ENPM640 - Rehabilitation Robotics
Final Exam

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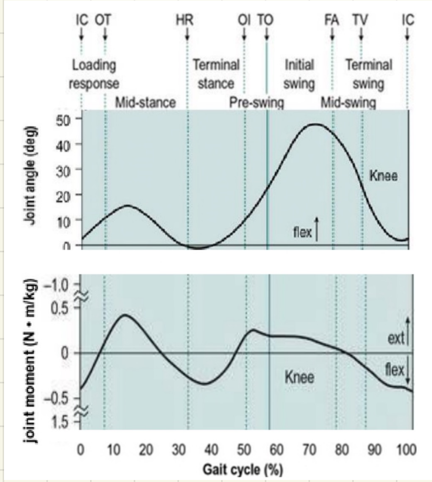
Part A : Rehab Robot Hardware Selection :

A1).



Robot to rehabilitate knee motion in the flexion - extension DoF.

$M = 100\ kg$

moment arm $= 5\ cm = 0.05\ m$

$M_{motor} + M_{act} \leq 1.75\ Kg$

Max linear dim. of act, "stroke" $\simeq 5\ cm = 1.9685\ in \simeq 2$ inches.
(lin. range of motion from min to max.)

Actuator Selection

Required:- max static thrust, stroke length

$M_{knee} = 0.5\ Nm/Kg$

$\tau_{p,knee} = 0.5 \times M$

$\tau_{p,knee} = 0.5 \times 100 = 50\ N.m$

Max thrust, $F_p = \tau_p / R \geq \frac{50}{0.05} = 1000\ N = 224.809\ lbf.$

Based on the max thrust required at the joint and the stroke length of the actuator, we narrow down on the following models.

FA-PO-150-12-XX & FA-PO-240-12-XX

To narrow down further, we choose the one with the highest transmission ratio FA-PO-240-12-XX. Since the stroke length is 2 inches, the chosen model is $\boxed{\text{FA-PO-240-12-2}}$

Motor Selection

α for the selected actuator is 30:1

Required end effector torque, $\tau_p = \frac{\tau_{p,knee}}{\alpha} = \frac{50}{30} = 1.66667$ N.m

110% of required torque = 1.83333 N.m

$\therefore \tau_{p,m} \geq 1.83333$ N.m

At peak torque we want the motor to consume $I \leq 15A$

Weight of chosen actuator is 2.7 lbs = 1.22467 Kg

W.K.T. $M_{motor} + M_{act} \leq 1.75$ Kg

$M_{motor} \leq 1.75 - 1.22467$

$M_{motor} \leq 0.5253 \simeq 0.53$ Kgs

Based on motor weight $\leq$ 0.53 Kgs we have the following models (frameless motors)

01810, 01510, 01511, 01512, 01513, 01210, 01211, 01212, 01213, 01214, 00710, 00711, 00712, 00713, 00714, 00510, 00511, 00512, 00513, 00410, 00411, 00412.

Shortlist further with peak torque $\geq$ 1.8333 N.m

& the one with the least weight is (0.428 Kg) → 01214

Narrowing it even further by the windings we get Model No. 01214

$\therefore$ The chosen model with peak current 13.4A is $\boxed{01214B}$

A-2) $C_{min} = 0.35\,um$; $\bar{F}t = 192.5\,N.s$

min count freq $> 5\,MHz$

$m = m_{body} + m_{act} + m_{motor}$

$= 100 + 1.22467 + 0.428$

$= 101.65267\,kg$

W.K.T. $F.t = m a.t$

Avg. F.t given = 192.5 N.s

$$192.5 = m \frac{\Delta V}{\Delta t} t \quad -(1)$$

Here t is total gait cycle. Δt is % of gait cycle for the paretic single support stance duration. From the figure we can see that stance is 40% of the gait cycle.

