CS447: Natural Language Processing

http://courses.engr.illinois.edu/cs447

Lecture 4: Language Models

(Intro to Probability Models for NLP)

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Last lecture’s key concepts

Dealing with words:

- Tokenization, normalization
- Zipf’s Law

Morphology (word structure):

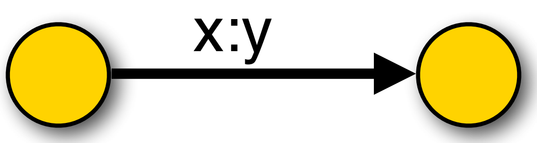
- Stems, affixes
- Derivational vs. inflectional morphology
- Compounding
- Stem changes
- Morphological analysis and generation

Finite-state methods in NLP

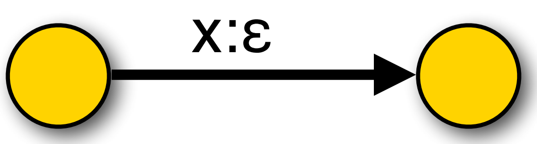
- Finite-state automata vs. finite-state transducers
- Composing finite-state transducers

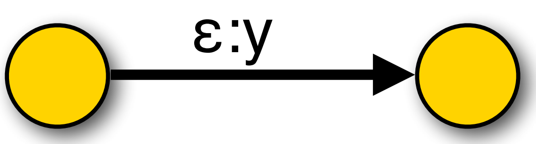
Finite-state transducers

– FSTs define a **relation** between two regular languages.
– Each state transition maps (**transduces**) a character from the input language to a character (or a sequence of characters) in the output language



– By using the **empty character** (ε), characters can be **deleted** (x:ε) or **inserted**(ε:y)





– FSTs can be composed (**cascaded**), allowing us to define **intermediate representations.**

Catching up:
Lecture 3
(Zipf's Law etc.)

How many different words are there in English?

How large is the **vocabulary** of English (or any other language)?

- **Vocabulary size** = the number of distinct word types
- Google N-gram corpus: 1 trillion tokens, 13 million word types that appear 40+ times

If you count words in text, you will find that…

- …a few words (mostly closed-class) are very frequent
  - (the, be, to, of, and, a, in, that,...)
- … most words (all open class) are very rare.
- … even if you've read a lot of text,
  - you will keep finding words you haven't seen before.
- **Word frequency**: the number of occurrences of a word type in a text (or in a collection of texts)

Vocabulary size and corpus size

The number of distinct word types (vocabulary size) increases with the size of the corpus

Herdan's Law/Heap's Law:

A corpus of N tokens has a vocabulary of size

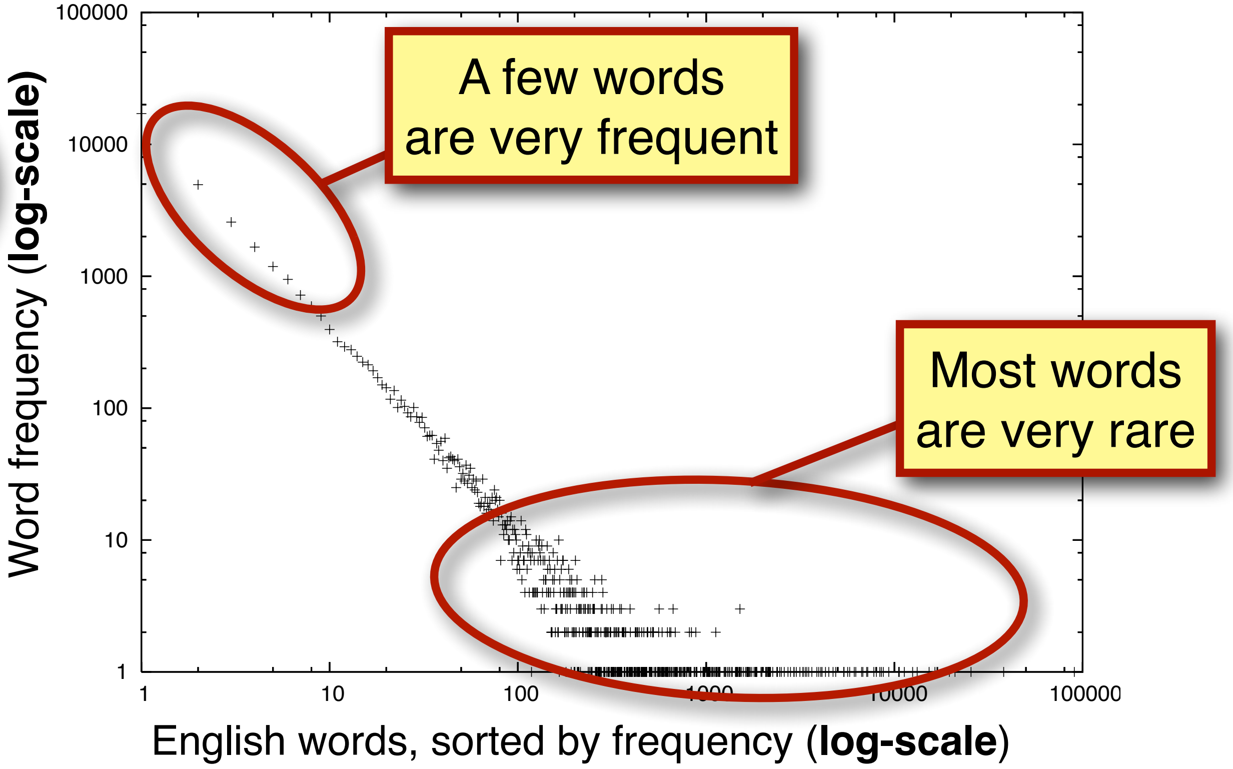
$$|V| = kN^{\beta}$$

for positive constants k and $0 < \beta < 1$

Zipf's law: the long tail

How many words occur once, twice, 100 times, 1000 times?

the r-th most common word w_r has $P(w_r) \propto 1/r$



$w_1 = \textit{the}$, $w_2 = \textit{to}$, …., $w_{5346} = \textit{computer}$, ...

