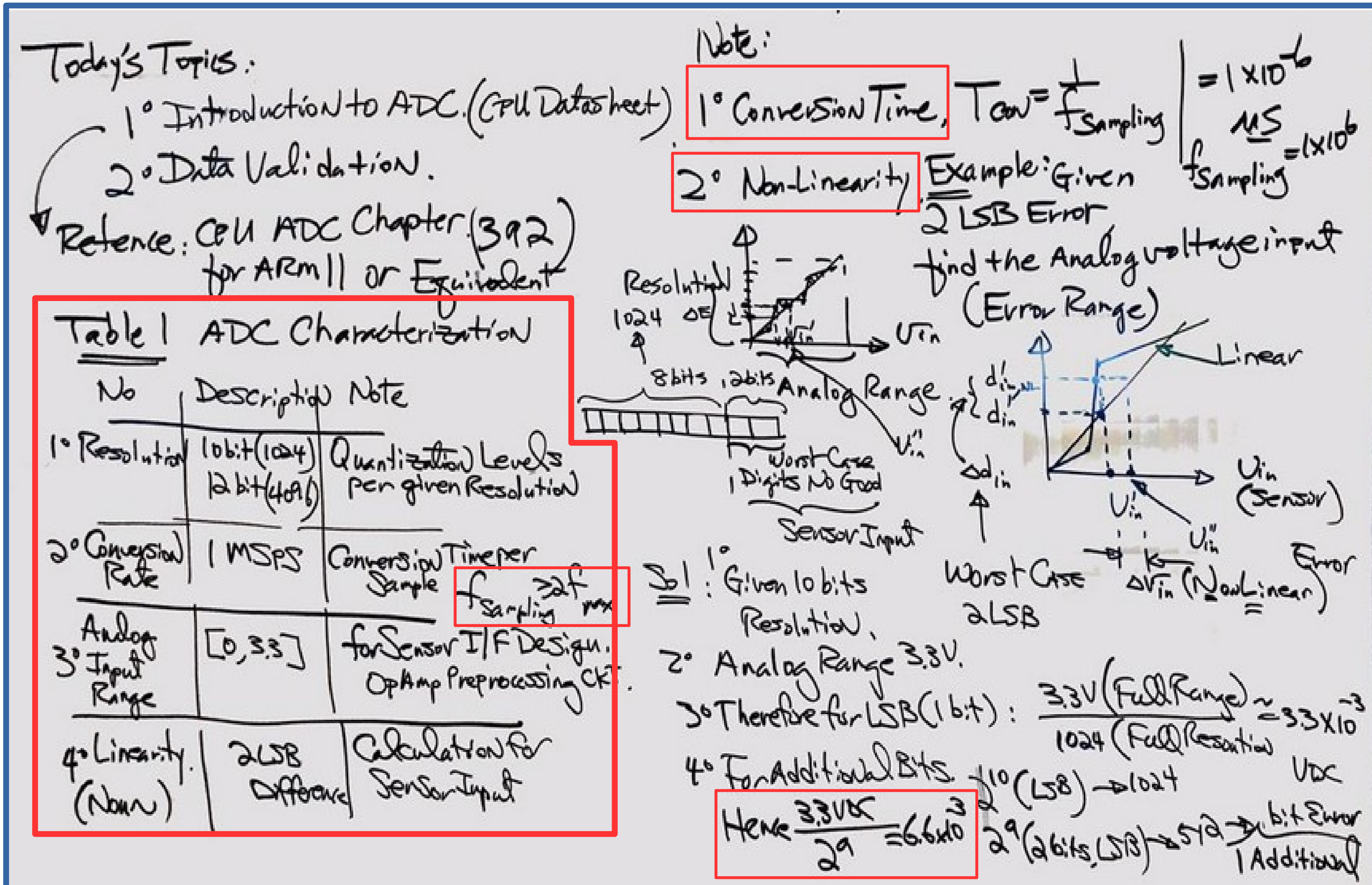
ADC Characterization



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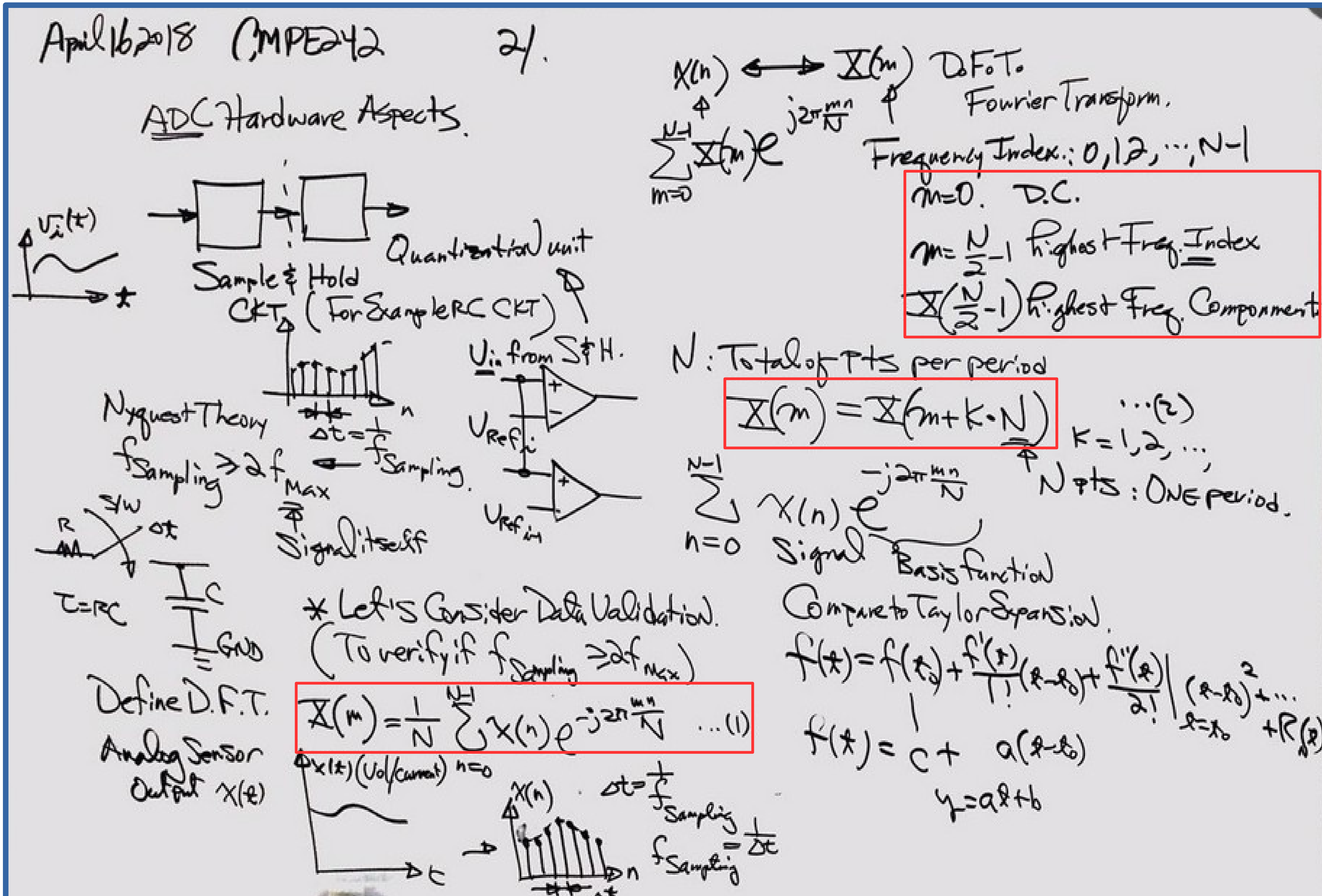
Introduction To DFT For Data Validation



Harry Li, Ph.D.

DFT Power Spectrum

April 18, 18. HL CMPE242 Embedded Hardware

Today's Topics: 1° Device Driver/CPU Datasheet.
Special Purpose Register →
* Define/Select Clock Rate $f_{Sampling}$

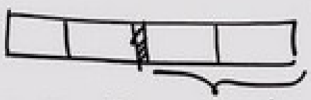
2° Data Validation (D.F.T.)

CPU Datasheet 39.6.2 Example: user App. Program.

fd = open("/dev/adc", 0);

