

CS 4644-DL / 7643-A: LECTURE 9

DANFEI XU

Topics:

- Convolutional Neural Networks Architectures (cont.)
- Training Neural Networks (Part 1)

Administrative

- PS1/HW1 due **today** (grace period till Sep 21st)
- PS2/HW2 out: **Difficult assignment. Start early!**
- Project proposal due **Sep 26th. No extension!**

CNN Architectures

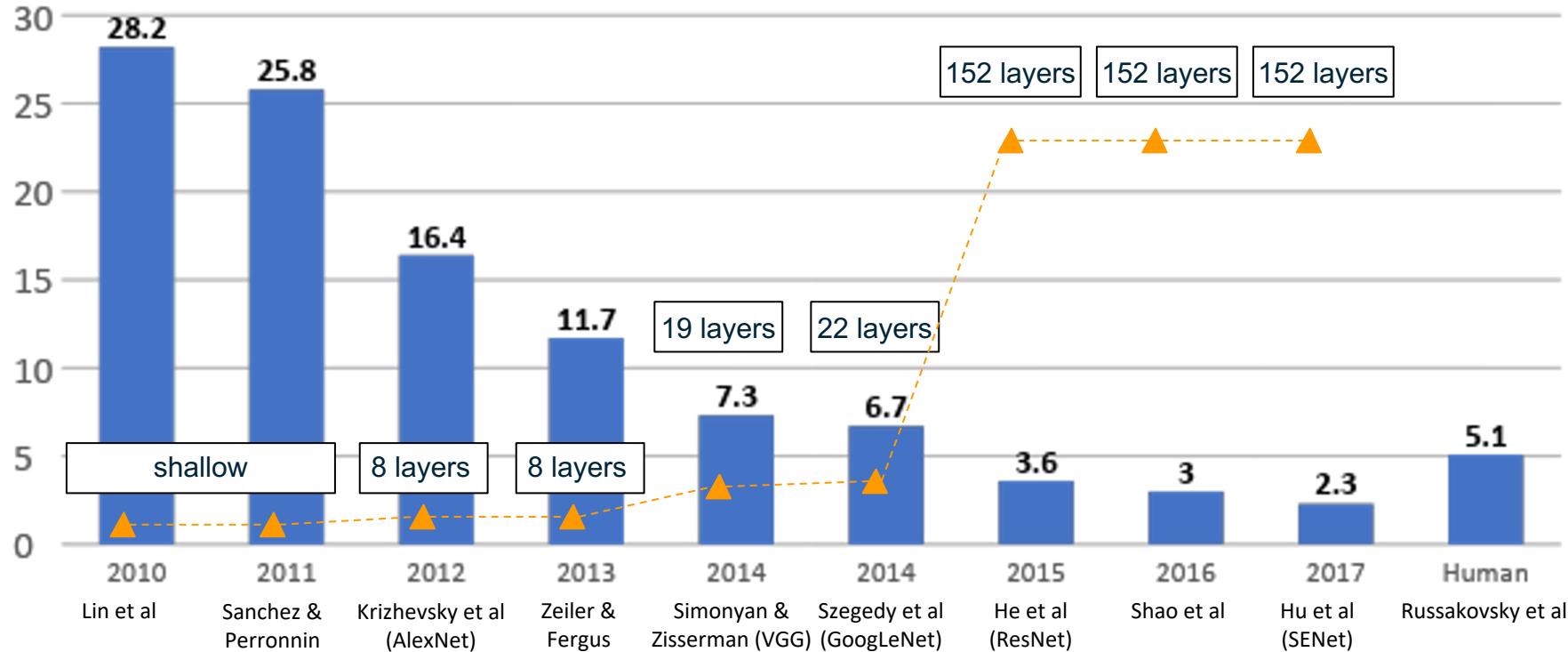
Case Studies

- AlexNet
- VGG
- GoogLeNet
- ResNet

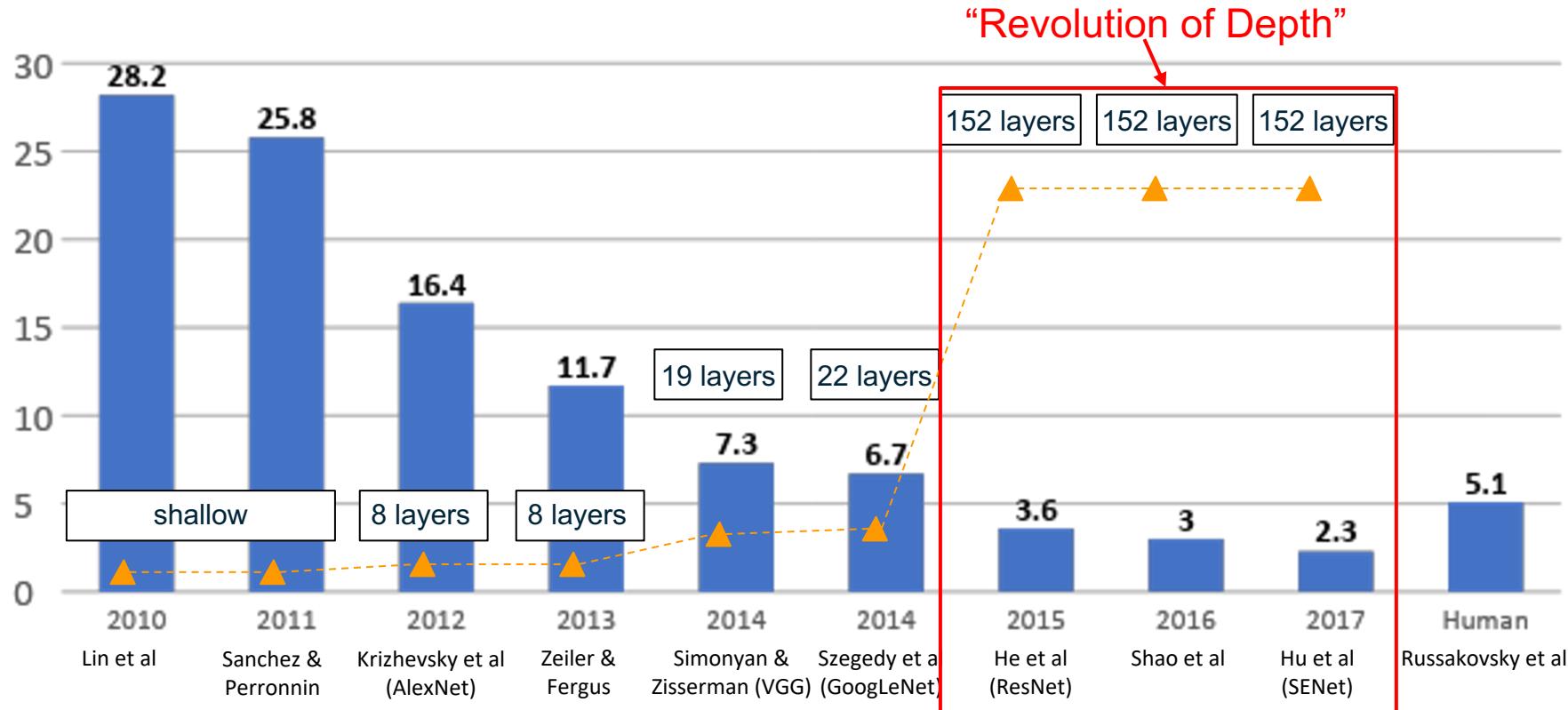
Also....

- SENet
- Wide ResNet
- ResNeXT
- DenseNet
- MobileNets
- NASNet
- EfficientNet

ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners

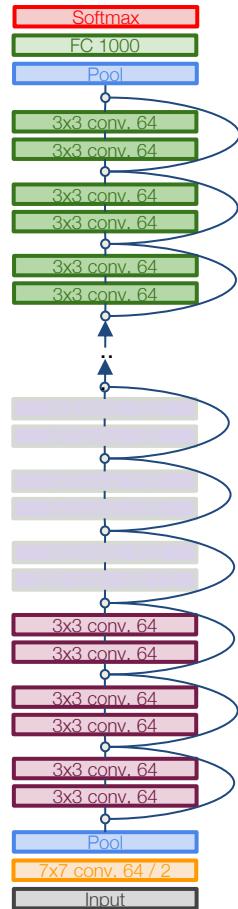
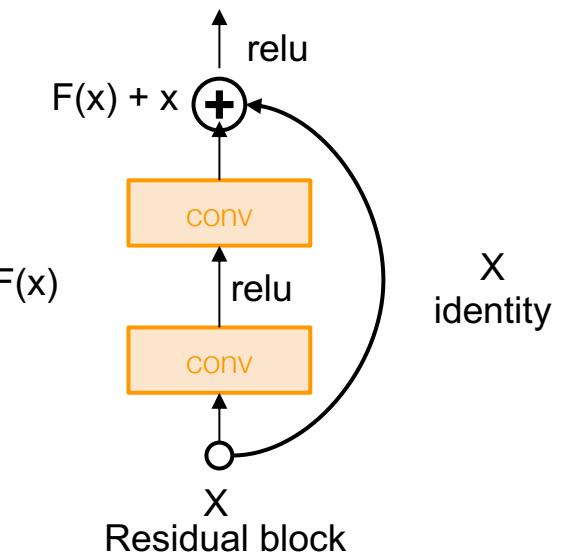


Case Study: ResNet

[He et al., 2015]

Very deep networks using residual connections

- 152-layer model for ImageNet
 - ILSVRC'15 classification winner (3.57% top 5 error)
 - Swept all classification and detection competitions in ILSVRC'15 and COCO'15!



Case Study: ResNet

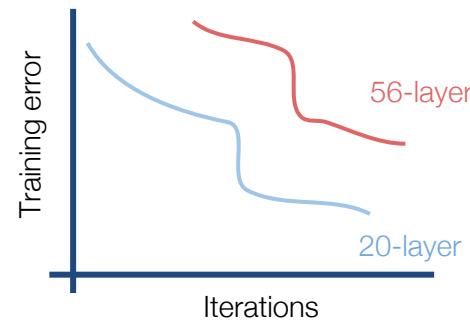
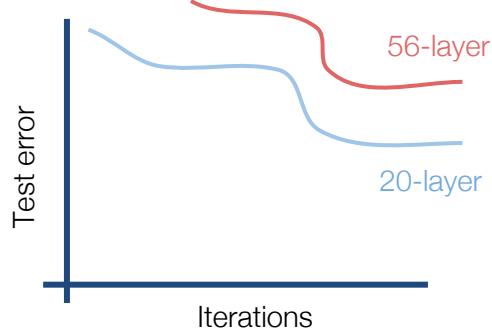
[He et al., 2015]

What happens when we continue stacking deeper layers on a “plain” convolutional neural network?

Case Study: ResNet

[He et al., 2015]

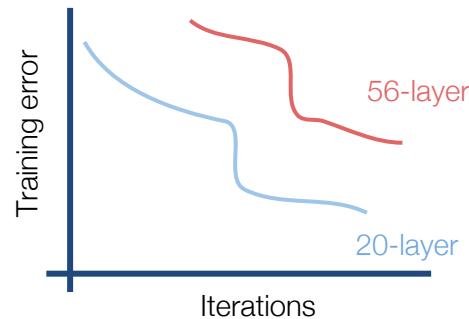
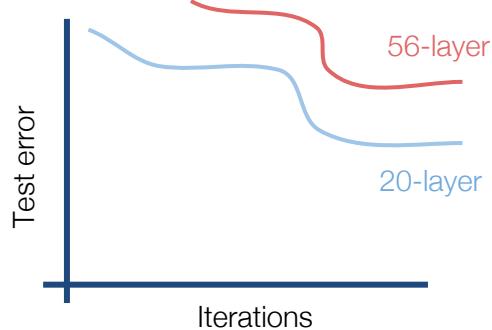
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Case Study: ResNet

[He et al., 2015]

What happens when we continue stacking deeper layers on a “plain” convolutional neural network?



56-layer model performs worse on both test and training error
-> The deeper model performs worse, but it's not caused by overfitting!

Case Study: ResNet

[He et al., 2015]

Fact: Deep models have more representation power
(more parameters) than shallower models.

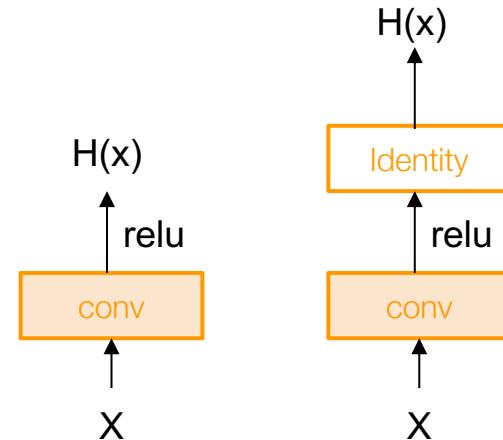
Hypothesis: the problem is an *optimization* problem,
deeper models are harder to optimize

Case Study: ResNet

[He et al., 2015]

A deeper model can **emulate** a shallower model: copy layers from shallower model, set extra layers to identity

Thus deeper models should do at least as good as shallow models



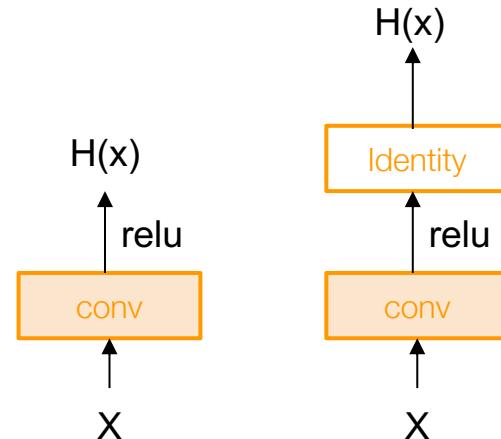
Case Study: ResNet

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A deeper model can **emulate** a shallower model: copy layers from shallower model, set extra layers to identity

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Deeper models are harder to optimize. They don't learn identity functions (no-op) to emulate shallow models



Case Study: ResNet

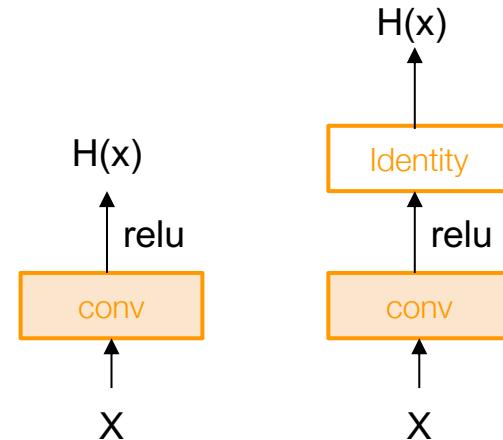
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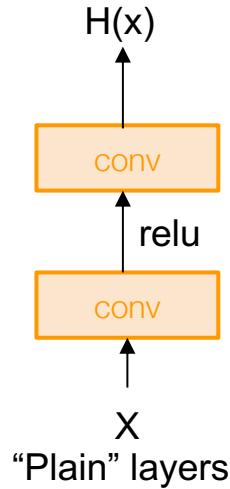
Solution: Change the network so learning identity functions (no-op) as extra layers is easy



Case Study: ResNet

[He et al., 2015]

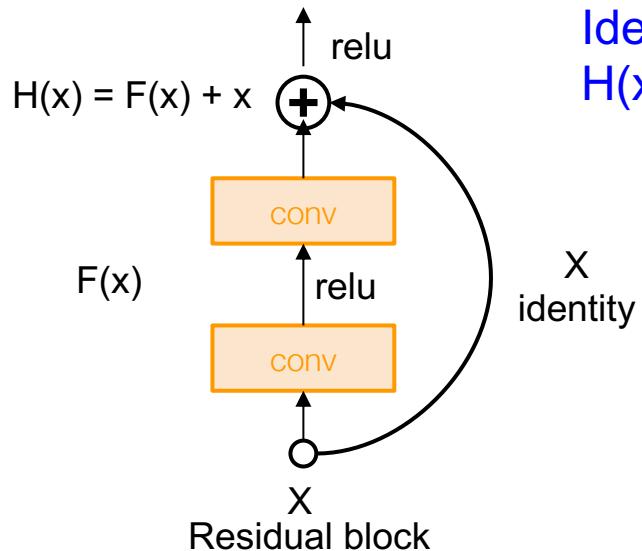
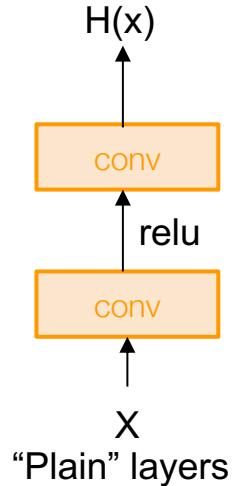
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Case Study: ResNet

[He et al., 2015]

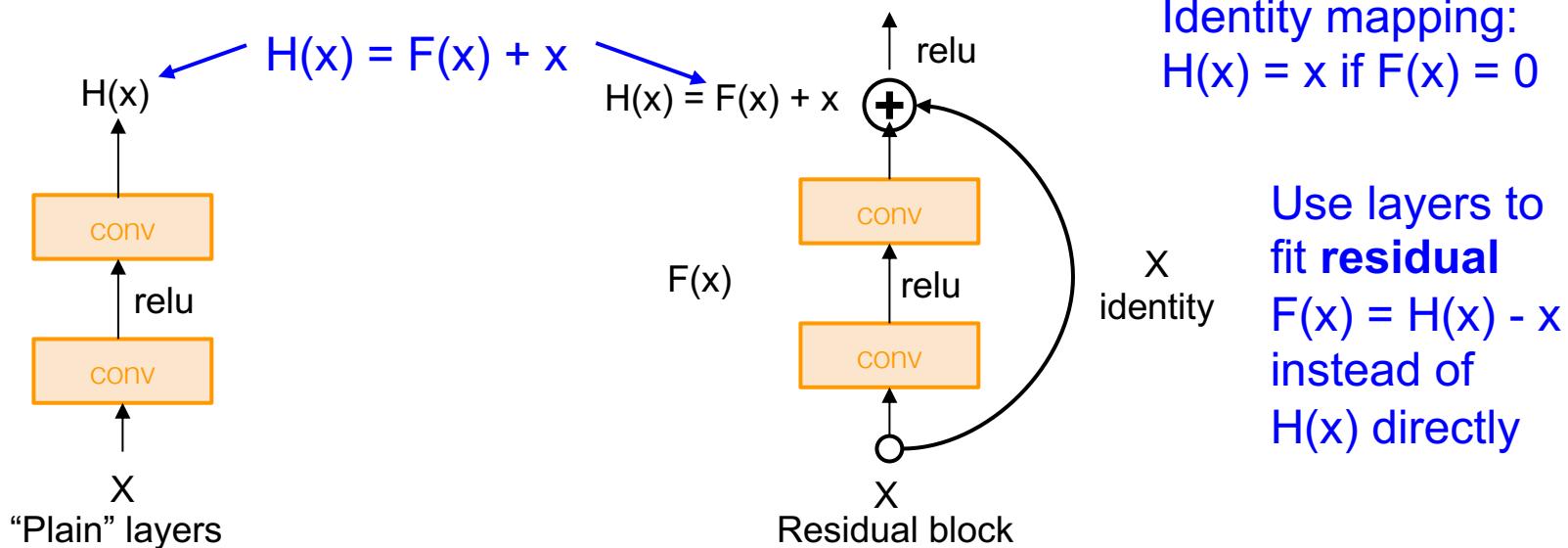
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Case Study: ResNet

[He et al., 2015]

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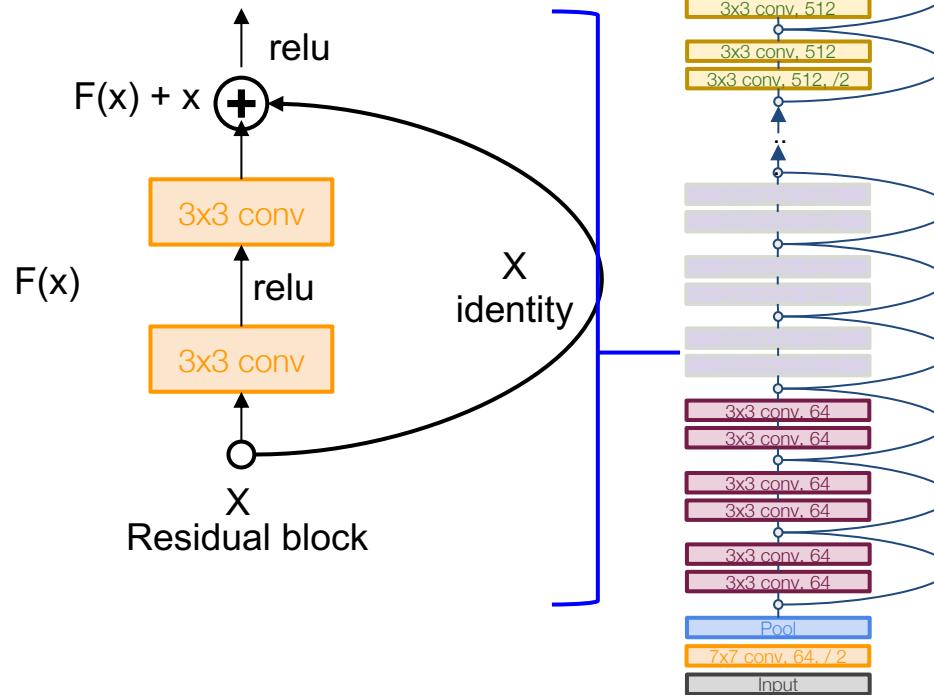


Case Study: ResNet

[He et al., 2015]

Full ResNet architecture:

- Stack residual blocks
- Every residual block has two 3x3 conv layers

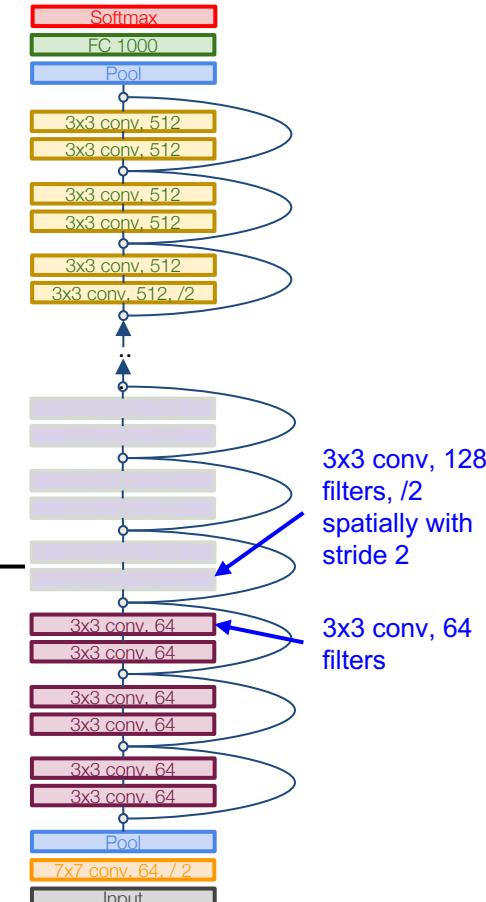
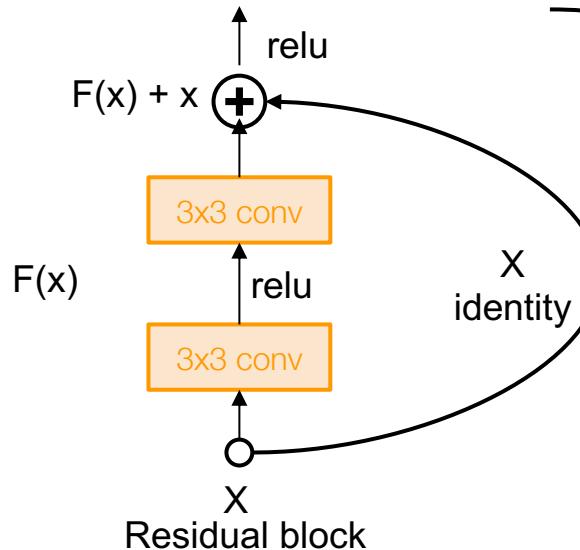


Case Study: ResNet

[He et al., 2015]

Full ResNet architecture:

- Stack residual blocks
- Every residual block has two 3x3 conv layers
- Periodically, double # of filters and downsample spatially using stride 2 (/2 in each dimension)
Reduce the activation volume by half.

