PHY114: QUANTUM PHYSICS (2024-25, 2nd Semester)
Homework-09

Q.1: (a) Prove that the operators x and p_x satisfy the commutation relation $[x, p_x] = i\hbar$.
(b) In one dimension for $H = \frac{p^2}{2m} + V(x)$ find $[x, H]$ and $[p, H]$.
(c) A measurable quantity corresponds to a Hermitian operator B which has eigen values β_i and corresponding eigen states $|u_i\rangle$. Prove that the average value of this quantity for a quantum state $|\psi\rangle$ is given by $\langle B\rangle_\psi = \langle\psi|B|\psi\rangle$.
(d) Use the Heisenberg equation of motion, to prove the following Ehrenfest theorems describing the semi-classical motion of a particle: $\frac{\partial}{\partial t}\langle x\rangle = \frac{\langle p\rangle}{m}$ and $\frac{\partial}{\partial t}\langle p\rangle = -\langle\frac{\partial V}{\partial x}\rangle$. Here, $\langle B\rangle$ denotes expectation value of operator B over a given time dependent wave function.

Q.2: Show that the expectation value of an operator A (Hermitian) over a wave function $\phi(x, t)$ is time independent if: (a) A commutes with Hamiltonian. (b) ϕ is an eigen-state of the Hamiltonian. (c) ϕ is an eigen-state of A. [Hint: use the Heisenberg equation of motion.]

Q.3: (a) Consider two operators: $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Prove that they commute. Thus, prove that if two operators are diagonal in certain common basis then they must commute with each other.

(b) Now, find these operators (i.e. $A' = SAS^\dagger$ and $B' = SBS^\dagger$) in another orthonormal basis described by the unitary transformation $S = \begin{pmatrix} 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{6} & 2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix}$.
(c) Will above found A' and B' commute with each other?
(d) Without working out explicitly, find the eigen values and eigen vectors of A' and B' operators and verify so.