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ScienceDirect

Procedia Computer Science 176 (2020) 330–338

Procedia Computer Science

www.elsevier.com/locate/procedia

24th International Conference on Knowledge-Based and Intelligent Information & Engineering Systems

Sentiment Analysis of Indian Stock Market Volatility

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Abstract

Traditional empirical models analyze impact of sentiments on financial market volatility using macroeconomic fundamentals or financial indicators. In this paper, recent methods of text based sentiment analysis of market from relevant news articles regarding economy and financial market are used. Two distinct market sentiments namely, positive and negative sentiments are constructed using different emotions which are identified through standard natural language processing methods. Further, the paper aims at proposing an augmented version of asymmetric GARCH model of conditional volatility for Indian stock exchange, Sensex, during the time period of April 19, 2007 to January 10, 2020 by incorporating aforementioned market sentiments. Empirical findings suggest dominant impact of negative market sentiment over positive one and it also provides evidence of noise trading in financially immature Indian stock market.

Keywords: risk modelling; stock market; NLP; sentiment analysis; GARCH;

Nomenclature

GARCH	Generalized Autoregressive Conditional Heteroscedasticity
NLP	Natural Language Processing
NLTK	Natural Language Toolkit
MA	Moving Average

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10.1016/j.procs.2020.08.035

AR	Autoregressive
ARMA	Autoregressive Moving Average

1. Introduction

Traditional financial modeling of market volatility using time series analysis has been experiencing a regime change in the domain and dominance of behavioral finance. Fundamental theories of finance like mean variance analysis of Markowitz (1952) posits the rational behaviour of investors and market fundamental is believed to be the sole factor that plays a decisive role in shaping the decision of such investors [1]. However recent literature in behavioural finance continues to challenge the notion of rational investor in the market since the impact of noise traders is found to dominate the market [2]. Unlike rational investors, noise traders impact market return and volatility due to their cognitive errors and emotional exuberance [3]. Transitory influence of noise traders were believed to be eradicated by rational investors of the market through the process of arbitrage but this argument of traditional financial theory is also challenged by some researchers [4]. The dynamic of trading in the financial market is observed to be linked with prevailing market sentiment [5]. Sentiment is conceptualized as the overall attitude of an investor towards the market or any specific stock and this is independent of market fundamentals [6]. Influence of such sentiment on market volatility has been gauged in literature using different proxies of market indices [7]. However, there is scarcity of literature in the context of Indian stock market's volatility using sentiment analysis. This paper attempts to frame volatility of Indian stock market using investor sentiment analysis. In this regard, we have blended traditional asymmetric GARCH models and proposed positive and negative sentiment factors which are generated from news articles by applying methods of natural language process (NLP) and NRC word-emotion association lexicon [8]. The paper departs from traditional approach of modeling conditional volatility using standard macroeconomic or financial indicators as sentiment factors to gauge their impact on conditional volatility of market. We employed a novel approach of using news driven sentiment analysis and further augment the volatility models with such sentiment factors to measure how different types of sentiments in Indian market. Major findings of the paper suggest dominant role of negative sentiments in shaping conditional volatility in Indian stock market. Findings also support evidence of noise traders which signifies immaturity of Indian financial market.

The paper is divided further in five sections. Second section consists of a brief literature review where existing related works are chronicled. In third section we explained construction of emotion based market sentiments using NLP methods and in the fourth section we detail the methodology and discussion of empirical findings from augmented conditional volatility model for Indian stock market. Finally in fifth section we put concluding remarks with future research scope.

2. Literature Review

Literature in empirical finance has perceived market sentiments in either absolute or relative sense. Absolute market sentiment is defined with reference to a longer base period (generally a quarter or bi-annual or annual) whereas relative market sentiment is conceptualized with shorter time period data like daily, weekly or monthly.

Market sentiment is generally calculated using temporal dynamics of different macroeconomic or financial variables like stock indices return, exchange rate movement or interest rate etc. For example, sentiments like fear or bearish market are captured through sharp fall or downward trend of index return for stipulated time period in a week or month [9]. Similarly, bullish and bubble sentiment in market can be identified by the significant upward swing in return or a sharp increase in price over a short period of time [10]. Recent literature has departed from these traditional approaches of gauging market sentiments based on macroeconomic or financial indicators. Owing to advent of Natural Language Process (NLP) and its widespread application in finance, financial analysts use text mining techniques to extract sentimental factors of market from contextual words and sentences in social media and other web sources. Seminal work of Bollen et al [11] had made significant contribution in this regard. These authors formulated a six dimensional representation of mood namely, calm, alert, sure, vital, kind, and happy by mining large number of tweets. The resulting characterizations of sentiments were utilized to predict the direction of movement of the Dow Jones index in US financial market. Similarly, Atkins et al. [12] prescribed usage of Stock Twits and Google Trends etc for capturing market sentiment and further predicting volatility in financial indices.