In natural language:

A small number of events (e.g. words) occur with high frequency

A large number of events occur with very low frequency

Implications of Zipf's Law for NLP

The good:

Any text will contain a number of words that are very **common**.

We have seen these words often enough that we know (almost) everything about them. These words will help us get at the structure (and possibly meaning) of this text.

The bad:

Any text will contain a number of words that are **rare**.

We know *something* about these words, but haven't seen them often enough to know everything about them. They may occur with a meaning or a part of speech we haven't seen before.

The ugly:

Any text will contain a number of words that are **unknown** to us.

We have *never* seen them before, but we still need to get at the structure (and meaning) of these texts.

Dealing with the bad and the ugly

Our systems need to be able to **generalize** from what they have seen to unseen events.

There are two (complementary) approaches to generalization:

— **Linguistics** provides us with insights about the rules and structures in language that we can exploit in the (symbolic) representations we use

E.g.: a finite set of grammar rules is enough to describe an infinite language

— **Machine Learning/Statistics** allows us to learn models (and/or representations) from real data that often work well empirically on unseen data

E.g. most statistical or neural NLP

How do we represent words?

Option 1: Words are **atomic symbols**

— Each (surface) word form is its own symbol

— Add some generalization by mapping different forms of a word to the same symbol

- — **Normalization:** map all variants of the same word (form) to the same canonical variant (e.g. lowercase everything, normalize spellings, perhaps spell-check)
- —**Lemmatization**: map each word to its lemma (esp. in English, the lemma is still a word in the language, but lemmatized text is no longer grammatical)
- — **Stemming**: remove endings that differ among word forms (no guarantee that the resulting symbol is an actual word)

How do we represent words?

Option 2: Represent the **structure** of each word

"books" => `“book N pl”` (or `“book V 3rd sg”`)

This requires a **morphological analyzer** (see Lecture 3)

The output is often a lemma (`“book”`)
plus morphological information (`“N pl”` i.e. plural noun)

This is particularly useful for highly inflected languages, e.g. Czech, Finnish, Turkish, etc. (less so for English or Chinese):

In Czech, you might need to know that *nejnezajímavějším*
is a regular, feminine, plural, dative adjective in the superlative.

How do we represent unknown words?

Many NLP systems assume a fixed vocabulary, but still have to handle **out-of-vocabulary (OOV)** words.

Option 1: the **UNK** token

Replace all rare words (with a frequency at or below a given threshold, e.g. 2, 3, or 5) in your training data with an UNK token (UNK = “Unknown word”).

Replace *all* unknown words that you come across after training (including rare training words) with the same UNK token

Option 2: **substring-based** representations

[often used in neural models]

Represent (rare and unknown) words [“Champaign”] as sequences of characters [‘C’, ‘h’, ‘a’,…,’g’, ’n'] or substrings [“Ch”, “amp”, “ai”, “gn”]

Byte Pair Encoding (BPE): learn which character sequences are common in the vocabulary of your language, and treat those common sequences as atomic units of your vocabulary

Lecture 4, Part 1: Overview

Today's lecture

How can we distinguish word salad, spelling errors and grammatical sentences?

Language models define probability distributions over the strings in a language.

N-gram models are the simplest and most common kind of language model.

We'll look at how these models are defined, how to estimate (learn) their parameters, and what their shortcomings are.

We'll also review some very basic probability theory.

Why do we need language models?

Many NLP tasks require **natural language output**:

- —**Machine translation**: return text in the target language
- —**Speech recognition**: return a transcript of what was spoken
- —**Natural language generation**: return natural language text
- —**Spell-checking**: return corrected spelling of input

Language models define **probability distributions over** (natural language) **strings or sentences**.

➔ We can use a language model to generate strings

➔ We can use a language model to score/rank candidate strings so that we can choose the best (i.e. most likely) one:
if $P_{LM}(A) > P_{LM}(B)$, return A, not B

Hmmm, but…

… what does it mean for a language model to "*define a probability distribution*"?

… *why* would we want to define probability distributions over languages?

… how can we construct a language model such that it *actually* defines a probability distribution?

… how do we know *how well* our model works?

You should be able to answer these questions after this lecture

Today's class

Part 1: **Overview**

Part 2: **Review of Basic Probability**

Part 3: **Language Modeling with N-Grams**

Part 4: **Generating Text with Language Models**

Part 5: **Evaluating Language Models**

Today’s key concepts

N-gram language models

- Independence assumptions
- Getting from n-grams to a distribution over a language
- Relative frequency (maximum likelihood) estimation
- Smoothing
- Intrinsic evaluation: Perplexity,
- Extrinsic evaluation: Word error rate (WER)

Today’s reading:

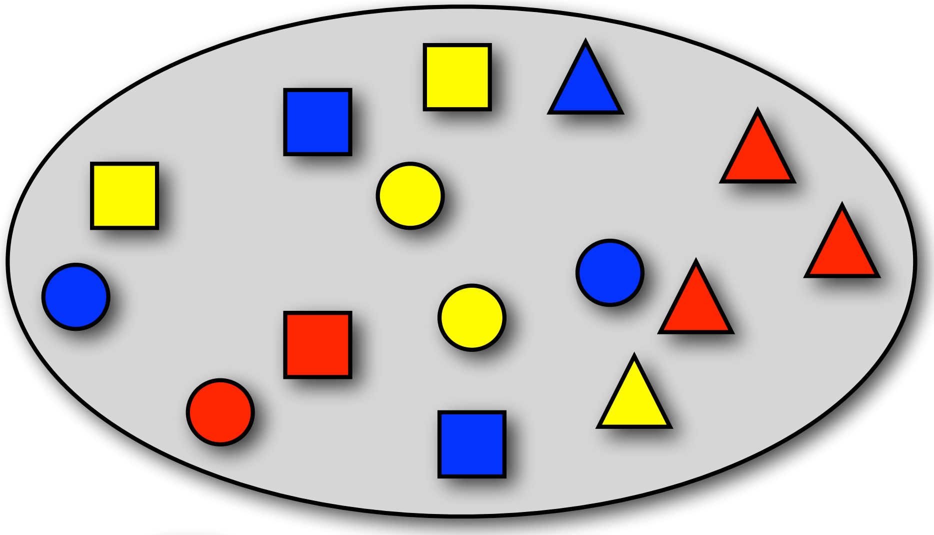
- Chapter 3 (3rd Edition)

