Random Numbers Assignmnet

Hema Sri Cheekatla, CS21BTECH11013

June 28, 2022

Question 1.1

Generate 10^6 samples of U using a C program and save into a file called uni.dat.

Solution:

Download the following files and execte the C program.

```
wget https://github.com/Hema−Sri−Ch/AI1110−
    Assignments/Assignment/codes/exrand.c
wget https://github.com/Hema−Sri−Ch/AI1110−
    Assignments/Assignment/codes/coeffs.h
```

And run the following commands in the terminal to execute the C program files

```
gcc −o out exrand.c coeffs.h −lm
./out
```

Then the corresponding "uni.dat" file will be created with 10^6 samples of U.

Question1.2

Load the uni.dat file into python and plot the emprical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \tag{1}$$

Solution

The following code plots Fig. 1 in the figs folder(Comment out or remove comments for some lines accordingly)

```
wget https://github.com/Hema−Sri−Ch/AI1110−
    Assignments/Assignment/codes/cdf_plot.py
```

And execute the following command in the terminal

```
python3 cdf_plot.py
```

Question 1.3

Find a theoretical expression for $F_U(x)$.

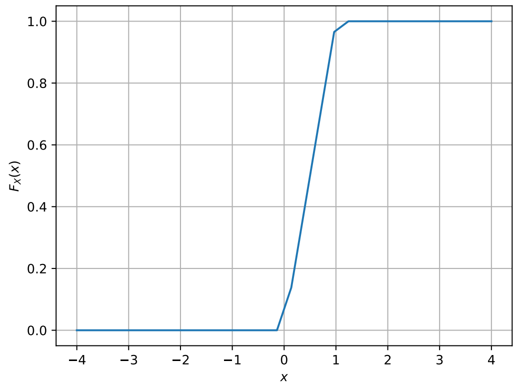

Figure 1: The CDF of U

Solution

We know that the PDF of a uniform distribution function in a particular intervel (a, b) is given by,

$$f(x) = \begin{cases} \frac{1}{b-a} & , \text{for } a \leq x \leq b \\ 0 & , otherwise \end{cases} \tag{2}$$

Hence, the CDF of a uniform distribution function is given as follows in a particular interval (a, b)

$$F_U(x) = \begin{cases} 0 & , \text{for } x < a \\ \frac{x-a}{b-a} & , \text{for } a \leq x \leq b \\ 1 & , \text{for } x > b \end{cases} \tag{3}$$

In Our case, the intervel is (0, 1). Hence the theoretical expression for $F_U(x)$ is given as follows,

$$F_U(x) = \begin{cases} 0 & , \text{for } x < 0 \\ x & , \text{for } 0 \leq x \leq 1 \\ 1 & , \text{for } x > 1 \end{cases} \tag{4}$$

Question 1.4

The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{5}$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{6}$$

Write a C program to find the mean and the variance of U

Solution

Download the following files and execte the C program.

```
wget https://github.com/Hema−Sri−Ch/AI1110−
      Assignments/Assignment/codes/exrand.c
wget https://github.com/Hema−Sri−Ch/AI1110−
      Assignments/Assignment/codes/coeffs.h
```

And run the following commands in the terminal to execute the C program files

```
gcc −o out exrand.c coeffs.h −lm
./out
```

The Mean and Variance of U is written as output in the terminal.

Question 1.5

Verify your result theoretically given that

$$E[U^k] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{7}$$

Solution

If k = 1, then E[U^1] is nothing but the mean of this uniform distribution,
Theoretically, we have

$$F_U(x) = \begin{cases} 0 & , \text{for } x < 0 \\ x & , \text{for } 0 \le x \le 1 \\ 1 & , \text{for } x > 1 \end{cases} \tag{8}$$

Hence $dF_U(x)$ can be written as follows,

$$dF_U(x) = f(x) = \begin{cases} 1 & , \text{for } 0 \le x \le 1 \\ 0 & , otherwise \end{cases} \tag{9}$$

$$\implies E[U] = \int_0^1 x\,dx \tag{10}$$

$$= 0.5 \tag{11}$$

Hence our theoretical mean is 0.5, where as we got our practical mean as 0.500007, which is almost same.
Hence our expression is verified for k = 1.
Similarly we can consider k = 2, then we get,

$$E[U^2] = \int_0^1 x^2\,dx \tag{12}$$

$$= 0.333333 \tag{13}$$

Hence our theoretical value of $E[U^2]$ is 0.333333, where we got our practical value of $E[U^2]$ as 0.333308, which is again almost same as that of theoretical value. Hence Verified.

