Assignment 7

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Outline

1. Question
2. Solution

Question

Show that if $y = x^2$,

$$f_y(y|x \geq 0) = \frac{U(y)}{1 - F_x(0)} \frac{f_x(\sqrt{y})}{2\sqrt{y}} \tag{1.0.1}$$

Solution

We know that,

$$f_y(y|x \geq 0) = \frac{d}{dy}(F_y(y|x \geq 0)) \quad (2.0.1)$$

We also know that,

$$F_y(y) = \Pr(\mathbf{y} \leq y) = \Pr\left(x^2 \leq y\right) \quad (2.0.2)$$

$$\implies F_y(y) = \Pr(-\sqrt{y} \leq x \leq \sqrt{y}) \quad (2.0.3)$$

$$\implies F_y(y|x \geq 0) = F_x(\sqrt{y}|x \geq 0) \text{ for y } > 0 \quad (2.0.4)$$

Solution

Hence,

$$F_y(y|x \geq 0) = \frac{\Pr\left(-\sqrt{y} \leq x \leq \sqrt{y}, x \geq 0\right)}{\Pr\left(x \geq 0\right)} \tag{2.0.5}$$

$$= \frac{\Pr\left(0 \leq x \leq \sqrt{y}\right)}{\Pr\left(x \geq 0\right)} \tag{2.0.6}$$

$$= \frac{F_x(\sqrt{y}) - F_x(0)}{1 - F_x(0)} \tag{2.0.7}$$

Solution

$$f_y(y|x \geq 0) = \frac{d}{dy}\left(\frac{F_x(\sqrt{y}) - F_x(0)}{1 - F_x(0)}\right) \tag{2.0.8}$$

$$= \frac{1}{1 - F_x(0)}\left(\frac{d}{dy}F_x(\sqrt{y}) - \frac{d}{dy}F_x(0)\right) \tag{2.0.9}$$

$$= \frac{1}{1 - F_x(0)}\left(\frac{1}{2\sqrt{y}}f_x(\sqrt{y}) - 0\right) \tag{2.0.10}$$

$$= \frac{1}{1 - F_x(0)}\frac{f_x(\sqrt{y})}{2\sqrt{y}} \tag{2.0.11}$$

Solution

Hence we proved that,

$$f_y(y|x \geq 0) = \frac{U(y)}{1 - F_x(0)} \frac{f_x(\sqrt{y})}{2\sqrt{y}} \tag{2.0.12}$$

where $U(y) = 1$ for $y \geq 0$