

UNIVERSITY OF MAURITIUS

FACULTY OF ENGINEERING



SECOND SEMESTER EXAMINATIONS

MAY 2016

PROGRAMME	BSc (Hons) Software Engineering - Full-Time BSc (Hons) Software Engineering - <u>MIXED MODE</u>		
MODULE NAME	Discrete Mathematics for Software Engineering		
DATE	Tuesday 24 May 2016	MODULE CODE	CSE1014Y(1)
TIME	09:30 - 12:30 Hrs	DURATION	3 hours
NO. OF QUESTIONS SET	6	NO. OF QUESTIONS TO BE ATTEMPTED	5

INSTRUCTIONS TO CANDIDATES

Answer any 5 questions.

All questions carry equal marks.

The Normal Distribution Table is attached.

Answer any 5 questions.

All questions carry equal marks.

Question 1

(a) Consider the following system of linear equations:

$$X + 2Y + Z = 3.$$

$$2X + 3Y + 3Z = 10$$

$$3X - Y + 2Z = 13$$

Solve the system of equations by

- (i) Gauss Elimination method
- (ii) Cramer's rule

[5 + 5 marks]

(b) Consider the following system of equations

$$X + 2Y - 1Z = 2$$

$$2X + 20Y - 2Z = 36$$

$$-X - 2Y + 10Z = 25$$

Perform five (5) iterations on the above system using the

- (i) Gauss Jacobi,
- (ii) Gauss Seidel method; and
- (iii) Compare your results.

Use $X=0, Y=0$, and $Z=0$ as the starting values, and provide your answers correct to 3 decimal places.

[4 + 4 + 2 marks]

Question 2

- (a) Consider a computer in a lab. The number of times the computer fails follows a Poisson distribution with a parameter (λ) of one (1) failure per four (4) hour slots.

The computer is on for eight (8) hours every day.

- (i) Determine the probability that the computer will have at least one failure in a day.
- (ii) A lab contains 20 such computers. Determine the probability that at least one computer will fail in that lab in a day.
- (iii) Determine the probability that between four (4) and sixteen (16) computers will fail during a particular day.

[3 + 4 + 4 marks]

- (b) A report on employment claims that the monthly salary of fresh graduates in the field of software engineering has a mean of Rs 16540.

To investigate this, the university sent 300 questionnaires for 300 randomly chosen graduates. 100 people replied and upon analysis of the 100 data, it was found to have a mean of Rs 16080 and a standard deviation of Rs 900.

Determine whether the claim of the report is justified by

- (i) Constructing a 95% confidence interval, and
- (ii) Doing a hypothesis test at the 5% significance level.

[4 + 5 marks]

Question 3

(a) Show that $A \cap B = B - (B - A)$

- (i) by using basic set identities, and
- (ii) by using membership/truth tables

[2 + 2 marks]

(b) Determine which of the following functions f are injections, surjections, bijections or none:

Also if f is invertible, determine its inverse.

- (i) $f: \mathbf{Z} \rightarrow \mathbf{R}$ is given by $f(x) = x^2$
- (ii) $f: \mathbf{Z} \rightarrow \mathbf{R}$ is given by $f(x) = 2x$
- (iii) $f: \mathbf{R} \rightarrow \mathbf{R}$ is given by $f(x) = x^3$
- (iv) $f: \mathbf{Z} \rightarrow \mathbf{N}$ is given by $f(x) = |x|$
- (v) $f: \{\text{people}\} \rightarrow \{\text{people}\}$ is given by $f(x) = \text{the father of } x$.

[5 * 1 marks]

(c) Compute $f \circ g$ where

- (i) $f: \mathbf{Z} \rightarrow \mathbf{R}, f(x) = x^2$ and $g: \mathbf{R} \rightarrow \mathbf{R}, g(x) = x^3$
- (ii) $f: \mathbf{Z} \rightarrow \mathbf{Z}, f(x) = x + 1$ and $g(x) = x - 1$
- (iii) $f: \{\text{people}\} \rightarrow \{\text{people}\}, f(x) = \text{the father of } x$, and $g = f$

[3*1 marks]

(d) Let the relations R, S and T be represented by the following zero-one matrices

$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	$M_S = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$	$M_T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
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Determine the matrices representing $R \cup S$, $R \cap S$ and T^2 .