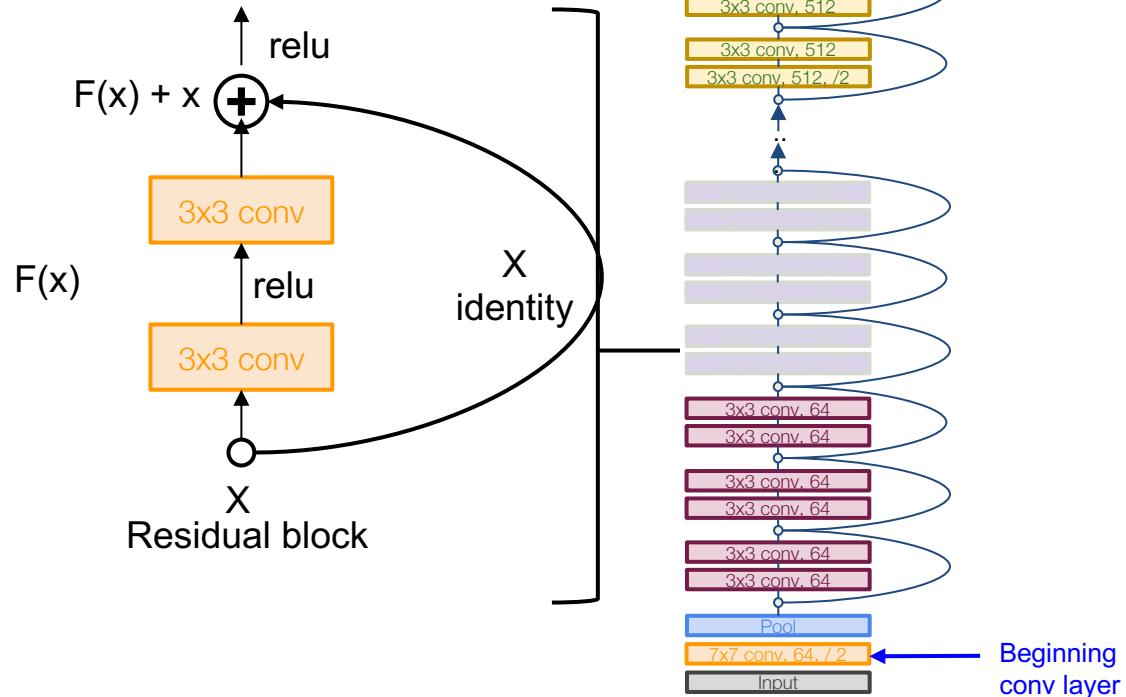


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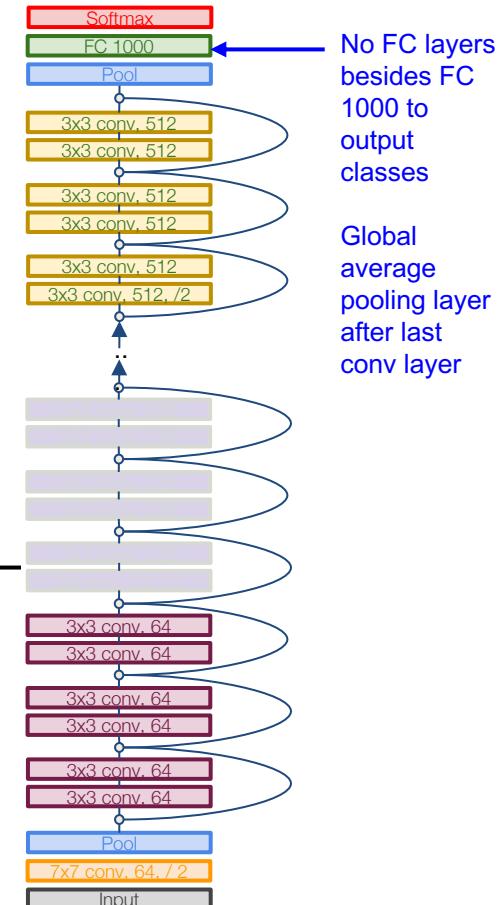
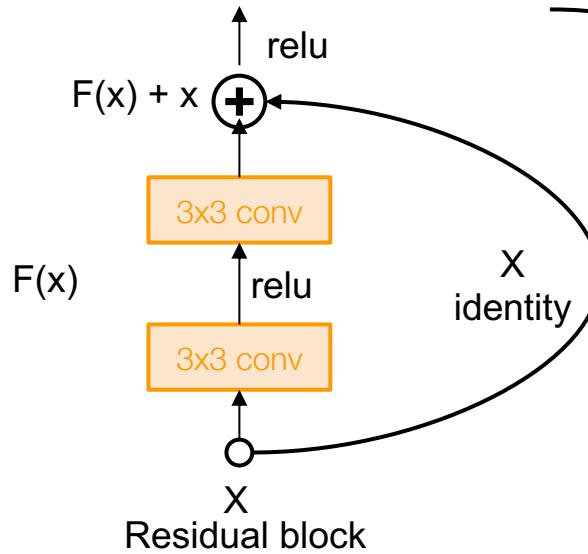


Case Study: ResNet

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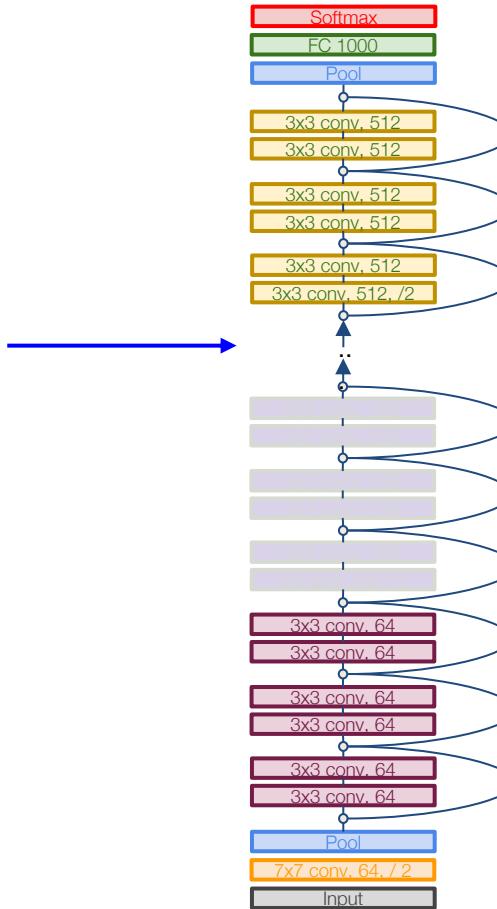
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- Periodically, double # of filters and downsample spatially using stride 2 (/2 in each dimension)
Reduce the activation volume by half.
- Additional conv layer at the beginning (stem)
- No FC layers at the end (only FC 1000 to output classes)



Case Study: ResNet

[He et al., 2015]

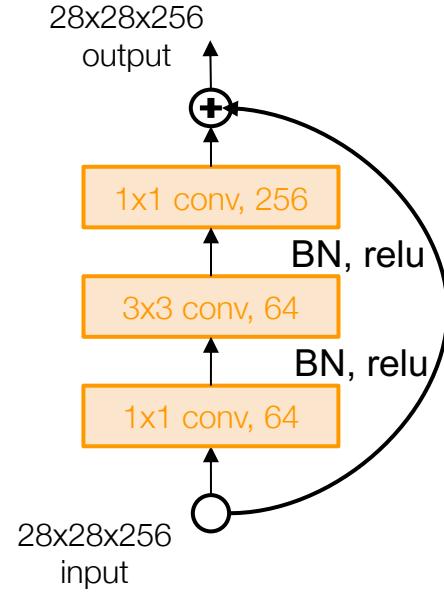
Total depths of 18, 34, 50,
101, or 152 layers for
ImageNet



Case Study: ResNet

[He et al., 2015]

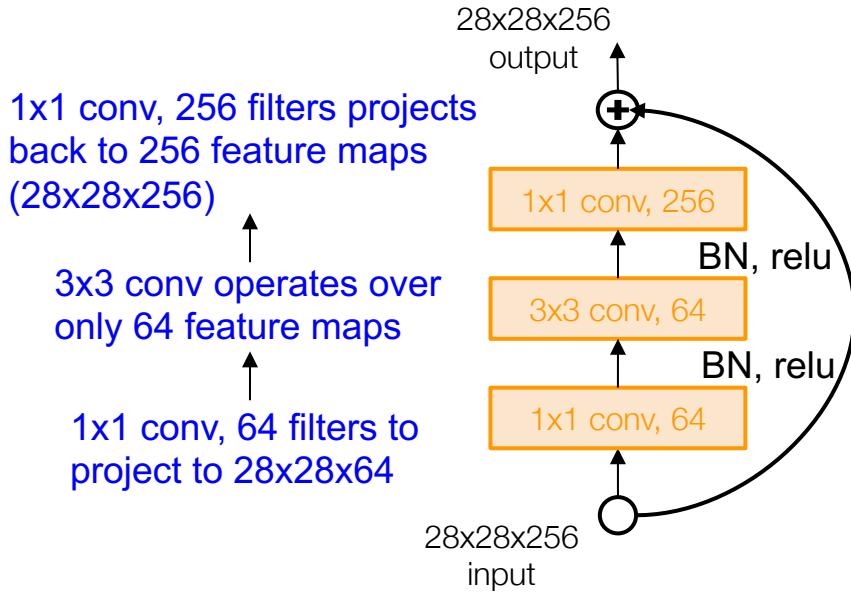
For deeper networks (ResNet-50+), use “bottleneck” layer to improve efficiency (similar to GoogLeNet)



Case Study: ResNet

[He et al., 2015]

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Case Study: ResNet

[He et al., 2015]

Training ResNet in practice:

- Batch Normalization after every CONV layer (this lecture)
- Xavier initialization from He et al. (this lecture)
- SGD + Momentum (this lecture)
- Learning rate: 0.1, divided by 10 when validation error plateaus
- Mini-batch size 256
- Weight decay of 1e-5
- No dropout used

Case Study: ResNet

[He et al., 2015]

Experimental Results

- Able to train very deep networks without degrading (152 layers on ImageNet, 1202 on Cifar)
- Deeper networks now achieve lower training error as expected
- Swept 1st place in all ILSVRC and COCO 2015 competitions

MSRA @ ILSVRC & COCO 2015 Competitions

- **1st places** in all five main tracks
 - ImageNet Classification: “Ultra-deep” (quote Yann) **152-layer** nets
 - ImageNet Detection: **16%** better than 2nd
 - ImageNet Localization: **27%** better than 2nd
 - COCO Detection: **11%** better than 2nd
 - COCO Segmentation: **12%** better than 2nd

Case Study: ResNet

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Experimental Results

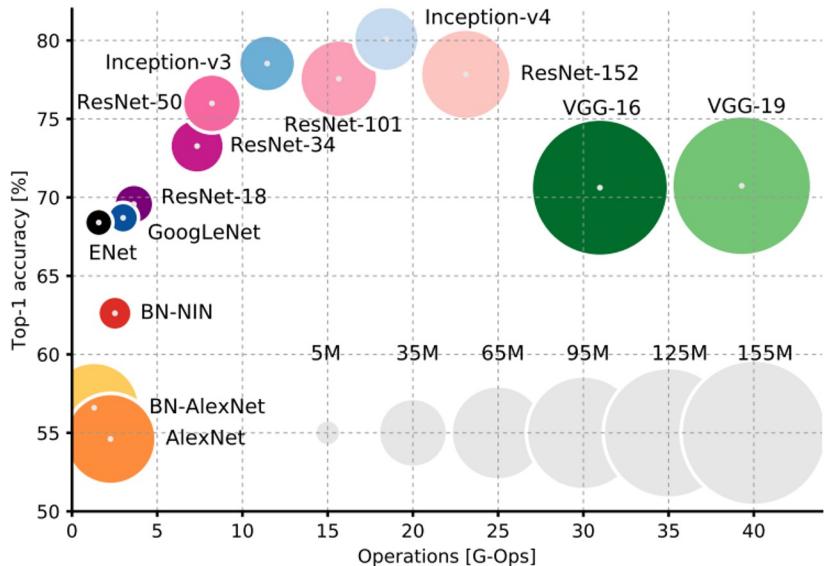
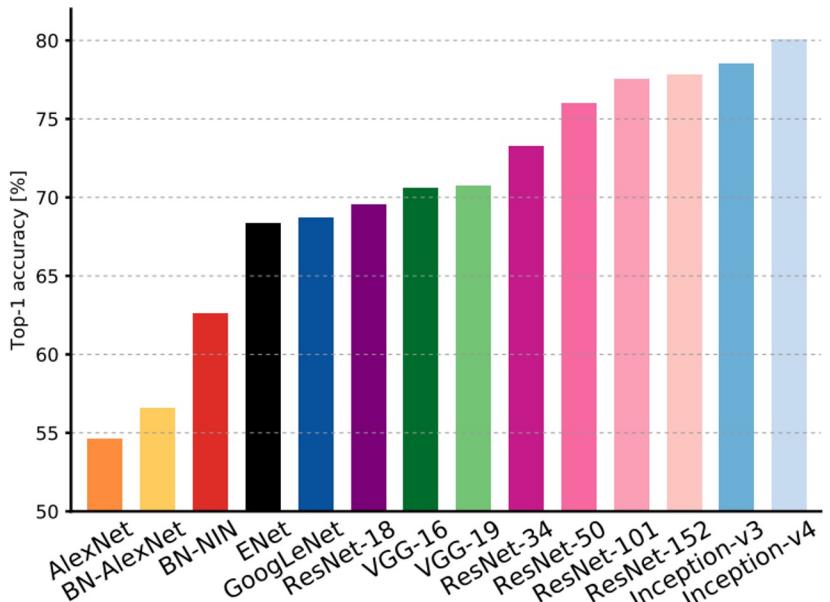
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ILSVRC 2015 classification winner (3.6% top 5 error) -- better than “human performance”! (Russakovsky 2014)

Comparing complexity...

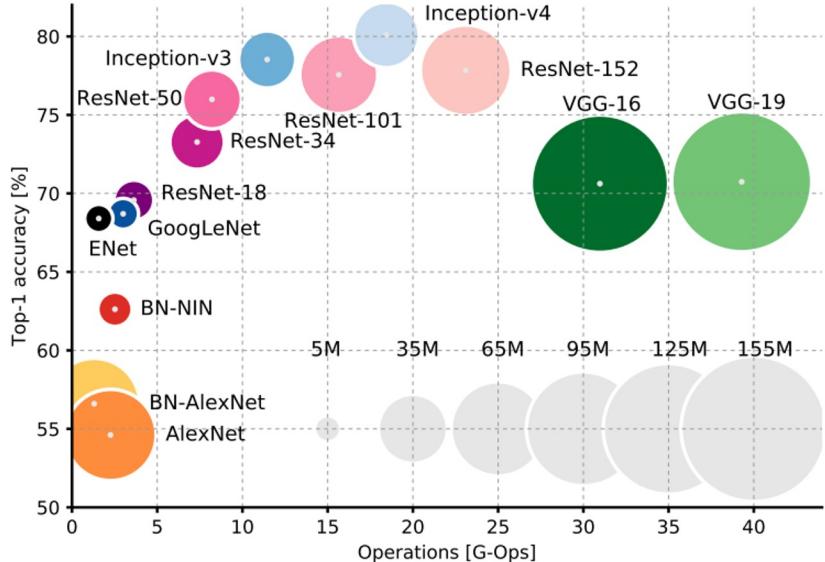
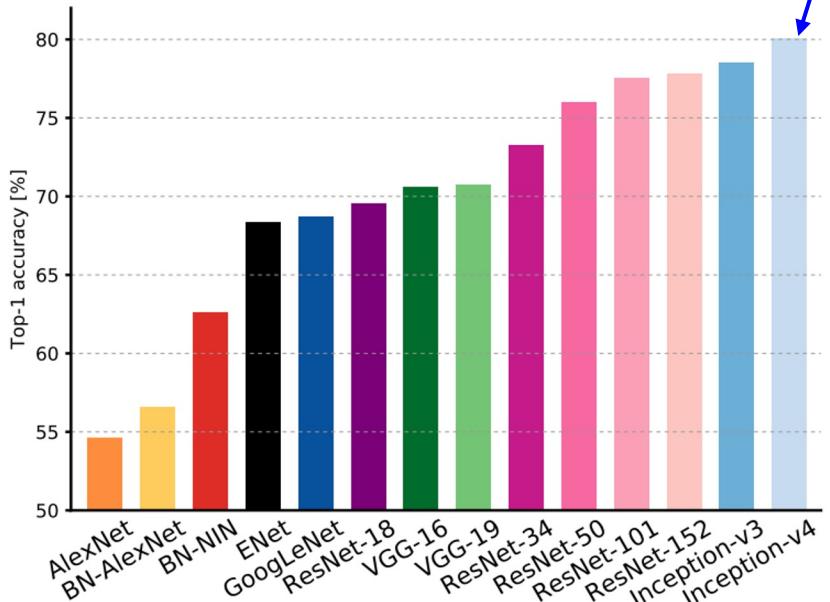


An Analysis of Deep Neural Network Models for Practical Applications, 2017.

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Comparing complexity...

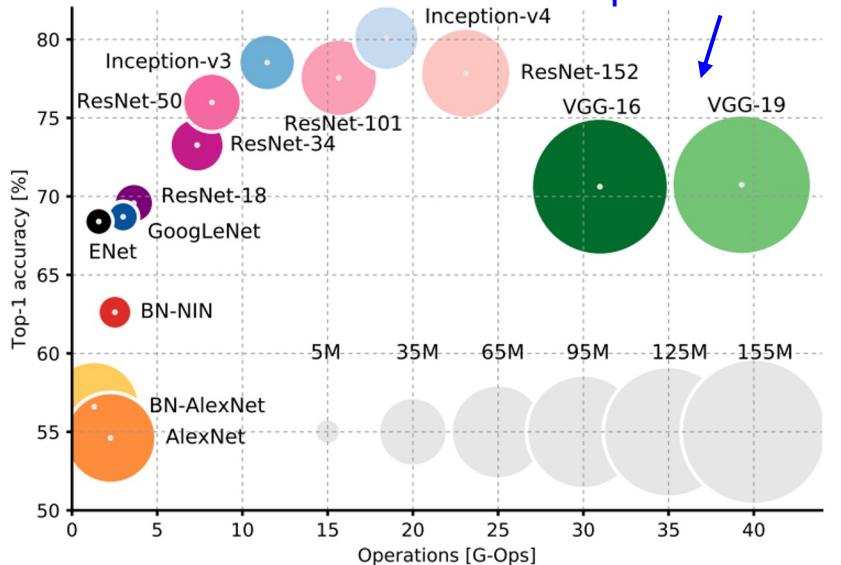
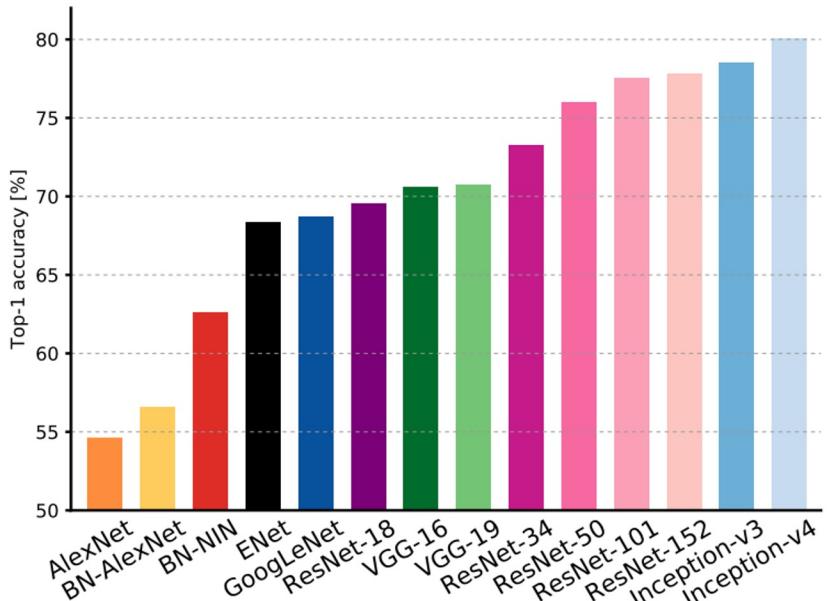
Inception-v4: Resnet + Inception!



An Analysis of Deep Neural Network Models for Practical Applications, 2017.