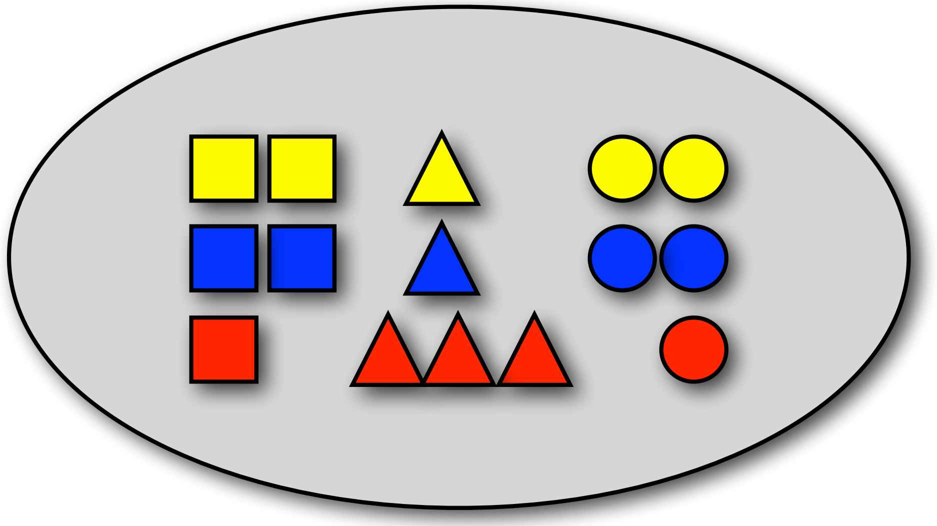
Next lecture: Basic intro to machine learning for NLP

Lecture 4, Part 2: Review of Basic Probability Theory

Sampling with replacement

Pick a random shape, then put it back in the bag.

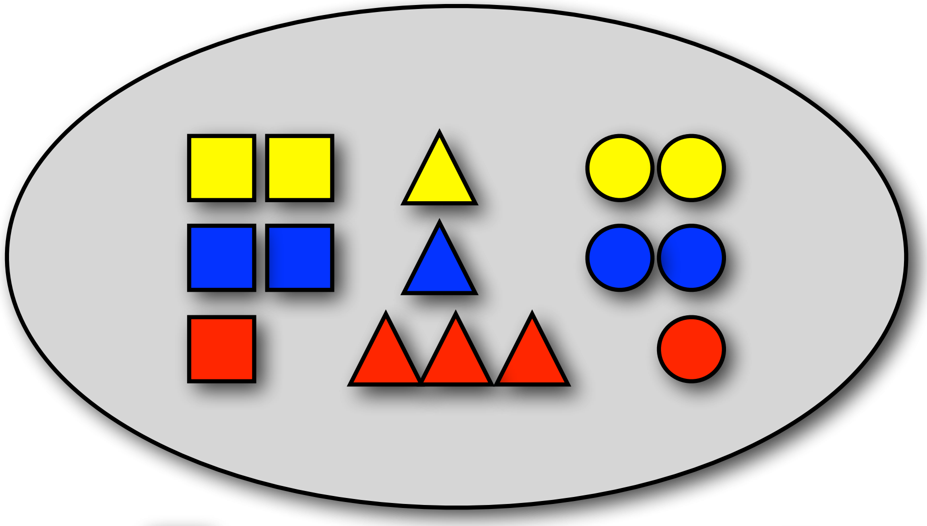




$P($🟦$) = 2/15$	$P($🟥$) = 1/15$	$P($🟥 or ▲$) = 2/15$
$P(\text{blue}) = 5/15$	$P(\text{red}) = 5/15$	$P($△$\mid \text{red}) = 3/5$
$P(\text{blue} \mid$ □$) = 2/5$	$P($□$) = 5/15$	

Sampling with replacement

Pick a random shape, then put it back in the bag.
What sequence of shapes will you draw?



$P($🔴▲▲🟦$) = 1/15 \times 1/15 \times 1/15 \times 2/15$
$= 2/50625$

$P($🔺🟡🔵🔺$) = 3/15 \times 2/15 \times 2/15 \times 3/15$
$= 36/50625$

$P($🟦$) = 2/15$ $P($🟥$) = 1/15$ $P($🟥 or ▲$) = 2/15$

$P($blue$) = 5/15$ $P($red$) = 5/15$ $P($△ | red$) = 3/5$

$P($blue | □$) = 2/5$ $P($□$) = 5/15$

Now let's look at natural language

Text as a bag of words

Alice was beginning to get very tired of sitting by her sister on the bank, and of having nothing to do: once or twice she had peeped into the book her sister was reading, but it had no pictures or conversations in it, 'and what is the use of a book,' thought Alice 'without pictures or conversation?'

