;
20         return u;
21     }
22     void extend(int i) {
23         int cur = get_link(last, i);
24         if (not at(cur).next[s[i]]) {
25             int now = size();
26             emplace_back(at(cur).len + 2);
27             back().link = at(get_link(at(cur).link, i)).next[s[i]];
28             back().sum = at(back().link).sum + 1;
29             at(cur).next[s[i]] = now;
30         }
31         last = at(cur).next[s[i]];
32     }
33 };

```

5 Number Theory

5.1 Gaussian Integer

```

1 i64 div_floor(i64 x, i64 y) {
2     return x / y - (x % y < 0);
3 }
4 i64 div_ceil(i64 x, i64 y) {
5     return x / y + (x % y > 0);
6 }
7 i64 div_round(i64 x, i64 y) {
8     return div_floor(2 * x + y, 2 * y);
9 }
10 struct Gauss {
11     i64 x, y;
12     i64 norm() { return x * x + y * y; }
13     bool operator!=(i64 r) { return y or x != r; }
14     Gauss operator~() { return {x, -y}; }
15     Gauss operator-(Gauss rhs) { return {x - rhs.x, y - rhs.y}; }
16     Gauss operator*(Gauss rhs) { return {x * rhs.x - y * rhs.y, x * rhs.y + y *
17         rhs.x}; }
18     Gauss operator/(Gauss rhs) {
19         auto [x, y] = operator*(~rhs);
20         return {div_round(x, rhs.norm()), div_round(y, rhs.norm())};
21     }
22     Gauss operator%(Gauss rhs) { return operator-(rhs*(operator/(rhs))); }
23 };

```

5.2 Modular Arithmetic

5.2.1 Sqrt

Find x such that $x^2 \equiv y \pmod{p}$.
Constraints: p is prime and $0 \leq y < p$.

```

1 i64 sqrt(i64 y, i64 p) {
2     static mt19937_64 mt;
3     if (y <= 1) {
4         return y;
5     };
6     if (power(y, (p - 1) / 2, p) != 1) {
7         return -1;
8     }
9     uniform_int_distribution uid(i64(0), p - 1);
10    i64 x, w;
11    do {
12        x = uid(mt);
13        w = (x * x + p - y) % p;
14    } while (power(w, (p - 1) / 2, p) == 1);
15    auto mul = [&](pair<i64, i64> a, pair<i64, i64> b) { return pair((a.first *
        b.first + a.second * b.second % p * w) % p, (a.first * b.second + a.
        second * b.first) % p); };

```

```

16 pair<i64, i64> a = {x, 1}, res = {1, 0};
17 for (i64 r = (p + 1) >> 1; r; r >>= 1, a = mul(a, a)) {
18     if (r & 1) {
19         res = mul(res, a);
20     }
21 }
22 return res.first;
23 }

```

5.2.2 Logarithm

Find k such that $x^k \equiv y \pmod{n}$.
Constraints: $0 \leq x, y < n$.

```

1 i64 log(i64 x, i64 y, i64 n) {
2     if (y == 1 or n == 1) {
3         return 0;
4     }
5     if (not x) {
6         return y ? -1 : 1;
7     }
8     i64 res = 0, k = 1 % n;
9     for (i64 d; k != y and (d = gcd(x, n)) != 1; res += 1) {
10         if (y % d) {
11             return -1;
12         }
13         n /= d;
14         y /= d;
15         k = k * (x / d) % n;
16     }
17     if (k == y) {
18         return res;
19     }
20     unordered_map<i64, i64> mp;
21     i64 px = 1, m = sqrt(n) + 1;
22     for (int i = 0; i < m; i += 1, px = px * x % n) {
23         mp[y * px % n] = i;
24     }
25     i64 ppx = k * px % n;
26     for (int i = 1; i <= m; i += 1, ppx = ppx * px % n) {
27         if (mp.count(ppx)) {
28             return res + i * m - mp[ppx];
29         }
30     }
31     return -1;
32 }

```

5.3 Chinese Remainder Theorem

```

1 tuple<i64, i64, i64> exgcd(i64 a, i64 b) {
2     i64 x = 1, y = 0, x1 = 0, y1 = 1;

```

