Assignment 6

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Outline

Question

A train and a bus arrive at the station at random between 9 A.M. and 10 A.M. The train stops for 10 minutes and the bus for k minutes. Find k so that the probability that the bus and the train will meet equals 0.5.

Solution

Consider the following events,
A = {the train arrives in the interval 9 A.M and 10 A.M, i.e., 60 minutes}
B = {the bus arrives in the interval 9 A.M and 10 A.M, i.e., 60 minutes}
Let us denote the time of arrival of train by a variable x and the time of arrival of bus by another variable y,
$\implies (x, y) = \{$All possible outcomes of these combined events$\}$
And we know that train stops for 10 minutes whereas the bus stops for k minutes

Solution

Now let us solve this problem graphically, Let the x-axis of this graph represents the time of arrival of the train in the interval of 60 minutes and the y-axis represents the time of arrival of the bus in the same interval of 60 minutes

Now there are two cases in it,

i Train comes before the bus, i.e., $x < y$

ii Bus comes before the train, i.e., $y < x$

Solution

In the first case where $y > x$, for the bus and train to meet, the bus must come within 10 minutes from the time of arrival of train.

$$\implies y \leq x + 10, y > x \tag{2.0.1}$$

Similarly in the second case where $x > y$, for the train to meet bus, it must come within k minutes from the time of arrival of bus.

$$\implies x \leq y + k, x > y \tag{2.0.2}$$

Solution

Now using above conditions we will obtain the following graph

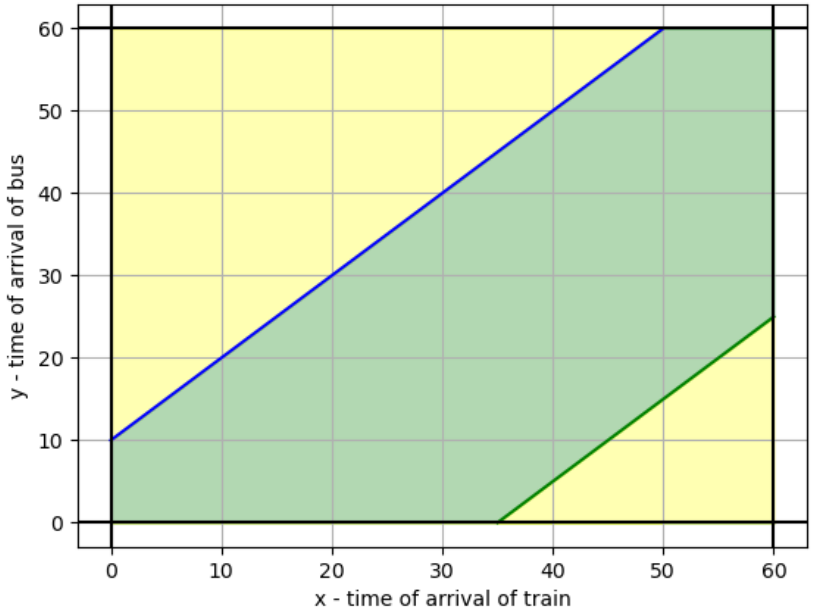

Figure: Graph between the train and bus timings

Solution

Now the points in the green coloured region are our favourable outcomes.
Hence the probablility of meeting of bus and train is given by

$$Probability = \frac{\text{Area of green region}}{\text{Area of the total region}} \tag{2.0.3}$$

We have Probability as 0.5
Area of the total region $= 60 \times 60 = 3600$

Solution

$$\text{Area of green region} = \text{Total Area - area of yellow region} \tag{2.0.4}$$

$$\text{Area of green region} = 3600 - \frac{1}{2}(60-10)^2 - \frac{1}{2}(60-k)^2 \tag{2.0.5}$$

$$= 3600 - 1250 - \frac{1}{2}(60-k)^2 \tag{2.0.6}$$

$$= 2350 - \frac{1}{2}(60-k)^2 \tag{2.0.7}$$

$$\implies \textit{Probability} = \frac{2350 - \frac{1}{2}(60-k)^2}{3600} \tag{2.0.8}$$

$$0.5 \times 3600 = 2350 - \frac{1}{2}(60-k)^2 \tag{2.0.9}$$

$$\implies k = 26.83 \tag{2.0.10}$$

Solution

Hence the bus must stop for arround 26.83 minutes to meet the train at probability of 0.5