$\therefore \Delta t = 40\%$ of entire gait cycle

plugging Δt back in (1), we get

$$192.5 = m \frac{\Delta V}{40\% \cancel{t}} \cdot \cancel{t}$$

$$192.5 = (101.65267) \frac{\Delta V}{(40/100)}$$

$$192.5 = 254.131675\,\Delta V$$

$$\Delta V = \frac{192.5}{254.131675} = 0.75748\ m/s$$

$\therefore$ The max. speed of actuator must be $\geq 0.75748\ m/s$

Based on the minimum resolution better than $0.35\,um$ & ΔV_{max}, we narrow down to ordering codes 13B & models K & A. From these the model with least edge separation of $0.07\,us$ is code 'K'.

So we are choosing the following encoder Model no. $\boxed{13B-K}$

Part B. Rehab. Robot Controller Design

B-1) 1-D slot controller for ~15% t_{ss}.

$t_{ss} = 0.35\,s$ (given in clarification announcement)

$\therefore T_1 = 15\%$ of $0.35 = 0.0525\,s$

initial commanded angle $\theta_0 = 5^\circ$

peak commanded angle $\theta^* = 20^\circ$ during T_1

initial, slot height $b_0 = 3^\circ$

final slot height $b_1 = 1^\circ$

Sampling freq = 10 KHz

$\theta_{ref}(i) = ?$

$$\theta_{ref}(i) = \theta_0 + \frac{(\theta^* - \theta_0)i}{T_1} \quad \forall\, i \in [0, T_1]$$

$$= 5 + \frac{(20-5)}{0.0525} i \implies \theta_{ref}(i) = 285.7142\,i + 5 \quad \text{— (1)}$$

bottom edge, $b(i) = \theta_{ref}(i) - h/2$ & top edge, $t(i) = \theta_{ref}(i) + h/2$

height of slot, $h = b_0 + \frac{(b_1 - b_0)i}{T_1} \quad \forall\, i \in [0, T_1]$

$$h = 3 + \frac{(1-3)i}{0.0525} \implies h = 3 - 38.0952\,i \quad \text{— (2)}$$

Use h from (2) to derive $b(i)$ & $t(i)$

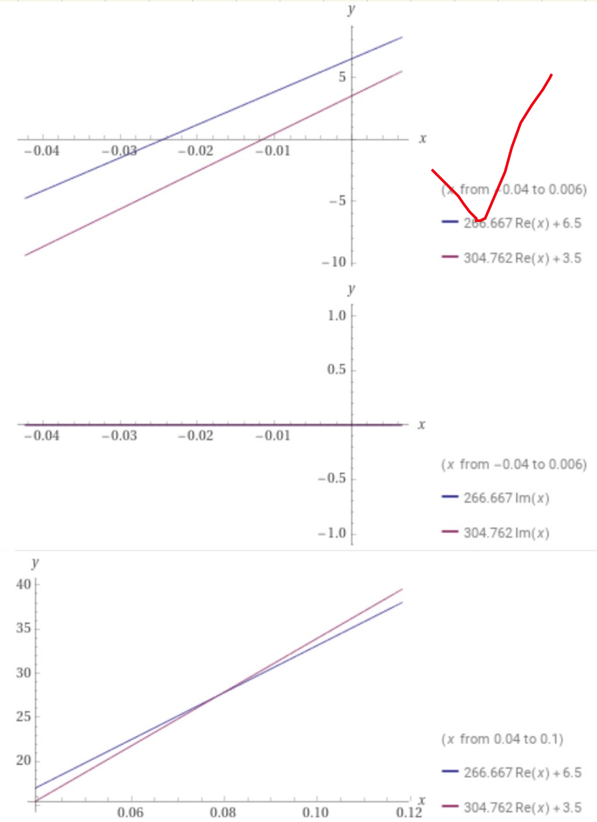
$$b(i) = (285.7142\,i + 5) - \frac{(3 - 38.0952)i}{2}$$

$$b(i) = 304.7618\,i + 3.5 \quad \text{— (3)}$$

$$t(i) = (285.7142\,i + 5) + \frac{(3 - 38.0952)i}{2}$$

$$t(i) = 266.6666\,i + 6.5 \quad \text{— (4)}$$

Plotting the top edge & the bottom edge as the function of 'i' using Wolfram Alpha, we get the following plot.



