PERFORMANCE

Term	Remarks	Staff Member's Signature
I	Completed	23/10/19
II	Completed 09/10	01/02/2020

Exam Seat No.____________

THAKUR COLLEGE OF SCIENCE & COMMERCE

NAAC Accredited with Grade "A" (3rd Cycle)

THAKUR TRUSTS

ISO 9001 : 2015 Certified

Degree College

Computer Journal

CERTIFICATE

SEMESTER 'I' **UID No.** ____________

Class F.Y.B.Sc **Roll No.** 1803 **Year** 2019 - 2020

This is to certify that the work entered in this journal is the work of ~~Mst.~~ / Ms. Hemant Saw

who has worked for the year 2019 - 2020 in the Computer Laboratory.

Singh 23/10/19

Teacher In-Charge

Head of Department

Date :____________

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PRACTICAL - 1

LIMITS & CONTINUITY

Q.1) $\lim_{x\to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

$\rightarrow$ $= \lim_{x\to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \dfrac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}} \times \dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$

$= \lim_{x\to a} \dfrac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$

$= \lim_{x\to a} \dfrac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+3\sqrt{x})}$

$= \dfrac{1}{3}\lim_{x\to a} \dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+3\sqrt{x}}$

$= \dfrac{1}{3}\,\dfrac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+3\sqrt{a}} = \dfrac{1}{3}\times\dfrac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+3\sqrt{a}}$

$= \dfrac{1}{3}\times\dfrac{4\sqrt{a}}{\sqrt{a}(\sqrt{3}+3)}$

$= \dfrac{2}{3\sqrt{3}}$

Q.2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$\rightarrow = \lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$

$= \lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$

$= \lim_{y \to 0} \frac{y}{y\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$

$= \lim_{y \to 0} \frac{1}{\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$

$= \frac{1}{\sqrt{a}\,(2\sqrt{a})} = \frac{1}{2a}$ //

Q.3) $\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

$\rightarrow$ By Substituting $x - \frac{\pi}{6} = h$

$x = h + \pi/6$

when $h \to 0$

$= \lim_{h \to 0} \frac{\cos(h+\pi/6) - \sqrt{3}\sin(h+\pi/6)}{\pi - 6 \times (h + \frac{\pi}{6})}$

{ Using $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$
$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$ }

$= \lim_{h \to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \cdot \sin\frac{\pi}{6} - \sqrt{3}\sin h \cdot \cos\frac{\pi}{6} + \cos h \cdot \sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$

$= \lim_{h \to 0} \frac{\cos h \cdot \frac{\sqrt{3}}{2} - \sin h \times \frac{1}{2} - \sqrt{3}\left(\sin h \times \sqrt{3}/2 + \cos h \times \frac{1}{2}\right)}{\pi - 6h - \pi}$

$= \lim_{h \to 0} \frac{\cos h \frac{\sqrt{3}}{2} h - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\frac{\sqrt{3}}{2}h}{-6h}$

$= \lim_{h \to 0} \frac{-\sin\frac{4h}{2}}{-6h} = \lim_{h \to 0} \frac{\sin 4h}{12h} = \frac{1}{3}\lim_{h \to 0} \frac{\sin h}{h}$

$= \frac{1}{3}$ //

Q.4) $\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

$\rightarrow$ By rationalizing Numerator & Denominator both

$= \lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \right]$

$= \lim_{x \to \infty} \left[\frac{(\cancel{x^2}+5 - \cancel{x^2}+3) \times \sqrt{x^2+3} + \sqrt{x^2+1}}{(\cancel{x^2}+3 - \cancel{x^2}+1) \times \sqrt{x^2+5} + \sqrt{x^2-3}} \right]$

$= \lim_{x \to \infty} \frac{8\,(\sqrt{x^2+3} + \sqrt{x^2+1})}{2\,(\sqrt{x^2+5} + \sqrt{x^2-3})}$

$= 4 \lim_{x \to \infty} \frac{\sqrt{x^2(1+3/x^2)} + x^2(1+1/x^2)}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2)}}$

after applying limit we get $4/$

Q(b) i) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \frac{\pi}{2}$

$= \dfrac{\cos x}{\pi - 2x}$, for $\frac{\pi}{2} < x < \pi$ } at $x = \frac{\pi}{2}$

$f(\frac{\pi}{2}) = \dfrac{\sin 2(\frac{\pi}{2})}{\sqrt{1-\cos(2\pi/2)}}$ $\therefore f(\pi/2) = 0$

f at $x = \frac{\pi}{2}$ define

$\lim\limits_{x \to \pi/2} f(x) = \lim\limits_{x \to \pi/2^+} \dfrac{\cos x}{\pi - 2x}$

By substituting method, $x - \frac{\pi}{2} = h$

$x = h + \frac{\pi}{2}$, where $h \to 0$

$= \lim\limits_{h \to 0} \dfrac{\cos(h+\pi/2)}{\pi - 2(h+\pi/2)} = \lim\limits_{h \to 0} \dfrac{\cos(h+\pi/2)}{\pi - 2(h+\frac{\pi}{2})}$

$= \lim\limits_{h \to 0} \dfrac{\cos(h+\pi/2)}{-2h}$

{By using $\cos(A+B) = \cos A \cos B - \sin A \cdot \sin B$}

$= \lim\limits_{h \to 0} \dfrac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \cdot \sin\frac{\pi}{2}}{-2h}$

$= \lim\limits_{h \to 0} \dfrac{-\sin h}{-2h} = \frac{1}{2} \lim\limits_{h \to 0} \dfrac{\sin h}{h} = 1/2$

ii) $\lim\limits_{x \to \pi/2} f(x) = \lim\limits_{x \to \pi/2^-} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$ {using $\sin 2x = 2\sin x \cdot \cos x$}

$= \lim\limits_{x \to \pi/2^-} \dfrac{2\sin x \cos x}{\sqrt{2}\ \sin^2 x}$

$= \lim\limits_{x \to \pi/2^-} \dfrac{2\cos x}{\sqrt{2}} = \lim\limits_{x \to \pi/2^-} \cos x$

$\therefore$ LHL $\ne$ RHL

$\therefore f$ is not Continue at $x = \pi/2$.

$f(x) = \dfrac{x^2-9}{x-3}$, $0 < x < 3$

$= x + 3$, $3 \le x \le 6$

$= \dfrac{x^2-9}{x+3}$, $6 \le x < 9$ } at $x = 3$ & $x = 6$

at $x = 3$

$\rightarrow$ ~~$\lim\limits_{x \to 6^+} (x-3) = 6 - 3 = 3$~~

$f(3) = \dfrac{x^2-9}{x-3} = 0$

$\therefore f$ at $x = 3$ define

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} x + 3$

$f(3) = x + 3 = 3 + 3 = 6$

f is define at $x = 3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3}$

$\therefore LHL = RHL$

$\therefore f$ is Continuous at $x=3$.

for $x=6$,

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

$\rightarrow \lim_{x\to 6^+} = \frac{x^2-9}{x+3} = \lim_{x\to 6^+} \frac{(x-3)\cdot\cancel{(x+3)}}{\cancel{x+3}}$

$\therefore \lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\therefore \lim_{x\to 6^-} x+3 = 3+6 = 9$

$\therefore LHL \neq RHL$

$\therefore$ fore is not Continuous.

