

# **UNIVERSITY OF MAURITIUS**

## **FACULTY OF INFORMATION COMMUNICATION AND DIGITAL TECHNOLOGIES**



### **SECOND SEMESTER EXAMINATIONS**

**MAY 2017**

<b>PROGRAMME</b>	<b>BSc(Hons) Software Engineering (Full-Time &amp; Mixed Mode)</b>		
<b>MODULE NAME</b>	<b>Discrete Mathematics for Software Engineering/ Discrete Structures</b>		
<b>DATE</b>	<b>Monday 22 May 2017</b>	<b>MODULE CODE</b>	<b>CSE1014Y(1)/ CSE1042Y(1)</b>
<b>TIME</b>	<b>13:30 – 16:30 Hrs</b>	<b>DURATION</b>	<b>3 Hours</b>
<b>NO. OF QUESTIONS SET</b>	<b>6</b>	<b>NO. OF QUESTIONS TO BE ATTEMPTED</b>	<b>5</b>

### **INSTRUCTIONS TO CANDIDATES**

**Answer any 5 questions.**

**All questions carry equal marks.**

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All questions carry equal marks.

### Question 1

- (a) A survey has been taken on methods of commuter travel. Each respondent was asked to check BUS, MOTORCYCLE or CAR as a major method of travelling to work. More than one answer was permitted. The results were as follows: BUS, 30 people; MOTORCYCLE, 35 people; CAR, 100 people; BUS and MOTORCYCLE, 15 people; BUS and CAR, 15 people; MOTORCYCLE and CAR, 20 people and all three methods, 5 people. How many people completed the survey form?
- (b) Evaluate  $\text{GCD}(30,220).\text{LCM}(30,200)$  and comment on your answer.
- (c) Use the Euclidean algorithm to find  $\text{GCD}(66,495)$ .
- (d) The product of three consecutive numbers is 720. What are these number; explain your reasoning?
- (e) Prove the following using mathematical induction.

$$3 \cdot 5^0 + 3 \cdot 5^1 + 3 \cdot 5^2 + \dots + 3 \cdot 5^n = 3(5^{n+1} - 1)/4 \quad n \geq 1$$

[4+3+4+4+5 marks]

### Question 2

- (a) (i) Using backtracking, find an explicit formula for:

$$G_n = nG_{n-1} \quad G_1 = 6.$$

- (b) Show that  $7^{644} \bmod 645 = 436$ .

- (c) Draw the truth table for

$$(P \iff Q) \equiv (P \implies Q) \wedge (Q \implies P)$$

(continued next page)

**(Question 2 continued)**

(d)  $A = \{1, 2, 3, 4\}$

$$R = \{(1,1), (1,2), (2,1), (2,2), (2,3), (2,4), (3,4), (4,1)\}$$

- (i) Show that  $R^2$  is a subset of  $A \times A$ .
- (ii) Draw the digraph of  $R^2$ .
- (iii) Calculate in degree and out degree of  $R^2$
- (iv) Find  $M^\infty$

(e)  $A = \{1, 2, 3, 4, 5, 6\}$

Compute  $(4, 1, 3, 5) \circ (5, 6, 2)$

(f) 
$$P = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 6 & 5 & 2 & 1 & 8 & 7 \end{bmatrix}$$

P represents a permutation on a set  $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Write P as a product of transposition cycles.

**[4+5+2+4\*1+2+3 marks]**

**Question 3**

- (a) There are 5 orange sweets, 7 red sweets and 2 yellow sweets in a bag. Peter takes a sweet at random, eats it and then takes another sweet at random. What is the probability that both sweets are of the same colour.

**(2 marks)**

- (b) In the population of Mauritian who drink tea, the average daily consumption is 3 cups per day. A particular company wants to know if its employees tend to drink more tea than the national average. They ask 50 employees how many cups of tea they drink each day and found  $\bar{x} = 3.8$  and  $s = 1.5$ . Does the company have evidence that its employees drink more than the national average?