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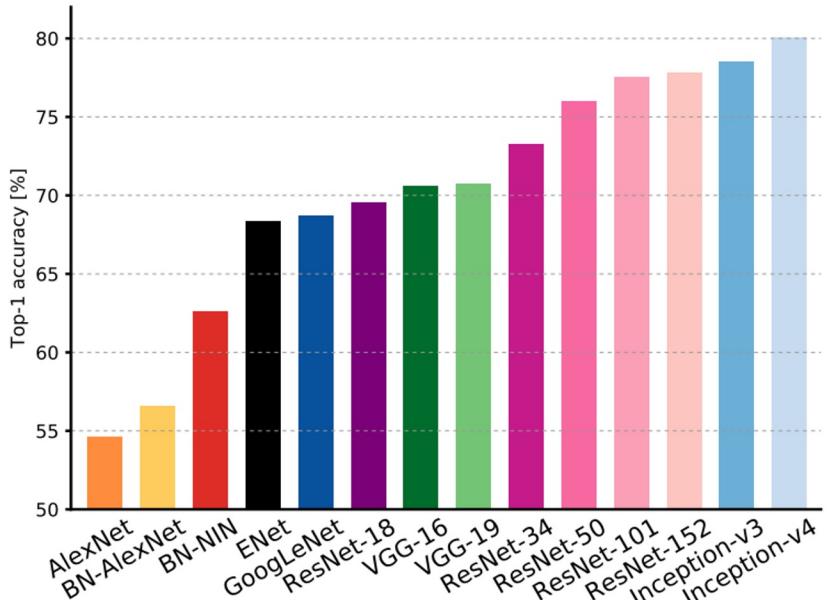
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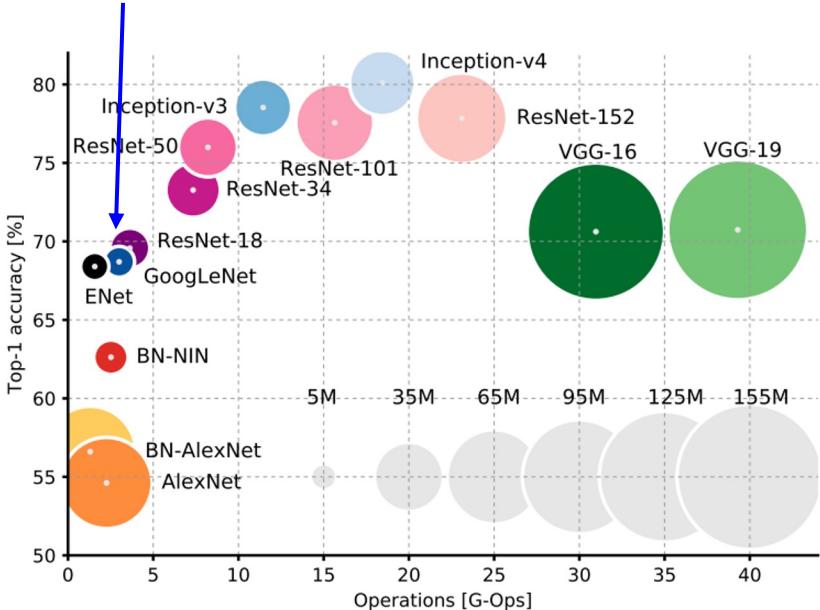
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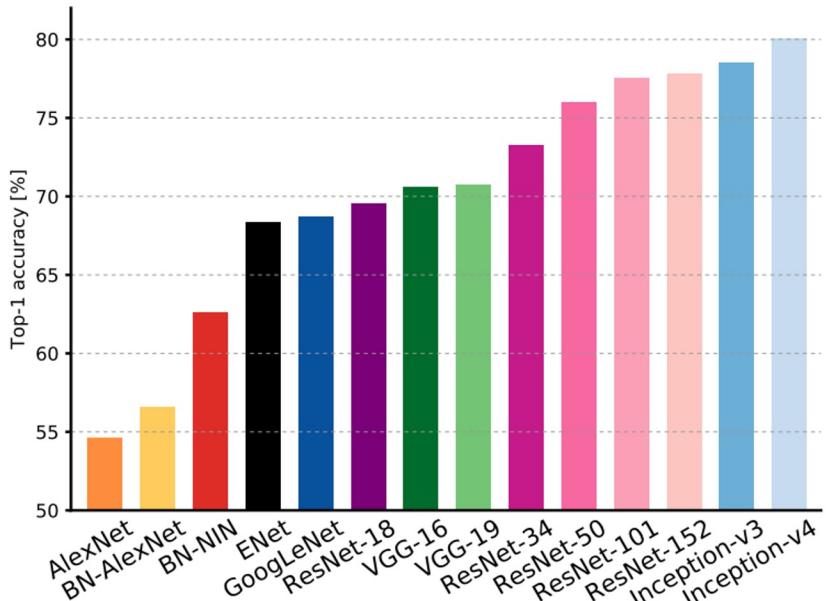
GoogLeNet:
most efficient



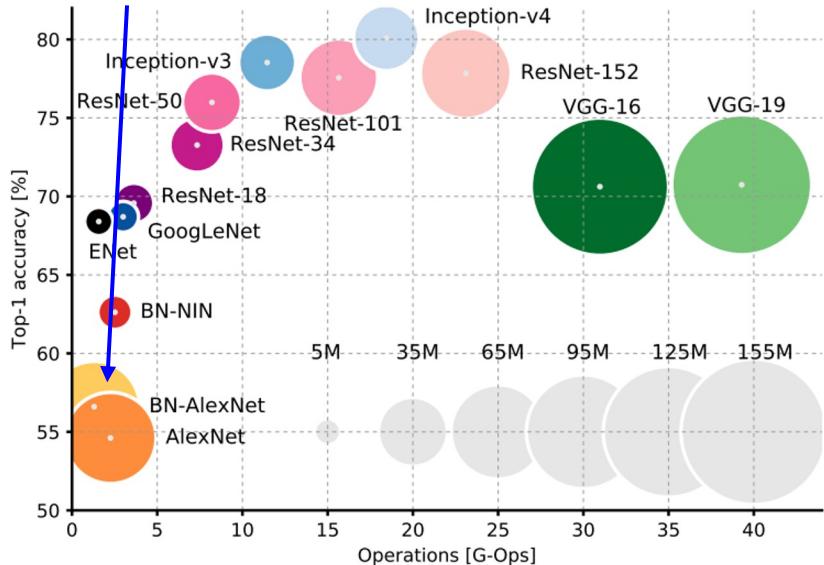
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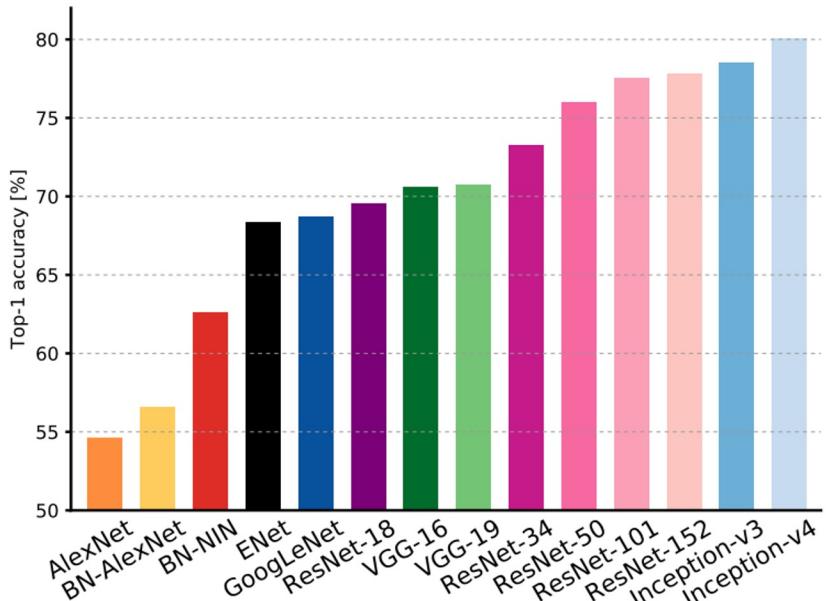
AlexNet:
Smaller compute, still memory heavy, lower accuracy



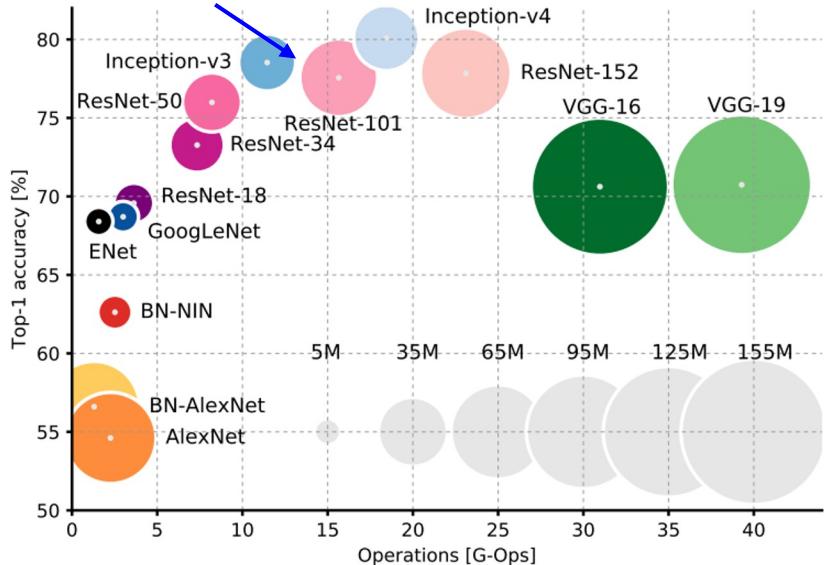
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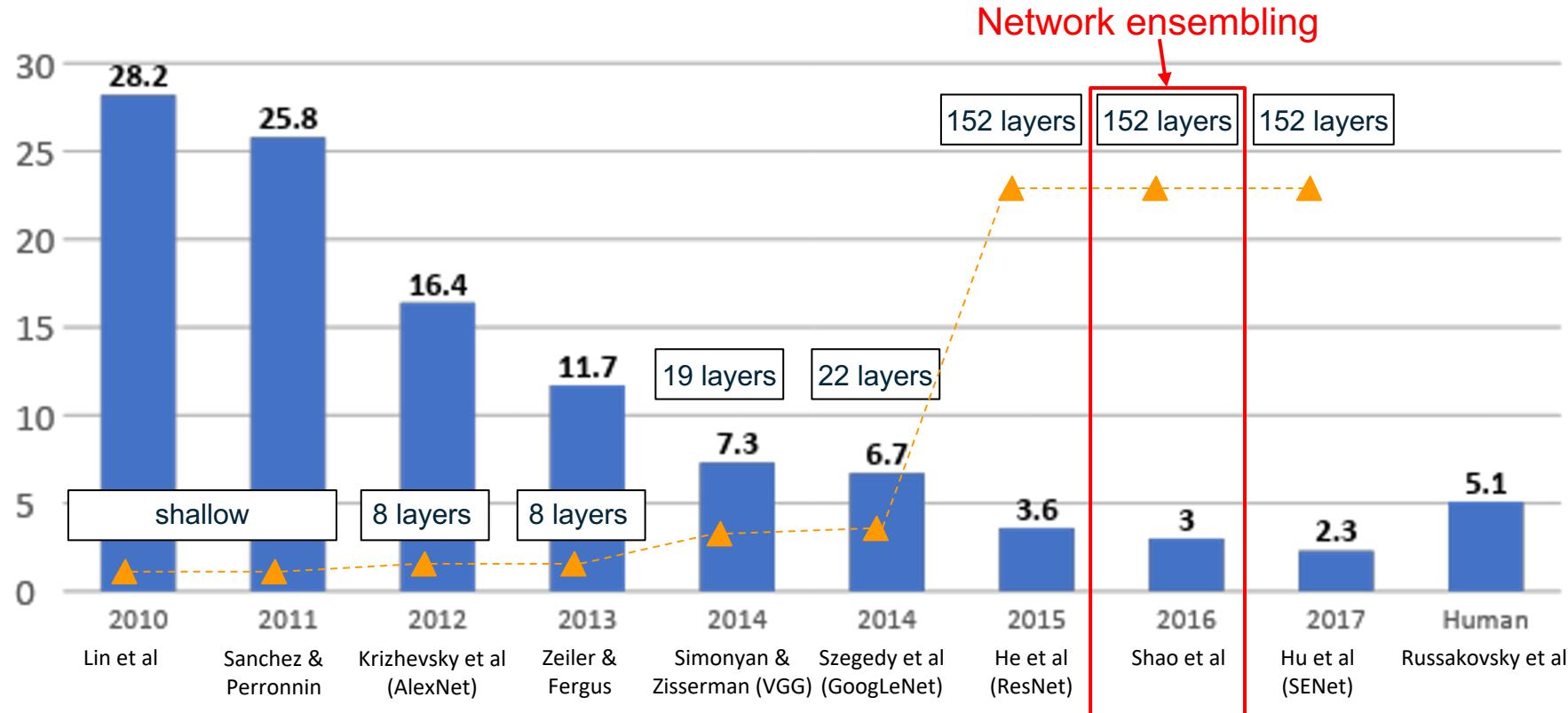
ResNet:
Moderate efficiency depending on
model, highest accuracy



An Analysis of Deep Neural Network Models for Practical Applications, 2017.

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ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



Improving ResNets...

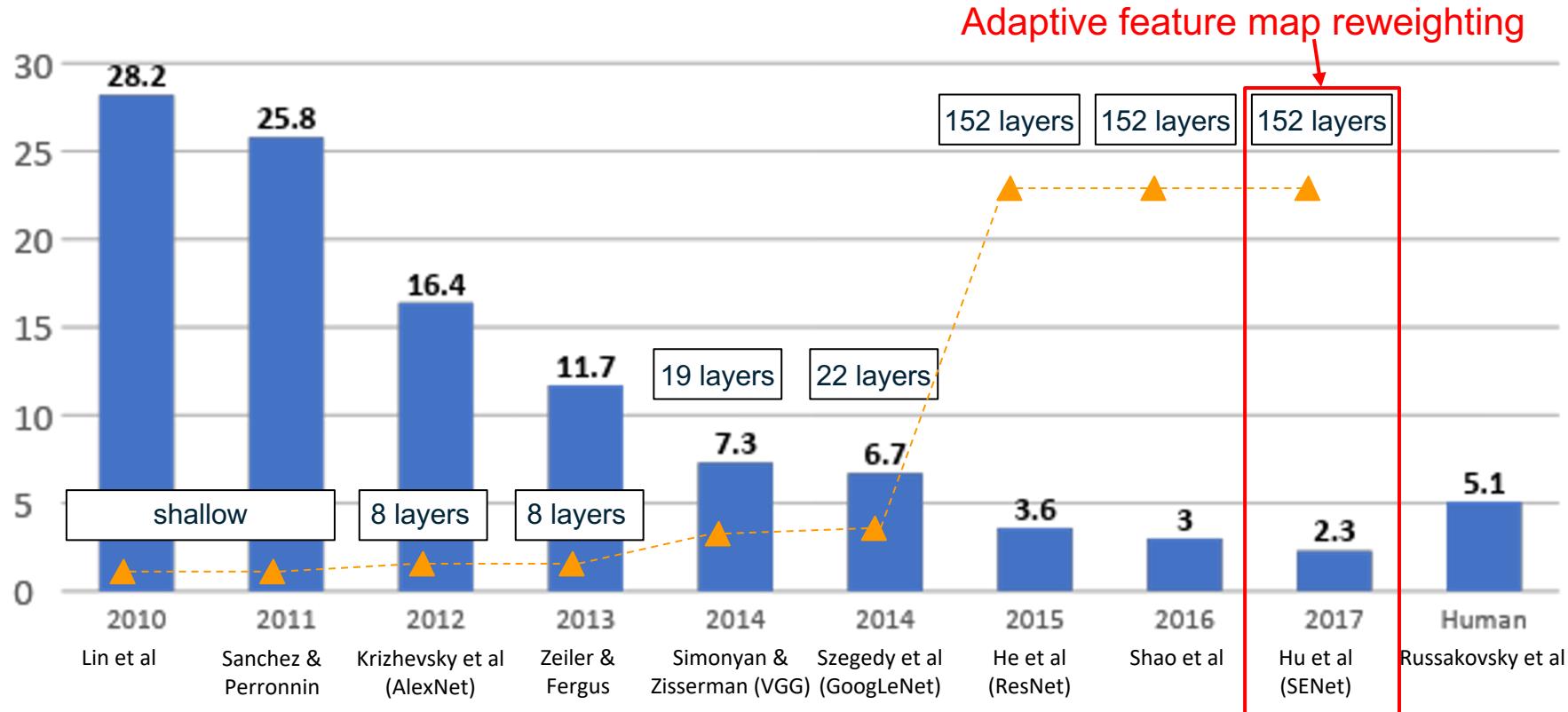
“Good Practices for Deep Feature Fusion”

[Shao et al. 2016]

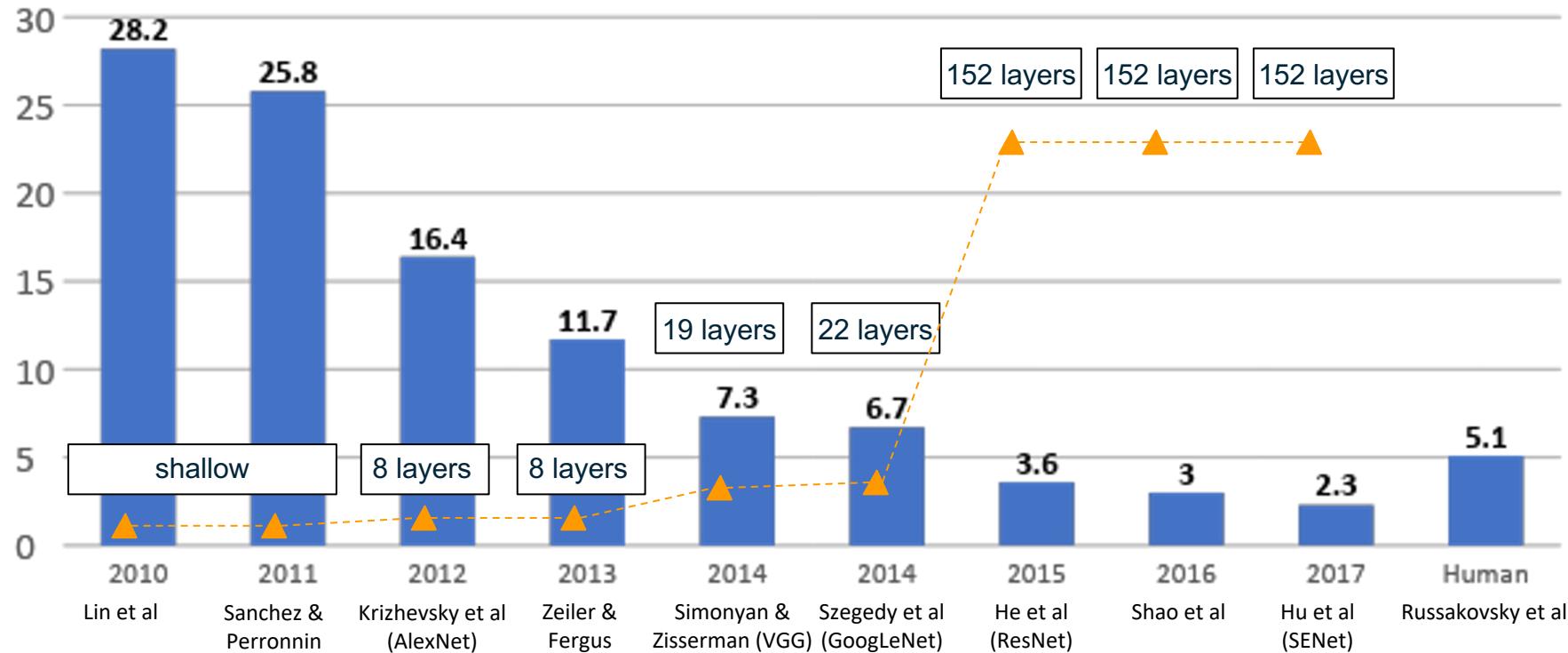
- Multi-scale ensembling of Inception, Inception-Resnet, Resnet, Wide Resnet models
- ILSVRC'16 classification winner

	Inception-v3	Inception-v4	Inception-Resnet-v2	Resnet-200	Wrn-68-3	Fusion (Val.)	Fusion (Test)
Err. (%)	4.20	4.01	3.52	4.26	4.65	2.92 (-0.6)	2.99

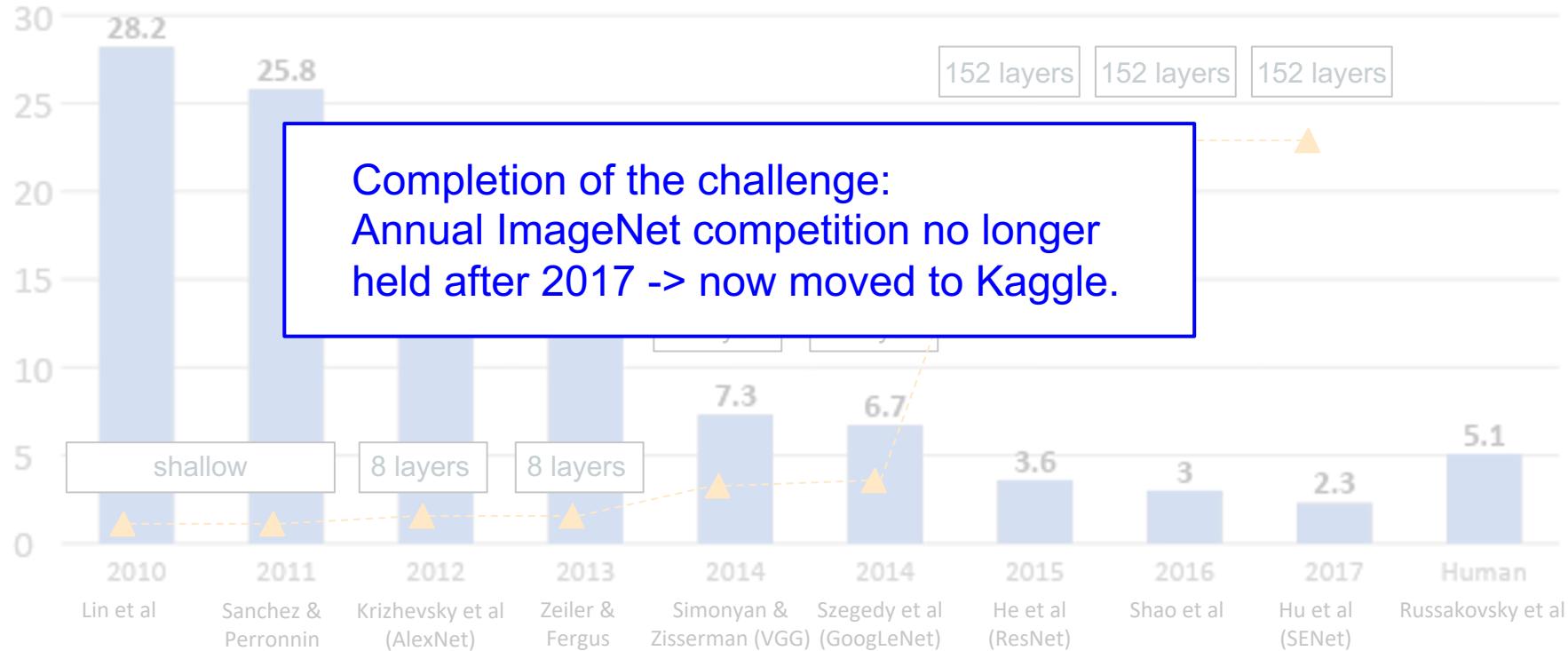
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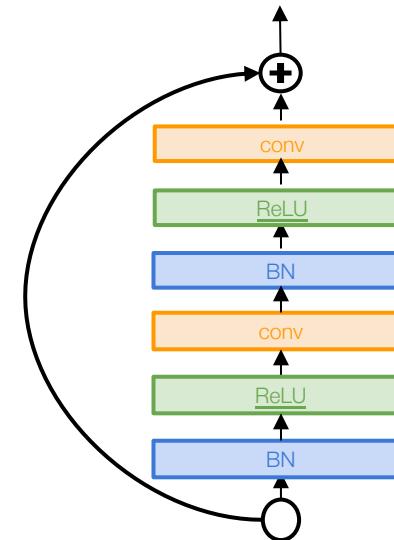
But research into CNN architectures is still flourishing

Improving ResNets...