```

3     while (b) {
4         i64 q = a / b;
5         tie(x, x1) = pair(x1, x - q * x1);
6         tie(y, y1) = pair(y1, y - q * y1);
7         tie(a, b) = pair(b, a - q * b);
8     }
9     return {a, x, y};
10 }
11 optional<pair<i64, i64>> linear_equations(i64 a0, i64 b0, i64 a1, i64 b1) {
12     auto [d, x, y] = exgcd(a0, a1);
13     if ((b1 - b0) % d) {
14         return {};
15     }
16     i64 a = a0 / d * a1, b = (i128)(b1 - b0) / d * x % (a1 / d);
17     if (b < 0) {
18         b += a1 / d;
19     }
20     b = (i128)(a0 * b + b0) % a;
21     if (b < 0) {
22         b += a;
23     }
24     return {{a, b}};
25 }

```

5.4 Miller Rabin

```

1 bool miller_rabin(i64 n) {
2     static constexpr array<int, 9> p = {2, 3, 5, 7, 11, 13, 17, 19, 23};
3     if (n == 1) {
4         return false;
5     }
6     if (n == 2) {
7         return true;
8     }
9     if (not(n % 2)) {
10         return false;
11     }
12     int r = countr_zero(u64(n - 1));
13     i64 d = (n - 1) >> r;
14     for (int pi : p) {
15         if (pi >= n) {
16             break;
17         }
18         i64 x = power(pi, d, n);
19         if (x == 1 or x == n - 1) {
20             continue;
21         };
22         for (int j = 1; j < r; j += 1) {
23             x = (i128)x * x % n;
24             if (x == n - 1) {
25                 break;
26             }

```

```

27     }
28     if (x != n - 1) {
29         return false;
30     }
31 }
32 return true;
33 };

```

5.5 Pollard Rho

```

1 vector<i64> pollard_rho(i64 n) {
2     static mt19937_64 mt;
3     uniform_int_distribution uid(i64(0), n);
4     if (n == 1) {
5         return {};
6     }
7     vector<i64> res;
8     function<void(i64)> rho = [&](i64 n) {
9         if (miller_rabin(n)) {
10             return res.push_back(n);
11         }
12         i64 d = n;
13         while (d == n) {
14             d = 1;
15             for (i64 k = 1, y = 0, x = 0, s = 1, c = uid(mt); d == 1; k <= 1, y =
16                 x, s = 1) {
17                 for (int i = 1; i <= k; i += 1) {
18                     x = ((i128)x * x + c) % n;
19                     s = (i128)s * abs(x - y) % n;
20                     if (not(i % 127) or i == k) {
21                         d = gcd(s, n);
22                         if (d != 1) {
23                             break;
24                         }
25                     }
26                 }
27             }
28             rho(d);
29             rho(n / d);
30         };
31         rho(n);
32         return res;
33     }

```

5.6 Primitive Root

Constraints: $n = 2, 4, p^k, 2p^k$ where p is odd prime.

```

1 i64 phi(i64 n) {
2     auto pd = pollard_rho(n);

```

```

3     ranges::sort(pd);
4     pd.erase(ranges::unique(pd).begin(), pd.end());
5     for (i64 pi : pd) {
6         n = n / pi * (pi - 1);
7     }
8     return n;
9 }
10 i64 minimum_primitive_root(i64 n) {
11     i64 pn = phi(n);
12     auto pd = pollard_rho(pn);
13     ranges::sort(pd);
14     pd.erase(ranges::unique(pd).begin(), pd.end());
15     auto check = [&](i64 r) {
16         if (gcd(r, n) != 1) {
17             return false;
18         }
19         for (i64 pi : pd) {
20             if (power(r, pn / pi, n) == 1) {
21                 return false;
22             }
23         }
24         return true;
25     };
26     i64 r = 1;
27     while (not check(r)) {
28         r += 1;
29     }
30     return r;
31 }

```

5.7 Sum of Floor

Returns $\sum_{i=0}^{n-1} \lfloor \frac{ai+b}{m} \rfloor$.

