

THE PARTICLE SWARM

OPTIMIZATION PROBLEM



CONTENTS

- Origins
- Concept
- PSO Algorithm

ORIGINS



- Inspired from the nature social behavior and dynamic movements with communications of insects, birds and fish.
- Proposed by James Kennedy & Russell Eberhart (1995)
- Combines self-experiences with social experiences



IN 1986, CRAIG REYNOLDS DESCRIBED THIS PROCESS IN 3 SIMPLE BEHAVIORS:

Separation

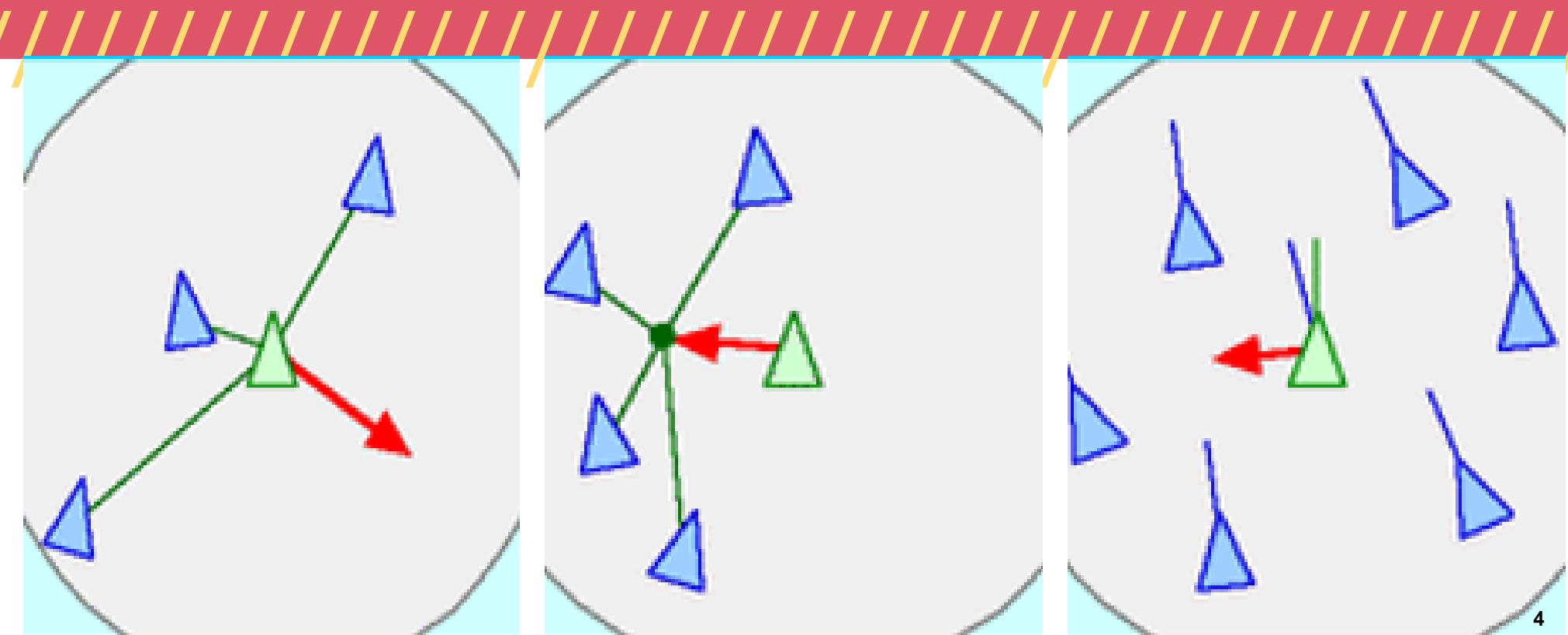
Alignment

Cohesion

avoid crowding local flockmates

move towards the average heading of local flockmates

move toward the average position of local flockmates



CONCEPT



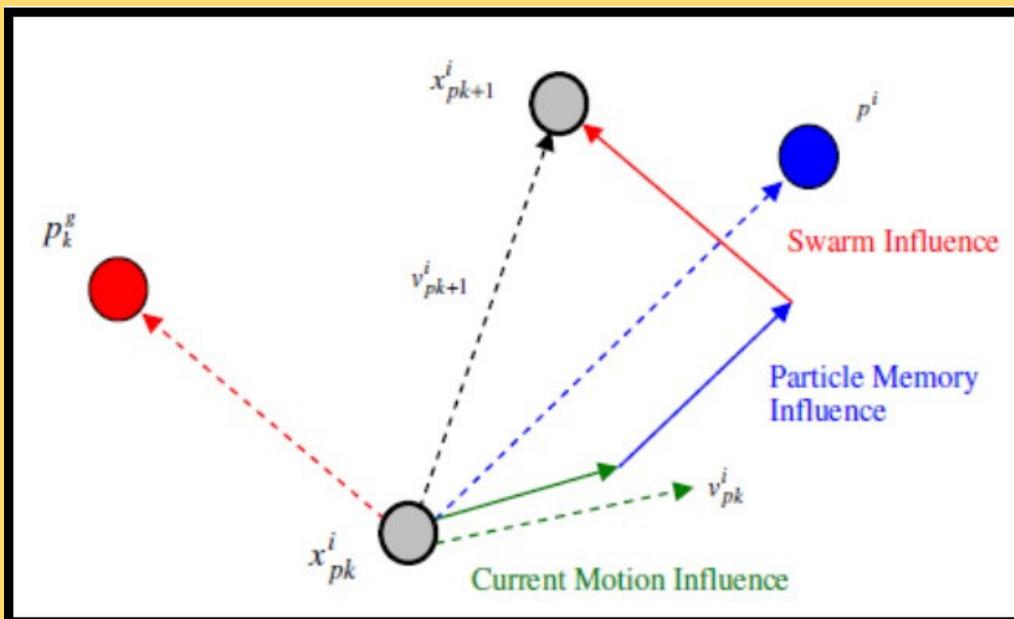
- Uses a number of agents (particles) that constitute a swarm moving around in the search space looking for the best solution
- Each particle in search space adjusts its “flying” according to its own flying experience as well as the flying experience of other particles

