

Convolutional Neural Networks

Danna Gurari

University of Colorado Boulder
Fall 2022



<https://home.cs.colorado.edu/~DrG/Courses/NeuralNetworksAndDeepLearning/AboutCourse.html>

Review

- Last class:
 - Universal approximation theorem
 - Selecting model capacity: avoid overfitting and underfitting
 - Selecting model hyperparameters
 - Learning efficiently: optimization methods
 - Programming tutorial
- Assignments (Canvas):
 - Lab assignment 1 due earlier today
 - Problem set 2 due next week
- Questions?

Today's Topics

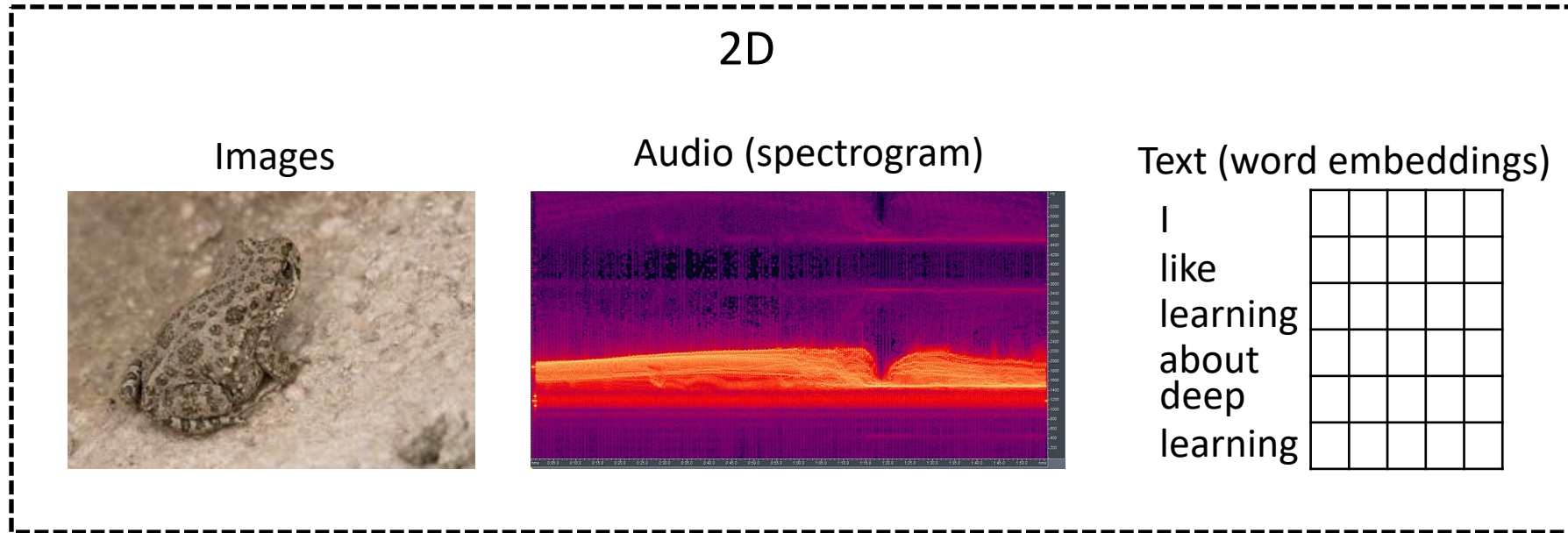
- Neural Networks for Spatial Data
- History of Convolutional Neural Networks (CNNs)
- CNNs – Convolutional Layers
- CNNs – Pooling Layers

Today's Topics

- Neural Networks for Spatial Data
- History of Convolutional Neural Networks (CNNs)
- CNNs – Convolutional Layers
- CNNs – Pooling Layers

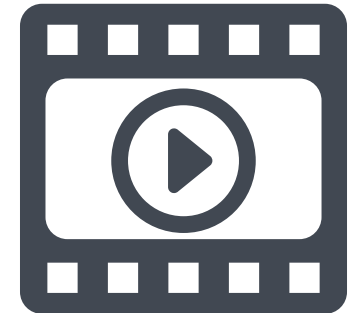
What is Spatial Data?

- Data where the order matters; e.g.,



3D

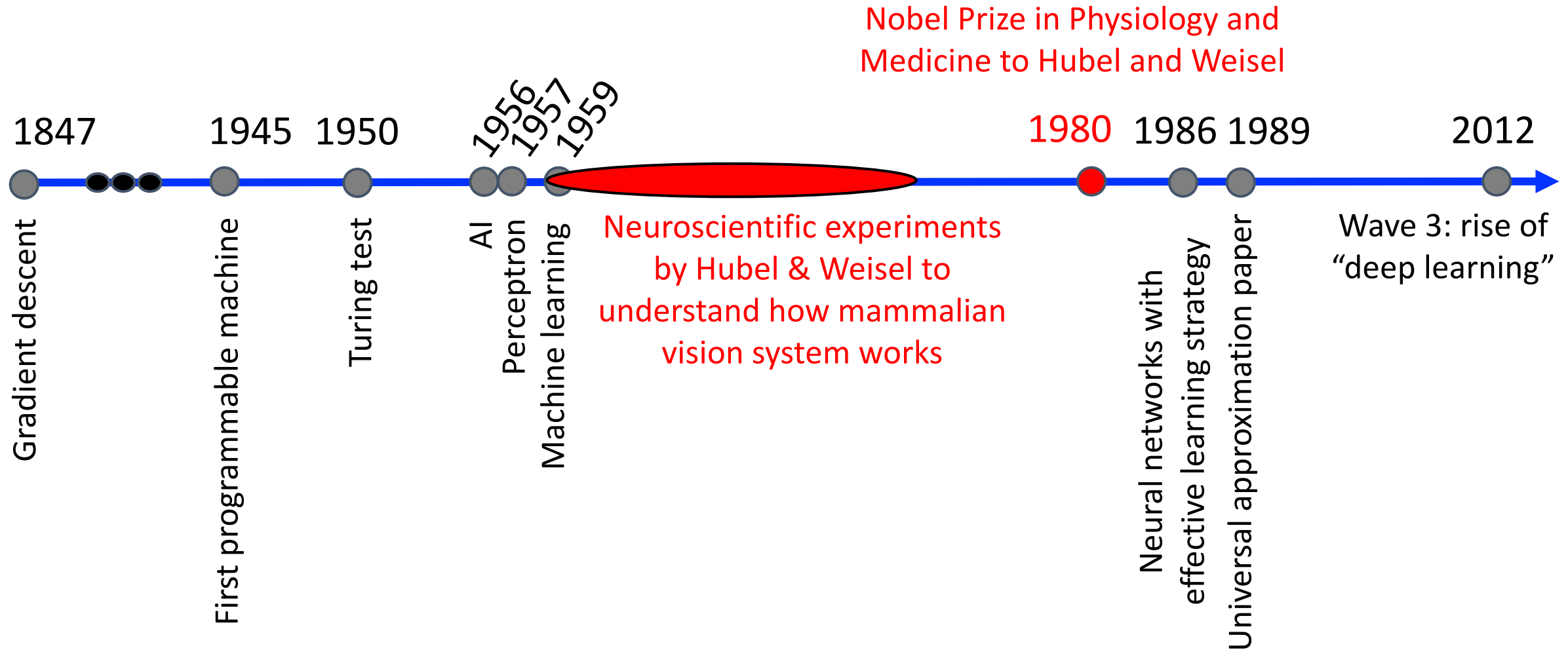
Video



Today's Topics

- Neural Networks for Spatial Data
- History of Convolutional Neural Networks (CNNs)
- CNNs – Convolutional Layers
- CNNs – Pooling Layers

Historical Context: Inspiration

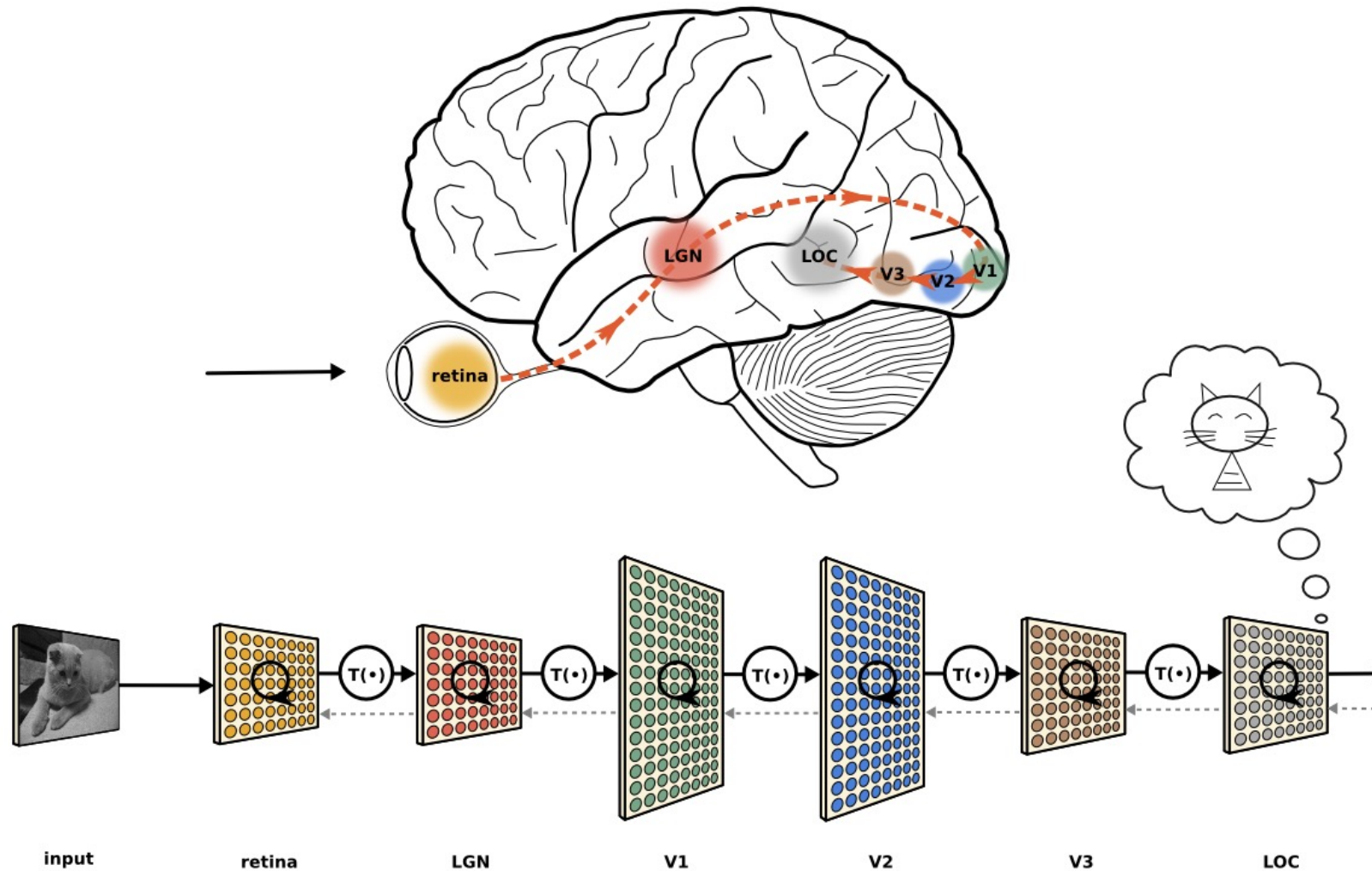


Motivation: How Vision System Works



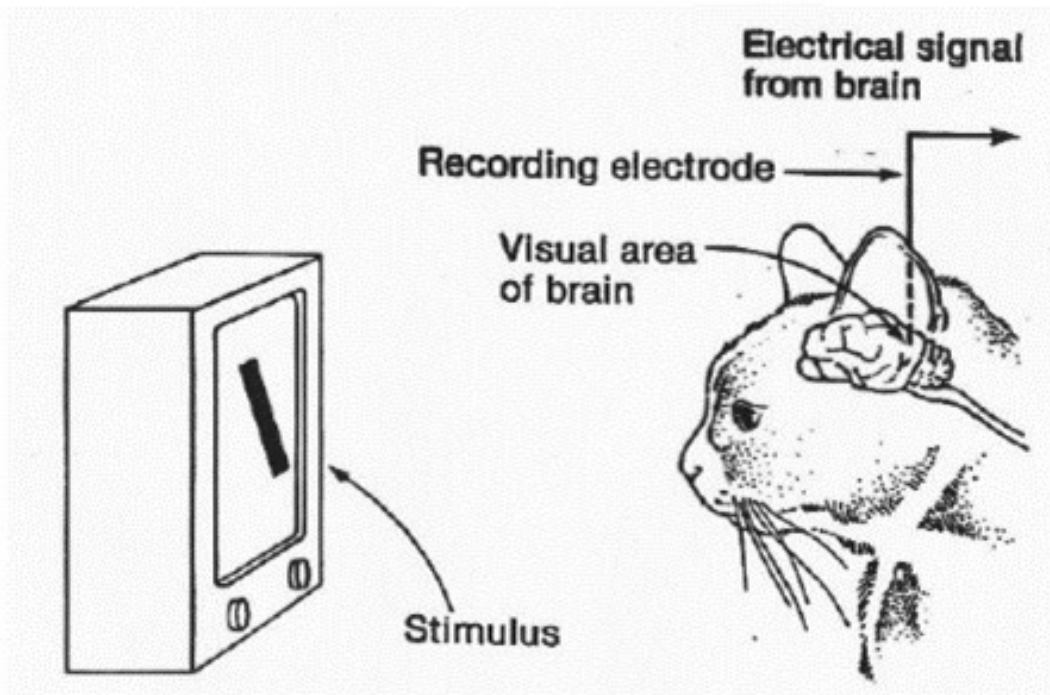
Image Source: <https://braintour.harvard.edu/archives/portfolio-items/hubel-and-wiesel>

Motivation: How Vision System Works



Motivation: How Vision System Works

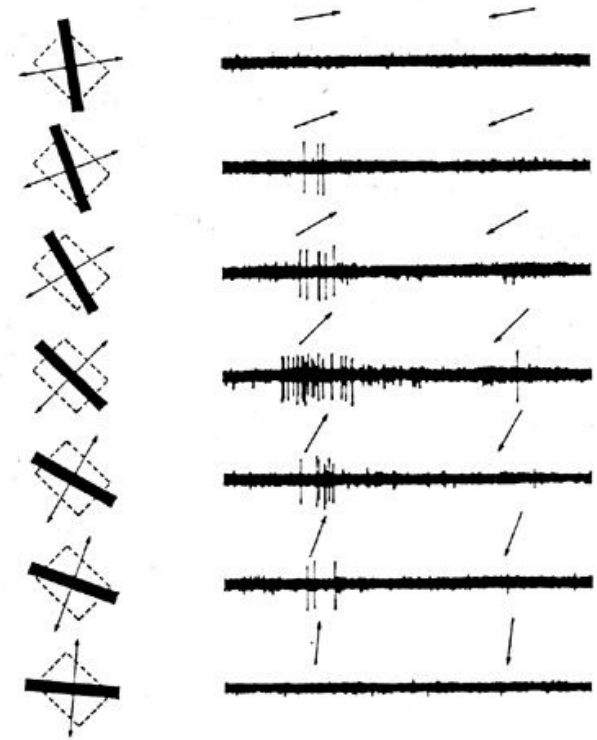
Experiment Set-up:



<https://www.esantus.com/blog/2019/1/31/convolutional-neural-networks-a-quick-guide-for-newbies>

Key Finding: initial neurons responded strongly only when light was shown in certain orientations

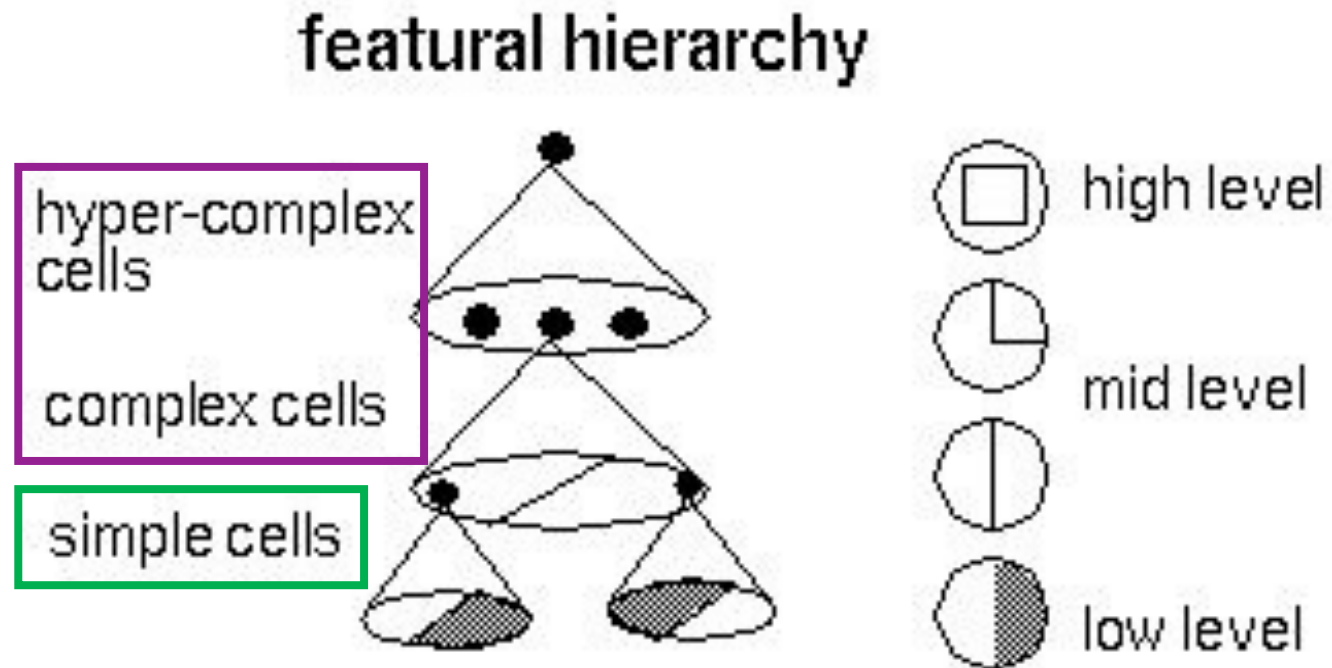
V1 physiology:
direction
selectivity



<https://www.cns.nyu.edu/~david/courses/perception/lecturenotes/V1/lgn-V1.html>

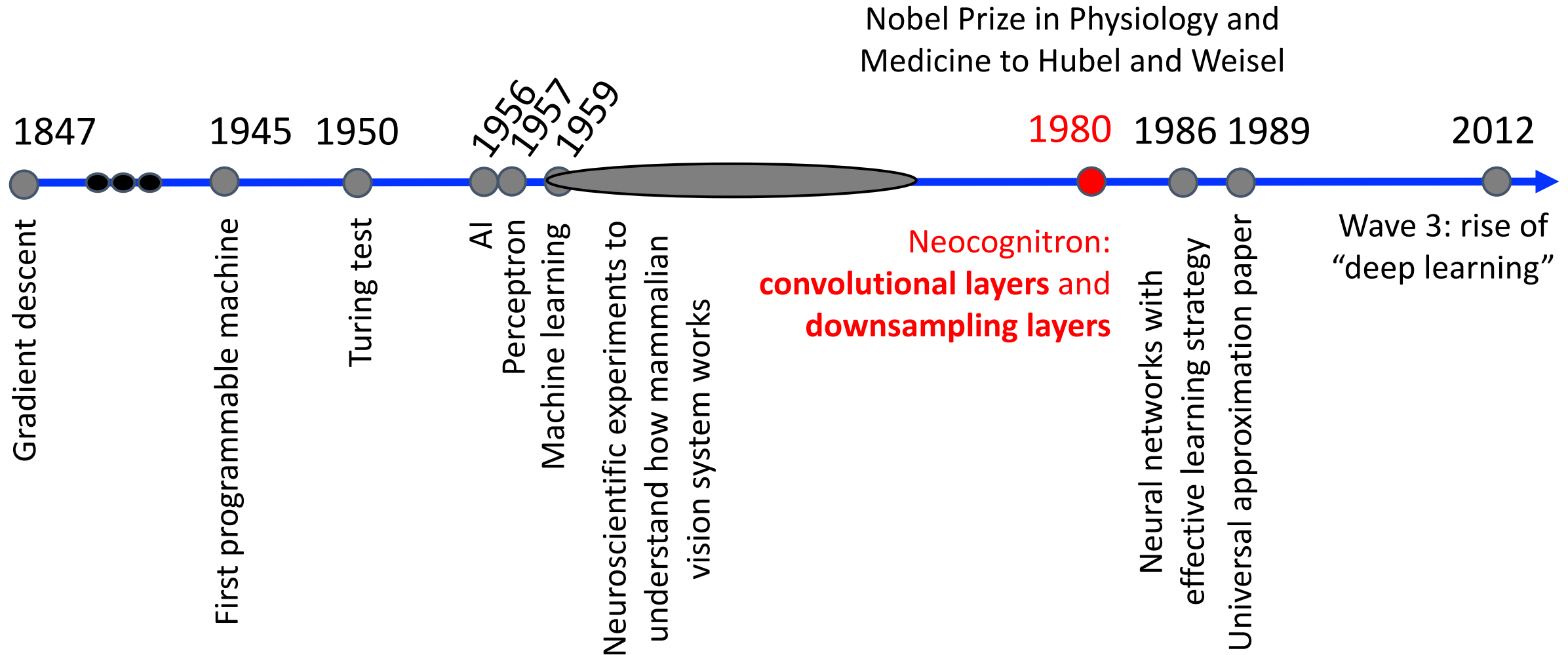
Motivation: How Vision System Works

Key Idea: cells are organized as a hierarchy of feature detectors, with **higher level features** responding to patterns of activation in **lower level cells**



Source: <https://bruceoutdoors.files.wordpress.com/2017/08/hubel.jpg>

Historical Context: Key Ingredients



Neocognitron: Key Ingredients



<http://personalpage.flsi.or.jp/fukushima/index-e.html>

“In this paper, we discuss how to synthesize a neural network model in order to endow it an ability of pattern recognition like a human being... the network acquires a similar structure to the hierarchy model of the visual nervous system proposed by Hubel and Wiesel.”