Identity Mappings in Deep Residual Networks

[He et al. 2016]

- Improved ResNet block design from creators of ResNet
- Creates a more direct path for propagating information throughout network
- Gives better performance

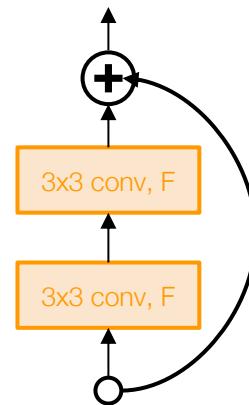


Improving ResNets...

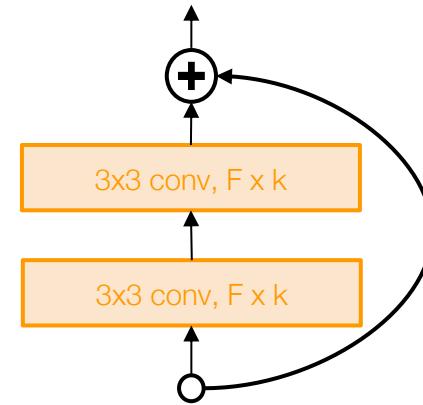
Wide Residual Networks

[Zagoruyko et al. 2016]

- Argues that residuals are the important factor, not depth
- Use wider residual blocks ($F \times k$ filters instead of F filters in each layer)
- 50-layer wide ResNet outperforms 152-layer original ResNet
- Increasing width instead of depth more computationally efficient (parallelizable)



Basic residual block



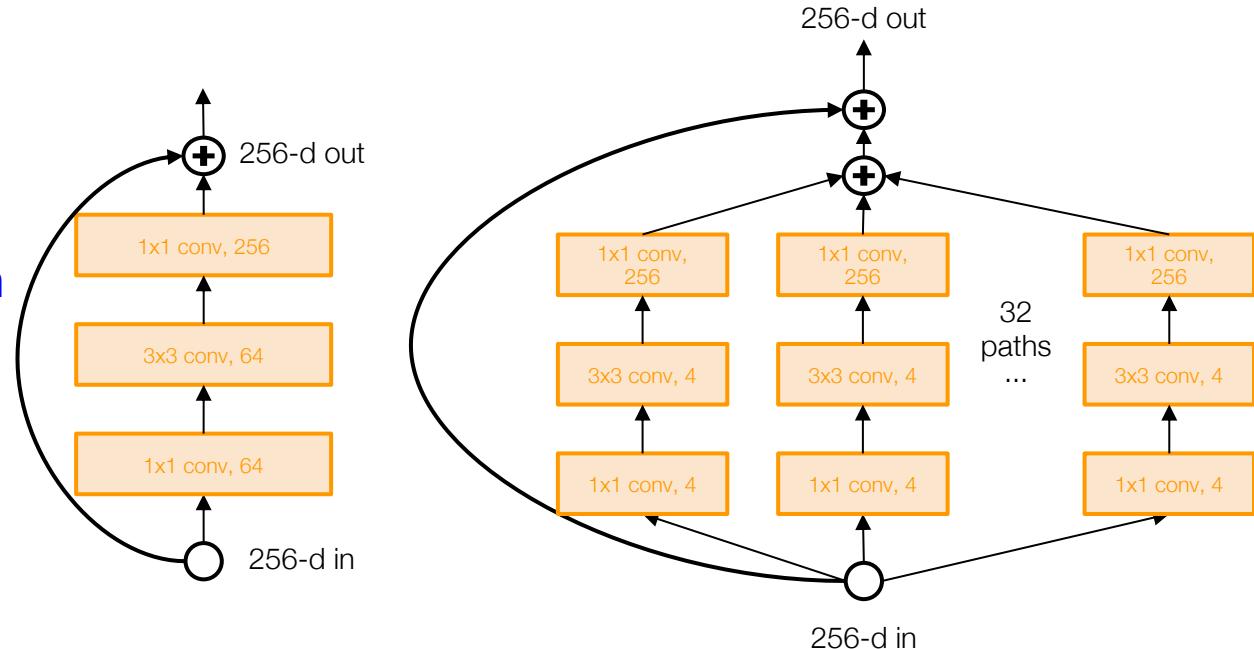
Wide residual block

Improving ResNets...

Aggregated Residual Transformations for Deep Neural Networks (ResNeXt)

[Xie et al. 2016]

- Also from creators of ResNet
- Increases width of residual block through multiple parallel pathways (“cardinality”)
- Parallel pathways similar in spirit to Inception module

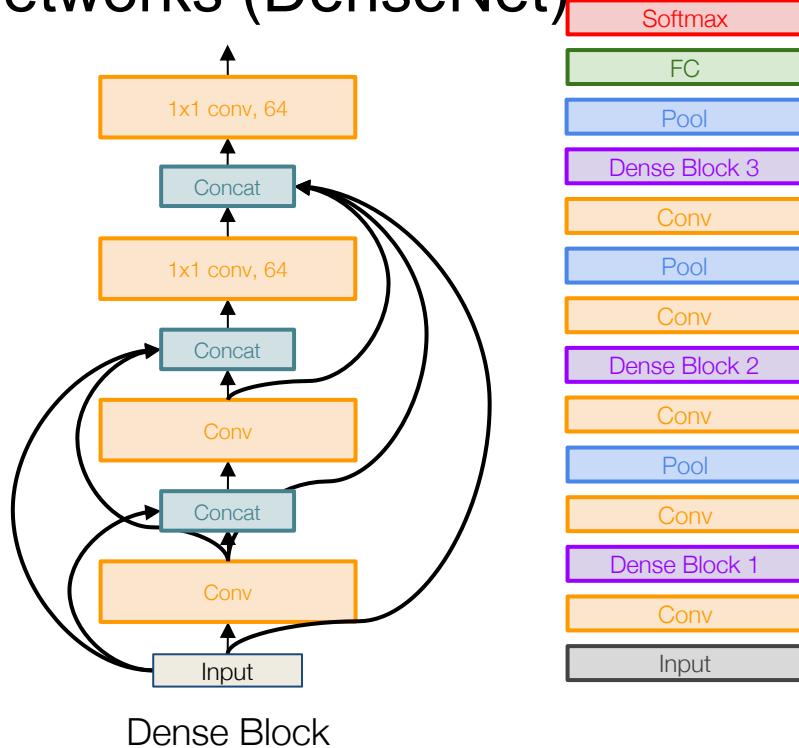


Other ideas...

Densely Connected Convolutional Networks (DenseNet)

[Huang et al. 2017]

- Dense blocks where each layer is connected to every other layer in feedforward fashion
- Alleviates vanishing gradient, strengthens feature propagation, encourages feature reuse
- Showed that shallow 50-layer network can outperform deeper 152 layer ResNet

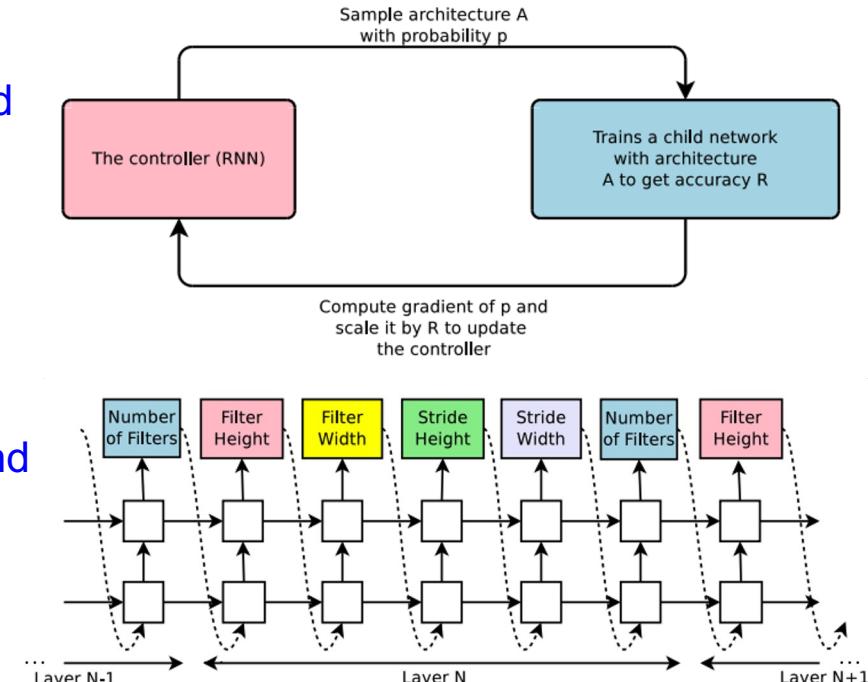


Learning to search for network architectures...

Neural Architecture Search with Reinforcement Learning (NAS)

[Zoph et al. 2016]

- “Controller” network that learns to design a good network architecture (output a string corresponding to network design)
- Iterate:
 - 1) Sample an architecture from search space
 - 2) Train the architecture to get a “reward” R corresponding to accuracy
 - 3) Compute gradient of sample probability, and scale by R to perform controller parameter update (i.e. increase likelihood of good architecture being sampled, decrease likelihood of bad architecture)

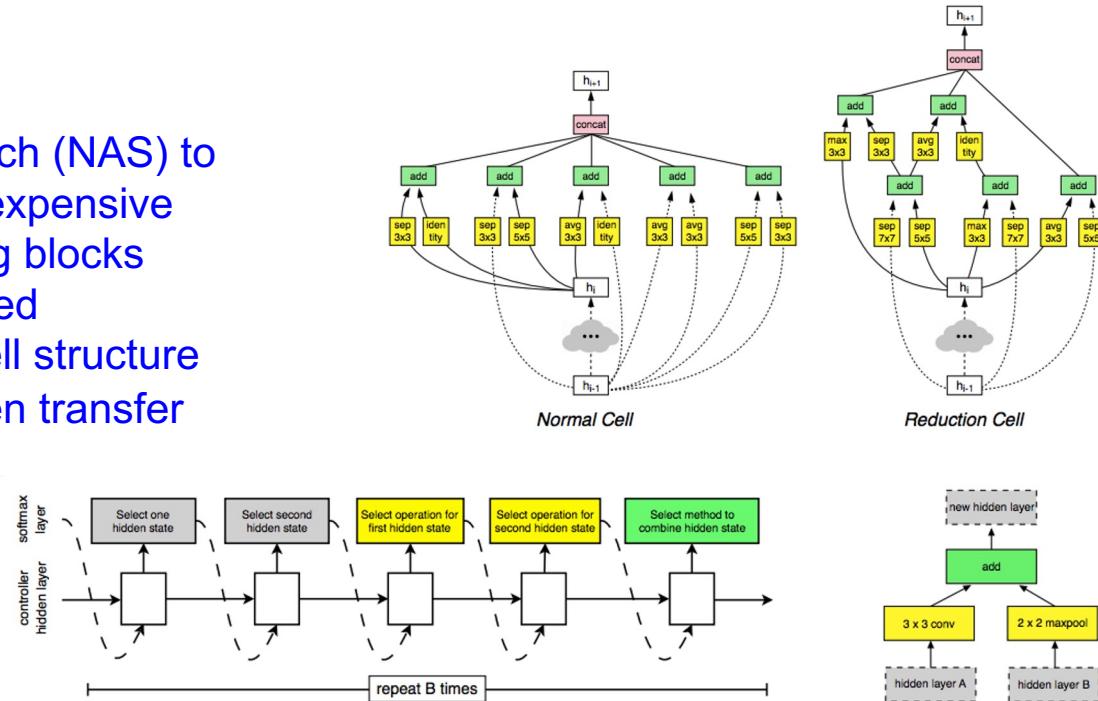


Learning to search for network architectures...

Learning Transferable Architectures for Scalable Image Recognition

[Zoph et al. 2017]

- Applying neural architecture search (NAS) to a large dataset like ImageNet is expensive
- Design a search space of building blocks (“cells”) that can be flexibly stacked
- NASNet: Use NAS to find best cell structure on smaller CIFAR-10 dataset, then transfer architecture to ImageNet
- Many follow-up works in this space e.g. AmoebaNet (Real et al. 2019) and ENAS (Pham, Guan et al. 2018)



But sometimes smart heuristic is better than NAS ...

EfficientNet: Smart Compound Scaling

[Tan and Le. 2019]

- Increase network capacity by scaling width, depth, and resolution, while balancing accuracy and efficiency.
- Search for optimal set of compound scaling factors given a compute budget (target memory & flops).
- Scale up using smart heuristic rules

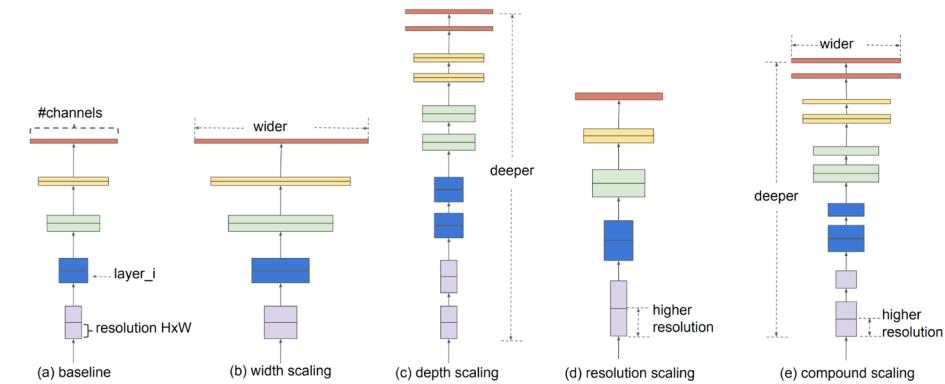
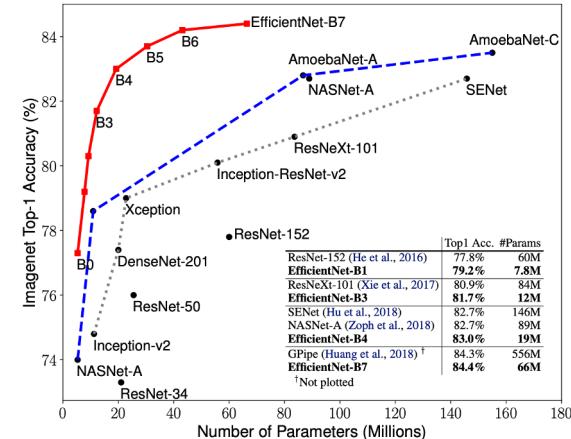
$$\text{depth: } d = \alpha^\phi$$

$$\text{width: } w = \beta^\phi$$

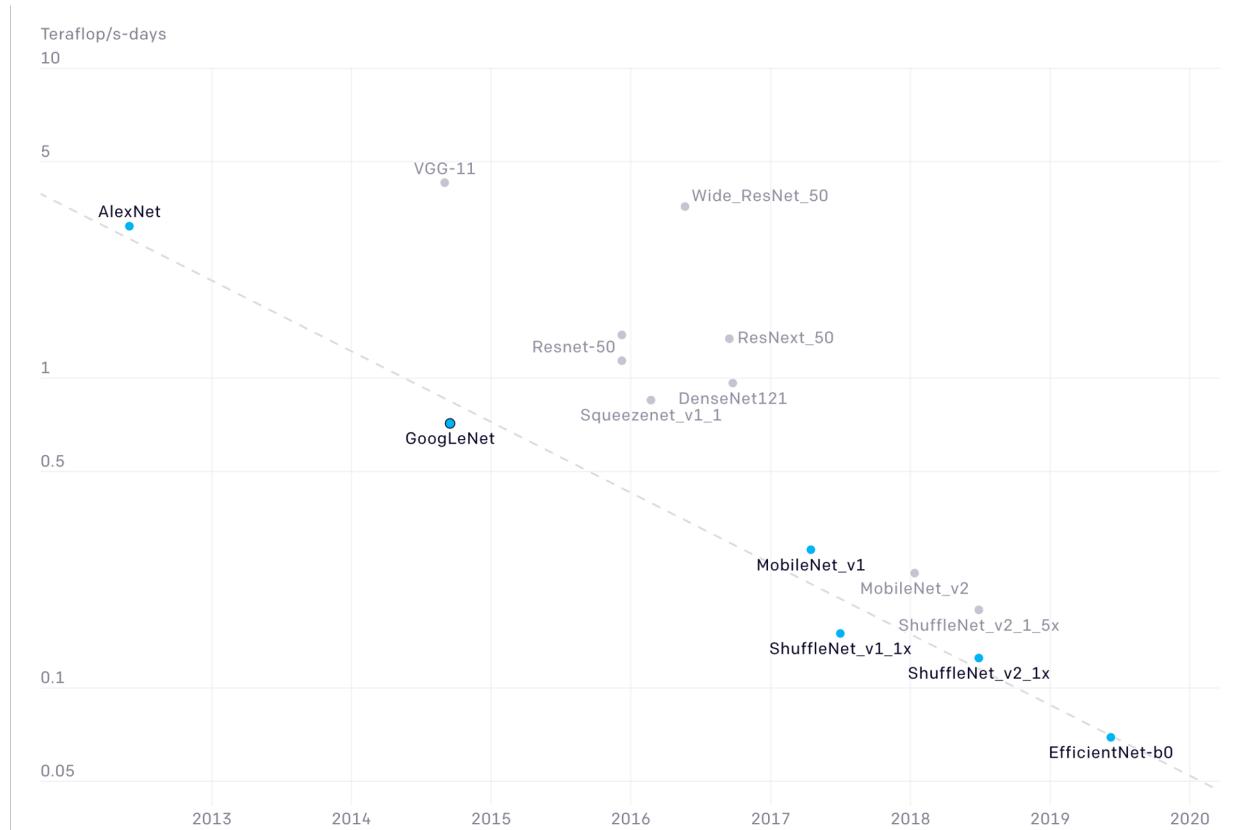
$$\text{resolution: } r = \gamma^\phi$$

$$\text{s.t. } \alpha \cdot \beta^2 \cdot \gamma^2 \approx 2$$

$$\alpha \geq 1, \beta \geq 1, \gamma \geq 1$$



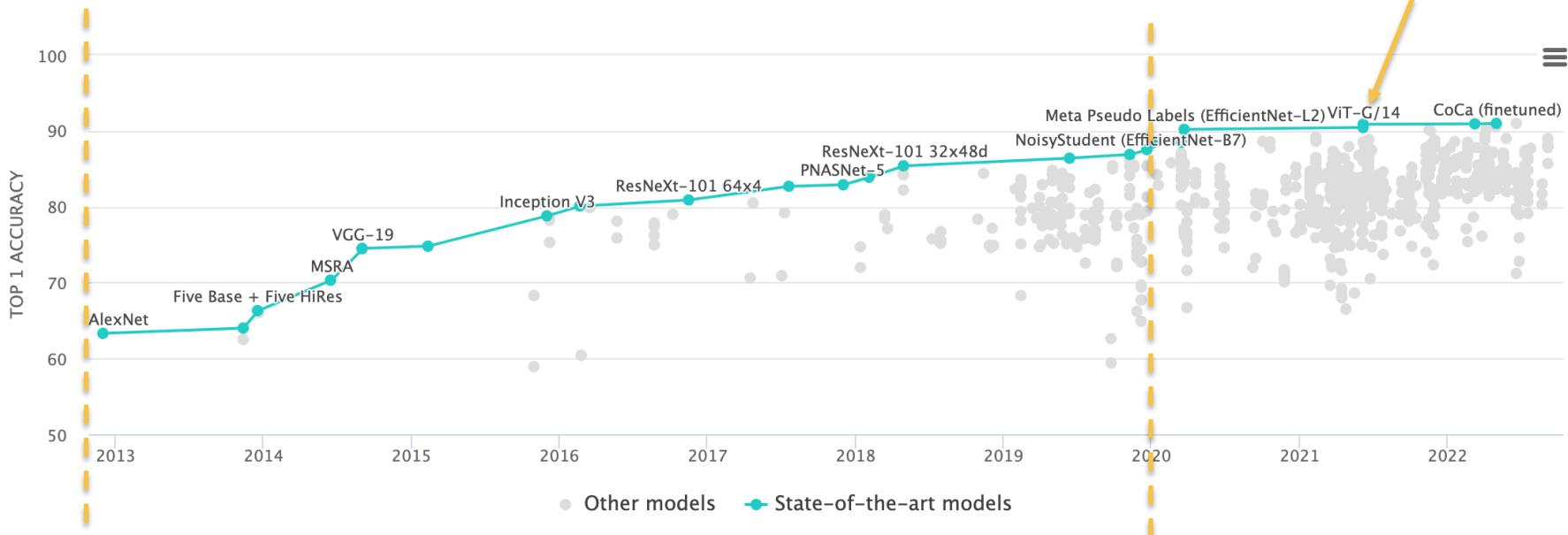
Amount of compute required to reach “AlexNet performance”



<https://openai.com/blog/ai-and-efficiency/>

This Lecture

Transformer
(later this sem.)



<https://paperswithcode.com/sota/image-classification-on-imagenet>

What we have learned so far ...

