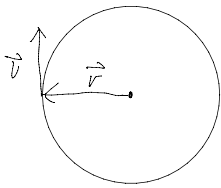


j = turn rate



$$I \quad \frac{360}{j} = T$$

$$II \quad 2\pi r/T = |\vec{v}| \cdot T$$

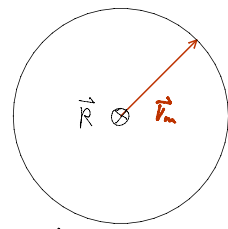
$$III \quad a_{\text{cent}} = \frac{|\vec{v}|^2}{|\vec{r}|}$$

$$\Rightarrow a_{\text{cent}} = \left(\frac{2\pi r/T}{360} \right)^2 \frac{1}{|\vec{r}|} \quad |\vec{v}|^2 = |\vec{r}|^2 \cdot j^2$$

$$= |\vec{r}| \left(\frac{2\pi}{360} \right)^2 \cdot j^2$$

Not in use!

Rate of rotation of \vec{r}_m in relation to \vec{a}_m :



rot axis = \vec{R}

\vec{v}_m

$$\vec{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}; q = (a+bi+cj+d\kappa)$$

$$I \quad \vec{a} = -|\vec{a}| \frac{\vec{r}}{|\vec{r}|} \Leftrightarrow \frac{\vec{a}}{|\vec{a}|} = -\frac{\vec{r}}{|\vec{r}|} \quad | \quad \vec{a} \text{ centripetal force}$$

$$II \quad |\vec{a}| = \frac{|\vec{v}|^2}{|\vec{r}|}$$

$$\Rightarrow |\vec{r}| = \frac{|\vec{v}|^2}{|\vec{a}|}$$

$$\Rightarrow |\vec{r}| \cdot \frac{\vec{r}}{|\vec{r}|} = \frac{|\vec{v}|^2}{|\vec{a}|} \cdot \frac{\vec{a}}{|\vec{a}|}$$

$$\Leftrightarrow \boxed{\vec{r} = - \frac{|\vec{v}|^2}{|\vec{a}|^2} \cdot \vec{a}} \quad 1*$$

$$III \quad \vec{v} = \vec{r} \times \vec{\omega}$$

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} v_y \omega_z - v_z \omega_y \\ v_z \omega_x - v_x \omega_z \\ v_x \omega_y - v_y \omega_x \end{pmatrix}$$

$$\begin{pmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \quad \text{under defined!!!}$$

$$\vec{v} = \vec{r} \times \vec{\omega} \quad | \times \vec{r}$$

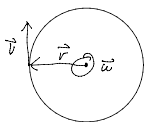
$$\Rightarrow \vec{v} \times \vec{r} = (\vec{r} \times \vec{\omega}) \times \vec{r} \quad | \quad (\vec{a} \times \vec{b}) \times \vec{c} = \vec{b}(\vec{c} \cdot \vec{a}) - \vec{a}(\vec{b} \cdot \vec{c})$$

$$\Rightarrow \vec{v} \times \vec{r} = \vec{\omega}(\vec{r} \cdot \vec{r}) - \vec{r}(\vec{\omega} \cdot \vec{r})$$

$$\Leftrightarrow \vec{v} \times \vec{r} = \vec{\omega} |\vec{r}|^2 - 0$$

$$\Leftrightarrow \vec{\omega} = \frac{\vec{v} \times \vec{r}}{|\vec{r}|^2}$$

Direct approach:



$$\frac{\vec{\omega}}{|\vec{\omega}|} = \frac{\vec{v} \times \vec{r}}{|\vec{v}| |\vec{r}|} \quad | \quad |\vec{v}| = |\vec{r}| \cdot |\vec{\omega}|$$

$$\Rightarrow \frac{\vec{\omega}}{|\vec{\omega}|} \cdot |\vec{\omega}| = \frac{\vec{v} \times \vec{r}}{|\vec{v}| |\vec{r}|} \cdot \frac{|\vec{r}|}{|\vec{r}|}$$

$$\Leftrightarrow \vec{\omega} = \frac{\vec{v} \times \vec{r}}{|\vec{r}|^2} \text{ g.e.d.}$$

$$1* \quad \vec{r} = - \frac{|\vec{v}|^2}{|\vec{a}|^2} \cdot \vec{a}$$

$$\Rightarrow \vec{\omega} = \vec{v} \times \vec{a} \cdot \frac{|\vec{v}|^2}{|\vec{a}|^2} \cdot \frac{1}{\left(\frac{|\vec{v}|^2}{|\vec{a}|^2} \cdot |\vec{a}| \right)^2}$$

$$\Rightarrow \vec{\omega} = \vec{v} \times \vec{a} \cdot \frac{|\vec{v}|^2}{|\vec{a}|^2} \cdot \frac{|\vec{a}|^3}{|\vec{v}|^4}$$

$$\Rightarrow \vec{\omega} = \vec{v} \times \vec{a} \cdot \frac{1}{|\vec{v}|^2} \quad 2*$$

APN

AUTOMATIC PROPORTIONAL NAVIGATION

$$\begin{aligned} \text{I } \vec{a} &= N(\vec{v}_v \times \vec{\Omega}) & N &\hat{=} \text{proportional const.} \\ \text{II } \vec{v}_r &= \vec{v}_t - \vec{v}_m & \vec{\Omega} &\hat{=} \text{rotation of line of sight} \\ \text{III } \vec{R} &= \vec{R}_t - \vec{R}_m & \vec{v}_t &\hat{=} \text{Velocity of target} \\ & & \vec{v}_m &\hat{=} \text{Velocity of missile} \\ \text{IV } \vec{\Omega} &= \frac{\vec{R} \times \vec{v}_r}{|\vec{R}|^2} & \vec{R}_t &\hat{=} \text{Position of target} \\ & & \vec{R}_m &\hat{=} \text{Position of missile} \end{aligned}$$

$$\Rightarrow \vec{a} = N(\vec{v}_t - \vec{v}_m) \times \left[(\vec{R}_t - \vec{R}_m) \times (\vec{v}_t - \vec{v}_m) \right] |\vec{R}_t - \vec{R}_m|^{-2}$$

$$2* \vec{\omega} = \vec{v} \times \vec{a} \cdot \frac{1}{|\vec{v}|^2}$$

$$\Rightarrow \vec{\omega} = \frac{1}{|\vec{v}|^2} \vec{v}_m \times \left[N(\vec{v}_t - \vec{v}_m) \times \left[(\vec{R}_t - \vec{R}_m) \times (\vec{v}_t - \vec{v}_m) \right] |\vec{R}_t - \vec{R}_m|^{-2} \right]$$