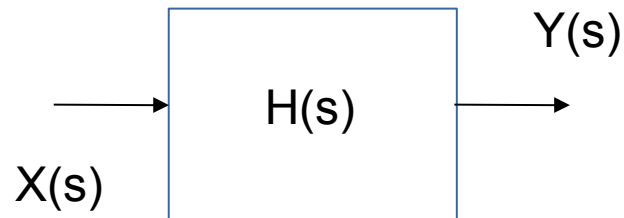


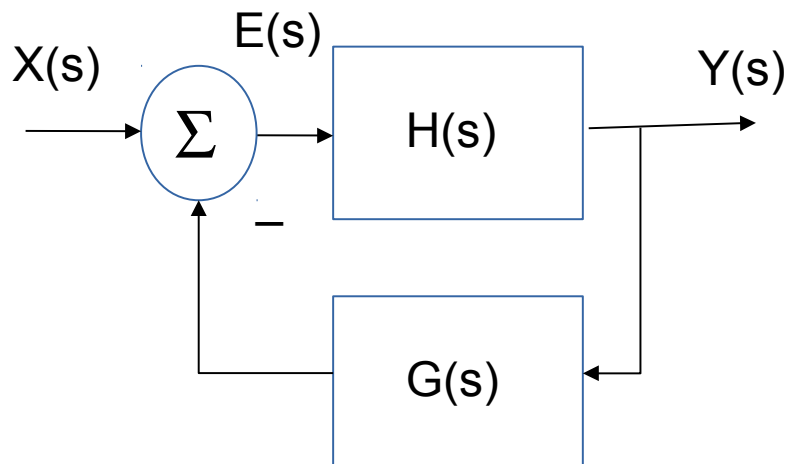
Close Loop Transfer Function



Start with open loop, where plant $H(s)$ can be a stepper motor for example, so we have

$$Y(s) = H(s) X(s) \quad \dots \quad (1)$$

Now we can add a sensor to form a feedback loop as follows



So, sensor $G(s)$ forms a feedback loop as shown in Figure 2.