- Fukushima, Neocognitron: A Self-organizing Neural Network Model for a Mechanism of Pattern Recognition Unaffected by Shift in Position. Biological Cybernetics, 1980.

Neocognitron: Key Ingredients

Cascade of **simple** and **complex** cells:

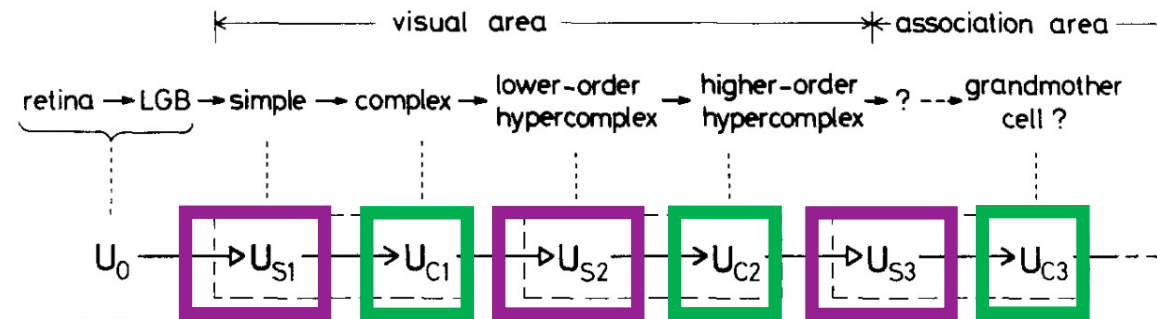


Fig. 1. Correspondence between the hierarchy model by Hubel and Wiesel, and the neural network of the neocognitron

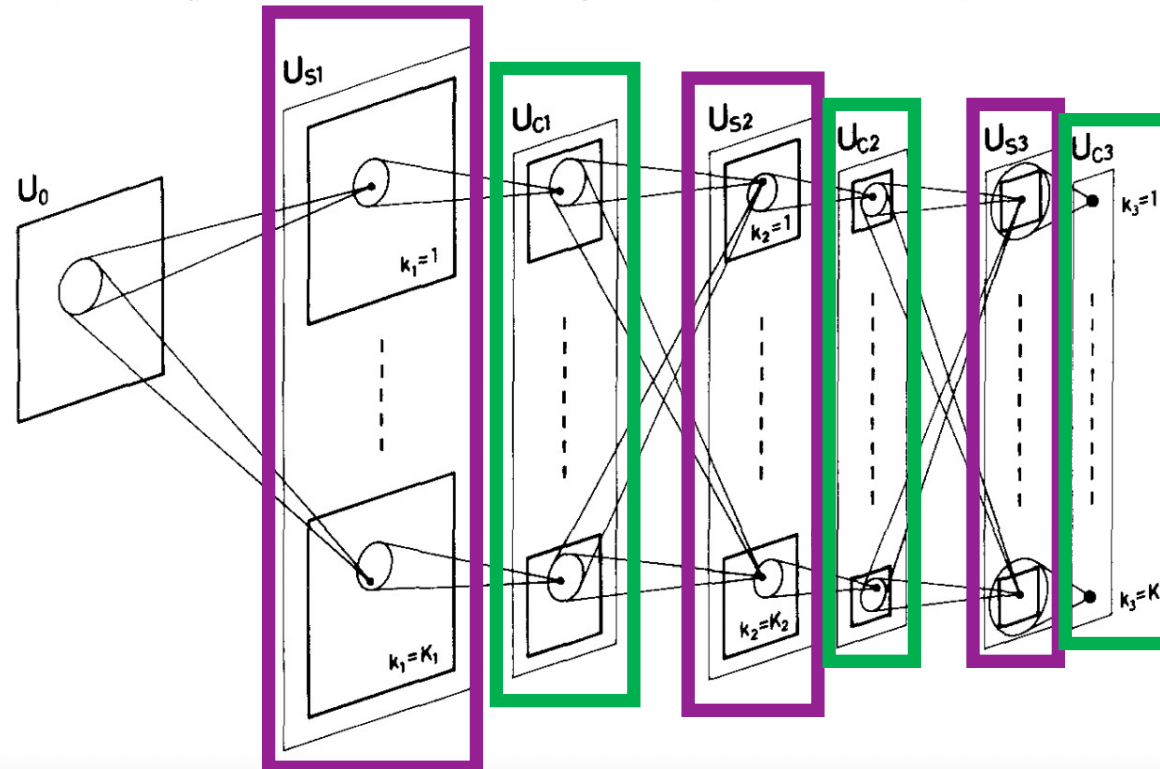


Fig. 2. Schematic diagram illustrating the interconnections between layers in the neocognitron
Fukushima, 1980.

Neocognitron: Key Ingredients

Simple cells extract local features using a sliding filter:

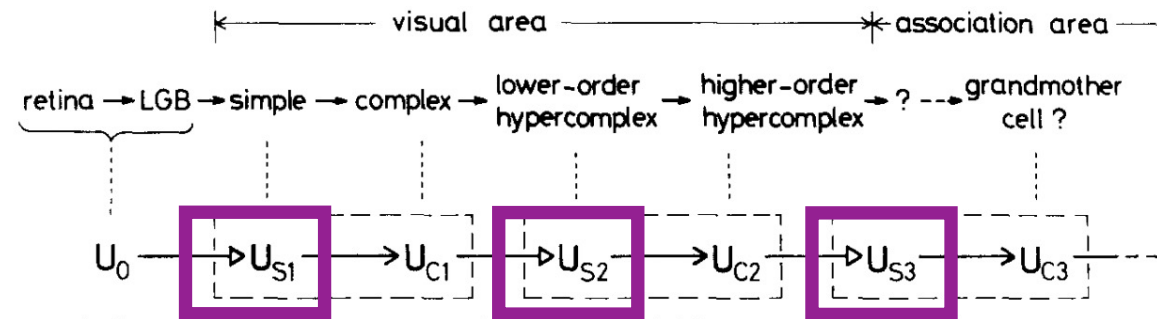
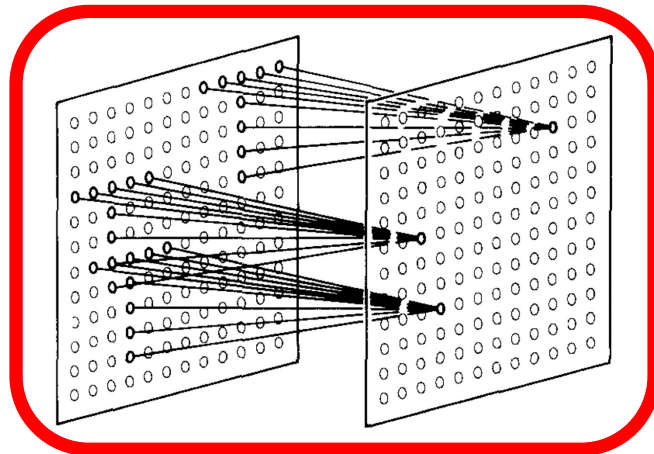


Fig. 1. Correspondence between the hierarchy model by Hubel and Wiesel, and the neural network of the neocognitron

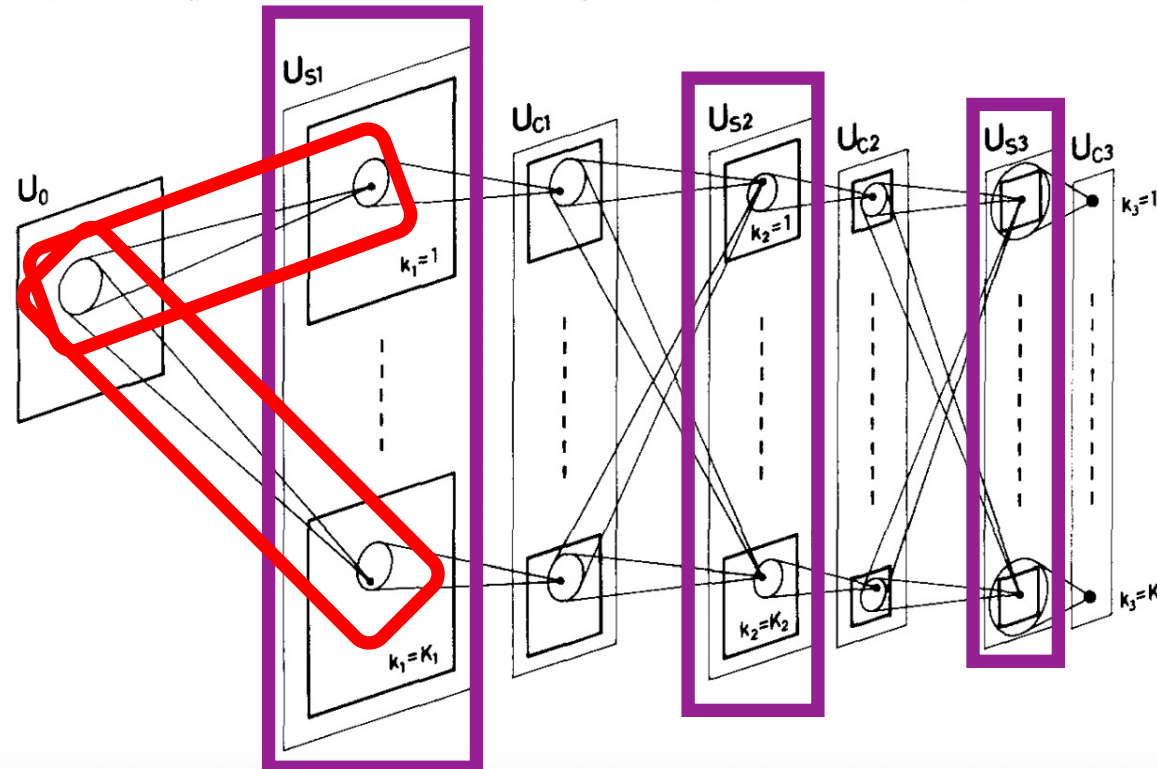


Fig. 2. Schematic diagram illustrating the interconnections between layers in the neocognitron
Fukushima, 1980.

Neocognitron: Key Ingredients

Complex cells fire when any part of the local region is the desired pattern

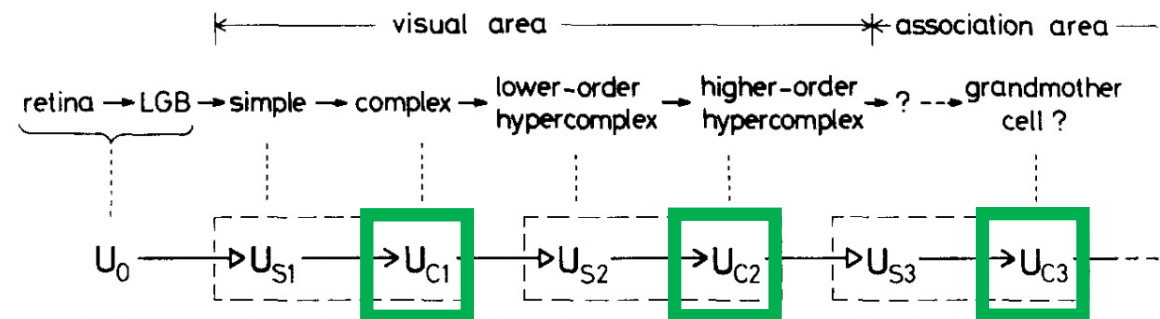


Fig. 1. Correspondence between the hierarchy model by Hubel and Wiesel, and the neural network of the neocognitron

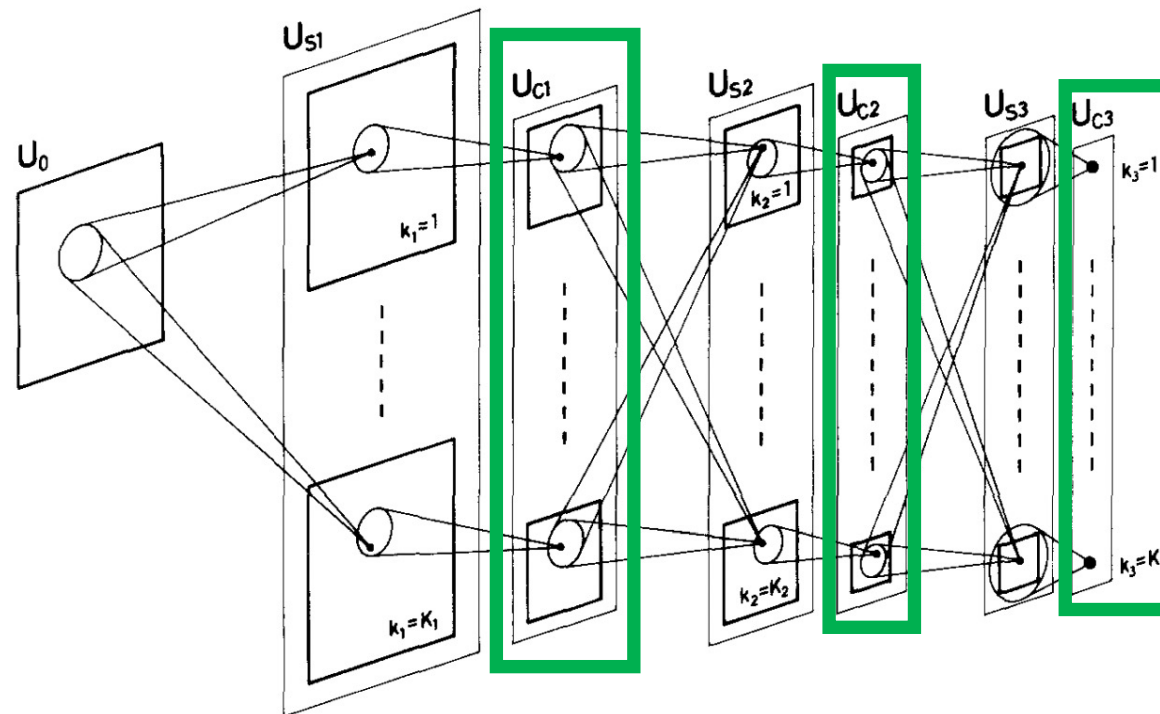


Fig. 2. Schematic diagram illustrating the interconnections between layers in the neocognitron
Fukushima, 1980.

Neocognitron: Key Ingredients

1. Convolutional layers

→ modifiable synapses

→ unmodifiable synapses

2. Pooling Layers

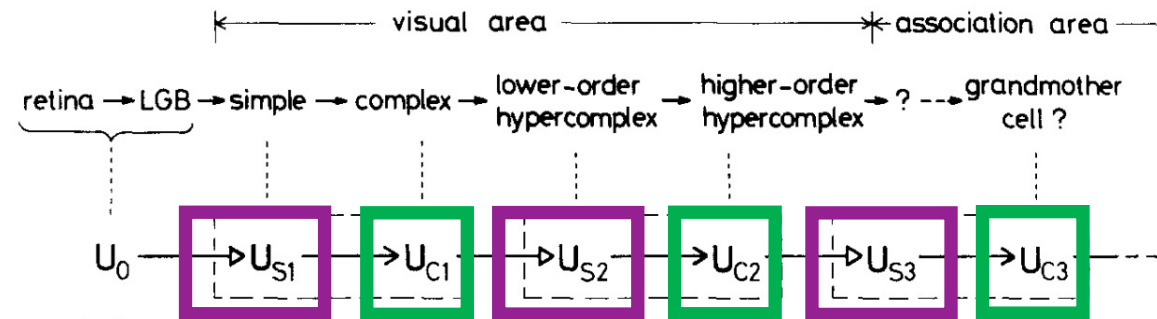


Fig. 1. Correspondence between the hierarchy model by Hubel and Wiesel, and the neural network of the neocognitron

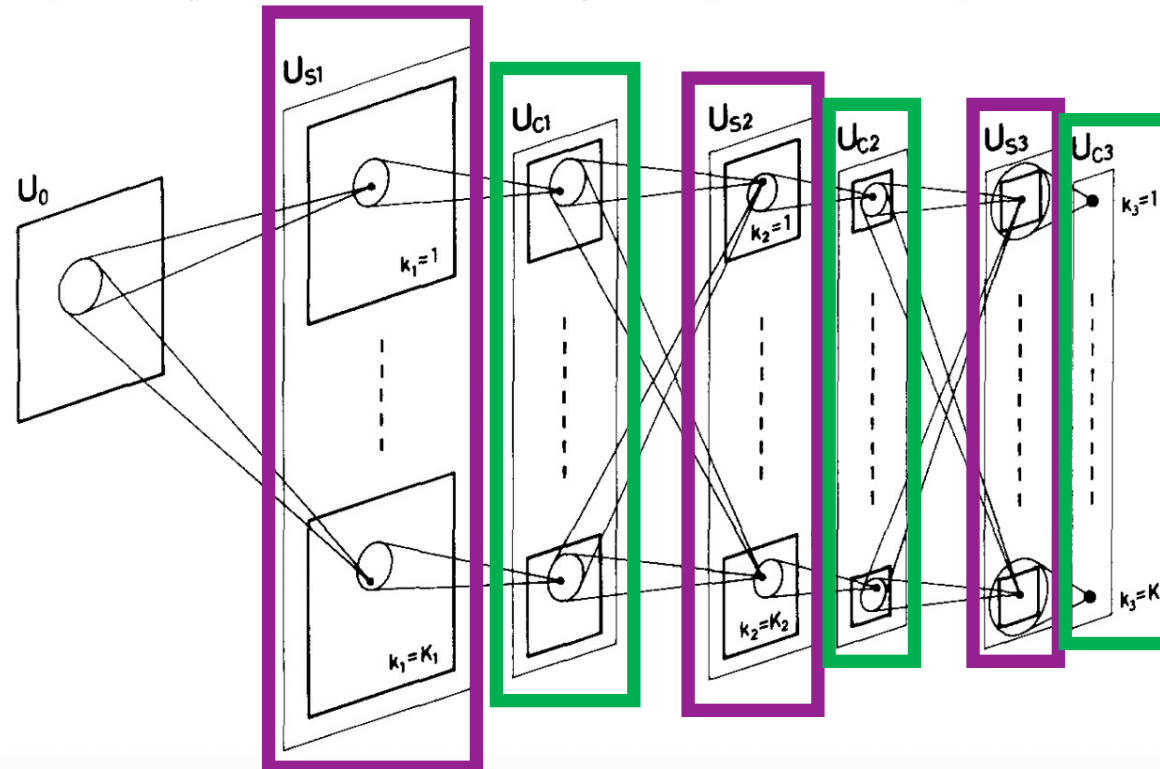
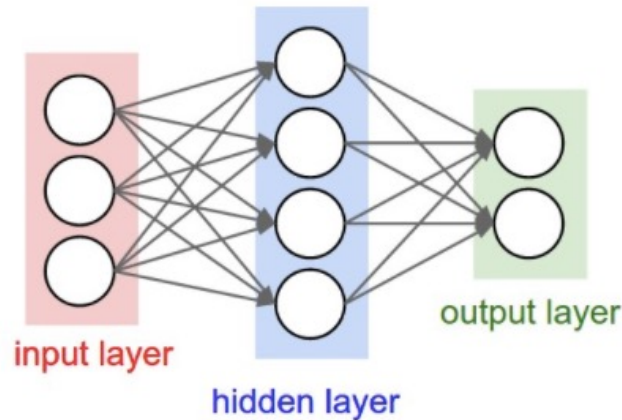


Fig. 2. Schematic diagram illustrating the interconnections between layers in the neocognitron
Fukushima, 1980.

