```r
# Q1 consistent estimator : c(mu,1) t1=mean,t2=median

#  Que. Based on the random samples generated from cauchy(μ, 1) check the
consistency of the estimators  T1 = X and T2= sample median for the
parameter μ . Construct the following table.

#                Sample size (n), P(|T1- μ|<0.1), P(|T2- μ|<0.1)



# Program

n=5

mu=1.3

sigma=1

eps=.1

x=rcauchy(n*n,mu,sigma)

data=matrix(x,n,n)

t1=apply(data,1,mean)

t2=apply(data,1,median)

prob_1=mean(abs(t1-mu)<eps)

prob_2=mean(abs(t2-mu)<eps)


#simulation study:

rm(list = ls(all=TRUE))

n=c(10,100,1000)

mu=2.1

sd=1

eps=.1

est_t1_prob=0

est_t2_prob=0

for (i in 1:length(n))

{

  x=rcauchy(n[i]*n[i],mu,sd)

  a=matrix(x,n[i],n[i])

  t1=apply(a,1,mean)

  t2=apply(a,1,median)

  est_t1_prob[i]=mean(abs(t1-mu)<eps)
```

```
  est_t2_prob[i]=mean(abs(t2-mu)<eps)
}
cbind(n,est_t1_prob,est_t2_prob)
```

```
##          n est_t1_prob est_t2_prob
## [1,]   10       0.100       0.400
## [2,]  100       0.070       0.450
## [3,] 1000       0.057       0.964
```

```
cat("EStimated prob of consistent estimator corresponding to mean",est_t1_p
rob,"\n")
```

```
## EStimated prob of consistent estimator corresponding to mean 0.1 0.07 0.
057
```

```
cat("EStimated prob of consistent estimator corresponding to mean",est_t2_p
rob,"\n")
```

```
## EStimated prob of consistent estimator corresponding to mean 0.4 0.45 0.
964
```

```
#Q2  Check the consistency of T1 = X(n),T2 = X and T3 = 2X in the
# case of Uniform(0,θ) based on the random sample. Suggest one more consist
ent
# estimator for θ. Compare T1,T2, and T3 in terms of MSE.
n=5
a=0
b=6
eps=.1
x=runif(n*n,a,b)
data=matrix(x,n,n)
t1=apply(data,1,max)
t2=apply(data,1,mean)
t3=2*t2
est_t1_prob=mean(abs(t1-b)<eps)
est_t2_prob=mean(abs(t2-b)<eps)
est_t3_prob=mean(abs(t3-b)<eps)
mse_1=mean((t1-b)^2)
mse_2=mean((t2-b)^2)
mse_3=mean((t3-b)^2)
cbind(n,est_t1_prob,est_t2_prob,est_t3_prob,mse_1,mse_2,mse_3)
```

```
##      n est_t1_prob est_t2_prob est_t3_prob    mse_1    mse_2    mse_3
## [1,] 5           0           0           0 1.418695 11.33218 3.051098
```

```
#simulation:
n=c(50,100,500,1000)
```

```r
a=0
b=6
eps=0.1
est_t1_prob=0;est_t2_prob=0;est_t3_prob=0;est_t4_prob=0
mse_1=0;mse_2=0;mse_3=0;mse_4=0
for (i in 1:length(n))
{
  x=runif(n[i]*n[i],a,b)
  data=matrix(x,n[i],n[i])
  t1=apply(data,1,max)
  t2=apply(data,1,mean)
  t3=2*t2
  est_t1_prob[i]=mean(abs(t1-b)<eps)
  est_t2_prob[i]=mean(abs(t2-b)<eps)
  est_t3_prob[i]=mean(abs(t3-b)<eps)
  mse_1[i]=mean((t1-b)^2)
  mse_2[i]=mean((t2-b)^2)
  mse_3[i]=mean((t3-b)^2)
}
cbind(n,est_t1_prob,est_t2_prob,est_t3_prob,mse_1,mse_2,mse_3,mse_4)
```

```
##         n est_t1_prob est_t2_prob est_t3_prob        mse_1    mse_2
mse_3
## [1,]   50       0.580           0       0.080 0.0269905620 9.024411 0.25
220601
## [2,]  100       0.800           0       0.170 0.0063386671 8.902444 0.12
105806
## [3,]  500       0.998           0       0.470 0.0003019584 9.034023 0.02
496796
## [4,] 1000       1.000           0       0.612 0.0000832190 8.995953 0.01
281578
##      mse_4
## [1,]     0
## [2,]     0
## [3,]     0
## [4,]     0
```