```

1 u64 sum_of_floor(u64 n, u64 m, u64 a, u64 b) {
2     u64 ans = 0;
3     while (n) {
4         ans += a / m * n * (n - 1) / 2;
5         a %= m;
6         ans += b / m * n;
7         b %= m;
8         u64 y = a * n + b;
9         if (y < m) {
10             break;
11         }
12         tie(n, m, a, b) = tuple(y / m, a, m, y % m);
13     }
14     return ans;
15 }

```

5.8 Minimum of Remainder

Returns $\min\{(ai + b) \bmod m : 0 \leq i < n\}$.

```
1 u64 min_of_mod(u64 n, u64 m, u64 a, u64 b, u64 c = 1, u64 p = 1, u64 q = 1) {
2     if (a == 0) {
3         return b;
4     }
5     if (c % 2) {
6         if (b >= a) {
7             u64 t = (m - b + a - 1) / a;
8             u64 d = (t - 1) * p + q;
9             if (n <= d) {
10                return b;
11            }
12            n -= d;
13            b += a * t - m;
14        }
15        b = a - 1 - b;
16    } else {
17        if (b < m - a) {
18            u64 t = (m - b - 1) / a;
19            u64 d = t * p;
20            if (n <= d) {
21                return (n - 1) / p * a + b;
22            }
23            n -= d;
24            b += a * t;
25        }
26        b = m - 1 - b;
27    }
28    u64 d = m / a;
29    u64 res = min_of_mod(n, a, m % a, b, c += 1, (d - 1) * p + q, d * p + q);
30    return c % 2 ? m - 1 - res : a - 1 - res;
31 }
```

5.9 Stern Brocot Tree

```
1 struct Node {
2     int a, b;
3     vector<pair<int, char>> p;
4     Node(int a, int b) : a(a), b(b) {
5         // gcd(a, b) == 1
6         while (a != 1 or b != 1) {
7             if (a > b) {
8                 int k = (a - 1) / b;
9                 p.emplace_back(k, 'R');
10                a -= k * b;
11            } else {
12                int k = (b - 1) / a;
13                p.emplace_back(k, 'L');
14                b -= k * a;
```

```
15        }
16    }
17 }
18 Node(vector<pair<int, char>> p, int _a = 1, int _b = 1) : p(p), a(_a), b(_b)
19 {
20     for (auto [c, d] : p | views::reverse) {
21         if (d == 'R') {
22             a += c * b;
23         } else {
24             b += c * a;
25         }
26     }
27 };
```

5.10 Nim Product

```
1 struct NimProduct {
2     array<array<u64, 64>, 64> mem;
3     NimProduct() {
4         for (int i = 0; i < 64; i += 1) {
5             for (int j = 0; j < 64; j += 1) {
6                 int k = i & j;
7                 if (k == 0) {
8                     mem[i][j] = u64(1) << (i | j);
9                 } else {
10                    int x = k & -k;
11                    mem[i][j] = mem[i ^ x][j] ^ mem[(i ^ x) | (x - 1)][(j ^ x) | (i & (x - 1))];
12                }
13            }
14        }
15    }
16    u64 nim_product(u64 x, u64 y) {
17        u64 res = 0;
18        for (int i = 0; i < 64 and x >> i; i += 1) {
19            if ((x >> i) % 2) {
20                for (int j = 0; j < 64 and y >> j; j += 1) {
21                    if ((y >> j) % 2) {
22                        res ^= mem[i][j];
23                    }
24                }
25            }
26        }
27        return res;
28    }
29 };
```

6 Numerical

6.1 Golden Search

```
1 template <int step>
2 f64 golden_search(function<f64(f64)> f, f64 l, f64 r) {
3     f64 ml = (numbers::phi - 1) * l + (2 - numbers::phi) * r;
4     f64 mr = l + r - ml;
5     f64 fml = f(ml), fmr = f(mr);
6     for (int i = 0; i < step; i += 1)
7         if (fml > fmr) {
8             l = ml;
9             ml = mr;
10            fml = fmr;
11            fmr = f(mr = (numbers::phi - 1) * r + (2 - numbers::phi) * l);
12        } else {
13            r = mr;
14            mr = ml;
15            fmr = fml;
16            fml = f(ml = (numbers::phi - 1) * l + (2 - numbers::phi) * r);
17        }
18    return midpoint(l, r);
19 }
```

6.2 Adaptive Simpson