CONCEPT



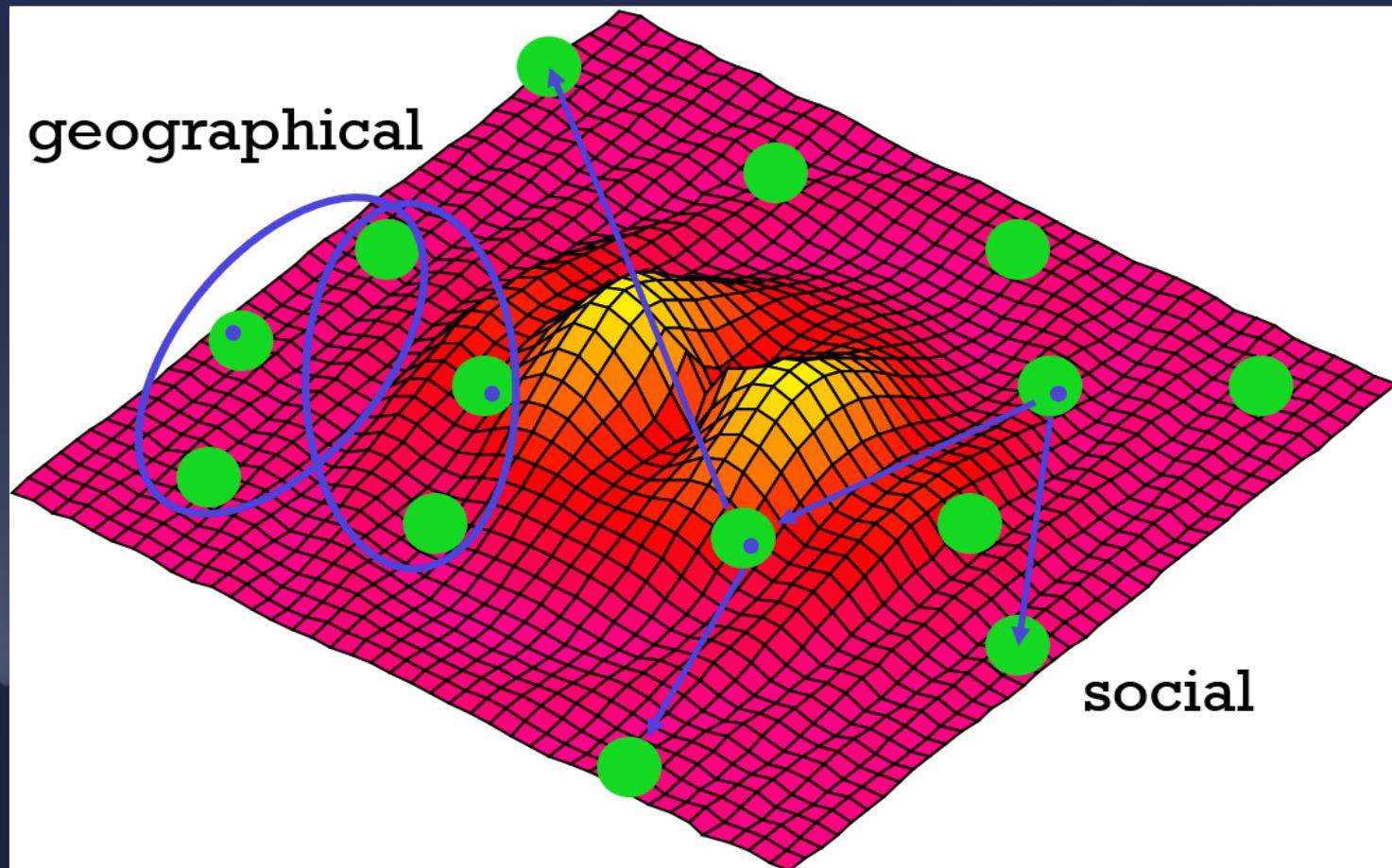
- Search area - Possible solutions
- Movement towards a promising area to get the global optimum
- Each particle keeps track:its best solution, personal best, pbest and the best value of any particle, global best, gbest