[3 * 1 marks]

(e) Let $A = \{a, b, c, d\}$ and let $R = \{(a, b), (b, c), (c, d), (d, b)\}$ be a relation on A

- i. Draw the directed graph representing R .
- ii. Determine the transitive closure R^* of R .

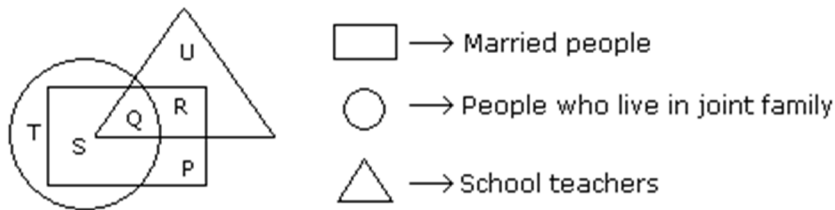
[2 + 3 marks]

Question 4

- (a) The product of 3 consecutive numbers is 504. Determine these three numbers. Explain your reasoning. **[6 marks]**
- (b) Use the Euclidean algorithm to
- (i) determine the greatest common divisor of 161 and 91 and
 - (ii) find integers s and t satisfying that $\gcd(161, 91) = s \cdot 161 + t \cdot 91$.

[4 + 4 marks]

(c)



Write letters corresponding to:

- (i) The married teachers who live in joint family.
- (ii) the married people who live in joint family but not are school teachers.
- (iii) the people who live in joint family but are neither married nor teachers.

[3*2 marks]

Question 5

- (a) A sequence is represented as follows
- $$A_n = A_{n-1} + 3, \quad A_0 = 0$$
- (i) Propose an explicit formula for A_n
 - (ii) Use proof by induction to verify your answer in part (i) above.
- [4 + 4 marks]**
- (b) Show that $f(x) = 2x^3 + x^2 + 5$ is $O(x^3)$. **[4 marks]**
- (c) Let f be a permutation function which operates on $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ to produce $\{2, 4, 8, 1, 5, 9, 3, 7, 6\}$. f can also be represented by

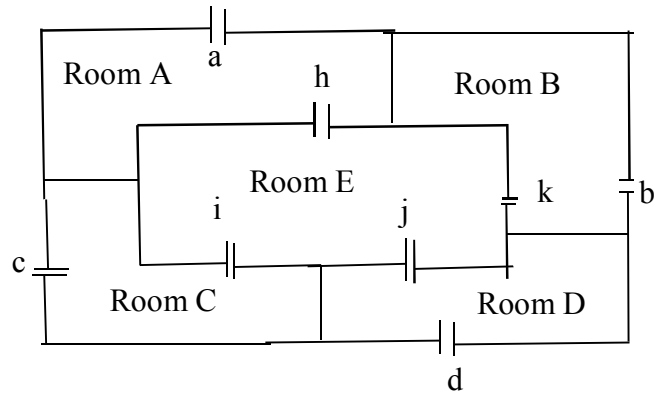
$$f = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 8 & 1 & 5 & 9 & 3 & 7 & 6 \end{bmatrix}$$

- (i) Write f as a product of cycles.
- (ii) Write down f^{-1}
- (iii) Let P be f^{-1} find P^{-1}
- (iv) P^{-1} is in fact f^{-2} . Compare f^{-2} to f and use that relationship to find f^{100}

[3+1+1+3 marks]

Question 6

a) The plan of a museum is shown below.



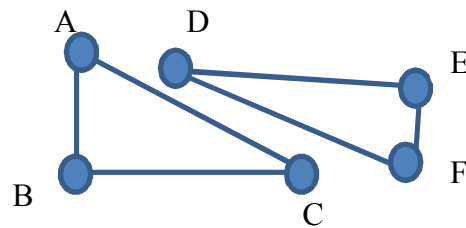
- (i) Determine whether it is possible to visit all the rooms by passing through each door exactly once using graph theory. Also state whether the visitor should start inside or outside. Explain your working.

[8 marks]

- (ii) State whether a Hamilton path exist for the graph above.

[4 marks]

- (b) (i) Derive the chromatic polynomial for the graph below.



[4 marks]

- (ii) Determine the number of ways the graph in part (b)(i) above can be coloured using 6 different colours. Explain your reasoning.

[4 marks]

END OF QUESTION PAPER