```
1 f64 simpson(function<f64(f64)> f, f64 l, f64 r) {
2     return (r - l) * (f(l) + f(r) + 4 * f(midpoint(l, r))) / 6;
3 }
4 f64 adaptive_simpson(const function<f64(f64)>& f, f64 l, f64 r, f64 eps) {
5     f64 m = midpoint(l, r);
6     f64 s = simpson(f, l, r);
7     f64 sl = simpson(f, l, m);
8     f64 sr = simpson(f, m, r);
9     f64 d = sl + sr - s;
10    if (abs(d) < 15 * eps) {
11        return (sl + sr) + d / 15;
12    }
13    return adaptive_simpson(f, l, m, eps / 2) + adaptive_simpson(f, m, r, eps /
14    2);
15 }
```

6.3 Simplex

Returns maximum of cx s.t. $ax \leq b$ and $x \geq 0$.

```
1 struct Simplex {
2     int n, m;
3     f64 z;
4     vector<vector<f64>> a;
```

```
5     vector<f64> b, c;
6     vector<int> base;
7     Simplex(int n, int m) : n(n), m(m), a(m, vector<f64>(n)), b(m), c(n), base(
8         n + m), z(0) { iota(base.begin(), base.end(), 0); }
9     void pivot(int out, int in) {
10        swap(base[out + n], base[in]);
11        f64 f = 1 / a[out][in];
12        for (f64& aij : a[out]) {
13            aij *= f;
14        }
15        b[out] *= f;
16        a[out][in] = f;
17        for (int i = 0; i <= m; i += 1) {
18            if (i != out) {
19                auto& ai = i == m ? c : a[i];
20                f64& bi = i == m ? z : b[i];
21                f64 f = -ai[in];
22                if (f < -eps or f > eps) {
23                    for (int j = 0; j < n; j += 1) {
24                        ai[j] += a[out][j] * f;
25                    }
26                    ai[in] = a[out][in] * f;
27                    bi += b[out] * f;
28                }
29            }
30        }
31    }
32    bool feasible() {
33        while (true) {
34            int i = ranges::min_element(b) - b.begin();
35            if (b[i] > -eps) {
36                break;
37            }
38            int k = -1;
39            for (int j = 0; j < n; j += 1) {
40                if (a[i][j] < -eps and (k == -1 or base[j] > base[k])) {
41                    k = j;
42                }
43            }
44            if (k == -1) {
45                return false;
46            }
47            pivot(i, k);
48        }
49        return true;
50    }
51    bool bounded() {
52        while (true) {
53            int i = ranges::max_element(c) - c.begin();
54            if (c[i] < eps) {
55                break;
56            }
57            int k = -1;
```



```

57     for (int j = 0; j < m; j += 1) {
58         if (a[j][i] > eps) {
59             if (k == -1) {
60                 k = j;
61             } else {
62                 f64 d = b[j] * a[k][i] - b[k] * a[j][i];
63                 if (d < -eps or (d < eps and base[j] > base[k])) {
64                     k = j;
65                 }
66             }
67         }
68     }
69     if (k == -1) {
70         return false;
71     }
72     pivot(k, i);
73 }
74 return true;
75 }
76 vector<f64> x() const {
77     vector<f64> res(n);
78     for (int i = n; i < n + m; i += 1) {
79         if (base[i] < n) {
80             res[base[i]] = b[i - n];
81         }
82     }
83     return res;
84 }
85 };

```

6.4 Green's Theorem

$$\oint_C (Pdx + Qdy) = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dxdy.$$

6.5 Double Integral

$$\iint_D f(x, y) dxdy = \iint_D f(x(u, v), y(u, v)) \left| \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} - \frac{\partial x}{\partial v} \frac{\partial y}{\partial u} \right| dudv.$$

7 Convolution

7.1 Fast Fourier Transform on \mathbb{C}