Market sentiment is broadly dominated by three types of investors namely, rational, emotional and noise traders. First two types of investors make their investment decision on the basis of market fundamentals and emotional factors respectively whereas noise traders behave erratically in terms of their investment decisions [13]. Traditional empirical finance has ample studies where first two types of investors are addressed for their role and influence in market dynamics. However recent studies are focusing on important role of noise traders in market. Noise traders are very much present in all stock markets but their overall impact on market depends considerably on the maturity of the respective market to absorb the shock and chaos created by such traders. These traders' dominance in market can lead to unanticipated return and risk which in turn may lead to an arbitrage opportunity [14]. Though their decisions are not sophisticated or well gauged, their actions play pivotal role to shape the market return and volatility in shorter time horizon [15].

Generally, volatility of stock return in earlier studies was calculated using standard deviation or variance but seminal work of Engle (1982) [16] proposed the notion of conditional volatility and since then a battery of extended models like GARCH, EGARCH(Exponential GARCH), GJR-GARCH etc. have been proposed in literature where the dynamic concept of volatility is explored. Unlike simple volatility, as measured by standard deviation, conditional volatility is expressed as random variable which is conditioned upon a given value of another variable and it is dynamic in nature. Advantages of using conditional volatility models are well documented in empirical finance and it is found to chronicle volatility of financial market more successfully [17].

GJR GARCH, being one of these conditional volatility models has an added advantage to examine asymmetric effect of sentiment on conditional volatility. In this model sentiments are classified as positive and negative and their respective importance on conditional volatility was assessed using error of return equation as proxy for sentiments. Positive errors were assumed to be proxy for good sentiment whereas negative errors were signifying negative sentiments. Further asymmetric impacts of these two sentiments were estimated in the model.

There are very few empirical studies which focus on the nexus between stock return and market sentiment in Indian financial market's context. Works of Pai et al. (2016) [18], Jana (2016) [19] and Narayan et al. (2015) [20] found strong impact of sentiments on return of India's stocks and indices like Sensex and Nifty. But no study is found to assess the impact of market sentiments on volatility of Indian stock market using NLP methods and conditional volatility models.

Since almost all works related to Indian market's sentiment analysis in literature are devoted to model return of stock prices or indices, we explored the conditional volatility aspect which remains unexplored till date. Novelty and contribution of this paper lie in text based emotions categorization from relevant news articles on Indian financial market and economy. Empirical attempt to construct market sentiment indices on the basis of these emotions and further augment the conditional volatility model of GJR GARCH [25] to assess the asymmetric impact of positive and negative market sentiments on risk dynamics of Indian financial market is also carried out. Finally, the study also addresses the existence and influence of noise traders in Indian stock market.

3. Data

Daily closing price of India's major market index, Sensex is considered for analysis which is the most suitable proxy for gauging the nation's financial system's performance. Daily data for the time period of April 19, 2007 to January 10, 2020 has been taken for empirical analysis. Return for Sensex is calculated using standard formula:

$$return = \ln(\frac{P_t}{P_{t-1}}) * 100 \quad (1)$$

Where P_t is Sensex index on day t and P_{t-1}refers to previous day's Sensex value.

The other variable of interest in this empirical work is market sentiment. Construction of a suitable market index for unobservable emotions and sentiments is an empirically challenging task. Some studies have constructed sentiment index using market fundamentals. Macroeconomic variables like daily exchange rate, interest rate, trading volume of stock etc. are used to construct proxies for market sentiment of investors as in [22] and [23]. However in recent past some alternative methods of gauging sentiments have been gathering attention in empirical finance. These are behavioural indices constructed on the basis of fluid information available in news tabloid and other online portals. Emerging machine learning mechanism and lexicon based approach have been used for measurement of market sentiment. In our study, emotional and sentiment indices are constructed using Natural Language Processing (NLP) techniques and NRC word-emotion association lexicon [8]. Most prominent web sources for India's business and economic policy related news have been considered for emotion classification and sentiment analysis. They include Reuters India (Business and Economic section), Livemint business news, The Hindu Business Line and Moneycontrol.com. Construction of the sentiment indices from different emotions follows following steps.