$P($of$) = 3/66$	$P($to$) = 2/66$	$P($,$) = 4/66$
$P($Alice$) = 2/66$	$P($her$) = 2/66$	$P($'$) = 4/66$
$P($was$) = 2/66$	$P($sister$) = 2/66$	

Sampling with replacement

A sampled sequence of words

beginning by, very Alice but was and? reading no tired of to into sitting sister the, bank, and thought of without her nothing: having conversations Alice once do or on she it get the book her had peeped was conversation it pictures or sister in, 'what is the use had twice of a book''pictures or' to

$P($of$) = 3/66$	$P($to$) = 2/66$	$P($,$) = 4/66$
$P($Alice$) = 2/66$	$P($her$) = 2/66$	$P($'$) = 4/66$
$P($was$) = 2/66$	$P($sister$) = 2/66$	

In this model, *P(English sentence) = P(word salad)*

Probability theory: terminology

Trial (aka “experiment”)

Picking a shape, predicting a word

Sample space Ω:

The **set of all possible outcomes**
(all shapes; all words in *Alice in Wonderland*)

Event $\omega \subseteq \Omega$:

An **actual outcome** (a subset of Ω)
(predicting ‘*the*’, picking a triangle)

Random variable X: $\Omega \rightarrow \mathrm{T}$

A function from the sample space (often the identity function)
Provides a ‘measurement of interest’ from a trial/experiment
(Did we pick ‘Alice’/a noun/a word starting with “x”/…?
How often does the word ‘Alice’ occur?
How many words occur in each sentence?)

What is a probability distribution?

$P(\omega)$ defines a **distribution** over Ω iff

1) *Every* event ω has a probability $P(\omega)$ between 0 and 1:

$$0 \leq P(\omega \subseteq \Omega) \leq 1$$

2) The *null* event $\varnothing$ has probability $P(\varnothing) = 0$:

$$P(\emptyset) = 0$$

3) And the probability of all *disjoint* events sums to 1.

$$\sum_{\omega_i \subseteq \Omega} P(\omega_i) = 1 \ \text{ if } \forall j \neq i : \omega_i \cap \omega_j = \emptyset$$
$$\text{and } \bigcup_i \omega_i = \Omega$$

Discrete probability distributions: Single Trials

'Discrete': a fixed (often finite) number of outcomes

Bernoulli distribution (Two possible outcomes (*head, tail*)

Defined by the probability of success (= *head*/yes)

The probability of *head* is p. The probability of *tail* is $1-p$.

Categorical distribution (N possible outcomes $c_1 \dots c_N$)

The probability of category/outcome c_i is p_i ($0 \leq p_i \leq 1$; $\sum_i p_i = 1$).

e.g. the probability of getting a six when rolling a die once

e.g. the probability of the next word (picked among a vocabulary of *N* words)

(NB: Most of the distributions we will see in this class are categorical.
Some people call them multinomial distributions, but those refer to *sequences* of trials, e.g. the probability of getting five sixes when rolling a die ten times)

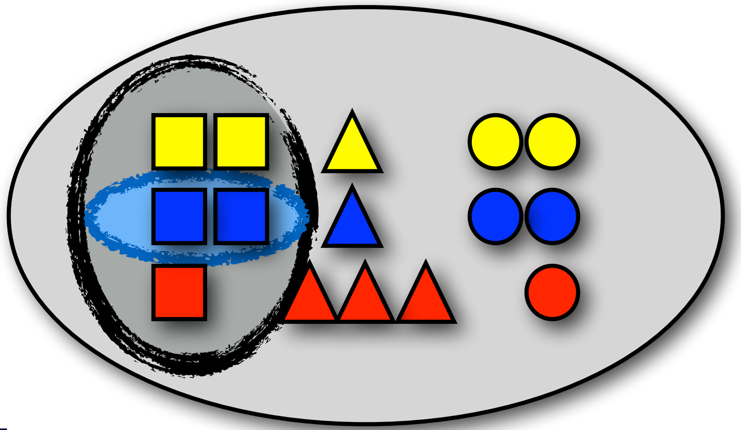
Joint and Conditional Probability

The **conditional probability** of X given Y, $P(X \mid Y)$, is defined in terms of the **probability** of Y, $P(Y)$, and the **joint probability** of X and Y, $P(X, Y)$:

$$P(X|Y) \quad = \quad \frac{P(X,Y)}{P(Y)}$$

What is the probability that we get a blue shape if we pick a square?

$P(\text{blue} \mid \square\ ) = 2/5$



The chain rule

The **joint probability** $P(X,Y)$ can also be expressed in terms of the **conditional probability** $P(X \mid Y)$

$$P(X,Y) \quad = \quad P(X|Y)P(Y)$$

Generalizing this to N joint events (or random variables) leads to the so-called **chain rule**:

$$\begin{aligned} P(X_1, X_2, \ldots, X_n) &= P(X_1)P(X_2|X_1)P(X_3|X_2, X_1)....P(X_n|X_1, ...X_{n-1}) \\ &= P(X_1)\prod_{i=2}^{n} P(X_i|X_1 \ldots X_{i-1}) \end{aligned}$$

Independence

Two events or random variables X and Y are **independent** if

$$P(X,Y) = P(X)P(Y)$$

If X and Y are independent, then $P(X \mid Y) = P(X)$:

$$\begin{aligned} P(X|Y) &= \frac{P(X,Y)}{P(Y)} \\ &= \frac{P(X)P(Y)}{P(Y)} \quad (X,\ Y \text{ independent}) \\ &= P(X) \end{aligned}$$

Probability models

Building a probability model consists of two steps:

1. Defining the model
2. Estimating the model's parameters (= training/learning)

Probability models (almost) always make
independence *assumptions.*

— Even though X and Y are not *actually* independent,
our model may treat them as independent.

— This can drastically reduce the number of parameters to estimate.

— Models without independence assumptions have (way)
too many parameters to estimate reliably from the data we have

— But since independence assumptions are often incorrect,
those models are often incorrect as well:
they assign probability mass to events that cannot occur

Lecture 4, Part 3: Language Modeling with N-Grams

Language modeling with N-grams

A **language model** over a vocabulary V assigns probabilities to strings drawn from V^*.

How do we compute the **probability of a string** $w^{(1)} \dots w^{(i)}$?