9) i) $f(x) = \left.\begin{array}{ll} \frac{1-\cos 4x}{x^2} & , x<0 \\ = k & , x=0 \end{array}\right\}$ at $x=0$

$\Rightarrow$ f is Continuous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$= \lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$

$= 2 \lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$

$= 2 \lim_{x\to 0} \left(\frac{\sin 2x}{x^2}\right) = k$

$\Rightarrow 2(2)^2 = k$

$\Rightarrow k = 8$

ii) $f(x) = \left.\begin{array}{ll} (\sec^2 x)^{\cot^2 x} & , x\neq 0 \\ = k & , x=0 \end{array}\right\}$ at $x=0$

Sol^n) $f(x) = (\sec^2 x)^{\cot^2 x}$

{By using $(\tan^2 x - \sec^2 x = 1)$

& $\cot^2 x = 1/\tan^2 x$

$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x}$

We know that, $\lim_{x\to 0} (1+Px)^{1/Px} = e$

$= e$

$\therefore k = e$

niii.

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$, $x \neq \pi/3$
$= K$, $x = \pi/3$ } at $x = \pi/3$

$x - \pi/3 = h \rightarrow x = h + \pi/3$; where $h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

{ By using, $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$ }

$$\lim_{h\to 0} \frac{\sqrt{3} - \frac{\tan \pi/3 + \tan h}{1 - \tan \frac{\pi}{3} \cdot \tan h}}{\cancel{\pi - \pi} - 3h}$$

$$\lim_{h\to 0} \frac{\frac{\sqrt{3}\left(1 - \tan\frac{\pi}{3} \cdot \tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3} \cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\frac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{1 - \sqrt{3} \cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)} = \lim_{h\to 0} \frac{4 - \tan h}{3h(1 - \sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{(1 - \sqrt{3}\tan h)}$$

$$= \frac{4}{3} \frac{1}{(1 - \sqrt{3}(0))}$$

$$= \frac{4}{3}$$

Q7) i) $f(x) = \frac{1 - \cos 3x}{x \tan x}$, $x \neq 0$
$= 9$, $x = 0$ } at $x = 0$.

$$f(x) = \frac{1 - \cos 3x}{x \tan x}$$

$$\lim_{x\to 0} \frac{-1 \sin^2 3/2\, x}{x \tan x}$$

$$\lim_{x\to 0} = \frac{\frac{2 \sin^2 \frac{3x}{2}}{x^2}}{x \cdot \frac{\tan x}{x^2}}$$

$$= 2 \lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1} = 2 \times \frac{9}{4} = \frac{9}{2}$$

$$\lim_{x\to 0} f(x) = \frac{9}{2} \;, \; 9 = f(0)$$

∴ f is not Continuous at $x = 0$.

Redefine function;

$$f(x) = \begin{cases} \frac{1 - \cos 3x}{x \tan x} , & x \neq 0 \\ \frac{9}{2} & , x = 0 \end{cases}$$

Now, $\lim_{x \to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$.

ii) $f(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x^{\circ}}{x^2}, & x \neq 0 \\ = \dfrac{\pi}{6}, & x = 0 \end{cases}$ at $x=0$.

$\rightarrow$ $$= \frac{\lim_{x \to 0} (e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$= \lim_{x \to 0} \frac{e^{3x}-1}{x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$= \lim_{x \to 0} \frac{3 \cdot e^{3x}-1}{3x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$= 3 \cdot \lim_{x \to 0} \frac{e^{3x}-1}{3x} \lim_{x \to 0} \sin\left(\frac{\pi x}{180}\right)/x$$

$$= 3 \log_e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

$\therefore$ f is Continuous at $x=0$.

Q.8) $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$, $x=0$

is Continuous at $x=0$

$\rightarrow$ Given:- $\lim_{x \to 0} f(x) = f(0)$

$$= \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$= \lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$$

$$= \lim_{x \to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x \to 0} \frac{1-\cos x}{x^2}$$

$$= \log e + \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$$

$$= \log e + 2\lim_{x \to 0} \left(\frac{\sin x/2}{x}\right)^2$$

Multiply with 2 on Num. & Deno.;

$$= 1 + 2 \times \frac{1}{4} = 3/2 = f(0)$$

No.
1.
2.
3.
4.
5.
6.

Q) $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$, $x \neq \pi/2$

$f(x)$ is Continuous at $x = \pi/2$

$= \lim_{x\to\frac{\pi}{2}} \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \dfrac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$= \lim_{x\to\pi/2} \dfrac{2 - 1 + \sin x}{\cos^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \dfrac{1 + \sin x}{1 - \sin^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \dfrac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}\,(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \dfrac{1}{(1-\sin x)\,(\sqrt{2} + \sqrt{1+\sin x})}$

$= \dfrac{1}{2(\sqrt{2} + \sqrt{2})}$

$= \dfrac{1}{4\sqrt{2}}$

$\therefore f(\pi/2) = \dfrac{1}{4\sqrt{2}}$

PRACTICAL - 2

Q.1) Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

i) $\cot x$.

$f(x) = \cot x$

$Df(a) = \lim_{x\to a} \dfrac{f(x) - f(a)}{x-a} = \lim_{x\to a} \dfrac{\cot x - \cot a}{x-a}$

$= \lim_{x\to a} \dfrac{1/\tan x - 1/\tan a}{x-a}$

$= \lim_{x\to 0} \dfrac{\tan a - \tan x}{(x-a)\tan x \cdot \tan a}$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$Df(h) = \lim_{h\to 0} \dfrac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$

$= \lim_{h\to 0} \dfrac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$

formula: $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A-B)\,(1 + \tan A \cdot \tan B)$

$= \lim_{h\to 0} \tan(a - a - h) \cdot (1 + \tan a + \tan(a+h))$

$$= \lim_{h\to 0} \frac{\tan h}{h} \times \frac{1+\tan a \cdot \tan(a+h)}{\tan(a+h)\cdot \tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= -\frac{\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore Df(a) = -\operatorname{cosec}^2 a$

$\therefore f$ is differentiable $\forall a \in R$.

ii) $\operatorname{cosec} x$.

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

Formula:

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \cdot \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)} = -\frac{\cos a}{\sin^2 a}$$

$$= -\cot a \cdot \operatorname{cosec} a$$

iii) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a} = \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cdot \cos x}$$

put $x-a = h$

$x = a+h$

as $x\to a$, $h\to 0$

$$Df(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cdot \cos(a+h)}$$

formula:- $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cdot \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)} = -\frac{1}{2} \times -2 \frac{\sin a}{\cos x \cos a}$$

$$= \tan a \cdot \sec a$$

Q.2) If $f(x) = 4x+1$, $x \le 2$

$= x^2+5$, $x > 2$ at $x=2$ then find function is differentiable or not.

Sol^n) LHD:

$$Dh(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-(4\cdot 2+1)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2} = \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4(x-2)}{x-2} = 4$$

$Df(2^-) = 4$

RHD

$$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2} = \lim_{x\to 2^+} \frac{(x+2)(x-2)}{x-2}$$

$= 2+2 = 4$

$\therefore$ RHD = LHD

f is differentiable at $x=2$.

