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CSA0669

1)

1) Calculate the number of ways to achieve a sum of 15 when rolling four six-sided dice. provide a detailed step-by-step solution.

No. of solutions

$x_1+x_2+x_3+x_4=15$ where $1 \le x_i \le 6$ into $0 \le y_i \le 5$.

This becomes.

$(y_1+1)+(y_2+1)+(y_3+1)+(y_4+1)=15$

$y_1+y_2+y_3+y_4+4=15$

$y_1+y_2+y_3+y_4=11$

using Inclusion - exclusion principle

"Stars and bars"

$\binom{11+4-1}{4-1} = \binom{14}{3}$

$\binom{14}{3} = \frac{14 \times 13 \times 12}{3 \times 2 \times 1} = 364$

set $y_1' = y_1 - 6$ then

$y_1' + y_2 + y_3 + y_4 = 5$

$\binom{5+4-1}{4-1} = \binom{8}{3}$

$\binom{8}{3} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

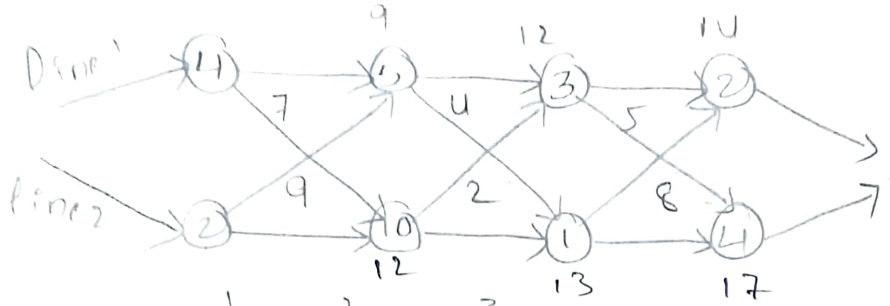
Since any of four variables $= 4 \times 56 = 224$.

set $y_1' = y_1 - 6$ and $y_2' = y_2 - 6$ then

$y_1' + y_2' + y_3 + y_4 = -1$

$\therefore$ The no. of valid solutions is $364 - 224 = 140$//.

2) Two assembly lines have station times as follows: line 1: [4,5,3,2], line 2: [2,10,1,4], transfer times b/w lines are: from line 1 to line 2: [7,4,5], from line 2 to line 1: [9,2,8]. Calculate the minimum time to assemble a product.



	1	2	3	4
$F_1[j]$	4	9	11	14
$F_2[j]$	2	12	13	17

	1	2	3	4
$L_1[j]$	1	1	1	1
$L_2[j]$	2	2	2	2

$$F_1[j] = \min\{(f_1(j-1) + a_1 j), (f_2(j-1) + (t_2, j-1) + a_1 j)\}$$

$$= \min\{9\}$$

$$= 9$$

$$F_2[j] = \min\{(f_1(j-1) + (t_1, j-1) + a_2 j), (f_2(j-1) + (a_2 j)\}$$

$$= \min\{21, 12\}$$

$$= 12.$$

3) Given keys {10,20,30,40} | {10,20,30,40} | {10,20,30,40} with access probabilities {0.1, 0.2, 0.4, 0.3} | {0.1, 0.2, 0.4, 0.3} | {0.1, 0.2, 0.4, 0.3} respectively

Construct the optimal binary search tree. Calculate the total cost of the tree.

Given keys with access probability.

$\{10, 20, 30, 40\}$ (keys numbered 1, 2, 3, 4)

$\{0.1, 0.2, 0.4, 0.3\}$

i \ j	0	1	2	3	4
0	0	0.1	0.4 [2]	[1.1] [3]	1.7 [3]
1		0	0.2	0.8 [3]	1.4 [2]
2			0	0.4	1.0 [3]
3				0	0.3
4					0

$j - i = 1$

$1 - 0 = 1\ (0,1)\ (1,1)$

$2 - 1 = 1\ (1,2)\ (2,2)$

$3 - 2 = 1\ (2,3)\ (3,3)$

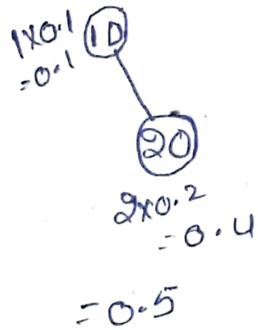
$4 - 3 = 1\ (3,4)\ (4,4)$

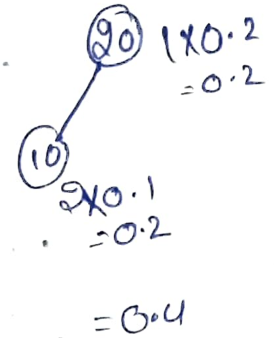
$j - i = 2$

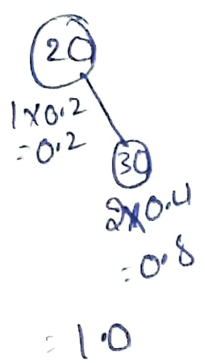
$2 - 0 = 2\ (0,2)\ (1,2)$

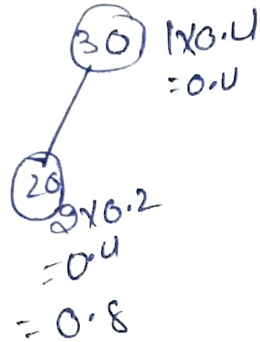
$3 - 1 = 2\ (1,3)\ (2,3)$

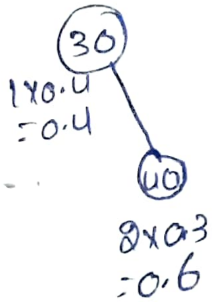
$4 - 2 = 2\ (2,4)\ (3,4)$

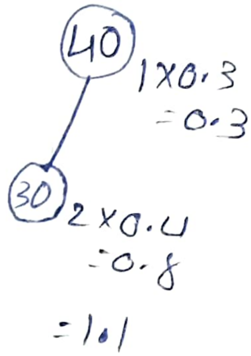












$j - i = 3$

$3 - 0 = 3\ (0,3)\ (1,3)$

$4 - 1 = 3\ (1,4)\ (2,4)$

$cost(i,j) = \min\{cost(i,k-1) + cost(k,j)\} + w_i$

$cost(0,3) = \min\{cost(0,0) + cost(1,3)\} + 0.7$

$$k=1,2,3 \left\{ \begin{array}{l} cost(0,1)+cost(2,3) \\ cost(0,2)+cost(3,3) \end{array} \right\}$$

$$= \min \left\{ \begin{array}{l} 0+0.8 \\ 0.1+0.4 \\ 0.4+0 \end{array} \right\} + 0.7$$

$$= \min \left\{ \begin{array}{l} 1.5 \\ 1.2 \\ 1.1 \end{array} \right\} = 1.1$$

$$cost(1,4) = \min_{k=2,3,4} \left\{ \begin{array}{l} cost(1,1)+cost(2,4) \\ cost(1,2)+cost(3,4) \\ cost(1,3)+cost(4,4) \end{array} \right\} + 0.9$$

$$= \min \left\{ \begin{array}{l} 0+1.0 \\ 0.2+0.3 \\ 0.8+0 \end{array} \right\} + 0.9$$

$$= \min \left\{ \begin{array}{l} 1.9 \\ 1.4 \\ 1.7 \end{array} \right\} = 1.4$$

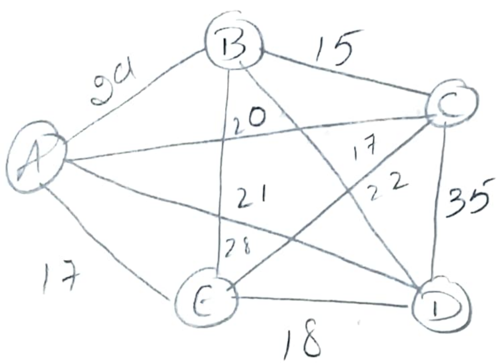
$$cost(0,4) = \min \left\{ \begin{array}{l} cost(0,0)+cost(1,4) \\ cost(0,1)+cost(2,4) \\ cost(0,2)+cost(3,4) \\ cost(0,3)+cost(4,4) \end{array} \right\} + 1.0$$

