ENGINEERING PHYSICS

JNTUH [R18]

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SYLLABUS

UNIT – I

Introduction to Mechanics: Transformation of scalars and vectors under Rotation transformation, Forces in Nature, Newton's laws and its completeness in describing particle motion, Form invariance of Newton's second law, Solving Newton's equations of motion in polar coordinates, Problems including constraints and friction, Extension to cylindrical and spherical coordinates.

UNIT – II

Harmonic Oscillations: Mechanical and electrical simple harmonic oscillators, Complex number notation and phasor representation of simple harmonic motion, Damped harmonic oscillator: heavy, critical and light damping, Energy decay in a damped harmonic oscillator, Quality factor, Mechanical
and electrical oscillators, Mechanical and electrical impedance, Steady state motion of forced damped harmonic oscillator, Power observed by oscillator.

UNIT – III

Waves in one dimension: Transverse wave on a string, The wave equation on a string, Harmonic waves, Reflection and transmission of waves at a boundary, Impedance matching, Standing waves and their Eigen frequencies, Longitudinal waves and the wave equations for them, Acoustic waves and speed of sound, Standing sound waves.

UNIT – IV

Wave Optics: Huygen's principle, Superposition of waves and interference of light by wave front splitting and amplitude splitting, Young's double slit experiment, Newton's rings, Michelson's interferometer, Mach-Zehnder interferometer, Frunhofer diffraction from a single slit and circular aperture, Diffraction grating- resolving power.

UNIT – V

Lasers and Fibre Optics:

Lasers: Introduction to interaction of radiation with matter, Coherence, Principle and working of Laser, Population inversion, Pumping, Types of Lasers: Ruby laser, Carbon dioxide (CO2) laser, He-Ne laser, Applications of laser.

Fibre Optics: Introduction, Optical fibre as a dielectric wave guide, Total internal reflection, Acceptance angle, Acceptance cone and Numerical aperture, Step and Graded index fibres, Losses associated with optical fibres, Applications of optical fibres.

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UNIT – I

INTRODUCTION TO MECHANICS

SYLLABUS

- Transformation of scalars and vectors under Rotation transformation.
- Forces in Nature.
- Newton's laws and its completeness in describing particle motion.
- Form invariance of Newton's second law.
- Solving Newton's equations of motion in polar coordinates.

BASIC TERMS

1. **Scalar quantity:** A quantity which has magnitude but no direction is called as scalar quantity. Examples: Distance, Mass, Time & Length etc.
2. **Vector quantity:** A quantity which has both magnitude & direction is called as Vector quantity. Examples: Velocity, Acceleration & Amplitude.

TRANSFORMATION OF SCALARS UNDER ROTATION TRANSFORMATION

A quantity which has magnitude but no direction is called as scalar quantity. For instance, consider a car moving on a road. The distance travelled by the car is a scalar quantity. Let $X - Y$ and $X' - Y'$ represent original and rotated frames respectively.

Let the car travels a distance of 1000 m in $X - Y$ frame. If the frame is rotated about z-axis i.e., $X' - Y'$ frames, then the distance travelled by the car is 1000 m as illustrated in figure 1.

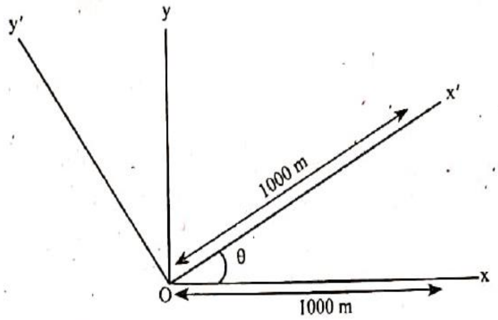


Figure (1)

The distance travelled is same when see from $X - Y$ frame or $X' - Y'$ frame i.e., 1000 m. Hence, a scalar quantity remains unchanged when seen from original or rotated frame.

TRANSFORMATION OF VECTORS UNDER ROTATION TRANSFORMATION

A quantity which has both magnitude and direction is called as vector quantity. For instance, consider a car moving 800 m along x-axis and 600 m along y axis as illustrated in figure (2).

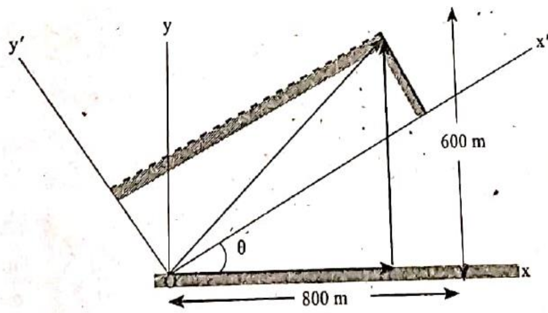


Figure 2

The total distance travelled by the car is 400 m in X–Y frame. The net displacement is a vector 1000 m in magnitude at an angle $\tan^{-1}\left(\frac{600}{800}\right)$ i.e., $\tan^{-1}\left(\frac{3}{4}\right)$ from x-axis.

In X'–Y' frame, the distance is 1400 m. As distance is a scalar quantity, it remains unchanged. It can be seen from figur 2 that the vector components X' and Y' are different. Hence, the vector components are different in rotated frame.

Equations for transformation of vectors under rotation about z-axis:

The equations for transformation of vectors under rotation about z-axis are,

$$A_{x'} = A_x \cos\theta + A_y \sin\theta$$

$$A_{y'} = -A_x \sin\theta + A_y \cos\theta$$

$$A_{z'} = A_z$$

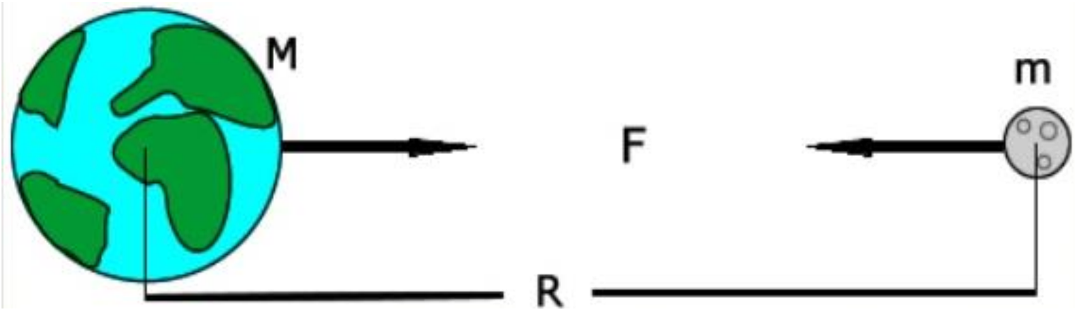
FORCES IN NATURE

Force: A force is a push or pull acting on an object that changes the motion of the object.

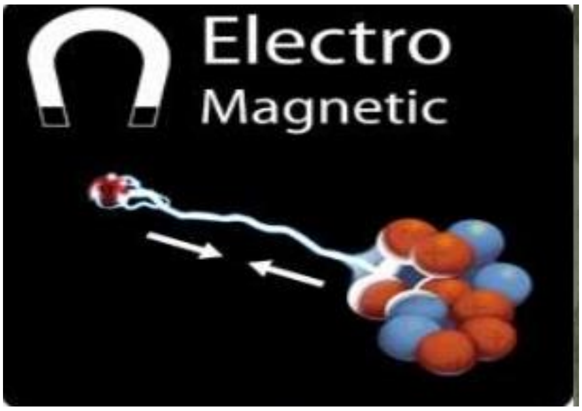
Types of forces: The four fundamental forces of nature are:

1. Gravitational force.
2. Electromagnetic force.
3. Weak nuclear force.
4. Strong force.

1. Gravitational force: This is the force which is always attractive. It acts between two masses. Each and every object in this universe applies this force on all other objects, This is the weakest force.

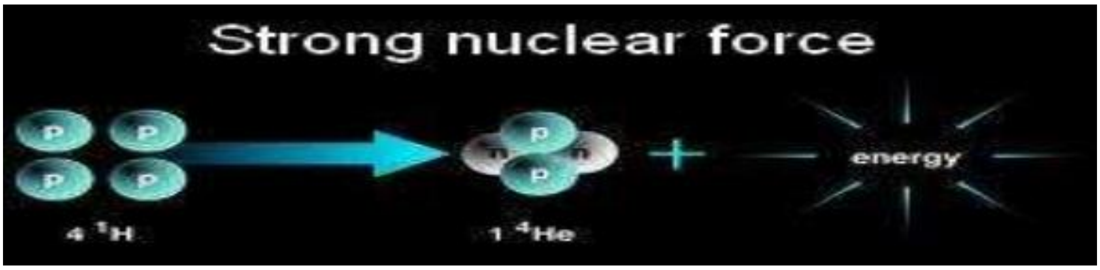


2. Electromagnetic force: When an electric charge is moving, it produces magnetic force besides the electric force. The combined effect of these two forces, is called the electromagnetic force. It is stronger than gravitational force.



3. Weak nuclear force: This is the force which is responsible for emission of beta particles from the nucleus. This force is more powerful than the gravitational force but it is weaker than the electromagnetic force.

4. Strong force: This is the strongest force in the nature. It acts on any two nucleons (i.e. proton and neutron) in the nucleus.



NEWTON'S LAWS AND ITS COMPLETENESS IN DESCRIBING PARTICLE MOTION

- Newton's laws of motion are three physical laws that, together, laid the foundation for classical mechanics.
- They describe the relationship between a body and the forces acting upon it, and its motion in response to those forces.
- More precisely, the first law defines the force qualitatively, the second law offers a quantitative measure of the force, and the third asserts that a single isolated force doesn't exist.
- These three laws have been expressed in several ways, over nearly three centuries, and can be summarised as follows:
  1. **First law:** In an inertial frame of reference, an object either remains at rest or continues to move at a constant velocity, unless acted upon by a force.
  2. **Second law:** In an inertial reference frame, the vector sum of the forces F on an object is equal to the mass m of that object multiplied by the acceleration a of the object: F = ma. (It is assumed here that the mass m is constant).
  3. **Third law:** When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

Mathematically,

$$\vec{F}_{12} = -\vec{F}_{21} \quad \ldots (4)$$

Where,

$\vec{F}_{12}$ – Force exerted on first body by the second body

$\vec{F}_{21}$ – Force exerted on second body by the first body.

FORM INVARIANCE OF NEWTON'S SECOND LAW

In frame S

$$\boldsymbol{F} = \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{p} = \frac{\mathrm{d}}{\mathrm{d}t}m\boldsymbol{v} = m\boldsymbol{a}$$

We can see that the direction of acceleration is the same as the force and that

$$m = \frac{|\boldsymbol{F}|}{|\boldsymbol{a}|}$$

is the **inertial mass**, *i.e.* the resistance of a body to it being accelerated.

Note that only **external** forces can change the state of motion (I cannot lift myself, for example).

In the frame S'

$$\boldsymbol{F}' = m\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{v} - \boldsymbol{u}) = m\dot{\boldsymbol{v}} - m\dot{\boldsymbol{u}}$$

As $\boldsymbol{u}$ is a constant, $\dot{\boldsymbol{u}} = \boldsymbol{0}$, and so

$$\boldsymbol{F}'(x', y', z') = \boldsymbol{F}(x, y, z)$$

which is what we expect from a vector: the force is the same but the individual components of the force change with a change of reference frame.

We will return to invariance later when we consider special relativity. For now, it is sufficient that we know that forces do not depend on our frame of reference.

SOLVING NEWTON'S EQUATIONS OF MOTION IN POLAR COORDINATES [OR] THE EQUATIONS OF MOTION WITH POLAR COORDINATES

- To finish our discussion of the equations of motion in two dimensions, we will examine Newton's Second law as it is applied to the polar coordinate system.
- In its basic form, Newton's Second Law states that the sum of the forces on a body will be equal to mass of that body times the rate of acceleration.
- For bodies in motion, we can write this relationship out as the equation of motion.

$$\sum \vec{F} = m * \vec{a}$$

- Just as we did with with rectangular and normal-tangential coordinates, we will break this single vector equation into two separate scalar equations.
- This involves identifying the r and theta directions and then using sines and cosines to break the given forces and accelerations down into components in those directions.

