Probability Exercise 13.5, Q2

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Outline

Question

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

Solution

On throwing a pair of dice, getting doublet is success and we are throwing the pair of dice 4 times.

Doublets: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)

Consider one event of getting success represented by a Bernoulli random variable X as shown

Table: Events for X

Event	Discription
$X = 0$	no doublet
$X = 1$	Doublet

$$P_X(1) = \frac{1}{6} \tag{2.0.1}$$

$$P_X(0) = \frac{5}{6} \tag{2.0.2}$$

Now consider another event of throwing this pair of dice 4 times and represent the corresponding outcomes by the Binomial Random Variable Y as shown

Table: Events for Y

Event	**Discription**
$Y = 0$	Getting 0 doublets
$Y = 1$	Getting 1 doublets
$Y = 2$	Getting 2 doublets
$Y = 3$	Getting 3 doublets
$Y = 4$	Getting 4 doublets

The corresponding Probabilities is given by,

$$P_Y(k) = \begin{cases} {}^nC_k p^k q^{n-k}, & k = 1, 2, \ldots, n \\ 0, & otherwise \end{cases} \tag{2.0.3}$$

Where n is number of trails $= 4$
p $= P_X(1) = \frac{1}{6}$
q $= P_X(0) = \frac{5}{6}$

Hence the probability of getting success twice is given by,

$$P_Y(2) = {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{4-2} \tag{2.0.4}$$

$$P_Y(2) = \frac{25}{216} \tag{2.0.5}$$

Hence the required probability is $\frac{25}{216}$

Now the Probability Mass Function of Y is as follows

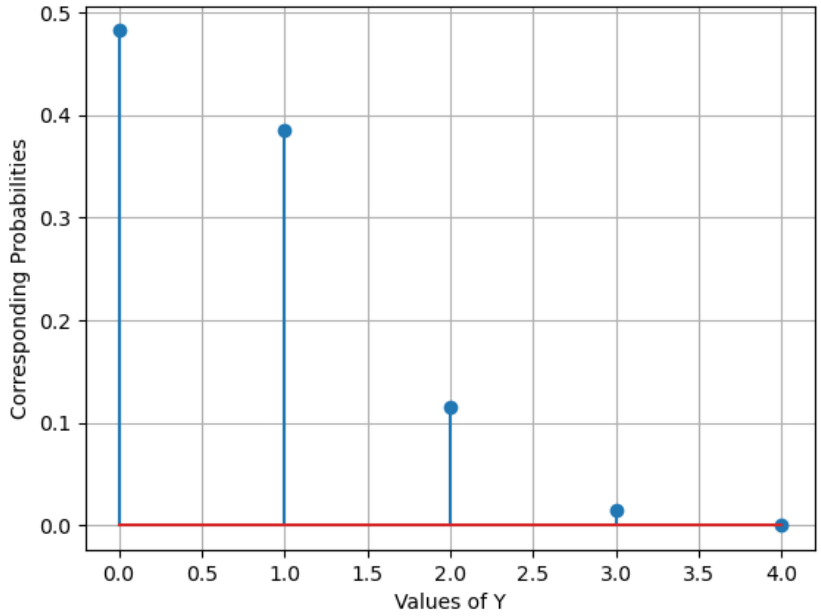

Figure: Probability Mass Function of Y