Today's Topics

- Neural Networks for Spatial Data
- History of Convolutional Neural Networks (CNNs)
- **CNNs – Convolutional Layers**
- CNNs – Pooling Layers

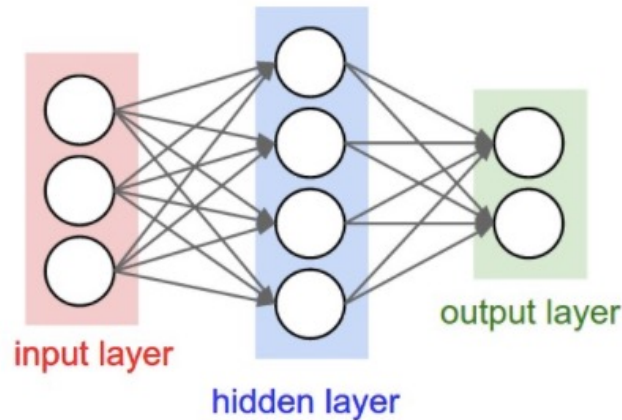
Motivation: Fully-Connected Layers Are Limited



Each node provides input to each node in the next layer

- Assume 2 layer model with 100 nodes per layer
 - e.g., how many weights are in a 640x480 grayscale image?
 - $640 \times 480 \times 100 + 100 \times 100 + 100 \times 1 = 30,730,100$
 - e.g., how many weights are in a 3.1 Megapixel grayscale image (2048X1536)?
 - $2048 \times 1536 \times 100 + 100 \times 100 + 100 \times 1 = 314,582,900$

Motivation: Fully-Connected Layers Are Limited



Issue: many model parameters
in fully connected networks

- Assume 2 layer model with 100 nodes per layer
 - e.g., how many weights are in a 640x480 grayscale image?
 - $640 \times 480 \times 100 + 100 \times 100 + 100 \times 1 = 30,730,100$
 - e.g., how many weights are in a 3.1 Megapixel grayscale image (2048X1536)?
 - $2048 \times 1536 \times 100 + 100 \times 100 + 100 \times 1 = 314,582,900$

Motivation: Fully-Connected Layers Are Limited

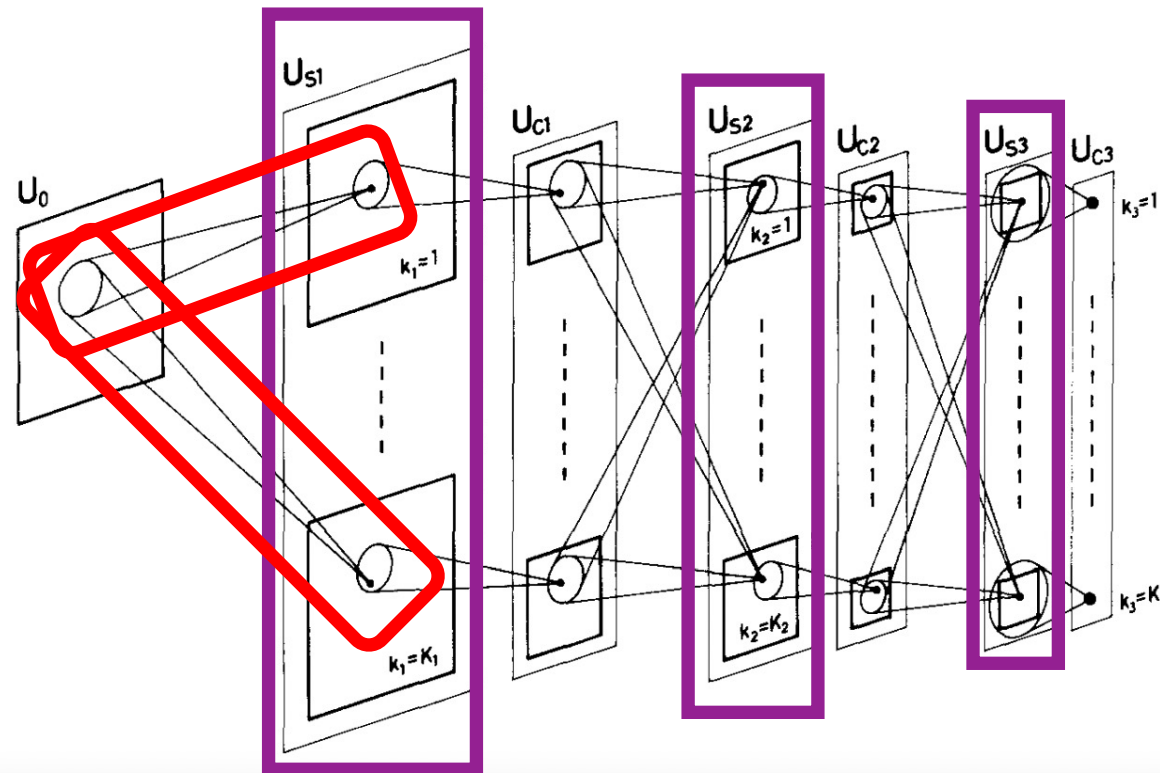
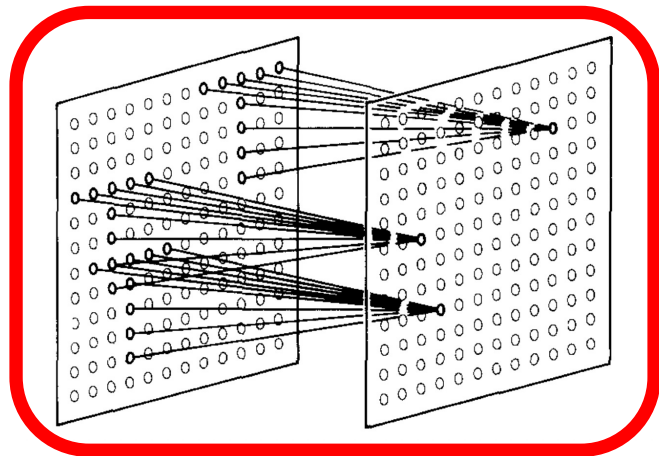
Many model parameters...

- increases chance of overfitting
- requires more training data
- increases training time

- Assume 2 layer model with 100 nodes per layer
 - e.g., how many weights are in a 640x480 grayscale image?
 - $640 \times 480 \times 100 + 100 \times 100 + 100 \times 1 = 30,730,100$
 - e.g., how many weights are in a 3.1 Megapixel grayscale image (2048X1536)?
 - $2048 \times 1536 \times 100 + 100 \times 100 + 100 \times 1 = 314,582,900$

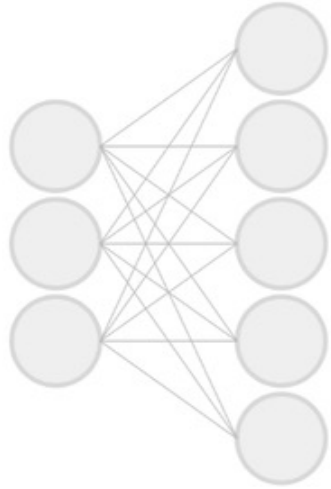
Convolutional Layer (Recall Neocognitron)

Idea: each node receives input only from a small neighborhood in previous layer and parameter sharing



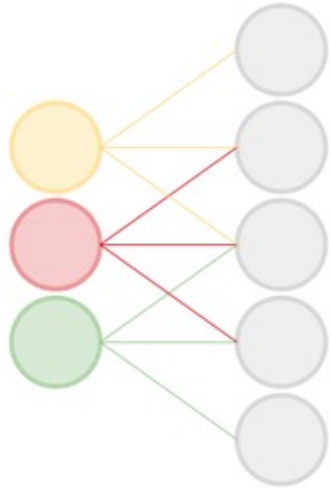
Fully-Connected vs Convolutional Layers

Fully-connected:



Rather than have each node provide input to each node in the next layer...

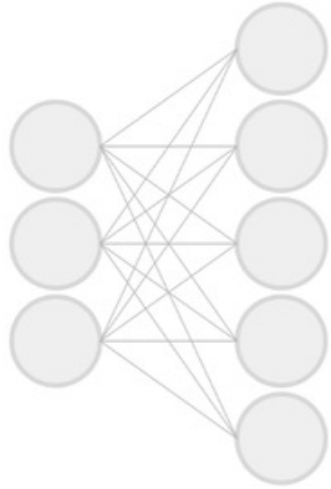
Convolutional:



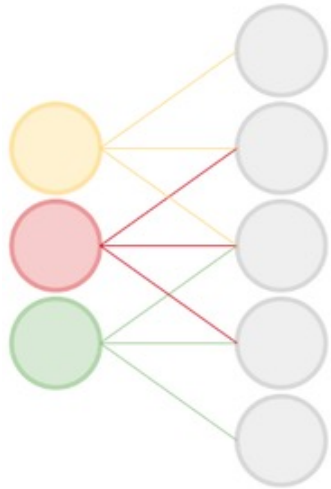
each node receives input only from a small neighborhood in previous layer (and there is parameter sharing)

Fully-Connected vs Convolutional Layers

Fully-connected:

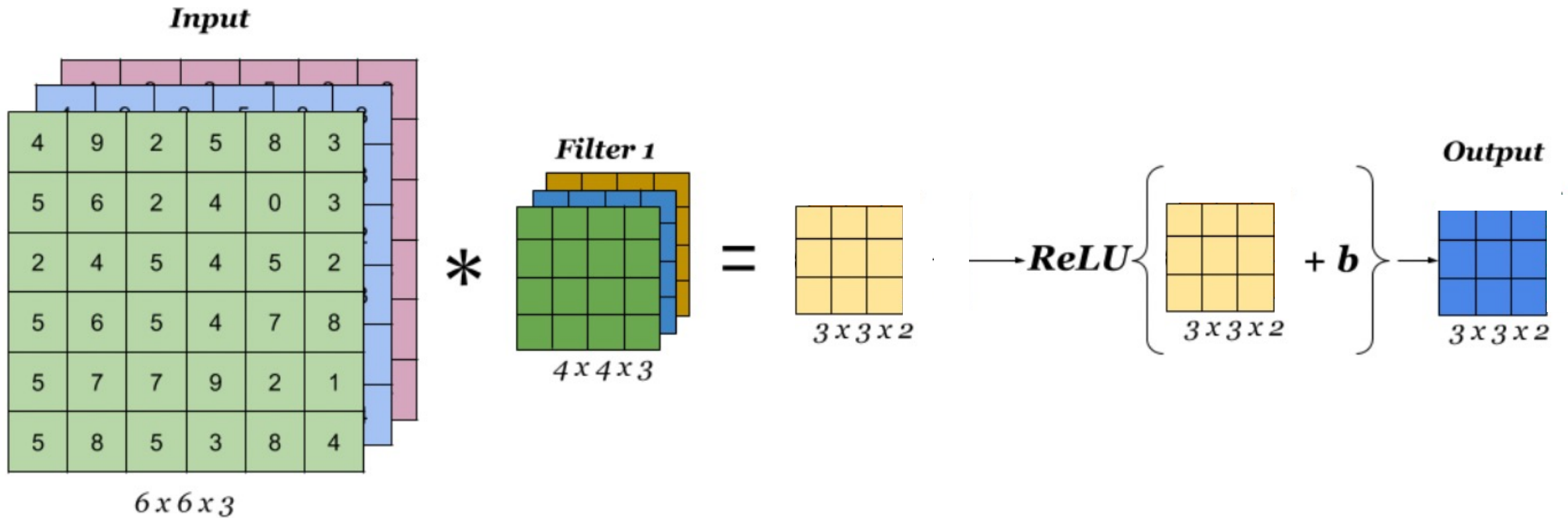


Convolutional:

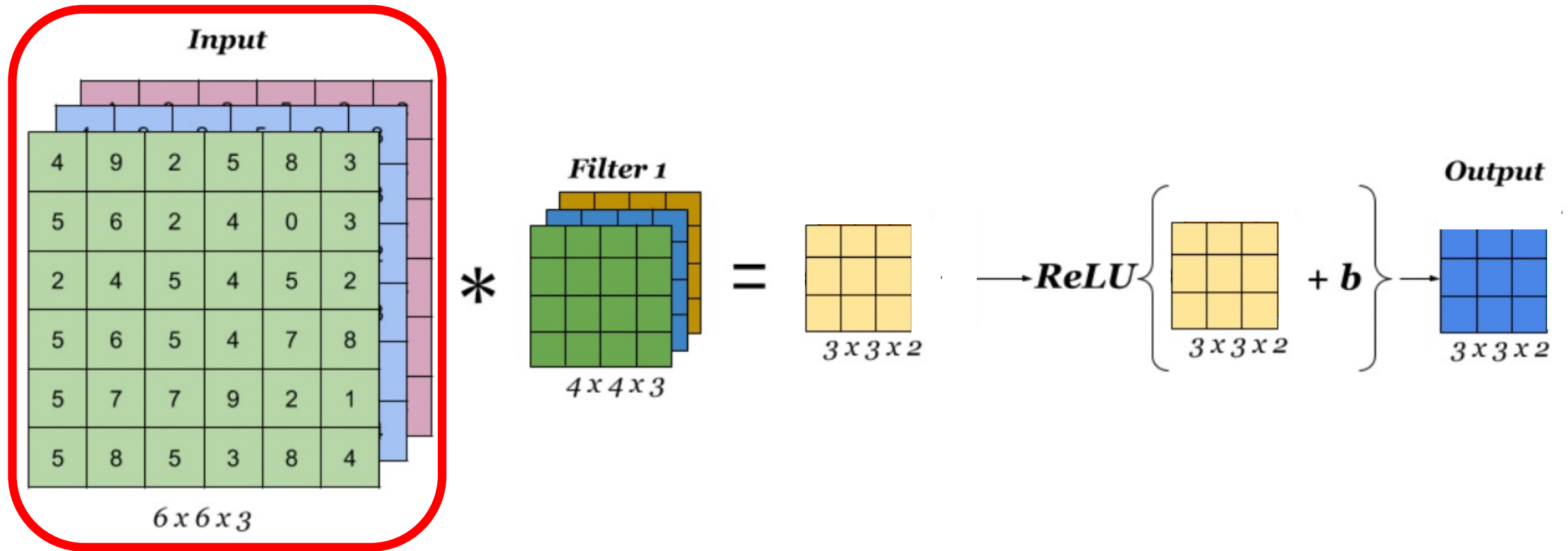


Convolutional layers dramatically
reduce number of model parameters!

Convolutional Layer Overview

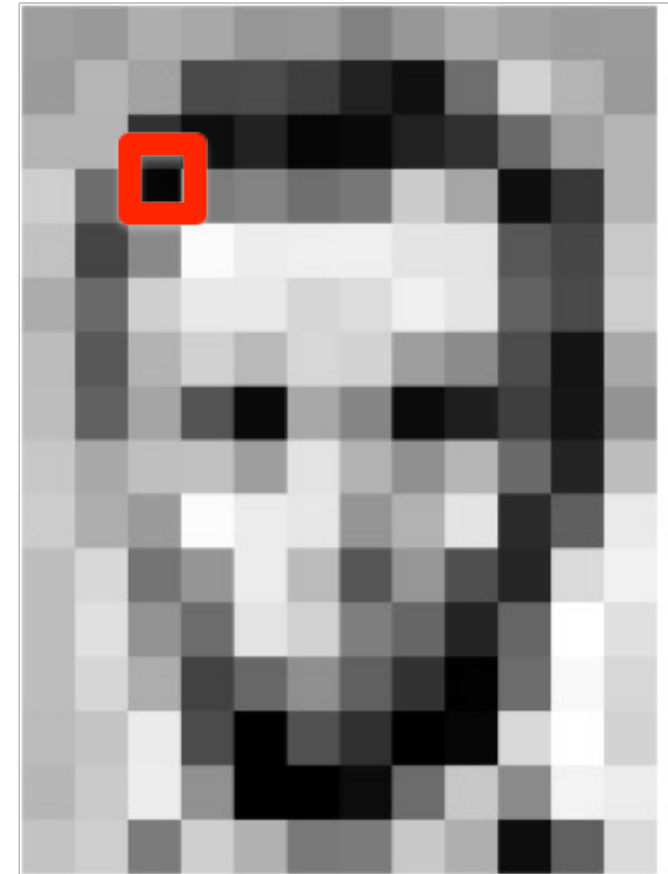


Convolutional Layer Overview

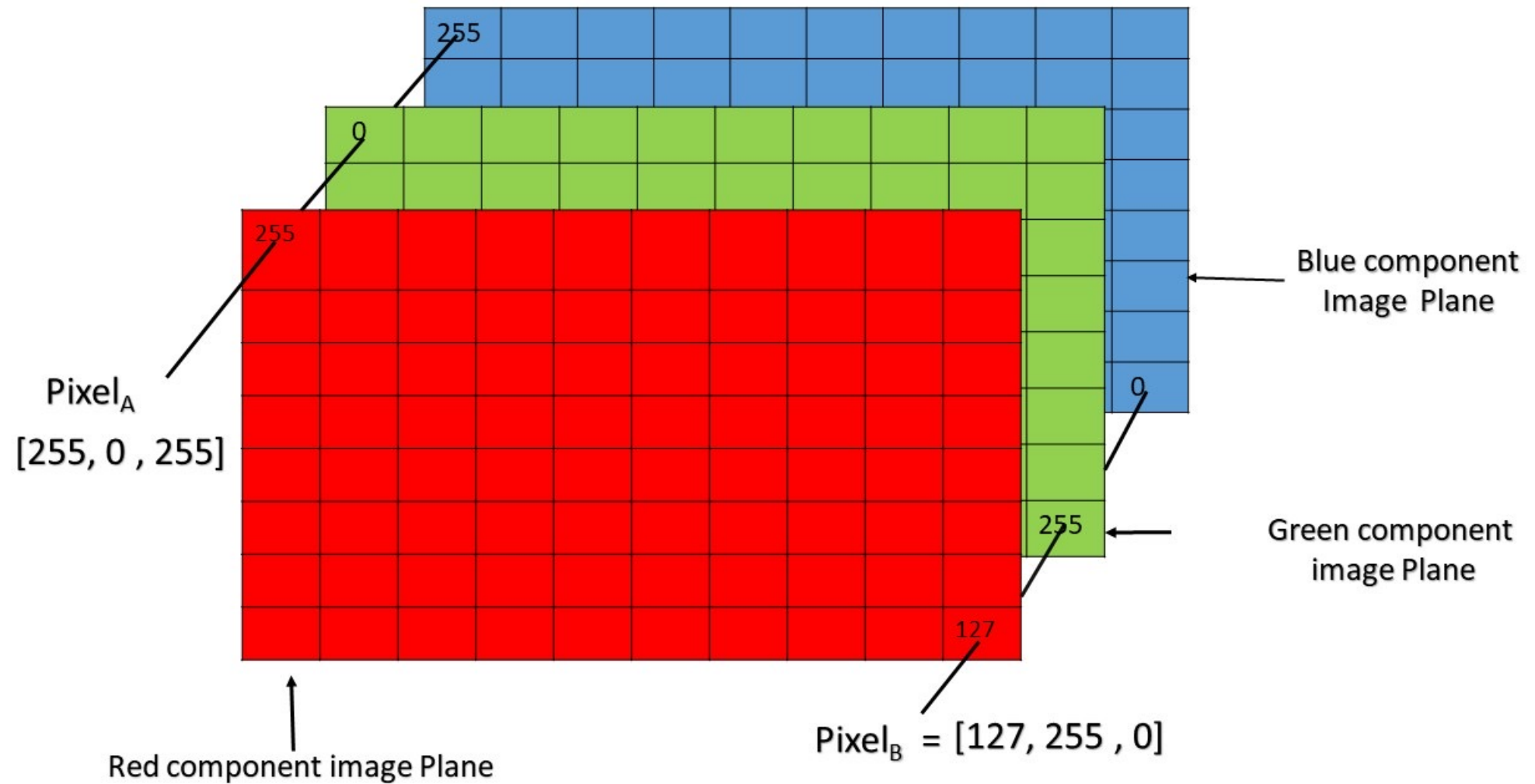


Recall: Image Representation (8-bit Grayscale)