```

1 void fft(vector<complex<f64>>& a, bool inverse) {
2     int n = a.size();
3     vector<int> r(n);
4     for (int i = 0; i < n; i += 1) {
5         r[i] = r[i / 2] / 2 | (i % 2 ? n / 2 : 0);
6     }
7     for (int i = 0; i < n; i += 1) {

```

```

8         if (i < r[i]) {
9             swap(a[i], a[r[i]]);
10        }
11    }
12    for (int m = 1; m < n; m *= 2) {
13        complex<f64> wn(exp((inverse ? 1.i : -1.i) * numbers::pi / (f64)m));
14        for (int i = 0; i < n; i += m * 2) {
15            complex<f64> w = 1;
16            for (int j = 0; j < m; j += 1, w = w * wn) {
17                auto &x = a[i + j + m], &y = a[i + j], t = w * x;
18                tie(x, y) = pair(y - t, y + t);
19            }
20        }
21    }
22    if (inverse) {
23        for (auto& ai : a) {
24            ai /= n;
25        }
26    }
27 }

```

7.2 Formal Power Series on \mathbb{F}_p

```

1 void fft(vector<i64>& a, bool inverse) {
2     int n = a.size();
3     vector<int> r(n);
4     for (int i = 0; i < n; i += 1) {
5         r[i] = r[i / 2] / 2 | (i % 2 ? n / 2 : 0);
6     }
7     for (int i = 0; i < n; i += 1) {
8         if (i < r[i]) {
9             swap(a[i], a[r[i]]);
10        }
11    }
12    for (int m = 1; m < n; m *= 2) {
13        i64 wn = power(inverse ? power(g, mod - 2) : g, (mod - 1) / m / 2);
14        for (int i = 0; i < n; i += m * 2) {
15            i64 w = 1;
16            for (int j = 0; j < m; j += 1, w = w * wn % mod) {
17                auto &x = a[i + j + m], &y = a[i + j], t = w * x % mod;
18                tie(x, y) = pair((y + mod - t) % mod, (y + t) % mod);
19            }
20        }
21    }
22    if (inverse) {
23        i64 inv = power(n, mod - 2);
24        for (auto& ai : a) {
25            ai = ai * inv % mod;
26        }
27    }
28 }

```

7.2.1 Newton's Method

$$h = g(f) \Leftrightarrow G(h) = f - g^{-1}(h) \equiv 0.$$

$$h = h_0 - \frac{G(h_0)}{G'(h_0)}.$$

7.2.2 Arithmetic

- For $f = pg + q$, $p^T = f^T g^T - 1$.
- For $h = \frac{1}{f}$, $h = h_0(2 - h_0 f)$.
- For $h = \sqrt{f}$, $h = \frac{1}{2}(h_0 + \frac{f}{h_0})$.
- For $h = \log f$, $h = \int \frac{df}{f}$.
- For $h = \exp f$, $h = h_0(1 + f - \log h_0)$.

7.2.3 Interpolation

$$g(x) = \prod_i (x - x_i)$$

$$f(x) = \sum_{i=0}^{n-1} y_i \left(\prod_{j \neq i} \frac{x - x_j}{x_i - x_j} \right).$$

$$f(x) = \sum_{i=0}^{n-1} \frac{y_i}{g'(x_i)} \prod_{j \neq i} (x - x_j).$$

7.2.4 Primes with root 3

$$469762049 = 7 \times 2^{26} + 1.$$

$$4179340454199820289 = 29 \times 2^{57} + 1.$$

7.3 Circular Transform

$$A_{ij} = w_k^{ij}, A_{ij}^{-1} = \frac{1}{k} w_k^{-ij}.$$

7.4 Truncated Transform

$$\sum_{j=0}^{n-1} \frac{i}{\prod_{k=0}^j m_k} \bmod n \quad \text{for } 0 \leq i < \prod_{j=0}^{n-1} m_k.$$

8 Geometry

8.1 Pick's Theorem

$$\text{Area} = \#\{\text{points inside}\} + \frac{1}{2} \#\{\text{points on the border}\} - 1.$$

8.2 2D Geometry

P: point, L: line, G: convex hull or polygon, C: Circle.