Q3) If $f(x) = 4x+7$, $x<3$

$= x^2+3x+1$, $x \geq 3$ at $x=3$, then find f is differentiable or not?

Sol: RHD

$$Df(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\cdot 3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3+6 = 9$$

$Df(3^+) = 9$

LHD = $Df(3^-)$

$$= \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3} = \lim_{x\to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x\to 3} \frac{4(x-3)}{(x-3)} = \lim_{x\to 3} \frac{4(x-3)}{(x-3)}$$

$Df(3^-) = 4$

RHD $\neq$ LHD

$\therefore f$ is not differentiable at $x=3$

4) If $f(x) = 3x-5$, $x<2$

$= 3x^2-4x+7$, $x\geq 2$ at $x=2$ then

find f is differentiable or not.

RHD:

$$Df(2^+) = \lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-6x+2x-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$$

$= 3 \times 2 + 2 = 8$

$\therefore Df(2^+) = 8$

LHD :

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$

$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$

$Df(2^-) = 8$

$\therefore$ LHD = RHD

$\therefore f$ is differentiable at $x = 3$.

PRACTICAL - 3

Topic:- Application of Derivative.

Q.1) Find the intervals in which function is increasing or decreasing

i) $f(x) = x^3 - 5x - 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = x^3 - 27x + 5$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$.

Q.2) Find the intervals in which function is concave upwards.

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

v) $y = 2x^3 + x^2 - 20x + 4$.

Soln :

Q1) i) $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\because f$ is increasing iff $f'(x) > 0$.

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x-\sqrt{5/3})(x+\sqrt{5/3})>0$

$+ \quad -\sqrt{5/3} \quad - \quad \sqrt{5/3} \quad +$ $\qquad x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x)<0$

$\therefore 3x^2-5<0$

$\therefore 3(x^2-5/3)<0$

$\therefore (x-\sqrt{5/3})(x+\sqrt{5/3})<0$

$+ \quad -\sqrt{5/3} \quad - \quad \sqrt{5/3} \quad +$ $\qquad x \in (-\sqrt{5/3}, \sqrt{5/3})$

ii) $f(x) = x^2-4x$

$f'(x) = 2x-4$

$\because f(x)$ is increasing iff $f'(x)>0$

$\therefore 2x-4>0$

$\therefore 2(x-2)>0$

$\therefore x-2>0$

$x \in (2,\infty)$

& f is decreasing iff $f'(x)<0$

$\therefore 2x-4<0$

$\therefore 2(x-2)<0$

$\therefore x-2<0$

$\therefore x \in (-\infty, 2)$

iii) $f(x) = 2x^3+x^2-20x+4$

$\therefore f'(x) = 6x^2+2x-20$

$\because f$ is increasing iff $f'(x)>0$

$\therefore 6x^2+2x-20>0$

$\therefore 2(3x^2+x-10)>0$

$\therefore 3x^2+x-10>0$

$\therefore 3x(x+2)-5(x+2)>0$

$\therefore (x+2)(3x-5)>0$

$+ \quad -2 \quad - \quad 5/3 \quad +$ $\qquad x \in (-\infty, -2) \cup (5/3, \infty)$

and f is increasing iff $f'(x)<0$

$\therefore 6x^2+2x-20<0$

$\therefore 3x^2+6x-5x-10<0$

$\therefore (x+2)(3x+5)<0$

$+ \quad -2 \quad - \quad 5/3 \quad +$ $\qquad x \in (-2, 5/3)$

iv) $f(x) = x^3-27x+5$

$f'(x) = 3x^2-27$

$\because f$ is increasing iff $f'(x)>0$

$\therefore 3(x^2-9)>0$

$\therefore (x-3)(x+3)>0$

$+ \quad -3 \quad - \quad 3 \quad +$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

& f is decreasing iff $f'(x)<0$

$\therefore 3x^2-27<0$

$\therefore 3(x^2-9)<0$

$\therefore (x-3)(x+3)<0$

$+ \quad -3 \quad - \quad 3 \quad +$

$\therefore x \in (-3, 3)$

v) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore (x-4)(x+1) > 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

& f is decreasing iff $f'(x) < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x-4)(x+1) < 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$\therefore x \in (-1, 4)$

9) i) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$x - \frac{1}{2} > 0$

$x > 1/2$

$\therefore f''(x) > 0$

$\therefore x \in (1/2, \infty)$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore (x-2)(x-1) > 0$

$+ \quad 1 \quad - \quad 2 \quad +$ $\quad x \in (-\infty, 1)$

iii) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty)$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12\,(x - 18/12) > 0$

$\therefore x - 3/2 > 0$

$\therefore x > 3/2$

$\therefore x \in (3/2, \infty)$

v) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore x + 1/6 > 0$

$\therefore x > -1/6$

$\because f''(x) > 0$

$\therefore$ There exist interval.

$\therefore x \in (-1/6, \infty)$

21/12/17

PRACTICAL-4

APPLICATION OF DERIVATIVE & NEWTON'S METHOD

Q1) Find maximum & minimum value of following functions.

i) $f(x) = x^2 + \frac{16}{x^2}$

$\rightarrow$ $f'(x) = 2x - \frac{32}{x^3}$

Now Consider,

$f'(x) = 0$

$\therefore 2x - \frac{32}{x^3} = 0$

$\therefore 2x - 32/x^3 = 0$

$\therefore x^4 = 16$

$\therefore x = \pm 2$

$f''(x) = 2x + \frac{96}{x^4} = 2 + \frac{96}{16} = 2 + 6$

$\therefore f''(x) = 8 > 0$

$\therefore f$ has maximum value at $x = 2$

$f(2) = 2^2 + \frac{16}{2^2} = 4 + \frac{16}{4} = 4 + 4$

$\therefore f(2) = 8$

$f''(-2) = 2 + \frac{96}{-2^4} = 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x=2$

$\therefore$ Function reaches minimum value at $x=2$ & $x=-2$

ii) $f(x) = 3 - 5x^3 + 3x^5$.

$\rightarrow$ $f'(x) = -15x^2 + 15x^4$.

Consider,

$f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$x^2 = 1$

$\therefore x = \pm 1$

Now,

$f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

$\therefore$ f has minimum value at $x=1$

$f(1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore$ f has maximum value at $x=-1$.

$f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

Therefore, f has the maximum value 5 at ~~and~~ $x=-1$ & has the minimum value 1 at $x=1$.

iii) $f(x) = x^3 - 3x^2 + 1$

$\rightarrow$ $f'(x) = 3x^2 - 6x$

Consider,

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

~~$\therefore 3x^2 = 6x$~~

~~$\therefore x = 2$~~

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

Now,

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$\therefore f''(0) = -6 < 0$

$\therefore$ f has maximum value at $x=0$.

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$f''(2) = 6(2) - 6 = 12 - 6 = 6 > 0$

$\therefore$ f has max value at 1 & $x=0$.

$\therefore$ f has maximum value -3 at $x=2$

(1).

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$.

$\rightarrow$ $f'(x) = 6x^2 - 6x - 12$

Consider,

$f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$\therefore f''(2) = 18 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6 = -18 < 0$

$\therefore f$ has maximum value at $x = -1$.