$$E(s) = X(s) - G(s) Y(s) \quad \dots \quad (2)$$

$$Y(s) = H(s) E(s) \quad \dots \quad (3)$$

From (2) we have

$$X(s) = E(s) + G(s) Y(s) \quad \dots \quad (4)$$

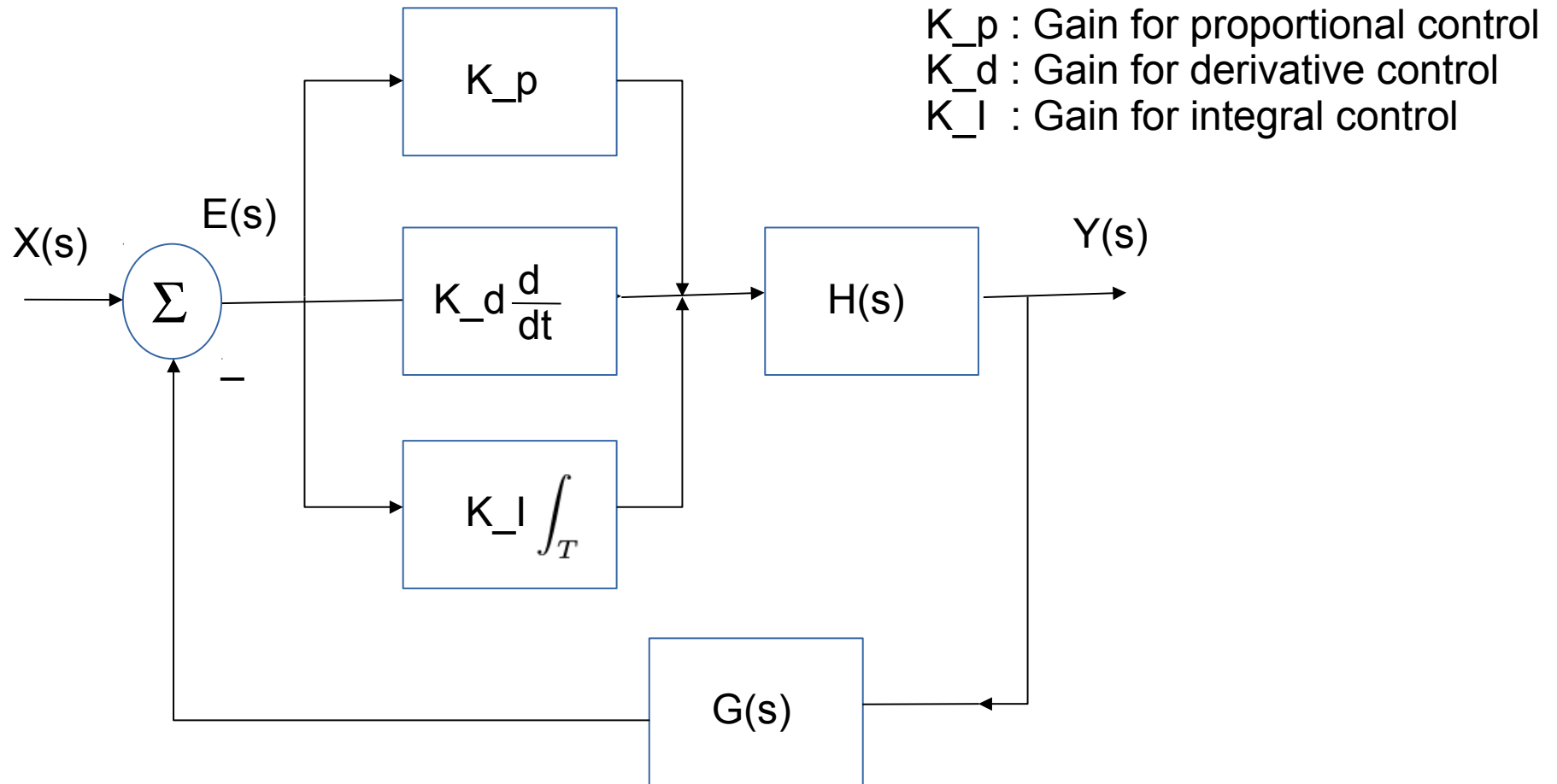
Substitute (3), we have

$$X(s) = E(s) + G(s) H(s) E(s)$$

Hence, the transfer function is

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s) H(s)} \quad \dots \quad (5)$$

PID Control

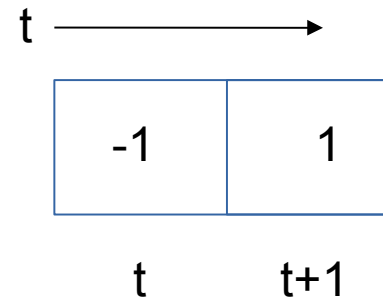


Central Difference And Its Kernel

Forward difference:

$$FD = \frac{de(t)}{dt} \approx e(t+1) - e(t)$$

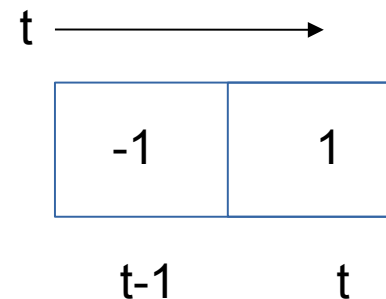
Map to a kernel



Backward difference:

$$BD = \frac{de(t)}{dt} \approx e(t) - e(t-1)$$

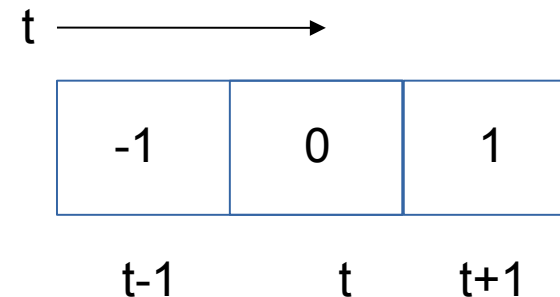
Map to a kernel



Central difference:

$$CD = \frac{1}{2}(FD + BD) = \frac{1}{2}[e(t+1) - e(t-1)]$$

Map to a kernel



LoG Kernel And Convolution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} \quad \dots(1)$$