Initially we gathered texts from headlines as well as summaries from relevant sections of each of aforementioned sources. Further we used NLTK (Natural Language Toolkit) to filter the data for emotion categorization of text. The steps included in pre-processing of text data and generation ofsuitable market index for unobservable emotionsof are as follows:

- Conversion of all texts to lower case to make it easy for NLTK to process.
- Tokenization is done to split the words in the string so as to process them further.
- Stop-words are removed from the text using NLTK. This is done to remove common words used in English language that do not help in generating any significant insights. For example, words like 'is',' am', 'are', 'on', 'always', etc.
- Lemmatization is done in the next step to convert various similar words to a common root form so that we can get better frequency insights. For example, converting words like 'causing', 'caused', 'causes', etc. to a single root word, i.e.'cause'.
- The pre-processed data is then converted to a frequency table using sklearn feature extractor of python. This table contains date wise frequency of each lemmatized word.
- These lemmatized words are further given sentiment scores based on standard NRC emotion lexicon (Emolex) consisting of 14,181 words with eight basic emotions (i.e. anger, fear, anticipation, trust, surprise, sadness, joy, and disgust) . Here we calculated the intensity index for each emotion, for each day ($IE_{e,t}$) by using following formula:

$$IE_{e,t} = \sum_{\forall w} \left(f_{w,t}\right)\left(I_{w,e}\right) \tag{2}$$

Where $IE_{e,t}$ refers to calculated index of emotion e on day t, $f_{w,t}$ refers to frequency of occurrence of word w on day t and $I_{w,e}$ refers to intensity of emotion e associated with word w according to the NRC word-emotion association lexicon. If for a word w no value of intensity was associated with a given emotion e then the value of $I_{w,e}$ was taken to be zero.

Aforementioned eight emotions are classified into two broad categories, namely positive sentiment and negative sentiment. Emotions like anger, fear, sadness and disgust are classified as factors for negative sentiment and rest of four emotions symbolize positive sentiment. Principal component analysis (PCA) is employed to respective groups of emotions where derived factor loading are assigned as suitable weights for each emotion to construct scores for the two sentiments. PCA is statistically more appropriate technique for construction of the sentiment scores since it uses orthogonal transformation method to transform set of related series into linearly independent ones. Finally relative share of each type of sentiment is calculated to measure relative dominance of two types of market sentiments. Relative share of each market sentiment is calculated by using following formula:

$$POS = \frac{S_p}{S_p + S_n} \tag{3}$$

Where *POS* refers to share of positive sentiment on a particular day, S_p refers to score of positive sentiment for that day and S_n refers to score of negative sentiment for that day. This implies that we can assign a share of negative market sentiment (*NEG*) as:

$$NEG = 1 - POS \tag{4}$$

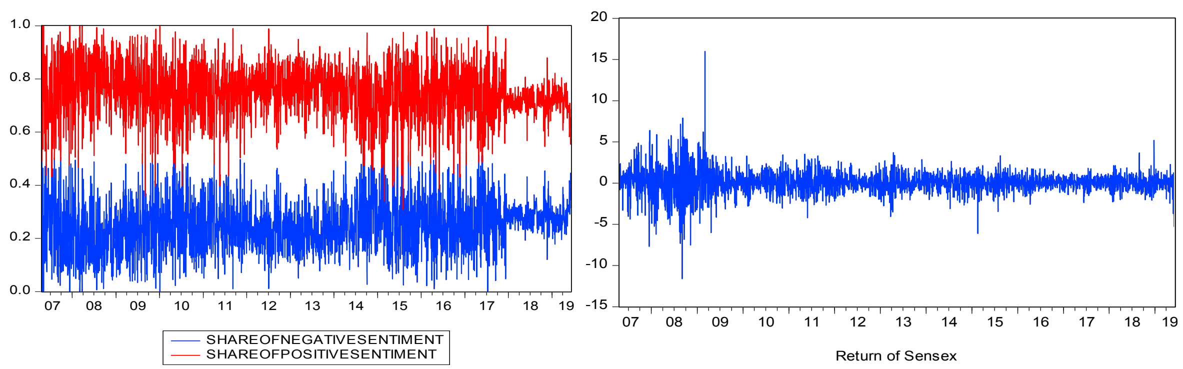
4. Volatility Modeling

We intend to gauge the impact of two opposite market sentiments on the conditional volatility of Indian stock market. Three different generalized autoregressive conditional heteroscedasticity (GARCH) models are used to analyze impact of market sentiments. Standard GARCH model along with two variants of it, namely, EGARCH or exponential GARCH model of Nelson [24] and GJR-GARCH model of Glosten, Jagannathan and Runkle [25] have been employed. All three variables under consideration in the model are initially tested for stationarity. We conducted two alternative unit root tests for checking non-stationarity in data. Results for both tests, Augmented Dickey-Fuller (1979) [29] and Kwiatkowski, Phillips, Schmidt, and Shin (1992) [28] are shown in Table 1.

Table 1.Unit root test result

Tests	Model specification	Return	Share of Positive Sentiment	Share of Negative Sentiment
ADF unit root test	Intercept	-52.54216*	-45.023*	-45.00*
	Trend and Intercept	-52.53*	-26.51*	-19.96*
KPSS unit root test	Intercept	0.517	0.200	0.200
	Trend and Intercept	0.049	0.1^{+}40	0.140

It is evident from Table 1 that both variables are stationary in level and need no further differencing for conversion to a stationary time series. ADF unit root test [29] and KPSS test [28] have contrasting null hypotheses regarding non-stationarity of variable under consideration. ADF test [29] proposes null hypothesis of unit root's presence in data whereas KPSS unit root test [28] hypothesizes stationarity of data. Both the tests confirm stationarity of all three series. Following Fig.1 shows the temporal dynamic of all three variables under consideration here.



* symbolizes statistically significant at 5% level of significance

Fig. 1.(a) Time series plot of POS and NEG; (b) Time Series plot of return of Sensex

GARCH models have been a very popular conditional volatility modeling since the seminal work of Engle (1982) [16] and it has been further extended by Bollerslev (1986) [27]. Although a simple GARCH model may successfully estimate a conditional variance in a model, it fails to depict the asymmetric impact of volatility. Distinct impact of positive and negative shocks led to conceptualization of leverage effect and this in turn causes extension to GARCH model. Glosten, Jagannathe and Runkle in 1993, proposed an augmented version of the GARCH model where they captured the asymmetric impact of two different types of shock on conditional volatility in financial data [25]. A standard GARCH(1,1) model consists of two equations, the mean equation (5) and conditional volatility equation (6).

The Mean Equation:

$$Y_t = \mu + \alpha * Y_{t-1} + \beta * \varepsilon_{t-1} + \varepsilon_t \tag{5}$$

Conditional Volatility Equation:

$$h_t = \rho + c * \varepsilon^2_{(t-1)} + \delta * h_{(t-1)} + \gamma * \varepsilon^2_{(t-1)} * D_{(t-1)} \tag{6}$$

Where Y_t is return of Sensex and follows an ARMA(1,1) process and ε_t is error term in model. Suitable order of an ARMA model is confirmed using autocorrelation and partial autocorrelation functions. Further, μ, α, β are parameters in mean equation and t suffix represents time. Unlike mean equation (5), conditional volatility equation of GJR-GARCH(1,1) model follows a deterministic path as conditional volatility, denoted by h_t depends on its own past as well as past error variance (ε^2_{t-1}). To assess asymmetric impact of error, a dummy variable (D_{t-1}) is introduced in the model which assumes value unity if past error , ε_{t-1} is negative and it assumes value zero otherwise. Coefficient γ signifies asymmetric impact of error on conditional volatility. A positive γ means negative shock has larger influence of conditional volatility whereas a negative γ symbolizes dominance of positive shock. Though traditional assumption of this GJR GARCH model to consider positive error ($\varepsilon_{t-1} > 0$) as a proxy for good news and its counterpart as bad news in market has been proven empirically very successful in literature but recent trend to capture market sentiment through news articles, online information, etc. has been gathering attention of researchers. This is why this paper attempts to augment the existing GJR GARCH model with previously proposed variables, share of positive market sentiment(*POS*) and share of negative market sentiment (*NEG*) to assess more rigorously how these two contradictory sentiments shape the dynamics of conditional volatility. Augmented conditional volatility equation can be expressed as:

$$h_t = c * \varepsilon^2_{(t-1)} + \delta * h_{(t-1)} + \gamma * \varepsilon^2_{(t-1)} * D_{(t-1)} + \varphi_1 * POS + \varphi_2 * NEG \tag{7}$$

Here the constant term of equation (6) is purposefully dropped since *POS* and *NEG* are linearly related and inclusion of the constant will lead to omission of either of these terms during estimation. Table 2 shows estimated coefficients of mean equation (5) and Table 3 shows estimated coefficients of augmented GJR GARCH model depicted in equation (7).