As we can see from the plot no. 3 in the image to the left, the top edge and the bottom edge merge hence signifying the collapse of the slot.

Absolute position of the top edge at 20ms

∴ From (4), we get

$$t(i) = 266.6666\, i + 6.5$$

$$= 266.6666\,(0.02) + 6.5$$

$$t(i) = 5.333332 + 6.5 = 11.8333$$

From , we get $b(i) = 304.7618\, i + 3.5$

$$b(i) = 304.7618\,(0.02) + 3.5 = 9.5952$$

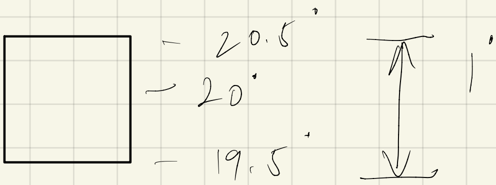
$$\boxed{\therefore \text{At } i = 20\text{ms} \quad t(i) = 11.8333^\circ \;\&\; b(i) = 9.5952^\circ}$$

B-2)

Final slot height = 1°

Peak Commanded angle = 20°

Drawing out, the final slot appears as below



$\therefore$ The θ of 10° is outside the slot

$\theta < b \Rightarrow 19.5^\circ$

Since $\theta < b$, we will use the following control law

$$\tau_a = K(b-\theta) + B(\dot{b}-\dot{\theta})$$

From eqn (3), we have

$$b(i) = 304.7618\, i + 3.5$$

$$\dot{b}(i) = 304.7618$$

From the question $K = 25$ N.m/rad & $B = 0.5$ N.m-rad/sec

$\dot{\theta} = V(T_1) = 57.3°/sec$

$$\therefore \tau_a = 25 \times \frac{\pi}{180}(19.5° - 10°) + 0.5 \times \frac{\pi}{180}(304.7618 - 57.3)$$

$$\tau_a = 4.14515 + 2.1595 = 6.30465 \text{ N.m}$$

$$\tau_m = \frac{\tau_a}{\lambda} = \frac{6.30465}{30} = 0.210155 \text{ N.m}$$

Using the motor we selected 01214B, the torque sensitivity value $K_t = 0.132$ N.m/Amp

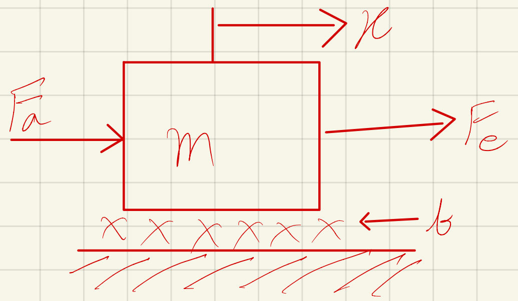
Since we're using SIC, the current at constant torque can be calculated as

$$\tau_m = K_T \cdot i_m$$

$$i_m = \frac{\tau_m}{K_T} = \frac{0.210155}{0.132} = 1.59208 \text{ A}$$

The commanded current to the motor at this time & for this scenario is, $\boxed{i_m = 1.59208 \text{ A}}$

B-3)



