PHY114: QUANTUM PHYSICS (2024-25, 2nd Semester)
Homework-11

Q.1: Suppose $\psi(x,t)$ is a general non-stationary state of a Simple Harmonic Oscillator (SHO) with natural frequency ω. Use the Ehrenfest relations (or otherwise) to show
(a) $\langle x(t)\rangle_\psi = \langle x(0)\rangle_\psi \cos\omega t + \frac{\langle p(0)\rangle_\psi}{m\omega}\sin\omega t$ and
(b) $\langle p(t)\rangle_\psi = \langle p(0)\rangle_\psi \cos\omega t - m\omega\langle x(0)\rangle_\psi \sin\omega t$.
Here $\langle A(t)\rangle_\psi$ is the expectation value of operator A over the time dependent wave-function $\psi(x,t)$.

Q.2: The normalized ground state for the SHO works out to be $\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega}{2\hbar}x^2\right)$ with energy $\frac{\hbar\omega}{2}$. Find the probability to find the particle in classically forbidden region for the ground state.

Q.3: (a) A particle of mass m is subjected to the potential $V(x) = \begin{cases} \infty & \text{for } x < 0 \\ \frac{1}{2}m\omega^2x^2 & \text{for } x > 0 \end{cases}$. Find the energy eigen values. **(b)** A diatomic molecule having atoms with charges $+e$ and $-e$ and reduced mass m experiences a uniform electric field E in addition to an inter-atomic harmonic potential $k(r-a)^2/2$. Here a is the equilibrium inter-atomic separation. Find the energy eigen values corresponding to small oscillations in this molecule.

Q.4: An HCl molecule has force constant 516 N/m, which is typical for single bond. Find the effective mass of this molecule and thus the energy difference between neighboring vibration levels. What wavelength and what region of the em-spectrum (visible, uv, IR, etc.) does this energy difference correspond to?

Q.5: What is the degeneracy of the 3rd excited state of an isotropic harmonic oscillator in 2D? What is it for an isotropic harmonic oscillator in 3D? Can you find the expression for the degeneracy for the general n^{th} state for the 2D & 3D isotropic harmonic oscillator?

Q.6: (a) Use the operators a_- & a_+ to find the matrix elements $\langle m|x|m\rangle$, $\langle m|p|m\rangle$, $\langle m|x^2|m\rangle$ and $\langle m|p^2|m\rangle$ with $|m\rangle$ as the m^{th} eigen-state of the SHO.
(b) Use these to find the uncertainty product $(\Delta x.\Delta p)$ for the n^{th} eigen-state of the SHO.

Q.7*: Consider the SHO with energy eigen-states $\phi_n(x)$ and $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$. Given that the state of the particle in this SHO potential at $t=0$ is described by a wave function: $\psi_\mu(x,0) = e^{-\frac{|\mu|^2}{2}}\sum_{n=0}^{\infty}\frac{\mu^n}{\sqrt{n!}}\phi_n(x)$ with μ as a complex number.
(a) Show that this state is normalized and it is an eigen-state of the a_- ladder operator. Find the corresponding eigen-value?
(b) Find the dependent expectation values of x and p for above state for a complex μ.
(c) Write the time dependent wave function for this state and find the time-dependent expectation values of x and p assuming μ to be real.