Table 2.Estimated coefficients of mean equation (5)

Parameters	Value (standard error)
M	0.0123(0.0266)
$\alpha^{\ddagger}$	0.958(0.0273)*
β	-0.938(0.033)*

Table 3.Estimated coefficients of augmented GJR GARCH model

Parameters	Value (standard error)	
C	0.144(0 .010) *	
δ	0.916 (0.005)*	
γ	-0.128 (0.011)*	
φ_1	-0.449 (0 .653)*	
φ_2	5.403 (1.849)*	
Log likelihood = -4746.99	Wald chi2(2) = 2490.27	Number of observation= 3166

Estimated coefficients of mean equation show that both AR and MA terms are statistically significant. In case of conditional volatility equation, (c+δ) is less then unity which confirms stability of the model and since (c+δ) is close to unity we can infer there exists volatility persistence in market. It is interesting to note that coefficient γ is negative and statistically significant. It implies that for Indian stock market excess positive return (presumed to be a crude proxy for good news for market) plays dominant role over its negative counterpart for specified time period. Here the threshold is decided on the basis of value zero of error term in mean equation of return. But we proposed sentiment on the basis of news information, a superior barometer to gauge the prevailing mood for the market. Coefficients of positive and negative sentiment on the basis of emotional quotients of news information enter the conditional volatility equation to gauge the degree of influence on overall market's risk profile. Coefficient φ_1 shows relative influence of positive sentiment on conditional volatility and it is found to bear a negative sign whereas parameter φ_2 measures relative importance of negative market sentiment on market volatility and found to be positive. We can infer that positive sentiment reduces market volatility but negative sentiment fuels the market volatility further. However, impact of negative sentiment is found to be much larger in size than its positive counterpart since estimated value of φ_2 is 5.403 in comparison to φ_1=-0.45. It indicates dominance of negative sentiments over positive sentiment in terms of impact on Indian stock market's volatility. It is evident from our empirical findings that noise traders are playing a dominant role. Contractionary impact of positive sentiment on market volatility indicates departure of noise traders from market but this effect is outweighed by amplifying influence of negative sentiment when dominance of such noise traders is observed in market. Our finding is in consonance with empirical works on Indian market by Naik and Padhi (2016) [26] and by Kumari and Mahakud (2015) [22]. However, both these papers used traditional macroeconomic and financial variables to gauge market sentiments whereas use of real stream of data which are available on a daily basis is a more appropriate way to assess market sentiments. This stream of data is more dynamic in nature than traditional monthly or quarterly indicators. Further, our model does not treat positive and negative errors in mean equation of the return series as proxies for respective market sentiments. Instead, we have generated separate market sentiments on the basis of filtered emotions from relevant news articles and finally assessed the impact of each of these sentiments on conditional volatility of Indian financial market. This approach is empirically more appealing and relevant in today's era of information abundance.

* symbolizes statistically significant at 5% level of significance

5. Conclusion

The paper offers an augmented asymmetric GARCH model where dominance of two contradictory investor sentiments, namely positive and negative sentiments is analyzed. Though, standard threshold choice of zero value of error gathered attention in literature to classify positive and negative values of errors as respective sentiments in market, this paper proposed news based sentiment analysis for Indian stock market. Empirical findings suggest that influence of negative sentiment is more pronounced than its positive counterpart and it also validates dominance of noise traders in Indian stock market. Findings of the study are useful from financial policy perspective since evidence and dominance of noise traders in Indian stock market necessitate to rethink about investment strategies for large number of rational investors. When market fundamentals are not key decisive factors and irrational exuberance shape the volatility of market then it also reflects the immaturity of an emerging financial market like India which needs to be contained through proper regulatory actions and practices.

The future scope of this study lies in a comparative analysis of different sectors of Indian stock market like energy, telecommunication or metal etc. which can be assessed and their respective impact on overall stock market can be analyzed to have a granular understanding of the market dynamic.

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