CONCEPT

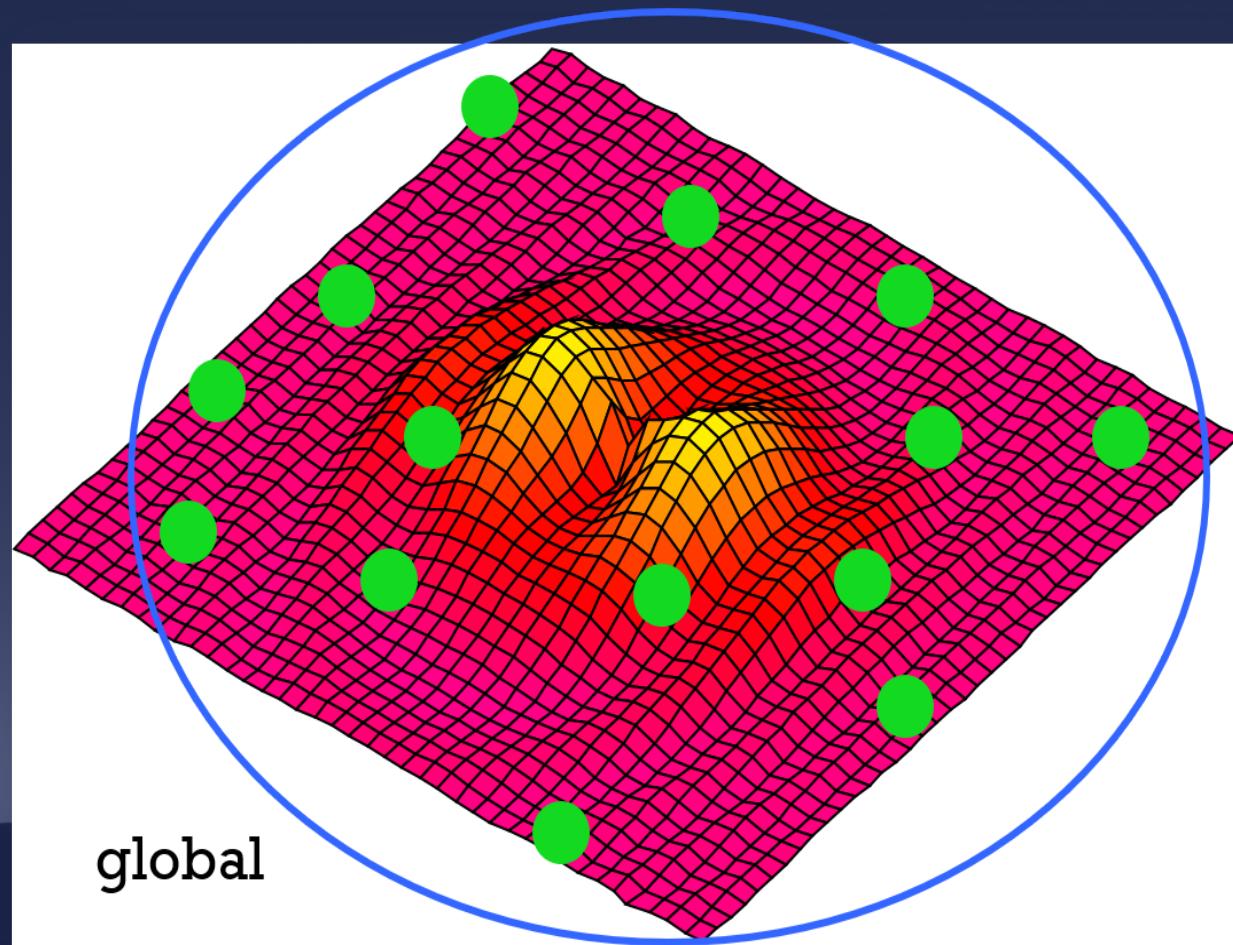


- Each particle adjusts its travelling speed dynamically corresponding to the flying experiences of itself and its colleagues. Each particle modifies its position according to:
 - its current position
 - its current velocity
 - the distance between its current position and pbest
 - the distance between its current position and gbest

ALGORITHM-NEIGHBORHOOD



ALGORITHM-NEIGHBORHOOD



ALGORITHM PARAMETERS

A : Population of agents

p_i : Position of agent a_i in the solution space

f : Objective function

v_i : Velocity of agent's a_i

$V(a_i)$: Neighborhood of agent a_i (fixed)

The neighborhood concept in PSO is not the same as the one used in other meta-heuristics search, since in PSO each particle's neighborhood never changes (is fixed)

ALGORITHM

```
[x*] = PSO()
P = Particle_Initialization();
For i=1 to it_max
    For each particle p in P do
        fp = f(p);
        If fp is better than f(pBest)
            pBest = p;
        end
    end
    gBest = best p in P;
    For each particle p in P do
        v = v + c1*rand*(pBest - p) + c2*rand*(gBest - p);
        p = p + v;
    end
end
```

ALGORITHM



Particle update rule

$$p = p + v$$

with

$$v = v + c1 * \text{rand} * (p\text{Best} - p) + c2 * \text{rand} * (g\text{Best} - p)$$

where

p: particle's position

v: path direction

c1: weight of local information

c2: weight of global information

pBest: best position of the particle

gBest: best position of the swarm

rand: random variable

ALGORITHM PARAMETERS



- Number of particles usually between 10 and 50
- C1 is the importance of personal best value
- C2 is the importance of neighborhood best value
- Usually $C1 + C2 = 4$ (empirically chosen value)
- If velocity is too low → algorithm too slow
- If velocity is too high → algorithm too unstable

ALGORITHM



- Create a 'population' of agents (particles) uniformly distributed over X
- Evaluate each particle's position according to the objective function
- If a particle's current position is better than its previous best position, update it
- Determine the best particle (according to the particle's previous best positions)

ALGORITHM

- Update particles' velocities:

$$\mathbf{v}_i^{t+1} = \underbrace{\mathbf{v}_i^t}_{inertia} + \underbrace{c_1 \mathbf{U}_1^t (\mathbf{pb}_i^t - \mathbf{p}_i^t)}_{personal\ influence} + \underbrace{c_2 \mathbf{U}_2^t (\mathbf{gb}^t - \mathbf{p}_i^t)}_{social\ influence}$$

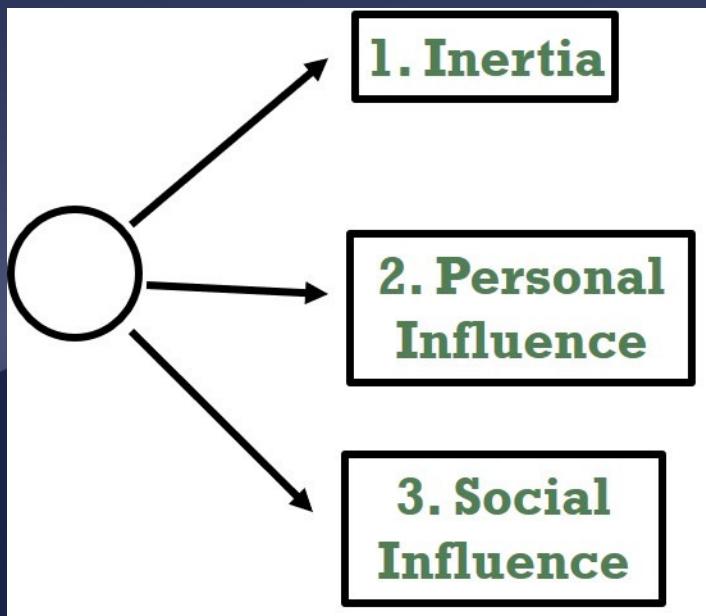
- Move particles to their new positions:

$$\mathbf{p}_i^{t+1} = \mathbf{p}_i^t + \mathbf{v}_i^{t+1}$$

- Go to step 2 until stopping criteria are satisfied.