**(6 marks)**

*(continued next page)*

**(Question 3 continued)**

- (c) A researcher assumes that there is a relationship between the amount of fertilizers supplied to potatoes and the subsequent yield of potatoes obtained. 8 potato plants of the same variety were chosen randomly and supplied weekly with a fixed volume of solution consisting of  $t$  grams of fertilizer and the same volume of water. The yield  $y$  kilograms of potatoes was recorded as follows:

Potato Plant	a	b	c	d	e	f	g	h
$t$	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
$y$	3.9	4.4	5.8	6.6	7.0	7.1	7.3	7.7

- a. Plot a scatter diagram of yield ( $y$ ) against amount of fertilizers used ( $t$ )  
(4 marks)
- b. Calculate the equation of the least squares regression line of  $y$  on  $t$   
(5 marks)
- c. Estimate the yield for a plant supplied weekly with a solution consisting of 3.2 grams of fertilizers  
(3 marks)

**[Total 20 marks]**

**Question 4**

- (a) Define the following terms

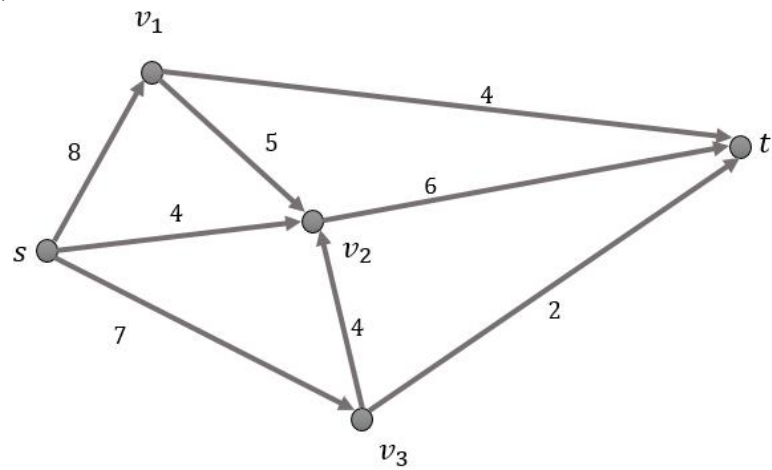
- i. Hamiltonian cycle
- ii. Euler circuit
- iii. A tree

**[3 \* 2 marks]**

- (b) A company in Mauritius wants to export some of its products in jars to a supermarket South Africa. There are different routes with some constraints through which the jars can be sent. The digraph below shows various routes with vertex  $s$  as the company's main factory and vertex  $t$  as the supermarket in South Africa. The other vertices represent intermediary locations where the jars can be stored. The number on an arc represents the maximum number of jars (in thousands) that can be transferred from  $A$  to  $B$  where  $\text{head}(e) = B$  and  $\text{tail}(e) = A$ . Find the route through which the maximum number of jars can be sent from the main factory to the supermarket in South Africa.

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**(Question 4 continued)**

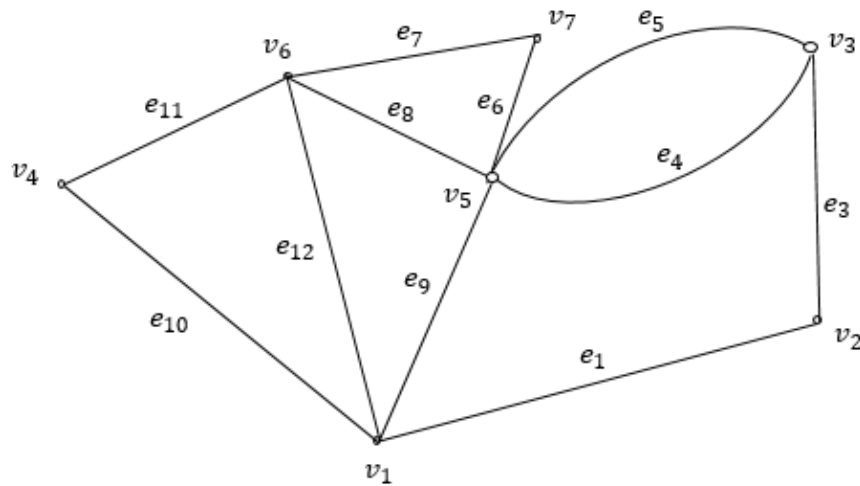


**[7 marks]**

- (c) Prove that in a tree with more than one vertex there is at least two vertices of degree 1.