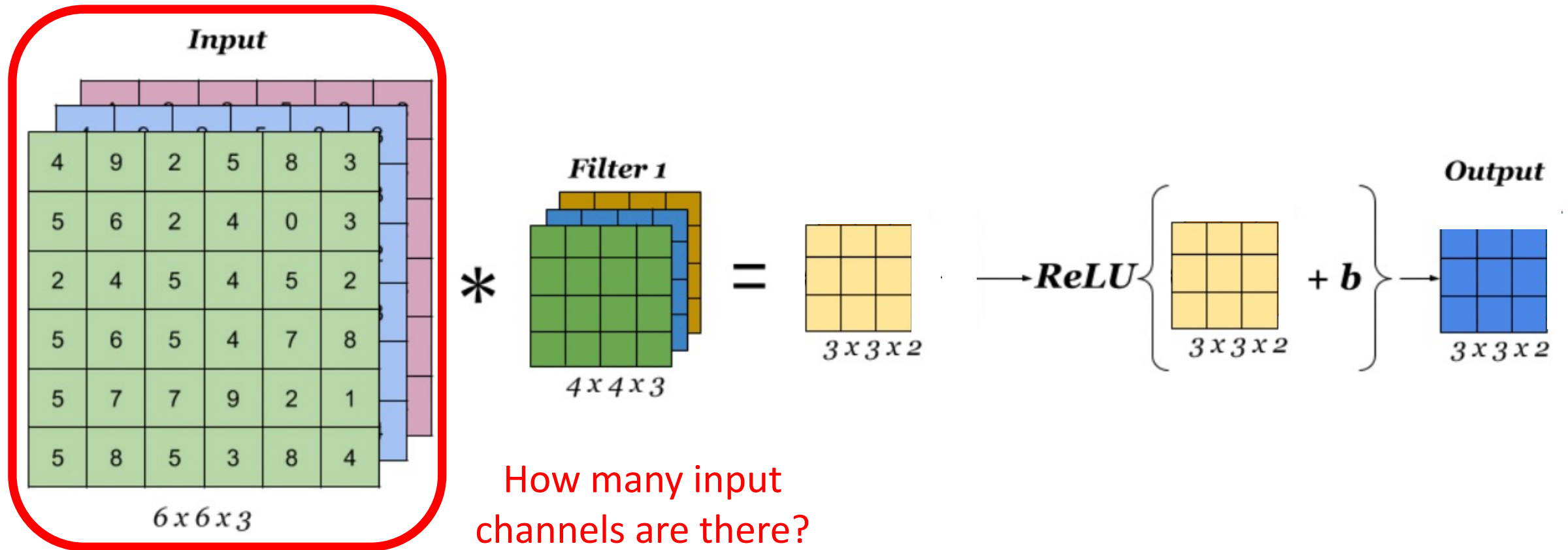
157	153	174	168	150	152	129	151	172	161	155	156
155	182	163	74	75	62	33	17	110	210	180	154
180	180	50	14	34	6	10	33	48	106	159	181
206	101	5	14	131	111	120	204	166	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	105	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	215	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	86	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	50	2	109	249	215
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218



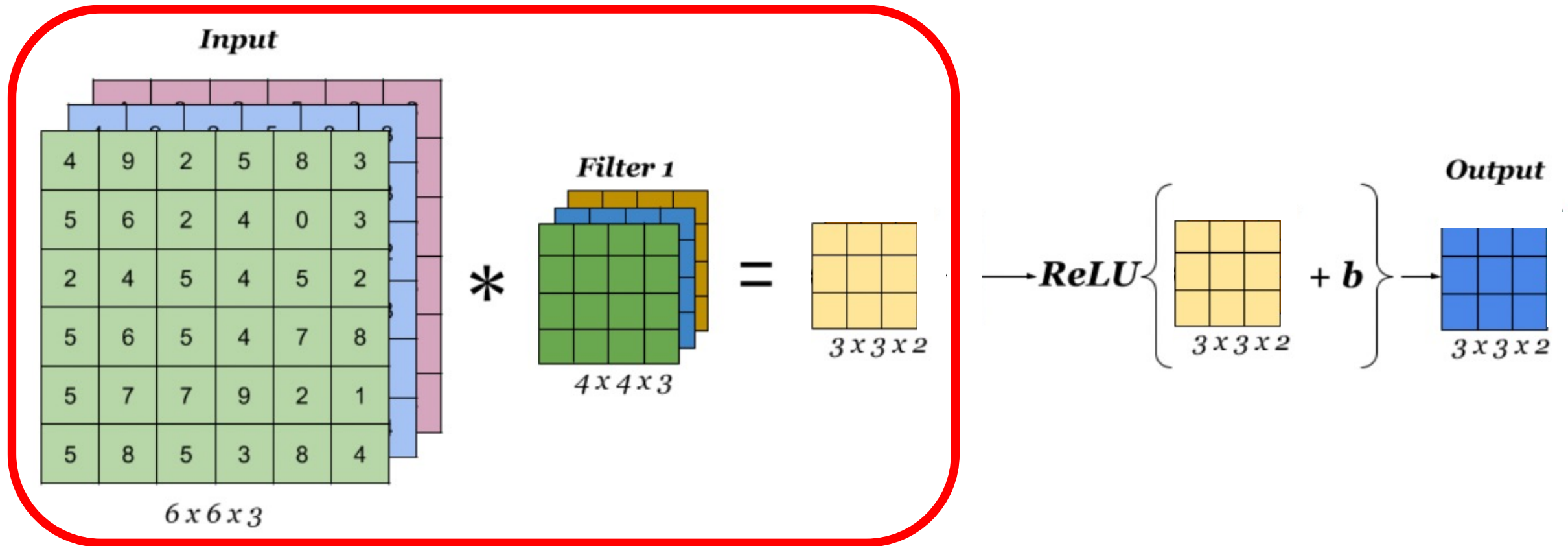
Color Images (e.g., 24-bit RGB image)



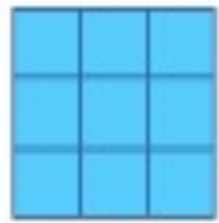
Convolutional Layer Overview



Convolutional Layer Overview



Convolution: Applies Linear Filter (e.g., 2D)



Input

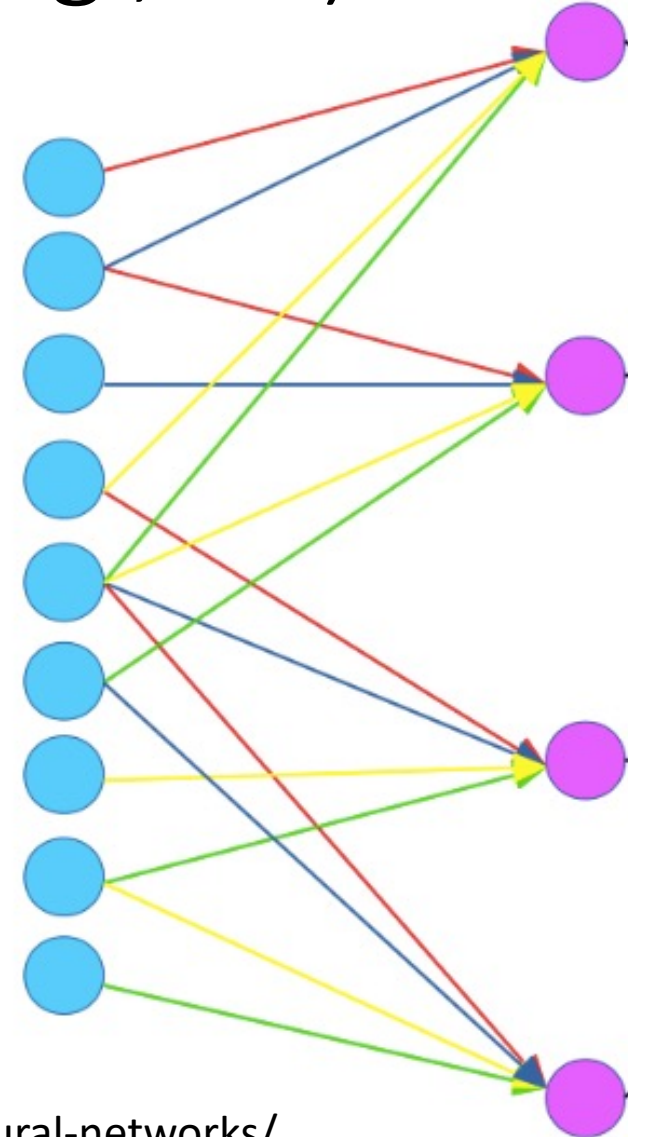


Filter
(aka – Kernel)



Feature
Map

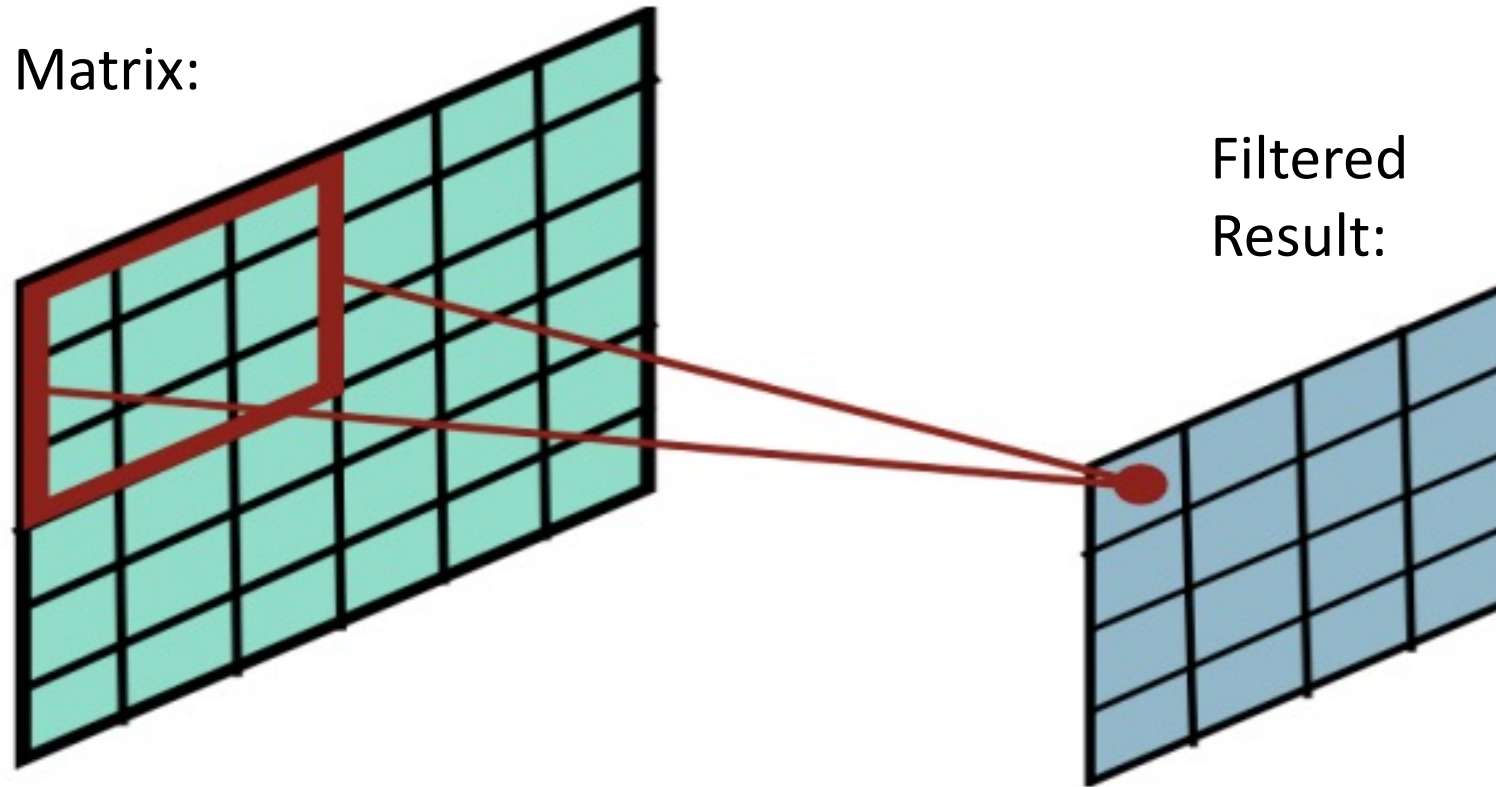
Way to Interpret
Neural Network



2D Filtering

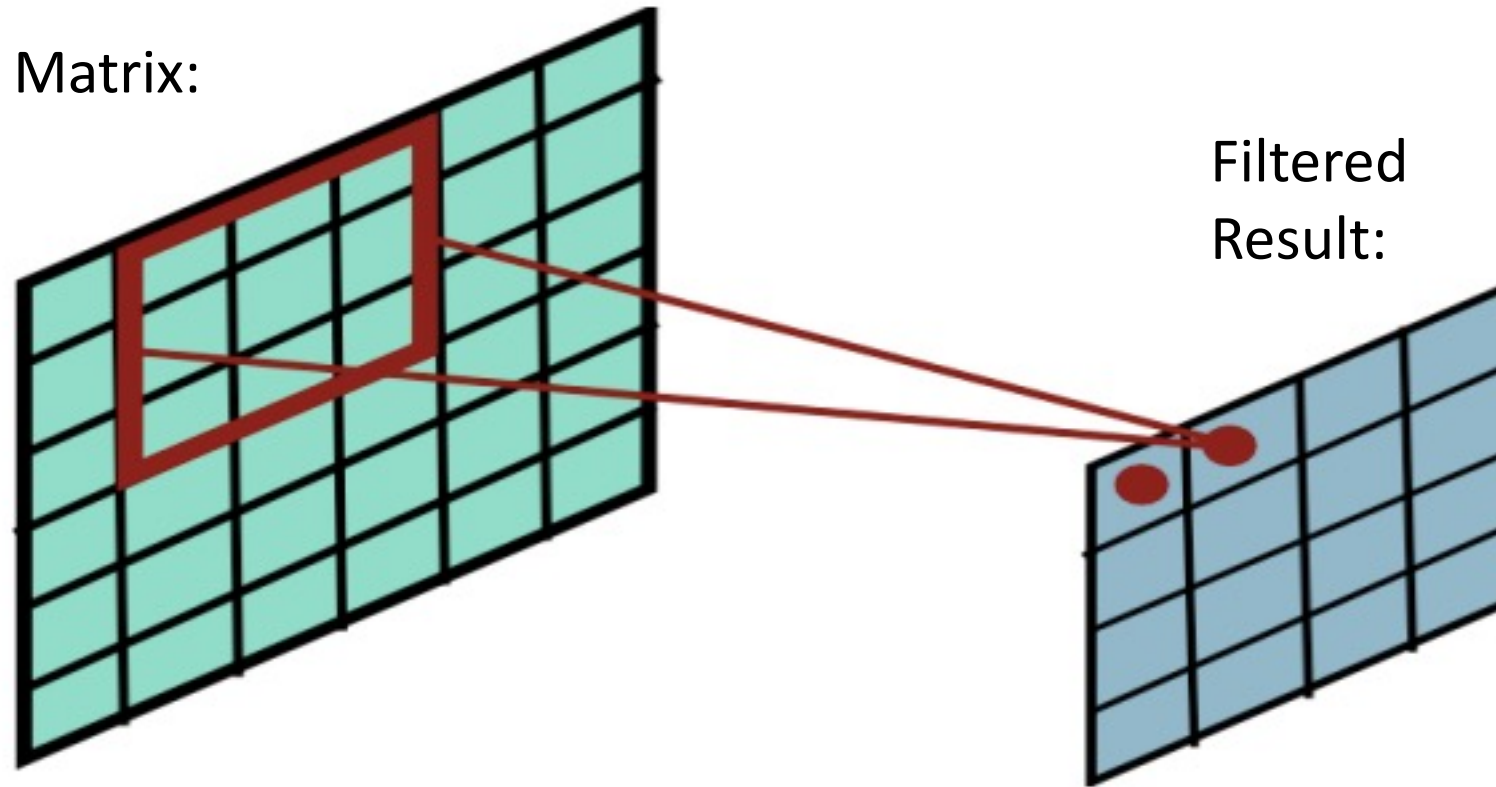
- Compute a **function of local neighborhood** for each location in matrix
- A **filter** specifies the function for how to combine neighbors' values

2D Filtering



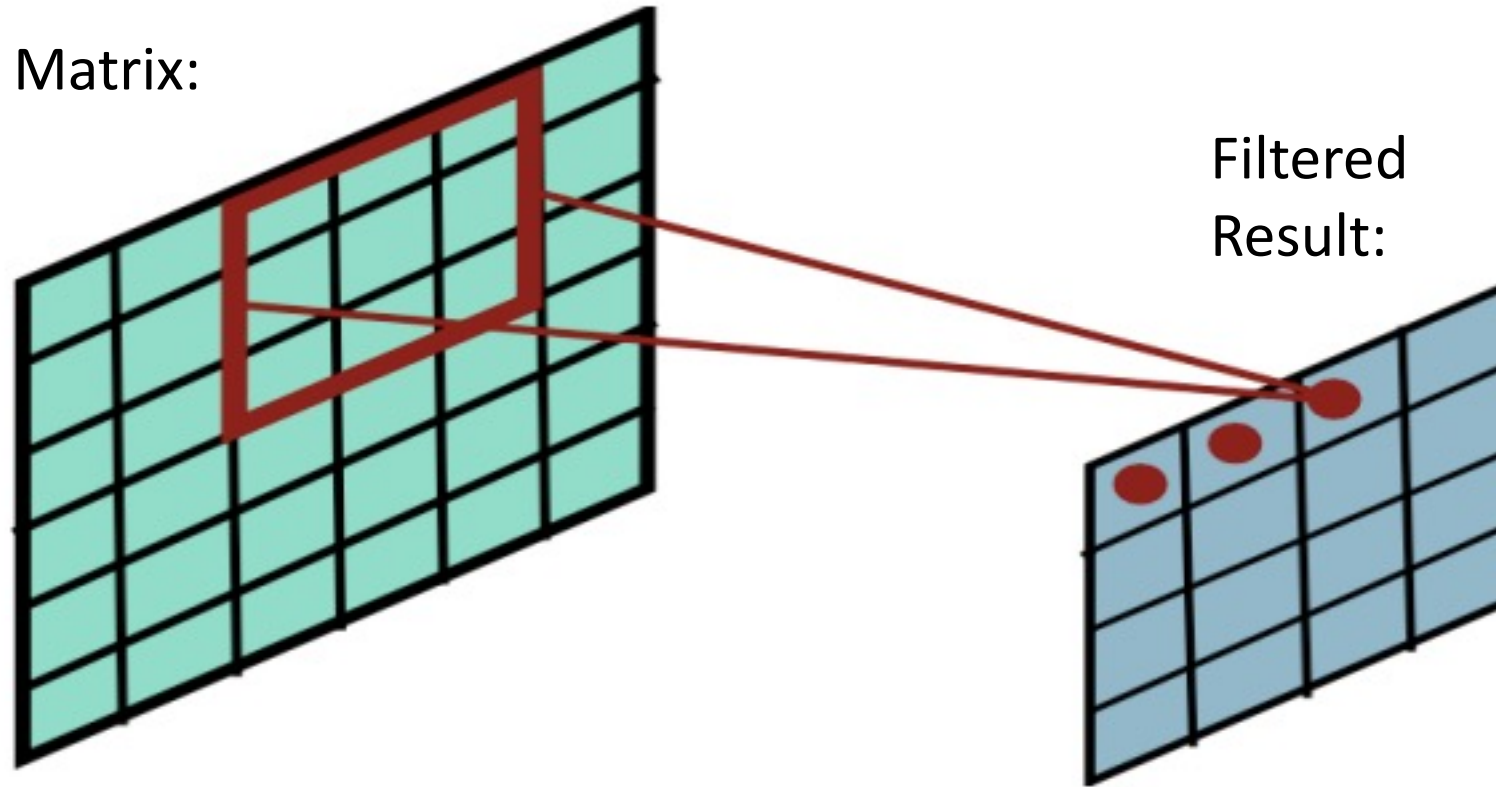
Slides filter over the matrix and computes dot products