Deep Neural Networks:

- What they are (composite parametric, non-linear functions)
- Where they come from (biological inspiration, brief history of ANN)
- How they are optimized, in principle (analytical gradient via computational graphs, backpropagation)
- What they look like in practice (Deep ConvNets for vision)

Next few lectures:

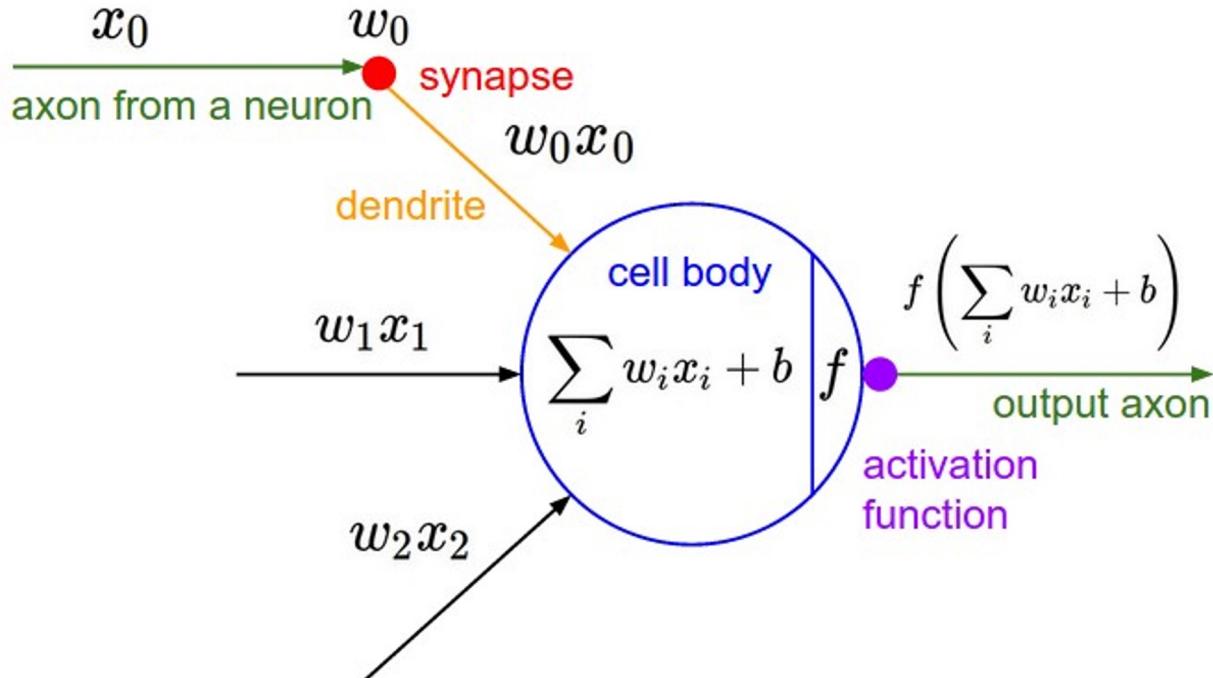
Training Deep Neural Networks

- Details of the non-linear activation functions
- Data normalization
- Weight Initialization
- Batch Normalization
- Regularization
- Advanced Optimization
- Data Augmentation
- Transfer learning
- Hyperparameter Tuning
- Model Ensemble

Today: Training Deep NNs (Part 1)

- Details of the non-linear activation functions
- Data normalization
- Weight Initialization

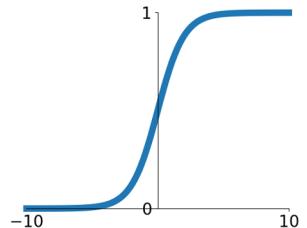
Activation Functions



Activation Functions

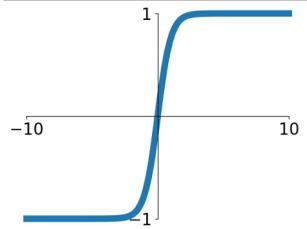
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



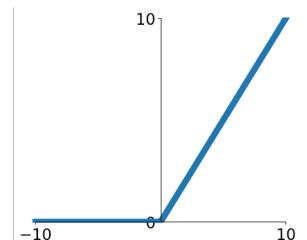
tanh

$$\tanh(x)$$



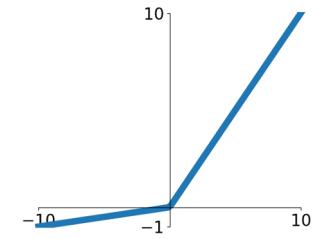
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

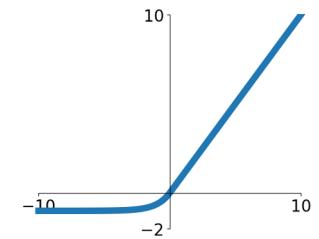


Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

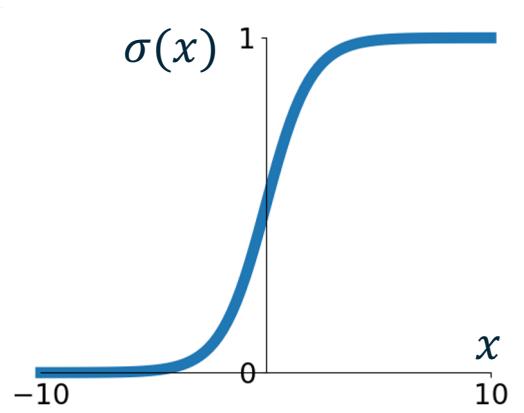
ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Activation Functions

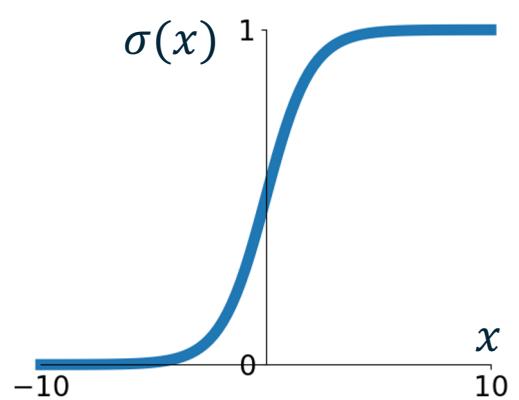
$$\sigma(x) = 1/(1 + e^{-x})$$



Sigmoid

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

Activation Functions



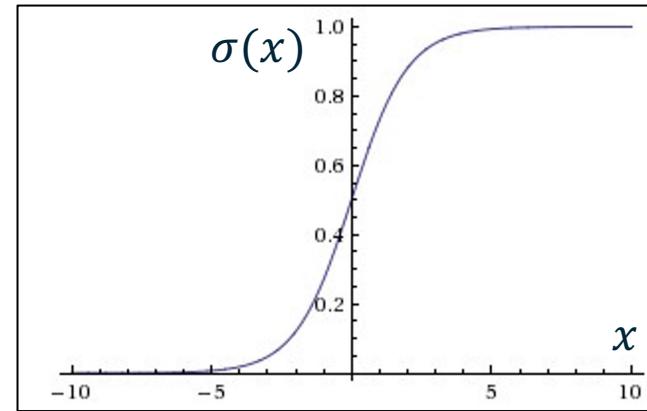
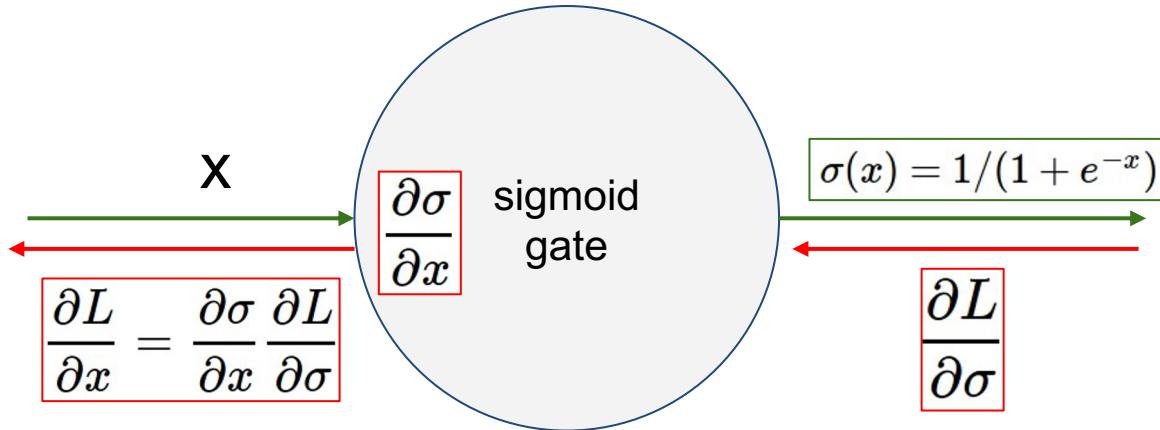
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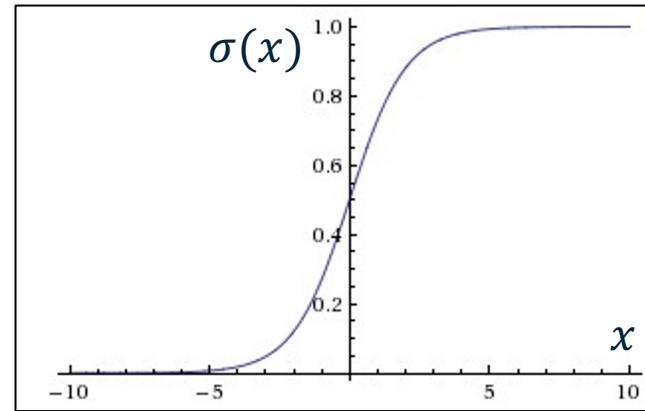
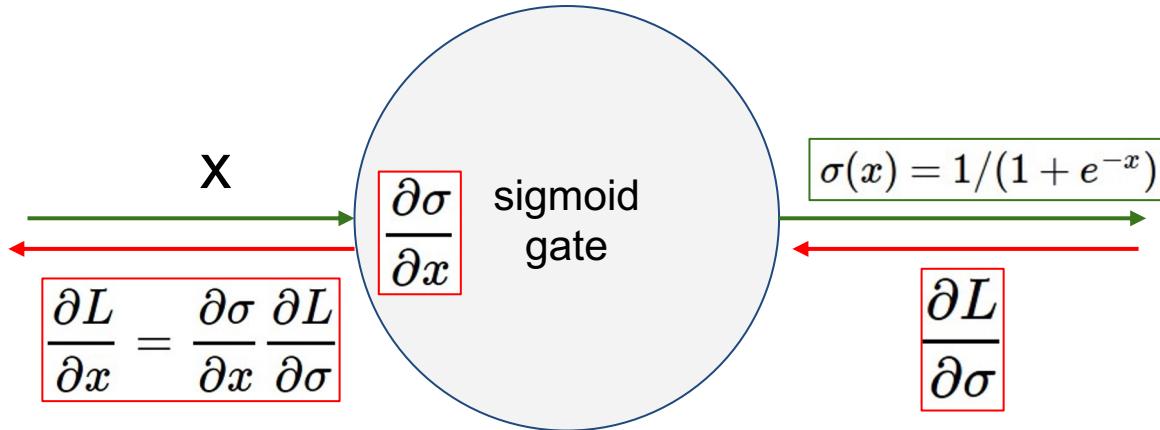
- Squashes numbers to range [0,1]
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Problems:

1. Saturated neurons “kill” the gradients

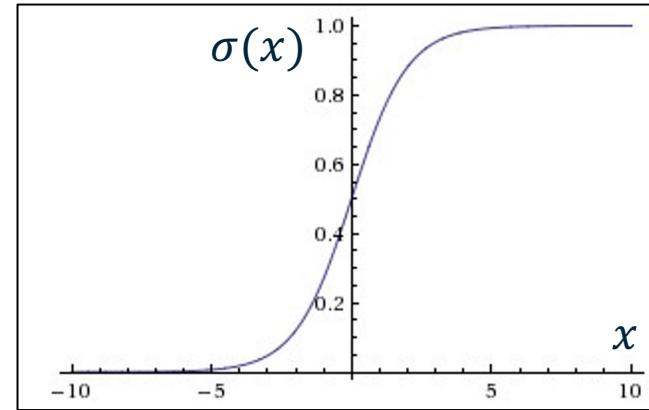
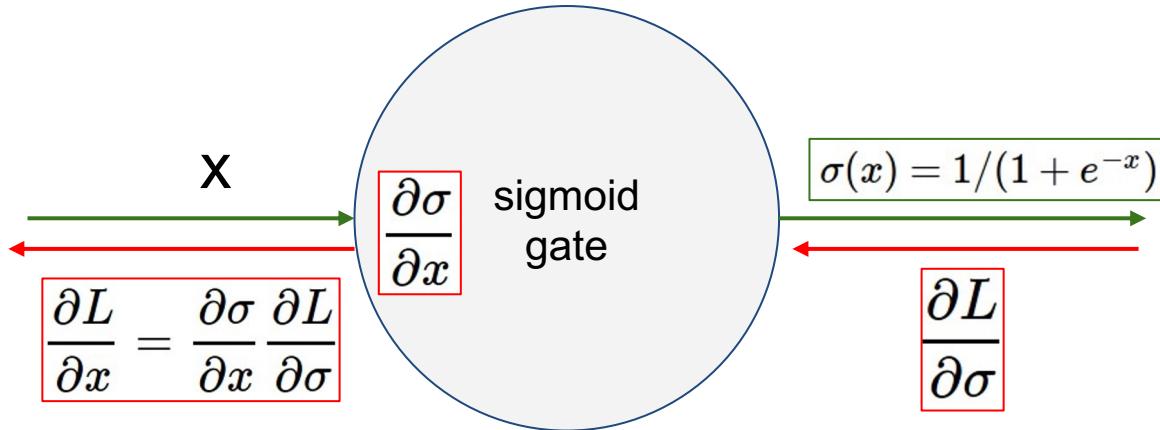


$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$



What happens when $x = -10$?

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$

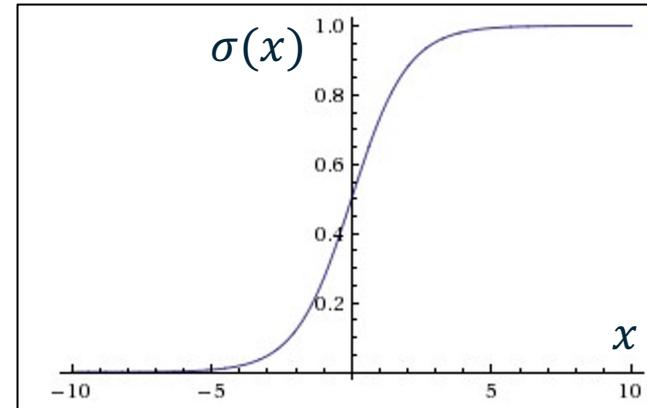
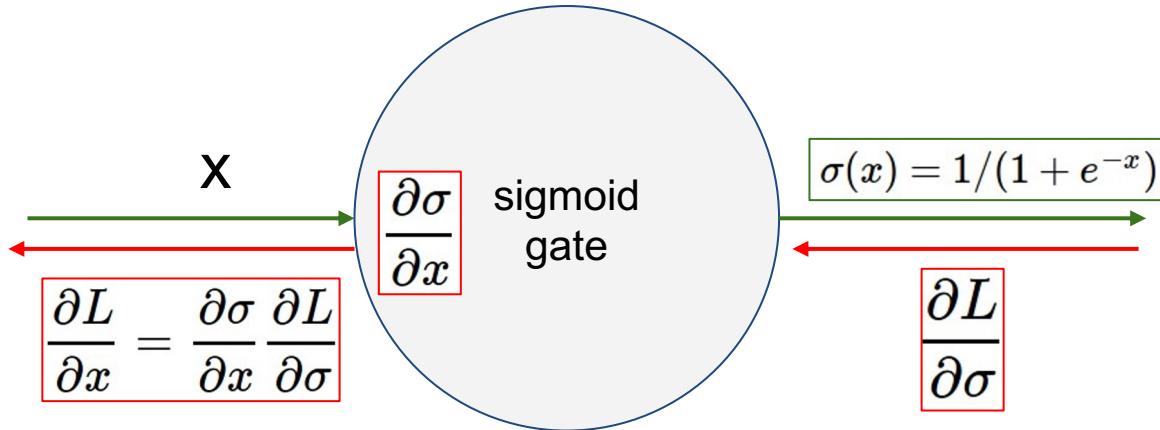


What happens when $x = -10$?

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$

$$\sigma(x) = \sim 0$$

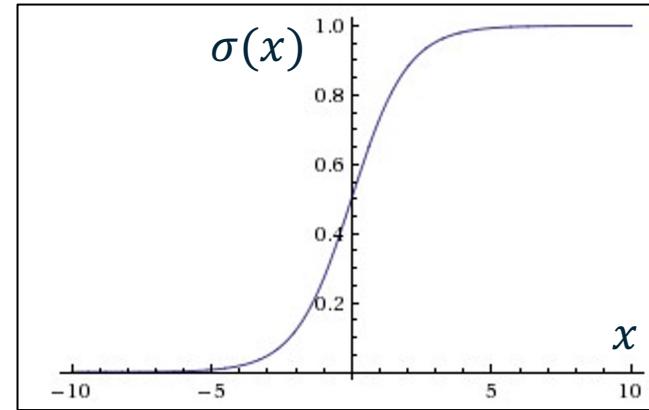
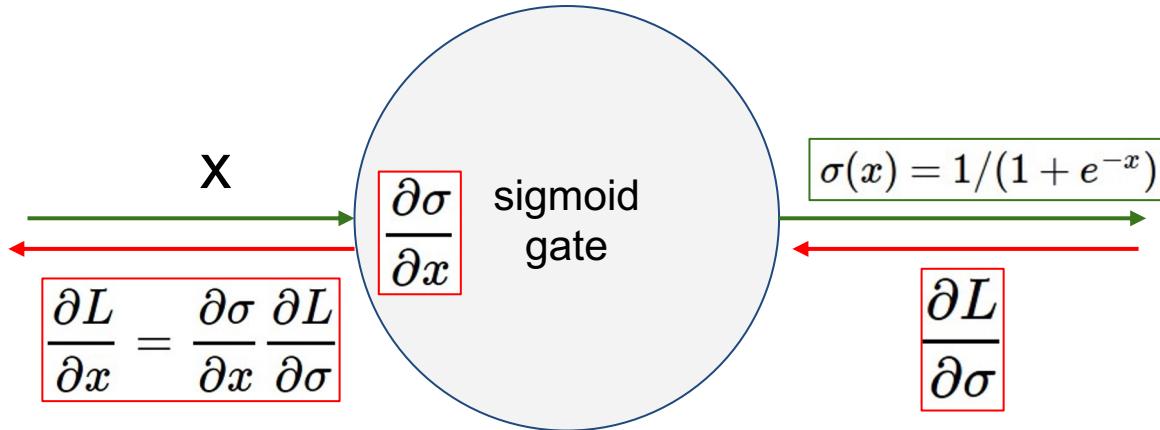
$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x)) = 0(1 - 0) = 0$$



What happens when $x = -10$?

What happens when $x = 10$?

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$

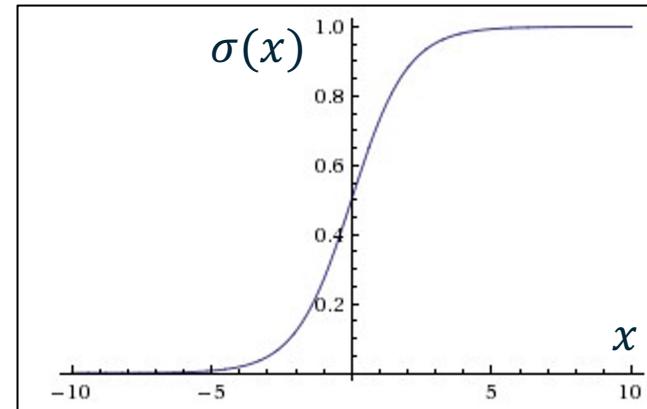
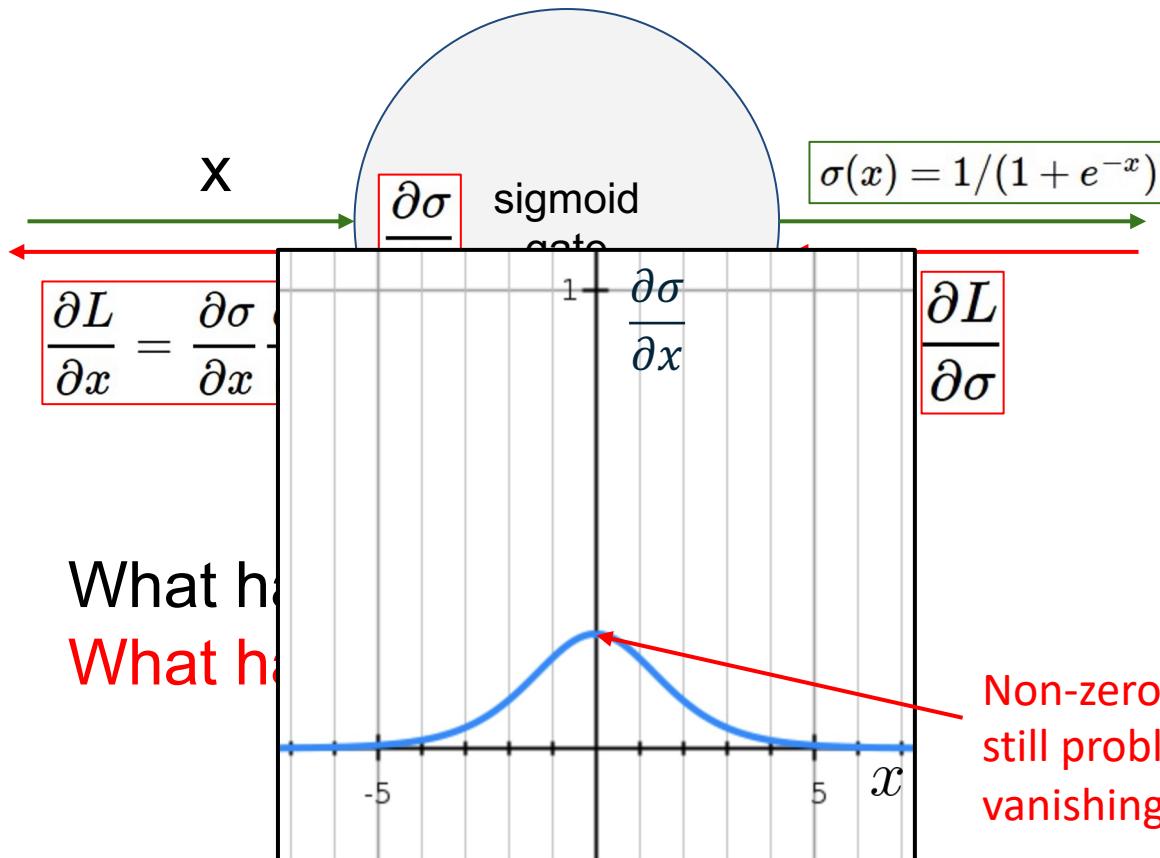


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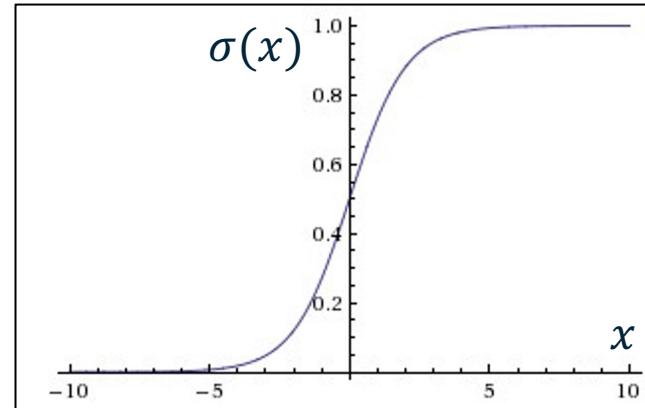
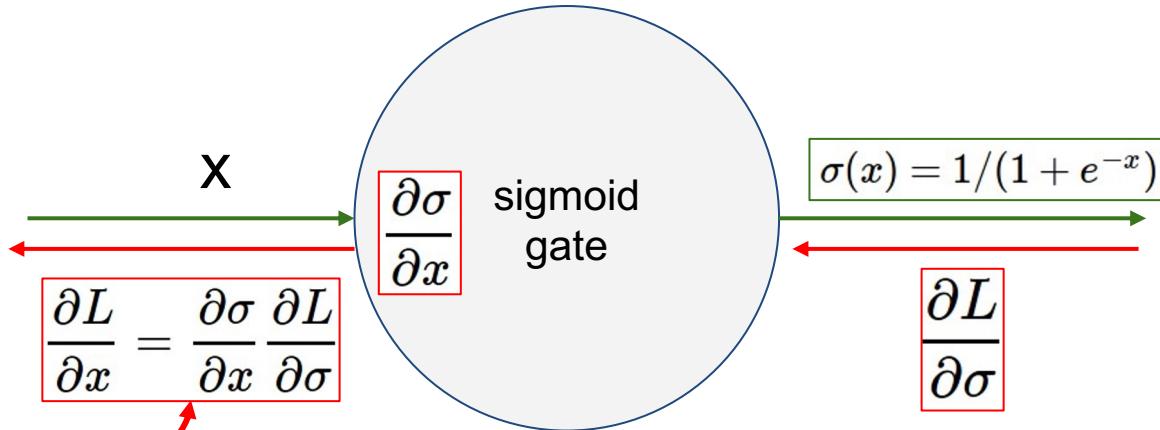
$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$

$$\sigma(x) = \sim 1 \quad \frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x)) = 1(1 - 1) = 0$$



$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)(1 - \sigma(x))$$

Non-zero but small:
still problematic, causes
vanishing gradient

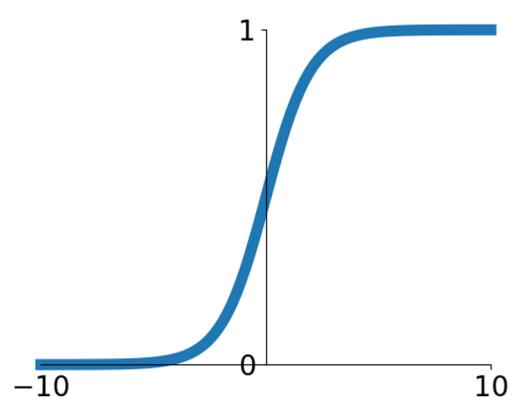


Why is this a problem?