**[3 marks]**

- (d) Consider the graph G. Find a spanning tree of the graph G.



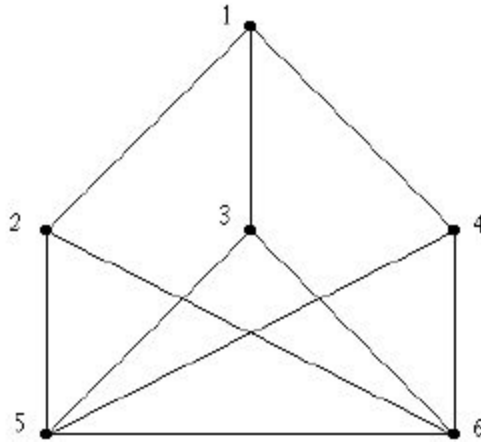
**[4 marks]**

**Question 5**

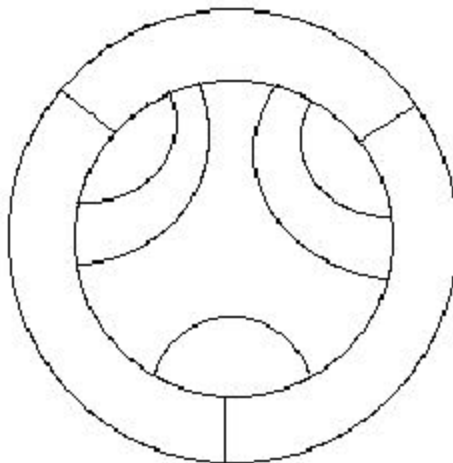
- (a) Using the O function state whether f or g grows faster or have the same order

$$f(n) = 3n^4 - 5n^2 \qquad g(n) = n^4$$

- (b) Determine whether the following graph is planar.



- (c) Find the minimum number of colours needed to colour the regions, including the infinite region, of the map in Fig 1, so that no adjacent regions have the same colour.

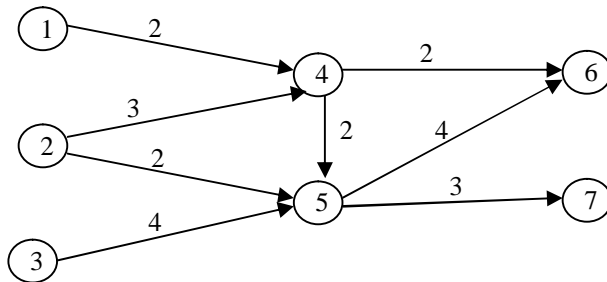


**Fig 1**

*(continued next page)*

**(Question 5 continued)**

- (c) Find the maximal flow for the network shown below where the capacity for each edge is shown. Use the labeling algorithm.



[4+ 3+3+10 marks]

**Question 6**

- (a) A is a symmetric matrix. Find the values of a,b and c.

$$A = \begin{bmatrix} 0 & -1 & 2 \\ c & 4 & b \\ a & -3 & 0 \end{bmatrix}$$

- (b) Using the properties of determinant show that

$$\begin{vmatrix} x+a & a & a \\ a & x+a & a \\ a & a & x+a \end{vmatrix} = x^2(x+3a)$$

*(continued next page)*

**(Question 6 continued)**

- (c) Decompose the matrix below into product of lower (L) and upper triangular (U) matrix.

$$\begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 6 \\ -1 & 0 & 1 \end{pmatrix}$$

- (d) Using the formula  $AX = B$  and the results obtained in (c) find the values of  $x_1$ ,  $x_2$  and  $x_3$ .

$$\begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 6 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \\ 2 \end{pmatrix}$$

**[5+5+5+5 marks]**

**END OF QUESTION PAPER**