Model Order Reduction Project

Heat Diffusion

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Problem 1

This system is non-linear and time-invariant.
According to description, this model is isotropic.
Therefor,

$$\rho(x,y)c(x,y)\frac{\partial T}{\partial t}(x,y,t) = \begin{bmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y}\end{bmatrix}\begin{bmatrix}\kappa(x,y) & 0 \\ 0 & \kappa(x,y)\end{bmatrix}\begin{bmatrix}\frac{\partial T}{\partial x}(x,y,t) \\ \frac{\partial T}{\partial y}(x,y,t)\end{bmatrix} + u(x,y,t) \tag{1}$$

i.e.

$$\rho(x,y)c(x,y)\frac{\partial T}{\partial t}(x,y,t) = (\frac{\partial \kappa}{\partial x}(x,y)\frac{\partial T}{\partial x}(x,y,t) + \frac{\partial \kappa}{\partial y}(x,y)\frac{\partial T}{\partial y}(x,y,t)) + u(x,y,t) \tag{2}$$

non-homogeneous

Non-linear

Firstly, just consider the left side of the equation(2),

$$\rho(x_1+x_2,y_1+y_2)c(x_1+x_2,y_1+y_2)\frac{\partial T}{\partial t}(x_1+x_2,y_1+y_2,t) \tag{3}$$

Since it's unknown whether ρ and c is linear(because the part of T is only associated with t, this part could be ignored), even though they are both linear,like equation (5), it is still non-linear.

$$\rho(x_1+x_2,y_1+y_2)c(x_1+x_2,y_1+y_2) = (\rho(x_1,y_1)+\rho(x_2,y_2))(c(x_1,y_1)+c(x_2,y_2)) \tag{4}$$

$$\rho(x_1,y_1)c(x_1,y_1)+\rho(x_1,y_1)c(x_2,y_2)+\rho(x_2,y_2)c(x_1,y_1)+\rho(x_2,y_2))c(x_2,y_2) \neq (\rho(x_1,y_1)c(x_1,y_1)+\rho(x_2,y_2))c(x_2,y_2)) \tag{5}$$

time-invariant

input delayed: $u_d(t) = u(t+\delta)$

$$\rho(x,y)c(x,y)\frac{\partial T}{\partial t}(x,y,t) - (\frac{\partial \kappa}{\partial x}(x,y)\frac{\partial T}{\partial x}(x,y,t) + \frac{\partial \kappa}{\partial y}(x,y)\frac{\partial T}{\partial y}(x,y,t)) = u(x,y,t+\delta) \tag{6}$$

output delayed:$T_d(t) = T(t+\delta)$

$$\rho(x,y)c(x,y)\frac{\partial T}{\partial t}(x,y,t+\delta) - (\frac{\partial \kappa}{\partial x}(x,y)\frac{\partial T}{\partial x}(x,y,t+\delta) + \frac{\partial \kappa}{\partial y}(x,y)\frac{\partial T}{\partial y}(x,y,t+\delta)) = u(x,y,t+\delta) \tag{7}$$

The right side of equation(6)and (7) is equal, so it is time-invariant.

homogeneous

Non-linear

When ρ and c is constant, consider the right side of equation(2),

$$\frac{\partial \kappa}{\partial x}(x_1+x_2, y_1+y_2)\frac{\partial T}{\partial x}(x_1+x_2, y_1+y_2, t)+\frac{\partial \kappa}{\partial y}(x_1+x_2, y_1+y_2)\frac{\partial T}{\partial y}(x_1+x_2, y_1+y_2, t)+u(x_1+x_2, y_1+y_2, t) \tag{8}$$

since κ is coupled with T, the first item

$$\frac{\partial \kappa}{\partial x}(x_1+x_2)\frac{\partial T}{\partial x}(x_1+x_2) \neq \frac{\partial \kappa}{\partial x}(x_1)\frac{\partial T}{\partial x}(x_1)+\frac{\partial \kappa}{\partial x}(x_2)\frac{\partial T}{\partial x}(x_2) \tag{9}$$

time-invariant

Because only u and T is associated with time that no matter the system is homogeneous or not, it is time-invariant. (Proof as equation(6) and (7))

Problem 2

Problem 3

Problem 4