ALGORITHM

$$\mathbf{v}_i^{t+1} = \underbrace{\mathbf{v}_i^t}_{inertia} + \underbrace{c_1 \mathbf{U}_1^t (\mathbf{pb}_i^t - \mathbf{p}_i^t)}_{personal\ influence} + \underbrace{c_2 \mathbf{U}_2^t (\mathbf{gb}^t - \mathbf{p}_i^t)}_{social\ influence}$$



Makes the particle move in the same direction and with the same velocity

Improves the individual, Makes the particle return to a previous position, better than the current, Conservative

Makes the particle follow the best neighbors direction

ALGORITHM



- Intensification: explores the previous solutions, finds the best solution of a given region
- Diversification: searches new solutions, finds the regions with potentially the best solutions

$$\mathbf{v}_i^{t+1} = \mathbf{v}_i^t + \underbrace{\mathbf{c}_1 \mathbf{U}_1^t (\mathbf{pb}_i^t - \mathbf{p}_i^t)}_{\text{Diversification}} + \underbrace{\mathbf{c}_2 \mathbf{U}_2^t (\mathbf{gb}^t - \mathbf{p}_i^t)}_{\text{Intensification}}$$

ALGORITHM CHARACTERISTICS



ADVANTAGES

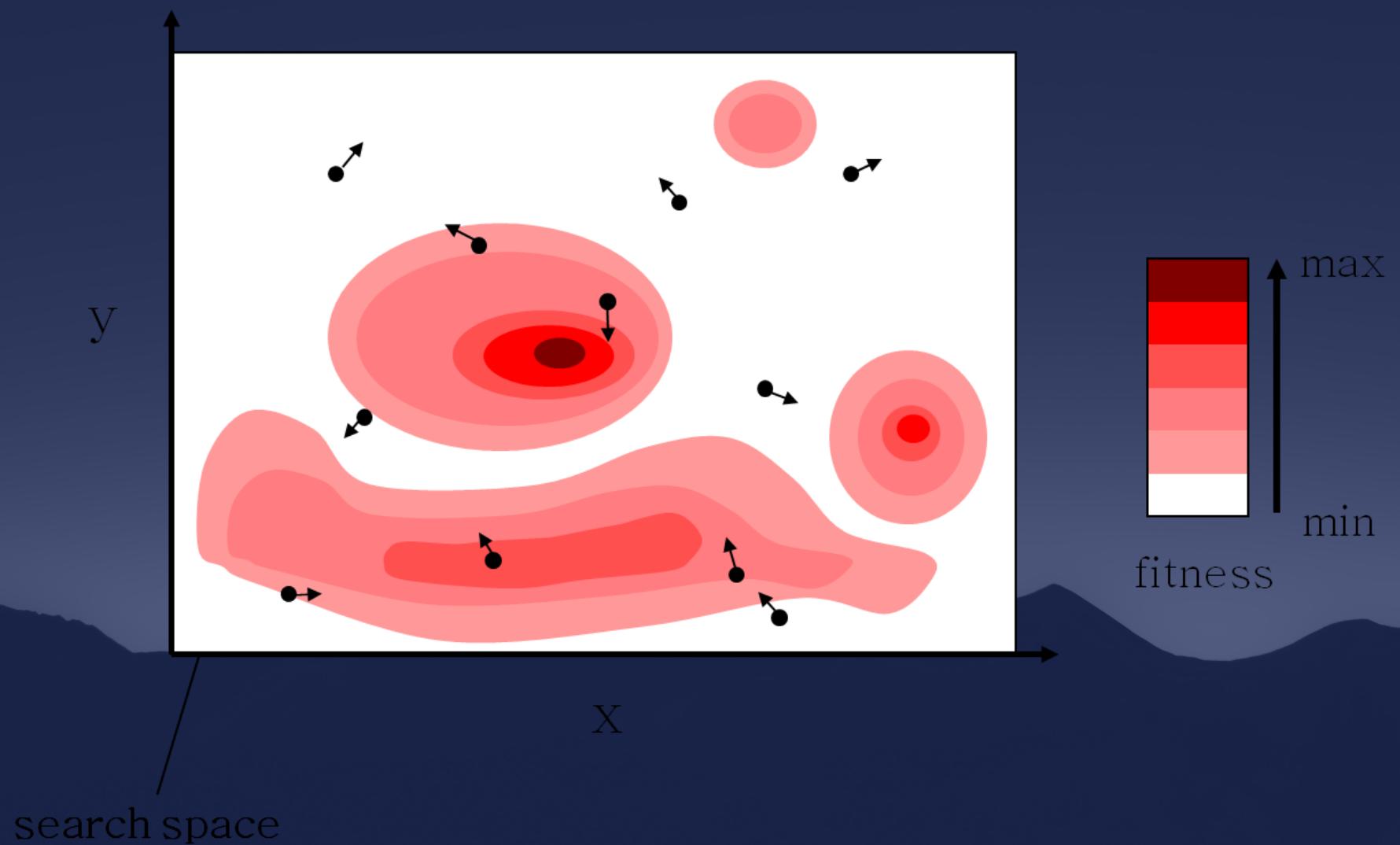
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- Inensitive to scaling of design variables
- Simple implementation
- Easily parallelized for concurrent processing
- Derivative free
- Very few algorithm parameters
- Very efficient global search algorithm