read(fd, buffer, sizeof buff);

1° ADCCON[13:6] Prescaler Range. ✓



8 bits

Formula for $f_{Sampling}$ is to be verified. HL

2° ADCCON[5:3] 3° Map Device Drivers.

Consider D.F.T. $\boxed{X(m)=X(m+kN)}$ → Power Spectrum.

$$\boxed{P(m)=\mathrm{Sqrt}\left(Re^2[X(m)]+Im^2[X(m)]\right) \quad \dots(3)}$$

$$\simeq Re^2[X(m)]+Im^2[X(m)]$$

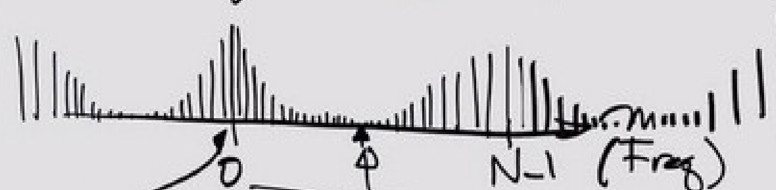
y. $\boxed{P(m)=P(m+kN)}$

$$\left(\because = Re^2(X(m+kN))+Im^2(X(m+kN))\right)$$

Example

$$= Re^2(X(m))+Im^2(X(m))$$

Distribution of the Energy v.s. freq. Index.