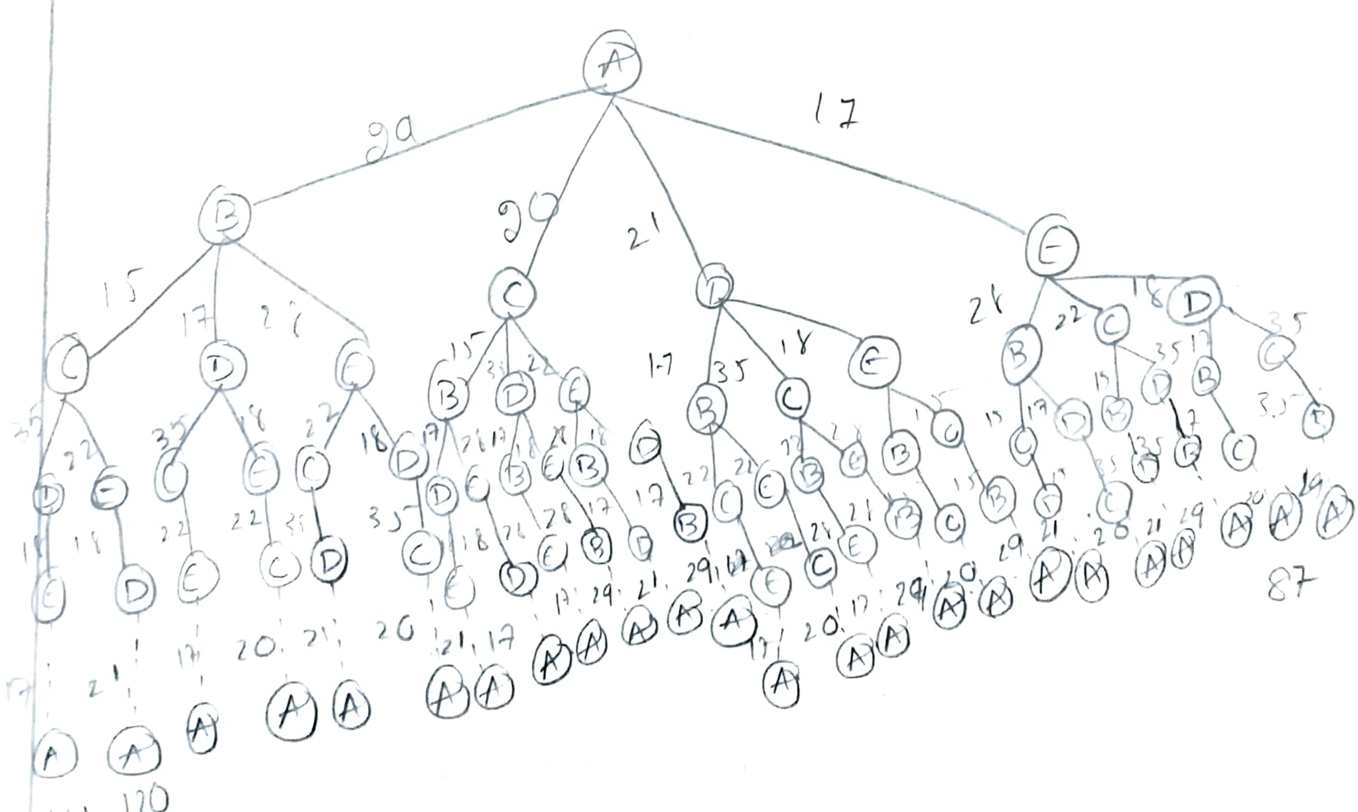
$$= \min \left\{ \begin{array}{l} 0+1.4 \\ 0.1+1.0 \\ 0.4+0.3 \\ 1.1+0 \end{array} \right\} + 1.0$$

$$= \min \left\{ \begin{array}{l} 2.4 \\ 2.1 \\ 1.7 \\ 2.1 \end{array} \right\} = 1.7$$

4) Solve the TSP for the following 5-city distance matrix using dp.

A : [0, 29, 20, 21, 17]
B : [29, 0, 15, 17, 28]
C : [20, 15, 0, 35, 22]
D : [21, 17, 35, 0, 18]
E : [17, 28, 22, 18, 0]





∴ The minimum cost = 87

A — E — D — B — C — A

5) You have a knapsack with a capacity of 50 units. There are 4 items with the following weights and values:

Item 1 :- weight = 10, value = 60

Item 2 :- weight = 20, value = 100

Item 3 :- weight = 30, value = 120

Item 4 :- weight = 40, value = 200.

Determine the maximum value that can be obtained using the 0/1 knapsack problem approach. Show the steps and the final solution.

Given capacity 50 units.

Item	weight	value (P)
1.	10	60
2.	20	100
3.	30	120
4.	40	200

v\w	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	160	180	220
4	0	60	100	160	200	260

Formula:-

$$V[i,w] = \max\{V[i-1,w], V[i-1, w-w[i]] + value[i]\}$$

$$V[4,50] = \max\{V[3,50], V[3,50-200] + value[4]\}$$

$$= \max\{220, 260\}$$

$$= 260$$

5) Given the following directed graph with vertices A, B, C, DA, B, C, D A, B, C, D and edges with weights.

A→BA\rightarrow BA→B with weight 1

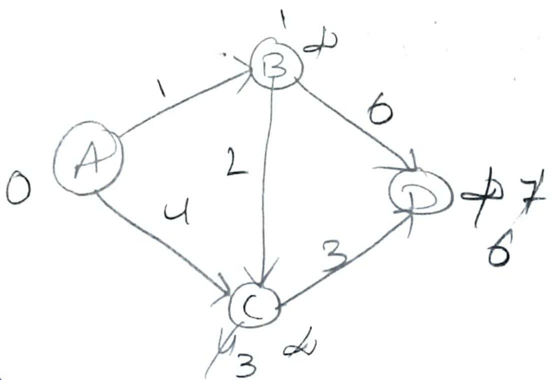
A→CA\rightarrow CA→C with weight 4

B→CB\rightarrow CB→C with weight 2

B→DB\rightarrow DB→D with weight 6

C→DC\rightarrow DC→D with weight 3

use the Bellman-Ford algorithm to find the shortest path from vertex AAA to all other vertices. Show the steps and the final distances.



A→B - 1

A→C · 4

B→C - 2

B→D · 6

C→D - 3

Initialize.