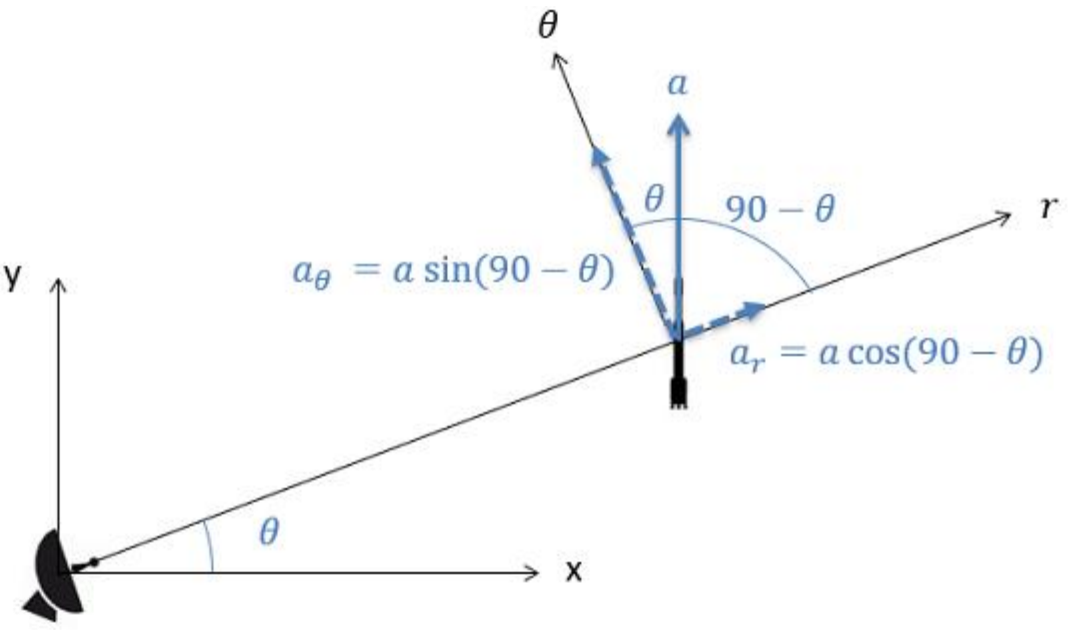


Fig: When working in the polar coordinate system, any given forces or accelerations can be broken down using sines and cosines assuming the angle of the force or acceleration is known relative to the r and theta directions.

$$\sum F_n = m * a_r$$

$$\sum F_t = m * a_\theta$$

- Just as with our other coordinate systems, the equations of motion are often used in conjunction with the kinematics equations, which relate positions, velocities and accelerations.
- In particular, we will often substitute the known values below for the r and theta components for acceleration.

$$a_r = \ddot{r} - r\dot{\Theta}^2$$

$$a_\theta = 2\dot{r}\dot{\Theta} + r\ddot{\Theta}$$

- Polar coordinates can be used in any kinetics problem, however they work best with problems where there is a stationary body tracking some moving body (such as a radar dish) or there is a particle rotating around some fixed point.
- These equations will also come back into play when we start examining rigid body kinematics.

Engineering Physics

Introduction to Mechanics

Numerical problems:-

①. Two bodies 50N & 30N are connected the two ends a light inextensible string to string is passing over a smooth pulley. Determine:-

(i). Acceleration. (ii). Tension.

Sol:- $W_1 = 50N$; $W_2 = 30N$; $\mu = 0$ (pulley is smooth)

(i) Acceleration:-

$$a = \frac{g(W_1 - W_2)}{W_1 + W_2}$$

$$= \frac{9.8(50-30)}{50+30}$$

$$= 9.8\left(\frac{20}{80}\right)$$

$$= \frac{9.8}{4}$$

$$\boxed{a = 2.45\, m/s^2}$$

(ii). Tension:-

$$T = \frac{2W_1 W_2}{W_1 + W_2}$$

$$= \frac{2 \times 50 \times 30}{50+30}$$

$$= \frac{3000}{80}$$

$$\boxed{T = 37.5N}$$

(Ans)

②. A lift carries a weight of 150N & is moving with a uniform acceleration of $2.45 m/s^2$. Determine the tension in the cable.

(i). When lift is moving upwards.

(ii). " " " " downwards.

Sol:- $W = 150N$; $a = 45 m/s^2$

(i). When lift is moving upwards:-

$$\text{Tension}(T) = W\left(1+\frac{a}{g}\right)$$

where, g = Acceleration due to gravity $= 9.8 m/s^2$

$$\Rightarrow T_{upwards} = W\left(1+\frac{a}{g}\right) \rightarrow (1)$$

$$= 150\left(1+\frac{2.45}{9.8}\right)$$

$$\boxed{T_{upwards} = 187.5N}$$

(ii). When lift is moving downwards:-

$$\text{Tension}(T) = W\left(1-\frac{a}{g}\right) \rightarrow (2)$$

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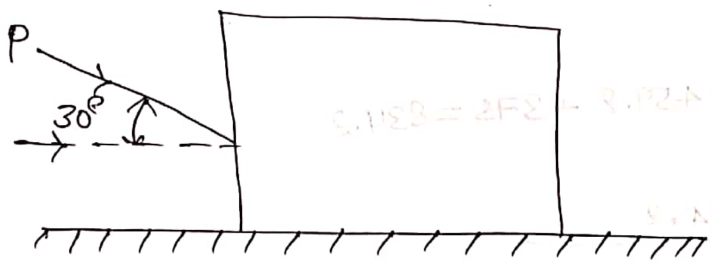
$$T_{downwards} = 150\left(1 - \frac{2.45}{9.8}\right)$$

$$= 150\,(0.75)$$

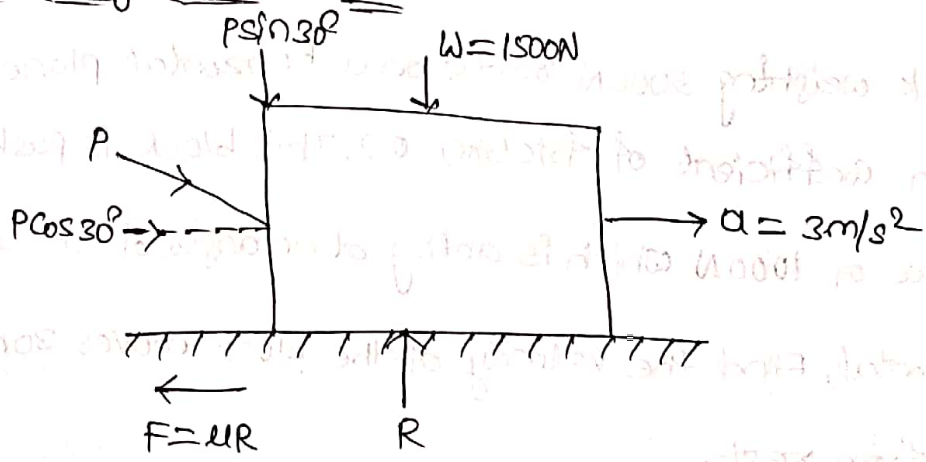
$$\boxed{T_{downwards} = 112.5\,N}$$ (Ans)

③. A block weighing 1.5KN rests on a horizontal plane as shown in figure. Find the magnitude of the force 'P' required to give the block an acceleration of $3\,m/s^2$ to the right. (Given $\mu = 0.25$).



Sol:- $W = 1.5KN = 500N$; $a = 3\,m/s^2$; $\mu = 0.25$

Free Body Diagram (FBD):-



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from FBD, $\Sigma F_y = ma_y$

$\Rightarrow P\sin 30^\circ + 1500 - R = m(0)$ [$\because a_y = 0$].

$R = P\sin 30^\circ + 1500$

$\therefore R = 1500 + \frac{P}{2}$

$\Sigma F_x = ma_x$

$\Rightarrow P\cos 30^\circ - \mu R = ma_x$

$\Rightarrow P\cos 30^\circ - 0.25\left(1500 + \frac{P}{2}\right) = \frac{1500}{9.8} \times 3$

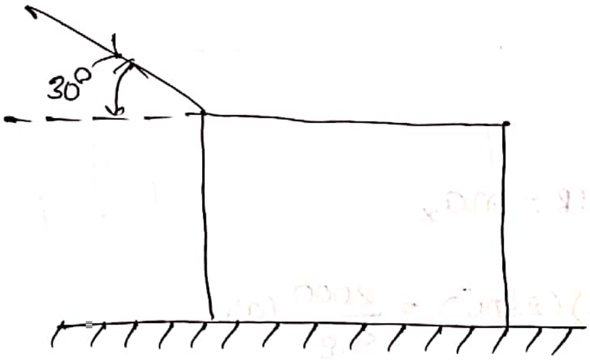
$\Rightarrow \frac{\sqrt{3}}{2}P - 375 - 0.125P = 459.2$

$\Rightarrow P\left(\frac{\sqrt{3}}{2} - 0.125\right) = 459.2 + 375 = 834.2$

$\Rightarrow$ ~~P(~~ $P = \frac{834.2}{(0.866 - 0.125)}$

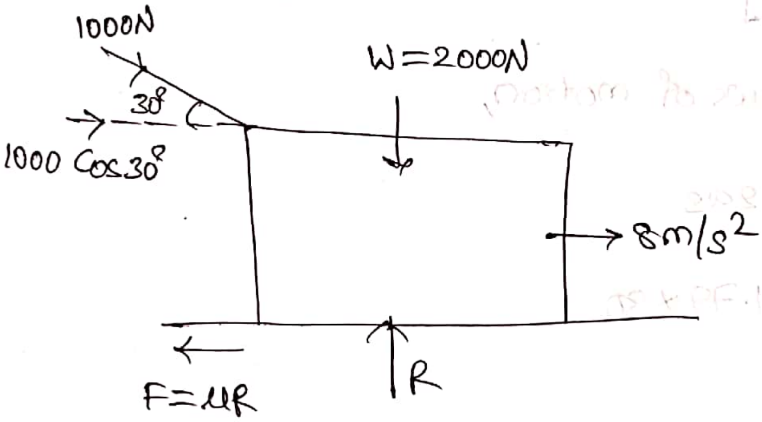
$\therefore \boxed{P = 1125.77N}$ (Ans)

④. A block weighting 2000N rests on a horizontal plane for which coefficient of friction 0.2. This block is pushed by a force of 1000N which is acting at an angle of 30° to the horizontal, Find the velocity of the block moves 30m, starting from rest.



Sol:— $W = 2000N$; $\mu = 0.2$; $F = 1000\,N$; $\theta = 30$

Free Body Diagram:—



From D-Alembert's principle,

$$\Sigma F_x = ma_x$$

$$1000\sin 30° + W - R = ma_x$$

$$1000\sin 30° + 2000 - R = \frac{2000}{9.8}.$$

$$\frac{1000}{2} + 2000 - R = 0$$

$$R = 2000 + 500$$

$$\therefore R = 2500N$$

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Consider,

$$\Sigma F_x = m a_x$$

$$1000 \cos 30^\circ - \mu R = m a_x$$

$$1000 \cos 30^\circ - (0.2)(2500) = \frac{2000}{9.8}(a)$$

$$a = \frac{1000(0.866) - 500}{204.1}$$

$$\boxed{a = 1.79 m/s^2}$$

According to laws of motion,

$$v^2 - u^2 = 2as$$

$$v^2 - 0^2 = 2 \times 1.79 \times 30$$

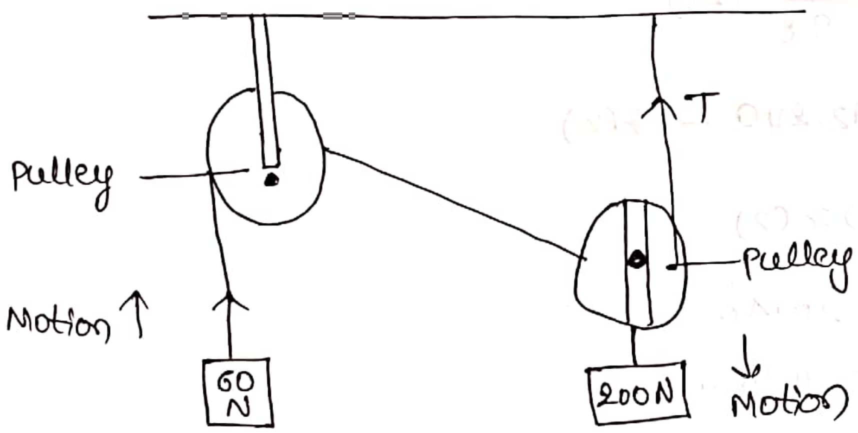
$$v^2 = 107.4$$

$$v = \sqrt{107.4}$$

$$\boxed{v = 10.36 m/s}$$

=(Ans)

⑤. Find the tension & acceleration of the two bodies of 900N & 60N connected by a string in figure.



Sol:- $W_1 = 200N$; $W_2 = 60N$

In the given system,

If W_1 moves a distance of x, then W_2 moves a distance of $2x$.