Knee robot is modelled as shown to the left

$$m = m_{act} + m_{motor} = 101.65267\ kg$$

$B = 2\ N\text{-}s/rad$; fric. loss, $b = 0.75\ N\text{-}s/rad$

Writing the equations of motion, we get

$$m\ddot{x} = -b\dot{x} + F_a + F_e$$

$$\Rightarrow m\ddot{x} = -b\dot{\theta} + F_a + F_e \quad \text{— (1)}$$

Since were use Natural Admittance Control characterized by a simple Impedance control, the Force from the actuator F_a is

$$(F_a)_{SIC} = K(\theta_{ref} - \theta) + B(\dot{\theta}_{ref} - \dot{\theta})$$

$$(F_a)_{NAC} = K_v\dot{\theta} + K_f F_e \leftarrow \text{external force}$$

$$F_a = (F_a)_{SIC} + (F_a)_{NAC}$$

Inserting F_a in eqn (1), we get

$$m\ddot{x} = -b\dot{\theta} + (F_a)_{SIC} + (F_a)_{NAC} + F_e$$

$$= -b\dot{\theta} + K(\theta_{ref} - \theta) + B(\dot{\theta}_{ref} - \dot{\theta}) + K_v\dot{\theta} + K_f F_e + F_e$$

$$\Rightarrow m\ddot{x} + (B - K_v + b)\dot{\theta} + K\theta = K\theta_{ref} + B\dot{\theta}_{ref} + (K_f + 1)F_e$$

Since we want the final system to be a simple mass-spring system, thus we need to equate the coeffs K_v term to 0

$$B - K_v + b = 0$$

$$\Rightarrow K_V = B + b$$
$$= 2 + 0.75$$
$$= 2.75$$

$$\boxed{\therefore K_V = 2.75 \text{ N-s/rad}}$$

Part C: (Bonus Question) Robot Arm Design

(c-1)

1 DoF elbow joint

h_i = 5% of final position

h_f = 2.5% of final position

Velocity gain = 1 N-s/m | position gain = 100 N/m

sampling frequency = 200 Hz

The jerk trajectory is as follows; for sampling instant (i).

$$x(i) = x_i + (x_f - x_i)\left(10\left(\frac{t_i}{d}\right)^3 - 15\left(\frac{t_i}{d}\right)^4 + 6\left(\frac{t_i}{d}\right)^5\right)$$

$$y(i) = y_i + (y_f - y_i)\left(10\left(\frac{t_i}{d}\right)^3 - 15\left(\frac{t_i}{d}\right)^4 + 6\left(\frac{t_i}{d}\right)^5\right)$$