If all the gradients flowing back will be zero and weights will never change (aka “Vanishing Gradient”)

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)(1 - \sigma(x))$$

Activation Functions



Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

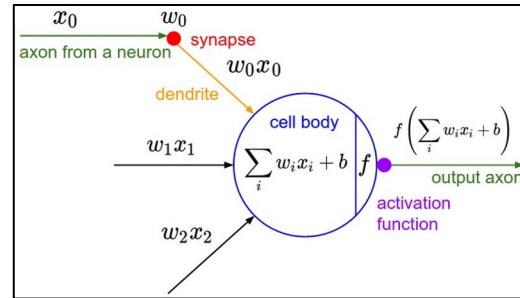
- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

Problems:

1. Saturated neurons “kill” the gradients
2. **Sigmoid outputs are not zero-centered**

Consider what happens when the input to a neuron is always positive...

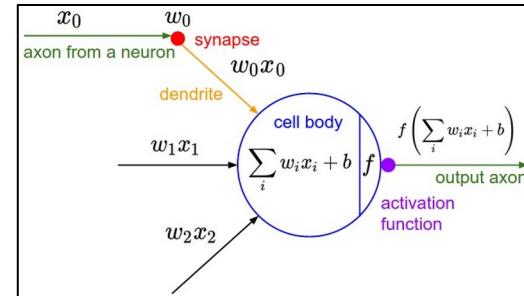
$$f \left(\sum_i w_i x_i + b \right)$$



What can we say about the gradients on w ?

Consider what happens when the input to a neuron is always positive...

$$f \left(\sum_i w_i x_i + b \right)$$

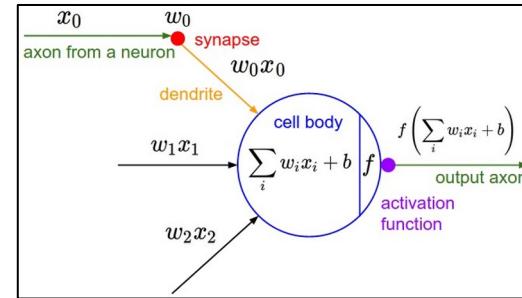


What can we say about the gradients on w ?

$$\frac{\partial L}{\partial w} = \sigma(\sum_i w_i x_i + b)(1 - \sigma(\sum_i w_i x_i + b))x \times \text{upstream_gradient}$$

Consider what happens when the input to a neuron is always positive...

$$f \left(\sum_i w_i x_i + b \right)$$



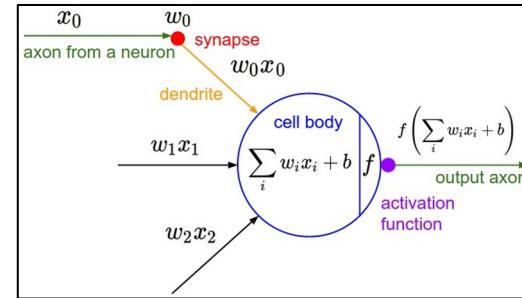
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We know that local gradient of sigmoid is always positive

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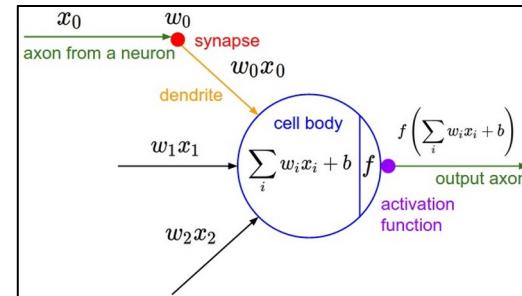
We know that local gradient of sigmoid is always positive

We are assuming x is positive

$$\frac{\partial L}{\partial w} = \boxed{\sigma(\sum_i w_i x_i + b)(1 - \sigma(\sum_i w_i x_i + b))x} \times \text{upstream_gradient}$$

Consider what happens when the input to a neuron is always positive...

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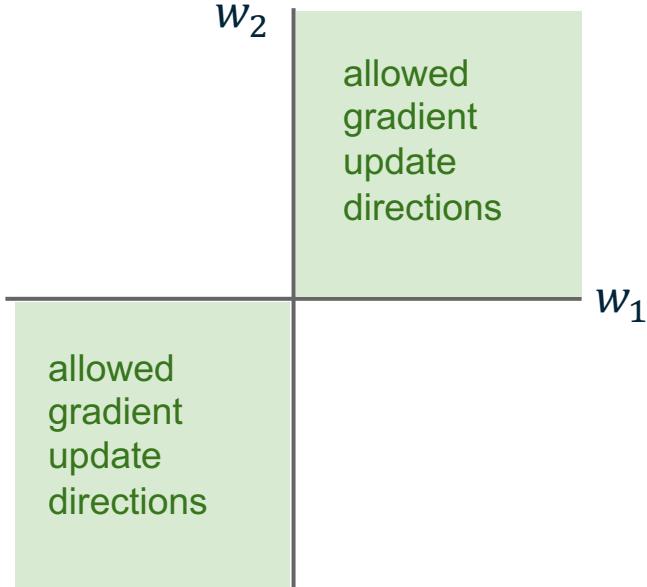
Sign of gradient for all w_i is the same as the sign of upstream gradient.

That is, local gradient cannot change the sign of global gradient

$$\frac{\partial L}{\partial w} = \sigma(\sum_i w_i x_i + b)(1 - \sigma(\sum_i w_i x_i + b))x \times \text{upstream_gradient}$$

Consider what happens when the input to a neuron is always positive...

$$f \left(\sum_i w_i x_i + b \right)$$



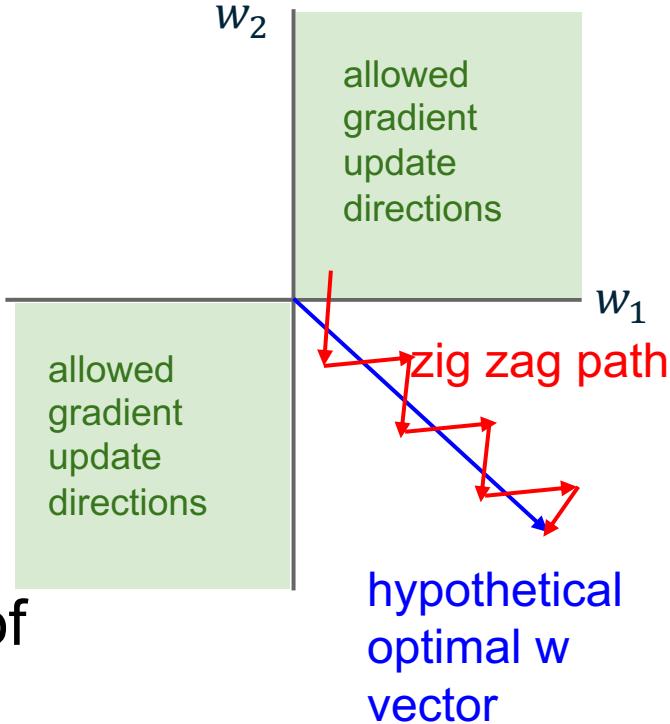
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$$f \left(\sum_i w_i x_i + b \right)$$

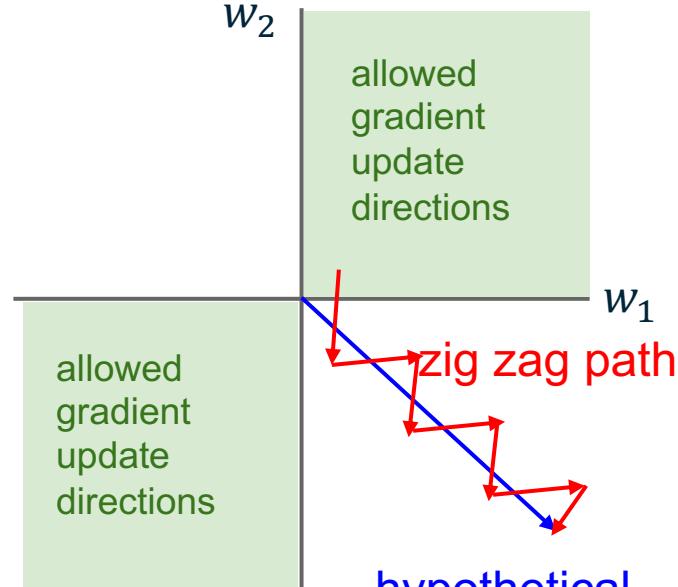
Local gradient cannot change the sign of global gradient.

Can easily lead to all-positive or all-negative gradient update (zig-zag).



Consider what happens when the input to a neuron is always positive...

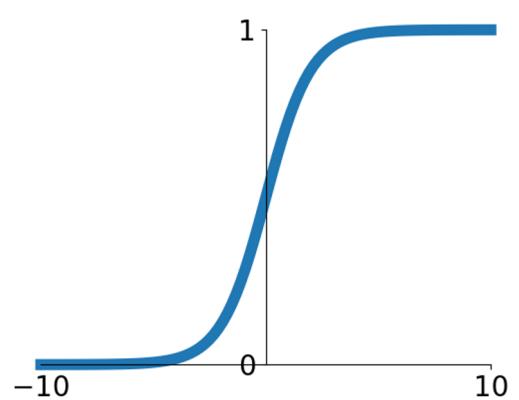
$$f \left(\sum_i w_i x_i + b \right)$$



Remark: both upstream gradient and local input can change the sign of gradient irrespective of the activation, but having a zero-centered activation function (output spans both positive and negative) can further minimize the “zig-zag” effect

hypothetical
optimal w
vector

Activation Functions



Sigmoid

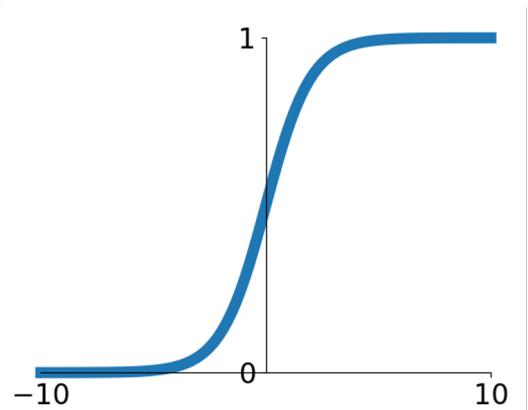
$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

Problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered (output does not span both positive and negative)

Activation Functions



Sigmoid

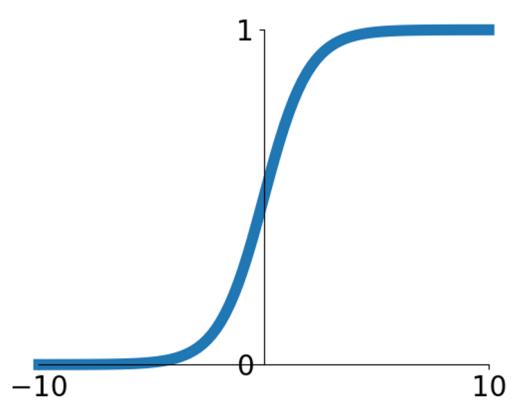
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Problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered (output does not span both positive and negative)
3. $\exp()$ is a bit compute expensive

Activation Functions



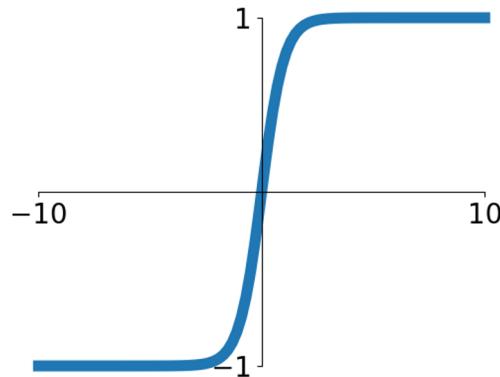
Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
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**Worst problem in practice:
Saturated neurons “kill” the
gradients / vanishing gradient**

Activation Functions

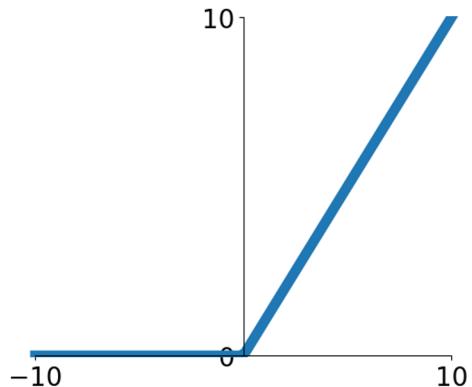


tanh(x)

- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

[LeCun et al., 1991]

Activation Functions

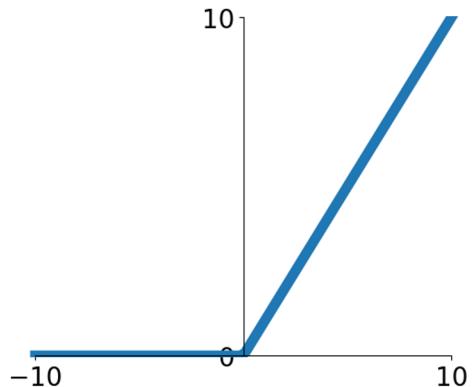


- Computes $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

ReLU
(Rectified Linear Unit)

[Krizhevsky et al., 2012]

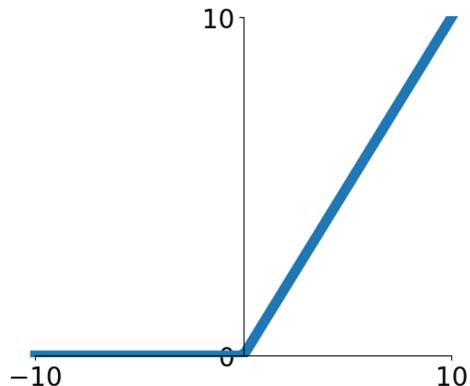
Activation Functions



ReLU
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- Not zero-centered output

Activation Functions



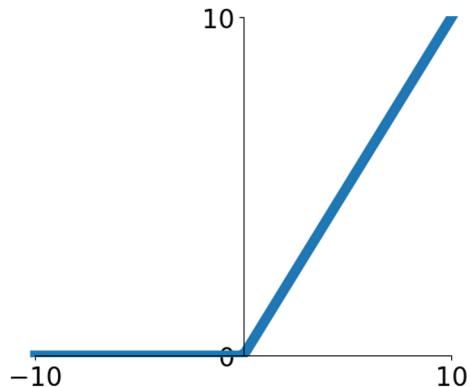
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- An annoyance:

hint: what is the gradient when $x < 0$?

Activation Functions



ReLU
(Rectified Linear Unit)

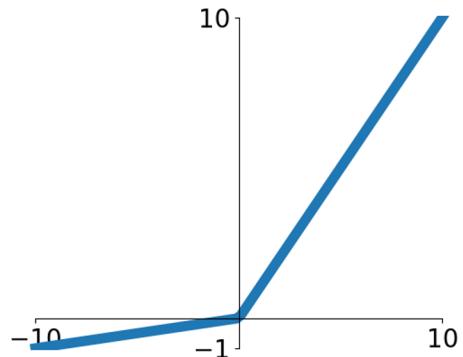
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- Not zero-centered output
- An annoyance:

hint: what is the gradient when $x < 0$?
Always 0 -> no update in weights ->
stays 0, A.K.A. “dead ReLU”

Activation Functions

[Mass et al., 2013]
[He et al., 2015]

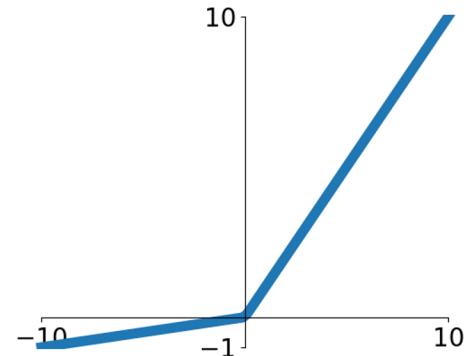


- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Activation Functions



Leaky ReLU

$$f(x) = \max(0.01x, x)$$

[Mass et al., 2013]
[He et al., 2015]

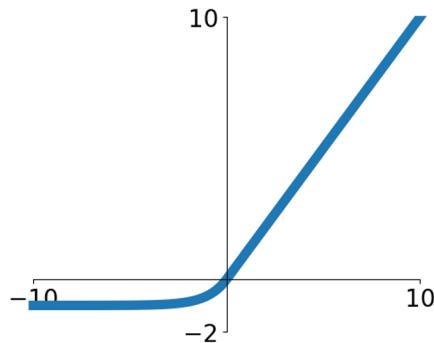
- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

Parametric Rectifier (PReLU)

$$f(x) = \max(\alpha x, x)$$

backprop into α
(parameter)

Exponential Linear Units (ELU)

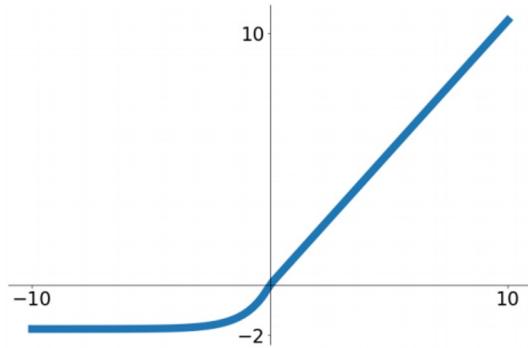


$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \leq 0 \end{cases}$$

(Alpha default = 1)

- All benefits of ReLU
- Negative saturation encodes presence of features (all goes to $-\alpha$), not magnitude
- Similar in backprop (αe^x when x is negative)
- Compared with Leaky ReLU: smooth gradient at 0 (no kink), better optimization landscape

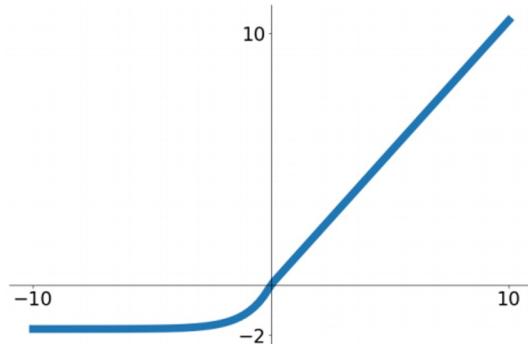
Scaled Exponential Linear Units (SELU)



$$f(x) = \begin{cases} \lambda x & \text{if } x > 0 \\ \lambda\alpha(e^x - 1) & \text{otherwise} \end{cases}$$

- Scaled version of ELU that works better for deep networks
- “Self-normalizing” property: under certain condition, the output of a feedforward network stays around zero-mean and unit variance

Scaled Exponential Linear Units (SELU)



$$f(x) = \begin{cases} \lambda x & \text{if } x > 0 \\ \lambda\alpha(e^x - 1) & \text{otherwise} \end{cases}$$

$$\alpha = 1.6732632423543772848170429916717$$
$$\lambda = 1.0507009873554804934193349852946$$

- Scaled version of ELU that works better for deep networks
- “Self-normalizing” property: under certain condition, the output of a feedforward network stays around zero-mean and unit variance