DISADVANTAGES

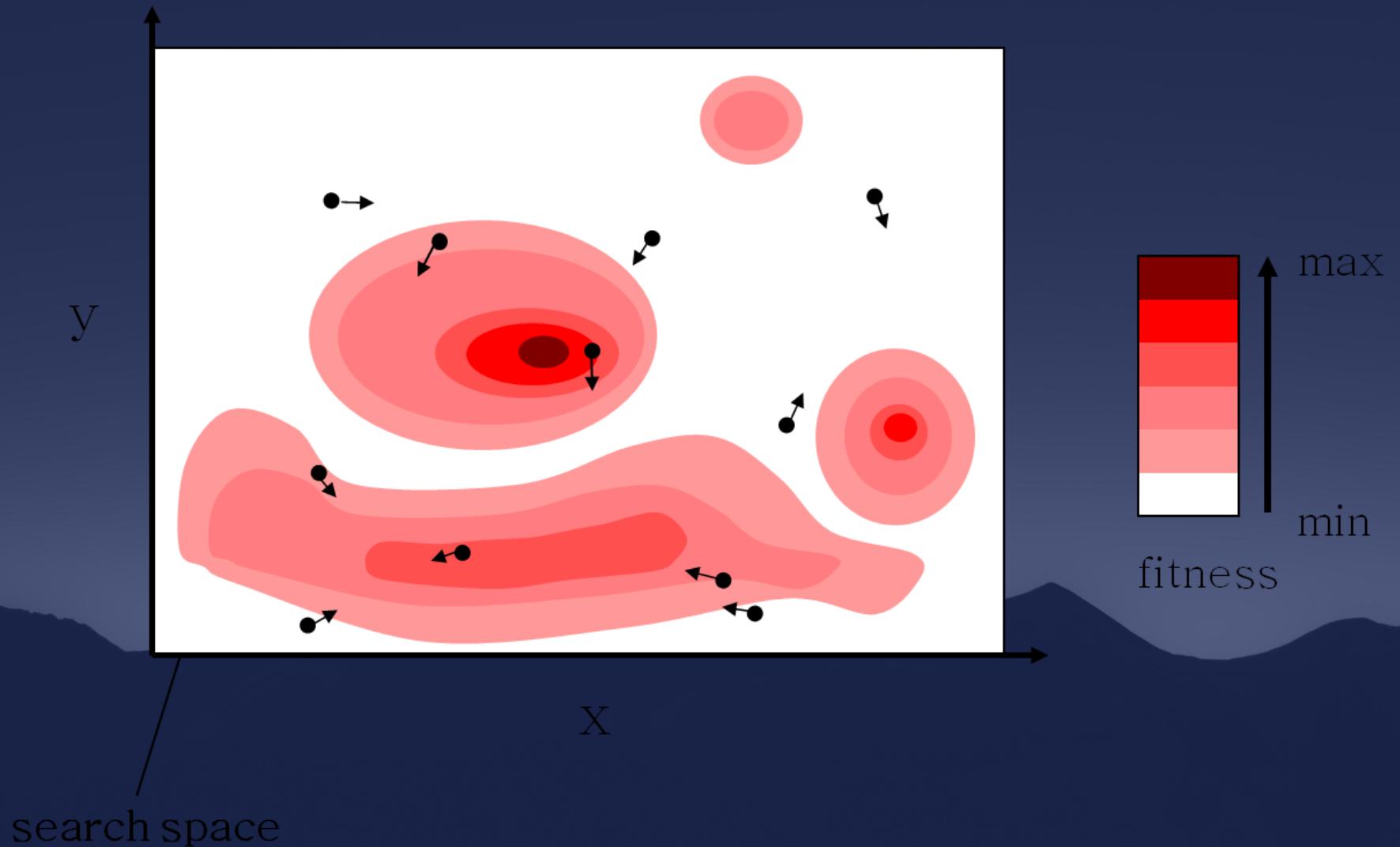


- Tendency to a fast and premature convergence in mid optimum points
- Slow convergence in refined search stage (weak local search ability)