V	A	B	C	D
d	0	∞	∞	∞
P	-	-	-	-

①

V	A	B	C	D
d	0	1	4	∞
P	-	A	A	B

② v	A	B	C	D
d	0	1	3	7
P	-	A	B	B

③ v	A	B	C	D
d	0	1	3	6
P	-	A	B	C

Path	shortest distance	shortest path
A-B	1	A-B
A-C	3	A-B-C
A-D	6	A-B-C-D

O/P → A → B → C → D.

7) Determine the probability of rolling five dice such that the sum is exactly 20. Include a combinatorial approach to arrive at the solution.

$6^5 = 7776$

$x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where $1 \le x_i \le 6$

$y_i = x_i - 1$ for $i = 1, 2, 3, 4, 5$

$(y_1+1) + (y_2+1) + (y_3+1) + (y_4+1) + (y_5+1) = 20$

$y_1 + y_2 + y_3 + y_4 + y_5 = 15$

where $0 \le y_i \le 5$

By "stars and bars"

$$\binom{15+5-1}{5-1} = \binom{19}{4}$$

$$\binom{19}{4} = \frac{19 \times 18 \times 17 \times 16}{4 \times 3 \times 2 \times 1} = 3876$$

If $y_1 \ge 6$, let $y_1' = y_1 - 6$

$y_1' + y_2 + y_3 + y_4 + y_5 = 9$

$$\binom{9+5-1}{5-1} = \binom{13}{4}$$

$$\binom{13}{4} = \frac{13\times12\times11\times10}{4\times3\times2\times1} = 715$$

There are 5 such variables.

$5\times715 = 3575$

If two variables $y_i, y_j \geq 6$, let $y_1' = y_1 - 6$ and $y_2' = y_2 - 6$.

$$y_1' + y_2' + y_3 + y_4 + y_5 = 3$$

$$\binom{3+5-1}{5-1} = \binom{7}{4}$$

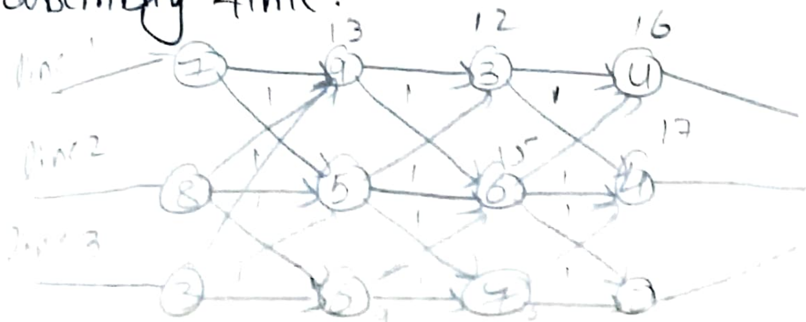
$$\binom{7}{4} = \frac{7\times6\times5\times4}{4\times3\times2\times1} = 35$$

$$\binom{5}{2}\times35 = 10\times35 = 350$$

$3876 - 3575 + 350 = 651$

$$\frac{651}{7776} = \frac{651}{7776} \approx 0.0837$$ //.

8) For three assembly lines with station times:
Line 1: [7, 9, 3, 4]; line 2: [8, 5, 6, 4], line 3: [3, 6, 7, 2]
and transfer times between lines given determine the optimal scheduling and the total minimum assembly time.



$F_1[j] = \min\{(f_1(j-1) + a_{1j}), (f_2(j-1) + (t_{2,j-1}) + a_{1j}), (f_3(j-1) + (t_{3,j-1}) + a_{1j}\}\}.$

$= \min\{(7+9), (8+1+9), (3+1+9)\} = 13$

	1	2	3	4
$F_1[j]$	7	13	12	16
$F_2[j]$	8	9	15	17
$F_3[j]$	3	9	16	15

	1	2	3	4
$L_1[j]$	1	3	1	1
$L_2[j]$	2	3	2	1
$L_3[j]$	3	3	3	1

9) Consider keys {15, 25, 35, 45, 55} | {15, 25, 35, 45, 55} | {15, 25, 35, 45, 55} with access probabilities {0.05, 0.15, 0.4, 0.25, 0.15}. Determine the structure of the optimal binary search tree and compute the expected cost.

$j - i = 1$

$1 - 0 = 1\ (0,1)(1,1)$

$2 - 1 = 1\ (1,2)(2,2)$

$3 - 2 = 1\ (2,3)(3,3)$

$4 - 3 = 1\ (3,4)(4,4)$

$5 - 4 = 1\ (4,5)(5,5)$

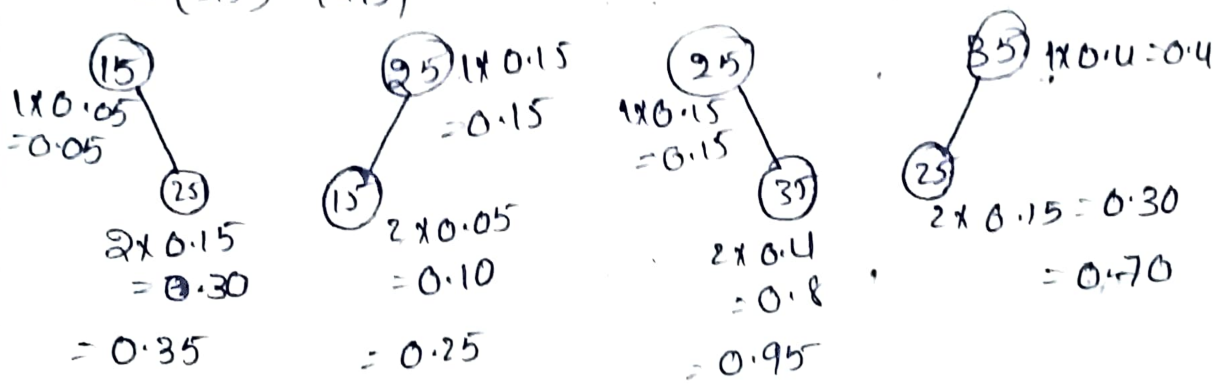
$j - i = 2$

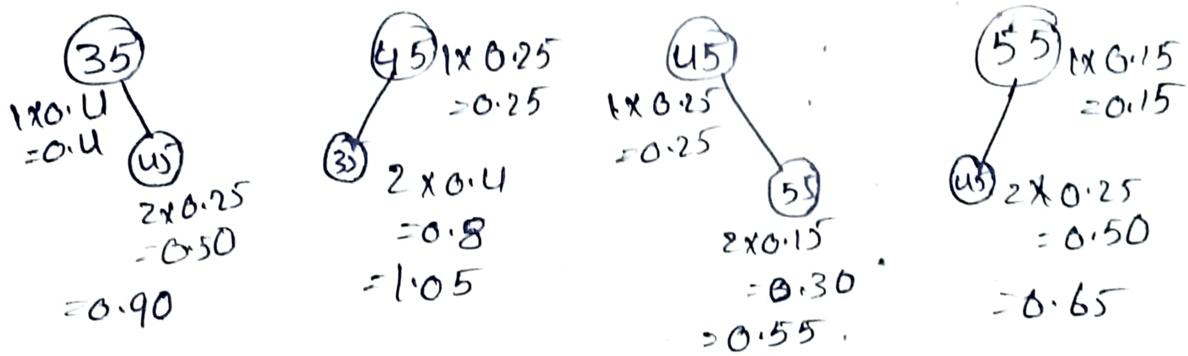
$2 - 0 = 2\ (0,2)(1,2)$

$3 - 1 = 2\ (1,3)(2,3)$

	0	1	2	3	4	5
0	0	0.05	0.25 [2]	0.85 [3]	1.35 [3]	1.80 [3]
1		0	0.15	0.70 [3]	1.20 [3]	1.80 [4]
2			0	0.4	0.90 [3]	1.35 [3,4]
3				0	0.25	0.55
4					0	0.15
5						0