If the acceleration of $W_1 = a$, then acceleration of $W_2 = 2a$

For 200N:-

From D'Alembert's principle

$\Sigma F = ma$

$200 - (T+T) = \frac{W_1}{g}(a)$

$200 - 2T = \frac{200}{9.8}(a)$

$200 - 2T = 20.4 \longrightarrow (1)$

For 60N:-

$T - 60 = m_2 a$

$T - 60 = \frac{W_2}{g}(2a)$

$$T - 60 = \frac{60}{9.8}(2a)$$

$$T - 60 = 12.24a \longrightarrow (2)$$

solving (1) & (2)

$$200 - \cancel{2T} = 20.4a$$
$$-120 + \cancel{2T} = 24.48a$$
$$80 = 44.48a$$

$$a = \frac{80}{44.88} = 1.78 m/s^2 \quad \text{(sub in eq (1))}$$

$$200 - 2T = 20.4(a)$$

$$2T = 200 - 20.4(1.78)$$

$$T = \frac{163.688}{2}$$

$$\boxed{T = 81.84N}$$

= Ans

THE END

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UNIT – II

HARMONIC OSCILLATIONS

SYLLABUS

- Mechanical and electrical simple harmonic oscillators.
- Damped harmonic oscillator: critical light damping.
- Energy decay in a damped harmonic oscillator.
- Quality factor.
- Mechanical and electrical oscillators.

INTRODUCTION

When the restoring force is directly proportional to the displacement from equilibrium, the resulting motion is known as simple harmonic motion and the oscillator is known as Harmonic Oscillator.

MECHANICAL AND ELECTRICAL SIMPLE HARMONIC OSCILLATORS

(i) The differential equation of mechanical oscillator is given as,

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Where, x – Displacement

ω – Angular frequency

(ii) The differential equation of electrical harmonic oscillator is given as,

$$\frac{d^2q}{dt^2} + \frac{1}{LC} q = 0$$

Where,

q – Charge

L – Inductance

C – Capacitance.

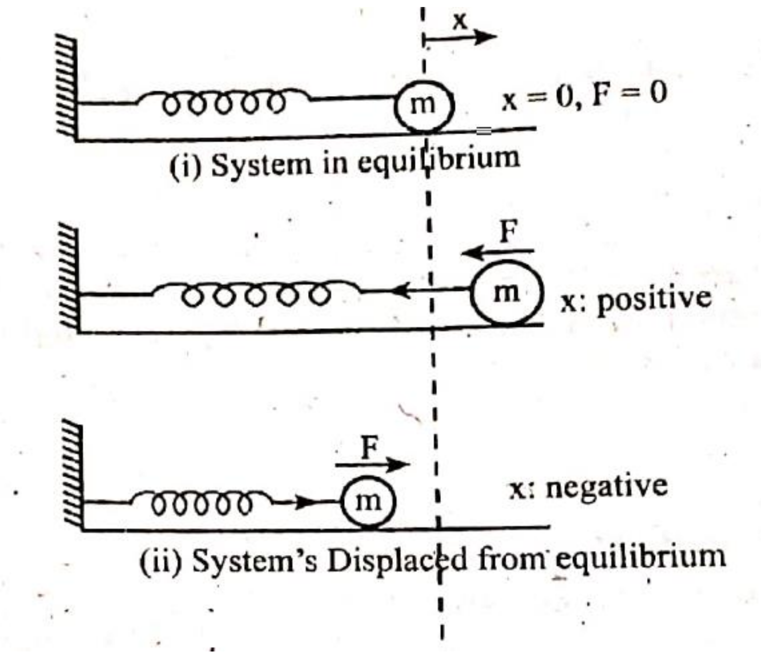


Fig: Mechanical simple harmonic oscillators

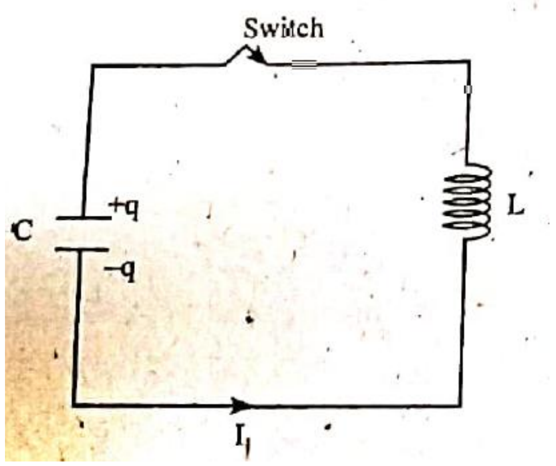


Fig: electrical simple harmonic oscillators

DAMPED OSCILLATION

- For a free oscillation the energy remains constant. Hence oscillation continues indefinitely.
- However in real fact, the amplitude of the oscillatory system gradually decreases due to experiences of damping force like friction and resistance of the media.
- The oscillators whose amplitude, in successive oscillations goes on decreasing due to the presence of resistive forces are called damped oscillators, and oscillation called damping oscillation.

- The damping force always acts in a opposite directions to that of motion of oscillatory body and velocity dependent.

$$F_{dam} \propto -v$$

$$F_{dam} = -bv$$

b = damping constant which is a positive quantity defined as damping force/velocity,

$$F_{net} = F_{res} + F_{dam}$$

$$F_{net} = -kx - bv$$

$$F_{net} = -kx - b\frac{dx}{dt}$$

$$M\frac{d2x}{dt2} + kx + b\frac{dx}{dt} = 0$$

$$\frac{d2x}{dt2} + \frac{b}{M}\frac{dx}{dt} + \frac{k}{M}x = 0$$

$$\frac{d2x}{dt2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0 \quad \ldots\ldots\ldots\ldots \ (2)$$

Where $\beta = \frac{b}{2M}$ is the damping co-efficient & $\omega_0 = \sqrt{\frac{k}{M}}$ is called the natural frequency of oscillating body.

The above equation is second degree linear homogeneous equation.

The general solution of above equation is found out by assuming x(t), a function which is given by

$$x(t) = Ae^{\alpha t}$$

$$\frac{dx}{dt} = A\alpha e^{\alpha t} = \alpha x$$

$$\frac{d2x}{dt2} = A\alpha^2 e^{\alpha t} = \alpha^2 x$$

Putting these values in equation

$$\alpha^2 x + 2\alpha^2\beta x + \omega_0^2 x = 0$$

$$\alpha^2 + 2\alpha^2\beta + \omega_0^2 = 0 \;\ldots\ldots\ldots\ldots\; (3)$$

$\alpha = -\beta \pm \sqrt{\beta 2 - }\omega_0^2$, is the general solution of above quadratic equation.

As we know,

$$x(t) = A_1 e^{\alpha t} + A_2 e^{\alpha t}$$

$$x(t) = A_1 e^{(-\beta + \sqrt{\beta 2 -}\ \omega 02)t} + A_2 e^{(-\beta - \sqrt{\beta 2 -}\ \omega 02)t}$$

$$x(t) = e^{-\beta t}\ (A_1 e^{\sqrt{\beta 2 -}\ \omega 02\ t} + A_2 e^{-\sqrt{\beta 2 -}\ \omega 02\ t}) \ldots (4)$$

Depending upon the strength of damping force the quantity (β^2-ω_0^2) can be positive /negative /zero giving rise to three different cases.

Case-1:- if $\beta < \omega_0^2$ => underdamping (oscillatory)

Case-2:- if $\beta > \omega_0^2$ => overdamping (non-oscillatory)

Case-3:- if $\beta = \omega_0^2$ => critical damping (non-oscillatory)

CRITICAL DAMPING

$$\beta^2 = \omega_0{}^2$$

The general solution of equation in this case,

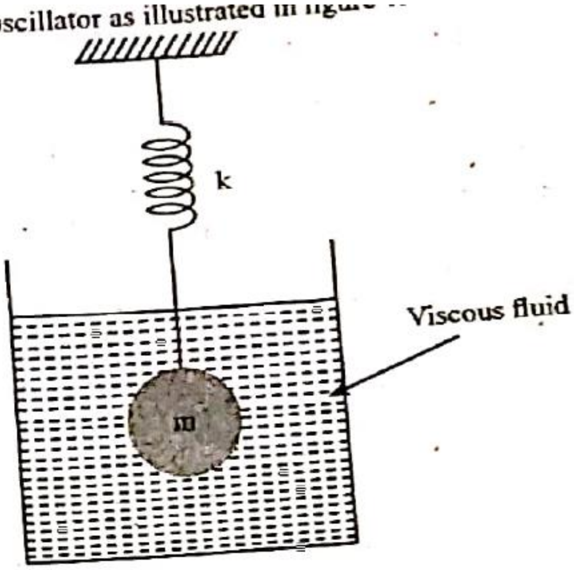
$$X(t) = (Ct+D)\, e^{-\beta t}$$

Here the displacement approaches to zero asymptotically for given value of initial position and velocity a critically damped oscillator approaches equilibrium position faster than other two cases.

Example: The springs of automobiles or the springs of dead beat galvanometer.

ENERGY DECAY IN A DAMPED HARMONIC OSCILLATOR

Consider a mechanical harmonic oscillator as illustrated in figure



The kinetic energy of oscillator is given as,

$$K = \frac{1}{2}mv^2$$

Where,

m – Mass

v – Velocity ... (2)

The potential energy of oscillator is given as,

$$U = \frac{1}{2}kx^2$$

Where,

K – Spring constant

x – Displacement ... (3)

The total energy of oscillator is given as,

$$E = K + U \qquad \text{... (4)}$$

Substituting equations (1) and (2) in equation (3)

Substituting

$$E = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$$

If very light damping is considered, i.e., $\frac{\gamma^2}{4} << \omega_0^2$, then the displacement is given as,

$$x = A_0 \exp\left(-\frac{\gamma t}{2}\right)\cos\omega_0 t \quad \text{... (5)}$$

Where,

A_0 – initial value of amplitude

ω_0 – Angular frequency

Differentiating equation (5) with respect to 't',

$$\frac{dx}{dt} = v = -A_0\omega_0 \exp\left(-\frac{\gamma t}{2}\right)\left[\sin\omega_0 t + \left(\frac{\gamma}{2\omega_0}\right)\cos\omega_0 t\right]$$

As $\frac{\gamma}{2} << \omega_0$, then second term can be neglected.

$$v = \frac{dx}{dt} = -A_0\omega_0 \exp\left(-\frac{\gamma t}{2}\right)\sin\omega_0 t$$

Substituting equations (5) and (6) in equation (4),

$$E = \frac{1}{2}m\left(-A_0\omega_0 \exp\left(-\frac{\gamma t}{2}\right)\sin\omega_0 t\right)^2 + \frac{1}{2}k\left(A_0 \exp\left(-\frac{\gamma t}{2}\right)\cos\omega_0 t\right)^2$$

$$= \frac{1}{2}mA_0^2\omega_0^2\left[\exp\left(-\frac{\gamma t}{2}\right)\right]^2 \sin^2\omega_0 t + \frac{1}{2}kA_0^2\left(\exp\left(-\frac{\gamma t}{2}\right)\right)^2 \cos^2\omega_0 t$$

$$\Rightarrow E = \frac{1}{2}A_0^2 \exp(-\gamma t)\left(m\omega_0^2 \sin^2\omega_0 t + k\cos^2\omega_0 t\right)$$

NGINEERING STUDENTS

QUALITY FACTOR

$$Q = 2\pi.\frac{\text{Energy stored in system}}{\text{Energy loss per period}} = 2\pi.\frac{<E>}{<P>T} = \frac{2\pi}{T}.\tau$$

$$\Rightarrow Q = \omega\tau$$

MECHANICAL AND ELECTRICAL OSCILLATORS

S.No.	Mechanical Oscillator	S.No.	Electrical Oscillator
1.	The impedence of mchanical oscillator is, $\lvert Z\rvert = \sqrt{b^2 + \left(\frac{k}{\omega} - \omega m\right)^2}$ Where, b – Constant; k – Spring constant; ω – Angular frequency; m – Mass	1.	The impedence of electrical oscillator is, $\lvert Z\rvert = \sqrt{R^2 + \left(\frac{1}{\omega c} - \omega L\right)^2}$ Where, R – Resistance; w – Angular frequency; C – Capacitance; L – Inductance
2.	The mechanical impedence has two reactances i.e., reactance due to stiffness $\left(\frac{k}{\omega}\right)$ and reactance due to mass (m)	2.	The electrical impedence hase resistance, R and reactance $\left(\text{i.e., } \frac{1}{\omega C} - \omega L\right)$
3.	The phase difference is given as, $\tan\phi = \frac{\left(\frac{k}{\omega} - \omega m\right)}{b}$	3.	The phase difference is given as, $\tan\phi = \frac{\left(\frac{1}{\omega C} - \omega L\right)}{R}$
4.	The frequency is given as, $n = \frac{1}{2\pi\sqrt{LC}}$	4.	The frequency is given as, $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
5.	The amplitude of oscillations is maximum at resonant frequency	5.	The current in the circuit is maximum at resonant frequency.