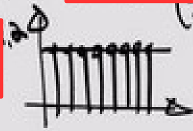


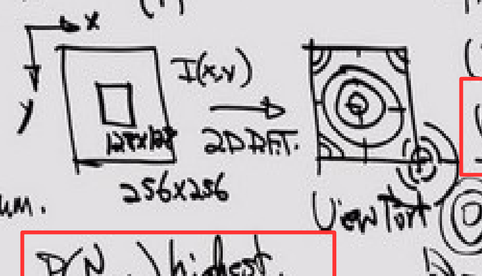
"m=0 "Low Pass" type

$\frac{N}{2}-1$

$\boxed{\text{N-pts, One Period}}$ (2)

$\boxed{P(m)|_{m=0} \text{ DC Comp.}}$ (1)





(3) periodic Function

$\boxed{(4)\ m_h = \frac{N}{2}-1 \text{ highest Freq. Index}}$

(5) Nyquist $f_{Sampling} \geq 2 f_M$.

$\boxed{P(\frac{N}{2}-1) \text{ highest freq. Comp.}}$

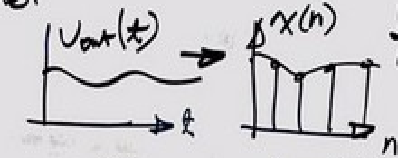
(6) verification: $\boxed{\sum_{m\in\Omega} P(m) \leq \text{Threshold}_{15\%} \quad \dots(4)}$

$\boxed{\text{Total } \sum_{m=0}^{N-1} P(m)}$

Reference: A.V. Oppenhaim, Digital Signal Processing

DFT Power Spectrum Calculation

Example:



$\{x(n) \mid n = 0, 1, 2, \cdots, N-1\}$

A.V. Oppenhaim "Digital Signal Processing"

$$X(m) = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j2\pi\frac{mn}{N}}$$

$$= \frac{1}{N}\left[x(0)e^{-j2\pi\frac{m\cdot 0}{N}} + x(1)e^{-j2\pi\frac{m\cdot 1}{N}} + \cdots + x(N-1)e^{-j2\pi\frac{m(N-1)}{N}}\right]$$

n (Time)

m (Freq)

$$X(p)\ \begin{bmatrix} X(0) \\ X(1) \\ \vdots \\ X(N-1) \end{bmatrix}_{N\times 1} = \underbrace{\begin{bmatrix} & & \vdots & \\ p & \cdots & \boxed{e^{-j2\pi\frac{p\cdot q}{N}}} & \cdots \\ & & \vdots & \end{bmatrix}}_{\boxed{N\times N}\ (1)} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}_{N\times 1} \quad \ldots(4)$$

(2) $e^{-j2\pi\frac{p\cdot q}{N}}$ m or p. freq Index; n or q Time Index

From Matrix Form,

$$\boxed{X(m) = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j2\pi\frac{mn}{N}}}$$

$$= \frac{1}{N}\left(x(0)e^{-j2\pi\frac{m\cdot 0}{N}} + x(1)e^{-j2\pi\frac{m\cdot 1}{N}} + \cdots + x(N-1)e^{-j2\pi\frac{m(N-1)}{N}}\right)$$

For m=0 $\quad X(0) = \frac{1}{N}\left(x(0)e^{-j2\pi\frac{0\cdot 0}{N}} + x(1)e^{-j2\pi\frac{0\cdot 1}{N}} + \cdots + x(N-1)e^{-j2\pi\frac{0\cdot(N-1)}{N}}\right)$

$= \frac{1}{N}\left(x(0) + x(1) + \cdots + x(N-1)\right)$, D.C. Comp.

m=1 $\quad X(1) = \frac{1}{N}\left(x(0)e^{-j2\pi\frac{1\cdot 0}{N}} + x(1)e^{-j2\pi\frac{1\cdot 1}{N}} + \cdots + x(N-1)e^{-j2\pi\frac{1(N-1)}{N}}\right)$