$4-2=2$ (2,4) (3,4)
$5-3=2$ (3,5) (4,5)





$j-i=3$
$3-0=3$ (0,3) (1,3)
$4-1=3$ (1,4) (2,4)
$5-2=3$ (2,5) (3,5)

$$\text{cost}(0,3) = \min_{k=1,2,3} \begin{Bmatrix} \text{cost}(0,0)+\text{cost}(1,3) \\ \text{cost}(0,1)+\text{cost}(2,3) \\ \text{cost}(0,2)+\text{cost}(3,3) \end{Bmatrix} + 0.6$$

$= 0.85$

$$\text{cost}(1,4) = \min_{k=2,3,4} \begin{Bmatrix} \text{cost}(1,1)+\text{cost}(2,4) \\ \text{cost}(1,2)+\text{cost}(3,4) \\ \text{cost}(1,3)+\text{cost}(4,4) \end{Bmatrix} + 0.8$$

$= 1.20$

$$\text{cost}(2,5) = \min_{k=3,4,5} \begin{Bmatrix} \text{cost}(2,2)+\text{cost}(3,5) \\ \text{cost}(2,3)+\text{cost}(4,5) \\ \text{cost}(2,4)+\text{cost}(5,5) \end{Bmatrix} + 0.8$$

= 1.35

$$\text{cost}(0,4) = \min_{k=1,2,3,4} \begin{Bmatrix} \text{cost}(0,0) + \text{cost}(1,4) \\ \text{cost}(0,1) + \text{cost}(2,4) \\ \text{cost}(0,2) + \text{cost}(3,4) \end{Bmatrix} + 0.85$$

= 1.35

$$\text{cost}(1,5) = \min_{k=2,3,4,5} \begin{Bmatrix} \text{cost}(1,1) + \text{cost}(2,5) \\ \text{cost}(1,2) + \text{cost}(3,5) \\ \text{cost}(1,3) + \text{cost}(4,5) \\ \text{cost}(1,4) + \text{cost}(5,5) \end{Bmatrix}$$

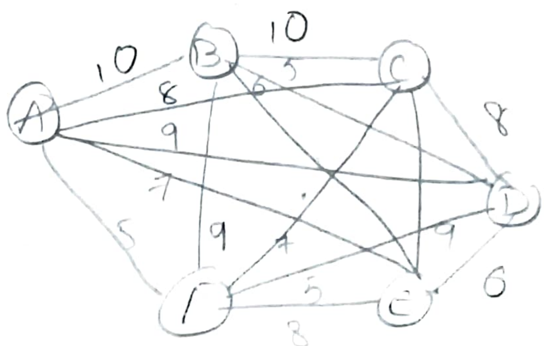
= 1.80

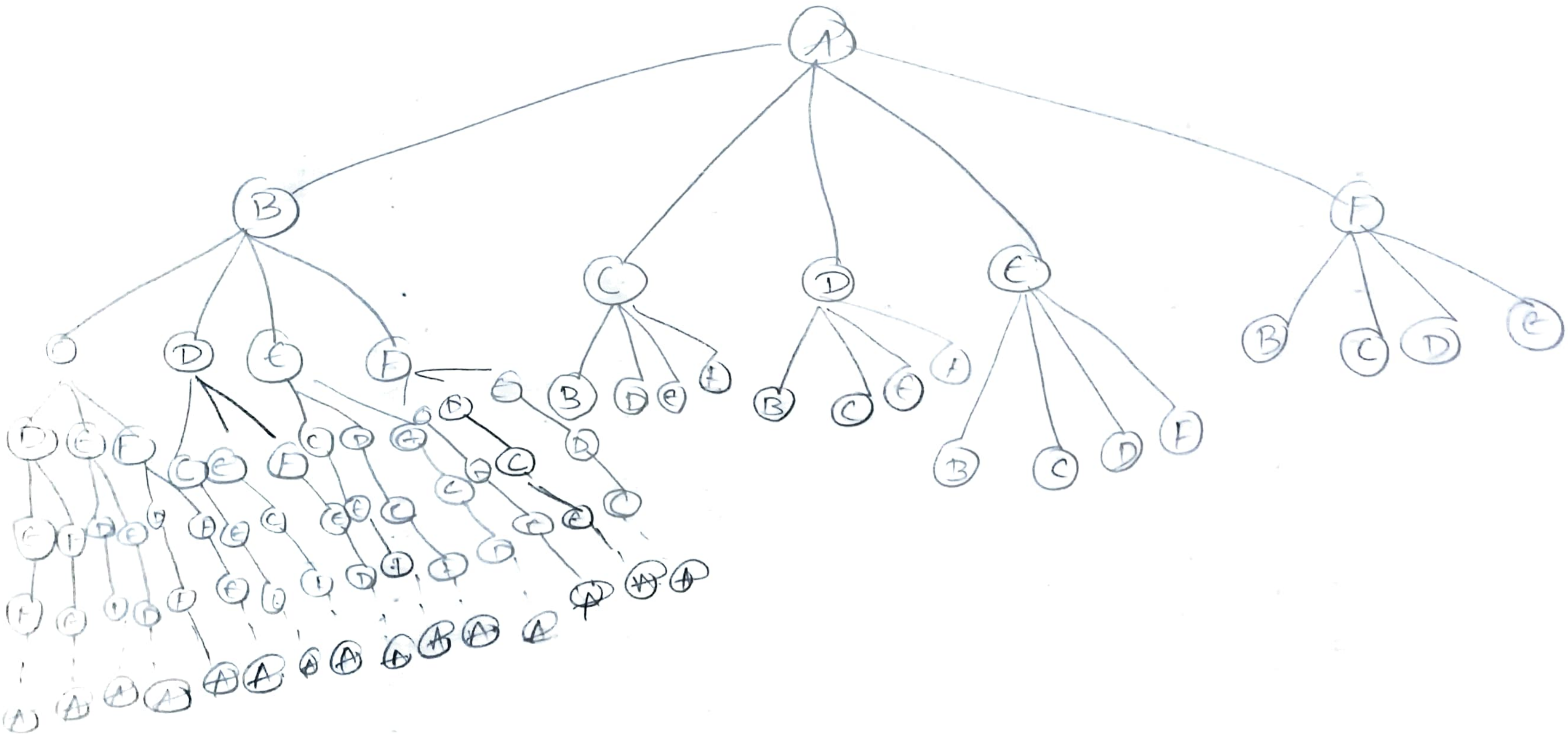
$$\text{cost}(0,5) = \min_{k=1,2,3,4,5} \begin{Bmatrix} \text{cost}(0,0) + \text{cost}(1,5) \\ \text{cost}(0,1) + \text{cost}(2,5) \\ \text{cost}(0,2) + \text{cost}(3,5) \\ \text{cost}(0,3) + \text{cost}(4,5) \\ \text{cost}(0,4) + \text{cost}(5,5) \end{Bmatrix} + 1$$

= 1.80.

10) Given a distance matrix for 6 cities, find the shortest path using nearest neighbour heuristic.