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1 = 8$

$\therefore f$ has minimum value -1 at $x = 2$

9.2) i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$\rightarrow$ $f'(x) = 3x^2 - 6x - 55$

By Newton's method,

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 0 + \frac{9.5}{55}$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= -55.9467$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.1727 - \frac{0.0829}{55.9467}$

$\therefore x_2 = 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= -55.9393$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.1712 + \frac{0.0011}{55.9393}$

$\therefore x_3 = 0.1712$

$\therefore$ The root of the equation is 0.1712

→ $f(x) = x^3 - 4x - 9$ $[2, 3]$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$
$= 8 - 8 - 9$
$= -9$

$f(3) = 3^3 - 4(3) - 9$
$= 27 - 12 - 9$
$= 6$

let $x_0 = 3$ be the initial approximate

∴ By Newton's method;

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{6}{3} = 2.7392$$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$
$= 0.5796$

$f'(x_1) = 3(2.7392)^2 - 4$
$= 22.5096 - 4$
$= 18.5096$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.7392 - \frac{0.5796}{18.5096}$$

∴ $x_2 = 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$
$= 19.8386 - 10.8284 - 9$
$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4$
$= 17.9851$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.7071 - \frac{0.0102}{17.9851}$$

$= 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$
$= 21.8943 - 4$
$= 17.8943$

$$x_4 = 2.7015 + \frac{0.0901}{17.8943}$$

$= 2.7015 + 0.0050$
$= 2.7065$

Q.3) $f(x) = x^3 - 1.8x^2 - 10x + 17$; $[1, 2]$

→ $f'(x) = 3x^2 - 3.6x - 10$ $= +5.2$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17$
$= -2.2$

Sd.

Let $x_0 = 2$ be real approximation

By Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{2.2}{5.2} = 2 - 0.4230$$

$$\therefore x_1 = 1.577$$

$$f(x_1) = (1.577)^3 - 1.5(1.577)^2 - 10(1.577) + 17$$

$$= 3.9219 - 4.4764 - 15.77 + 17$$

$$= -8.2164$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1577 + \frac{0.1755}{8.2164}$$

$$= 1.577 + 0.0822$$

$$\therefore x_2 = 1.6592$$

$$f(x_2) = (1.6592)^3 - 1.5(1.6592)^2 - 10(1.6592) + 17$$

$$= 0.0204$$

$$f'(x_2) = 3(1.6592)^2 - 3.1(1.6592) - 10$$

$$= 8.2588 - 5.97312 - 10$$

$$\therefore f'(x_2) = -7.7143$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.6592 + \frac{0.0204}{7.7143}$$

$$= 1.6618$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.6618 + \frac{0.0004}{7.6977}$$

$$= 1.6618$$

Therefore, the root of equation is 1.6618

PRACTICAL - 15

INTEGRATION

Q. Solve the following integration.

1) $\int \frac{dx}{\sqrt{x^2+2x+3}}$

2) $\int (4e^{3x}+1)\,dx$

3) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

5) $\int t^7 \sin(2t^4)\,dt$

6) $\int \sqrt{x}\,(x^2-1)\,dx$

7) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

8) $\int \frac{\cos x}{\sqrt[3]{\sin x}}\,dx$

9) $\int e^{\cos^2 x}\cdot \sin 2x\,dx$

10) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right)dx$

1) Soln) $\int \frac{1}{x^2+2x-3}\,dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}\,dx \quad \ldots \{a^2+2ab+b^2 =$

$= \int \frac{1}{(x+1)^2-4}\,dx$

Substitute, put $x+1=t$

$\therefore dx = dt$

$= \int \frac{1}{t^2-4}\,dt$

By using, $\int \frac{1}{x^2-a^2}\,dx = \ln\left(|x+\sqrt{x^2-a^2}|\right)$

$= \ln\left(|t+\sqrt{t^2-4}|\right)$

$= \ln\left(|x+1+\sqrt{(x+1)^2-4}|\right)$

$= \ln\left(|x+1+\sqrt{x^2+2x-3}|\right)$

$= \ln\left(|x+1+\sqrt{x^2+2x-3}|\right)+c$

2) Soln) $\int (4e^{3x}+1)\,dx$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx \quad \{ \int e^{ax}\,dx = \frac{1}{a}e^{ax}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c$

3) Sol) $\int 2x^2 - 3\sin x + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin x + 5x^{1/2}\,dx \qquad \left\{ \sqrt[n]{a^m} = a^{m/n} \right\}$

$= \int 2x^2 dx - \int 3\sin x\,dx + \int 5x^{1/2}dx$

$= \frac{2x^3}{3} - 3\cos x + \frac{5 \times 2x\sqrt{x}}{3} + c$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$

4) Sol) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\,dx$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}}\,dx = \int \frac{x^3}{x^{1/2}}dx + \frac{3x}{x^{1/2}}dx + \frac{4}{x^{1/2}}dx$

$= \int x^{5/2}dx + 3x^{1/2} + \frac{4}{x^{1/2}}dx\ dx$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5) $\int t^4 \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^4 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du = \frac{t^4 \times \sin(2t^4)}{8}\,du$

Substitute t^4 with $\frac{u}{2}$

$= \int \frac{u/2 \times \sin u}{8}\,du = \int \frac{u \times \sin u}{2} / 8\,du$

$= \int \frac{u \times \sin u}{16}\,du$

$= \frac{1}{16} \int u \times \sin(u)\,du$

$= \frac{1}{16} \int (u \times (-\cos u)) + \int \cos u\,du$

$\cdots \{ \cos x\,dx = \sin x \}$

$= \frac{1}{16} \times (u \times (-\cos u)) + \sin u)$

$= \frac{1}{16} \times 2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$

6) $\int \sqrt{x}\,(x^2 - 1)\,dx$

$= \int \sqrt{x}\,x^2 - \sqrt{x}\,dx = \int x^{1/2}x^2 - \int x^{1/2}dx$

$= \left[\frac{x^{5/2+1}}{\frac{5}{2}+1} - \frac{x^{1/2+1}}{1/2+1} \right] = \frac{2x^{7/2}}{7} - \frac{2x^{3/2}}{3}$

$= \frac{2x^3\sqrt{x}}{7} - \frac{2\sqrt{x^3}}{3} = \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$

9) $I = \int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx = \int \frac{\cos x}{\sin x^{2/3}}\,dx$

put,

$t = \sin x$

$t = \cos x$

$= \int \frac{\cos x}{\sin x^{2/3}} \times \frac{1}{\cos x}\,dt$

$= \left[\int \frac{1}{\sin x^{2/3}}\,dt\right]$

$= \frac{1}{t^{2/3}}\,dt$

$I = \int \frac{1}{t^{2/3}}\,dt = \frac{-1}{-2/3\,t^{2/3-1}} = \frac{1}{1/3\,t^{-1/3}} = \frac{t^{1/3}}{1/3}$