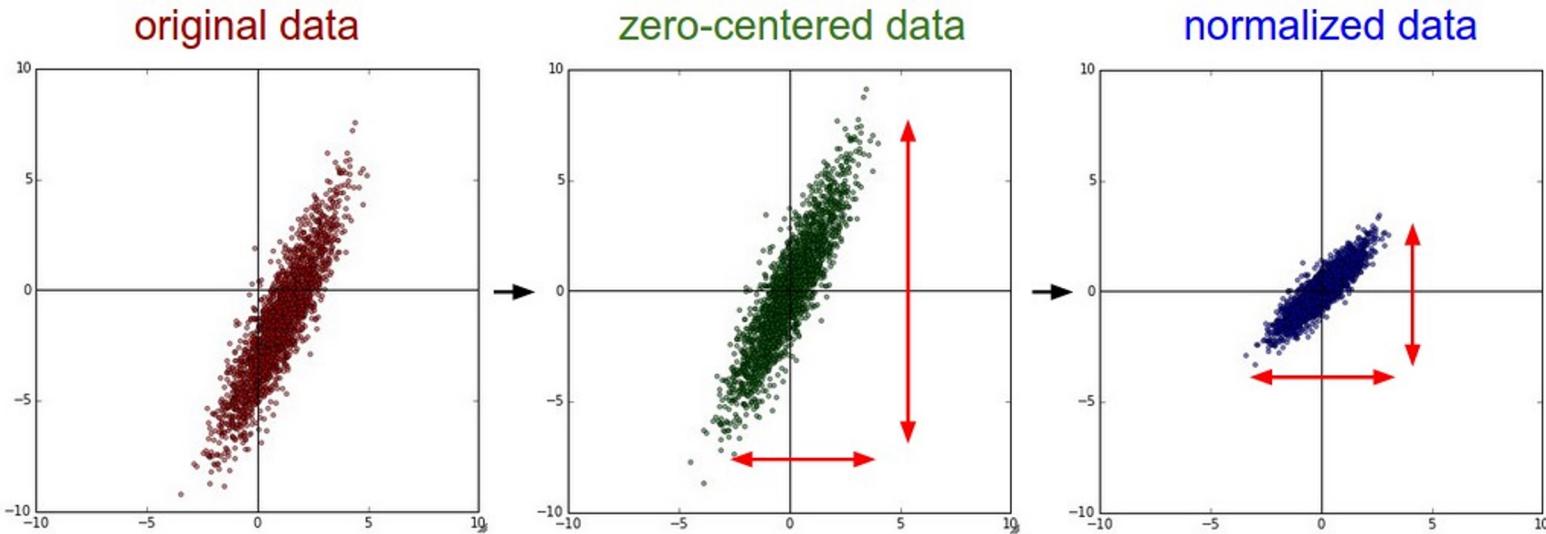
(Klambauer et al, Self-Normalizing Neural Networks, ICLR 2017)

TLDR: In practice:

- Many possible choices beyond what we've talked here, but ...
- Use ReLU. Be careful with your learning rates
- Try out Leaky ReLU / ELU / SELU
 - To squeeze out some marginal gains
- Don't use sigmoid or tanh

Data Preprocessing

Data Preprocessing



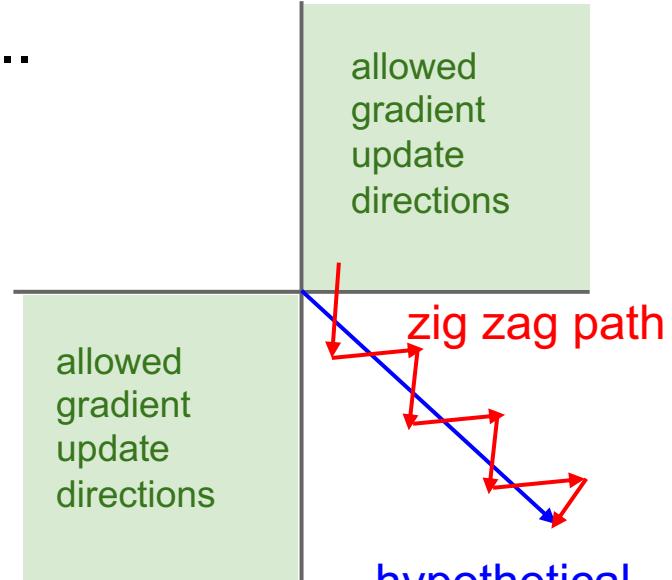
```
X -= np.mean(X, axis = 0)
```

```
X /= np.std(X, axis = 0)
```

(Assume $X [NxD]$ is data matrix,
each example in a row)

Remember: Consider what happens when the input to a neuron is always positive...

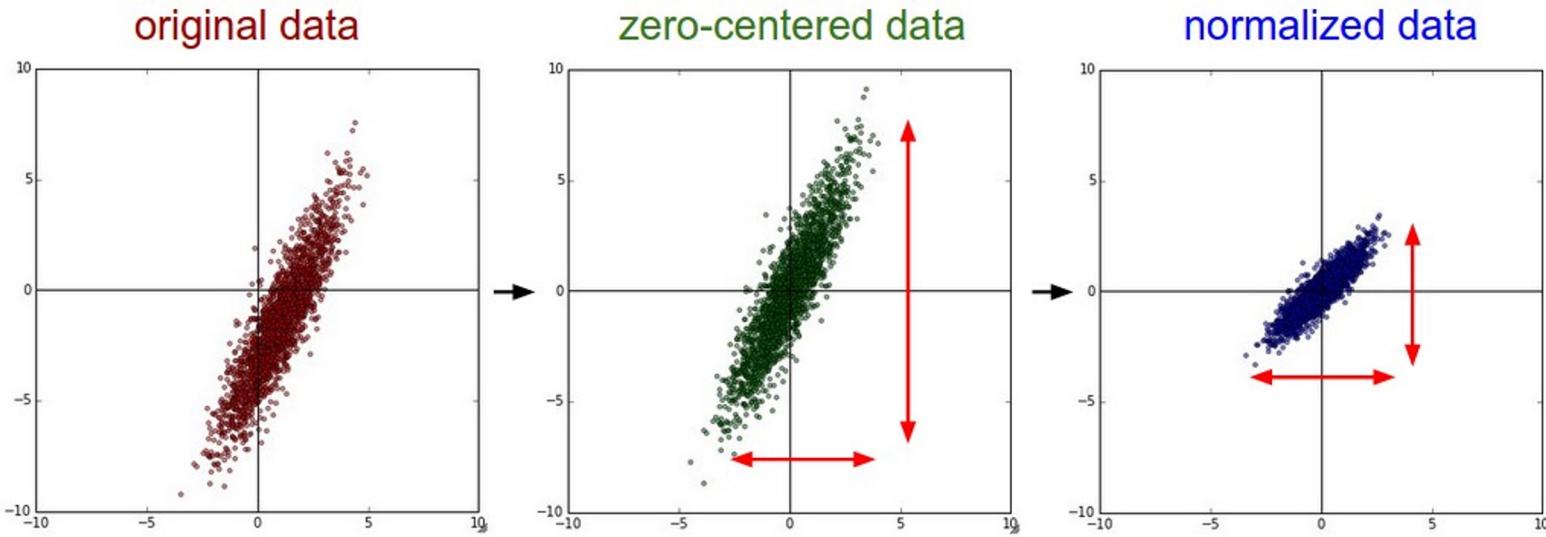
$$f \left(\sum_i w_i x_i + b \right)$$



In addition to upstream and local gradient, input also determines the sign of the gradient.

To reduce biases in gradient, we want the input to span both positive and negative value

Data Preprocessing



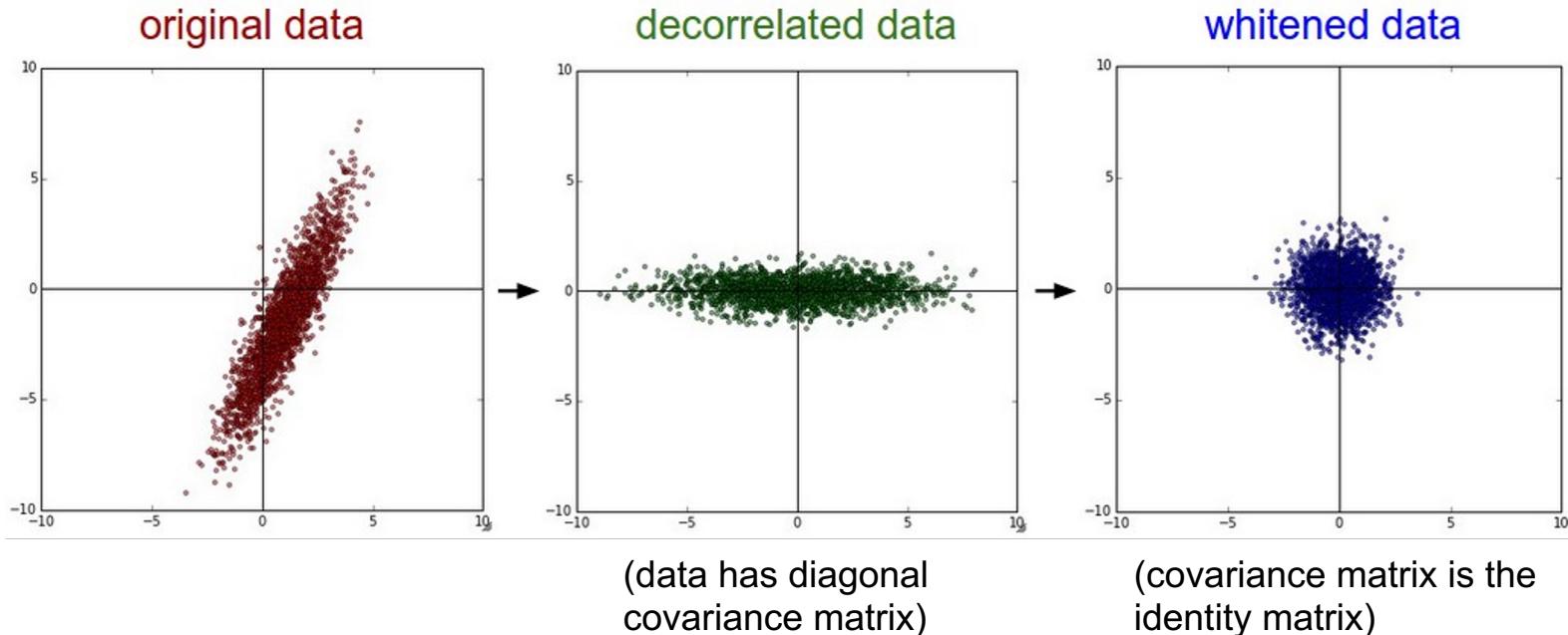
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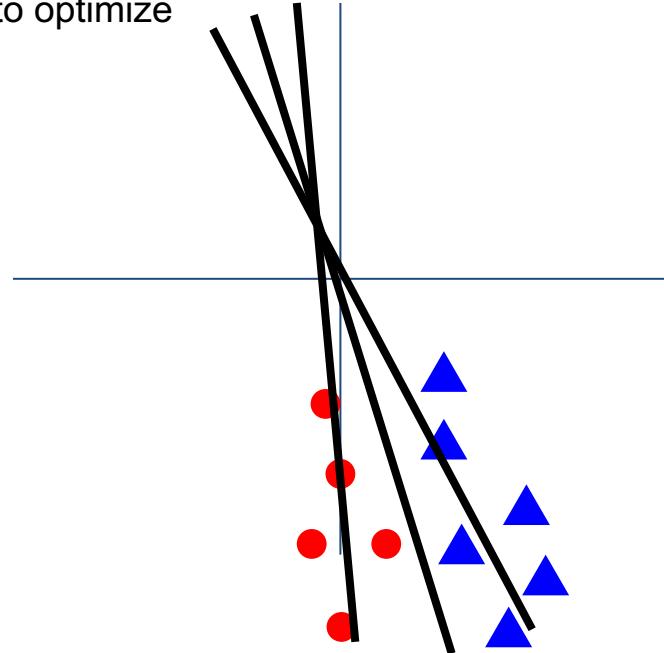
Data Preprocessing

In practice, you could also **PCA** and **Whitening** of the data

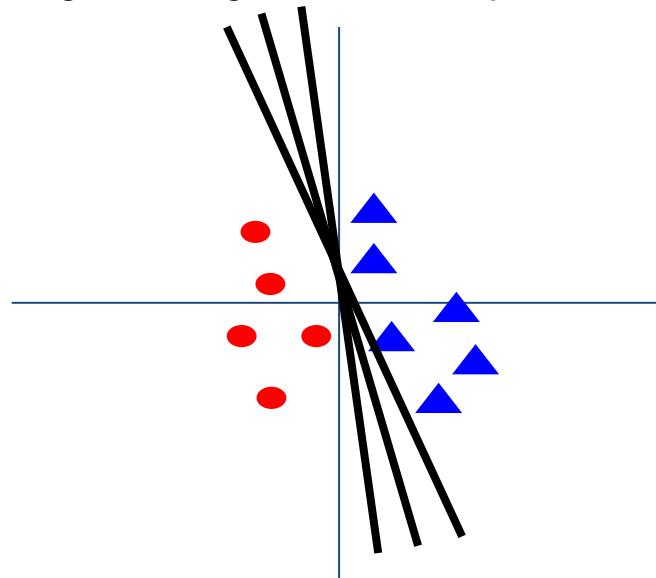


Data Preprocessing

Before normalization: classification loss very sensitive to changes in weight matrix; hard to optimize



After normalization: less sensitive to small changes in weights; easier to optimize



Examples: images

e.g. consider CIFAR-10 example with [32,32,3] images

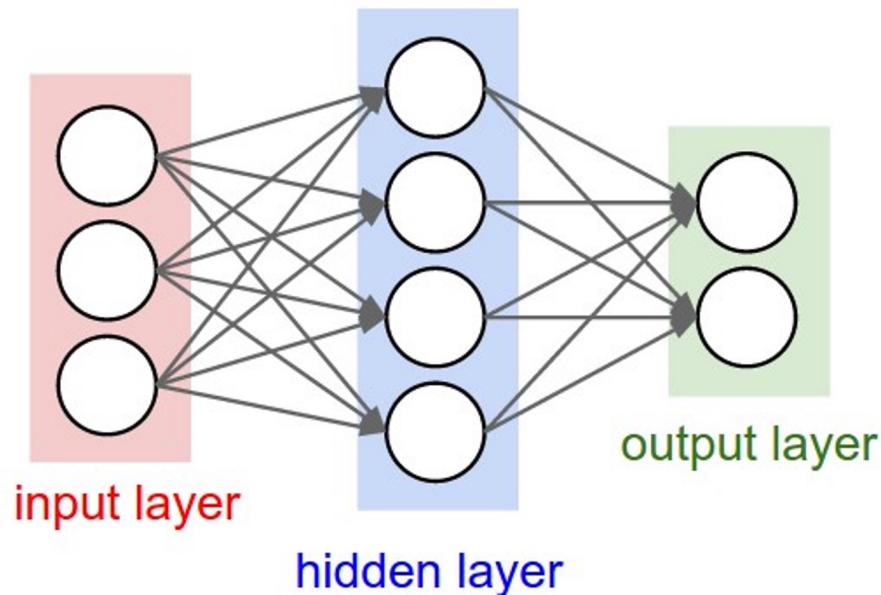
- Subtract the per-pixel mean (e.g. AlexNet)
(mean image = [32,32,3] array)
- Subtract per-channel mean (e.g. VGGNet)
(mean along each channel = 3 numbers,)
- Subtract per-channel mean and
Divide by per-channel std (e.g. ResNet)
(mean along each channel = 3 numbers)

Examples: other domains

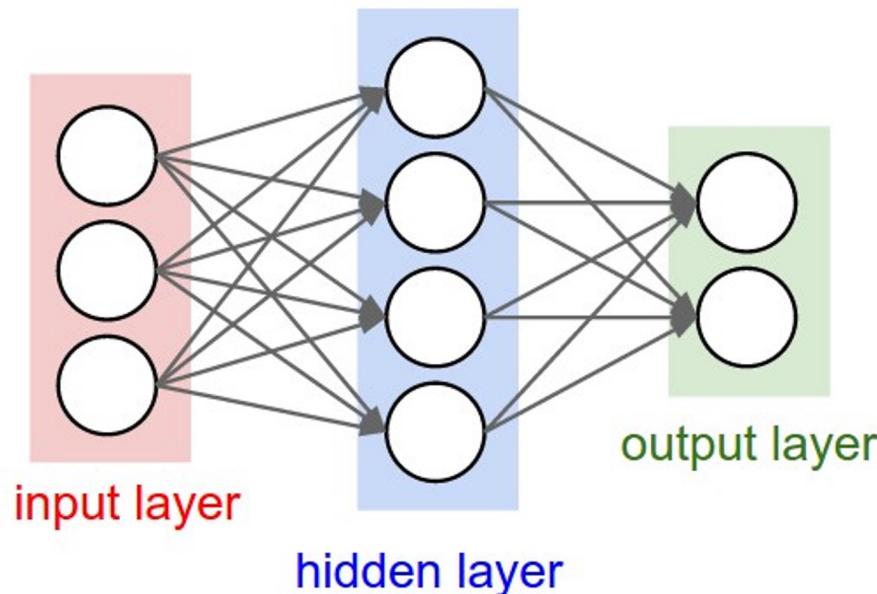
- **Natural language processing:** Normalize word embeddings like Word2Vec or GloVe vectors so that they have a unit norm
- **Graph Neural Networks (GNN):** the feature vector of a node might be scaled by the inverse of its degree or the square root of its degree.
- **Audio data:** Spectral normalize waveforms to ensure that the frequency components are on a similar scale.
- **Reinforcement learning:** reward can be normalized to have zero mean and unit variance to stabilize learning.

Weight Initialization

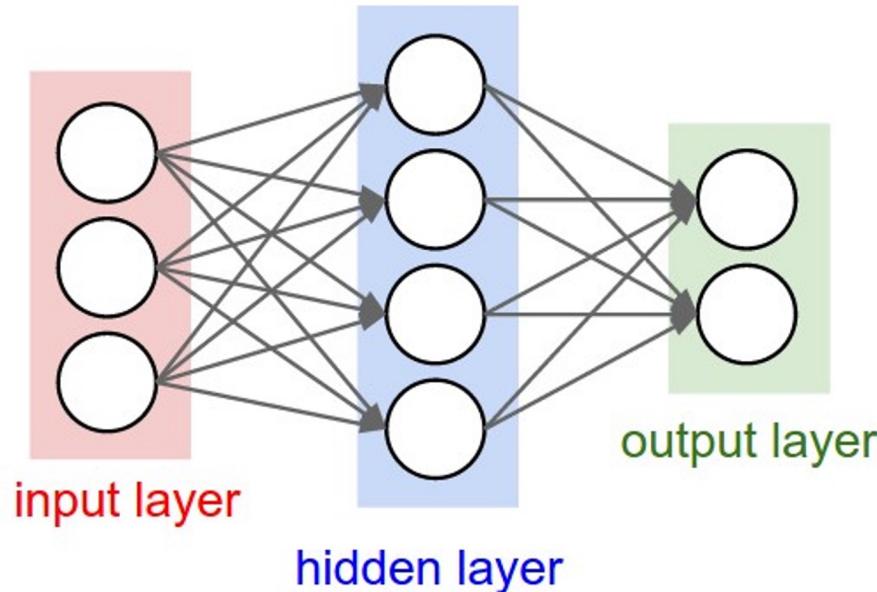
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- A: All output will be the same! $w_1^T x = w_2^T x$ if $w_1 = w_2$



- Q: what happens when $W=\text{same}$ initial value is used?
- A: All output will be the same! $w_1^T x = w_2^T x$ if $w_1 = w_2$
- Want to **maintain variance** through the layers.



- First idea: **Small random numbers**
(gaussian with zero mean and 1e-2 standard deviation)

```
W = 0.01 * np.random.randn(Din, Dout)
```

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(gaussian with zero mean and 1e-2 standard deviation)

```
W = 0.01 * np.random.randn(Din, Dout)
```

Works ~okay for small networks, but problems with deeper networks.

Weight Initialization: Activation statistics

```
dims = [4096] * 7      Forward pass for a 6-layer
hs = []                  net with hidden size 4096
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = 0.01 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

What will happen to the activations for the last layer?

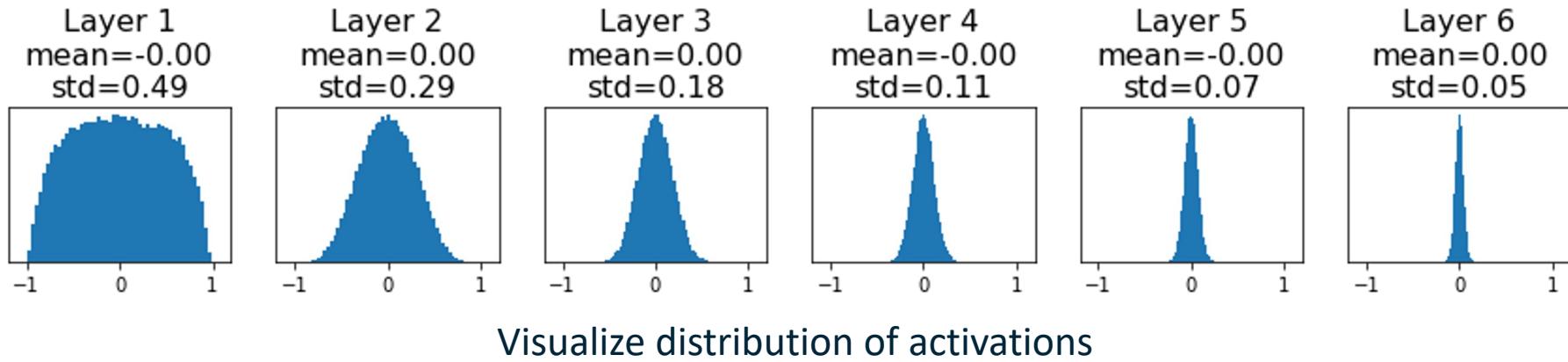
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for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = 0.01 * np.random.randn(Din, Dout)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

All activations tend to zero for deeper network layers

Q: What do the gradients dL/dW look like?

Hint: $\frac{\partial L}{\partial w} = x^T \left(\frac{\partial L}{\partial y} \right)$



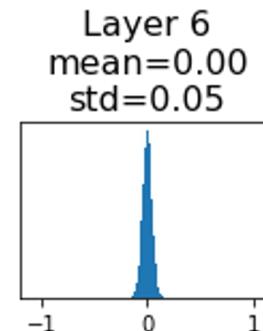
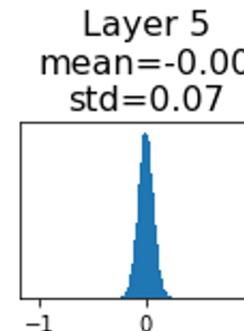
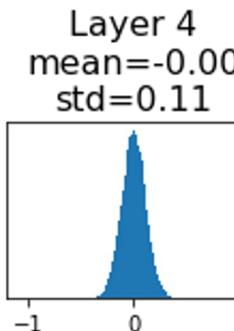
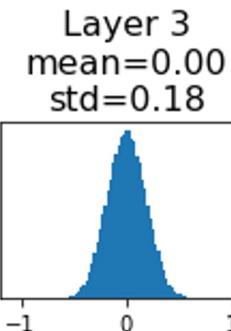
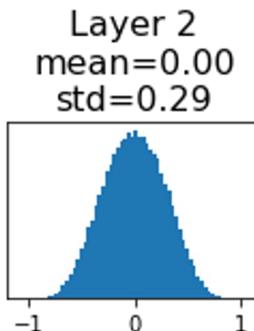
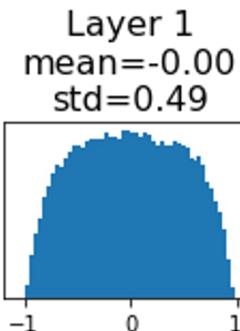
Weight Initialization: Activation statistics

```
dims = [4096] * 7      Forward pass for a 6-layer  
hs = []                  net with hidden size 4096  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = 0.01 * np.random.randn(Din, Dout)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

All activations tend to zero for deeper network layers

Q: What do the gradients dL/dW look like?