A: [0, 10, 8, 9, 7, 5]
B: [10, 0, 10, 5, 6, 9]
C: [8, 10, 0, 8, 9, 7]
D: [9, 5, 8, 0, 6, 5]
E: [7, 6, 9, 6, 0, 8]
F: [5, 9, 7, 5, 8, 0]





o/p - A - F - E - D - C - B - A = 47/.

11) solve the fractional knapsack problem for a knapsack with a capacity of 60 units and the following items:

Item 1: $w=20$, $v=100$

item 2: $w=30$, $v=120$

item 3: $w=10$, $v=60$

calculate the maximum value that can be achieved and describe the fractions of items taken.

Item	weight	value (p)	$w = 60$ units
1.	20	100	
2.	30	120	
3.	10	60	

v|w	0	10	20	30	40	50	60
0	0	0	0	0	0	0	0
1	0	0	100	100	100	100	100
2	0	0	100	120	120	220	220
3	0	10	100	120	180	220	230

Formula:-

$$v[i,w] = \max\{v[i-1,w], v[i-1, w-w(i)] + value[i]\}$$

12) consider a directed graph with 5 vertices v_1, v_2, v_3, v_4, v_5 v_1, v_2, v_3, v_4, v_5 v_1, v_2, v_3, v_4, v_5 and the following edges with weights:

$v_1 \to v_2$ v_1 \ oriented arrow $v_2 v_1 \to v_2$ with $w-3$

$v_1 \to v_3$ v_1 $w-8$

$V_2 \to V_3 V_2$ $w-2$

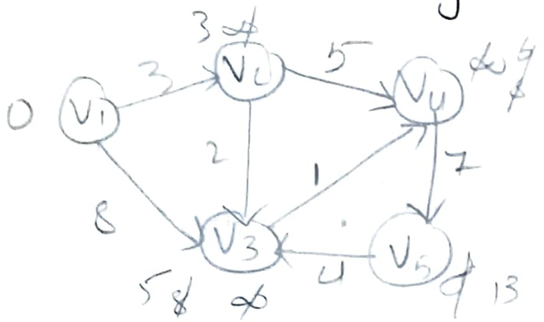
$V_2 \to V_4 V_2$ $w-5$

$V_3 \to V_4 V_3$ $w-1$

$V_4 \to V_5 V_4$ $w-7$

$V_5 \to V_3 V_5$ $w-4$

Apply Bellman-Ford algorithm.



$V_1 \to V_2$ 3

$V_1 \to V_3$ 8

$V_2 \to V_3$ 2

$V_2 \to V_4$ 5

$V_3 \to V_4$ 1

$V_4 \to V_5$ 7

$V_5 \to V_3$ 4

Initialize

V	V_1	V_2	V_3	V_4	V_5
d	0	∞	∞	∞	∞
P	-	-	-	-	-

(1)

V	V_1	V_2	V_3	V_4	V_5
d	0	3	8	∞	∞
P	-	V_1	V_1	V_2	V_4

(2)

V	V_1	V_2	V_3	V_4	V_5
d	0	3	5	8	∞
P	-	V_1	V_2	V_2	V_4

(3)

V	V_1	V_2	V_3	V_4	V_5
d	0	3	5	6	15
P	-	V_1	V_2	V_3	V_4

v	v_1	v_2	v_3	v_4	v_5
d	0	3	5	6	13
P	-	v_1	v_2	v_3	v_4

o/p $\to v_1 \to v_2 \to v_3 \to v_4 \to v_5$

13) Given two eight-sided dice, compute the no. of ways to achieve a sum of 10. Then, extend this to three dice and find the new no. of ways to get the same sum.

We need to count the pairs (x,y) such that $x+y=10$. where $1 \le x, y \le 8$

$(x,y) = (2,8)$
$(x,y) = (3,7)$
$(x,y) = (4,6)$
$(x,y) = (5,5)$
$(x,y) = (6,4)$
$(x,y) = (7,3)$
$(x,y) = (8,2)$

Sum of 10 is: 7.

1) $x=1$
$y+z=9$
$(1,1,8), (1,2,7), (1,3,6), (1,4,5), (1,5,4) (1,4,5), (1,6,3), (1,7,2), (1,8,1)$

2) $x=2$:
$y+z=8$
$(2,1,7), (2,2,6), (2,3,5), (2,4,4), (2,5,3), (2,6,2), (2,7,1)$

3) $x = 3$:
$y + z = 7$:
$(3,1,6), (3,2,5), (3,3,4), (3,4,3), (3,5,2), (3,6,1)$

4) $x = 4$:
$y + z = 6$:
$(4,1,5), (4,2,4), (4,3,3), (4,4,2), (4,5,1)$

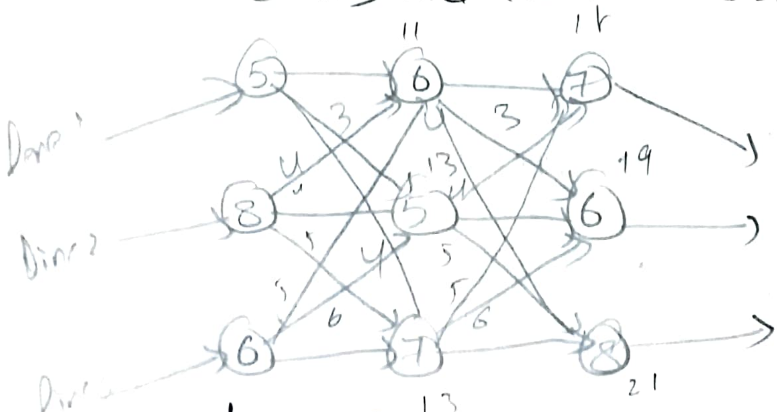
5) $x = 5$
$y + z = 5$
$(5,1,4), (5,2,3), (5,3,2), (5,4,1)$

6, $x = 6$
$y + z = 4$:
$(6,1,3)\ (6,2,2), (6,3,1)$

7. $x = 7$
$y + z = 3$
$(7,1,2), (7,2,1)$

8. $x = 8$
$y + z = 2$: $(8,1,1)$
Sum $= 8+7+6+5+4+3+2+1 = 36$ //.

14) Given station times for line 1: [5,6,7], line 2: [8,5,6] and line 3: [6,7,8] and transfer times b/w lines.