Then taking the 2nd order derivative of equation (1), as shown below (note I have done 2D example on LoG, in the case of 1D, LoG, just let 2nd independent variable $y = 0$).

$$\frac{\partial^2}{\partial x^2} G(x,y) = -\frac{1}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{x^2}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \dots (4)$$

Based on the result from equation (4) (let $y = 0$ for 1D case), so we can construct K by K kernel (K as an odd number) as illustrated below:

Example: Illustration of K by K kernel



Evaluate each coefficient of the K by K kernel, by setting the $x = 0$ at the center of the kernel, and then you can find coefficient for both $x = +1$ and $x = -1$ (symmetric, they are equal, and so on.)

LoG Kernel And Convolution

Derive LoG Kernel

Example: LoG stands for Laplace of Gaussian
First, Laplace operator is given as:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \dots (1)$$

First, 1D Gaussian function

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Then, 2D Gaussian function,

$$G(x, y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu_x)^2 + (y-\mu_y)^2}{2\sigma^2}} \quad \dots (5)$$

Assume $\mu_x = \mu_y = 0$

$$\frac{\partial}{\partial x} G(x, y) = -\frac{x}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \dots (3)$$

Hence

$$\frac{\partial^2}{\partial x^2} G(x, y) = -\frac{1}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{x^2}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \dots (4)$$

$$\frac{\partial^2}{\partial y^2} G(x, y) = -\frac{1}{\sqrt{2\pi}\sigma^3} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{y^2}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad \dots (5)$$

$$\nabla^2 G(x, y) = \frac{x^2+y^2-2\sigma^2}{\sqrt{2\pi}\sigma^5} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

1D LoG Kernel

March 5th, 2018 ONPE242 Embedded Hardware Harry LI.

- 1) Midterm Exam: March 21st. (Wed)
- 2) Review 1 (ONGPP), Review 2 (ON PWM Timing) Posted ON github/hualili
- 3) Lab 1 (Close Loop w/ FWA Motor Drive) Due March 19th. (Monday)



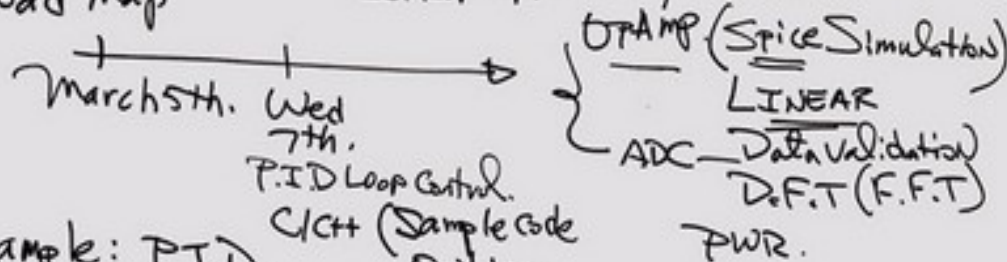
$$G(x,y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (\text{see P.P.T.})$$

- ① σ^2 More Noisy Data
- ② Kernel size \uparrow , to process more noisy data.

Let $y=0$, then $\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$... (2)

Road Map

Sensor I/F IOT/Embedded.



Example: P.I.D.
 K_P, K_I

$$K_I \sum e^2(\tau) \quad \text{C.D.} = \frac{1}{2} [F.D. + B.D.] \rightarrow \text{Kernel} \begin{bmatrix} -1 & +1 \end{bmatrix}$$

Robust Kernel Log

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) G(x,y) \dots (1)$$

$$\frac{\partial}{\partial x} G(x,y) = \frac{\partial}{\partial x} \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2}{2\sigma^2}} \right)$$

Where "Chain" Rule

$$\frac{\partial}{\partial x} G(x,y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2}{2\sigma^2}} \cdot \frac{d}{dx} \left(-\frac{x}{\sigma^2} \right)$$

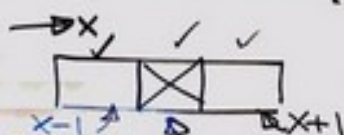
(See eqn (3) ON P.P.T.)