$= 3t^{1/3} = 3\sqrt[3]{t}$

Return sub $t = \sin x$

$I = 3\sqrt[3]{\sin x} + C$

10) $I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\,dx$

put $x^3 - 3x^2 + 1 = dt$

$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3(2x)}\,dt$

$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x}\,dt$

$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 - 2x)}\,dt$

$= \int \frac{1}{x^3 - 3x^2 + 1} \times \frac{1}{3}\,dt$

$= \int \frac{1}{3(x^3 - 3x^2 + 1)}\,dt$

$= \frac{1}{3t}\,dt$

$= \frac{1}{3}\int \frac{1}{t}\,dt \quad \cdots \left\{ \int \frac{1}{x}\,dx = \ln|x| \right\}$

$= \frac{1}{3}\ln|t| + c$

$I = \frac{1}{3} \times \ln(|x^3 - 3x^2 + 1|) + c$

04/01/2020

PRACTICAL - 6

APPLICATION OF INTEGRATION AND NUMERICAL INTEGRATION

Q.1) Find the length of the following curve:-

1) $x = t\sin t$, $y = 1 - \cos t$, $t \in [0, 2\pi]$
2) $y = \sqrt{4 - x^2}$, $x \in [-2, 2]$
3) $y = 3/2$ in $[0, 4]$
4) $x = 3\sin t$, $y = 3\cos t$; $y \in [0, 2\pi]$
5) $x = \frac{1}{6} y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q.2) Using Simpson's Rule solve the following.

1) $\int_0^2 e^{x^2} dx$ with $n = 4$
2) $\int_0^4 x^2 \cdot dx$ with $n = 4$
3) $\int_0^{\pi/3} \sqrt{\sin x} \cdot dx$, with $n = 6$

1) $y = \sqrt{4-x^2}$, $x \in [-2, 2]$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}}(-2x) = \frac{-2x}{\sqrt{4-x^2}}$

$= \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_{-2}^{2} \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{4-x^2}}\, dx = \int_{-2}^{2} \sqrt{\frac{2}{\sqrt{4-x^2}}}\, dx$

$= \int_{-2}^{2} \frac{2}{\sqrt{4-x^2}}\, dx = 2\left[\sin^{-1}(x/2)\right]_{-2}^{2}$

$= \sin^{-1}(1) - \sin^{-1}(-1) = 2\left[\frac{\pi}{2} + \frac{\pi}{2}\right]$

$= 2\pi$

2) if $x^{3/2}$ in $[0, 4]$

$\frac{dy}{dx} = \frac{3}{2} x^{3/2 - 1}$

$L = \int_{0}^{4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_{0}^{4} \sqrt{1 + \left(\frac{3\sqrt{x}}{2}\right)^2}\, dx = \int_{0}^{4} \sqrt{1 + 9x/4}\, dx$

$= \frac{1}{2}\int_0^4 \left[\frac{(4+9x)^{1/2+1}}{3/2}\right] = 1/2\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$

$= \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4 = \frac{1}{27}\left[(4)^{3/2} - (40)^{3/2}\right]$

$= \frac{1}{27}\left[(9)^{3/2} - 1\right]$ units.

3) $x = 3\sin t$, $y = 3\cos t$

$\frac{dx}{dt} = 3\cos t$, $\frac{dy}{dt} = -3\sin t$

$= 2\pi \int_0 \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= 2\pi \int_0 \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} \sqrt{9(1)}\, dt = \int_0^{2\pi} 3\, dt$

$= 3\,[x]_0^{2\pi} = 3\,(2\pi - 0) = 6\pi$

4) $dx/dy = \frac{1}{6}\frac{d}{dy}(y^3) - \frac{1}{2}\frac{dx}{dy}(1/y)$

$= y^2/2 - 1/2y^2 = y^2 - 1/2y^2$

$= \int_1^2 \sqrt{1 + (dx/dy)^2} = \int_1^2 \sqrt{1 + \left(\frac{y^2-1}{2y^2}\right)^2}\, dy$

$= \int_1 \sqrt{1 + \frac{(y^2-1)}{(4y^4)}}\, dy$

$$- \int_1^2 y^4/2y \cdot dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy \ \uparrow - \int_1^2 \frac{1}{2y^2} \cdot dy$$

$$= \frac{1}{2} \int_1^2 y\, dy - 1/2 \int 1/y^2 \cdot dy$$

$$= \frac{1}{2} \left[\frac{2}{3} - \frac{1}{2} - \frac{1}{3} + 1 \right] = \frac{1}{2} \left[7/3 + 1/2 \right]$$

$$= 17/12 \ //$$

5) Curve length $\int_0^{2\pi} \sqrt{(dx/dt)^2 + (dy/dt)^2}$

$$dt = \int_0^{2\pi} \sqrt{(1-\cos t)^2 + \cos^2 t + \sin^2 t} + dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt = \int_0^{2\pi} 2\sin t\, dt$$

$$= \left[-4\cos t/2 \right]_0^{2\pi} = (-4\cos\pi) + 4\cos 0$$

$$= 8 \text{ units} \ //$$

9.2) 1) $\int_0^2 e^{x^2} dx$ with $n = 4$

$a = 0, \ b = 2, \ n = 4$

$h = \frac{b-a}{n} = 0.5$

x	0	0.5	1	1.5	2
	1	1.284	2.718	9.487	54.598

$$= \int_0^2 e^{x^2} = \frac{h}{3} \left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + x_5 \right]$$

$$= \frac{0.5}{3} \left[(1) + 4(1.284) + 2(2.718) + 4(9.487) \right]$$

$$= \frac{0.5}{3} \left[104.148 \right] = 17.358 \ //$$

2) $\int_0^4 x^2 dx$, $n = 4$, $a = 0$, $b = 4$

$h = \frac{b-a}{n} = \frac{4-0}{4} = 1$

x	0	1	2	3	4
	0	1	4	9	16

$$\int_0^4 x^2 dx = \frac{h}{3} \left[0 + 4(1) + 2(4) + 4(9) + 16 \right]$$

$$= \frac{1}{3} \left[0 + 4 + 8 + 36 + 16 \right]$$

$$= \frac{1}{3} \left[64 \right] = 21.333 \ //$$

3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$

$h = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi/3}{6} = \frac{\pi}{18}$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$\frac{6\pi}{18} \cdot (\pi/3)$
y	0	0.4167	0.7071	0.8017	0.8752	0.9306	
	y_0	y_1	y_2	y_3	y_4	y_5	

$$\Rightarrow \int_0^{\pi} \sqrt{\sin x}\, dx = \frac{h}{3}\Big[0.4167 + 0.9306 + 4(0.4367 + 0.7071 + 0.7\ldots) + 2(0.5847 + 0.5017)\Big]$$

$$= \frac{\pi}{54}\left[1.3473 + 4(1.599) + 2(1.3865)\right]$$

$$= \frac{\pi}{54} \times 12.1163$$

$$= \int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049$$

AR 25/01/2022

PRACTICAL-7

DIFFERENTIAL EQUATION

Q.1) Solve the following differential equation

i) $x\frac{dy}{dx} + y = e^x$

Soln) Dividing by x

$$\frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$$

& Comparing with $dy/dx + P(x)y = Q(x)$

$$I.F. = e^{\int P dx} = e^{\int 1/x\, dx} = e^{\int \log x\, dx} = x$$

$$y(I.F.) = \int Q\,(I.F.)\, x\, dx + c$$

$$y(x) = \int \frac{e^x}{x} \cdot x\, dx + c$$

$$y(x) = \int e^x + c = e^x + c$$

ii) $e^x \frac{dy}{dx} + 2y = 1$

Dividing by e^x

$$dy/dx + 2y = 1/e^x$$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$$I.F. = e^{\int P dx} = e^{\int 2 dx} = e^{2x}$$

$$y(e^{2x}) = \int 1/e^x \cdot e^{2x} \cdot dx + c$$

$= \int e^{2x} \cdot e^{-x}\, dx + c$

$= \int e^{x} dx + c$

$\therefore y e^{2x} = e^{x} + c$

iii) $x\, dy/dx = \frac{\cos x}{x} - 2y$

sol) $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} = \frac{\cos x}{x^2} - \frac{2y}{x}$

