AI1110 Assignment 1

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Question 4a

Solve the following inequation, write down the solution set and represent it on the real number line:

$$-2 + 10x \leq 13x + 10 < 24 + 10x, x \in Z$$

Solution

$$-2 + 10x \leq 13x + 10 < 24 + 10x, x \in Z \tag{1}$$

Let us solve the above expression geometrically.

now consider each equation in this expression as a line,i.e., L_1 L_2 ans L_3

$$L_1 \equiv 10x - y - 2 \tag{2}$$
$$L_2 \equiv 13x - y + 10 \tag{3}$$
$$L_3 \equiv 10x - y + 24 \tag{4}$$

Clearly slopes of L_1 and L_3 are same i.e., $slope = 10$ and $L_1 \leq L_2 < L_3$, so the integral values of x on x-axis satisfying this inequality are the required solution set. In vector form,

$$L_1 \equiv \begin{pmatrix} 10 & -1 \end{pmatrix} \mathbf{x} = 2 \tag{5}$$
$$L_2 \equiv \begin{pmatrix} 13 & -1 \end{pmatrix} \mathbf{x} = -10 \tag{6}$$
$$L_3 \equiv \begin{pmatrix} 10 & -1 \end{pmatrix} \mathbf{x} = -24 \tag{7}$$

So we need to find the range of x at where the line L_2 lies between between line L_1 and the line L_3

We can obtain the intersection point of L_1 and L_2 by the following way,

$$\begin{pmatrix} 10 & -1 \\ 13 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ -10 \end{pmatrix} \tag{8}$$

The augmented matrix for the above matrix equation is

$$\left(\begin{array}{cc|c} 10 & -1 & 2 \\ 13 & -1 & -10 \end{array}\right) \tag{9}$$
$$\xleftrightarrow{R_2 \leftarrow 10R_2 - 13R_1} \left(\begin{array}{cc|c} 10 & -1 & 2 \\ 0 & 3 & -126 \end{array}\right) \tag{10}$$
$$\xleftrightarrow{R_1 \leftarrow 3R_1 + R_2} \left(\begin{array}{cc|c} 30 & 0 & -120 \\ 0 & 3 & -126 \end{array}\right) \tag{11}$$
$$\xleftrightarrow{R_1 \leftarrow R_1/30} \left(\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 3 & -126 \end{array}\right) \tag{12}$$
$$\xleftrightarrow{R_2 \leftarrow R_2/3} \left(\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & -42 \end{array}\right) \tag{13}$$
$$\implies \mathbf{x} = \begin{pmatrix} -4 \\ -42 \end{pmatrix} \tag{14}$$

Hence the point of intersection of lines L_1 and L_2 is $\begin{pmatrix} -4 \\ -42 \end{pmatrix}$.
Similarly we get the x value at intersection point of lines L_2 and L_3

$$\begin{pmatrix} 10 & -1 \\ 13 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -24 \\ -10 \end{pmatrix} \tag{15}$$

The augmented matrix for the above matrix equation is

$$\left(\begin{array}{cc|c} 10 & -1 & -24 \\ 13 & -1 & -10 \end{array}\right) \tag{16}$$
$$\xleftrightarrow{R_2 \leftarrow 10R_2 - 13R_1} \left(\begin{array}{cc|c} 10 & -1 & -24 \\ 0 & 3 & 212 \end{array}\right) \tag{17}$$
$$\xleftrightarrow{R_1 \leftarrow 3R_1 + R_2} \left(\begin{array}{cc|c} 30 & 0 & 140 \\ 0 & 3 & 212 \end{array}\right) \tag{18}$$
$$\xleftrightarrow{R_1 \leftarrow R_1/30} \left(\begin{array}{cc|c} 1 & 0 & 4.67 \\ 0 & 3 & 212 \end{array}\right) \tag{19}$$
$$\xleftrightarrow{R_2 \leftarrow R_2/3} \left(\begin{array}{cc|c} 1 & 0 & 4.67 \\ 0 & 1 & 70.67 \end{array}\right) \tag{20}$$
$$\implies \mathbf{x} = \begin{pmatrix} 4.67 \\ 70.67 \end{pmatrix} \tag{21}$$

Hence the point of intersection of lines L_2 and L_3 is $\begin{pmatrix} 4.67 \\ 70.67 \end{pmatrix}$.

Since $L_1 \leq L_2 < L_3$, this implies the corresponding x-coordinates follows, $-4 \leq x < 4.67$

Now let us draw the corresponding lines

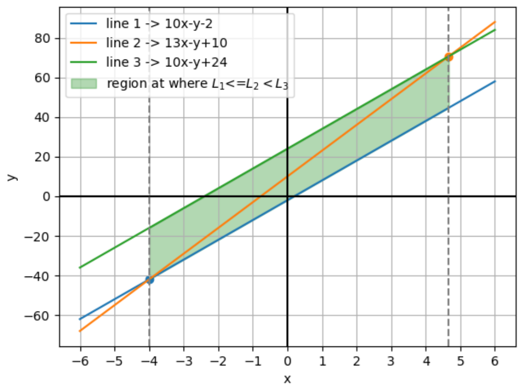

Figure 1: lines L_1, L_2 and L_3

If we observe this graph, it is clear that the lines L_1 and L_2 are intersecting at $x = -4$ and the lines L_2 and L_3 are intersecting at some point where $x > 4$

Hence the required range of x is $[\,-4, 4.67)$

Therefore the integers in this range are,

$$\boxed{\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}}$$

Here is the plot of corresponding points on the real number line

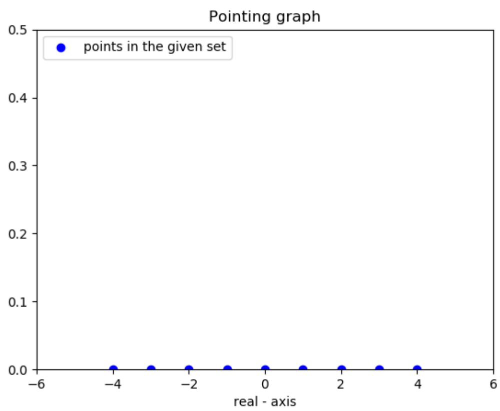

Figure 2: set of points that obey given expression on real number line