AI1110 Assignment 1

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Question 4a

Solve the following inequation, write down the solution set and represent it on the real number line:

$$-2 + 10x \leq 13x + 10 < 24 + 10x, x \in \mathbb{Z}$$

Solution

$$-2 + 10x \leq 13x + 10 < 24 + 10x, x \in \mathbb{Z}$$

Let us solve the above expression geometrically.

now consider each equation in this expression as a line equation

$y_1 = 10x - 2$

$y_2 = 13x + 10$

$y_3 = 10x + 24$

Clearly slopes of y_1 and y_3 are same i.e., $slope = 10$

and $y_1 \leq y_2 < y_3$, so the integral values of x on x-axis satisfying this inequality are the required solution set.

So we need to find the range of x at where the line y_2 lies between between line y_1 and the line y_3

We can obtain the x value at intersection point of y_1 and y_2 by equating them that is,

$$\begin{aligned} & y_1 = y_2 \\ \Rightarrow\ & 10x - 2 = 13x + 10 \\ \Rightarrow\ & -10 - 2 = 13x - 10x \\ \Rightarrow\ & -12 = 3x \\ \Rightarrow\ & x = -4 \end{aligned}$$

Similary we get the x value at intersection point of y_2 and y_3 by equating y_2 and y_3

$$\begin{aligned} & y_3 = y_2 \\ \Rightarrow\ & 10x + 24 = 13x + 10 \\ \Rightarrow\ & 24 - 10 = 13x - 10x \\ \Rightarrow\ & 14 = 3x \\ \Rightarrow\ & x = 4.67 \end{aligned}$$

Since $y_1 \leq y_2 < y_3$, this implies $-4 \leq x < 4.67$

Now let us draw the corresponding lines

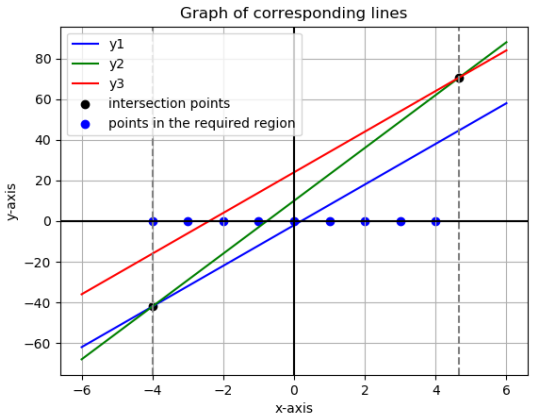

Figure 1: lines y_1, y_2 and y_3

If we observe this graph, it is clear that the lines y_1 and y_2 are intersecting at $x = -4$ and the lines y_2 and y_3 are intersecting at some point where $x > 4$

Hence the required range of x is $[4, 4.67)$

Therefore the integers in this range are,

$$\boxed{\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}}$$

Here is the plot of corresponding points on the real number line

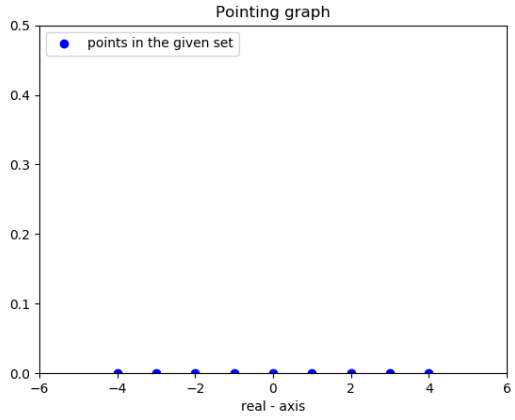

Figure 2: set of points that obey given expression on real number line