$I.F = e^{\int P dx} = e^{\int 2/x\, dx} = e^{2 \ln x} = e^{\ln x^2} = x^2$

$y\,(I.F) = \int g(x)\,(I.F)\,dx + c$

$\therefore x^2 y = \int \frac{\cos x}{x^2} \cdot x^2\, dx + c$

$= \int \cos x + c$

$\therefore x^2 y = \sin x + c$

iv) $\frac{dy}{dx} + 3y/x = \frac{\sin x}{x^3}$ (÷ by x on B.S.)

$\rightarrow P(x) = \frac{3}{x}$, $g(x) = \sin x / x^3$

$I.F = e^{\int P dx} = e^{\int 3/x\, dx} = e^{3 \ln x} = e^{\ln x^3} = x^3$.

$y(I.F) = \int g(x)\,(I.F)\,dx + c$

$= \int \frac{\sin x}{x^3} \cdot x^3 dx + c$

$= \int \sin x\, dx + c$

$x^3 y = -\cos x + c$

v) $e^{2x} \frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$\rightarrow P(x) = 2$, $g(x) = 2x/e^{2x} = 2x e^{-2x}$

$I.F = e^{\int 2 dx} = e^{2x}$

Now,

$y e^{2x} = \int 2x e^{-2x} \cdot e^{2x}\, dx + c$

$= \int 2x\, dx + c$

$\therefore y \cdot e^{2x} = x^2 + c$

vi) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$.

sol) $\sec^2 x \tan y\, dx = -\sec^2 y \tan x\, dy$

$\therefore \frac{\sec^2 x\, dx}{\tan x} = -\frac{\sec^2 y\, dy}{\tan y}$

$\therefore \int \frac{\sec^2 x\, dx}{\tan x} = -\int \frac{\sec^2 y\, dy}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\therefore \log|\tan x \cdot \tan y| = c$

$\therefore \tan x \cdot \tan y = e^{c}$

vii) $\frac{dy}{dx} = \sin^2 (x-y+1)$

put $x-y+1 = v$

Diff on Both the sides;

$x - y + 1$

$\therefore 1 - \frac{dy}{dx} = dv/dx$

$\therefore 1 - \frac{dv}{dx} = dy/dx$

$\therefore 1 - \frac{dv}{dx} = \sin^2 v$

$\therefore \frac{dy}{dx} = 1 - \sin^2 v$

$\therefore \frac{dv}{dx} = \cos^2 v$

$\therefore dv / \cos^2 x = dx$

$\int \sec^2 v \, dv = \int dx$

$\therefore \tan v = x + c$

$\therefore \tan (x + y - 1) = x + c$

viii) $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

→ put $2x + 3y - 1 = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$= \frac{v - 1 + 2v + 4}{v+2} = \frac{3v+3}{v+2}$

$= \frac{3(v+1)}{v+2}$

$\int \left(\frac{v+2}{v+1}\right) dv = 3dx$

$= \int \frac{v+1}{v} dx + 1 \left(\frac{1}{v+1}\right) dv = 3x$

$v + \log(x) = 3x + c$

$2x + 3y + \log |2x + 3y + 1| = 3x + c$

$3y = x - \log |2x + 3y + 1| + c.$

AP
21/01/2020

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PRACTICAL-8

EULER'S METHOD

Q.1> Using euler's method find the following.

a) $\frac{dy}{dx} = y + e^x - 2$, $y_0 = 2$, $h = 0.5$

find $y(2)$.

Ans> Here, $x_0 = 0$, $y_0 = 2$, $h = 0.5$

$f(x,y) = y + e^x - 2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5000
1	0.5	2.5	2.1487	3.5744
2	1	3.544	4.2924	5.7208
3	1.5	5.7208	8.2025	9.8221
4	2	9.8221		

$\therefore y(2) = 9.8221$

b) $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, $h = 0.2$, find $y(1)$

Ans>

n	x_n	y_n	$f(x_n, y_n)$	$y_{(n+1)}$
0	0	0	1	0.2000
1	0.2	0.2000	1.1665	0.4080
2	0.4	0.4080	1.4813	0.6413
3	0.6	0.6143	1.8590	0.9231
4	0.8	0.9236	1.0400	1.2942
5	1	1.2942		

$\therefore y(1) = 1.2942$

c) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$, find $y(1)$.

Soln>

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2106
3	0.6	1.2106	0.7040	1.3514
4	0.8	1.3514	0.7694	1.5053
5	1	1.5053		

$\therefore y(1) = 1.5053$

d) $\frac{dy}{dx} = 3x^2+1$, $y(1)=2$, find $y(2)$

for $h=0.5$, $h=0.25$

Sol) $h=0.5$,

h	x_n	y_n	$f(x_n,y_n)$	y_{n+1}
0	1	2	4.000	4.000
1	1.5	4	7.7500	7.8750
2	2	7.8750		

$\therefore y(2) = 7.8750$

For $h=0.25$;

h	x_n	y_n	$f(x_n,y_n)$	y_{n+1}
0	1	2	4	4.000
1	1.25	4	5.6875	5.4219
2	1.50	5.4219	7.7500	7.3594
3	1.75	7.3594	10.1875	9.9063
4	2	9.9063		

$\therefore y(2) = 9.9063$

e) $dy/dx = \sqrt{xy}+2$, $y(1)=1$, find $y(1.2)$ with $h=0.2$

Sol) Here,

$x_0=1$, $y_0=1$, $f(x_n,y_n) = \sqrt{xy}+2$, $h=0.2$

h	x_n	y_n	$f(x_n,y_n)$	y_{n+1}
0	1	1	3	1.6000
1	2	1.6000		

$\therefore y(1.2) = 1.6000$

Atish 01/02/20

PRACTICAL - 9

LIMITS AND PARTIAL ORDER DERIVATIVES

Q1) i) $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

→ At $(-4,-1)$ denominator $\neq 0$

∴ By applying limit

$= \frac{(-4)^3-3(-1)+(-4)^2-1}{(-4)(-1)+5} = \frac{64+3+16-1}{4+5}$

$= \frac{-61}{9}$

ii) $\lim_{(x,y)\to(-2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

$\lim_{(x,y)\to(-2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

At $(-2,0)$ denominator $\neq 0$

∴ By applying limit

$= \frac{(0+1)(4+0-8)}{2}$

$= \frac{-4}{2}$

$= -2$

1Q2

3) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^2-x^2y^2}$

→ ~~x~~ (1,1,1) denominator = 0

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^2-x^2y^2}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$

on applying limits

$= \frac{1+1(1)}{1^2} = 1$ //

Q.4) $f(x,y) = xy\,e^{x^2+y^2}$

$f_x = \frac{\delta f(x,y)}{\delta x}$

$= \frac{\delta(xye^{x^2+y^2})}{\delta x}$

$= y e^{x^2+y^2}(2x)$

$\therefore f_x = 2xye^{x^2+y^2}$ //

$f_y = \frac{\delta f(x,y)}{\delta y}$

$= \frac{\delta}{\delta x}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$= 2yxe^{x^2+y^2}$ //