Question 2.1

Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{14}$$

usign a C program, where $U_i, i = 1, 2, ..., 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat

Solution

Download the following files and execte the C program.

```
wget https://github.com/Hema−Sri−Ch/AI1110−
      Assignments/Assignment/codes/exrand.c
wget https://github.com/Hema−Sri−Ch/AI1110−
      Assignments/Assignment/codes/coeffs.h
```

And run the following commands in the terminal to execute the C program files

```
gcc −o out exrand.c coeffs.h −lm
./out
```

Then the corresponding ”gau.dat” file will be created with 10^6 samples of X.

Question 2.2

Load gau.dat in python and plot the emprical CDF of X using the samples in gau.dat. What properties does a CDF have?

Solution

The following code plots Fig. 2 in the figs folder(Comment out or remove comments for some lines accordingly)

```
wget https://github.com/Hema−Sri−Ch/AI1110−
      Assignments/Assignment/codes/cdf_plot.py
```

And execute the following command in the terminal

```
python3 cdf_plot.py
```

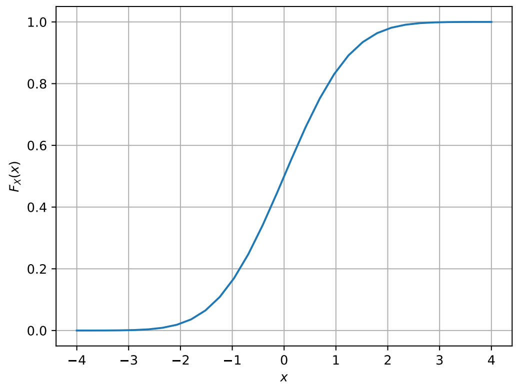

Figure 2: The CDF of X

Question 2.3

Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{15}$$

What properties does the PDF have?

Solution

The following code plots Fig. 3 in the figs folder(Comment out or remove comments for some lines accordingly)

```
wget https://github.com/Hema−Sri−Ch/AI1110−
     Assignments/Assignment/codes/pdf_plot.py
```

And execute the following command in the terminal

```
python3 pdf_plot.py
```

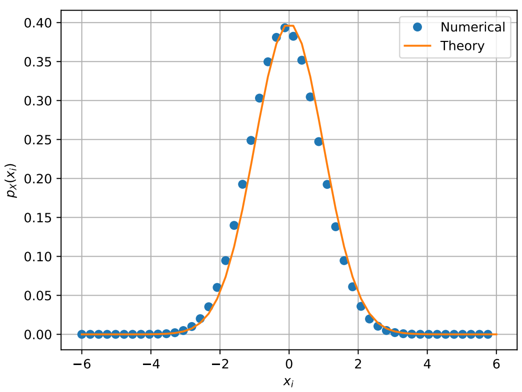

Figure 3: The PDF of X

Question 2.4

Find the mean and variance of X by writing a C program

Solution

Download the following files and execte the C program.

```
wget https://github.com/Hema−Sri−Ch/AI1110−
     Assignments/Assignment/codes/exrand.c
wget https://github.com/Hema−Sri−Ch/AI1110−
     Assignments/Assignment/codes/coeffs.h
```

And run the following commands in the terminal to execute the C program files

```
gcc −o out exrand.c coeffs.h −lm
./out
```

The Mean and Variance of X is written as output in the terminal.