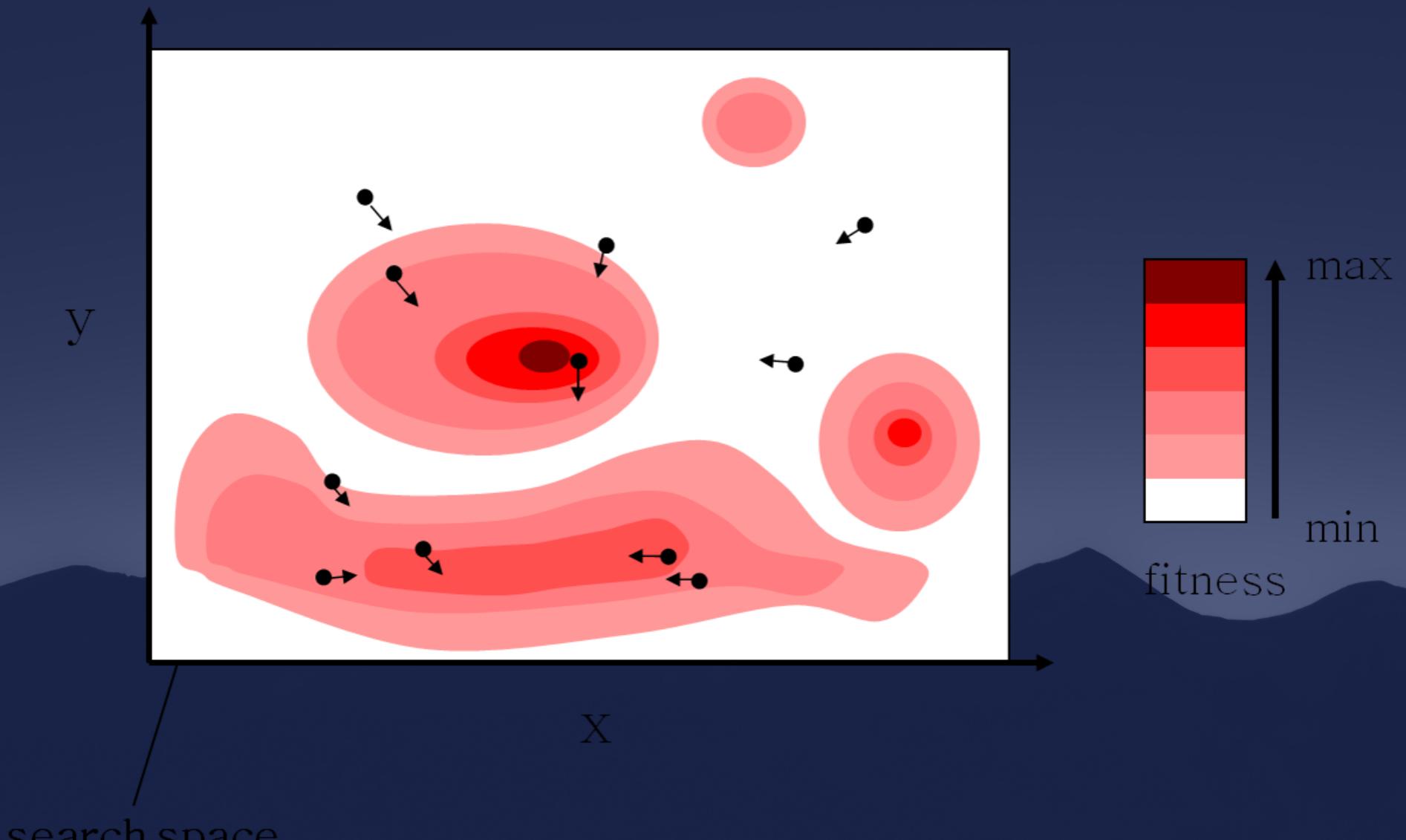
EXAMPLE



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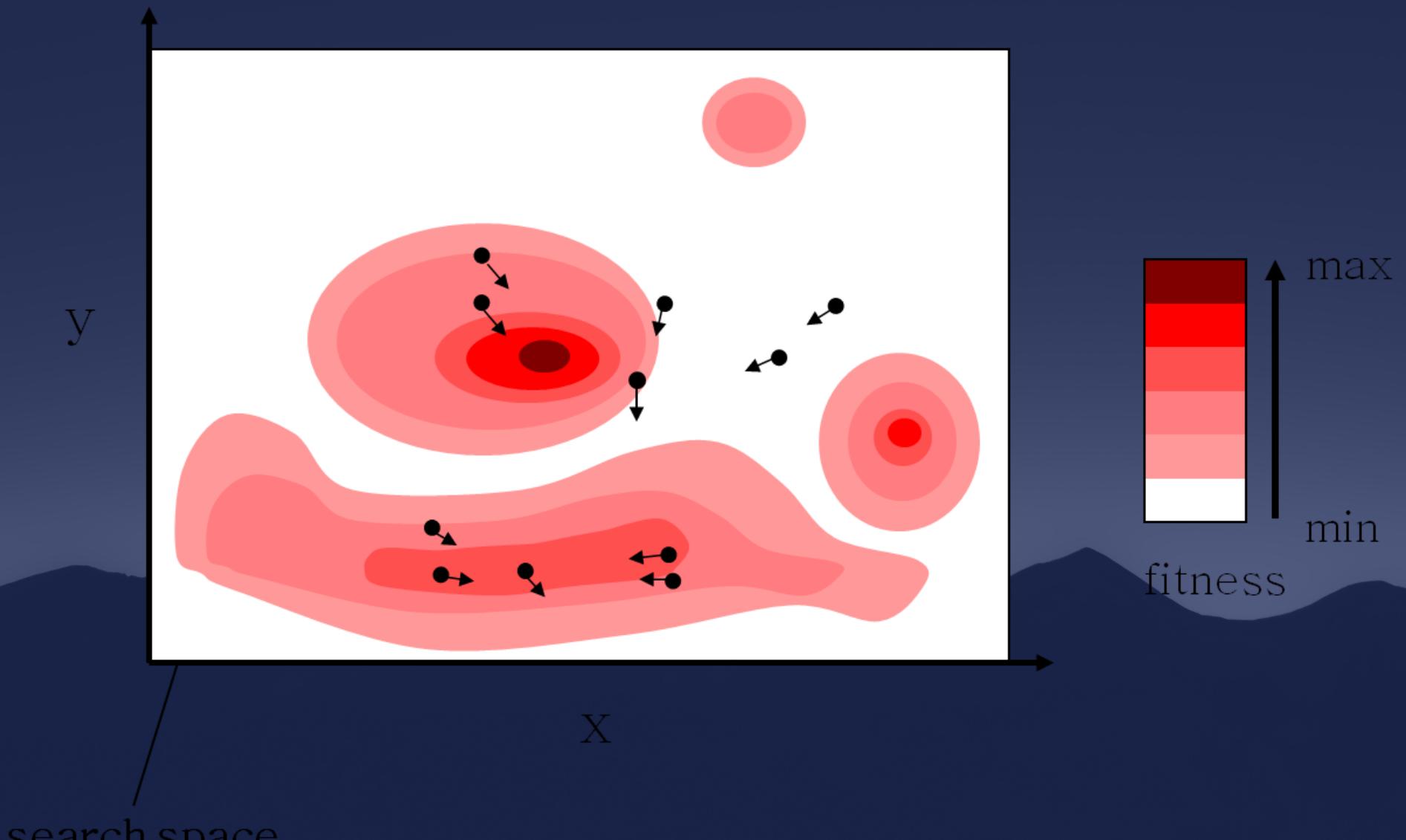