```r
# Q.3  Based on the random samples, check the consistency of empirical
# distribution function at t=2 for the FX (t) , where F(.) is cdf of
# Exponential distribution with parameter θ .
# Program:
```

```r
n=5
t=2
theta=1
eps=.05
est_t1_prob=0
n=c(100,200,500)
f_x=1-exp(-t*theta)
for (i in 1:length(n))
{
  x=rexp(n[i]*n[i],theta)
  y=matrix(x,n[i],n[i])
  y1=y<=t
  t1=apply(y1,1,mean)
  est_t1_prob[i]=mean(abs(t1-f_x)<eps)
}
cbind(n,est_t1_prob)
```

```
##         n est_t1_prob
## [1,] 100        0.83
## [2,] 200        0.96
## [3,] 500        1.00
```

```r
#4 Let X be a random variable having poisson distribution with parameter λ.
# Suggest one consistent estimator for e^(-λ). Based on the random samples,
# demonstrate the consistency of the estimator suggested by you.
# Program:
rm(list = ls(all=T))
n=5
eps=.06
lambda=2
est_t1_prob=0
est_t2_prob=0
n=c(50,100,200,500)
for (i in 1:length(n))
{
  x=matrix(rpois(n[i]*100,lambda = lambda),100,n[i])
  t1=apply(x,1,mean)
  t2=exp(-t1)
```

```
  t3=apply(x,1,var)
  t4=exp(-t3)
  est_t1_prob[i]=mean(abs(t2-exp(-lambda))<eps)
  est_t2_prob[i]=mean(abs(t4-exp(-lambda))<eps)

}
cbind(n,est_t1_prob,est_t2_prob)
```

```
##          n est_t1_prob est_t2_prob
## [1,]   50        0.94        0.71
## [2,]  100        1.00        0.91
## [3,]  200        1.00        0.96
## [4,]  500        1.00        1.00
```

---

AC

```
# Q.1  let X be a random variable having Poisson(λ). Based on the random
#      sample of size n= 20 from Poisson(λ), obtain 95% ACI for λ,
#      using:  a)VST      b) Pivotal method.
# Program:

rm(list=ls(all=T))
n=100; lambda=2;
x=rpois(n,lambda)
m=mean(x)
z=qnorm(0.025,lower.tail=F)
L_1=((2*m+z^2/n)-sqrt((2*m+z^2/n)^2-4*m^2))/2
U_1=((2*m+z^2/n)+sqrt((2*m+z^2/n)^2-4*m^2))/2
cbind(L_1,U_1)
```

```
##           L_1     U_1
## [1,] 1.657655 2.20076
```

```
length_1=U_1-L_1
L_2=(sqrt(m)-z/(2*sqrt(n)))^2
U_2=(sqrt(m)+z/(2*sqrt(n)))^2
cbind(L_2,U_2)
```

```
##           L_2      U_2
```

```
## [1,] 1.648731 2.190476
```

```r
length_2=U_2-L_2

# Q.2 let X be a random variable having B(1, θ). Based on the random sample of          size n= 25 from B(1,0.4), obtain 95% ACI for θ,