	1	2	3
$F_1(i)$	5	11	18
$F_2(i)$	8	13	19
$F_3(i)$	6	13	21

$L_1[j]$ 1 ← 1 ← 1 ←

$L_2[j]$ 2 2 2

$L_3[j]$ 3 3 3

$$F_1[j] = \min\{(f_1(j-1) + a_{1,j}), (f_2(j-1) + (t_2, j-1) + a_{1,j}), (f_3(j-1) + (t_3, j-1) + a_{1,j})\}$$

$$= \min\{11, 18, 17\} = 11 //$$

15) Given keys {5, 15, 25, 35, 45, 55} with access probabilities {0.1, 0.05, 0.2, 0.25, 0.3, 0.1}.

j - i = 1

1 - 0 = 1

2 - 1 = 1

3 - 2 = 1

4 - 3 = 1

5 - 4 = 1

6 - 5 = 1

j - i = 2

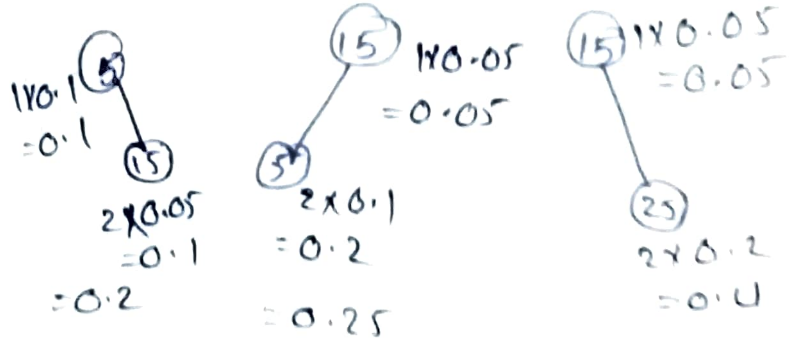
2 - 0 = 2 (0,2) (1,2)

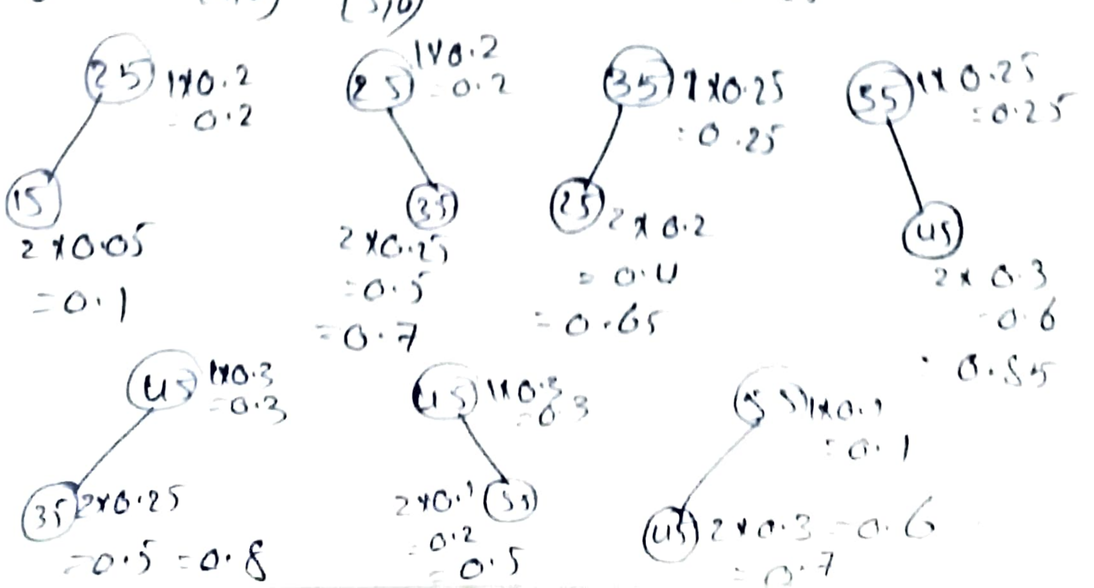
3 - 1 = 2 (1,3) (2,3)

4 - 2 = 2 (2,4) (3,4)

5 - 3 = 2 (3,5) (4,5)

6 - 4 = 2 (4,6) (5,6)





$j - i = 3$

$3 - 0 = 3$ (0,3) (1,3)

$4 - 1 = 3$ (1,4) (2,4)

$5 - 2 = 3$ (2,5) (3,5)

$6 - 3 = 3$ (3,6) (4,6)

$$cost(0,3) = \min_{k=1,2,3} \begin{Bmatrix} cost(0,0) + cost(1,3) \\ cost(0,1) + cost(2,3) \\ cost(0,2) + cost(3,3) \end{Bmatrix} + 0.35$$

$$= 0.55$$

$$cost(1,4) = \min_{k=2,3,4} \begin{Bmatrix} (1,1) + (2,4) \\ (1,2) + (3,4) \\ (1,3) + (4,4) \end{Bmatrix} + 0.5$$

$$= 0.8$$

$$cost(2,5) = \min_{k=3,4,5} \begin{Bmatrix} (2,2) + (3,5) \\ (2,3) + (4,5) \\ (2,4) + (5,5) \end{Bmatrix} + 0.75$$

$$= 1.25$$

$$cost(3,6) = \min_{k=4,5,6} \begin{Bmatrix} (3,3) + (4,6) \\ (3,4) + (5,6) \\ (3,5) + (6,6) \end{Bmatrix} + 0.65$$

$$= 1.$$

$j - i = 4$

$4 - 0 = 4$ (0,4) (1,4)

$5 - 1 = 4$ (1,5) (2,5)

$6 - 2 = 4$ (2,6) (3,6)

$$\text{cost}(0,4) = \min_{k=1,2,3,4} \begin{Bmatrix} (0,0) + (1,4) \\ (0,1) + (2,4) \\ (0,2) + (3,4) \\ (0,3) + (4,4) \end{Bmatrix} + 0.6$$

$= 1.05$

$$\text{cost}(1,5) = \min_{k=2,3,4,5} \begin{Bmatrix} (1,1) + (2,5) \\ (1,2) + (3,5) \\ (1,3) + (4,5) \\ (1,4) + (5,5) \end{Bmatrix} + 0.8$$

$= 1.4.$

$\text{cost}(2,6) = 1.55$

$\text{cost}(0,5) = 1.75$

$\text{cost}(1,6) = 1.7$

$$\text{cost}(0,6) = \min \begin{Bmatrix} (0,0) + (1,6) \\ (0,1) + (2,6) \\ (0,2) + (3,6) \\ (0,3) + (4,6) \\ (0,4) + (5,6) \\ (0,5) + (6,6) \end{Bmatrix} + 1$$

$= 2.05.$

16) Extend the following distance matrix to 7 cities

A: [0, 12, 10, 19, 8, 16]

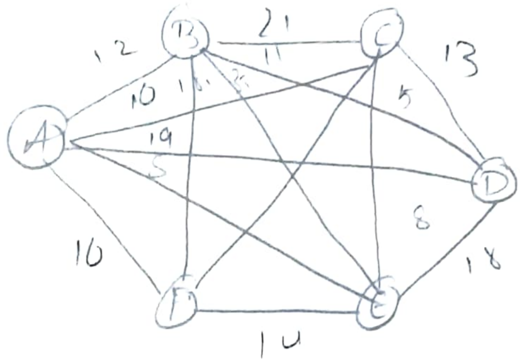
B: [12, 0, 21, 11, 15, 10]

C: [10, 21, 0, 13, 5, 7]

D: [19, 11, 13, 0, 18, 8]

E: [8, 15, 5, 18, 0, 14]

F: [16, 10, 7, 8, 14, 0]



O/P -> A - B - C - D - E - F = 83//.

17) Given a knapsack capacity of 70 units and the following items.

Item 1 :- W = 25, V = 80

Item 2 :- W = 35, V = 90

Item 3 :- W = 45, V = 120

Item 4 :- W = 30, V = 70

V/W	0	25	35	45	30	70
0	0	0	0	0	0	0
1	0	80	80	80	80	80
2	0	80	90	90	90	90
3	0	80	90	120	120	90
4	0	80	90	80	120	150

18) For a graph

A -> BA, W - 1

A -> CA, W - 4

B -> CB, W - 3

B->DB, w-2

B->EB, w=2 use Bellman-ford.