Recall the **chain rule**:

$$P(w^{(1)} \dots w^{(i)}) = P(w^{(1)}) \cdot P(w^{(2)} \,|\, w^{(1)}) \cdot \ldots \cdot P(w^{(i)} \,|\, w^{(i-1)}, \dots, w^{(1)})$$

An **n-gram** language model assumes each word $w^{(i)}$ depends only on the **last n−1 words** $w^{(i-1)}, \dots, w^{(i-(n+1))}$

$$P_{ngram}(w^{(1)} \dots w^{(i)}) = P(w^{(1)}) \cdot P(w^{(2)} \,|\, w^{(1)}) \cdot \ldots \cdot P(w^{(i)} \,|\, w^{(i-1)}, \dots, w^{(i-(n+1))})$$

N-gram models

N-gram models *assume* each word (event) depends only on the previous n–1 words (events):

Unigram model: $P(w^{(1)} \dots w^{(N)}) = \prod_{i=1}^{N} P(w^{(i)})$

Bigram model: $P(w^{(1)} \dots w^{(N)}) = \prod_{i=1}^{N} P(w^{(i)} \mid w^{(i-1)})$

Trigram model: $P(w^{(1)} \dots w^{(N)}) = \prod_{i=1}^{N} P(w^{(i)} \mid w^{(i-1)}, w^{(i-2)})$

NB: Independence assumptions where the n-th event in a sequence depends only on the last n-1 events are called Markov assumptions (of order n–1).

How many parameters do n-gram models have?

Given a vocabulary V of |V| word types: so, for $|V| = 10^4$:

Unigram model: |V| parameters — 10^4 parameters

(one distribution $P(w^{(i)})$ with |V| outcomes
[each $w \in V$ is one outcome])

Bigram model: $|V|^2$ parameters — 10^8 parameters

(|V| distributions $P(w^{(i)} \mid w^{(i-1)})$, one distribution for each $w \in V$
with |V| outcomes each [each $w \in V$ is one outcome])

Trigram model: $|V|^3$ parameters — 10^{12} parameters

($|V|^2$ distributions $P(w^{(i)} \mid w^{(i-1)}, w^{(i-2)})$, one per bigram w'w'',
with |V| outcomes each [each $w \in V$ is one outcome])

A bigram model for Alice

Alice was beginning to get very tired of sitting by her sister on the bank, and of having nothing to do: once or twice she had peeped into the book her sister was reading, but it had no pictures or conversations in it, 'and what is the use of a book,' thought Alice 'without pictures or conversation?'

$P(w^{(i)} = \texttt{of} \mid w^{(i-1)} = \texttt{tired}) = 1$
$P(w^{(i)} = \texttt{of} \mid w^{(i-1)} = \texttt{use}) = 1$
$P(w^{(i)} = \texttt{sister} \mid w^{(i-1)} = \texttt{her}) = 1$
$P(w^{(i)} = \texttt{beginning} \mid w^{(i-1)} = \texttt{was}) = 1/2$
$P(w^{(i)} = \texttt{reading} \mid w^{(i-1)} = \texttt{was}) = 1/2$

$P(w^{(i)} = \texttt{bank} \mid w^{(i-1)} = \texttt{the}) = 1/3$
$P(w^{(i)} = \texttt{book} \mid w^{(i-1)} = \texttt{the}) = 1/3$
$P(w^{(i)} = \texttt{use} \mid w^{(i-1)} = \texttt{the}) = 1/3$

Using a bigram model for Alice

English

Alice was beginning to get very tired of sitting by her sister on the bank, and of having nothing to do: once or twice she had peeped into the book her sister was reading, but it had no pictures or conversations in it, 'and what is the use of a book,' thought Alice 'without pictures or conversation?'

Word Salad

beginning by, very Alice but was and? reading no tired of to into sitting sister the, bank, and thought of without her nothing: having conversations Alice once do or on she it get the book her had peeped was conversation it pictures or sister in, 'what is the use had twice of a book''pictures or' to

Now, $P(English) \gg P(word\ salad)$

$P(w^{(i)} = \texttt{of} \mid w^{(i-1)} = \texttt{tired}) = 1$

$P(w^{(i)} = \texttt{of} \mid w^{(i-1)} = \texttt{use}) = 1$

$P(w^{(i)} = \texttt{sister} \mid w^{(i-1)} = \texttt{her}) = 1$

$P(w^{(i)} = \texttt{beginning} \mid w^{(i-1)} = \texttt{was}) = 1/2$

$P(w^{(i)} = \texttt{reading} \mid w^{(i-1)} = \texttt{was}) = 1/2$

$P(w^{(i)} = \texttt{bank} \mid w^{(i-1)} = \texttt{the}) = 1/3$

$P(w^{(i)} = \texttt{book} \mid w^{(i-1)} = \texttt{the}) = 1/3$

$P(w^{(i)} = \texttt{use} \mid w^{(i-1)} = \texttt{the}) = 1/3$

From n-gram probabilities to language models

Recall: a language $\mathrm{L} \subseteq \mathrm{V}^*$ is a (possibly infinite) set of strings over a (finite) vocabulary V.

$P(\mathrm{w}^{(\mathrm{i})} \mid \mathrm{w}^{(\mathrm{i-1})})$ defines a distribution over the words in V:

$$\forall w \in V : \left[\sum_{w' \in V} P\left(w^{(i)} = w' \mid w^{(i-1)} = w\right)\right] = 1$$

By multiplying this distribution N times, we get
one distribution over all strings of the same length N (V^{N}):

Prob. of one N-word string: $P\left(w_1 \ldots w_N\right) = \prod_{i=1 \ldots N} P\left(w^{(i)} = w_i \mid w^{(i-1)} = w_{i-1}\right)$

Prob. of all N-word strings $P(V^N) = \sum_{w, w' \in V} \left[\prod_{i=1 \ldots N} P\left(w^{(i)} = w \mid w^{(i-1)} = w'\right)\right] = 1$

But instead of N separate distributions…

…we want one distribution over strings of *any* length

From n-gram probabilities to language models

We have just seen how to use n-gram probabilities to define *one* distribution $P(V^N)$ *for each string length* N.