$\vdots$

$$X(N-1) = \frac{1}{N}\left(x(0)e^{-j2\pi\frac{(N-1)\cdot 0}{N}} + x(1)e^{-j2\pi\frac{(N-1)\cdot 1}{N}} + \cdots + x(N-1)e^{-j2\pi\frac{(N-1)(N-1)}{N}}\right)$$

$e^{-j2\pi\frac{p\cdot q}{N}}$ where ~~i~~ p for freq. index, ~~j~~ q for time index $\quad\cdots(5)$

Eular Formula,

$$\boxed{e^{-j2\pi\frac{pq}{N}} = \cos 2\pi\frac{pq}{N} - j\sin 2\pi\frac{pq}{N} \quad (5*)}$$

Reference: A.V. Oppenhaim, Digital Signal Processing

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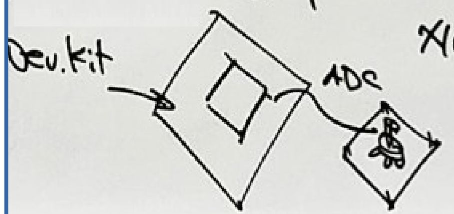
Power Spectrum Example

Example: Given from my ADC. $\{x(n) \mid n=0,1,2,\ldots,N-1\}$



$x(0)=2,\ x(1)=3,\ x(2)=4,\ x(3)=4$

$N=4.$

1st Step

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}_{4\times 4}$$

for $m=1,\ n=1$, hence from (5)

$$e^{-j2\pi\frac{m\cdot n}{N}} = e^{-j2\pi\frac{1\cdot 1}{4}} = e^{-j\cancel{2}\pi/2}$$

$N=4$

$$= \cancel{\cos\pi/2}^{\,0} - j\sin\pi/2 = 0 - j = -j$$

for $m=1,\ n=2$, $e^{-j2\pi\frac{1\cdot 2}{4}} = e^{-j\pi}$

$$= \cos\pi - j\sin\pi = -1 - j0 = -1$$

for $m=1,\ n=3$, $e^{-j2\pi\frac{1\cdot 3}{4}} = e^{-j\frac{3\pi}{2}} =$

$$= \cancel{\cos 3\pi/2}^{\,0} - j\sin 3\pi/2 = 0 - j(-1) = j$$

For $m=2,\ n=2$, $e^{-j2\pi\frac{2\cdot 2}{4}} = e^{-j2\pi} = \cos 2\pi - j\sin 2\pi = 1 - j\cdot 0 = 1$

For $m=2,\ n=3$, $e^{-j2\pi\frac{2\cdot 3}{4}} = e^{-j3\pi} = \cos 3\pi - j\sin 3\pi = -1$

For $m=3,\ n=3$, $e^{-j2\pi\frac{3\cdot 3}{4}} = e^{-j9\pi/2} = e^{-j\left(\frac{8\pi}{2}+\frac{\pi}{2}\right)}$

$$= e^{-j\frac{8\pi}{2}} e^{-j\frac{\pi}{2}} = 1\cdot e^{-j\frac{\pi}{2}} = \cancel{\cos\frac{\pi}{2}}^{\,0} - j\sin\frac{\pi}{2} = -j$$

2nd Step

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix}_{4\times 1} = \frac{1}{4} E_{4\times 4} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 4 \end{bmatrix}$$

$P(m)|_{m=\frac{N}{2}-1} = P(1)$ "Bigger"

$f_{sampling} \geq 2 f_{max}$

Hence;

$X(0) = \frac{1}{4}(1\cdot 2 + 1\cdot 3 + 1\cdot 4 + 1\cdot 4) = \frac{1}{4}\times 13 = 13/4$

$X(1) = \frac{1}{4}(1\cdot 2 + (-j)3 + (-1)4 + j\cdot 4) = \frac{1}{4}(-2+j)$

$X(2) = \frac{1}{4}(1\cdot 2 + (-1)3 + 1\cdot 4 + (-1)4) = \frac{1}{4}(-1)$

$X(3) = \frac{1}{4}(2 + 3j - 4 - 4j) = \frac{1}{4}(-2-j)$

$P(m)|_{m=0} = Re^2[X(m)] + Im^2[X(m)] = Re^2[X(0)] + Im^2[X(0)] = \left(\frac{13}{4}\right)^2 + 0 = \left(\frac{13}{4}\right)^2$

$P(1) = \left(-\frac{2}{4}\right)^2 + \left(\frac{1}{4}\right)^2;\ P(2) = \left(-\frac{1}{4}\right)^2 + 0 = \left(\frac{1}{4}\right)^2;\ P(3) = \left(-\frac{2}{4}\right)^2 + \left(-\frac{1}{4}\right)^2$

Reference: A.V. Oppenhaim, Digital Signal Processing