2D Filtering



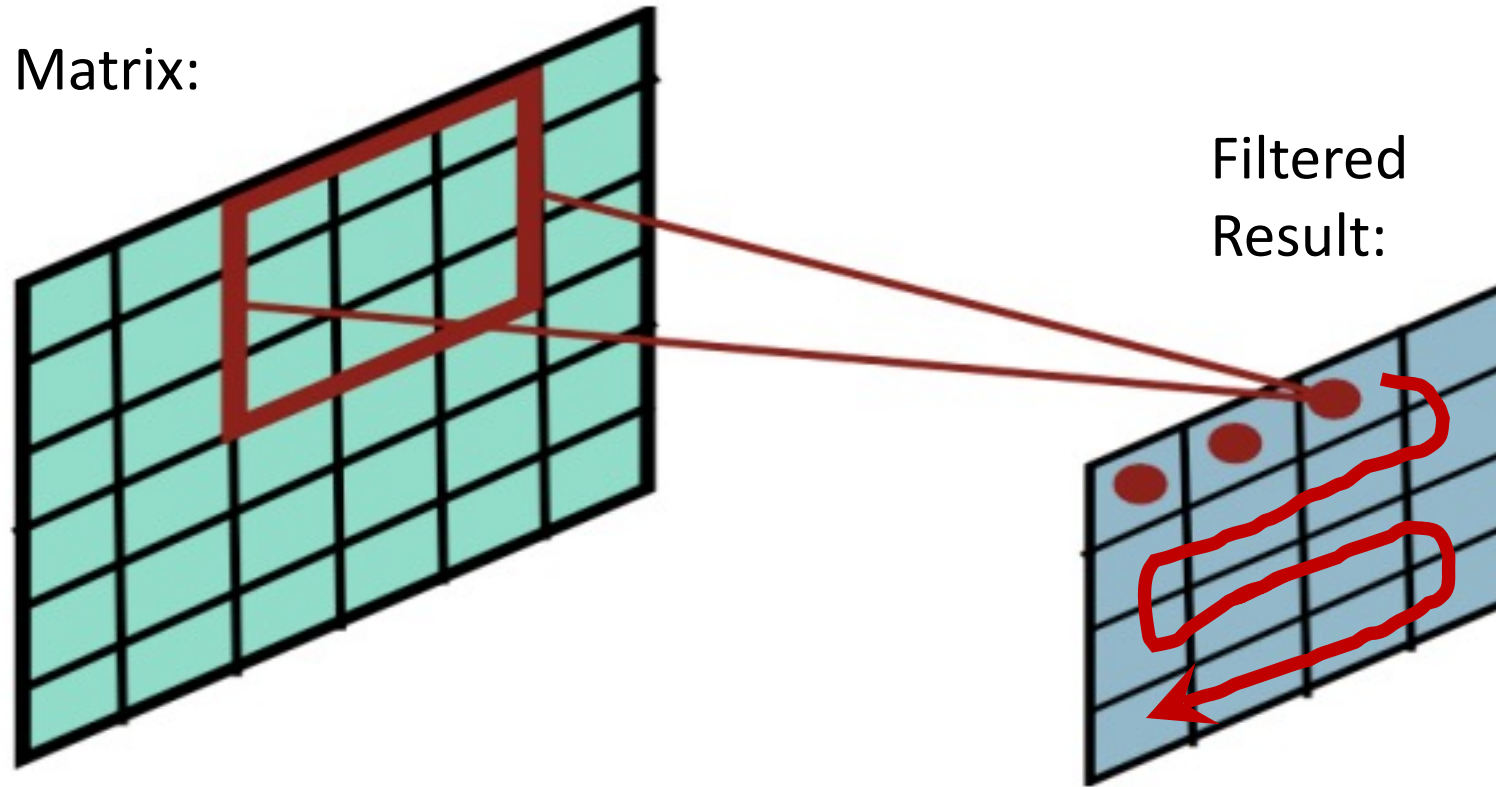
Slides filter over the matrix and computes dot products

2D Filtering



Slides filter over the matrix and computes dot products

2D Filtering



Slides filter over the matrix and computes dot products

2D Filtering: Toy Example

Input

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Filter

1	0	1
0	1	0
1	0	1

Feature Map

?	?	?
?	?	?
?	?	?

Dot Product = $1*1 + 1*0 + 1*1 + 0*0 + 1*1 + 1*0 + 0*1 + 0*1 + 0*0 + 0*0 + 1*1$

Dot Product = 4

2D Filtering: Toy Example

Input

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Filter

1	0	1
0	1	0
1	0	1

Feature Map

4	?	?
?	?	?
?	?	?

2D Filtering: Toy Example

Input

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Filter

1	0	1
0	1	0
1	0	1

Feature Map

4	3	?
?	?	?
?	?	?

2D Filtering: Toy Example

Input

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Filter

1	0	1
0	1	0
1	0	1

Feature Map

4	3	4
?	?	?
?	?	?

2D Filtering: Toy Example

Input

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Filter

1	0	1
0	1	0
1	0	1

Feature Map

4	3	4
2	?	?
?	?	?

2D Filtering: Toy Example

Input

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Filter

1	0	1
0	1	0
1	0	1

Feature Map

4	3	4
2	4	?
?	?	?

2D Filtering: Toy Example

Input

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Filter

1	0	1
0	1	0
1	0	1

Feature Map

4	3	4
2	4	3
?	?	?

2D Filtering: Toy Example

Input

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Filter

1	0	1
0	1	0
1	0	1

Feature Map

4	3	4
2	4	3
2	?	?

2D Filtering: Toy Example

Input

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Filter

1	0	1
0	1	0
1	0	1

Feature Map

4	3	4
2	4	3
2	3	?

2D Filtering: Toy Example

Input

1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Filter

1	0	1
0	1	0
1	0	1

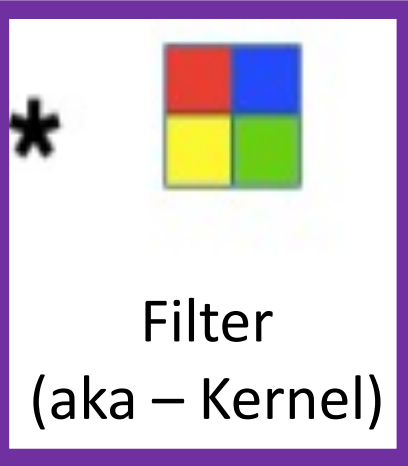
Feature Map

4	3	4
2	4	3
2	3	4

Convolutional Layer: Parameters to Learn

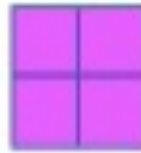


Input



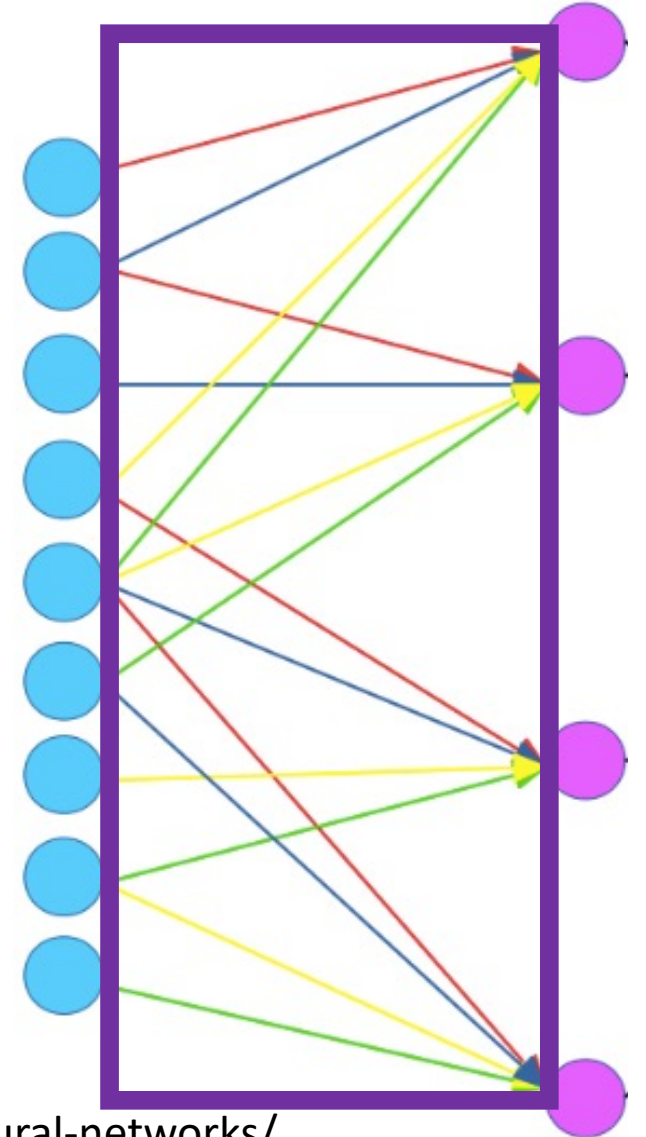
Filter
(aka – Kernel)

=



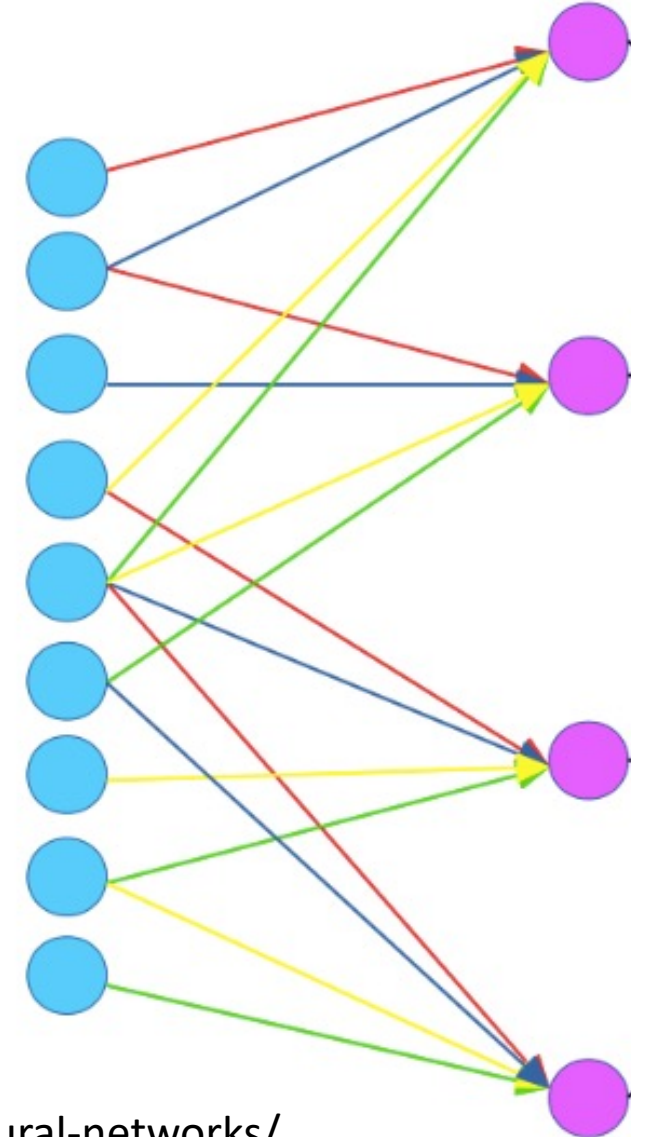
Feature
Map

Way to Interpret
Neural Network



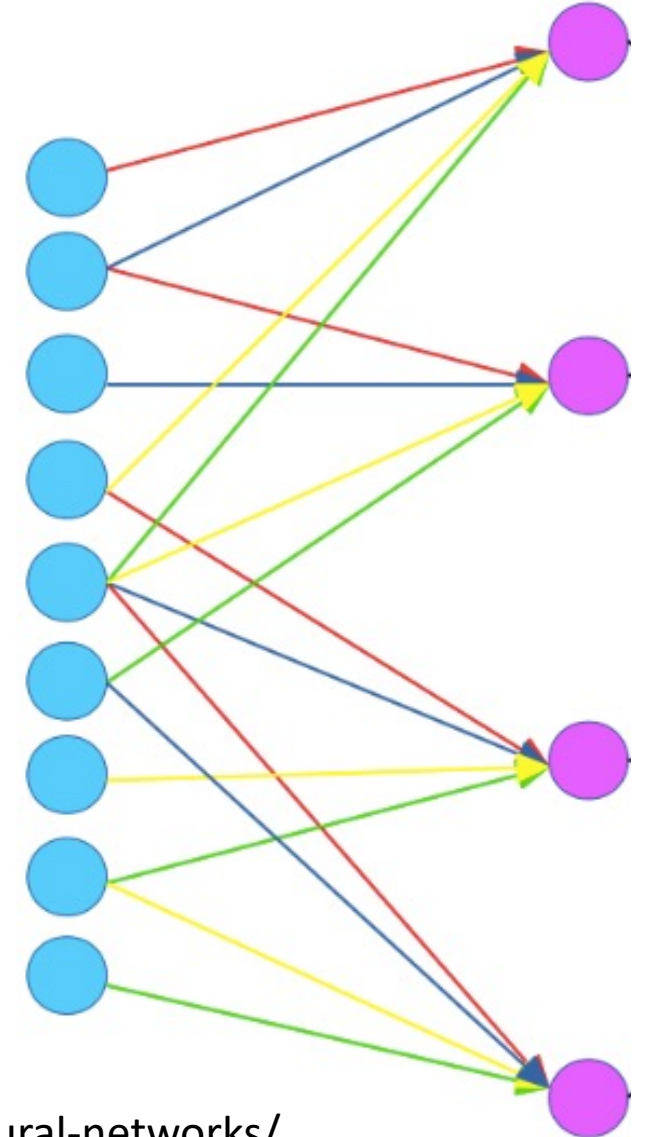
Convolutional Layer: Parameters to Learn

- For shown example, how many weights must be learned?
 - 4 (red, blue, yellow, and green values)
- If we instead used a fully connected layer, how many weights would need to be learned?
 - 36 (9 turquoise nodes x 4 magenta nodes)
- For shown example, how many parameters must be learned?
 - 5 (4 weights + 1 bias)
- If we instead used a fully connected layer, how many parameters would need to be learned?
 - 40 (36 weights + 4 bias)



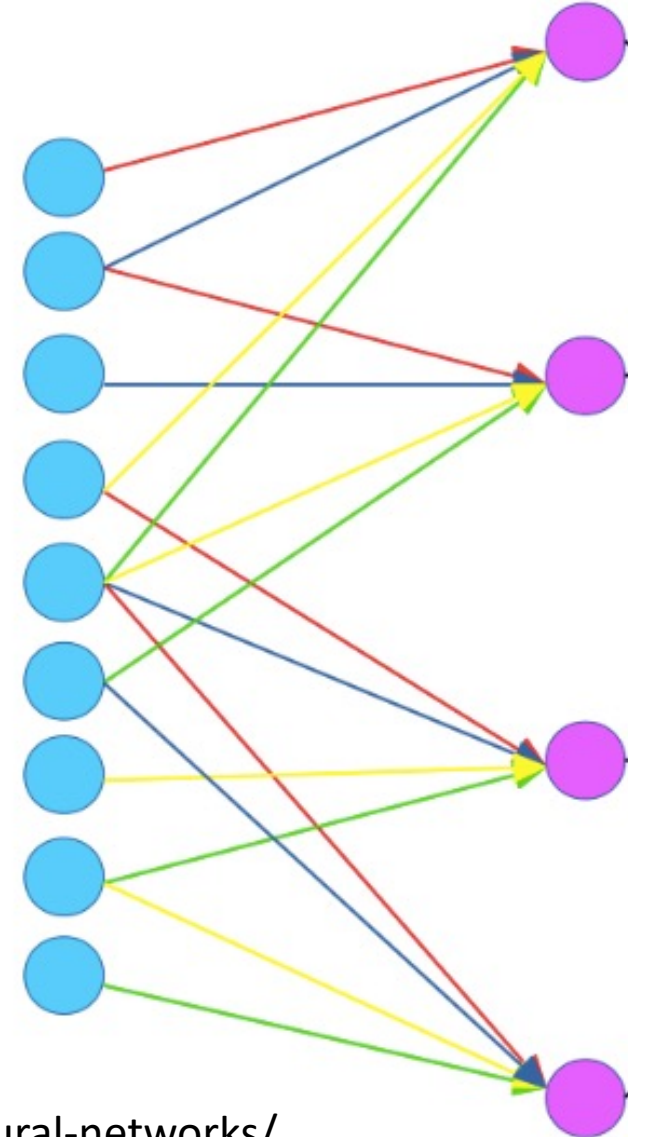
Convolutional Layer: Parameters to Learn

- Parameter sharing significantly reduces number of parameters to learn and so storage requirements
- Sparse (rather than full) connectivity also significantly reduces the number of computational operations required



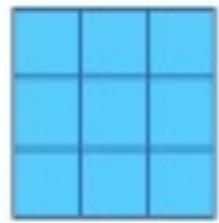
Convolutional Layer: Parameters to Learn

Neocognitron hard-coded filter values...
we will cover models that learn the filter
values in the next lecture



Convolutional Layer

- Many neural network libraries use “convolution” interchangeably with “cross correlation”; for mathematicians, these are technically different
- Examples in these slides show the “cross-correlation” function



Input

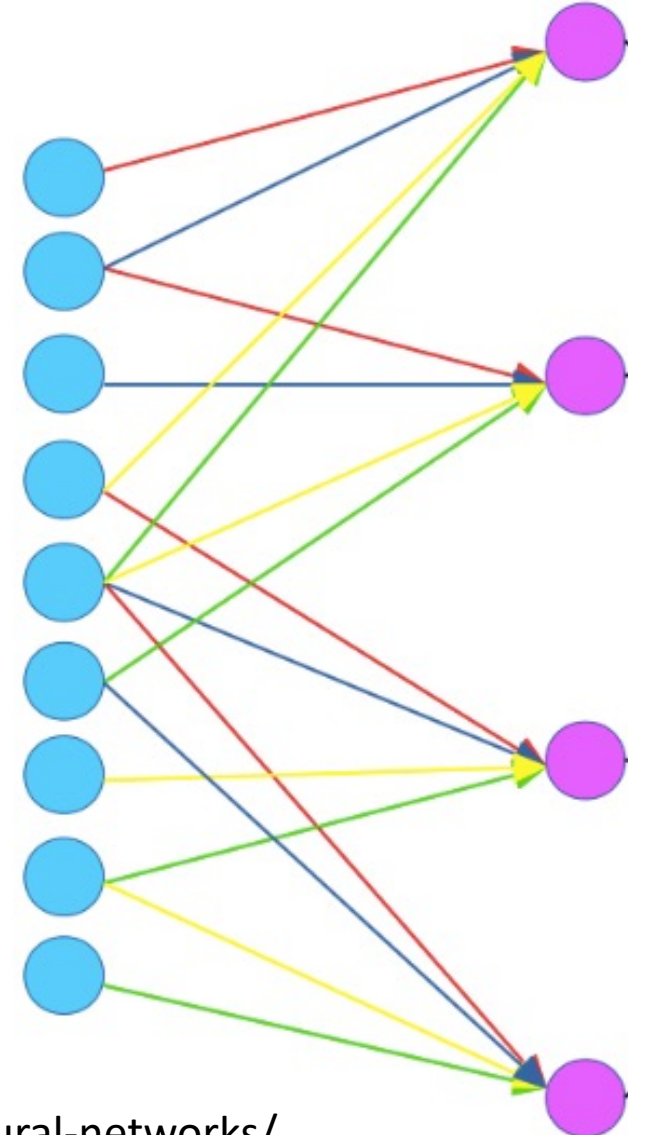


Filter
(aka – Kernel)



Feature
Map

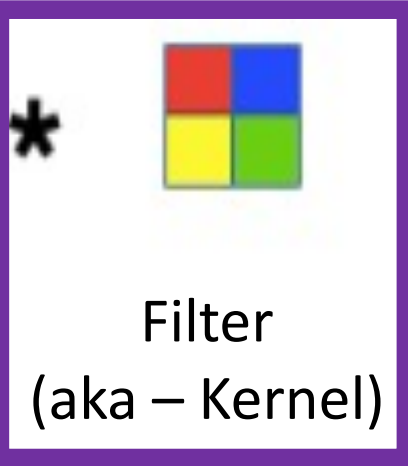
Way to Interpret
Neural Network



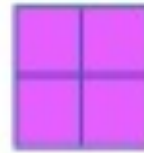
Convolutional Layer: What Does The Filter Do?



