

# Assignment 10

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# Outline

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# Question

On observing 100 samples of  $x$ , find the 0.95 confidence interval of the median  $x_{0.5}$  of  $x$

# Concept

## **u Percentile:**

The  $u$  percentile of a random variable  $x$  is by definition a number  $x_u$  such that  $F(x_u) = u$ . Thus  $x_u$  is the inverse function  $F^{-1}(u)$  of the distribution  $F(x)$  of  $x$ .

Let  $y_k$  be the  $k^{th}$  number assuming that all the numbers are arranged in ascending order

$y_k < x_u < y_{k+r}$ , iff atleast  $k$  and atmost  $k + r - 1$  of the samples  $x_i$  are less than  $x_u$

## Concept

The event  $\{y_k < x_u < y_{k+r}\}$  occurs iff number of success of event  $\{x \leq x_u\}$  in  $n$  repetitions of the experiment is atleast  $k$  and atmost  $k + r - 1$ .

And since  $\Pr(x \leq x_u) = u$  which means success probability  $p = u$ , we can obtain the following

$$\Pr(y_k < x_u < y_{k+r}) = \sum_{m=k}^{k+r-1} \binom{n}{m} u^m (1-u)^{n-m} \quad (2.0.1)$$

## Concept

On approximation of above equation and assuming that  $X$  is a Normal or Gaussian Random Variable and  $n$  is large, for specific value of  $u$ , we obtain the following

$$\Pr(y_k < x_u < y_{k+r}) = G\left(\frac{k+r-0.5-nu}{\sqrt{nu(1-u)}}\right) - G\left(\frac{k-0.5-nu}{\sqrt{nu(1-u)}}\right) = \gamma \quad (2.0.2)$$

Where,  $G$  is a Gaussian distribution function

$$G(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy \quad (2.0.3)$$

## Concept

From these equations, for a specific  $\gamma$ ,  $r$  is minimum if  $nu$  is near the center of the interval  $(k, k + r)$ . This yields,

$$k \simeq nu - z_{1-\delta/2} \sqrt{nu(1-u)} \quad (2.0.4)$$

$$k + r \simeq nu - z_{1-\delta/2} \sqrt{nu(1-u)} \quad (2.0.5)$$

where  $z_u$  is the standard normal percentile with ,

$$u = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_u} e^{-z^2/2} dz \quad (2.0.6)$$

## Solution:

Since we are supposed to find the median of  $\mathbf{x}$  which is nothing but  $x_{0.5}$   
We have ,

$$n = 100 \quad (3.0.1)$$

$$u = 0.5 \quad (3.0.2)$$

$$\gamma = 1 - \delta = 0.95 \quad (3.0.3)$$

$$\implies \delta/2 = 0.025 \quad (3.0.4)$$

$$\implies z_{1-\delta/2} \simeq 2 \quad (3.0.5)$$



## Solution

Hence  $k$  and  $k+r$  are given by as follows,

$$k \simeq (100)(0.5) - 2(\sqrt{(100)(0.5)(1 - 0.5)}) \quad (3.0.6)$$

$$k = 40 \quad (3.0.7)$$

$$\text{Similarly, } k + r \simeq (100)(0.5) + 2(\sqrt{(100)(0.5)(1 - 0.5)}) \quad (3.0.8)$$

$$\implies k + r = 60 \quad (3.0.9)$$

Therefore the median of  $x$  is inbetween  $y_{40}$  and  $y_{60}$

That is the median is inbetween 40th and 60th number.