

AI1110 Assignment 1

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Question 4a

Solve the following inequation, write down the solution set and represent it on the real number line:

$$-2 + 10x \leq 13x + 10 < 24 + 10x, x \in Z$$

Solution

$$-2 + 10x \leq 13x + 10 < 24 + 10x, x \in Z$$

Let us solve the above expression geometrically.

now consider each equation in this expression as a line, i.e., L_1 , L_2 and L_3

$$L_1 \equiv 10x - y - 2$$

$$L_2 \equiv 13x - y + 10$$

$$L_3 \equiv 10x - y + 24$$

Clearly slopes of L_1 and L_3 are same i.e., $\text{slope} = 10$ and $L_1 \leq L_2 < L_3$, so the integral values of x on x -axis satisfying this inequality are the required solution set. In vector form,

$$L_1 \equiv \begin{pmatrix} 10 \\ -1 \end{pmatrix} \mathbf{x} = 2$$

$$L_2 \equiv \begin{pmatrix} 13 \\ -1 \end{pmatrix} \mathbf{x} = -10$$

$$L_3 \equiv \begin{pmatrix} 10 \\ -1 \end{pmatrix} \mathbf{x} = -24$$

So we need to find the range of x at where the line L_2 lies between line L_1 and the line L_3

We can obtain the intersection point of L_1 and L_2 by the following way,

$$\begin{pmatrix} 10 & -1 \\ 13 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ -10 \end{pmatrix}$$

The augmented matrix for the above matrix equation is

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$$\begin{aligned} & \begin{pmatrix} 10 & -1 & | & 2 \\ 13 & -1 & | & -10 \end{pmatrix} \\ \xleftrightarrow{R_2 \leftarrow 10R_2 - 13R_1} & \begin{pmatrix} 10 & -1 & | & 2 \\ 0 & 3 & | & -126 \end{pmatrix} \\ \xleftrightarrow{R_1 \leftarrow 3R_1 + R_2} & \begin{pmatrix} 30 & 0 & | & -120 \\ 0 & 3 & | & -126 \end{pmatrix} \\ \xleftrightarrow{R_1 \leftarrow R_1/30} & \begin{pmatrix} 1 & 0 & | & -4 \\ 0 & 3 & | & -126 \end{pmatrix} \\ \xleftrightarrow{R_2 \leftarrow R_2/3} & \begin{pmatrix} 1 & 0 & | & -4 \\ 0 & 1 & | & -42 \end{pmatrix} \\ \implies & \mathbf{x} = \begin{pmatrix} -4 \\ -42 \end{pmatrix} \end{aligned}$$

Hence the point of intersection of lines L_1 and L_2 is $\begin{pmatrix} -4 \\ -42 \end{pmatrix}$.

Similarly we get the x value at intersection point of lines L_2 and L_3

$$\begin{pmatrix} 10 & -1 \\ 13 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -24 \\ -10 \end{pmatrix}$$

The augmented matrix for the above matrix equation is

$$\begin{aligned} & \begin{pmatrix} 10 & -1 & | & -24 \\ 13 & -1 & | & -10 \end{pmatrix} \\ \xleftrightarrow{R_2 \leftarrow 10R_2 - 13R_1} & \begin{pmatrix} 10 & -1 & | & -24 \\ 0 & 3 & | & 212 \end{pmatrix} \\ \xleftrightarrow{R_1 \leftarrow 3R_1 + R_2} & \begin{pmatrix} 30 & 0 & | & 140 \\ 0 & 3 & | & 212 \end{pmatrix} \\ \xleftrightarrow{R_1 \leftarrow R_1/30} & \begin{pmatrix} 1 & 0 & | & 4.67 \\ 0 & 3 & | & 212 \end{pmatrix} \\ \xleftrightarrow{R_2 \leftarrow R_2/3} & \begin{pmatrix} 1 & 0 & | & 4.67 \\ 0 & 1 & | & 70.67 \end{pmatrix} \\ \implies & \mathbf{x} = \begin{pmatrix} 4.67 \\ 70.67 \end{pmatrix} \end{aligned}$$

Hence the point of intersection of lines L_2 and L_3 is $\begin{pmatrix} 4.67 \\ 70.67 \end{pmatrix}$.

Since $L_1 \leq L_2 < L_3$, this implies the corresponding x -coordinates follows, $-4 \leq x < 4.67$

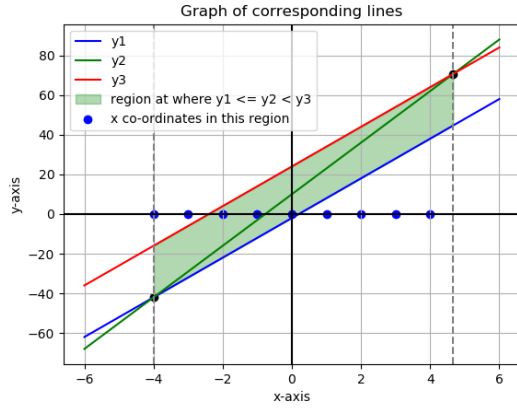


Figure 1: lines L_1 , L_2 and L_3

Now let us draw the corresponding lines

If we observe this graph, it is clear that the lines L_1 and L_2 are intersecting at $x = -4$ and the lines L_2 and L_3 are intersecting at some point where $x > 4$

Hence the required range of x is $[-4, 4.67)$

Therefore the integers in this range are,

$$\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

Here is the plot of corresponding points on the real number line

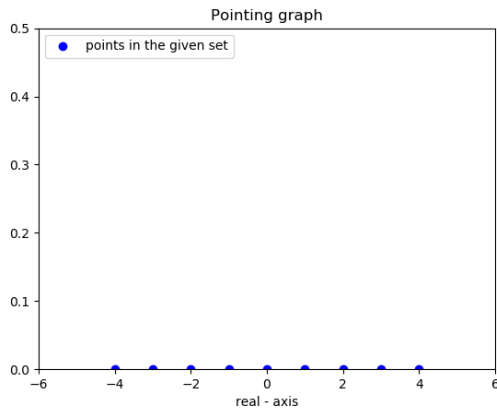


Figure 2: set of points that obey given expression on real number line