Then $\frac{\partial}{\partial x} \left(\frac{x}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2}{2\sigma^2}} \right)$ See Eqn (4) ON P.P.T. for Y

Calculate PID of Error

CMPE242 March 5th 2018 2/ Harry Li

Derive the kernel. (K by 1)



1° Identify the center of the kernel.

2° Kernel weights (coefficients)

$$\nabla^2 G(x, y)|_{y=0} = \nabla^2 G(x) \text{ (from Eqn (a))}$$

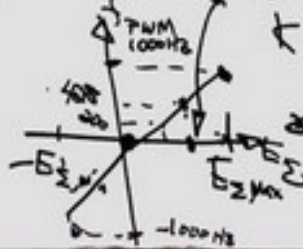
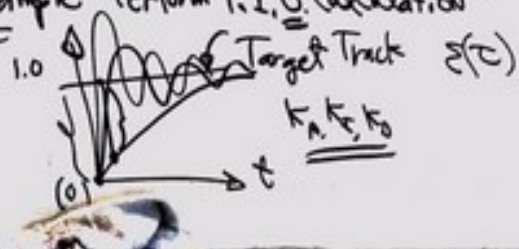
① Kernel Location, $x=0$, $\nabla^2 G(x)|_{x=0} = \frac{1}{\sqrt{2\pi}\sigma^2} (x-\delta^2) e^{-\frac{x^2}{2\sigma^2}}$

$$= \frac{1}{\sqrt{2\pi}} (0-\delta^2) e^{-\frac{0}{2\sigma^2}} = -\frac{\delta^2}{\sqrt{2\pi}\sigma^2} \cdot 1 = -\frac{1}{\sqrt{2\pi}\sigma^2}$$

② Kernel Location, $x=1$, $\nabla^2 G(x)|_{x=1} = \frac{1}{\sqrt{2\pi}\sigma^2} (1-\delta^2) e^{-\frac{1^2}{2\sigma^2}}$

③ Kernel Location, $x=-1$, $\nabla^2 G(x)|_{x=-1} = \frac{1}{\sqrt{2\pi}\sigma^2} (-1-\delta^2) e^{-\frac{1^2}{2\sigma^2}}$

Example: Perform P.I.D. Calculation



Sensor Current Location \rightarrow Compare w/ the target desired Location \rightarrow Error $e(t)$

Location	$E(t)$	$K_p E(t)$	$K_i \int E(t) dt$	$K_d \frac{dE(t)}{dt}$
$L(0)$	$D(0) - L(0) = E(0)$	$K_p E(0)$	—	—
$L(1)$	$D(1) - L(1) = E(1)$	$K_p E(1)$	$K_i (E(0) + E(1))$	$K_d (E(1) - E(0))$
$L(2)$	$D(2) - L(2) = E(2)$	$K_p E(2)$	$K_i (E(0) + E(1) + E(2))$	$K_d (E(2) - E(1))$
\vdots	\vdots	\vdots	\vdots	\vdots
$L(k)$	$D(k) - L(k) = E(k)$	$K_p E(k)$	$K_i (E(0) + E(1) + \dots + E(k))$	$K_d (E(k) - E(k-1))$

$x=0$

replace by C.D.

$$\frac{1}{2} K_d (E(2) - E(1))$$

$$\frac{1}{2} K_i (E(0) + E(1))$$

$$\frac{1}{2} K_d (E(1) - E(0))$$

$$E(s) = \sum_{t=0}^{\infty} E(t) e^{-st}$$

$$K_p E(s) + K_i \frac{1}{s} E(s) + K_d s E(s)$$