Engineering Physics

Harmonic Oscillations

Numerical problems:-

①. The electron in an excited atom behaves like a damped harmonic oscillator when the atom radiates light. The lifetime of an excited atomic state is 10^{-8}s and the wavelength of the emitted light is 500nm. Deduce a value for the quality factor.

Sol:-

$\tau = 10^{-8}s$; $\lambda = 500nm = 500 \times 10^{-9}m$

Quality factor $(Q) = \omega\tau = 2\pi n\tau \rightarrow (1)$

But, $n = \frac{c}{\lambda}$

where, $c = 3 \times 10^{8} m/s$

sub in eq(1),

$$Q = \frac{2\pi \times 3 \times 10^{8} \times 10^{-8}}{500 \times 10^{-9}}$$

$$= \frac{6\pi}{5} \times 10^{7}$$

$$\boxed{Q = 3.77 \times 10^{7}}$$ (Ans)

(2). A mass of 1.5kg rests on a horizontal table & is attached to one end of a spring of spring constant $150 Nm^{-1}$, The other end of the spring is moved in the horizontal direction according to $x = a\cos\omega t$. where, $a = 5\times10^{-3} m$ & $\omega = 6\pi$ rad/sec. The damping constant $b = 3.2 Nm^{-1}s$. Determine the amplitude & relative phase of the steady state oscillations of the mass. Show that if the applied frequency were adjusted for resonance, the mass would oscillate with an amplitude of approximately, $2.5\times10^{-2} m$.

Sol:-

$m = 1.5 kg$

$K = 150 Nm^{-1}$

$b = 3.0 Nm^{-1}s$

$\omega = 6\pi$ rad/s

$a = 5\times10^{-3} m$

Amplitude of a forced oscillator,

$$A = \frac{aK/m}{\left[(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2\right]^{1/2}} \longrightarrow (1)$$

Consider,

$$a, \frac{k}{m} = \frac{150 \times 5 \times 10^{-3}}{1.5} = \frac{1}{2}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{150}{1.5}}$$

$$= 10.\text{rad}.s^{-1}$$

$$\gamma = \frac{b}{m} = \frac{3}{1.5} = 2s^{-1}$$

Sub, the corresponding values in eq (1)

$$A(\omega) = \frac{1}{2\sqrt{(10^2-(6\pi)^2)^2 + (2)^2(6\pi)^2}}$$

$$\therefore \boxed{A(\omega) = 1.96 \times 10^{-3} m}$$

Relative phase of the steady state oscillations:-

$$\text{Relative phase}, \tan\alpha = \frac{\omega\gamma}{(\omega_0^2 - \omega^2)}$$

$$= \frac{6\pi \times 2}{10^2 - (6\pi)^2}$$

$$= \frac{12\pi}{100 - 355.3}$$

$\tan\alpha = -0.15$

since, $\omega > \omega_0$, the phase angle must lie b/w $\frac{\pi}{2}$ & π

when the frequency is adjusted to resonance,

$\omega = \omega_0$

$$\Rightarrow A(\omega) = \frac{a\omega_0^2}{(\omega_0)2b}$$

$$= \frac{a\omega_0}{2b}$$

$$= \frac{5\times10^{-3}\times10}{2}$$

$$\boxed{A(\omega) = 2.5\times10^{-2}m}$$

Ans

THE END

Prepared By:-

Riyaz Mohammed

Prasha

UNIT – III

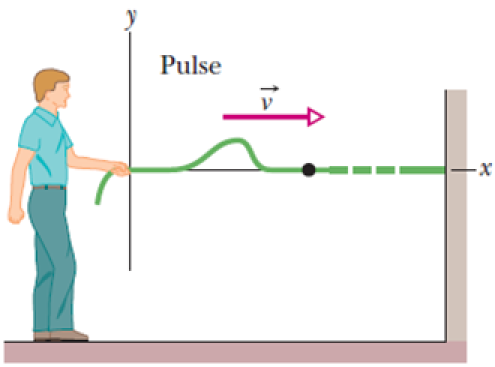
WAVES IN ONE DIMENSION

SYLLABUS

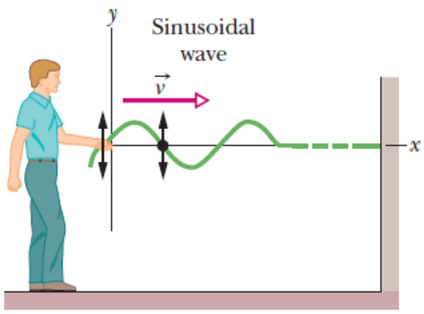
- Transverse wave on a string.
- The wave equation on a string.
- Standing waves.
- Longitudinal waves.
- Speed of sound.

TRANSVERSE WAVE ON A STRING

- A wave sent along a stretched, that string is the simplest mechanical wave.
- If you give one end of a stretched string a single up-and-down jerk, a wave in the form of a single pulse travels along the string. This pulse and its motion can occur because the string is under tension.
- When you pull your end of the string upward, it begins to pull upward on the adjacent section of the string via tension between the two sections.
- As the adjacent section moves upward, it begins to pull the next section upward, and so on. Meanwhile, you have pulled down on your end of the string.
- As each section moves upward in turn, it begins to be pulled back downward by neighbouring sections that are already on the way down.
- The net result is that a distortion in the string's shape (a pulse, as in Fig. a) moves along the string at some velocity v:



(a)



(b)

Fig: (a) A single pulse is sent along a stretched string. A typical string element (marked with a dot) moves up once and then down as the pulse passes. The element's motion is perpendicular to the wave's direction of travel, so the pulse is a transverse wave. (b) A sinusoidal wave is sent along the string. A typical string element moves up and down continuously as the wave passes. This too is a transverse wave.

- If you move your hand up and down in continuous simple harmonic motion, a continuous wave travels along the string at velocity.
- Because the motion of your hand is a sinusoidal function of time, the wave has a sinusoidal shape at any given instant, that is, the wave has the shape of a sine curve or a cosine curve.
- We consider here only an "ideal" string, in which no friction-like forces within the string cause the wave to die out as it travels along the string.
- In addition, we assume that the string is so long that we need not consider a wave rebounding from the far end.
- One way to study the waves of Fig is to monitor the wave forms (shapes of the waves) as they move to the right.
- Alternatively, we could monitor the motion of an element of the string as the element oscillates up and down while a wave passes through it.
- We would find that the displacement of every such oscillating string element is perpendicular to the direction of travel of the wave.
- This motion is said to be transverse, and the wave is said to be a transverse wave.

THE WAVE EQUATION ON A STRING

- As a wave passes through any element on a stretched string, the element moves perpendicularly to the wave's direction of travel (we are dealing with a transverse wave).
- By applying Newton's second law to the element's motion, we can derive a general differential equation, called the wave equation, that governs the travel of waves of any type.
- Figure 16-11a shows a snapshot of a string element of mass and length as a wave travels along a string of linear density m that is stretched along a horizontal x axis.
- Let us assume that the wave amplitude is small so that the element can be tilted only slightly from the x axis as the wave passes.
- The force 2 on the right end of the element has a magnitude equal to tension t in the string and is directed slightly upward.
- The force 1 on the left end of the element also has a magnitude equal to the tension t but is directed slightly downward.
- Because of the slight curvature of the element, these two forces are not simply in opposite direction so that they cancel.
- Instead, they combine to produce a net force that causes the element to have an upward acceleration ay.
- Newton's second law written for y components:

$$F_{2y} - F_{1y} = dm\, a_y. \tag{16-34}$$

Let's analyze this equation in parts, first the mass dm, then the acceleration component a_y, then the individual force components F_{2y} and F_{1y}, and then finally the net force that is on the left side of Eq. 16-34.

Mass. The element's mass dm can be written in terms of the string's linear density μ and the element's length ℓ as $dm = \mu\ell$. Because the element can have only a slight tilt, $\ell \approx dx$ (Fig. 16-11a) and we have the approximation

$$dm = \mu\, dx. \tag{16-35}$$

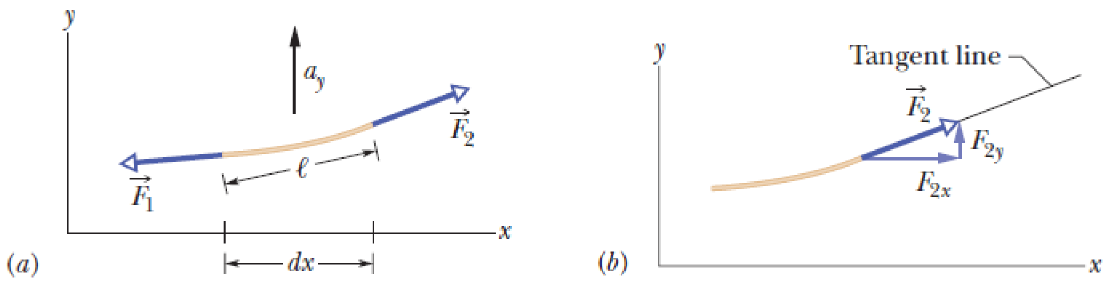


Figure 16-11 (*a*) A string element as a sinusoidal transverse wave travels on a stretched string. Forces $\vec{F}_1$ and $\vec{F}_2$ act at the left and right ends, producing acceleration $\vec{a}$ having a vertical component a_y. (*b*) The force at the element's right end is directed along a tangent to the element's right side.

Acceleration. The acceleration a_y in Eq. 16-34 is the second derivative of the displacement y with respect to time:

$$a_y = \frac{d^2y}{dt^2}. \qquad (16\text{-}36)$$

Forces. Figure 16-11*b* shows that $\vec{F}_2$ is tangent to the string at the right end of the string element. Thus we can relate the components of the force to the string slope S_2 at the right end as

$$\frac{F_{2y}}{F_{2x}} = S_2. \qquad (16\text{-}37)$$

We can also relate the components to the magnitude F_2 $(= \tau)$ with

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2}$$

or

$$\tau = \sqrt{F_{2x}^2 + F_{2y}^2}. \qquad (16\text{-}38)$$

However, because we assume that the element is only slightly tilted, $F_{2y} \ll F_{2x}$ and therefore we can rewrite Eq. 16-38 as

$$\tau = F_{2x}. \qquad (16\text{-}39)$$

Substituting this into Eq. 16-37 and solving for F_{2y} yield

$$F_{2y} = \tau S_2. \qquad (16\text{-}40)$$

Similar analysis at the left end of the string element gives us

$$F_{1y} = \tau S_1. \qquad (16\text{-}41)$$

Net Force. We can now substitute Eqs. 16-35, 16-36, 16-40, and 16-41 into Eq. 16-34 to write

$$\tau S_2 - \tau S_1 = (\mu\, dx)\frac{d^2y}{dt^2},$$

or

$$\frac{S_2 - S_1}{dx} = \frac{\mu}{\tau}\frac{d^2y}{dt^2}. \qquad (16\text{-}42)$$

Because the string element is short, slopes S_2 and S_1 differ by only a differential amount dS, where S is the slope at any point:

Because the string element is short, slopes S_2 and S_1 differ by only a differential amount dS, where S is the slope at any point:

$$S = \frac{dy}{dx}. \tag{16-43}$$

First replacing $S_2 - S_1$ in Eq. 16-42 with dS and then using Eq. 16-43 to substitute dy/dx for S, we find

$$\frac{dS}{dx} = \frac{\mu}{\tau}\frac{d^2y}{dt^2},$$

$$\frac{d(dy/dx)}{dx} = \frac{\mu}{\tau}\frac{d^2y}{dt^2},$$

and

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{\tau}\frac{\partial^2 y}{\partial t^2}. \tag{16-44}$$

In the last step, we switched to the notation of partial derivatives because on the left we differentiate only with respect to x and on the right we differentiate only with respect to t. Finally, substituting from Eq. 16-26 ($v = \sqrt{\tau/\mu}$), we find

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \text{(wave equation)}. \tag{16-45}$$

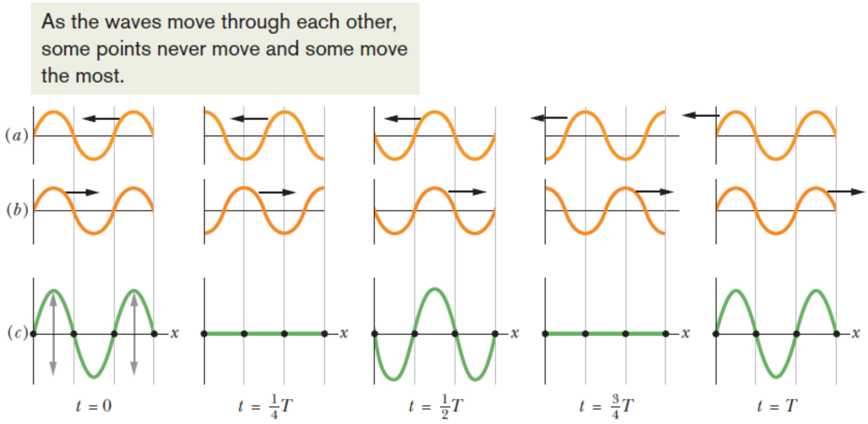
This is the general differential equation that governs the travel of waves of all types.