But a language model $P(L) = P(V^*)$ should define one distribution $P(V^*)$ that sums to one over *all* strings in $L \subseteq V^*$, *regardless of their length*:

$$P(L) = P(V) + P(V^2) + P(V^3) + \ldots P(V^n) + \ldots = 1$$

Solution:

Add an End-of-Sentence (EOS) token to V

Assume a) that each string ends in EOS and
b) that EOS can only appear at the end of a string.

From n-gram probabilities to language models with EOS

Think of a language model as a **stochastic process:**

— At each time step, randomly pick one more word.
— **Stop** generating more words when the word you pick is a special end-of-sentence (EOS) token.

To be able to pick the EOS token, we have to **modify our training data** so that each sentence ends in EOS.

This means our vocabulary is now $V^{EOS} = V \cup \{EOS\}$

We then get an actual language model,
i.e. a distribution over strings of *any* length

Technically, this is only true because P(EOS | …) will be high enough that we are always guaranteed to stop after having generated a finite number of words
A leaky or inconsistent language model would have P(L) < 1. That could happen if EOS had a very small probability (but doesn't really happen in practice).

Why do we want *one* distribution over L?

*Why do we care about having **one probability distribution for all lengths?***

This allows us to *compare the probabilities* of strings of different lengths, because they're computed by the same distribution.

This allows us to *generate* strings of arbitrary length with one model.

Parameter Estimation

Or: Where do we get the probabilities from?

Learning (estimating) a language model

Where do we get the parameters of our model (its actual probabilities) from?

$$P(w^{(i)} = \text{'}the\text{'} \mid w^{(i-1)} = \text{'}on\text{'}) = ???$$

We need (a large amount of) text as training data to estimate the parameters of a language model.

The most basic parameter estimation technique: relative frequency estimation (frequency = counts)

$$P(w^{(i)} = \text{'}the\text{'} \mid w^{(i-1)} = \text{'}on\text{'}) = C(\text{'}on\ the\text{'}) / C(\text{'}on\text{'})$$

Also called Maximum Likelihood Estimation (MLE)

$C(\text{'}on\ the\text{'})$ [or f('on the') for frequency]:

How often does 'on the' appear in the training data?

NB: $C(\text{'}on\text{'}) = \sum_{w \in V} C(\text{'}on\text{'}\, w)$

Handling unknown words: UNK

Training:

— Define a fixed vocabulary V such that all words in V appear at least *n* times in the training data
(e.g. all words that occur at least *5* times in the training corpus, or the most common 10,000 words in training)

— Add a new token UNK to V, and replace all other words in the corpus that are not in V by this token UNK

— Estimate the model on this modified training corpus.

Testing (when computing the probability of a string):

Replace any words not in the vocabulary by UNK

Refinements:

Use different UNK tokens for different types of words
(numbers, capitalized words, lower-case words, etc.)

What about the beginning of the sentence?

In a trigram model

$$P(w^{(1)}w^{(2)}w^{(3)}) = P(w^{(1)})P(w^{(2)} \mid w^{(1)})P(w^{(3)} \mid w^{(2)}, w^{(1)})$$

only the third term $P(w^{(3)} \mid w^{(2)}, w^{(1)})$ is an actual trigram probability. What about $P(w^{(1)})$ and $P(w^{(2)} \mid w^{(1)})$?

If this bothers you:

Add n–1 **beginning-of-sentence** (BOS) symbols to each sentence for an n–gram model:

```
BOS₁ BOS₂ Alice was …
```

Now the unigram and bigram probabilities involve only BOS symbols.

Summary: Estimating a bigram model with BOS (<s>), EOS (</s>) and UNK using MLE

1. Replace all words not in V in the training corpus with UNK
2. Bracket each sentence by special start and end symbols:

   `<s> Alice was beginning to get very tired … </s>`
3. Define the Vocabulary V' = all tokens in modified training corpus (all common words, UNK, <s>, </s>)
4. Count the frequency of each bigram….

   $C(\text{<s> Alice}) = 1$, $C(\text{Alice was}) = 1, \ldots$
5. .... and normalize these frequencies to get probabilities:

$$P(\textit{was} \mid \textit{Alice}) = \sum_{w_i \in V'} \frac{C(\textit{Alice was})}{C(\textit{Alice } w_i)}$$

Lecture 4, Part 4: Generating text with Language Models

How do we use language models?

Independently of any application, we could use a language model as a random sentence *generator*

(we sample sentences according to their language model probability)

NB: There are very few real world use cases where you want to actually generate language randomly, but understanding how to do this and what happens when you do so will allow us to do more interesting things later.

We can use a language model as a sentence *ranker*.

Systems for applications such as machine translation, speech recognition, spell-checking, generation, etc. often produce many candidate sentences as output.

We prefer output sentences S_{Out} that have a higher language model probability.