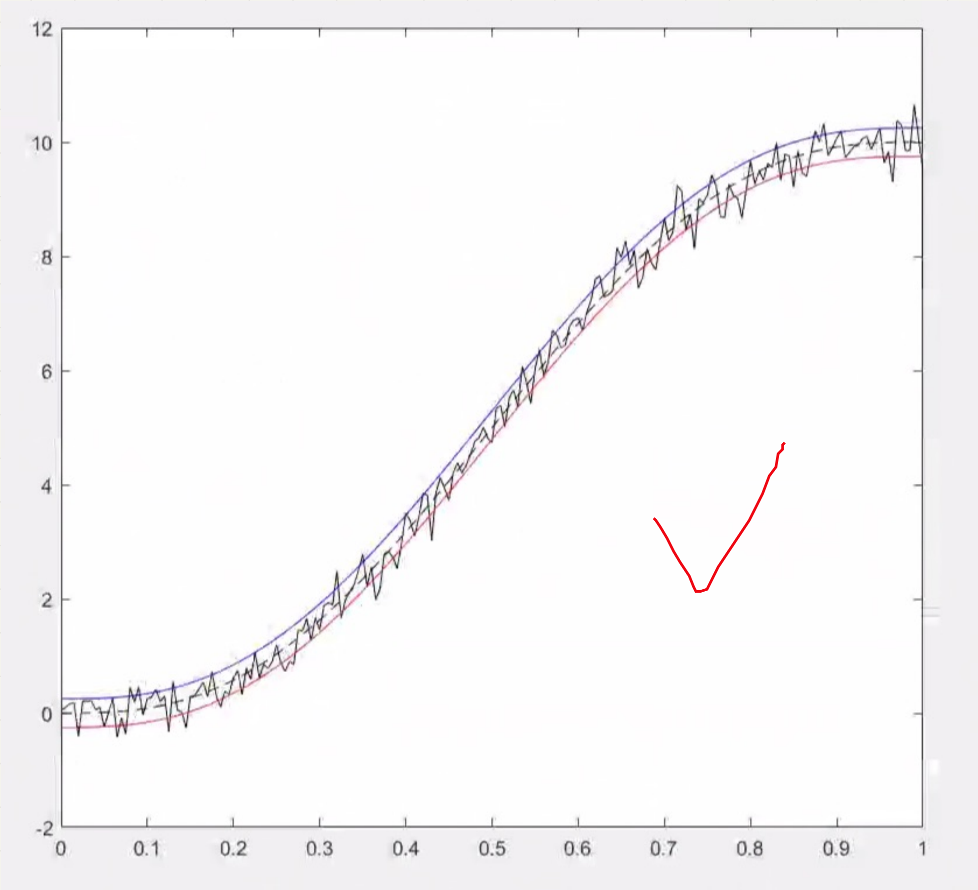
the top & bottom edge is given by

$$t(i) = \theta_{ref}(i) + h/2 \;\&\; b(i) = \theta_{ref}(i) - h/2$$

height of slot for each instance is

$$h(i) = b_0 + \left(\frac{b_1 - b_0}{T_1}\right) i$$

We generate random values for the trajectory and plot it using MATLAB, we get



Adding noise to the reference trajectory to replicate human hand's oscillation during movement.

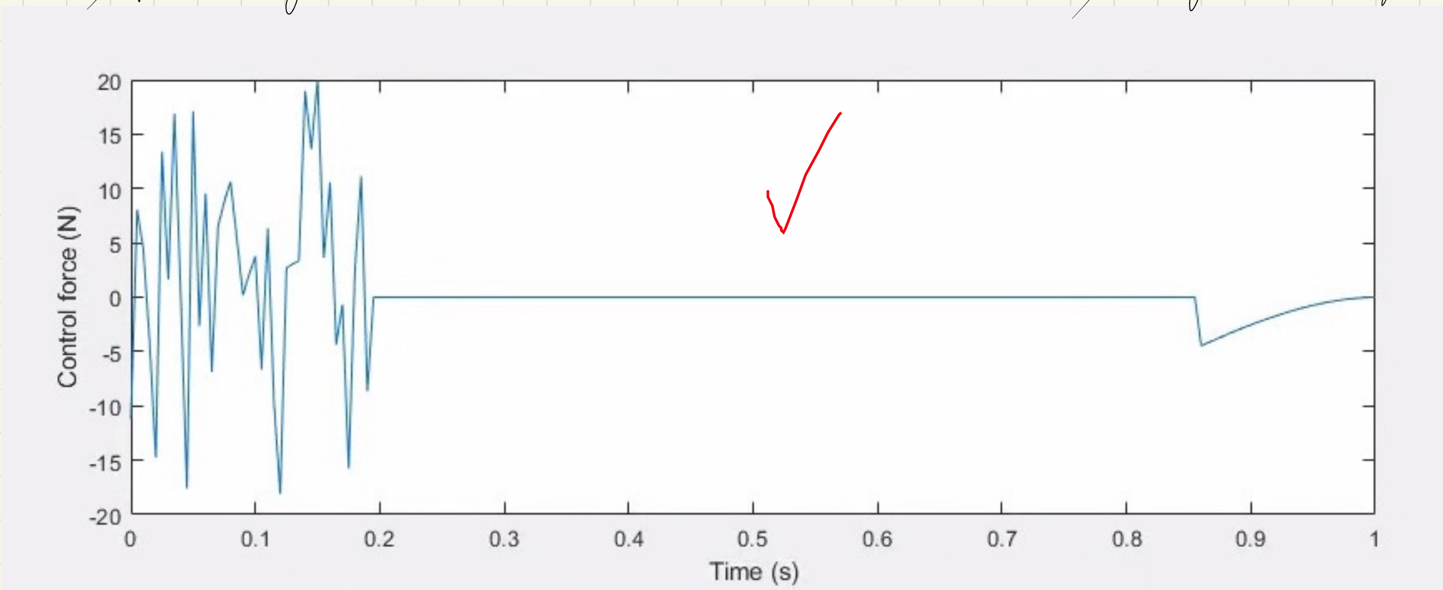
(c-2) Position error is given by $P_{err} = \theta_{noiseref}(i) - \theta_{ref}(i)$