D->BD, w-1

D->CD, w-5

E->DE, w-3

A->B = -1 B->D = 2 D->C = 5

A->C = 4 B->E = 2 E->D = -3

B->C = 3 D->B = 1

v	A	B	C	D	E
d	0	∞	∞	∞	∞
p	-	-	-	-	-

①

v	A	B	C	D	E
d	0	-1	4	∞	∞
p	-	A	A	-	-

②

v	A	B	C	D	E
d	0	-1	4	∞	1
p	-	A	A	-	B

③

v	A	B	C	D	E
d	0	-1	4	3	1
p	-	A	A	E	B

④

v	A	B	C	D	E
d	0	-1	4	3	1
p	-	A	A	E	B

19) Find the expected value of the sound outcomes when rolling 3 four-sided dice.

Sum - 3 (1+1+1)

Sum 4 = $\frac{3}{64}$ (1+1+2, 1+2+1, 2+1+1)

5 = $\frac{6}{64}$ (1+1+3, 1+2+2, 1+3+1, 2+1+2, 2+2+1, 3+1+1)

6 = $\frac{10}{64}$ (1+2+3, 1+3+2, 2+1+3, 2+2+2, 2+3+1, 3+1+2, 3+2+1, 1+4+1, 2+2+2, 2+3+1)

$\frac{12}{64}$ = 7 (1+3+3, 2+2+3, 2+3+2, 3+1+3, 3+2+2, 3+3+1, 1+4+2,
2+3+2, 2+4+1, 3+2+2, 3+3+1, 4+1+2)

8 = 12/64

9 = 10/64

10 = 6/64

11 = 3/64

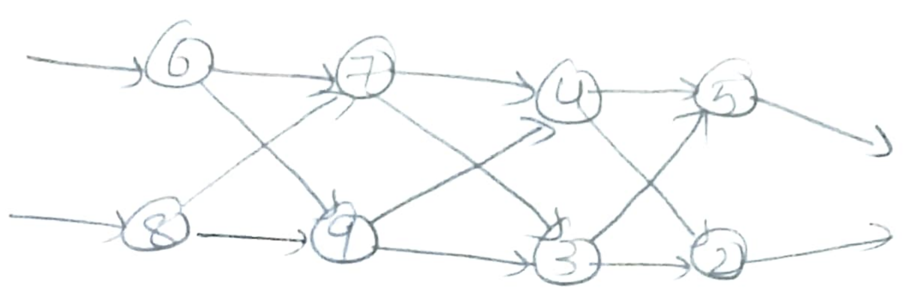
12 = 1/64

Σ (sum * probability).

$= \left(3 \times 1/64\right) + \left(4 \times \frac{3}{64}\right) + \left(5 \times \frac{6}{64}\right) + \left(6 \times \frac{10}{64}\right) + \left(7 \times \frac{12}{64}\right) + \left(8 \times \frac{12}{64}\right) + \left(9 \times \frac{10}{64}\right) + \left(10 \times \frac{6}{64}\right) + \left(11 \times \frac{3}{64}\right) + \left(12 \times \frac{1}{64}\right)$

$= \frac{480}{64} = 7.5$ //

20) calculate min.time for line1: [6, 7, 4, 5], line2: [8, 9, 3, 2] with transfer lines [4, 5, 6] 1 to 2 and [6, 5, 4] 2 to 1.



	1	2	3	4
$F_1(i)$	6	13	17	12
$F_2(i)$	8	17	20	22
$l_1(i)$	1	1	1	1
$l_2(i)$	2	2	2	2

21) Keys {10, 20, 30} have probabilities {0.2, 0.5, 0.3} do OBST.

K = {10, 20, 30}

V = {0.2, 0.5, 0.3}

$j - i = 3$

$3 - 0 = (0.3)$

$\text{cost}(0,3) = \min \begin{Bmatrix} 2.1 \\ 1.5 \\ 1.1 \end{Bmatrix}$

	0	1	2	3
0	0	0.2	0.7	1.1
1		0	0.5	1.1
2			0	0.3
3				0

22) using 5 cities

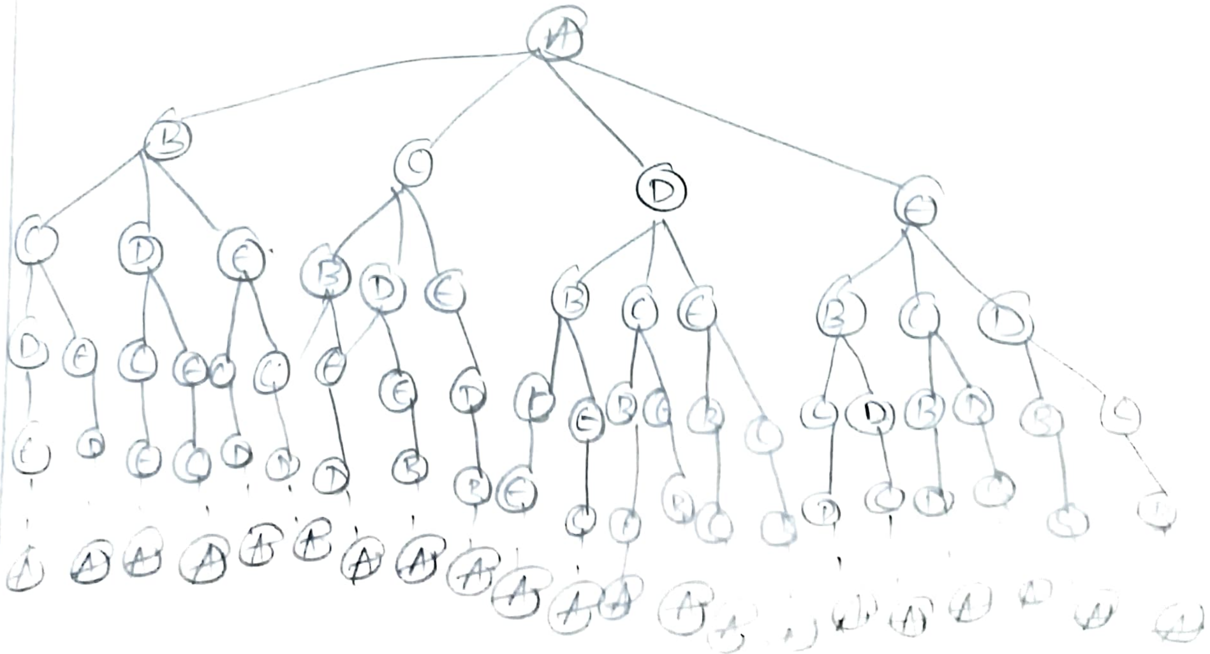
A: [0, 14, 4, 10, 20]

B: [14, 0, 7, 8, 7]

C: [4, 7, 0, 12, 6]

D: [10, 8, 12, 0, 15]

E: [20, 7, 6, 15, 0]



23) Knapsack 0/1 50 units.

I - 1 = $w=10, V=50$

I - 2 = $w=20, V=70$

I - 3 = $w=30, V=90$

I - 4 = $w=25, V=60$

I - 5 = $w=15, V=40$.