We can use a language model $P(S_{Out})$ to score and rank these different candidate output sentences, e.g. as follows:

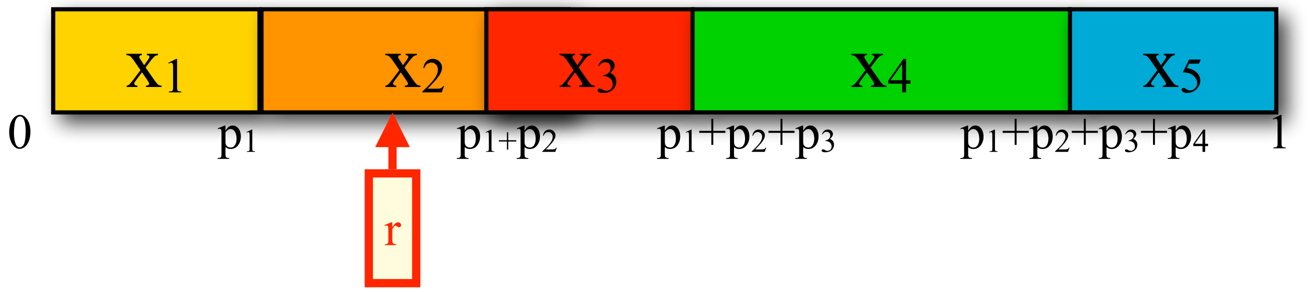
$$\mathrm{argmax}_{S_{Out}}\, P(S_{Out} \mid \mathrm{Input}) = \mathrm{argmax}_{S_{Out}}\, P(\mathrm{Input} \mid S_{Out}) P(S_{Out})$$

Generating from a distribution

How do you generate text from an *n*-gram model?

That is, how do you sample from a distribution $P(X \mid Y=y)$?

- Assume X has N possible outcomes (values): $\{x_1, \ldots, x_N\}$ and $P(X=x_i \mid Y=y) = p_i$
- Divide the interval [0,1] into N smaller intervals according to the probabilities of the outcomes
- Generate a random number r between 0 and 1.
- Return the x_1 whose interval the number is in.



Generating the Wall Street Journal

unigram: Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives

bigram: Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her

trigram: They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

Generating Shakespeare

Unigram	• To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have • Every enter now severally so, let • Hill he late speaks; or! a more to leg less first you enter • Are where exeunt and sighs have rise excellency took of.. Sleep knave we. near; vile like
Bigram	• What means, sir. I confess she? then all sorts, he is trim, captain. •Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. •What we, hath got so she that I rest and sent to scold and nature bankrupt, nor the first gentleman? •Enter Menenius, if it so many good direction found'st thou art a strong upon command of fear not a liberal largess given away, Falstaff! Exeunt
Trigram	• Sweet prince, Falstaff shall die. Harry of Monmouth's grave. • This shall forbid it should be branded, if renown made it empty. • Indeed the duke; and had a very good friend. • Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.
Quadrigram	• King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in; • Will you not tell me who I am? • It cannot be but so. • Indeed the short and the long. Marry, 'tis a noble Lepidus.

Shakespeare as corpus

The Shakespeare corpus has N=884,647 word tokens for a vocabulary of V=29,066 word types

Shakespeare used 300,000 bigram types out of V^2= 844 million possible bigram types.
99.96% of possible bigrams don't occur in this corpus.

Corollary: A relative frequency estimate based on this corpus assigns non-zero probability to only 0.04% of possible bigrams

  That percentage is even lower for trigrams, 4-grams, etc.

  4-grams *look* like Shakespeare because they *are* Shakespeare!

The UNK token

What would happen if we used an UNK token on a corpus the size of Shakespeare's?

1. If we set the frequency threshold for which words to replace too high, a very large fraction of tokens become UNK.

2. Even with a low threshold, UNK will have a very high probability, because in such a small corpus, many words appear only once.

3. But we would still only observe a small fraction of possible bigrams (or trigrams, quadrigrams, etc.)

MLE doesn’t capture unseen events

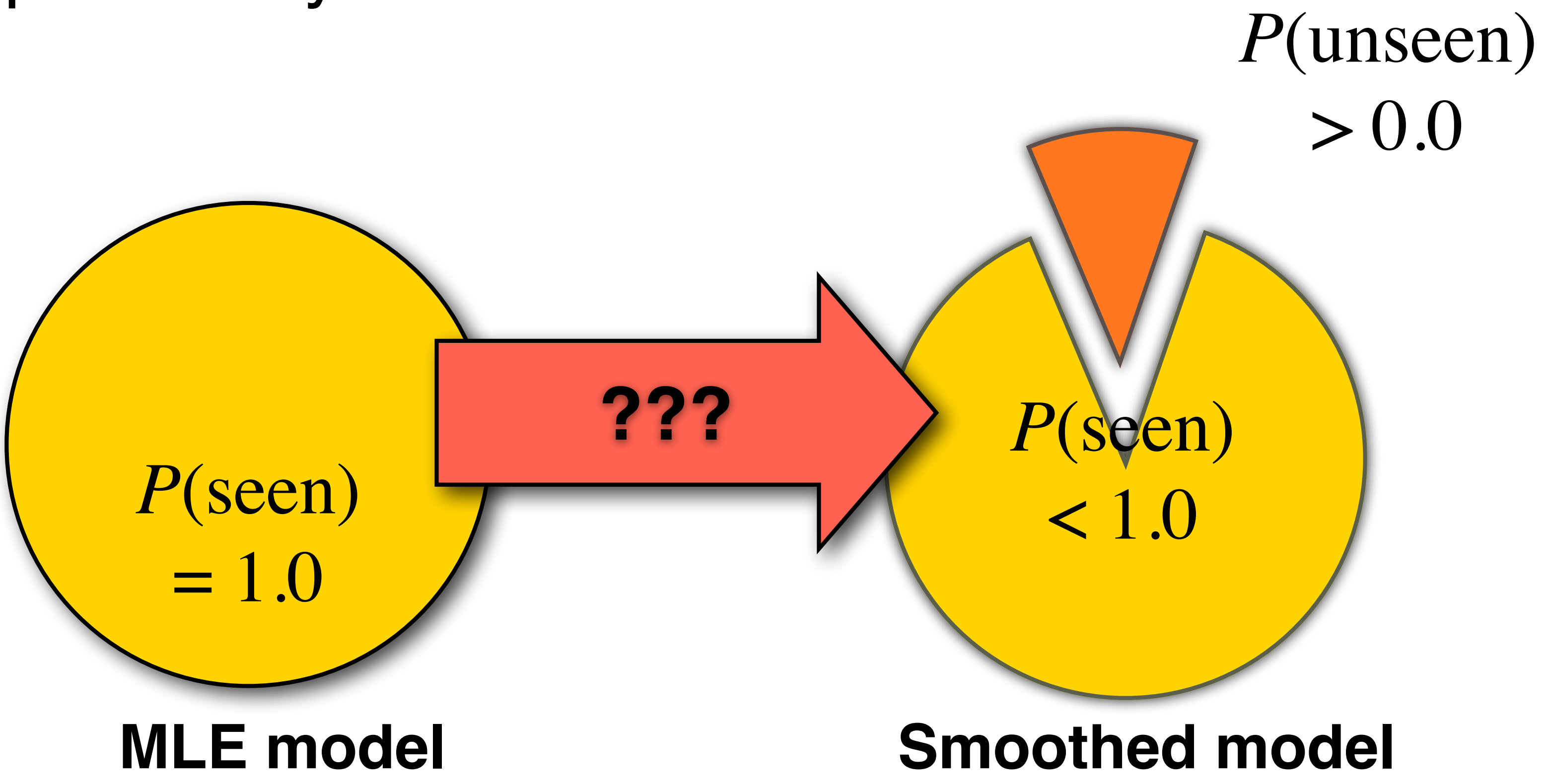
We estimated a model on 884K word tokens, but:

