AI1110 Assignment 4

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Probability Exercise 16.3, 5
Question: A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of the numbers that turn up is

(i) 3
(ii) 12

Solution: Let us denote the outcomes of the event A of tossing the given coin by a random variable X such that $X \in \{0, 1\}$ and the outcomes of the event B of tossing the die by another random variable Y such that $Y \in \{0, 1, 2, 3, 4, 5\}$
The corresponding probabilities of each outcome

TABLE I
EVENT TABLE

Random Variable	Event
X = 0	Coin tossing 1
X = 1	Coin tossing 6
Y = 0	Die showing 1
Y = 1	Die showing 2
Y = 2	Die showing 3
Y = 3	Die showing 4
Y = 4	Die showing 5
Y = 5	Die showing 6

is clearly given as followed in the given table since both the die and coin are fair, all outcomes are equally probable in the corresponding event

(i) Sum of the outcomes on tossing the die and the coin is 3
Clearly this is possible only when coin shows up 1 and die shows up 2 i.e., X = 0 and Y = 1;
Since these two events are quiet independent events,

$$\Pr(AB) = \Pr(BA) \tag{1}$$
$$= \Pr(A)\Pr(B) \tag{2}$$

TABLE III
PROBABILITY VALUES

Probability	Value
Pr(X = 0)	1/2 = 0.5
Pr(X = 1)	1/2 = 0.5
Pr(Y = 0)	1/6 = 0.167
Pr(Y = 1)	1/6 = 0.167
Pr(Y = 2)	1/6 = 0.167
Pr(Y = 3)	1/6 = 0.167
Pr(Y = 4)	1/6 = 0.167
Pr(Y = 5)	1/6 = 0.167

Hence the Probability of the sum to be 3 is given by $\Pr(X = 0|Y = 2)$

$$\Pr(X_0Y_1) = \Pr(X_0)\Pr(Y_1) \tag{3}$$
$$\Pr(X_0Y_1) = \frac{1}{2} \times \frac{1}{6} \tag{4}$$
$$\Pr(X_0Y_1) = \frac{1}{12} \tag{5}$$
$$\Pr(X_0Y_1) = 0.0833 \tag{6}$$

Therefore the probability that the sum of the numbers that turn up is 3 = $\frac{1}{12}$ = 0.0833

(ii) Sum of the outcomes on tossing the die and the coin is 12
Similarly this is possible only when coin shows up 6 and die shows up 6 i.e., X = 1 and Y = 5;
Since these two events are quiet independent events,

$$\Pr(AB) = \Pr(BA) \tag{7}$$
$$= \Pr(A)\Pr(B) \tag{8}$$

Hence the Probability of the sum to be 12 is

given by $\Pr(X = 1|Y = 5)$

$$\Pr(X_1 Y_5) = \Pr(X_1)\Pr(Y_5) \tag{9}$$

$$\Pr(X_1 Y_5) = \frac{1}{2} \times \frac{1}{6} \tag{10}$$

$$\Pr(X_1 Y_5) = \frac{1}{12} \tag{11}$$

$$\Pr(X_1 Y_5) = 0.0833 \tag{12}$$

Therefore the probability that the sum of the numbers that turn up is 12 = $\frac{1}{12}$ = 0.0833