AI1110 Assignment 2

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Question 21a:
The cost function of a product is given by $C(x) = \frac{x^3}{3} - 45x^2 - 900x + 36$ where x is number of units produced. How many units should be produced to minimise the marginal cost?
Solution:
The marginal cost $MC = \frac{\Delta C}{\Delta Q}$ where,
ΔC is change in cost
ΔQ is change in quantity

TABLE I
TABLE CONSISTING SYMBOLS, FORMULAE, DESCRIPTION

Symbol	Formula	Description
x	-	No. of units
$C(x)$	$\frac{x^3}{3} - 45x^2 - 900x + 36$	Cost function
$MC or f(x)$	$\frac{d}{dx}(C(x))$	Marginal Cost
c	$f(c) = \min_x f(x)$	No. of units for min MC

Hence Marginal Cost MC is given by,

$$MC = \frac{d}{dx}(C(x)) \tag{1}$$
$$= \frac{d}{dx}\left(\frac{x^3}{3} - 45x^2 - 900x + 36\right) \tag{2}$$
$$= \frac{3x^2}{3} - 2 \times 45x - 900 + 0 \tag{3}$$
$$\implies MC = x^2 - 90x - 900 \tag{4}$$

Now let $MC = y = f(x)$

We need to find the x at where f(x) is minimum by using gradient descent method. Let us find $\nabla f(x)$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 - 90x - 900\right) \tag{5}$$
$$\implies f'(x) = 2x - 90 \tag{6}$$

We will be able to find the corresponding x value of the minimum of $f(x)$ by iterating the following equation till $(f'(x_{k-1}))$ approaches zero.

$$x_k = x_{k-1} - (\alpha \times f'(x_{k-1})) \tag{7}$$

where x_{k-1} is initial assumed value/ previous obtained value
x_k is updated assumed value
α represents the step size we are taking according to the slope $(f'(x_{k-1}))$
At first, let us randomly choose x_{k-1} as 0. Then, $f'(0) = -90$.
Since the slope is too far from zero and for manual purpose, we can take large step size.
Hence let us choose α as 0.25
Lets go through couple of iterations

$$x_k = 0 - 0.25 \times (-90)$$
$$= 22.5 \tag{8}$$
$$x_k = 22.5 - 0.25 \times (-45)$$
$$= 33.75 \tag{9}$$
$$x_k = 33.75 - 0.25 \times (-22.5)$$
$$= 39.375 \tag{10}$$
$$x_k = 39.375 - 0.25 \times (-11.25)$$
$$= 42.1875 \tag{11}$$
$$x_k = 42.1875 - 0.25 \times (-5.625)$$
$$= 43.59375 \tag{12}$$
$$x_k = 43.59375 - 0.25 \times (-2.8125)$$
$$= 44.296875 \tag{13}$$
$$x_k = 44.296875 - 0.25 \times (-1.40625)$$
$$= 44.6484375 \tag{14}$$
$$x_k = 44.6484375 - 0.25 \times (-0.703125)$$
$$= 44.82421875 \tag{15}$$
$$x_k = 44.82421875 - 0.25 \times (-0.3515625)$$
$$= 44.912109375 \tag{16}$$
$$x_k = 44.912109375 - 0.25 \times (-0.17578125)$$
$$= 44.9560546875 \tag{17}$$
$$x_k = 44.9560546875 - 0.25 \times (-0.087890625)$$
$$= 44.978027344 \tag{18}$$

Hence as the slope $f'(x_{k-1})$ is tending to zero, x_k is tending to 45. Hence the possible whole number at where the minimum of $f(x)$ exists is $c = 45$.

Note: For solving the minimum using gradient descent method with algorithms, we can iterate through a lot of times to obtain the more precise value and we can take small step size too.

Here is the corresponding graph obtained by using algorithm

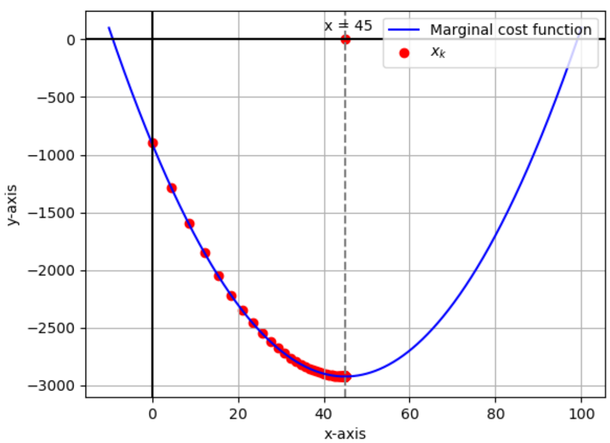

Fig. 0. Finding minimum of $f(x)$ by gradient descent method

Therefore the number of units that are to be produced to get minimum marginal cost is 45 units.