Only 30,000 word types occur in the training data
Any word that does not occur in the training data has zero probability!

Only 0.04% of all possible bigrams (for 30K word types) occur in the training data
Any bigram that does not occur in the training data has *zero* probability (even if we have seen both words in the bigram by themselves)

How can you assign non-zero probability to unseen events?

We have to “smooth” our distributions to assign some probability mass to unseen events



We won’t talk much about smoothing this year.

Smoothing methods

Add-one smoothing:
Hallucinate counts that didn’t occur in the data

Linear interpolation:

$$\tilde{P}(w \mid w', w'') = \lambda \hat{P}(w \mid w', w'') + (1 - \lambda)\tilde{P}(w \mid w')$$

Interpolate n-gram model with (n–1)-gram model.

Absolute Discounting: Subtract constant count from frequent events and add it to rare events

Kneser-Ney: AD with modified unigram probabilities

Good-Turing: Use probability of rare events to estimate probability of unseen events

Add-One (Laplace) Smoothing

A really simple way to do smoothing:
Increment the actual observed count of every *possible* event (e.g. bigram) by a hallucinated count of 1
(or by a hallucinated count of some k with $0 < k < 1$).

Shakespeare bigram model (roughly):

0.88 million actual bigram counts
+ 844.xx million hallucinated bigram counts

Oops. Now almost *none* of the counts in our model come from actual data. We're back to word salad.

K needs to be really small. But it turns out that that still doesn't work very well.

Lecture 4, Part 5: Evaluating language models

Intrinsic vs Extrinsic Evaluation

How do we know whether one language model is better than another?

There are two ways to evaluate models:

- intrinsic evaluation measures how well the model captures what it is supposed to capture (e.g. probabilities)
- extrinsic (task-based) evaluation measures how useful the model is in a particular task.

Both cases require an evaluation metric that allows us to measure and compare the performance of different models.

Intrinsic Evaluation of Language Models: Perplexity

Intrinsic evaluation

Define an evaluation metric (scoring function).

We will want to measure how similar the predictions of the model are to real text.

Train the model on a 'seen' training set

Perhaps: tune some parameters based on held-out data (disjoint from the training data, meant to emulate unseen data)

Test the model on an unseen test set

(usually from the same source (e.g. WSJ) as the training data)

Test data must be disjoint from training and held-out data

Compare models by their scores (more on this in the next lecture).

Perplexity

The perplexity of a language models is defined as the inverse ($\frac{1}{P(\ldots)}$) of the probability of the test set, normalized ($\sqrt[N]{\ldots}$) by the # of tokens (N) in the test set.

If a LM assigns probability $P(w_1, \ldots, w_N)$ to a test corpus $w_1 \ldots w_N$, the LM's perplexity, $PP(w_1 \ldots w_N)$, is

$$PP(w_1 ... w_N) = \sqrt[N]{\frac{1}{P(w_1 ... w_N)}}$$

A LM with **lower** perplexity is **better** because it assigns a higher probability to the unseen test corpus.

LM$_1$ and LM$_2$'s perplexity can only be compared if they use the same vocabulary

— Trigram models have lower perplexity than bigram models;

— Bigram models have lower perplexity than unigram models, etc.

Practical issues: Use logarithms!

Since language model probabilities are very small, multiplying them together often yields to underflow.

It is often better to use logarithms instead, so replace

$$PP(w_1 \ldots w_N) =_{def} \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_{i-1}, \ldots, w_{i-n+1})}}$$

with

$$PP(w_1 \ldots w_N) =_{def} \exp\left(-\frac{1}{N} \sum_{i=1}^{N} \log P(w_i | w_{i-1}, \ldots, w_{i-n+1} \right)$$

Extrinsic (Task-Based) Evaluation of LMs: Word Error Rate

Intrinsic vs. Extrinsic Evaluation

Perplexity tells us which LM assigns a higher probability to unseen text

This doesn't necessarily tell us which LM is better for our task (i.e. is better at scoring candidate sentences)

Task-based evaluation:

- Train model A, plug it into your system for performing task T
- Evaluate performance of system A *on task T*.
- Train model B, plug it in, evaluate system B on same task T.
- Compare scores of system A and system B on task T.

Word Error Rate (WER)

Originally developed for speech recognition.

How much does the *predicted* sequence of words differ from the *actual* sequence of words in the correct transcript?

$$\text{WER} = \frac{\text{Insertions} + \text{Deletions} + \text{Substitutions}}{\text{Actual words in transcript}}$$

Insertions: “eat lunch” → “eat **a** lunch”

Deletions: “see **a** movie” → “see movie”

Substitutions: “drink **ice** tea”→ “drink **nice** tea”

The End