AI1110 Assignment 2

Hema Sri Cheekatla, CS21BTECH11013

Question 21a:
The cost function of a product is given by $C(x) = \frac{x^3}{3} - 45x^2 - 900x + 36$ where x is number of units produced. How many units should be produced to minimise the marginal cost?
Solution:
The marginal cost $MC = \frac{\Delta C}{\Delta Q}$ where,
ΔC is change in cost
ΔQ is change in quantity

TABLE I
TABLE CONSISTING SYMBOLS, FORMULAE, DESCRIPTION

Symbol	Formula	Description
x	-	No. of units
$C(x)$	$\frac{x^3}{3} - 45x^2 - 900x + 36$	Cost function
MC	$\frac{d}{dx}(C(x))$	Marginal Cost
c	$\frac{d}{dx}(C(x)) = 0$, at $x = c$	No. of units at min. MC

Hence Marginal Cost MC is given by,

$$MC = \frac{d}{dx}(C(x)) \tag{1}$$
$$= \frac{d}{dx}\left(\frac{x^3}{3} - 45x^2 - 900x + 36\right) \tag{2}$$
$$= \frac{3x^2}{3} - 2 \times 45x - 900 + 0 \tag{3}$$
$$\implies MC = x^2 - 90x - 900 \tag{4}$$

Now let $MC = y = f(x)$
We need to find the number of units to be produced such that the Marginal cost is minimum.This Marginal cost $(y = f(x))$ is minimum for some $x = c$, at where it obeys the following conditions

(i) $\frac{dy}{dx} = 0$ at x=c
(ii) $\frac{d^2y}{dx^2} > 0$ at x=c

Now let us consider $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 - 90x - 900\right) \tag{5}$$
$$\implies \frac{dy}{dx} = 2x - 90 = f'(x) \tag{6}$$

From first condition, at x=c, $\frac{dy}{dx} = 0$

$$i.e., f'(c) = 0 \tag{7}$$
$$2c - 90 = 0 \tag{8}$$
$$\implies c = 45 \tag{9}$$

Hence at x = 45, $\frac{dy}{dx} = 0$
Now let us consider the second condition at x=45

$$\frac{d^2y}{dx^2} > 0 \tag{10}$$
$$\frac{d}{dx}\left(\frac{dy}{dx}\right) > 0 \tag{11}$$
$$\frac{d}{dx}(2x - 90) > 0 \tag{12}$$
$$2 > 0 \tag{13}$$

Hence the second condition is also valid at x = 45
Therefore at $x = 45$, $y = f(x)$ is minimum, i.e., Marginal cost is minimum if we produce 45 units.

Hence the Number of units that are to be produced to minimise the marginal cost is 45 units.