EXAMPLE



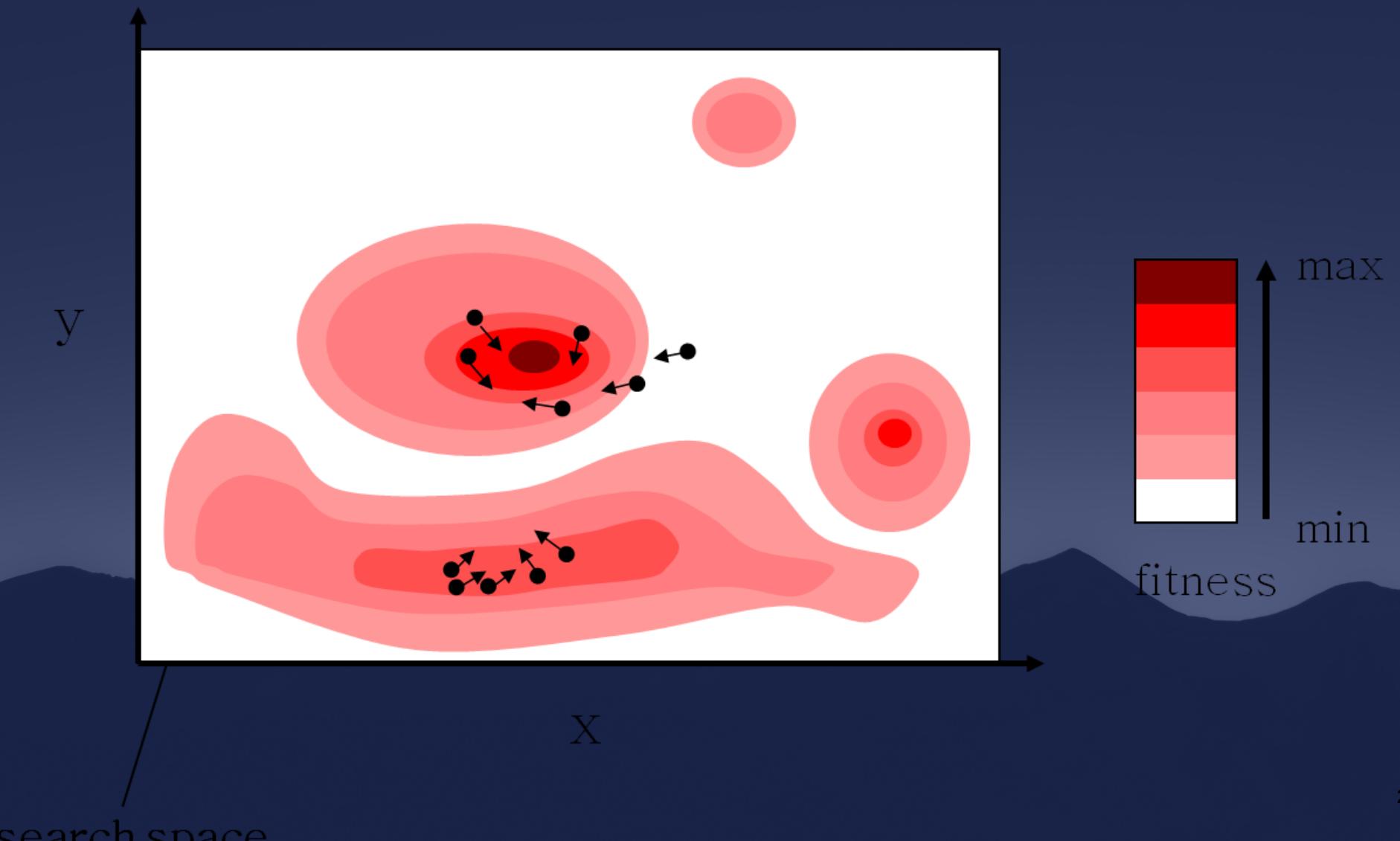
search space

EXAMPLE

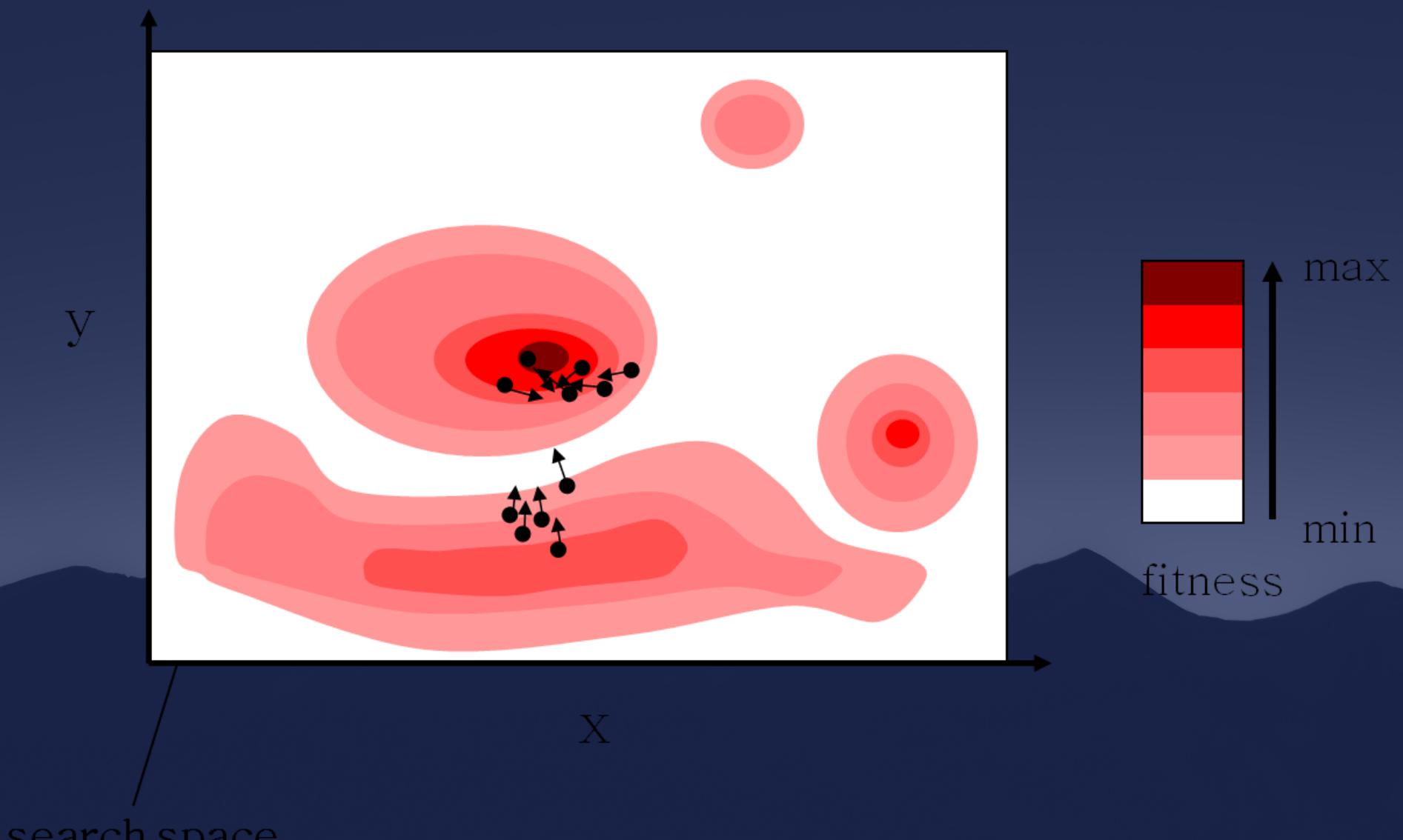


