Assignment 9

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Outline

1. Question:
2. Solution:

Question

x, y are independent uniformly distributed random variables in (0, 1). Let

$$w = max(x, y) \tag{1.0.1}$$

$$z = min(x, y) \tag{1.0.2}$$

Find the p.d.f of

(a) $r = w - z$

(b) $s = w + z$

Solution(a)

$$R = W - Z \tag{2.0.1}$$
$$= max(X, Y) - min(X, Y) \tag{2.0.2}$$
$$= \begin{cases} X - Y & , X \geq Y \\ Y - X & , X < Y \end{cases} \tag{2.0.3}$$
$$\text{Hence, } F_R(r) = \Pr\left(R \leq r\right) \tag{2.0.4}$$
$$= \Pr\left(X - Y \leq r, X \geq Y\right) + \Pr\left(Y - X \leq r, X < Y\right) \tag{2.0.5}$$

Solution(a)

The corresponding regions are shaded as below, where we are given that x,y are in (0,1)

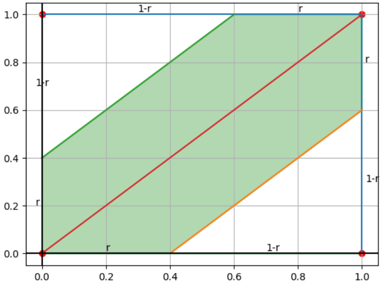

Figure: The possible region

Solution(a)

From the plot, The area is given by,

$$F_R(r) = 1 - 2\frac{(1-r)^2}{2}, 0 \leq r \leq 1 \tag{2.0.6}$$

$$= 1 - (1-r)^2 \tag{2.0.7}$$

Hence the p.d.f is written as follows,

$$p_R(r) = \frac{d}{dr} F_R(r) \tag{2.0.8}$$

$$p_R(r) = \frac{d}{dr}(1 - (1-r)^2) \tag{2.0.9}$$

$$p_R(r) = \begin{cases} 2(1-r) & , 0 \leq r \leq 1 \\ 0 & , otherwise \end{cases} \tag{2.0.10}$$

Solution(b)

$$S = W + Z \tag{2.0.11}$$

$$= max(X, Y) + min(X, Y) \tag{2.0.12}$$

$$= \begin{cases} X + Y & , X \geq Y \\ Y + X & , X < Y \end{cases} \tag{2.0.13}$$

$$S = X + Y, X, Y \in (0, 1) \tag{2.0.14}$$

$$\text{Hence, } F_S(s) = \Pr\left(S \leq s\right) \tag{2.0.15}$$

$$= \Pr\left(X + Y \leq s\right), 0 < s < 2 \tag{2.0.16}$$

$$\tag{2.0.17}$$

Solution(b)

For $0 < s < 1$, the range is given below,

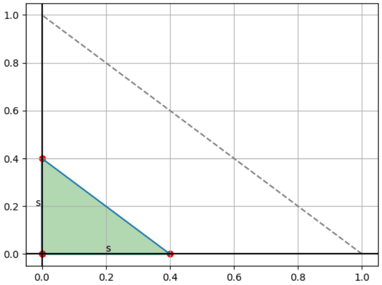

Figure: for $0 < s < 1$

$$F_S(s) = \frac{1}{2}s^2, 0 < s < 1 \tag{2.0.18}$$

Solution(b)

for $1 < s < 2$, the possible range is given as follows,

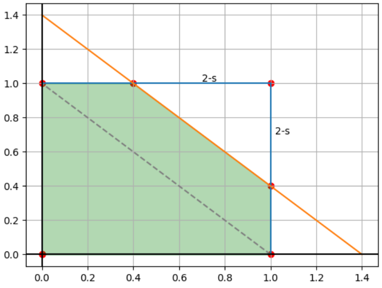

Figure: For $1 < s < 2$

$$\implies F_s(s) = 1 - \frac{1}{2}(2-s)^2, 1 < s < 2 \tag{2.0.19}$$

Solution(b)

$$F_S(s) = \begin{cases} \frac{s^2}{2} & ,0 < s < 1 \\ 1 - \frac{(2-s)^2}{2} & ,1 < s < 2 \end{cases} \tag{2.0.20}$$

Hence the p.d.f of S is given as follows,

$$p_S(s) = \frac{d}{ds} F_S(s) \tag{2.0.21}$$

$$= \begin{cases} \frac{d}{ds}(\frac{s^2}{2}) & ,0 < s < 1 \\ \frac{d}{ds}(1 - \frac{(2-s)^2}{2}) & ,1 < s < 2 \end{cases} \tag{2.0.22}$$

$$= \begin{cases} s & ,0 < s < 1 \\ 2 - s, & 1 < s < 2 \\ 0 & ,otherwise \end{cases} \tag{2.0.23}$$

Solution

pdf of R,

$$p_R(r) = \begin{cases} 2(1-r) & , 0 \le r \le 1 \\ 0 & , otherwise \end{cases} \tag{2.0.24}$$

pdf of S,

$$p_S(s) = \begin{cases} s & , 0 < s < 1 \\ 2 - s, & 1 < s < 2 \\ 0 & , otherwise \end{cases} \tag{2.0.25}$$