A: All zero, no learning =(



Visualize distribution of activations

Weight Initialization: Activation statistics

```
dims = [4096] * 7    Increase std of initial  
hs = []                weights from 0.01 to 0.05  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = 0.05 * np.random.randn(Din, Dout)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

Initialize with higher values

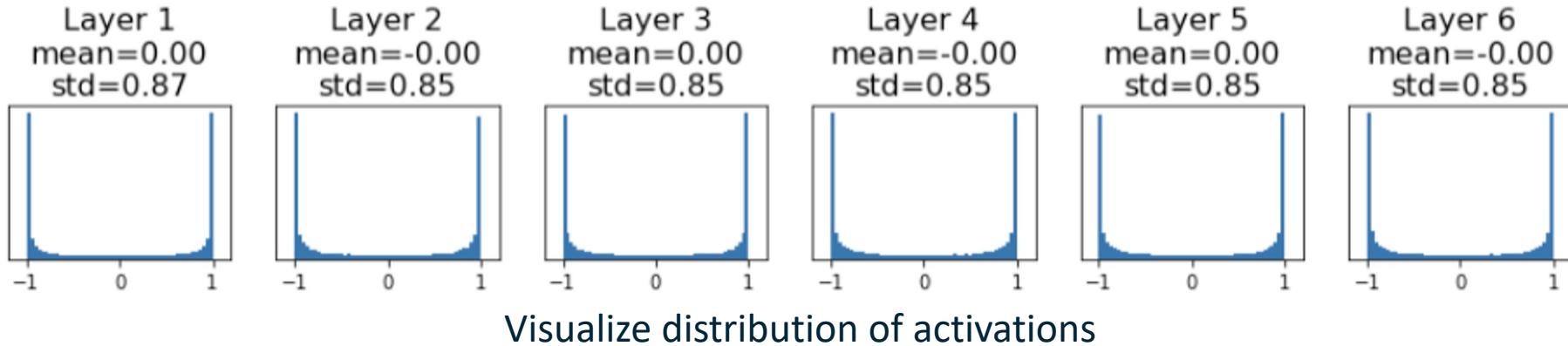
What will happen to the activations for the last layer?

Weight Initialization: Activation statistics

```
dims = [4096] * 7      Increase std of initial  
hs = []                  weights from 0.01 to 0.05  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = 0.05 * np.random.randn(Din, Dout)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

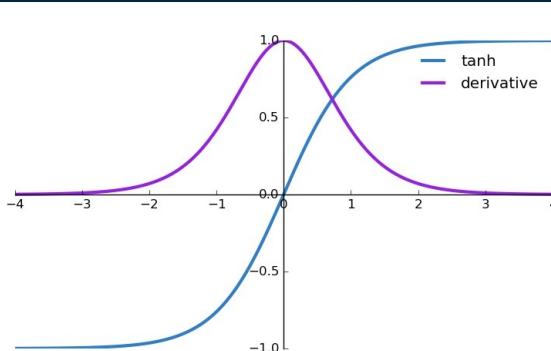
All activations saturate

Q: What do the gradients look like?



Weight Initialization: Activation statistics

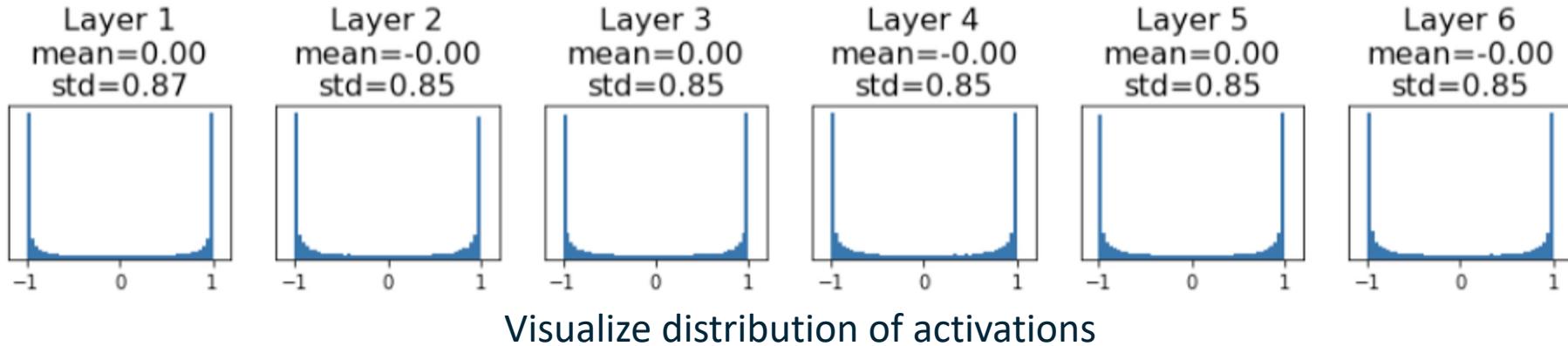
```
dims = [4096] *  
hs = []  
x = np.random.r  
for Din, Dout i  
    W = 0.05 *  
    x = np.tanh  
    hs.append(x)
```



All activations saturate

Q: What do the gradients look like?

A: Local gradients all zero,
no learning =(



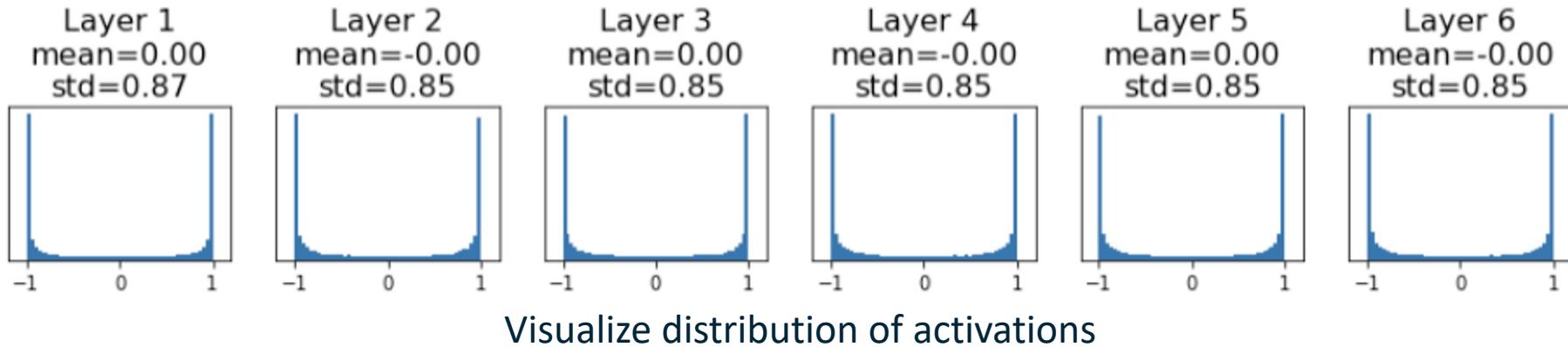
Weight Initialization: Activation statistics

```
dims = [4096] * 7      Increase std of initial  
hs = []                  weights from 0.01 to 0.05  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = 0.05 * np.random.randn(Din, Dout)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

All activations saturate

Q: What do the gradients look like?

More generally, *gradient explosion* (high w \rightarrow high output \rightarrow high gradient).



Weight Initialization: “Xavier” Initialization

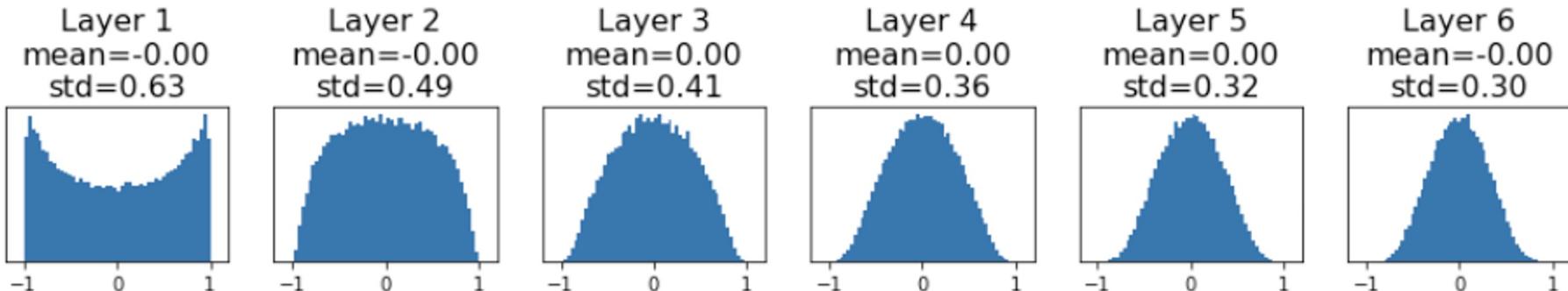
```
dims = [4096] * 7           "Xavier" initialization:  
hs = []                      std = 1/sqrt(Din)  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = np.random.randn(Din, Dout) / np.sqrt(Din)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

Glorot and Bengio, “Understanding the difficulty of training deep feedforward neural networks”, AISTAT 2010

Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7          "Xavier" initialization:  
hs = []                      std = 1/sqrt(Din)  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = np.random.randn(Din, Dout) / np.sqrt(Din)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

“Just right”: Activations are nicely scaled for all layers!



Glorot and Bengio, “Understanding the difficulty of training deep feedforward neural networks”, AISTAT 2010

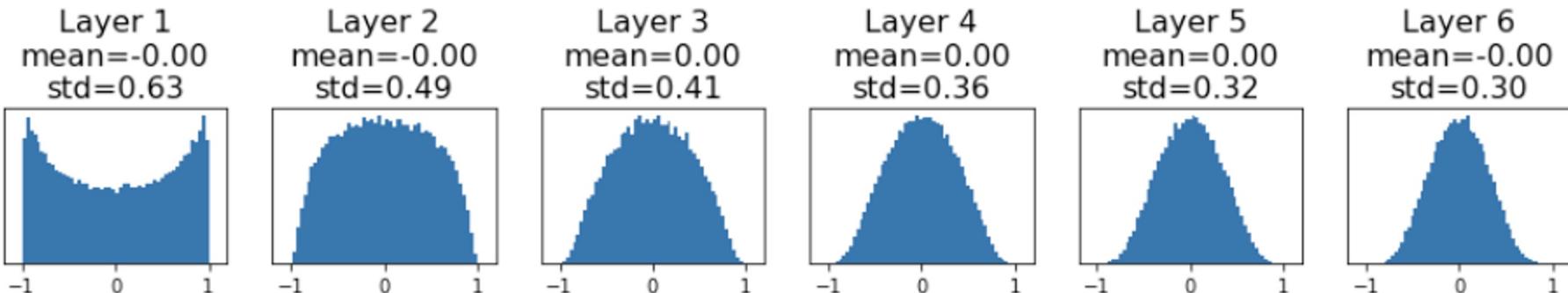
Visualize distribution of activations

Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7          "Xavier" initialization:  
hs = []                      std = 1/sqrt(Din)  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = np.random.randn(Din, Dout) / np.sqrt(Din)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

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For conv layers, Din is $\text{filter_size}^2 * \text{input_channels}$



Glorot and Bengio, “Understanding the difficulty of training deep feedforward neural networks”, AISTAT 2010

Visualize distribution of activations

Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7          "Xavier" initialization:  
hs = []                      std = 1/sqrt(Din)  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = np.random.randn(Din, Dout) / np.sqrt(Din)  
    x = np.tanh(x.dot(W))  
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```

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For conv layers, Din is $\text{filter_size}^2 * \text{input_channels}$

Let: $y = x_1w_1 + x_2w_2 + \dots + x_{Din}w_{Din}$

Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7           "Xavier" initialization:  
hs = []                      std = 1/sqrt(Din)  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = np.random.randn(Din, Dout) / np.sqrt(Din)  
    x = np.tanh(x.dot(W))  
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Let: $y = x_1w_1 + x_2w_2 + \dots + x_{Din}w_{Din}$

Assume: $\text{Var}(x_1) = \text{Var}(x_2) = \dots = \text{Var}(x_{Din})$

Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7          "Xavier" initialization:  
hs = []                      std = 1/sqrt(Din)  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = np.random.randn(Din, Dout) / np.sqrt(Din)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

“Just right”: Activations are nicely scaled for all layers!

For conv layers, Din is $\text{filter_size}^2 * \text{input_channels}$

Let: $y = x_1 w_1 + x_2 w_2 + \dots + x_{Din} w_{Din}$

Assume: $\text{Var}(x_1) = \text{Var}(x_2) = \dots = \text{Var}(x_{Din})$

We want: $\text{Var}(y) = \text{Var}(x_i)$

Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7          "Xavier" initialization:  
hs = []                      std = 1/sqrt(Din)  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = np.random.randn(Din, Dout) / np.sqrt(Din)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

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For conv layers, Din is $\text{filter_size}^2 * \text{input_channels}$

Let: $y = x_1w_1 + x_2w_2 + \dots + x_{Din}w_{Din}$

Assume: $\text{Var}(x_1) = \text{Var}(x_2) = \dots = \text{Var}(x_{Din})$

We want: $\text{Var}(y) = \text{Var}(x_i)$

$$\text{Var}(y) = \text{Var}(x_1w_1 + x_2w_2 + \dots + x_{Din}w_{Din})$$

[substituting value of y]

Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7          "Xavier" initialization:  
hs = []                      std = 1/sqrt(Din)  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = np.random.randn(Din, Dout) / np.sqrt(Din)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

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For conv layers, Din is $\text{filter_size}^2 * \text{input_channels}$

Let: $y = x_1w_1 + x_2w_2 + \dots + x_{Din}w_{Din}$

Assume: $\text{Var}(x_1) = \text{Var}(x_2) = \dots = \text{Var}(x_{Din})$

We want: $\text{Var}(y) = \text{Var}(x_i)$

$$\begin{aligned}\text{Var}(y) &= \text{Var}(x_1w_1 + x_2w_2 + \dots + x_{Din}w_{Din}) \\ &= \sum \text{Var}(x_i w_i) = \text{Din} \text{Var}(x_i w_i)\end{aligned}$$

[Assume all x_i, w_i are iid] $\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2$

Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7          "Xavier" initialization:  
hs = []                      std = 1/sqrt(Din)  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = np.random.randn(Din, Dout) / np.sqrt(Din)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

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For conv layers, Din is $\text{filter_size}^2 * \text{input_channels}$

Let: $y = x_1w_1 + x_2w_2 + \dots + x_{Din}w_{Din}$

Assume: $\text{Var}(x_1) = \text{Var}(x_2) = \dots = \text{Var}(x_{Din})$

We want: $\text{Var}(y) = \text{Var}(x_i)$

$$\begin{aligned}\text{Var}(y) &= \text{Var}(x_1w_1 + x_2w_2 + \dots + x_{Din}w_{Din}) \\ &= \text{Din} \text{Var}(x_i w_i) \\ &= \text{Din} \text{Var}(x_i) \text{Var}(w_i)\end{aligned}$$

[Assume all x_i, w_i are zero mean]

$$\begin{aligned}\text{Var}(XY) &= E(X^2Y^2) - (E(XY))^2 = \text{Var}(X)\text{Var}(Y) + \cancel{\text{Var}(X)(E(Y))^2} \\ &\quad + \cancel{\text{Var}(Y)(E(X))^2}\end{aligned}$$

Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7          "Xavier" initialization:  
hs = []                      std = 1/sqrt(Din)  
x = np.random.randn(16, dims[0])  
for Din, Dout in zip(dims[:-1], dims[1:]):  
    W = np.random.randn(Din, Dout) / np.sqrt(Din)  
    x = np.tanh(x.dot(W))  
    hs.append(x)
```

“Just right”: Activations are nicely scaled for all layers!

For conv layers, Din is $\text{filter_size}^2 * \text{input_channels}$

Let: $y = x_1w_1 + x_2w_2 + \dots + x_{Din}w_{Din}$

Assume: $\text{Var}(x_1) = \text{Var}(x_2) = \dots = \text{Var}(x_{Din})$

We want: $\text{Var}(y) = \text{Var}(x_i)$

$$\begin{aligned}\text{Var}(y) &= \text{Var}(x_1w_1 + x_2w_2 + \dots + x_{Din}w_{Din}) \\ &= \text{Din} \text{Var}(x_i w_i) \\ &= \text{Din} \text{Var}(x_i) \text{Var}(w_i)\end{aligned}$$

[Assume all x_i, w_i are iid]

So, $\text{Var}(y) = \text{Var}(x_i)$ only when $\text{Var}(w_i) = 1/\text{Din}$

Glorot and Bengio, “Understanding the difficulty of training deep feedforward neural networks”, AISTAT 2010

Weight Initialization: What about ReLU?

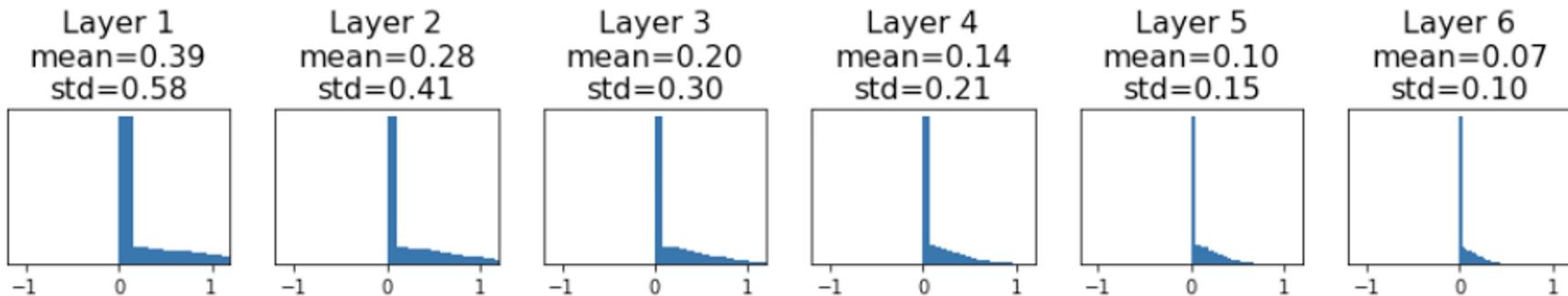
```
dims = [4096] * 7      Change from tanh to ReLU
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

Weight Initialization: What about ReLU?

```
dims = [4096] * 7      Change from tanh to ReLU
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

Xavier assumes zero centered activation function

Activations collapse to zero again, no learning =(



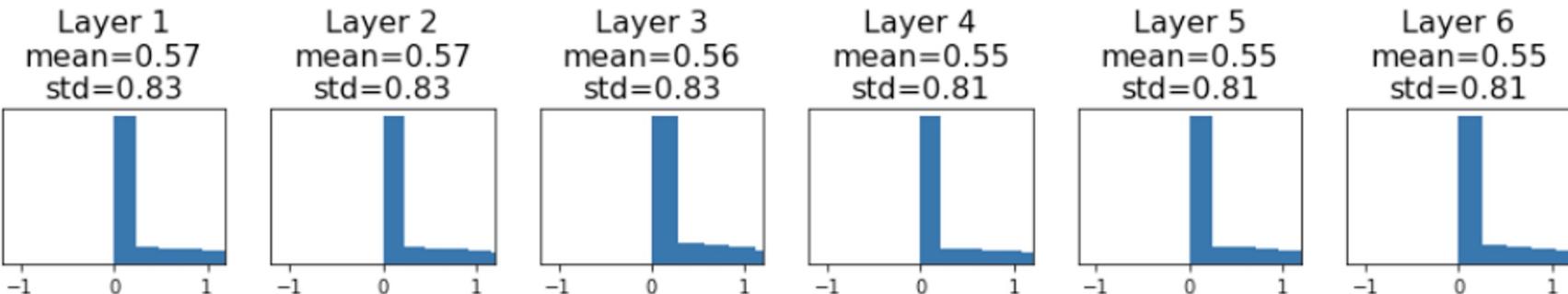
Visualize distribution of activations

Weight Initialization: Kaiming / MSRA Initialization

```
dims = [4096] * 7    ReLU correction: std = sqrt(2 / Din)
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) * np.sqrt(2/Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

Issue: Half of the activation get killed.

Solution: make the non-zero output variance twice as large as input



He et al, "Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification", ICCV 2015

Visualize distribution of activations

Proper initialization is an active area of research...

Understanding the difficulty of training deep feedforward neural networks

by Glorot and Bengio, 2010

Exact solutions to the nonlinear dynamics of learning in deep linear neural networks by Saxe et al, 2013

Random walk initialization for training very deep feedforward networks by Sussillo and Abbott, 2014

Delving deep into rectifiers: Surpassing human-level performance on ImageNet classification by He et al., 2015

Data-dependent Initializations of Convolutional Neural Networks by Krähenbühl et al., 2015

All you need is a good init, Mishkin and Matas, 2015

Fixup Initialization: Residual Learning Without Normalization, Zhang et al, 2019

The Lottery Ticket Hypothesis: Finding Sparse, Trainable Neural Networks, Frankle and Carbin, 2019

Summary

Training Deep Neural Networks

- Details of the non-linear activation functions
 - Sigmoid, Tanh, ReLU, LeakyRELU, ELU, SELU
- Data normalization
 - Zero-centering, decorrelation, image normalization
- Weight Initialization
 - Constant init, random init, Xavier Init, Kaiming Init

Next time:

Training Deep Neural Networks

- Details of the non-linear activation functions
- Data normalization
- Weight Initialization
- Batch Normalization
- Advanced Optimization
- Regularization
- Data Augmentation
- Transfer learning
- Hyperparameter Tuning
- Model Ensemble