STANDING WAVES

- Figure 16-17 suggests the situation graphically.
- It shows the two combining waves, one traveling to the left in Fig. 16-17a, the other to the right in Fig. 16-17b. Figure 16-17c shows their sum, obtained by applying the superposition principle graphically.
- The outstanding feature of the resultant wave is that there are places along the string, called nodes, where the string never moves.
- Four such nodes are marked by dots in Fig. 16-17c.
- Halfway between adjacent nodes are antinodes, where the amplitude of the resultant wave is a maximum.
- Wave patterns such as that of Fig. 16-17c are called standing waves because the wave patterns do not move left or right; the locations of the maxima and minima do not change.

- If two sinusoidal waves of the same amplitude and wavelength travel in opposite directions along a stretched string, their interference with each other produces a standing wave.

Figure 16-17 (*a*) Five snapshots of a wave traveling to the left, at the times t indicated below part (*c*) (T is the period of oscillation). (*b*) Five snapshots of a wave identical to that in (*a*) but traveling to the right, at the same times t. (*c*) Corresponding snapshots for the superposition of the two waves on the same string. At $t = 0, \frac{1}{2}T$, and T, fully constructive interference occurs because of the alignment of peaks with peaks and valleys with valleys. At $t = \frac{1}{4}T$ and $\frac{3}{4}T$, fully destructive interference occurs because of the alignment of peaks with valleys. Some points (the nodes, marked with dots) never oscillate; some points (the antinodes) oscillate the most.



To analyze a standing wave, we represent the two waves with the equations

$$y_1(x,t) = y_m \sin(kx - \omega t) \tag{16-58}$$

and

$$y_2(x,t) = y_m \sin(kx + \omega t). \tag{16-59}$$

The principle of superposition gives, for the combined wave,

$$y'(x,t) = y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t).$$

Applying the trigonometric relation of Eq. 16-50 leads to Fig. 16-18 and

$$y'(x,t) = [2y_m \sin kx] \cos \omega t. \tag{16-60}$$

This equation does not describe a traveling wave because it is not of the form of Eq. 16-17. Instead, it describes a standing wave.

The quantity $2y_m \sin kx$ in the brackets of Eq. 16-60 can be viewed as the amplitude of oscillation of the string element that is located at position x. However, since an amplitude is always positive and $\sin kx$ can be negative, we take the absolute value of the quantity $2y_m \sin kx$ to be the amplitude at x.

In a traveling sinusoidal wave, the amplitude of the wave is the same for all string elements. That is not true for a standing wave, in which the amplitude *varies with position*. In the standing wave of Eq. 16-60, for example, the amplitude is zero for values of kx that give $\sin kx = 0$. Those values are

$$kx = n\pi, \quad \text{for } n = 0, 1, 2, \ldots. \tag{16-61}$$

Substituting $k = 2\pi/\lambda$ in this equation and rearranging, we get

$$x = n\frac{\lambda}{2}, \quad \text{for } n = 0, 1, 2, \ldots \quad \text{(nodes)}, \tag{16-62}$$

as the positions of zero amplitude—the nodes—for the standing wave of Eq. 16-60. Note that adjacent nodes are separated by $\lambda/2$, half a wavelength.

The amplitude of the standing wave of Eq. 16-60 has a maximum value of $2y_m$, which occurs for values of kx that give $|\sin kx| = 1$. Those values are

$$kx = \tfrac{1}{2}\pi, \tfrac{3}{2}\pi, \tfrac{5}{2}\pi, \ldots$$
$$= (n + \tfrac{1}{2})\pi, \qquad \text{for } n = 0, 1, 2, \ldots . \tag{16-63}$$

Substituting $k = 2\pi/\lambda$ in Eq. 16-63 and rearranging, we get

$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, \qquad \text{for } n = 0, 1, 2, \ldots \qquad \text{(antinodes)}, \tag{16-64}$$

as the positions of maximum amplitude—the antinodes—of the standing wave of Eq. 16-60. Antinodes are separated by $\lambda/2$ and are halfway between nodes.

LONGITUDINAL WAVES

- Fig shows how a sound wave can be produced by a piston in a long, air-filled pipe.
- If you suddenly move the piston rightward and then leftward, you can send a pulse of sound along the pipe.
- The rightward motion of the piston moves the elements of air next to it rightward, changing the air pressure there.
- The increased air pressure then pushes rightward on the elements of air somewhat farther along the pipe.
- Moving the piston leftward reduces the air pressure next to it.
- As a result, first the elements nearest the piston and then farther elements move leftward.
- Thus, the motion of the air and the change in air pressure travel rightward along the pipe as a pulse.
- If you push and pull on the piston in simple harmonic motion, as is being done in Fig, a sinusoidal wave travels along the pipe.
- Because the motion of the elements of air is parallel to the direction of the wave's travel, the motion is said to be longitudinal, and the wave is said to be a longitudinal wave.
- Both a transverse wave and a longitudinal wave are said to be traveling waves because they both travel from one point to another, as from one end of the string to the other end.

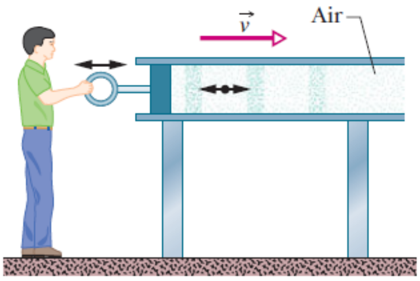


Fig: A sound wave is set up in an airfilled pipe by moving a piston back and forth. Because the oscillations of an element of the air (represented by the dot) are parallel to the direction in which the wave travels, the wave is a longitudinal wave.

THE SPEED OF SOUND

- The speed of any mechanical wave, transverse or longitudinal, depends on both an inertial property of the medium (to store kinetic energy) and an elastic property of the medium (to store potential energy).
- Thus,

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}, \qquad (17\text{-}1)$$

where (for transverse waves) t is the tension in the string and m is the string's linear density.

$$B = -\frac{\Delta p}{\Delta V/V} \quad \text{(definition of bulk modulus).} \qquad (17\text{-}2)$$

Here $\Delta V/V$ is the fractional change in volume produced by a change in pressure Δp. As explained in Module 14-1, the SI unit for pressure is the newton per square meter, which is given a special name, the *pascal* (Pa). From Eq. 17-2 we see that the unit for B is also the pascal. The signs of Δp and ΔV are always opposite: When we increase the pressure on an element (Δp is positive), its volume decreases (ΔV is negative). We include a minus sign in Eq. 17-2 so that B is always a positive quantity. Now substituting B for τ and ρ for μ in Eq. 17-1 yields

$$v = \sqrt{\frac{B}{\rho}} \quad \text{(speed of sound)} \qquad (17\text{-}3)$$

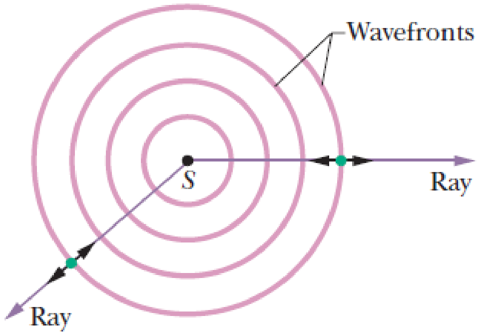


Fig: A sound wave travels from a point source S through a three-dimensional medium. The wavefronts form spheres centered on S; the rays are radial to S. The short, double-headed arrows indicate that elements of the medium oscillate parallel to the rays.

Table 17-1 The Speed of Sound[a]

Medium	Speed (m/s)
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater[b]	1522
Solids	
Aluminum	6420
Steel	5941
Granite	6000

[a]At 0°C and 1 atm pressure, except where noted.
[b]At 20°C and 3.5% salinity.

Engineering physics

Waves in one Dimension

Numerical problems:—

①. A wire of mass 0.001kg & length 2.5m is under tension of 1N. Find the fundamental frequency of the wire.

Sol:—

Mass (M) = 0.001kg

Length (l) = 2.5m

then, mass per unit length (m) = $\frac{0.001\,kg}{2.5m}$

Tension in the spring (T) = 1N

Fundamental frequency (n) = $\frac{1}{2l}\sqrt{\frac{T}{m}}$

$$= \frac{1}{2(2.5)}\sqrt{\frac{1}{\left(\frac{0.001}{2.5}\right)}}$$

$$= \frac{1}{5}\sqrt{\left(\frac{2.5}{0.001}\right)}$$

$$= \frac{1}{5}\sqrt{\frac{25\times10^{-1}}{10^{-3}}}$$

$$= \frac{1}{5}\sqrt{5^2\times10^2}$$

$$= \frac{1}{\cancel{5}}\times\cancel{5}\times10$$

$\therefore$ $\boxed{n = 10}$ (Ans)

②. A string of linear density 0.1 kg/m & tension 10N fixed at one end. Calculate the power required to oscillate the other end with amplitude 0.1m & with 10Hz frequency.

Sol:- $\mu = 0.1\,kg/m$; $A = 0.1\,m$

$f = 10\,Hz$; $T = 10\,N$

wave velocity $(V) = \sqrt{\frac{T}{\mu}}$

$= \sqrt{\frac{10}{0.1}}$

$= \sqrt{100}$

$\boxed{V = 10}$

Power $(P) = 2\pi^2 f^2 A^2 V \mu$

$= 2\pi^2 (10)^2 (0.1)^2 (10)(0.1)$

$\boxed{P = 19.74\,watt}$

Ans

③. A flexible string of length 1m & mass 1gm is stretched to a Tension T. The string is found to vibrate in the ~~stres~~ three segments at a frequency 612Hz. Calculate the tension of the spring.

Sol:-

$f = 612 Hz$

$l = 1m$

$m = \frac{1gm}{1m} = 10^{-1} kg/m$

The expression for frequency of the string vibrating in three segments,

$$f = \frac{3}{2l}\sqrt{\frac{T}{m}}$$

$$612 = \frac{3}{2(1)}\sqrt{\frac{T}{10^{-3}}}$$

$$\frac{612 \times 2}{3} = \sqrt{\frac{T}{10^{-3}}}$$

S.O.B.S

$$\left(\frac{612\times 2}{3}\right)^2 = \frac{T}{10^{-3}}$$

$$T = 10^{-3} \times 408^2$$

$$\boxed{T = 166.46N}$$

(Ans)

④ The velocity of a transverse wave on a stretched string is 500m/s, when it is stretched under a tension of 19.6N, If the tension is altered to a value of 78.4N, what will be the velocity of the wave?

Sol:- $T_1 = 19.6N$; $V_1 = 500 m/s$; T_2 (altered) $= 78.4N$

$$\text{velocity } (v) = \sqrt{\frac{T}{m}}$$

then,

$$V_1 = \sqrt{\frac{T_1}{m}} \quad \& \quad V_2 = \sqrt{\frac{T_2}{m}}$$

$$\frac{V_2}{V_1} = \frac{\sqrt{\frac{T_2}{m}}}{\sqrt{\frac{T_1}{m}}}$$

$$V_2 = V_1 \sqrt{\frac{T_2}{T_1}}$$

$$V_2 = 500 \times \sqrt{\frac{78.4}{19.6}}$$

$$V_2 = 500 \times \sqrt{4}$$

$$V_2 = 500 \times 2$$

$$\boxed{V_2 = 1000 m/s}$$

$\therefore$ The velocity of the transverse wave altered is 1000m/s.

(Ans)

⑤. A flexible string of length 1m & mass 1gm is stretched to a tension T. The string is found to vibrate in the three segments at a frequency 900Hz. Calculate the tension of the string.

Sol:- $n = 800Hz$; $l = 1m$; $m = \frac{1gm}{1m} = 10^{-3} kg/m$

The expression for frequency of the string vibrating in three segments,

$$n = \frac{3}{2l}\sqrt{\frac{T}{m}}$$

$$900 = \frac{3}{2(1)}\sqrt{\frac{T}{10^{-3}}}$$

$$\frac{900 \times 2}{3} = \sqrt{\frac{T}{10^{-3}}}$$

S.O.B.S

$$\left(\frac{900 \times 2}{3}\right)^2 = \frac{T}{10^{-3}}$$

$$T = 10^{-3} \times (600)^2$$

$$\boxed{T = 360N}$$

= Ans

6. Two closed pipes, one filled with O_2 & the other with H_2, have the same fundamental frequency. Find the ratio of their lengths.