search space

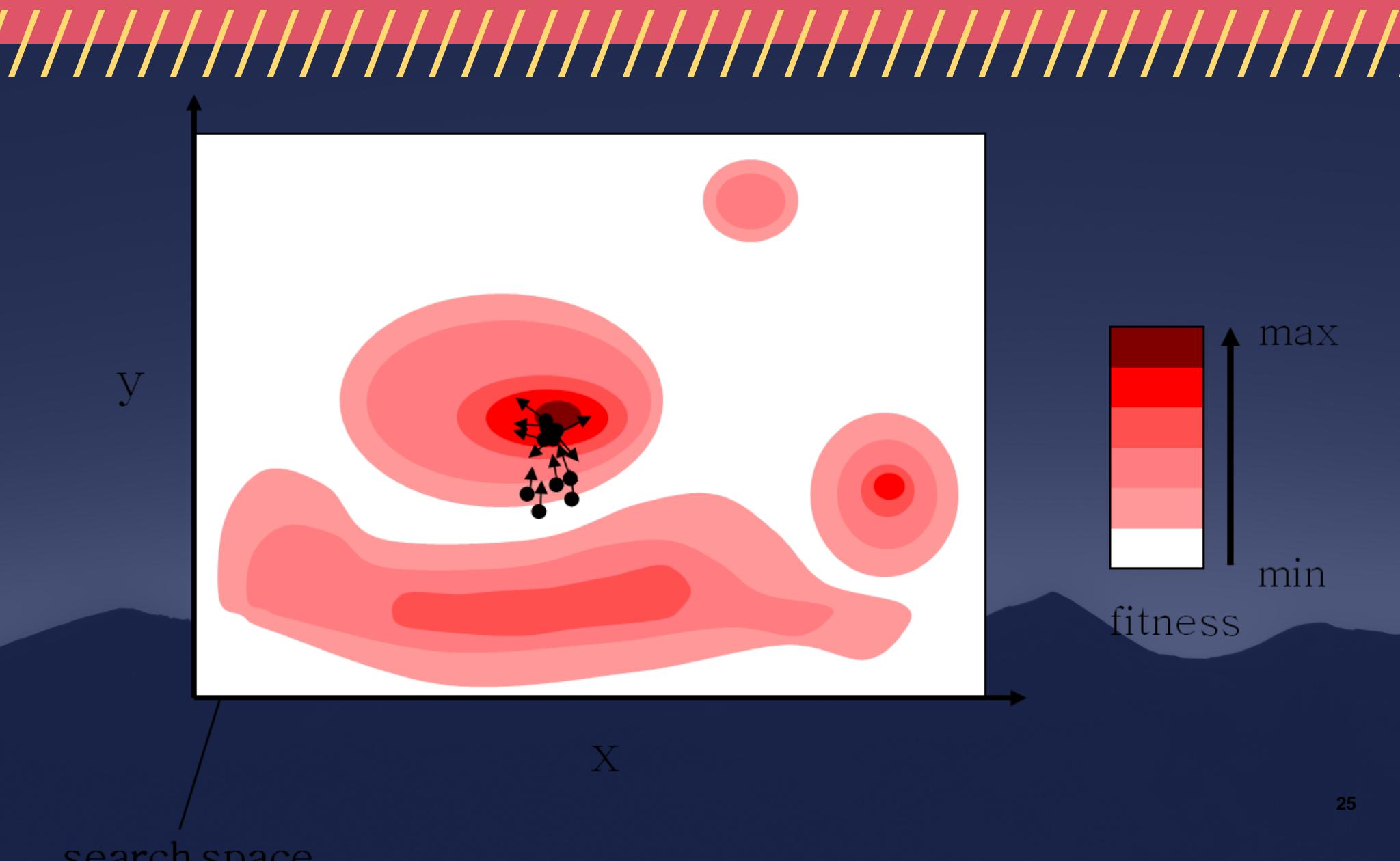
EXAMPLE



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