Question 2.5

Given that,

$$p_X(x) = \frac{1}{\sqrt{2\pi}} exp(-\frac{x^2}{2}) \quad , -\infty < x < \infty \tag{16}$$

Find the mean and variance of this Gaussian distribution theoretically

Solution

We know that,

$$Mean = E[U] = \int_{-\infty}^{\infty} x p_X(x)\, dx \tag{17}$$

$$= \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}} exp(-\frac{x^2}{2})\, dx \tag{18}$$

Since the function $x\frac{1}{\sqrt{2\pi}}exp(-\frac{x^2}{2})$ is an odd function, its integral in the interval $(-\infty, \infty)$ is zero. Hence the Theoretical Mean is 0, whereas the practical Mean we have obtained is 0.000326 which is almost as same as that of Theoretical Mean

We know that,

$$Varience = E[U^2] - E[U]^2 \tag{19}$$
$$\implies Variance = E[U^2] \tag{20}$$
$$Variance = E[U^2] = \int_{-\infty}^{\infty} x^2 p_X(x)\,dx \tag{21}$$
$$= \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx \tag{22}$$
$$= \frac{2\sqrt{2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-x^2}\,dx \tag{23}$$
$$\text{since, } \int_{-\infty}^{\infty} x^2 e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2} \tag{24}$$
$$\implies Variance = 1 \tag{25}$$

Hence the Theoretical Variance is 1, whereas the practical Varaince we have obtained is 1.000907 which is almost as same as that of the Theoretical Varaince

Question 3.1

Generate samples of

$$V = -2\ln(1-U) \tag{26}$$

and plot its CDF.

Solution

Download the following files and execte the C program.

```
wget https://github.com/Hema−Sri−Ch/AI1110−
    Assignments/Assignment/codes/exrand.c
wget https://github.com/Hema−Sri−Ch/AI1110−
    Assignments/Assignment/codes/coeffs.h
```

And run the following commands in the terminal to execute the C program files

```
gcc −o out exrand.c coeffs.h −lm
./out
```

Then the corresponding "req.dat" file will be created with 10^6 samples of V. The following code plots Fig. 4 in the figs folder.(Comment out or remove comments for some lines accordingly)

```
wget https://github.com/Hema−Sri−Ch/AI1110−
    Assignments/Assignment/codes/cdf_plot.py
```

And execute the following command in the terminal

```
python3 cdf_plot.py
```

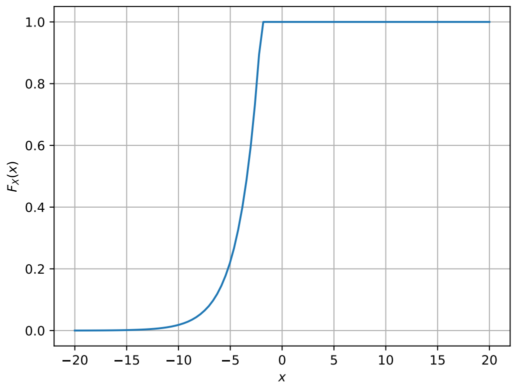

Figure 4: The CDF of V

Question 3.2

$$V = -2\ln(1-U) \tag{27}$$

Find a theoretical expression for $F_V(x)$.

Solution

We know that,

$$F_V(x) = P(V \le x) \tag{28}$$
$$F_V(x) = P(-2\ln(1-U) \le x) \tag{29}$$
$$F_V(x) = P(\ln(1-U) \ge -\frac{x}{2}) \tag{30}$$
$$F_V(x) = P(1-U \ge e^{-\frac{x}{2}}) \tag{31}$$
$$F_V(x) = P(U \le 1-e^{-\frac{x}{2}}) \tag{32}$$
$$F_V(x) = F_U(1-e^{-\frac{x}{2}}) \tag{33}$$
$$\text{Hence, } F_V(x) = \begin{cases} 0 & ,x < a \\ 1-e^{-\frac{x}{2}} & ,a \le x \le b \\ 1 & ,x > b \end{cases} \tag{34}$$