Stanford University

AA228/CS238: Decision Making Under Uncertainty

Fall 2023

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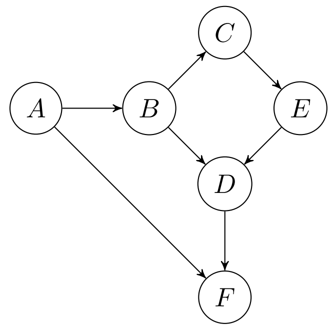
PROBLEM SESSION 1: BAYESIAN NETWORKS **October 4, 2023 4:00pm PT**

Question 1. Joint Probabilities of Bayesian Networks.

a) Given is the following Bayesian network.



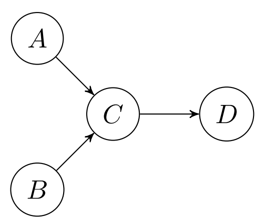
Find an expression for the joint probability $p(A, B, C, D, E, F)$ using the structure of the Bayesian network.

b) You are given the following expression for the joint probability of an unknown Bayesian Network:

$$p(A, B, C, D, E, F, G) = p(A)p(B \mid A)p(C \mid A, B)p(D \mid B, C)p(E \mid C)p(F \mid D, E)p(G).$$

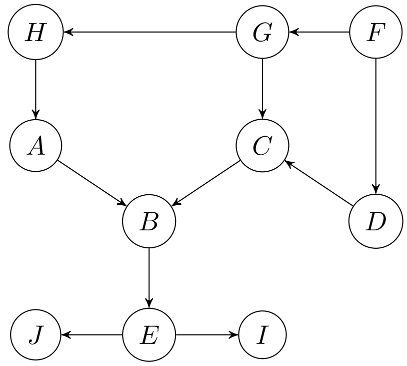
Draw the Bayesian network.

c) Consider the following Bayesian network



where A is a discrete random variable that can take 3 values, B is a discrete random variable that can take 4 values, C is a discrete random variable that can take 5 values, and D is a continuous random variable that is normally (i.e., Gaussian) distributed. How many independent parameters are necessary to specify the full probability distribution represented by the Bayesian network?

Question 2. D-Separation for Bayesian Networks.
For this question, consider the following slightly more complex Bayesian network:



a) Check whether D is conditionally independent of H if we observe I and G, i.e., check if $(D \perp H \mid I, G)$.

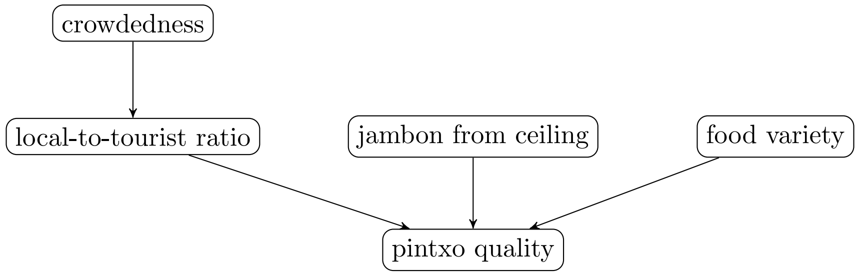
b) What is the Markov blanket of D? What variable(s) need to be added to the evidence variables (from the Markov blanket) to solve the problem you encountered in part a)? Verify that the added evidence variable(s) solve the problem!

Question 3. Sum-Product Variable Elimination.
The Basque country in northern Spain with its charming cities of Bilbao and San Sebastián is known for a culinary specialty called *Pintxos*. Those bread-based delicacies are often topped with regional specialties such as *Jambon de Bayonne*, cheese, or cod.

(a) San Sebastián.

(b) Pintxos.

Numerous pintxo bars can be found in the narrow alleys of the cities' old towns, each offering their own unique creations. Thus, the tradition of a *pintxo crawl* was born where much like in a regular bar crawl, one samples the pintxos from different establishments throughout the evening. However, the pintxo quality at each of the bars is not equal and one would like to maximize the quality of the food consumed throughout the evening. There are certain factors like the crowdedness of the place, the variety of food offerings, whether or not they have jambon hanging from the ceiling (several sources mention this as an indicator for good food quality), and the ratio of local people to tourists that can help us to make an informed decision. For this question, we consider the following Bayesian nework:



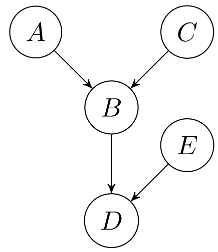
We can assume that we have knowledge of the following factors: ϕ_1(crowdedness), ϕ_2(jambon from ceiling), ϕ_3(food variety), ϕ_4(crowdedness, local-to-tourist ratio), and ϕ_5(local-to-tourist ratio, jambon from ceiling, food variety, pintxo quality). Furthermore, the food variety and the local-to-tourist ratio is not accessible by an outside inspection only, so we treat those as hidden variables while we directly observe the crowdedness and whether or not there is jambon hanging from the ceiling.

For this problem, use the sum-product variable elimination algorithm to infer:

$$p(\text{pintxo quality}^1 \mid \text{crowdedness}^1, \text{jambon from ceiling}^1),$$

that is the probability for a good pintxo experience, given that we observe a crowded bar that has jambon hanging from the ceiling. Choose the elimination order *food variety* $\rightarrow$ *local-to-tourist ratio*.

Question 4. Likelihood Weighted Sampling.
For this question, we consider the following Bayesian network:



a) Similar to most other sampling methods, likelihood weighted sampling requires a topological sort. Give a topological sort for the given Bayesian network. Is this sort unique?

b) For the subsequent problems, the goal is to find an estimate for $p(d^2 \mid b^0, e^1)$. Assume that A, B, C, and E are binary, while D can take 3 values. Generate 5 samples for likelihood weighted sampling.

c) Use the samples from part b) to calculate the weights w_i and finally an estimate for $p(d^2 \mid b^0, e^1)$. Assume that $p(e^0) = 0.3$. The following conditional probability table might be helpful:

A	B	C	$p(B \mid A, C)$
0	0	0	0.2
0	0	1	0.7
1	0	0	0.3
1	0	1	0.9