Input

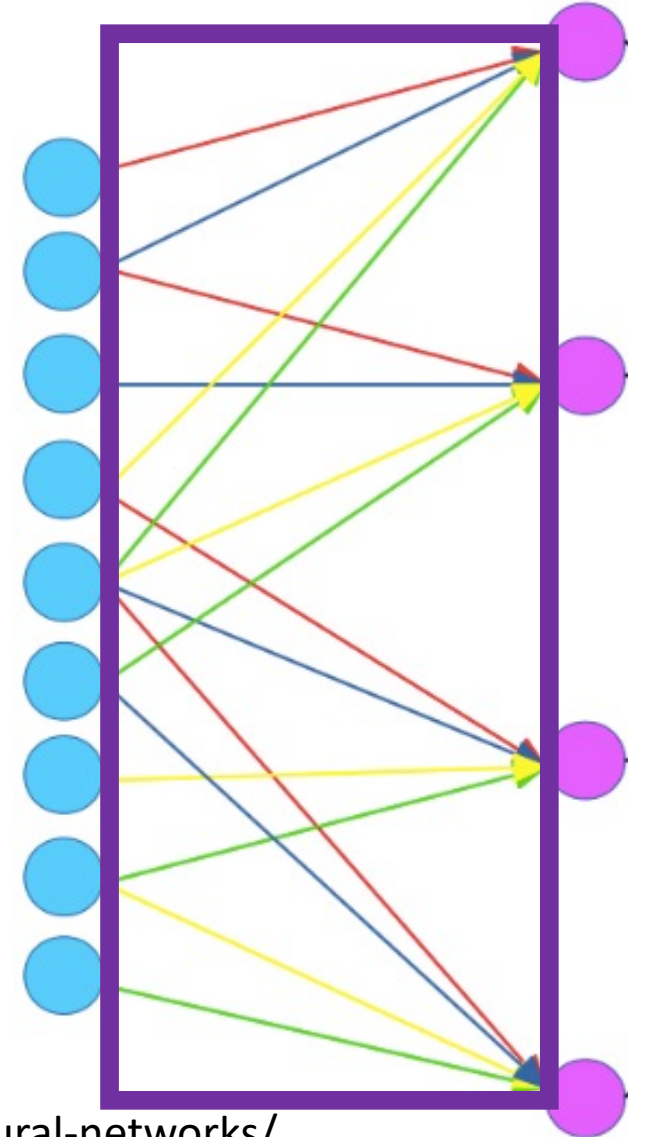


=



Feature
Map

Way to Interpret
Neural Network



Filter: What Does It Do?

Filter



Filter: What Does It Do?

- e.g.,

Filter

0	0	0	0	0	30	0
0	0	0	0	30	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	0	0	0	0

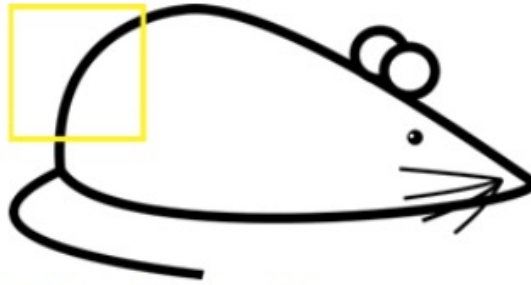
Visualization of Filter



Filter: What Does It Do?

- e.g.,

Filter Overlaid on Image



Image

0	0	0	0	0	0	30
0	0	0	0	50	50	50
0	0	0	20	50	0	0
0	0	0	50	50	0	0
0	0	0	50	50	0	0
0	0	0	50	50	0	0
0	0	0	50	50	0	0

*

Filter

0	0	0	0	0	30	0
0	0	0	0	30	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	0	0	0	0

Weighted Sum = ?

Weighted Sum = $(50 \times 30) + (20 \times 30) +$
 $(50 \times 30) + (50 \times 3) + (50 \times 30)$

Weighted Sum = 6600 (**Large Number!!**)

Filter: What Does It Do?

- e.g.,

Filter Overlaid on Image



Image

0	0	0	0	0	0	0
0	40	0	0	0	0	0
40	0	40	0	0	0	0
40	20	0	0	0	0	0
0	50	0	0	0	0	0
0	0	50	0	0	0	0
25	25	0	50	0	0	0

Filter

*

0	0	0	0	0	30	0
0	0	0	0	30	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	0	0	0	0

Weighted Sum = ?

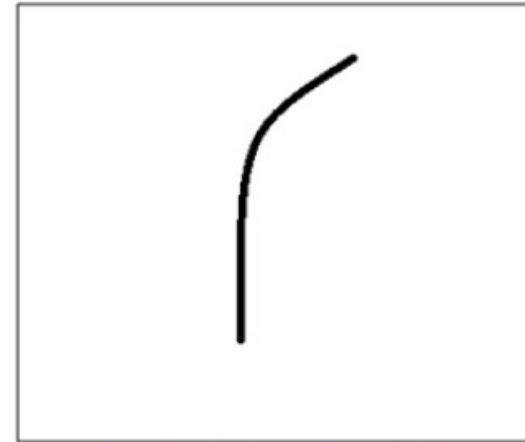
Weighted Sum = 0 (**Small Number!!**)

Filter: What Does It Do?

This Filter is a Curve Detector!

- e.g.,

0	0	0	0	0	30	0
0	0	0	0	30	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	0	0	0	0





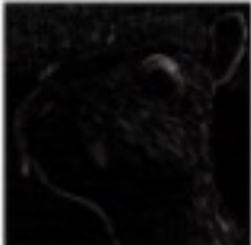

Filter Overlaid on Image (**Big Response!**)






Filter Overlaid on Image (**Small Response!**)




Filters Detect Different Features

	Filter	Feature Map
Identity	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	
Edge detection	$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$	
	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	
	$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	

	Filter	Feature Map
Sharpen	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
Box blur (normalized)	$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	
Gaussian blur (approximation)	$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$	

Different Filters Detect Different Features



Filter:
Sharpen

Image:
Bell

0	-3	0
-3	21	-3
0	-3	0

Divisor: 9

The Matrix

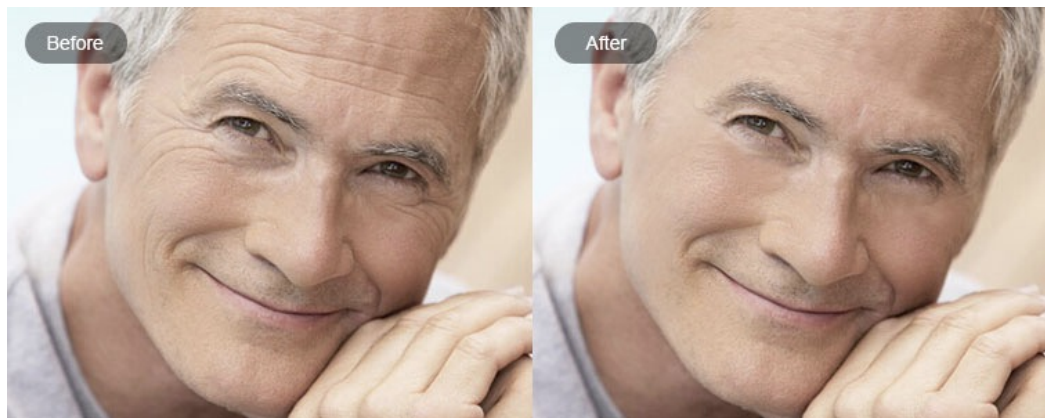
Demo: <http://beej.us/blog/data/convolution-image-processing/>

Group Discussion

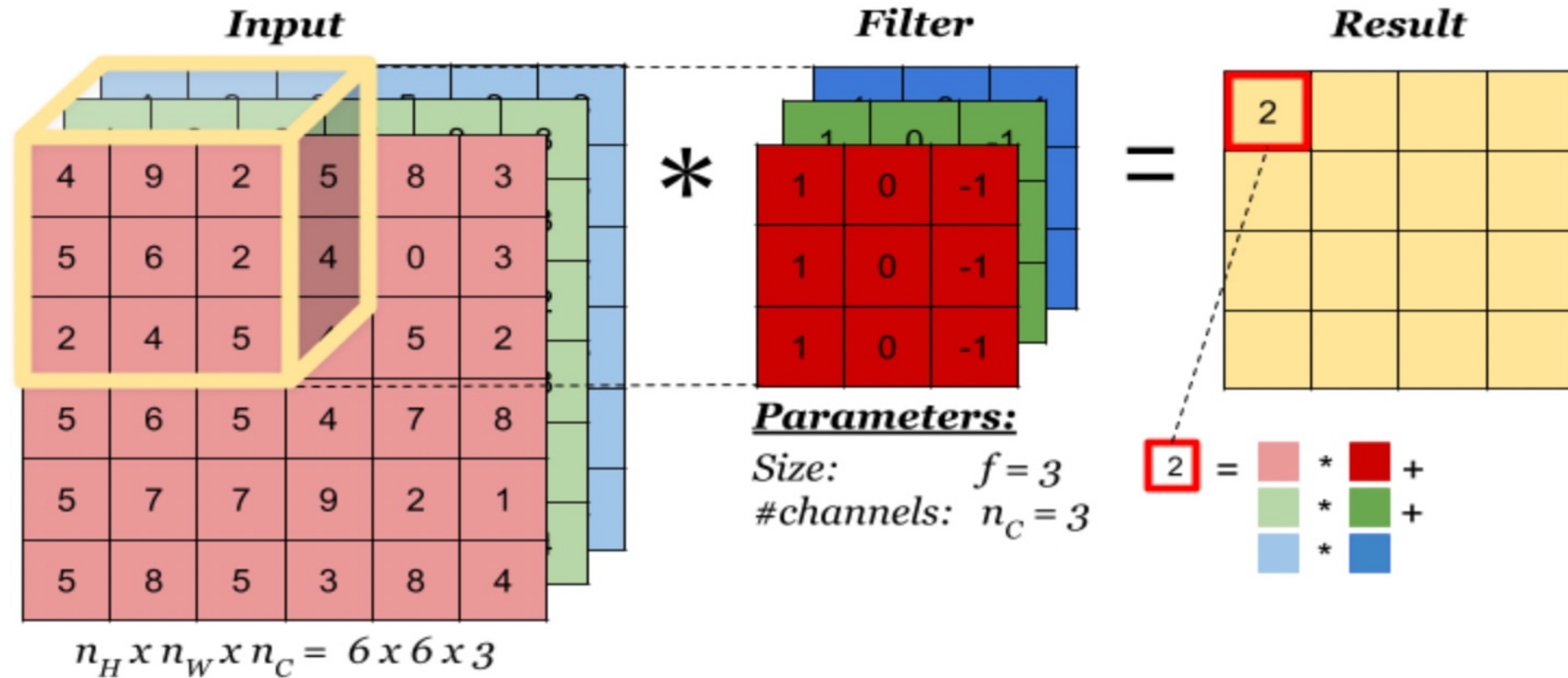
1. How would you design a filter to “brighten” an image?



2. How would you design a filter to remove wrinkles/blemishes?

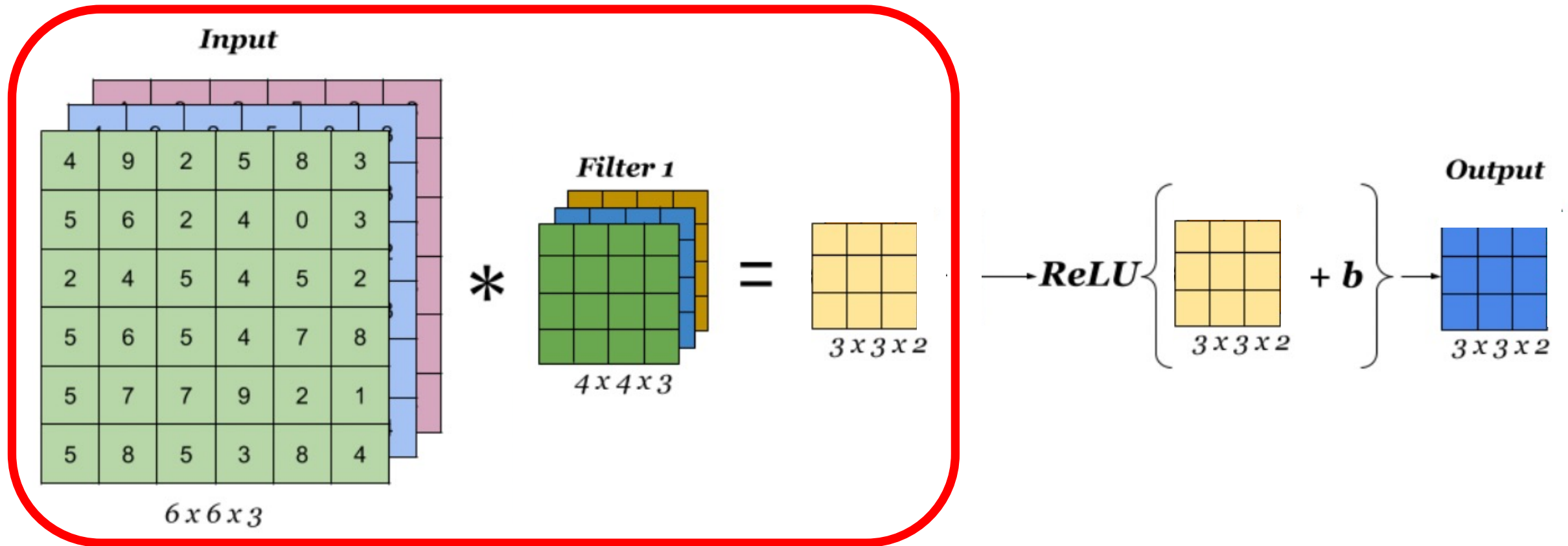


How Filters Are Applied to Multi-Channel Inputs



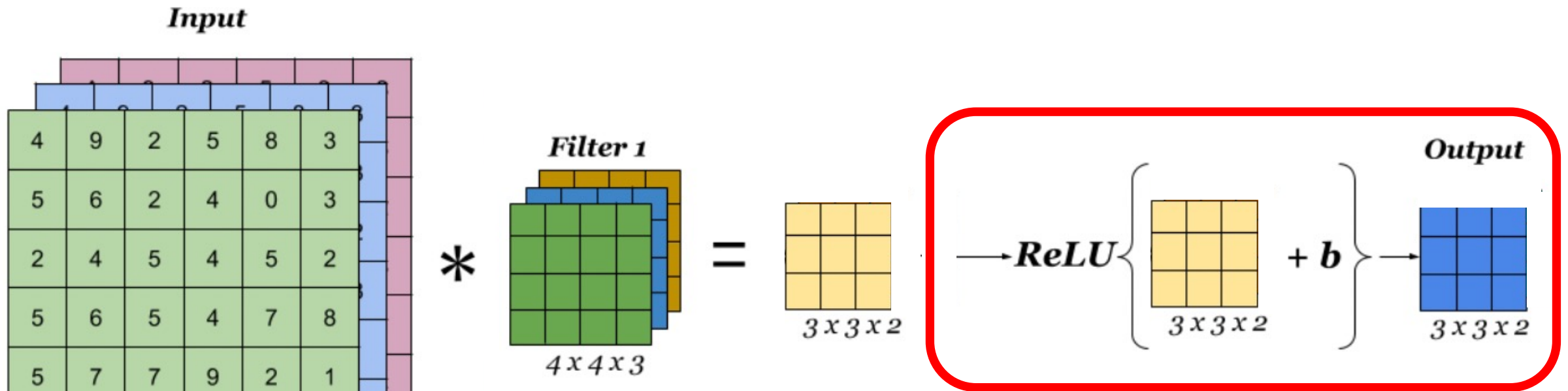
Number of channels in a filter matches that of the input

Convolutional Layer Overview



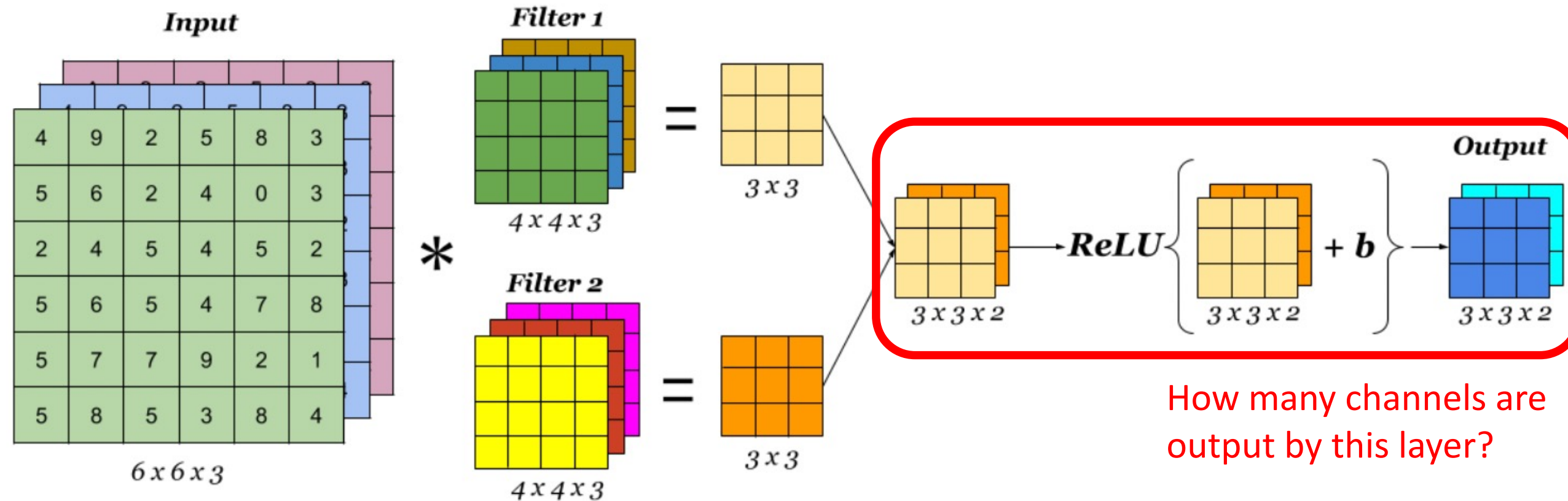
Can choose filters of any size to support feature learning!