```
1 template <typename T>
2 T eps = 0;
3 template <>
4 f64 eps<f64> = 1e-9;
5 template <typename T>
6 int sign(T x) {
7     return x < -eps<T> ? -1 : x > eps<T>;
8 }
9 template <typename T>
10 struct P {
11     T x, y;
12     explicit P(T x = 0, T y = 0) : x(x), y(y) {}
13     P operator*(T k) { return P(x * k, y * k); }
14     P operator+(P p) { return P(x + p.x, y + p.y); }
15     P operator-(P p) { return P(x - p.x, y - p.y); }
16     P operator-() { return P(-x, -y); }
17     T len2() { return x * x + y * y; }
18     T cross(P p) { return x * p.y - y * p.x; }
19     T dot(P p) { return x * p.x + y * p.y; }
20     bool operator==(P p) { return sign(x - p.x) == 0 and sign(y - p.y) == 0; }
21     int arg() { return y < 0 or (y == 0 and x > 0) ? -1 : x or y; }
22     P rotate90() { return P(-y, x); }
23 };
24 template <typename T>
25 bool argument(P<T> lhs, P<T> rhs) {
26     if (lhs.arg() != rhs.arg()) {
27         return lhs.arg() < rhs.arg();
28     }
29     return lhs.cross(rhs) > 0;
30 }
31 template <typename T>
32 struct L {
33     P<T> a, b;
34     explicit L(P<T> a = {}, P<T> b = {}) : a(a), b(b) {}
35     P<T> v() { return b - a; }
36     bool contains(P<T> p) { return sign((p - a).cross(p - b)) == 0 and sign((p - a).dot(p - b)) <= 0; }
37     int left(P<T> p) { return sign(v().cross(p - a)); }
38     optional<pair<T, T>> intersection(L l) {
39         auto y = v().cross(l.v());
40         if (sign(y) == 0) {
41             return {};
42         }
43         auto x = (l.a - a).cross(l.v());
44         return y < 0 ? pair(-x, -y) : pair(x, y);
45     }
46 };
47 template <typename T>
48 struct G {
49     vector<P<T>> g;
```