Velocity error is given by $V_{err} = \frac{d}{dt}[P_{err}(i) - P_{err}(i-1)]$

If θ_{ref} is under the slot, $F = K_p P_{err} + K_v V_{err}$

If θ_{ref} is over the slot, $F = B \cdot V_{err}$

Now, plotting the Force over time instances, we get this plot.



Part D. Bonus Question (Ankle bot performance)

Considering Roy et.al (2009).

Avg. act. length = 30 cm = $x_{av-act,\,len}$ = 0.3 m

transverse length, $X_{tr,\,len}$ = 3 cm = 0.03 m

Moment arm = 60% of foot length

Avg. foot length from anthropometric data = 0.152 H

Avg. human height = 180 cm

$\therefore L_{foot} = 0.152H = 0.152 \times 180 = 27.36$ cm

$\therefore R = 60\%$ of $27.36 = 16.416$ cm $= x_{len} = 0.6 \times 0.152H$
$= 0.0912 H$

Ankle angle, $\theta_{dp} = 30°$

We set the θ_{dp}, ~~offset~~ to 0

From the paper, we have the following eqn

$\theta_{dp} = \sin^{-1}(x) \Rightarrow x = \sin(\theta_{dp}) = \sin(30°) = \frac{1}{2}$

$\therefore x = 0.5$ mts $= 50$ cm

From the paper, we have the following eqn

$$x = \left(\frac{(x_{tr,\,len})^2 + (L_{shank})^2 - (x_{link,\,disp})^2}{2 \cdot x_{len} L_{shank}} \right) \quad (1)$$

We need to find the length of the shank using the data from the anthropometric figure.

W.K.T. Length of leg, $L_{leg} = 0.285H$
from the knee to ground

Length of the foot $L_{ankle} = 0.039H$
from ankle to ground)

$\therefore$ Length of shank, $L_{shank} = L_{leg} - L_{ankle}$

$$= 0.285H - 0.039H$$

$$L_{shank} = 0.246H$$

Insert this value to eqn ①, we get

$$0.5 = \frac{(0.03)^2 + (0.246H)^2 - (x_{link, disp})^2}{2(0.0912H)(0.246H)}$$ ✓

$$0.5 \times 2 \times (0.0912H)(0.246H) = 0.0009 + 0.060516H^2 - (x_{link, disp})^2$$

$$0.0224352H^2 = 0.0009 + 0.060516H^2 - (x_{link, disp})^2$$

$$\Rightarrow (x_{link, disp})^2 = 0.0380808H^2 + 0.0009$$

Sq root on BS & taking only +ve values since the displacement during dorsiflexion is +ve (as per the paper)

$$x_{link, disp} = \sqrt{0.0380808H^2 + 0.0009} \quad \text{— ②}$$

From the paper, we also have the following eqn.

$$x_{link, disp} = \left(\frac{x_{aw-ad, len} - x_{right}}{2}\right) + \left(\frac{x_{aw-ad, len} - x_{left}}{2}\right)$$

$$x_{link,disp} = \frac{2x_{av\text{-}act,len} - x_{right} - x_{left}}{2}$$

$$x_{link,disp} = x_{av\text{-}act,len} - \left(\frac{x_{right} + x_{left}}{2}\right)$$

$$\therefore \frac{x_{right} + x_{left}}{2} = x_{av\text{-}act,len} - x_{link,disp} \quad \text{--- (3)}$$