Sol:-

In a closed pipe,

one pipe is filled with O_2 (Density, $\rho_1 = 16$)

other " " " " H_2 (" , $\rho_2 = 1$)

frequency of vibration $(n) = \frac{v}{4l} \rightarrow (1)$

where, $n_1 = \frac{v_1}{4l_1}$; $n_2 = \frac{v_2}{4l_2} \rightarrow (3)$

$\downarrow$
(2)

Comparing eq (2) & (3)

$$\frac{v_1}{4l_1} = \frac{v_2}{4l_2} \quad (\because n_1 = n_2 = n)$$

$$\frac{l_1}{l_2} = \sqrt{\frac{d_2}{d_1}} \quad (\because v \propto \frac{1}{d})$$

$$= \sqrt{\frac{1}{16}}$$

$$\frac{l_1}{l_2} = \frac{1}{4}$$

$\therefore l_1 : l_2 = 1:4$ (Ans)

THE END

Prepared By:-
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UNIT – IV

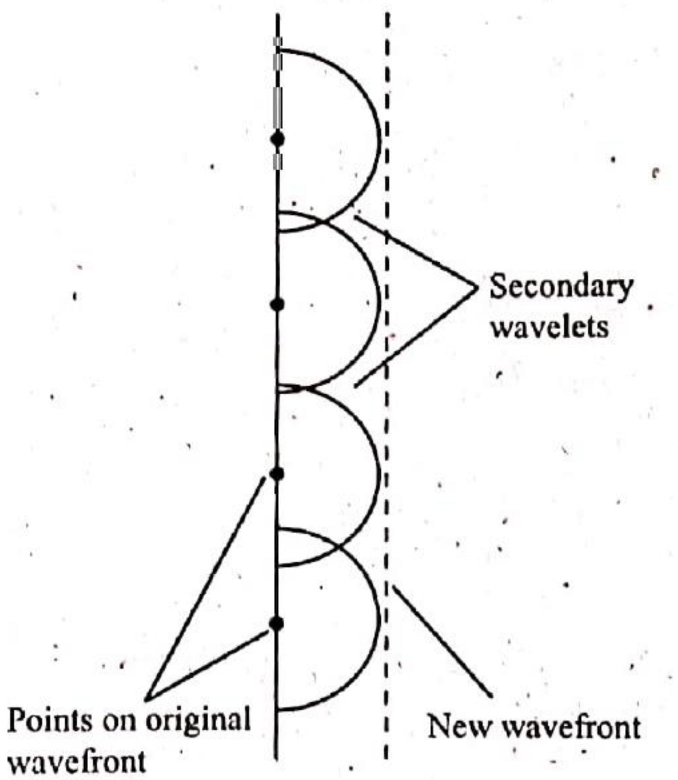
WAVE OPTICS

SYLLABUS

- Huygen's principle.
- Superposition of waves.
- Interference of light.
- Newton's rings.
- Michelson's interferometer.
- Mach-Zehnder interferometer.
- Frunhofer diffraction from a single slit and circular aperture.
- Diffraction grating- resolving power.

HUYGEN'S PRINCIPLE

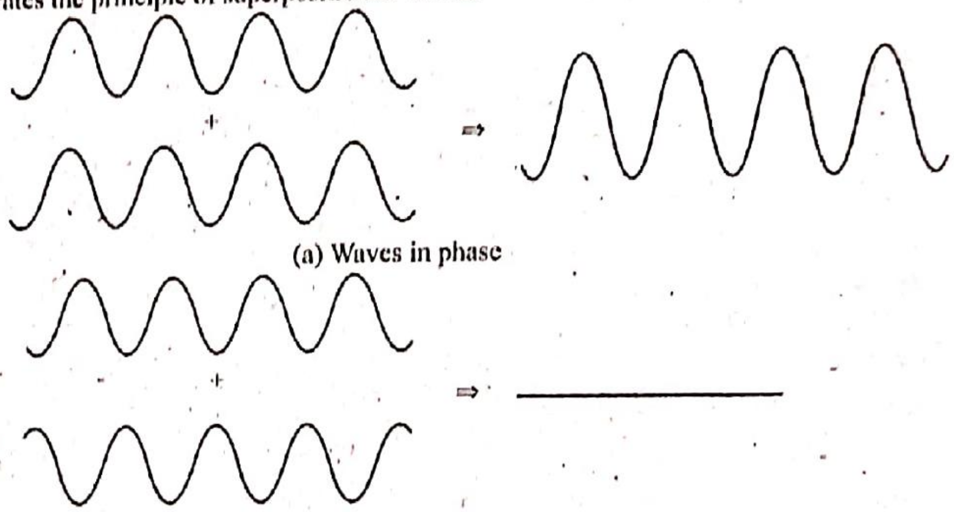
It states that, "Every point on a wavefront acts as a secondary source of spherical wavelets having the same frequency & same initial phase". The wavefront formed is a linear superposition of these wavelets as shown in below fig:



PRINCIPLE OF SUPERPOSITION OF WAVES

The principle of superposition of waves states that, "The resultant displacement of a particle of the medium acted upon by two or more waves simultaneously is the algebraic sum of the displacements of the same particle due to individual waves in the absence of other wave".

Figure below illustrates the principle of superposition of waves.



(b) Waves out of phase

Figure: Superposition of Waves

INTERFERENCE OF LIGHT

Definition: When two or more light waves superimpose in the medium then according to the principle of super position the two waves added with each other. The resultant wave amplitude is equal to the sum or difference of individual waves. It results variation of intensity in the region of superposition. This phenomenon is known as Interference. It can produce fringes or bands.

Introduction:

- Wave Theory of light attempts to understand the various optical phenomena exhibited by light waves.
- Interference constituted the first proof of the wave nature of light.
- Thomas Young first experimentally demonstrated interference in light waves.
- The superposition principle forms the conceptual basis for the explanation of interference.
- To produce interference, the light waves should be coherent, i.e., the light waves should have constant phase difference and same frequencies.

Classification [or] Types: It is classified into two types:

1. Constructive Interference.
2. Destructive Interference.

1. Constructive Interference: When the resultant wave amplitude is equal to the sum of amplitude of individual waves then such interference is known as constructive Interference.

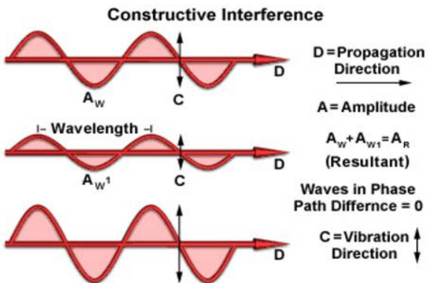


Figure 2

2. Destructive Interference: When the resultant wave amplitude is equal to the difference of amplitude of individual waves then such interference is known as destructive Interference.

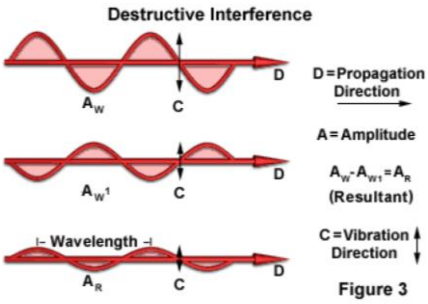


Figure 3

NEWTON'S RINGS

- Newton's rings are one of the best examples for the interference in a nonuniform thin film.
- When a Plano-convex lens with its convex surface is placed on a plane glass plate, an air film of increasing thickness is formed between the two.
- The thickness of the film at the point of contact is zero.
- If monochromatic light is allowed to fall normally and the film is viewed in the reflected light, alternate dark and bright rings concentric around the point of contact between the lens and glass plate are seen.
- These circular rings were discovered by Newton and are called Newton's rings.

- To study the interference in a non-uniform thin film Newton's rings experiment is used.

Experimental Arrangement:

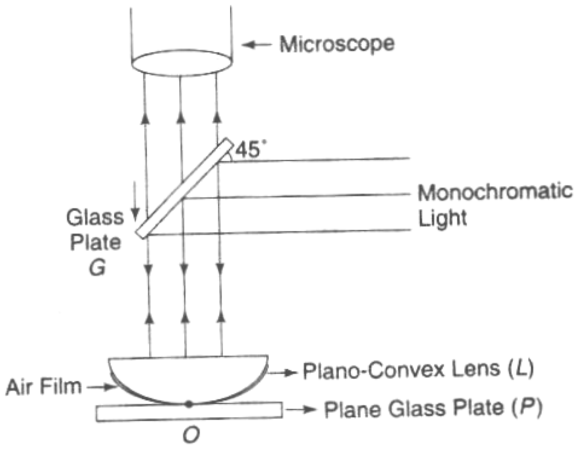


Fig Newton's rings (Experimental set up)

- The experimental arrangement is as shown in fig.
- The Plano-convex lens (L) of large radius of curvature is placed with its convex surface on a plane glass plate P).
- The lens makes the contact with the plate at 'O'.
- The monochromatic light falls on a glass plate G held at an angle of 45° with the vertical.
- The glass plate G reflects normally a part of the incident light towards the air film enclosed by the lens L and the glass plate P.
- A part of the light is reflected by the curved surface of the lens L and a part is transmitted which is reflected back from the plane surface of the plate.
- These reflected rays interfere and give rise to an interference pattern in the form of circular rings.
- These rings are seen near the upper surface of the air film through the microscope.

Explanation of Newton's Rings:

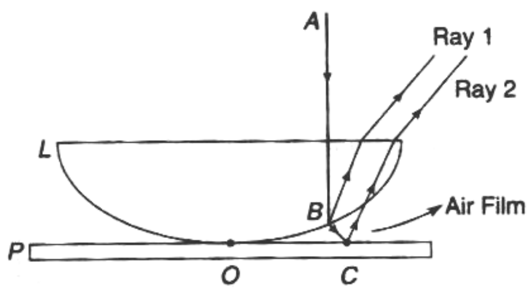


Fig Formation of Newton's rings

- Newton's rings are formed due to interference between the light rays reflected from the top and bottom surfaces of air film between the plate and the lens.
- The formation of Newton's rings can be explained with the help of Fig.
- A part of the incident monochromatic light AB is reflected at B (glass-air boundary) in the form of the ray (1) with any additional phase (or path) change.
- The other part of light is refracted along BC.
- Then at C (air-glass boundary), it is again reflected in the form of the ray (2) with additional phase change of π or path change of $\lambda/2$.

Path difference = $\delta = 2\ \mu t \cos r + \lambda/2$

For air, μ=1 then path difference = $\delta = 2t \cos r + \lambda/2$

The rays are incidenting normally, r = 0 then = $\delta = 2t \cos r + \lambda/2$

Path difference = $\delta = 2t + \lambda/2$

At point of contact, t = 0, so $\delta = \lambda/2$

i.e, at point of contact t = 0, path difference = $\lambda/2$, i.e., the reflected light rays at the point of contact has a phase change of π. Hence, the incident and reflected light rays are out of phase. So, it will form dark region at the center.

The condition for bright ring is:

$$2t + \lambda/2 = n\lambda$$

$$2t = (2n - 1)\ \lambda/2\ [where\ n = 1, 2, 3, \ldots.]$$

The condition for dark ring is:

$$2t + \lambda/2 = (2n + 1)\ \lambda/2$$

$$2t = n\lambda,\ [where\ n = 0, 1, 2, 3, \ldots.]$$

- For monochromatic light, the bright and dark rings depend on thickness of the air film.
- For a Newton's rings system, the focus of points having same thickness lie on a circle having its centre at the point of contact.
- Thus, we get bright and dark circular rings with the point of contact as the centre.

Theory of Newton's Rings [or] Diameter of Newtons Rings:

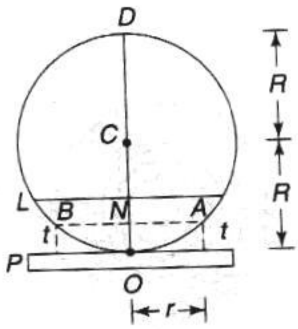


Fig: Theory of Newton's rings

- To find the diameters of dark and bright rings, let L be a lens placed on a glass plate P.
- The convex surface of the lens is the part of spherical surface (Fig) with center at C.
- Let R be the radius of curvature and r be the radius of Newton's ring corresponding to the film thickness t.
- From the property of a circle, NA * NB = NO * ND
- From fig. NA = NB = r and NO = t using these in the above fig. we get,

$$r * r = t * (2R - t)$$

$$r^2 = 2Rt - t^2$$

if thickness is very small, t^2 becomes very very small t so it can be neglected.

$$r^2 = 2Rt$$

$$t = r^2/2R$$

For bright ring, the condition is

$$2t = (2n - 1)\frac{\lambda}{2}$$

$$2\frac{r^2}{2R} = (2n - 1)\frac{\lambda}{2}$$

$$r^2 = \frac{(2n - 1)\lambda R}{2}$$

Replacing r by $\frac{D}{2}$, the diameter of n^{th} bright ring will be

$$\frac{D^2}{4} = \frac{(2n - 1)}{2}\lambda R$$

$$D = \sqrt{2n - 1}\sqrt{2\lambda R}$$

$$D \propto \sqrt{2n - 1}$$

$$D \propto \sqrt{\text{odd natural numbers}}$$

Thus the diameter of the bright rings are proportional to the square root of odd natural numbers. For dark ring, the condition is

$$2t = n\lambda$$

$$2\frac{r^2}{2R} = n\lambda$$

$$r^2 = n\lambda R$$

$$D^2 = 4n\lambda R$$

$$D = 2\sqrt{n\lambda R}$$

$$D \propto \sqrt{n}$$

$$D \propto \sqrt{\text{natural numbers}}$$

Thus, the diameters of dark rings are proportional to the square root of natural numbers.

