AI1110 Assignment 2

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Question 21a:
The cost function of a product is given by $C(x) = \frac{x^3}{3} - 45x^2 - 900x + 36$ where x is number of units produced. How many units should be produced to minimise the marginal cost?
Solution:
The marginal cost $MC = \frac{\Delta C}{\Delta Q}$ where,
ΔC is change in cost
ΔQ is change in quantity

TABLE I
TABLE CONSISTING SYMBOLS, FORMULAE, DESCRIPTION

Symbol	Formula	Description
x	-	No. of units
$C(x)$	$\frac{x^3}{3} - 45x^2 - 900x + 36$	Cost function
$MC or f(x)$	$\frac{d}{dx}(C(x))$	Marginal Cost
c	$f(c) = \min_x f(x)$	No. of units for min MC

Hence Marginal Cost MC is given by,

$$MC = \frac{d}{dx}(C(x)) \tag{1}$$
$$= \frac{d}{dx}\left(\frac{x^3}{3} - 45x^2 - 900x + 36\right) \tag{2}$$
$$= \frac{3x^2}{3} - 2 \times 45x - 900 + 0 \tag{3}$$
$$\implies MC = x^2 - 90x - 900 \tag{4}$$

Now let $MC = y = f(x)$
We need to find the number of units to be produced (c) such that the Marginal cost is minimum. Since $x = c$ is the required number of units that are to be produced, it must be a whole number.

$$f(c) = \min_x \{f(x)\}, x \in \mathbf{W} \tag{5}$$
$$= \min_x \left\{x^2 - 90x - 900\right\} \tag{6}$$
$$= \min_x \left\{x^2 - 2(45)x + 2025 - 2925\right\} \tag{7}$$
$$= \min_x \left\{(x-45)^2 - 2925\right\}, x \in \mathbf{W} \tag{8}$$

But we know that,

$$(x-45)^2 \geq 0, \tag{9}$$
$$\implies (x-45)^2 - 2925 \geq -2925 \tag{10}$$
$$\implies \min\left\{(x-45)^2 - 2925\right\} = -2925 \tag{11}$$
$$\implies f(c) = -2925 \tag{12}$$

We obtain the minimum at where,

$$(x-45)^2 = 0 \tag{13}$$
$$\implies x = 45 \tag{14}$$

Since $45 \in \mathbf{W}$ and $f(45) = -2925$, $c = 45$.
Therefore the number of units that are to be produced to get the minimum value of Marginal cost is 45 units.

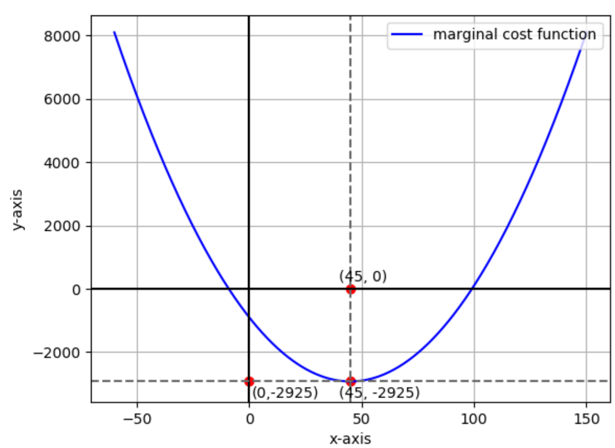

Fig. 0. Marginal cost function $MC = x^2 - 90x - 900$