Convolutional Layer Overview



Filtered results are passed, with a bias term, through an activation function to create **activation/feature maps**

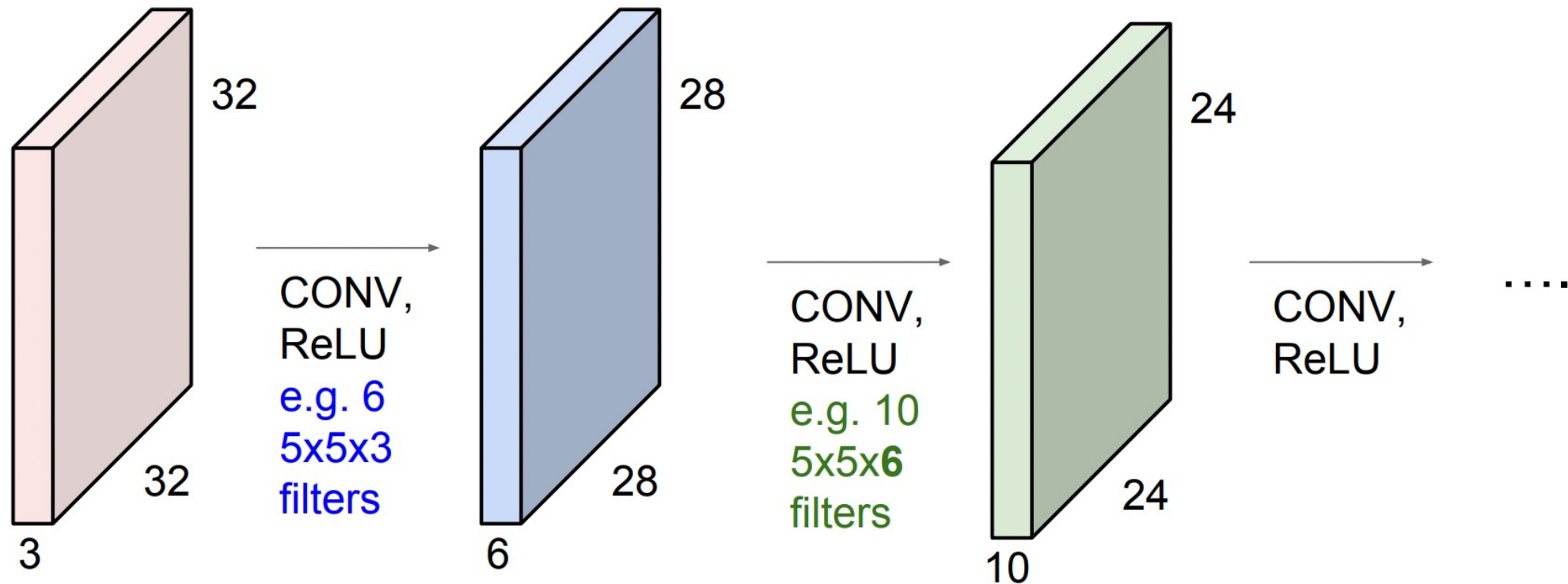
Convolutional Layer Overview – Multiple Filters



Can have multiple filters (note: there is a unique bias parameter per filter)

Convolutional Layers Stacked

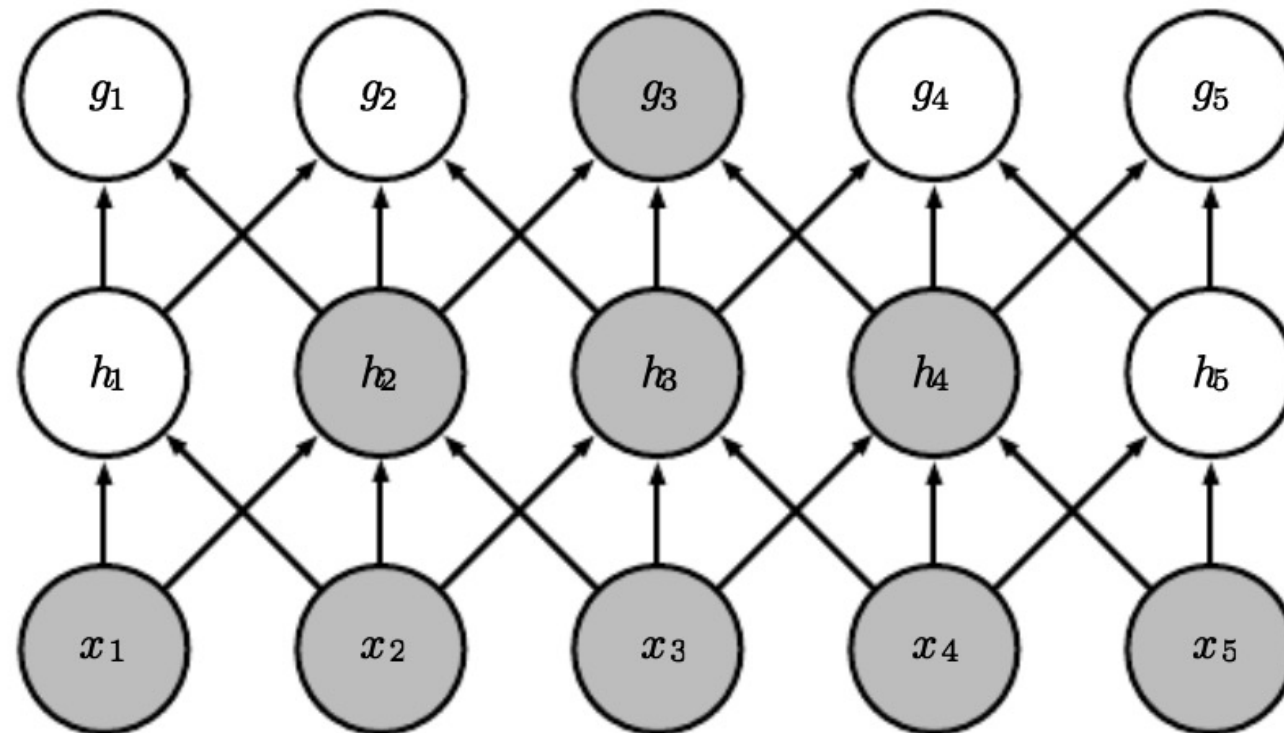
Can then stack a sequence of convolution layers; e.g.,



(Recall each filter's number of channels matches that of its input)

Convolutional Layers Stacked

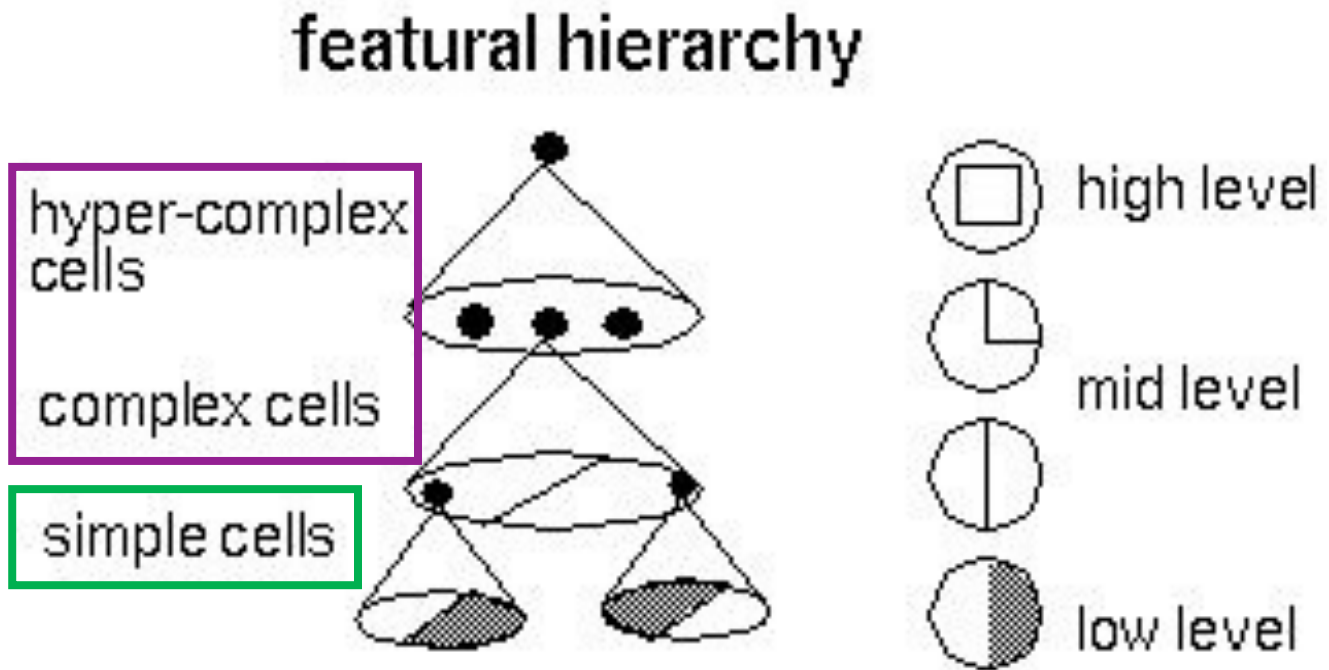
Can then stack a sequence of convolution layers, which leads to identifying patterns in increasingly **larger regions of the input (e.g., pixel) space**:



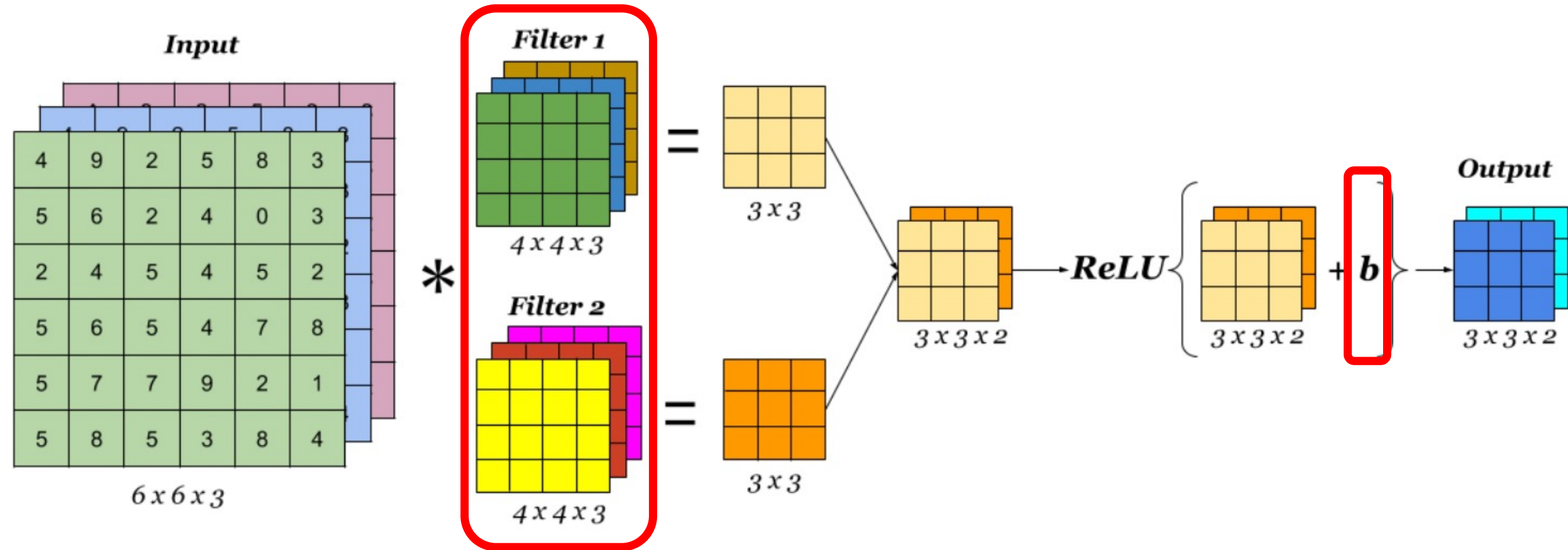
Convolutional Layers Stacked

Can then stack a sequence of convolution layers, which leads to identifying patterns in increasingly **larger regions of the input (e.g., pixel) space** and **mimicking vision system**:

Higher level features are constructed by combining lower level features



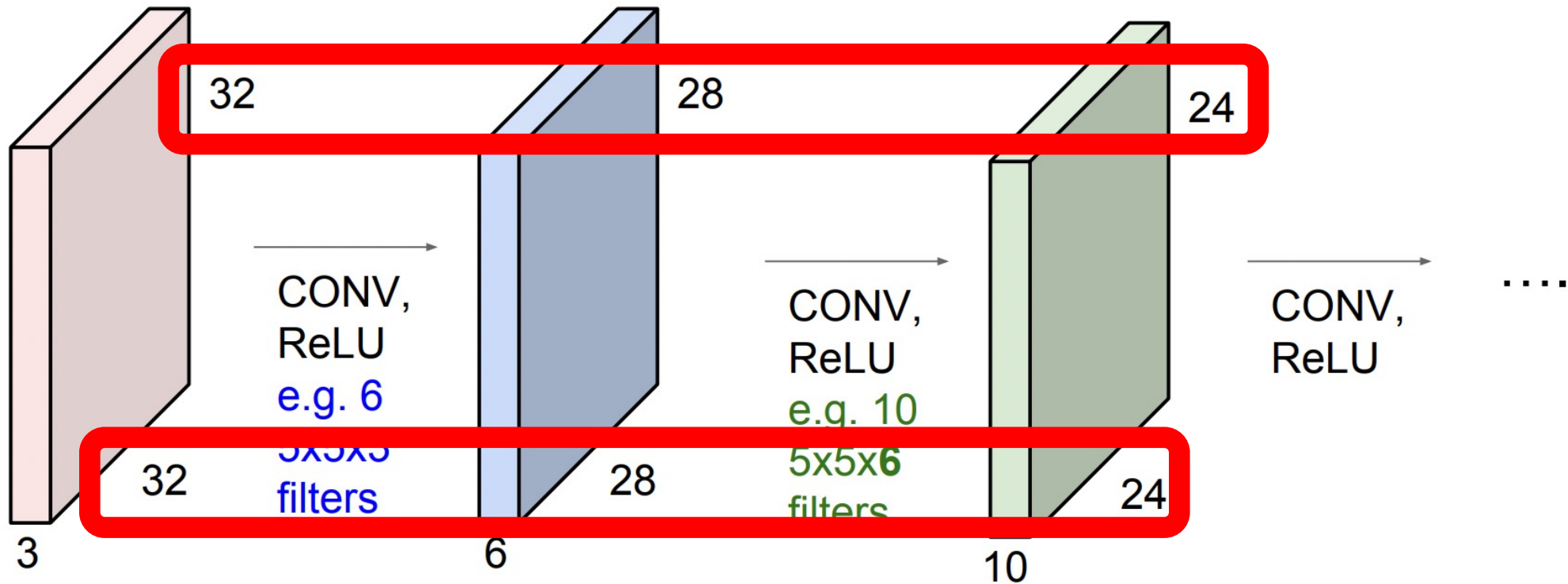
Convolutional Layer: Parameters to Learn



Parameters: bank of filters and biases used to create the activation maps (aka – feature maps)

Problem: Input Shrinks

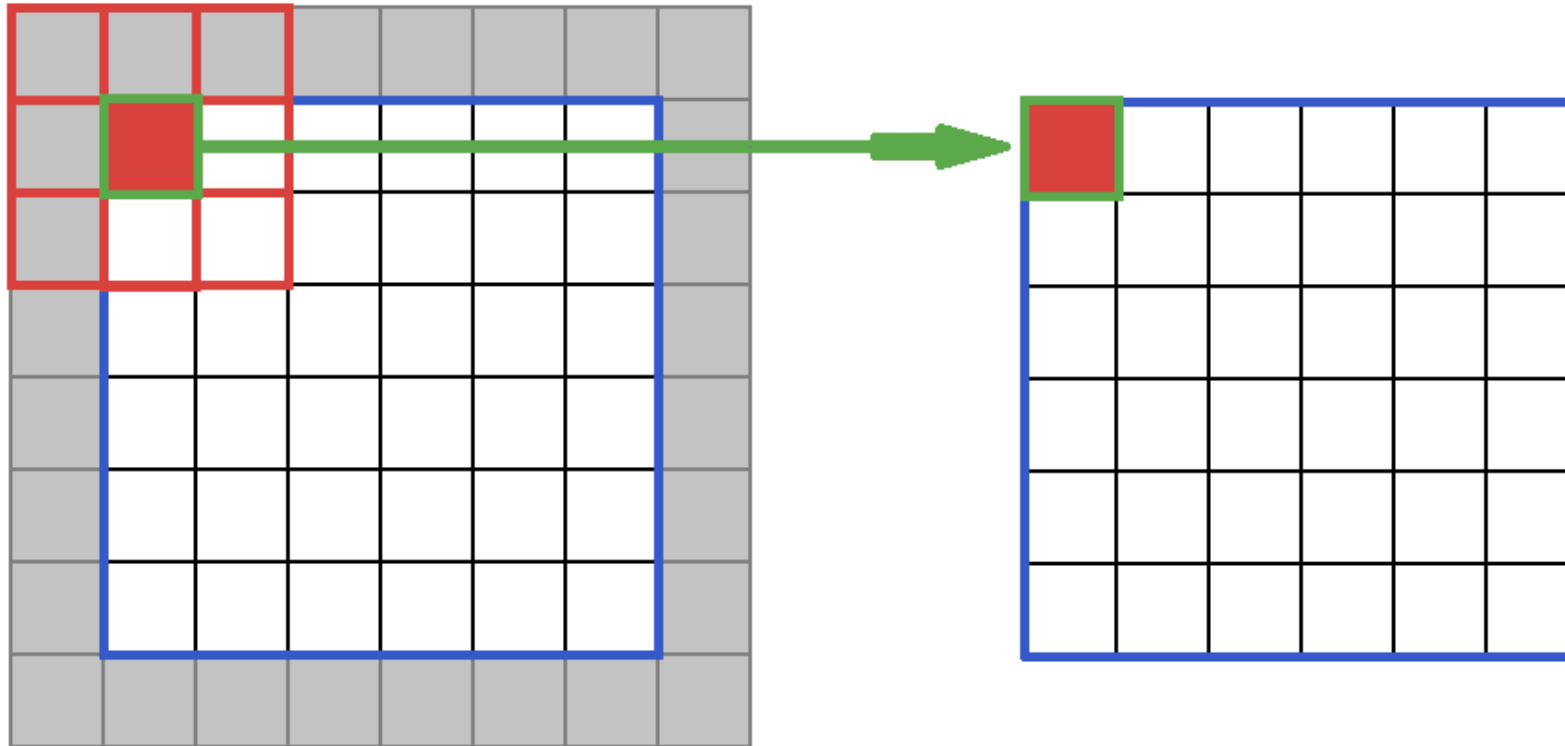
Why do the dimensions shrink with each convolutional layer?



Information is lost around boundary of the input!