With increase in the order (n), the rings get closer and the fringe width decreases and is shown in Fig. 1.6.

Figure 1.6 Newton's ring pattern

Applications of Newtons Rings:

1. Determination of Wavelength of a Light Source:

Let R be the radius of curvature of a Plano-convex lens, λ be the wavelength of light used. Let D_m and D_n are the diameters of m[th] and n[th] dark rings respectively. Then

D_m^2=4mλR

$$\text{And } D_n^2=4(n)\,\lambda R$$

$$D_n^2 - D_m^2 = 4(m-n)\,\lambda R$$

$$\lambda = \frac{D_n^2 - D_m^2}{4(m-n)R}$$

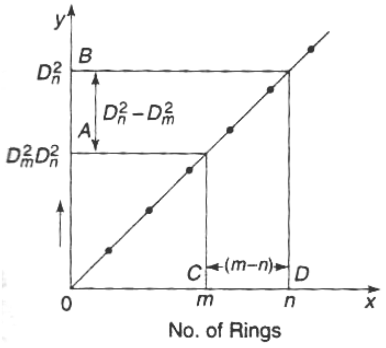


Fig Plot of D² with respect to number of rings

Newton's rings are formed with suitable experimental setup. With the help of travelling microscope, the readings for different orders of dark rings were noted from one edge of the rings to the other edge. The diameters of different orders of the rings can be known. A plot between D^2 and the number of rings gives a straight line as shown in the fig. From the graph,

$$\frac{D_n^2 - D_m^2}{(m-n)} = \frac{AB}{CD}$$

The radius R of the Plano-convex lens can be obtained with the help of a Spherometer. Substituting these values in the formula, λ can be calculated.

2. Determination of Refractive Index of a Liquid:

The experiment is performed when there is an air film between glass plate and the Plano-convex lens. The diameters of m^{th} and n^{th} dark rings are determined with the help of travelling microscope. We have

$$D_n^2 - D_m^2 = 4(m-n)\,\lambda R$$

The system is placed into the container which consists of the liquid whose refractive index (μ) is to be determined. Now, the air film is replaced by the liquid film. Again, the diameters of the same m^{th} and n^{th} dark rings are to be obtained. Then we have

$$D_n'^2 - D_m'^2 = \frac{4(m-n)\,\lambda R}{\mu}$$

$$\mu = \frac{D_n^2 - D_m^2}{D_n'^2 - D_m'^2}$$

Using the above formula, 'μ' can be calculated.

INTERFEROMETER

An interferometer is a measuring instrument, which makes use of light wave interference. The use of interferometer is faster & easier than that of optical flats, & these are considered to be the most accurate measuring instrument.

MICHELSON'S INTERFEROMETER

Michelson's interferometer is the older device. It utilizes monochromatic light from an extended source & works on the principle of interference.

Construction:

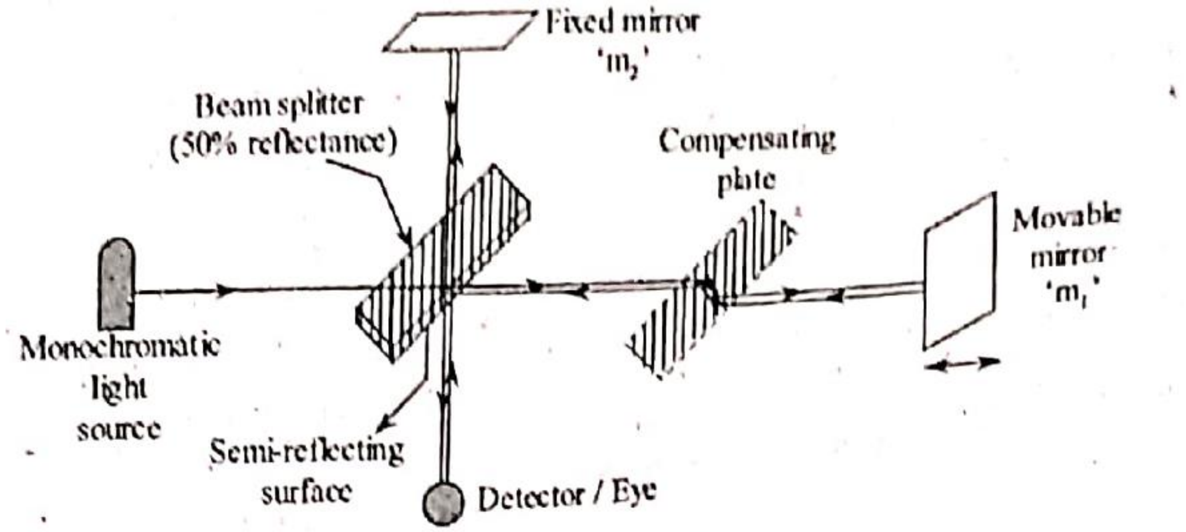


Figure: Michelson Interferometer

MACH – ZEHNDER INTERFEROMETER

It is a simple device which works on the principle of interference by division of amplitude.

Construction:

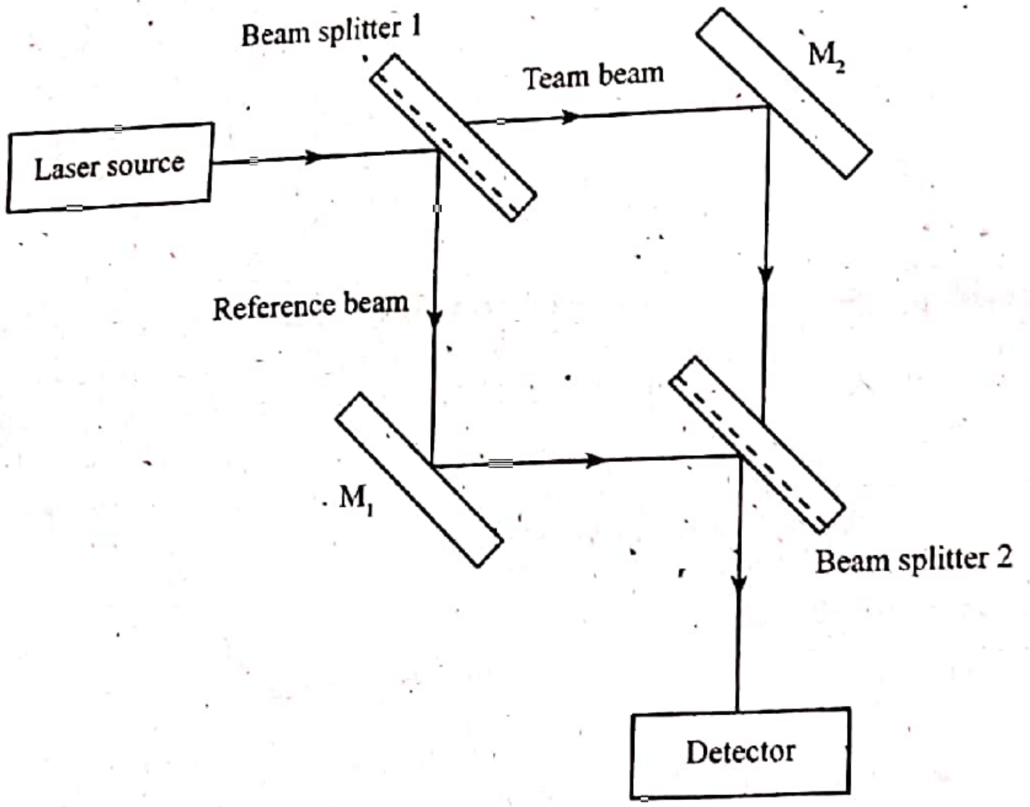


Figure: Mach-Zehnder Interferometer Schematic Diagram

Phase shift:

$$\delta = 2\pi\left(\frac{l_1 - l_2}{\lambda}\right)$$

DIFFRACTION

Definition: When light is incident on an obstacle whose size is comparable with the wavelength of light then the incident light bend around the edges (or) corners. This bending phenomenon of light is known as diffraction. It results bright and dark shadow regions known as diffraction pattern.

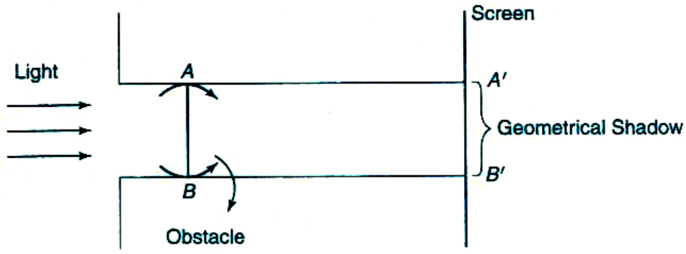


Fig: Diffraction

Introduction to Diffraction:

- The wave nature of light is further confirmed by the optical phenomenon of diffraction.
- The word diffraction is derived from the Latin word diffractus which means to break to pieces.
- It is common experience that waves bend around obstacles placed in their path.
- When light waves encounter an obstacle, they bend round the edges of the obstacle.
- This bending is predominant when the size of the obstacle is comparable to the wavelength of light.
- The bending of light waves around the edge of an obstacle is diffraction.
- It was first observed by Gremaldy.

Classification [or] types: It is classified into two types. They are:

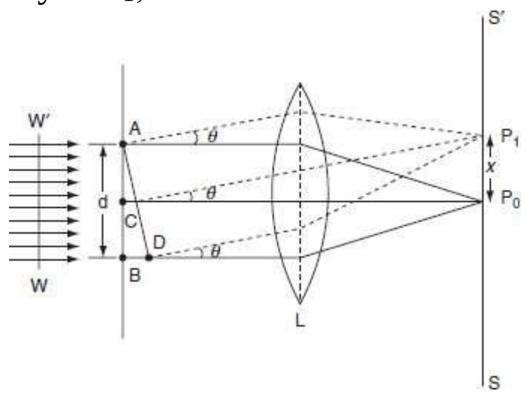
1. Fresnel diffraction.
2. Fraunhofer diffraction.

Distinction Between Fresnel & Fraunhoffer Diffraction:

SNO.	Fresnel Diffraction	Fraunhoffer Diffraction
1.	In this diffraction, the source and screen are kept at finite distances from the obstacle.	In this diffraction, the source and screen are kept at finite distances from the obstacle.
2.	Hence, lenses are not required to see the diffraction.	Hence, lenses are required to see the diffraction.
3.	It can be studied in the direction of propagation of light.	It can be studied in any direction.
4.	Here the incident wave fronts are either spherical or cylindrical.	Here the incident wave fronts are plane wave fronts.