Substituting eqn (2) in eqn (3) we get

$$\frac{x_{right} + x_{left}}{2} = x_{av\text{-}act,len} - \sqrt{0.0380808H^2 + 0.0009}$$

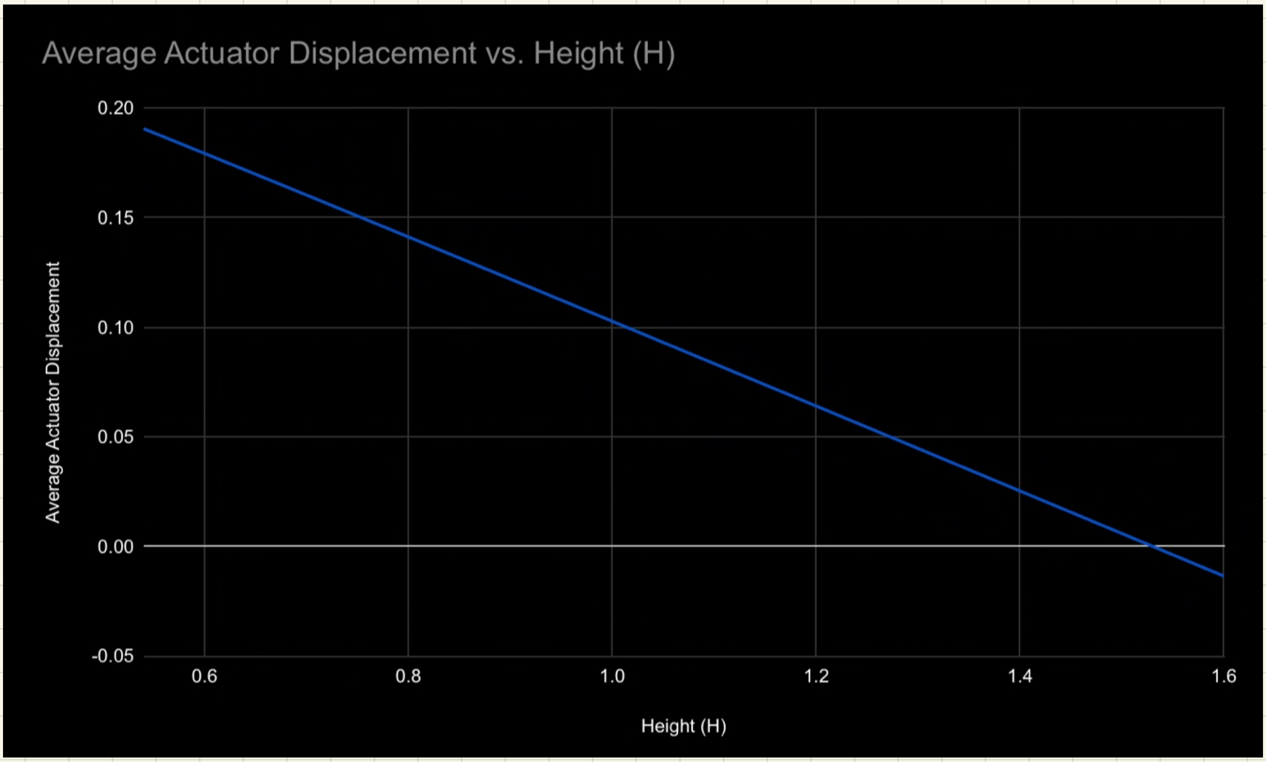
$$\frac{x_{right} + x_{left}}{2} = 0.3 - \sqrt{0.0380808\,H^2 + 0.0009}$$

$\Downarrow$

Avg. actuator displacement

$$\boxed{x_{av\text{-}act\,disp} = 0.3 - \sqrt{0.0380808H^2 + 0.0009}}$$

✓



✓

From this plot, we can see that the average displacement decreases with increase in Height & after 1.4 mtr it goes into the non-negative constraint for it.