1) $f(x,y) = e^x \cos y$

→ $f_x = \frac{\delta f(x,y)}{\delta x}$

$= \frac{\delta}{\delta x}(e^x \cos y)$

$\therefore f_x = e^x \cos y$ //

Now,

$f_y = \frac{\delta f(x,y)}{\delta y} = \frac{\delta (e^x \cos y)}{\delta y}$

$\therefore f_y = -e^x \sin y$ //

3) $f(x,y) = x^3y^3 - 3x^2y + y^3 + 1$

→ $f_x = \frac{\delta f(x,y)}{\delta x}$

$= \frac{\delta f}{\delta x}(x^3y^3 - 3x^2y + y^3 + 1)$

$= 3x^2y^3 - 6xy$ //

$f_y = \frac{\delta f(x,y)}{\delta y}$

$= \frac{\delta f}{\delta y}(x^3y^3 - 3x^2y + y^3 + 1)$

$= (2x^3y - 3x^2 + 3y^2)$ //

Q.3) 1) $f(x,y) = \frac{2x}{1+y^2}$

$\rightarrow f_x = \frac{\delta f}{\delta x}(x,y)$

$= \frac{\delta f}{\delta x}(2x)/(1+y^2)$

$= \frac{(1+y^2)\frac{\delta(2x)}{\delta x} - 2x\frac{\delta}{\delta x}(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2}{(1+y^2)^2}$

$\therefore f_x = \frac{2}{1+y^2}$ //

At $f(0,0) \rightarrow \frac{2}{1+0} = 2$

$f_y = \frac{\delta f}{\delta y}(2x)/(1+y^2)$

$= \frac{(1+y^2)\frac{\delta}{\delta x}(2x) - \frac{\delta}{\delta x}(1+y^2)}{(1+y^2)^2}$

$= \frac{-4xy}{(1+y^2)^2}$

At $(0,0) \rightarrow \frac{-4(0)(0)}{(1+0)^2} = 0$ //

Q.4) 1) $f(x,y) = \frac{y^2 - xy}{x^2}$

$f_x = \frac{x^2\frac{\delta}{\delta x}(y^2 - xy) - (y^2 - xy)\frac{\delta}{\delta x}(x^2)}{(x^2)^2}$

$= \frac{-x^2y - 2x(y^2 - xy)}{x^4}$

$\rightarrow f_y = \frac{2y - x}{x^2}$

$f_{xx} = \frac{\delta}{\delta x}\frac{(-x^2y - 2x(y^2 - xy))}{x^4}$

$= \frac{x^4\left(\frac{\delta}{\delta x}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)\frac{\delta}{\delta x}(x^4)}{(x^4)^2}$

$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^8}$ ... (i)

$\rightarrow f_{yy} = \frac{\delta}{\delta y}\left(\frac{2y - x}{x^2}\right)$

$= \frac{2-0}{x^2} = \frac{2}{x^2}$ ... (ii)

$f_{xy} = \frac{\delta}{\delta y}\frac{(-x^2y - 2xy^2 + 2x^2y)}{x^4}$

$= \frac{-x^2 - 4xy + 2x^2}{x^4}$ ... (iii)

$f_{yx} = \frac{\delta}{\delta x}\left(\frac{2y - x}{x^2}\right)$

$= x^2\frac{\delta}{\delta x}(2y - x) - (2y - x)\frac{\delta}{\delta x}(x^2)$

$= \frac{x^2 - 4xy - 2x^2}{x^4}$ ... (iv)

from (iii) & (iv)

$f_{xy} = f_{yx}$ //

2)
$f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$\rightarrow f_x = \frac{\delta}{\delta x}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$

$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$f_y = \frac{\delta}{\delta y}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$

$= 6x^2y$

$f_{xx} = 6x + 6y^2 \frac{(x^2+1)\frac{\delta}{\delta x}(2x) - 2x\left(\frac{\delta}{\delta x}(x^2+1)\right)}{(x^2+1)^2}$

$= \frac{6x + 6y^2 - \left(2(x^2+1) - 4x^2\right)}{(x^2+1)^2}$ — (i)

$f_{yy} = \frac{\delta}{\delta y}(6x^2y)$

$= 6x^2$

$f_{xy} = \frac{\delta}{\delta y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 0 + 12xy - 0$

$= 12xy$ ..... (iii)

$f_{yx} = \frac{\delta}{\delta x}(6x^2y)$

$= 12xy$ ... (iv)

from (iii) & (iv)

$\therefore f_{xy} = f_{yx}$

3) $\rightarrow$
$f(x,y) = \sin(xy) + e^{x+y}$

$f_x = y\cos(xy) + e^{x+y}(1)$ — (i)

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$f_{xx} = \frac{\delta}{\delta x}\left(y\cos(xy) + e^{x+y}\right)$

$= -y\sin(xy)(y) + e^{x+y}$

$= -y^2\sin(xy) + e^{x+y}$ ... (i)

$f_{yy} = \frac{\delta}{\delta y}\left(x\cos(xy) + e^{x+y}\right)$

$= -x\sin(xy)(x) + e^{x+y}$

$= -x^2\sin(xy) + e^{x+y}$ .... (ii)

$f_{xy} = \frac{\delta}{\delta y}\left(y\cos(xy) + e^{x+y}\right)$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ ... (iii)

$f_{yx} = \frac{\delta}{\delta x}\left(x\cos(xy) + e^{x+y}\right)$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ ... (iv)

from (iii) & (iv)

$f_{xy} \neq f_{yx}$

Q.

9) 1) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$.