	0	10	20	30	25	15	50
0	0	0	0	0	0	0	0
1	0	50	50	50	50	50	50
2	0	50	70	70	70	70	70
3	0	50	70	90	90	90	160
4	0	50	70	90	90	90	160
5	0	50	70	90	90	90	160

24) Bellman-Ford

$1 \to 21, w=4$

$1 \to 31, w=5$

$2 \to 32, w=-2$

$3 \to 43, w=3$

$4 \to 24, w=-10$

V	1	2	3	4
d	0	∞	∞	∞
P	-	-	-	-

$1 \to 2 - 4$

$1 \to 3 - 5$

$2 \to 3 - 3$

$4 \to 2 = -10$

v	1	2	3	4
d	0	4	5	∞
p	-	1	1	-

v	1	2	3	4
d	0	4	2	∞
p	-	1	2	-

v	1	2	3	4
d	0	4	2	5
p	-	1	2	3

vertex	Dist	path
1	0	1
2	4	1->2
3	2	1->2->3
4	5	1->2->3->4

25) Roll six six-sided dice. Determine the no. of ways to get a sum of 18.

$x+x^2+x^3+x^4+x^5+x^6.$

$\Rightarrow x(1+x+x^2+x^3+x^4+x^5)$

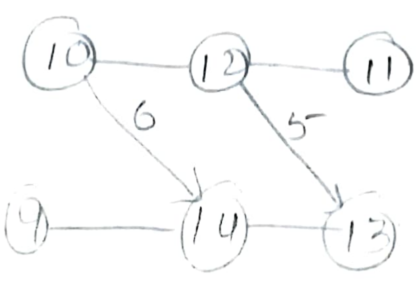
$= \dfrac{x(1+x^6)}{1-x}$

For six dice.

$\left(\dfrac{x(1-x^6)}{1-x}\right)^6 = x^6(1-x^6)^6(1-x)^{-6} = (x^{n18})$

$= 340$//

26) Given line 1: [10, 12, 11], line 2: [9, 14, 13], Transfer lines [6, 5], by 2 units.



Before reduction			After Red(2)		
l_1	6	5	l_1	4	5
l_2	30	30	l_2	28	27

27) For keys (8, 12, 16, 20, 24) with access possibilities {0.2, 0.05, 0.4, 0.25, 0.1}.

(8, 12, 16, 20, 24)

(0.2, 0.05, 0.4, 0.25, 0.1)

$j - i = 0$

$j - i = 1$

$j - i = 2$

$2 - 0 = [0, 2]$

$3 - 1 = [1, 3]$

$4 - 2 = [2, 4]$

$5 - 3 = [3, 5]$

	0	1	2	3	4	5
0	0	0.2	0.3	0.7	1.45	1.8
1		0	0.05	0.5	1	1.3
2			0	0.4	0.9	1.2
3				0	0.25	0.05
4					0	0.1
5						0

$$\text{Cost}(1,5) = \min \begin{Bmatrix} \text{Cost}(1,1) + \text{cost}(2,5) \\ \text{cost}(1,2) + \text{cost}(3,5) \\ \text{cost}(1,3) + \text{cost}(4,5) \\ \text{cost}(1,4) + \text{cost}(5,5) \end{Bmatrix} + 0.8$$

$$= \min \begin{Bmatrix} 0 + 1.2 \\ 0.05 + 0.45 \\ 0.5 + 0.1 \\ 1 + 0 \end{Bmatrix} + 0.8$$

$$= 1.3$$

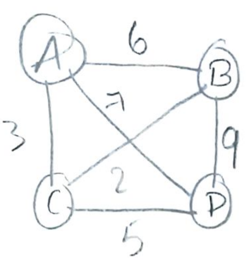
28) Solve TSP 4 cities

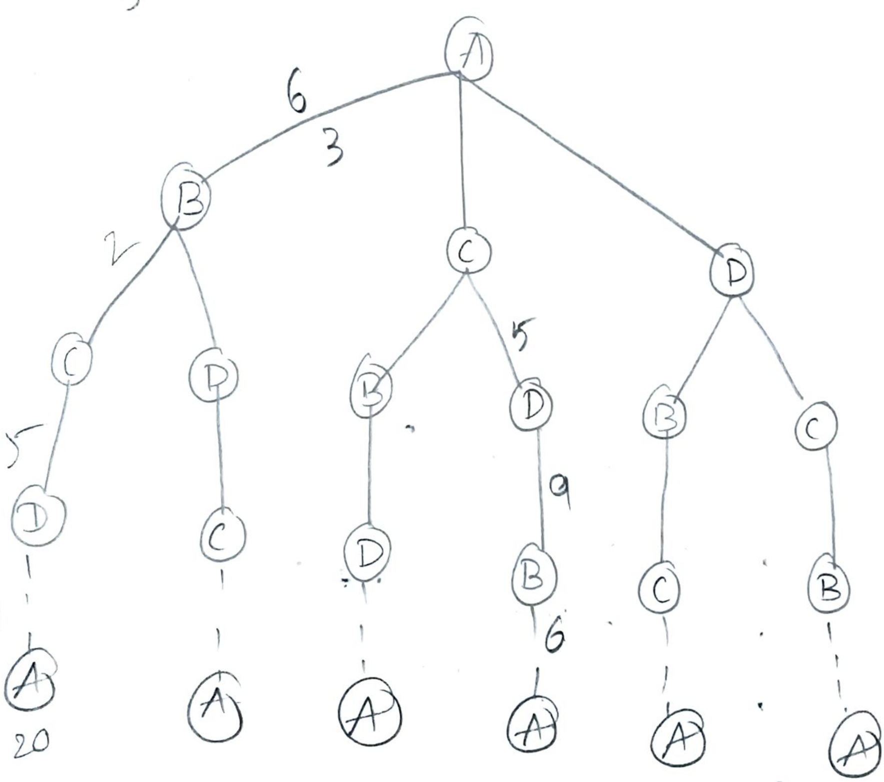
A : [0,6,3,7]

B : [6,0,2,9]

C : [3,2,0,5]

D : [7,9,5,0]





A - B - C - D - A - 20 } min optimal path.
A - D - C - B - A - 20 }

29) Knapsack 0/1 50 units

I_1, W=10, V=60

I_2, W=20, V=100

I_3, $w=30$, $V=120$

I_4, $w=40$, $V=200$

v\w	0	10	20	30	40	50
0	0	0	0	0	0	0
1	0	60	60	60	60	60
2	0	60	100	160	160	160
3	0	60	100	120	180	180
4	0	60	100	120	200	260

30) Bellman- Ford

A -> BA, w=6

A -> DA, w=7

A -> CB, w=5

B -> EB, w=4

B -> DB, w=8

C -> BC, w=-2

D -> CD, w=-3

D -> ED, w=-9

E -> FE, w=7

F -> CF, w=2

v	A	B	C	D	E	F
d	0	∞	∞	∞	∞	∞
p	-	-	-	-	-	-

1)

v	A	B	C	D	E	F
d	0	6	4	7	2	9
p	-	A	D	A	B	E

(2)

V	A	B	C	D	E	F
d	0	2	u	7	2	9
P	-	C	D	A	B	E

(3)

V	A	B	C	D	E	F
d	0	2	u	7	2	9
P	-	C	D	A	B	E

(4)

V	A	B	C	D	E	F
d	0	2	u	7	2	9
P	-	C	D	A	B	E

(5)

V	A	B	C	D	E	F
d	0	2	4	7	2	9
P	-	C	D	A	B	E

Vertex	Dist	path
A	0	A
B	2	A-D-C-B
C	u	A-D-C
D	7	A-D
E	2	A-D-C-B-E
F	9	A-D-C-B-E-F