```

50 explicit G(int n) : g(n) {}
51 explicit G(const vector<P<T>>& g) : g(g) {}
52 optional<int> winding(P<T> p) {
53     int n = g.size(), res = 0;
54     for (int i = 0; i < n; i += 1) {
55         auto a = g[i], b = g[(i + 1) % n];
56         L l(a, b);
57         if (l.contains(p)) {
58             return {};
59         }
60         if (sign(l.v().y) < 0 and l.left(p) >= 0) {
61             continue;
62         }
63         if (sign(l.v().y) == 0) {
64             continue;
65         }
66         if (sign(l.v().y) > 0 and l.left(p) <= 0) {
67             continue;
68         }
69         if (sign(a.y - p.y) < 0 and sign(b.y - p.y) >= 0) {
70             res += 1;
71         }
72         if (sign(a.y - p.y) >= 0 and sign(b.y - p.y) < 0) {
73             res -= 1;
74         }
75     }
76     return res;
77 }
78 G convex() {
79     ranges::sort(g, {}, [&](P<T> p) { return pair(p.x, p.y); });
80     vector<P<T>> h;
81     for (auto p : g) {
82         while (ssize(h) >= 2 and sign((h.back() - h.end()[-2]).cross(p - h.back
83             ())) <= 0) {
84             h.pop_back();
85         }
86         h.push_back(p);
87     }
88     int m = h.size();
89     for (auto p : g | views::reverse) {
90         while (ssize(h) > m and sign((h.back() - h.end()[-2]).cross(p - h.back
91             ())) <= 0) {
92             h.pop_back();
93         }
94         h.push_back(p);
95     }
96     return G(h);
97 }
98 // Following function are valid only for convex.
99 T diameter2() {
100     int n = g.size();
101     T res = 0;
102     for (int i = 0, j = 1; i < n; i += 1) {
103         auto a = g[i], b = g[(i + 1) % n];
104         while (sign((b - a).cross(g[(j + 1) % n] - g[j])) > 0) {
105             j = (j + 1) % n;
106         }
107         res = max(res, (a - g[j]).len2());
108         res = max(res, (a - g[j]).len2());
109     }
110     return res;
111 }
112 optional<bool> contains(P<T> p) {
113     if (g[0] == p) {
114         return {};
115     }
116     if (g.size() == 1) {
117         return false;
118     }
119     if (L(g[0], g[1]).contains(p)) {
120         return {};
121     }
122     if (L(g[0], g[1]).left(p) <= 0) {
123         return false;
124     }
125     if (L(g[0], g.back()).left(p) > 0) {
126         return false;
127     }
128     int i = *ranges::partition_point(views::iota(2, ssize(g)), [&](int i) {
129         return sign((p - g[0]).cross(g[i] - g[0])) <= 0; });
130     int s = L(g[i - 1], g[i]).left(p);
131     if (s == 0) {
132         return {};
133     }
134     return s > 0;
135 }
136 int most(const function<P<T>(P<T>>& f) {
137     int n = g.size();
138     auto check = [&](int i) { return sign(f(g[i]).cross(g[(i + 1) % n] - g[i
139         ])) >= 0; };
140     P<T> f0 = f(g[0]);
141     bool check0 = check(0);
142     if (not check0 and check(n - 1)) {
143         return 0;
144     }
145     return *ranges::partition_point(views::iota(0, n), [&](int i) -> bool {
146         if (i == 0) {
147             return true;
148         }
149         bool checki = check(i);
150         int t = sign(f0.cross(g[i] - g[0]));
151         if (i == 1 and checki == check0 and t == 0) {
152             return true;
153         }
154         return checki ^ (checki == check0 and t <= 0);
155     });

```

<pre> 152 }); 153 } 154 pair<int, int> tan(P<T> p) { 155 return {most([&](P<T> x) { return x - p; }), most([&](P<T> x) { return p - x; })}}; 156 } 157 pair<int, int> tan(L<T> l) { 158 return {most([&](P<T> _) { return l.v(); }), most([&](P<T> _) { return -l .v(); })}}; 159 } 160 }; 161 162 template <typename T> 163 vector<L<T>> half(vector<L<T>> ls, T bound) { 164 // Ranges: bound ^ 6 165 auto check = [](L<T> a, L<T> b, L<T> c) { 166 auto [x, y] = b.intersection(c).value(); 167 a = L(a.a * y, a.b * y); 168 return a.left(b.a * y + b.v() * x) < 0; 169 }; 170 ls.emplace_back(P(-bound, (T)0), P(-bound, -(T)1)); 171 ls.emplace_back(P((T)0, -bound), P((T)1, -bound)); 172 ls.emplace_back(P(bound, (T)0), P(bound, (T)1)); 173 ls.emplace_back(P((T)0, bound), P(-(T)1, bound)); 174 ranges::sort(ls, [&](L<T> lhs, L<T> rhs) { 175 if (sign(lhs.v().cross(rhs.v())) == 0 and sign(lhs.v().dot(rhs.v())) >= 0) { 176 return lhs.left(rhs.a) == -1; 177 } </pre>	<pre> 178 return argument(lhs.v(), rhs.v()); 179 }); 180 deque<L<T>> q; 181 for (int i = 0; i < ssize(ls); i += 1) { 182 if (i and sign(ls[i - 1].v().cross(ls[i].v())) == 0 and sign(ls[i - 1].v ().dot(ls[i].v())) == 1) { 183 continue; 184 } 185 while (q.size() > 1 and check(ls[i], q.back(), q.end()[-2])) { 186 q.pop_back(); 187 } 188 while (q.size() > 1 and check(ls[i], q[0], q[1])) { 189 q.pop_front(); 190 } 191 if (not q.empty() and sign(q.back().v().cross(ls[i].v())) <= 0) { 192 return {}; 193 } 194 q.push_back(ls[i]); 195 } 196 while (q.size() > 1 and check(q[0], q.back(), q.end()[-2])) { 197 q.pop_back(); 198 } 199 while (q.size() > 1 and check(q.back(), q[0], q[1])) { 200 q.pop_front(); 201 } 202 return vector<L<T>>(q.begin(), q.end()); 203 } </pre>
---	---