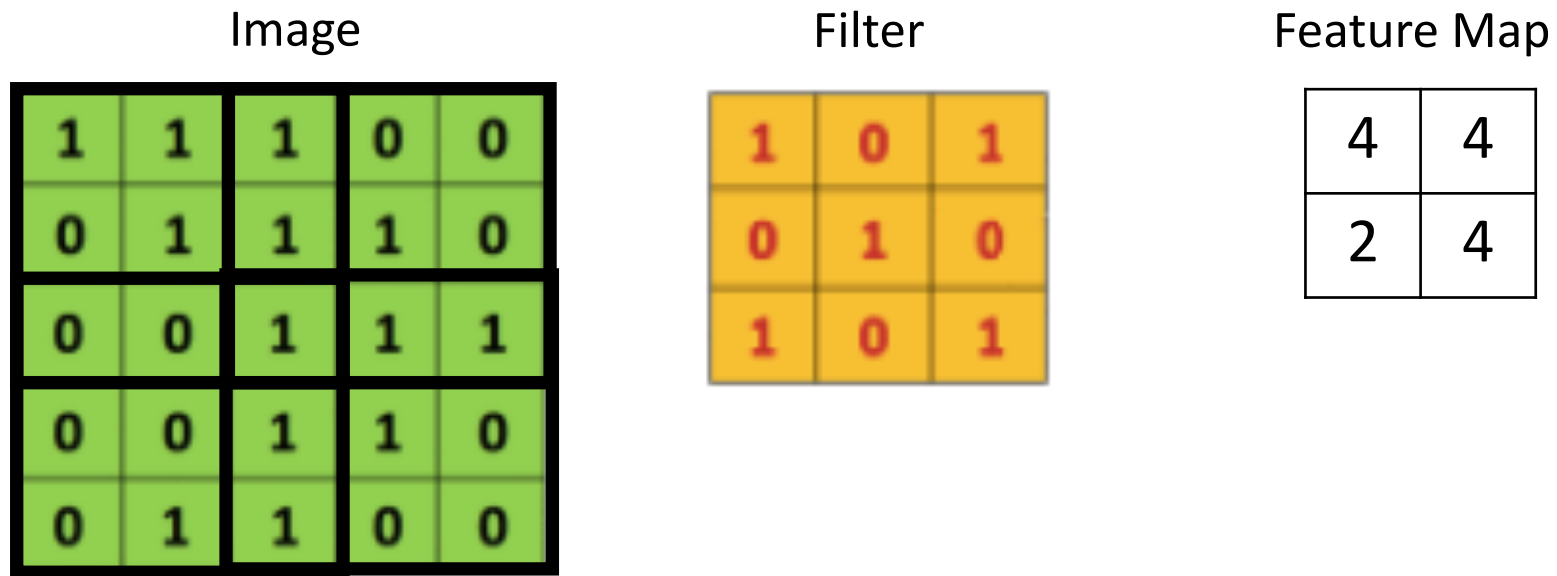
Idea: Control Output Size with Padding

- **Padding:** add values at the boundaries



Convolution: Implementation Detail

- **Stride:** how many steps taken spatially before applying a filter
 - e.g., 2x2



Convolution: Implementation Details

- Demo:
 - https://theano-pymc.readthedocs.io/en/latest/tutorial/conv_arithmetic.html

Convolutional Layers: Parameters vs Hyperparameters

- Parameters
 - Weights
 - Biases
- Hyperparameters:
 - Number of filters, including height and width of each
 - Strides
 - Padding type
 - Activation function

Today's Topics

- Neural Networks for Spatial Data
- History of Convolutional Neural Networks (CNNs)
- CNNs – Convolutional Layers
- CNNs – Pooling Layers

Pooling Layer: Summarizes Neighborhood

- **Max-pooling:** partitions input into a set of non-overlapping rectangles and outputs the maximum value for each chunk

Single depth slice

1	1	2	4
5	6	7	8
3	2	1	0
1	2	3	4

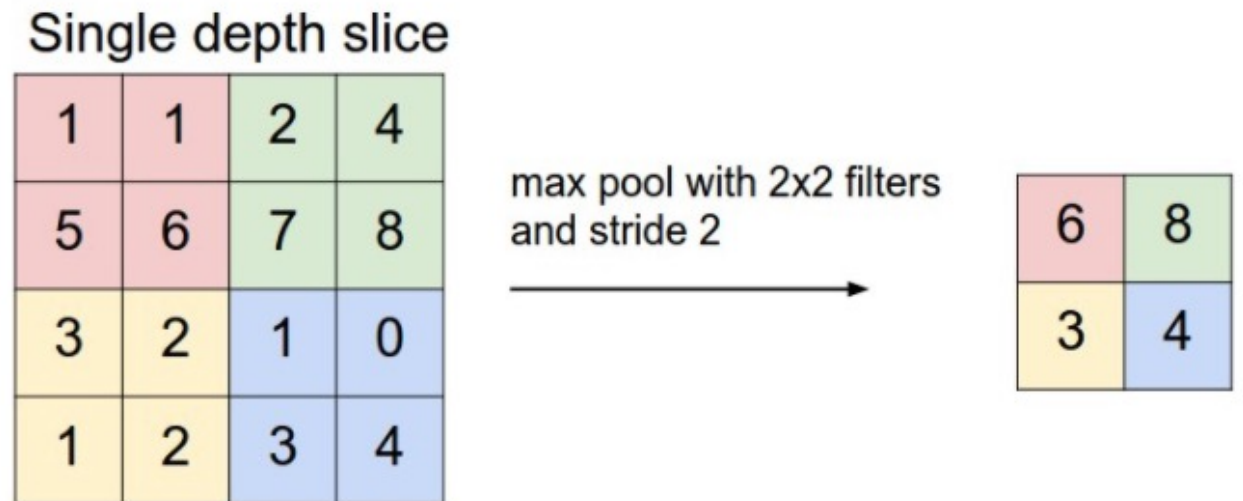
max pool with 2x2 filters
and stride 2



?	?
?	?

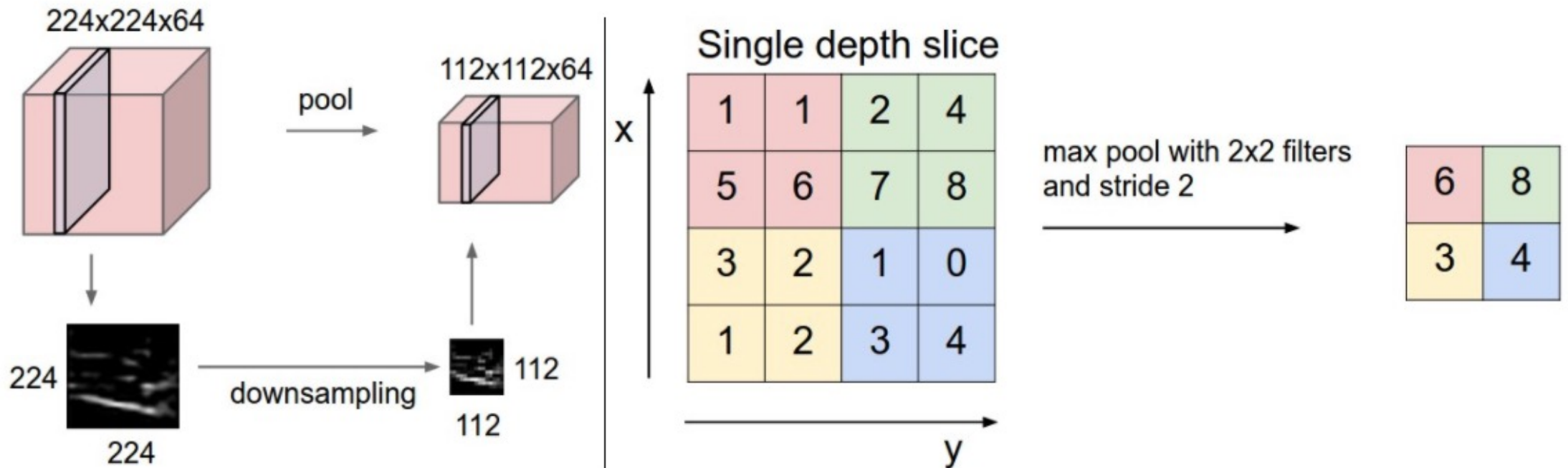
Pooling Layer: Summarizes Neighborhood

- **Max-pooling:** partitions input into a set of non-overlapping rectangles and outputs the maximum value for each chunk



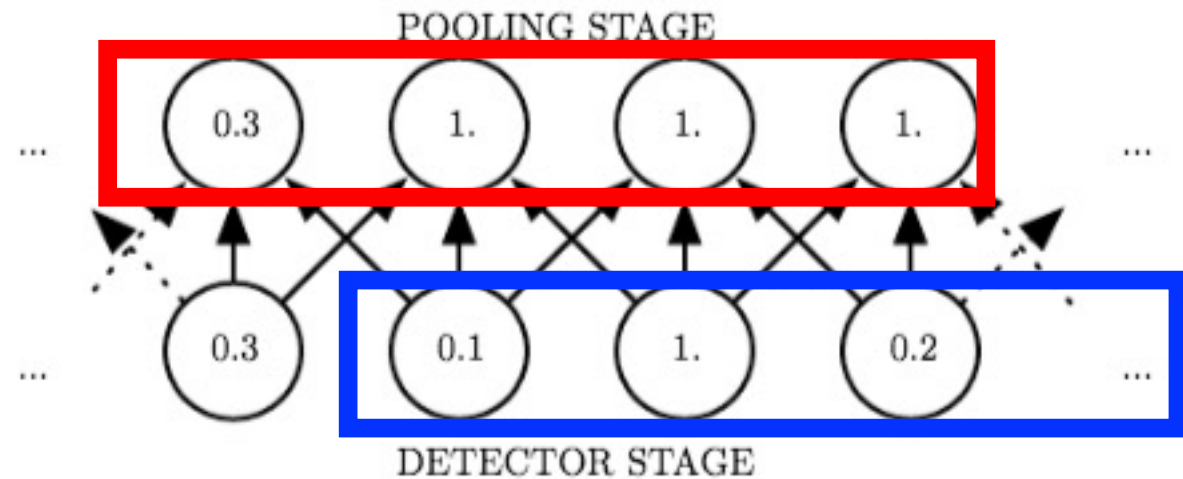
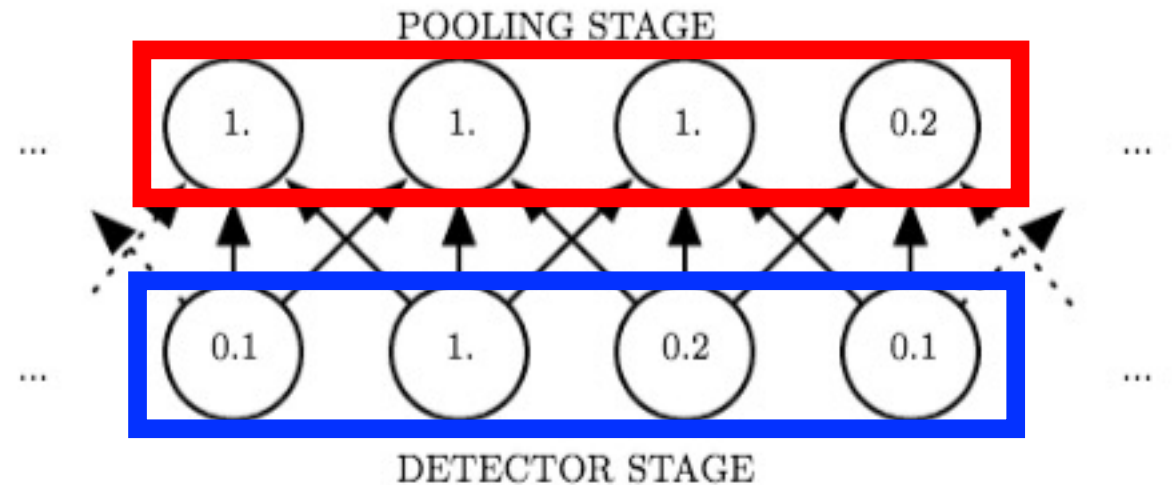
Pooling Layer: Summarizes Neighborhood

- **Max-pooling:** partitions input into a set of non-overlapping rectangles and outputs the maximum value for each chunk



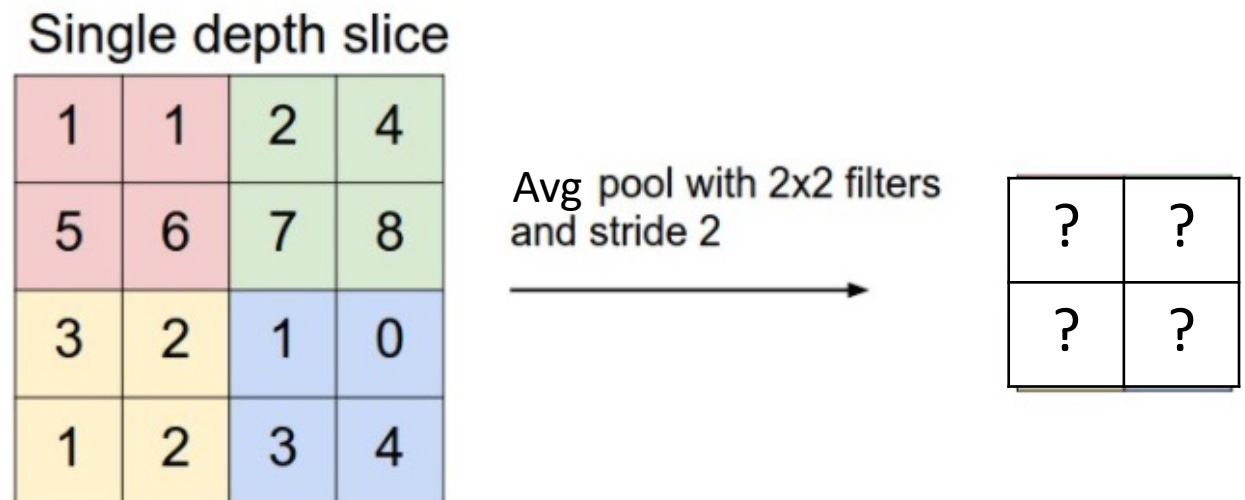
Pooling Layer

- Resilient to small translations
- e.g.,
 - Input: all values change (shift right)
 - Output: only half the values change



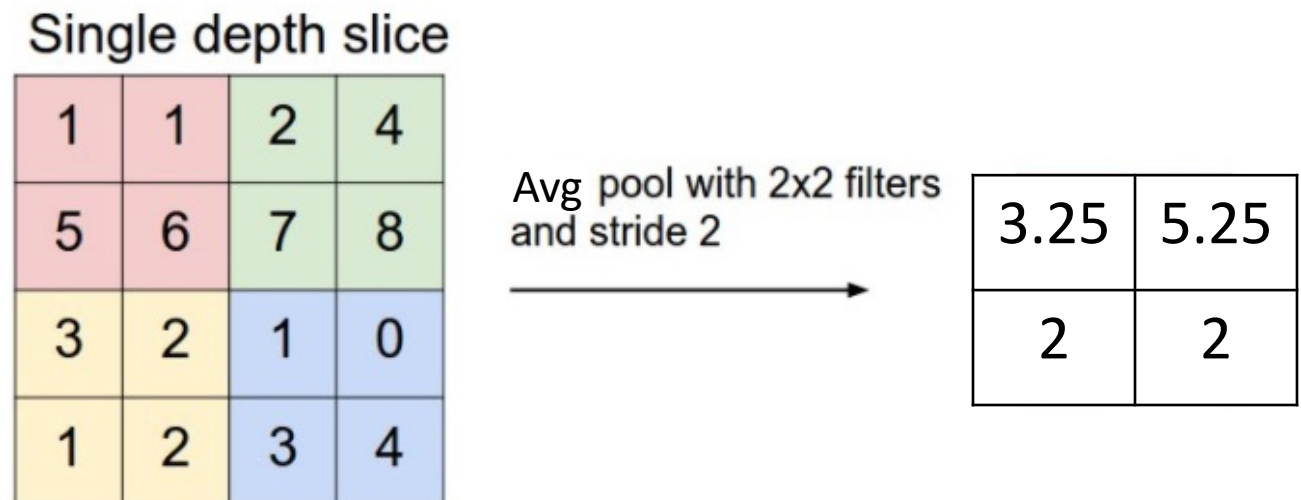
Pooling Layer: Summarizes Neighborhood

- **Max-pooling:** partitions input into a set of non-overlapping rectangles and outputs the maximum value for each chunk
- **Average-pooling:** partitions input into a set of non-overlapping rectangles and outputs the average value for each chunk

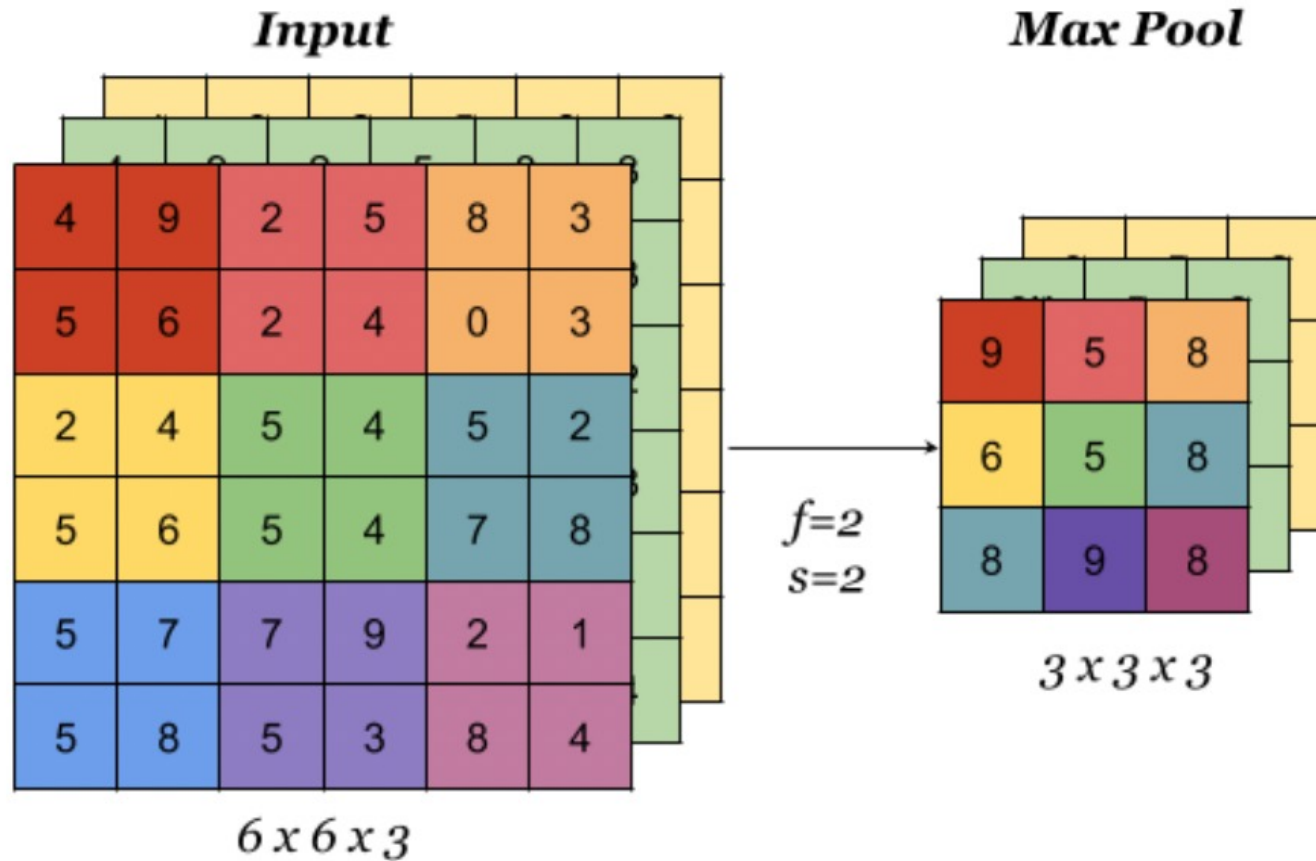


Pooling Layer: Summarizes Neighborhood

- **Max-pooling:** partitions input into a set of non-overlapping rectangles and outputs the maximum value for each chunk
- **Average-pooling:** partitions input into a set of non-overlapping rectangles and outputs the average value for each chunk



Pooling for Multi-Channel Input



Pooling is applied to each input channel separately

Pooling Layer: Benefits

- How many parameters must be learned?
 - None
- Benefits?
 - Builds in invariance to translations of the input
 - Reduces memory requirements
 - Reduces computational requirements

Today's Topics

- Neural Networks for Spatial Data
- History of Convolutional Neural Networks (CNNs)
- CNNs – Convolutional Layers
- CNNs – Pooling Layers



The End