PHY114: QUANTUM PHYSICS (2024-25, 2nd Semester)
Homework-07

Q.1: A 1D potential consists of an infinite potential well with a delta function at its center, i.e.

$$V(x) = \begin{cases} \gamma\delta(x) & \text{for } |x| < L/2 \\ \infty & \text{otherwise} \end{cases}.$$

(a) With $V(x)$ being symmetric about the origin, write possible solutions of the TISE in different regions together with the boundary conditions.
(b) Find the eigen-energies and eigen-state wave-functions for the anti-symmetric solutions. Plot the lowest energy antisymmetric wave-function.
(c) Find the equation that determines the eigen-energy for the symmetric case. Plot the ground state wave-function. Argue that the ground state energy for $\gamma > 0$ is higher than that for $\gamma = 0$ and for $\gamma < 0$ it is lower.
(d) Find the ground state energy values in small (but non-zero) and large positive γ ($\frac{L\gamma m}{\hbar^2} \gg 1$) limits.

Q.2: Consider an attractive double δ-function potential given by $V(x) = -\gamma\delta(x-a) - \gamma\delta(x+a)$ with $\gamma > 0$. This potential will have at most two bound states ($E < 0$) and our objective is to analyze these bound states for different values of the dimensionless parameter $\lambda = 2a\gamma m/\hbar^2$.
(a) Plot the two possible wave-functions qualitatively considering the symmetry of the potential and the known single δ-function bound-state wave-function. (Note bonding and anti-bonding nature.)
(b) Write possible solutions (symmetric and antisymmetric) of the TISE in different regions together with the boundary conditions.
(c) Find the transcendental equations that dictate the eigen energies.
(d) Make appropriate plots for graphical solutions for eigen energies.
(e) Discuss the two states and their wave-functions in the limits: $\lambda \gg 1$ and $\lambda \ll 1$.

Q.3: In a 3D cartesian system an orthonormal basis of unit vectors is given by $\{\hat{u}_i\} = (\hat{x}, \hat{y}, \hat{z})$. You are given another rotated basis-set of unit vectors $(\hat{x}', \hat{y}', \hat{z}')$ such that $\hat{x}' = (\hat{x} - \hat{y} + \hat{z})/\sqrt{3}$, $\hat{y}' = (\hat{x} + \hat{y})/\sqrt{2}$ and $\hat{z}' = (-\hat{x} + \hat{y} + 2\hat{z})/\sqrt{6}$.

(a) Find if the basis set $(\hat{x}', \hat{y}', \hat{z}')$ is orthonormal.
(b) We can write a vector as a column matrix with its elements as the three components. Write the transformation matrix T between these two given bases so that one can find a vector $\vec{V}$ (given in the un-primed basis) in the primed basis $\vec{V}'$, i.e. $\vec{V}' = T\vec{V}$.
(c) Prove that T is an orthogonal matrix, i.e. $TT^T = I$. Here I represents identity matrix.
(d) Given $\vec{V} = \hat{x} + \hat{y} + \hat{z}$, find the corresponding $\vec{V}'$.
(e) Prove that the scalar product of two vectors written as $\vec{V}_1.\vec{V}_2$ is same in the two bases.
(f) Suppose the moment of inertia tensor of a rigid body in the unprimed basis is given by $I = I_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. Find this tensor I' in the primed basis.