#       using:    a) Pivotal method.

# Program:



rm(list=ls(all=T))

n=25

th=0.4

x=rbinom(n,1,th)

m=mean(x)

z=qnorm(0.025,lower.tail = F)

L_1=((2*n*m+z^2)-sqrt((4*z^2*n*m*(1-m)+z^4)))/(2*(z^2+n))

U_1=((2*n*m+z^2)+sqrt((4*z^2*n*m*(1-m)+z^4)))/(2*(z^2+n))

cbind(L_1,U_1,length_1=U_1-L_1)
```

```
##             L_1       U_1 length_1

## [1,] 0.3003129 0.6650148 0.364702
```

```r
# Q.3 . let X be a random variable having Cauchy(μ,1). Based on the random sample of size n= 25 from  Cauchy(4,1), obtain 95% ACI for μ,

#        using:    a) Pivotal method.

# Program:



rm(list=ls(all=T))

n=25

mu=4

x=rcauchy(n,mu,1)

u=c(median(x),rep(0,10))

v=c(median(x),rep(0,10))

I=1/2

z=qnorm(0.025,lower.tail = F)

for(i in 1:10)

{

  y=x-u[i]

  d_1=2*sum((y)/(1+y^2))

  d2_1=2*sum((2*y^2-1-y^2)/(1+y^2)^2)
```

```
  u[i+1]=u[i]-d_1/d2_1

  v[i+1]=u[i]+d_1/(n*I)

}

u
```

```
##  [1] 3.510699 3.609152 3.605713 3.605710 3.605710 3.605710 3.605710 3.60
5710

##  [9] 3.605710 3.605710 3.605710
```

```
v
```

```
##  [1] 3.510699 3.592675 3.606081 3.605710 3.605710 3.605710 3.605710 3.60
5710

##  [9] 3.605710 3.605710 3.605710
```

```
L_1=v-(z*sqrt(2/n))

U_1=v+(z*sqrt(2/n))

cbind(L_1,U_1)
```

```
##              L_1      U_1

##  [1,] 2.956337 4.065060

##  [2,] 3.038313 4.147036

##  [3,] 3.051720 4.160443

##  [4,] 3.051348 4.160072

##  [5,] 3.051348 4.160071

##  [6,] 3.051348 4.160071

##  [7,] 3.051348 4.160071

##  [8,] 3.051348 4.160071

##  [9,] 3.051348 4.160071

## [10,] 3.051348 4.160071

## [11,] 3.051348 4.160071
```

```
# Q.4 Let X be a random variable having Exp(mean=?).

#      Based on the random sample of size n= 25 from  Exp(??), obtain 95% A
CI          for ?  using: a)VST b) Pivotal method.

# Program:

{

rm(list=ls(all=T))

n=25

theta=3

x=rexp(n,theta)

z=qnorm(0.025,lower.tail = F)

m=mean(x)

m1=log(m)
```

```r
L_1=exp(m1-z/sqrt(n))
U_1=exp(m1+z/sqrt(n))}
cbind(L_1,U_1,length_1=U_1-L_1)
```

```
##              L_1      U_1  length_1
## [1,] 0.2259846 0.494948 0.2689633
```

```r
s1=1+(z/sqrt(n))
s2=1-(z/sqrt(n))
L_2=mean(x)/s1
U_2=mean(x)/s2
cbind(L_2,U_2,length_2=U_2-L_2)
```

```
##              L_2       U_2  length_2
## [1,] 0.2402604 0.5500606 0.3098001
```

```r
# Q.5 Let X be a random variable having Laplace(θ, 1). Based on the random
sample of size n= 25 from Laplace(2.5,1), obtain 95% ACI for θ , using
Pivotal method.
#   Program:
rm(list=ls(all=T))
n=25
t=2.5
z=qnorm(0.025,lower.tail = F)
Y=runif(n,0,1)
X=t+log(2*Y)*(Y<=0.5)-log(2*(1-Y))*(Y<=0.5)
L1=median(X)-z/sqrt(n)
U1=median(X)+z/sqrt(n)
cbind(L1,U1,length1=U1-L1)
```

```
##              L1       U1   length1
## [1,] 2.108007 2.891993 0.7839856
```