$\rightarrow$ $f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x) = \frac{x}{\sqrt{x^2+y^2}}$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = 1/\sqrt{2}$

f_y at $(1,1) = 1/\sqrt{2}$

$f(x,y) = f(x,y) + f_x(a,b)(x-a) - f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \left(\frac{1}{\sqrt{2}}\right)(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$

3) $f(x,y) = \log x + \log y$. at $(1,1)$

$f(1,1) = \log(1) + \log(1)$

$f_x = \frac{1}{x}$ $\quad$ $f_y = 1/y$

f_x at $(1,1) = 1$ $\quad$ f_y at $(1,1) = 1$

$f(x,y) = f(a,b)(x-a) - f_y(a,b)(y-b)$

$= 0 + (1(x-1) + 1(y-1))$

$= x-1+y-1$

$\therefore f(x,y) = x+y-2$

AK
15/01/2020

PRACTICAL - 10

Q.1) Find the directional derivative of the following functions at given points & in the direction of given vector.

i) $f(x,y) = x+2y-3$, $a=(1,-1)$, $u=3i-j$

Here,

$u = 3i-j$ is not a unit vector

$|u| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

Unit vector along u is :

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = 1+2(-1)-3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\,2\left(1+\frac{3}{\sqrt{10}}\right), \left(-1, -\frac{h}{\sqrt{10}}\right)$

$= 1 + \frac{3}{\sqrt{10}} + 2\left(-1-\frac{h}{\sqrt{10}}\right)$

$\therefore f(a+hu) = -1 + \frac{h}{\sqrt{10}}$

$$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{-4+h \quad +4}{\sqrt{26}}$$

$$\therefore D_u f(a) = \frac{1}{\sqrt{26}}$$

ii) $f(x) = y^2 - 4x + 1$

Here,

$u = i + 5j$ is not a unit vector

$|\vec{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$f(a) = f(3, 4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, h + \frac{5h}{\sqrt{26}}\right)$$

$$f_{xy}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{25h^2}{\sqrt{26}} \frac{-4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$= \frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$$

$$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

iii) $2x + 3y$; $a = (1, 2)$; $u = (3i + 4j)$

→ Here, $u = 3i + 4j$ is not a unit vector

$|\vec{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3, 4)$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

$f(a) = f(1, 2) = 2(1) + 3(2) = 8$

$$f(a+hu) = f(1, 2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$$

$$= f\left(1 + \frac{3h}{5}, 2 + \frac{4h}{5}\right)$$

$$f(a+ha) = 2\left(3+\frac{3h}{5}\right) + 2\left(2+\frac{4h}{5}\right)$$

$$= 2 + \frac{6h}{3} + 6 + \frac{8h}{5}$$

$$= \frac{14h}{5} + 8$$

$$D_u f(a) = \lim_{h \to 0} \frac{14h + 8 - 8}{h} = \frac{14}{5}$$

Q) Find Gradient vector for the following function at given point

i) $f(x,y) = x^y + y^x$, $a = (1,1)$

$$f(x) = y \cdot x^{y-1} + y^x \log y$$

$$f(y) = x^y \log x + x y^{x-1}$$

$$\nabla f(x,y) = (f_x, f_y)$$

$$= (y x^{y-1} + y^x \log y,\ x^y \log x + x y^{x-1})$$

$$f(1,1) = (1+0,\ 0+1) = (1,1).$$

ii) $f(x,y) = (\tan^{-1} x) y^2$, $a = (1,-1)$

$$f_x = \frac{1}{1+x^2} \cdot y^2$$

$$f_y = 2y \tan^{-1} x$$

$$\nabla f(x,y) = (f_x, f_y)$$

$$= \frac{y^2}{1+x^2},\ 2y (\tan^{-1} x)$$

$$f(1,-1) = \left(\frac{1}{2},\ \tan^{-1}(1)(-1)\right) = \left(\frac{1}{2},\ \frac{\pi}{4}(-1)\right)$$

$$= \left(\frac{1}{2},\ -\frac{\pi}{4}\right)$$

iii) $f(x,y,z) = f_x, f_y, f_z$

$$f_x = yz - e^{x+y+z}$$

$$f_y = xz - e^{x+y+z}$$

$$f_z = xy - e^{x+y+z}$$

$$\nabla f(x,y,z) = f_x, f_y, f_z$$

$$= yz - e^{x+y+z},\ xz - e^{x+y+z},\ xy - e^{x+y+z}$$

$$f(1,-1,0) = \left((-1)(0) - e^{(1+(-1)+0)},\ (1)(0) - e^{(1+(-1)+0)},\ (1)(-1) - e^{(1+(-1)+0)}\right)$$

$$= (0 - e^0,\ 0 - e^0,\ -1 - e^0)$$

$$= -1, -1, -2$$

Q) Find the equation of tangent & normal to each of the following curves at given points.

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$$f_x = \cos y\ 2x + e^{xy} y$$

$$f_y = x^2(-\sin y) + e^{xy}(x)$$

$$(x_0, y_0) = (1,0)$$

$$x_0 = 1,\ y_0 = 0$$

Eq. of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$f_x(x_0,y_0) = 1(2) + 0$

$= 2$

$f_y(x_0,y_0) = 1$

$2(x-1) + (y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ → It is the required eq. of tangent

Eqn of ~~tangent~~ Normal

$ax + by + c = 0$

$bx + ay + d = 0$

$\therefore 1(x) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1,0)$

$d + 1 = 0$

$d = -1$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

→ $f_x = 2x + 0 - 2 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2,-2)$ ∴ $x_0 = 2$, $y_0 = -2$

$f_x(x_0,y_0) = 2(2) - 2 = 2$

$f_y(x_0,y_0) = 2(-2) + 3 = -1$

Eqn of tangent:-

$f_x(x-x_0) + f_y(y-y_0) = 0$

$\therefore 2(x-2) - (y+2) = 0$

$\therefore 2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ → It is required eq. of tangent

Eqn of normal:-

$ax + by + c = 0$

$bx - ay + d = 0$

$\therefore (-1)x + 2y + d = 0$

$\therefore -x + 2y + d = 0$ at $(2,-2)$

$\therefore -2 + 2(-2) + d = 0$

$\therefore -2 - 4 + d = 0$

$\therefore -6 + d = 0$

$\therefore d = 6$

Q.5) Find the local maxima & minima for the following function

1) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$2x - y + 2 = 0$

$2x - y = -2 \quad \cdots (i)$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \quad \cdots (ii)$

Multiply eqn (i) with 2

$\therefore 4x - 2y = -4$

$2y - 3x = 4$

$x = 0$

Substitute value of x in eqn (i)

$2(0) - y = -2$

$-y = -2$

$\therefore y = 2$

$\therefore$ Critical point are $(0,2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here, $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2 = 3 > 0$

$\therefore f$ has maxima at $(0,2)$

$\therefore 3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$\therefore 0 + 4 - 0 + 0 - 8 = -4$

2) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 - 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0 \quad \cdots (i)$

$f_y = 0$

$3x^2 - 2y = 0 \quad \cdots (ii)$

Multiplying eqn (i) with 3 & eqn (ii) with 4

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$\therefore 1y = 0$

$\therefore y = 0$

Subs. value of y in eqn (i)

$4x^2 + 3(0) = 0$

$\therefore 4x^2 = 0$

$\therefore x = 0$

Critical point is $(0,0)$.

$r = f_{xx} = 3x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6 - 0 = 6x = 6(0) = 0$

r at $(0,0)$

$= 3x(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (s)^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(nothing to say)

$f(x,y)$ at $(0,0)$

$3(0)^3 + 3(0)^2 \; 6(0) - (0)$

$= 0 + 0 - 0$

$= 0$ //

iii) $f(x,y) = x^2 - y^2 + 8y - 70$

$\therefore f_x = 2x + 2$

$\therefore f_y = -2y + 8$

$f_x = 0 \quad 2x + 2 = 0$

$x = \frac{-2}{3} \quad \therefore x = -1$

$f_y = 0 \quad -2y + 8 = 0$

$y = \frac{-8}{-2} = 4$

$\therefore$ Critical point is $(-1,4)$.

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1,4)$

$= (-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= (-1)^2 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70$

$= 33$ //

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