FRAUNHOFER DIFFRACTION AT A SINGLE SLIT

- Consider a plane wave front coming from monochromatic light and incident on the slit AB of width ‘e’.
- The incident light get diffracted and focused on the screen.
- According to Hiegen’s principle, every point on the plane wavefront in the plane of slit is a secondary wavelet.
- These wavelets travelling in all directions.
- The waves travelling normal to the slit i.e., along ‘OP’ focus at ‘P’.
- It produce a bright image at ‘O’ on the screen; while the waves diffracted or making an angle ‘θ’ focus at P_1.
- Depending on the path difference between ‘OP’ and ‘OP_1’, the point P_1 may have maximum or minimum intensity.
- So, to find intensity at P_1, draw a normal line ‘AC’ to the line ‘BC’.



path difference = BC =AB sin θ

slit width = AB = e then BC = e sin θ

$$phase\ difference = \frac{2\pi}{\lambda} \times path\ difference$$

$$phase\ difference = \frac{2\pi}{\lambda} \times e \sin\theta$$

If the slit width is divides into 'n' equal parts then amplitude of each wave is 'a'. Then the phase difference between two continuous waves is written as

$$\frac{1}{n}[total\ phase\ difference = \frac{1}{n}\left[\frac{2\pi}{\lambda} e\ sin\theta\right] = d$$

According to vector addition of amplitude, the resultant wave amplitude is,

Using the method of vector addition of amplitudes, the resultant amplitude R is given by

$$R = \frac{a \sin nd/2}{\sin d/2}$$

$$= \frac{a \sin (\pi e \sin \theta/\lambda)}{\sin (\pi e \sin \theta/n\lambda)}$$

$$= a\frac{\sin \alpha}{\sin \alpha/n} \quad \text{where } \alpha = \pi e \sin \theta/\lambda$$

$$= a\frac{\sin \alpha}{\alpha/n} \quad (\because \alpha/n \text{ is very small})$$

$$= n\frac{a \sin \alpha}{\alpha} \quad (\because na = A)$$

$$= A\frac{\sin \alpha}{\alpha}$$

Intensity $$I = R^2 = A^2\left(\frac{\sin \alpha}{\alpha}\right)^2 \quad (1)$$

2.2.1 Principal Maximum

The resultant amplitude R can be written in ascending powers of α as

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \ldots\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \ldots\right]$$

I will be maximum, when the value of R is maximum. For maximum value of R, the negative terms must vanish, i.e., $\alpha = 0$

$$\frac{\pi e \sin \theta}{\lambda} = 0$$

$$\sin \theta = 0$$

$$\theta = 0$$

Then

$$R = A \tag{2}$$

$$I_{max} = R^2 = A^2 \tag{3}$$

The condition $\theta = 0$ means that the maximum intensity is formed at P_0 and is known as *principal maximum.*

2.2.2 Minimum Intensity Positions

I will be minimum, when $\sin \alpha = 0$

$$\alpha = \pm \pi, \pm 2\pi, \pm 3\pi$$

$$\alpha = \pm m\pi$$

$$\frac{\pi e \sin \theta}{\lambda} = \pm m\pi$$

Thus, we obtain minimum intensity positions on either side of principal maximum. For m = 0, sin θ = 0. It represents principal maximum positions.

Secondary Maxima: In between secondary minima, we can get secondary maxima. The positions can be obtained by differentiating the Eq. (1) and equating to zero.

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2 \left(\frac{sin\alpha}{\alpha}\right)^2\right] = 0$$

$$A^2 . \frac{2sin\alpha}{\alpha} . \frac{\alpha cos\alpha - sin\alpha}{\alpha^2} = \frac{d}{d\alpha}\left[A^2 \left(\frac{sin\alpha}{\alpha}\right)^2\right] = 0$$

In this Eq, either sin α = 0 or α cos α – sin α = 0

If sin α = 0, it gives the positions of minima.

So, the positions of secondary maxima can be obtained by:

$$\alpha \cos \alpha - \sin \alpha = 0$$

$$\alpha = \tan \alpha$$

the values of α satisfying the above equations are determined graphically by drawing the curves y = α and y = tan α. The curves are obtained as shown in the fig. The intersection of two curves gives values of α.

 Riyaz Mohammed

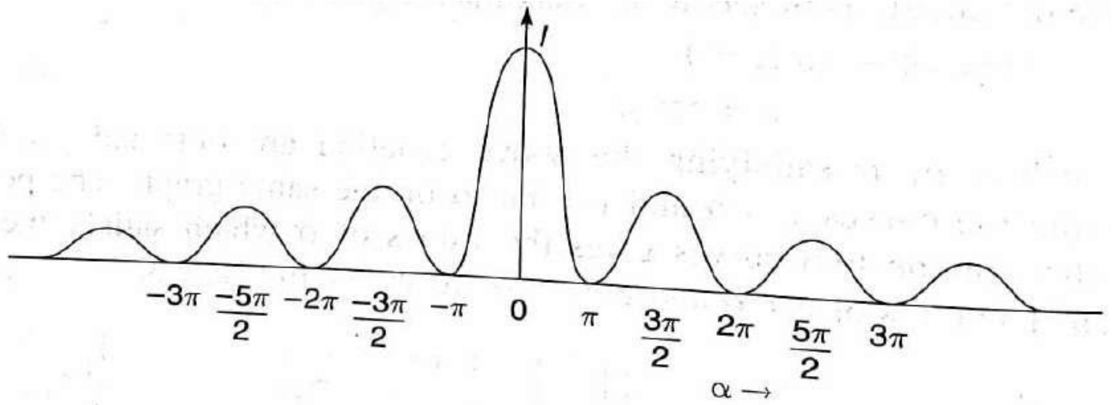
$\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} \ldots \ldots \ldots$

Using the values in Eq. (1), the minimum intensity positions can be obtained.

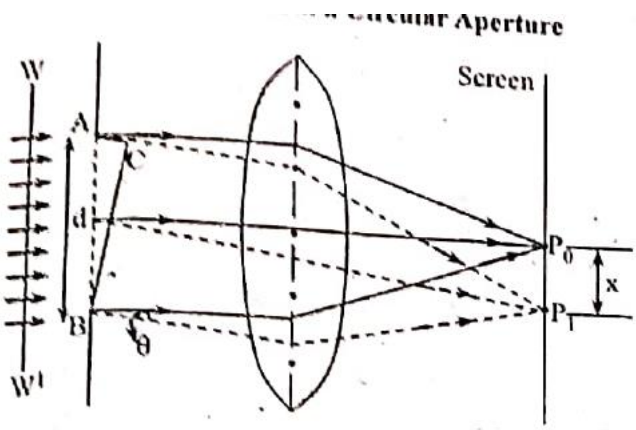
If $\alpha = 0$, then $I_0 = A^2$ gives principal maximum

$$If\ \alpha = \pm\frac{3\pi}{2}, then\ I_1 = A^2\left[\frac{\sin\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}}\right]^2 \approx \frac{A^2}{22}\ give\ 1st\ secondary\ maxima$$

$$If\ \alpha = \pm\frac{5\pi}{2}, then\ I_2 = A^2\left[\frac{\sin\left(\frac{5\pi}{2}\right)}{\frac{5\pi}{2}}\right]^2 \approx \frac{A^2}{62}\ give\ 2nd\ secondary\ maxima$$



FRUNHOFER DIFFRACTION FROM A CIRCULAR APERTURE



The secondary waves meet the screen at 'P_1' if they travel in a direction inclined at an angle 'θ' with the normal to the aperture. From points A and B, the path difference between the extreme waves is given by,

$$AC = AB \sin\theta$$

From Figure (1), $AB = d$ then,

$$AC = d\sin\theta \quad \text{... (1)}$$

Similarly to a single slit, the point P_1 is of minimum intensity if the path difference is an integral multiple of λ' and of maximum intensity if the path difference is an odd multiple of $\lambda/2$'. Then,

$$d\sin\theta = n\lambda \text{ [Minima]} \quad \text{... (2)}$$

$$\Rightarrow \quad d\sin\theta = (2n+1)\frac{\lambda}{2} \text{ [Maxima]} \quad \text{... (3)}$$

Where, $n = 1, 2, 3, \ldots\ldots\ldots\ldots$

If $n = 0$, the central maximum at P_0 is obtained.

If the collecting lens is placed near to the circular aperture or the screen is far away from the lens, then

$$\sin\theta = \frac{x}{f} \quad \text{....(4)}$$

Where, f – Focal length of lens.

For $n = 1$,

$$d\sin\theta = 1.\lambda \Rightarrow \sin\theta = \theta = \frac{\lambda}{d} \quad \text{....(5)}$$

[$\because$ First secondary minimum]

From equations (4) and (5),

$$\frac{x}{f} = \frac{\lambda}{d}$$

$$\therefore x = \frac{\lambda f}{d}$$

Where, x – Radius of the Airy's Disc

$\Rightarrow$ Airy has given the exact value of 'x', i.e.,

$$\therefore x = \frac{1.22\lambda f}{d}$$

From the above relation, it can be noticed that as the diameter of the aperture increases, the radius of the central disc decreases.

Fraunhofer diffraction due to a circular aperture method is employed to find the resolving powers of telescopes and microscopes.

DIFFRACTION GRATING

- An arrangement which consists of a large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating.
- Fraunhofer used the first grating consisting of a large number of parallel wires placed side by side very closely at regular intervals.
- Now gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass with a fine diamond point.
- The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit.
- Commercial gratings are produced by taking the cast of an actual grating on a transparent film like that of cellulose acetate.
- Solution of cellulose acetate is poured on the ruled surface and allowed to dry to form a thin film, detachable from the surface.
- These impressions of a grating are preserved by mounting the film between two glass sheets.

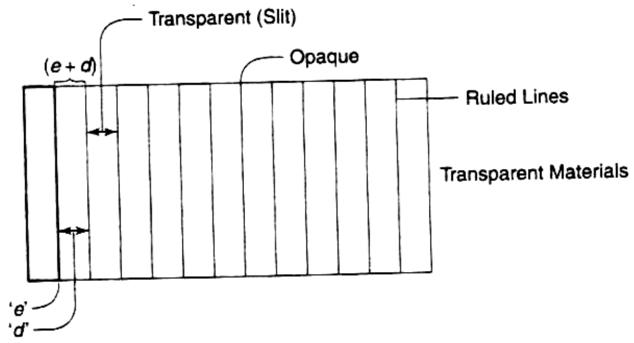


Fig: Diffraction Grating

Let ‘e’ be the width of the line and ‘d’ be the width of the slit. Then (e + d) is known as grating element. If ‘N’ is the number of lines per inch on the grating, then

$$N\,(e+d) = 1'' = 2.54 \text{ cm}$$
$$e+d = \frac{2.54}{N} \text{ cm}$$

There will be nearly 30,000 lines per inch of a grating. Due to the above fact, the width of the slit is very narrow and is comparable to the wavelength of light. When light falls on the grating, the light gets diffracted through each slit. As a result, both diffraction and interference of diffracted light gets enhanced and forms a diffraction pattern. This pattern is known as diffraction spectrum.

Grating Spectrum: The condition to form the principal maxima in a grating is given by

$$(e + d) \sin \Theta = n\lambda$$

Where (e + d) is the grating element and the above equation is known as grating equation.

From the grating equation, the following is clear.

1. For a particular wavelength λ, the angle of diffraction Θ is different for principal maxima of different orders.
2. As the number of lines in the grating is large, maxima appear as sharp, bright parallel lines and are termed as spectral lines.
3. For white light and for a particular order of n, the light of different wavelengths will be diffracted in different directions.
4. At the center, $\Theta = 0$ gives the maxima of all wavelengths which coincides to form the central image of the same colour as that of the light source. This forms zero order.
5. The principal maxima of all wavelengths forms the first, second, etc order spectra for n=1,2, …
6. The longer the wavelength, greater is the angle of diffraction. Thus, the spectrum consists of violet being in the innermost position and red being in the outermost positions.

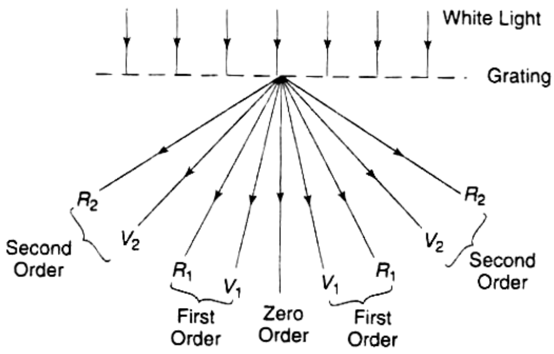


Fig: Grating Spectrum

7. Most of the intensity goes to zero order and the rest is distributed among other orders.
8. Spectra of different orders are situated symmetrically on both sides of zero order.
9. The maximum number of orders available with the grating is $n_{max} = e+d/\lambda$

RESOLVING POWER OF A GRATING

The ability of n diffraction grating to separate the maxima of diffraction of two neighbouring wavelength is called resolving power of a grating.

Mathematically,

Resolving power of a grating, $\frac{\lambda}{d\lambda} = nN$

Where,

λ – Wavelength of a wave

$\lambda + d\lambda$ – Wavelength of a neighbouring wave

n – Order of wavelength

N – An integer.

Engineering physics

Wave optics

Numerical problems;—

①. A soap film ($n=1.33$) in air is 320nm thick. If it is illuminated with white light at normal incidence, what colour will it appear to be in reflected light?

Sol:— $n=1.33$; thickness $(d) = 320mm$

Let, $\lambda = \frac{2dn}{m-\frac{1}{2}} \longrightarrow (1)$

In visible region, $m=2$

$$(1) \Rightarrow \lambda = \frac{2 \times 320 \times 10^{-9} \times 1.33}{2-\frac{1}{2}}$$

$$\therefore \boxed{\lambda = 567nm}$$

Light with a wavelength of 567nm appears yellow-green.

—(Ans)

②. A parallel beam of light $\lambda = 5890 \times 10^{-10} m$ is incident on a thin glass plate $\mu=1.5$ such that the angle of refraction into the plate is 60°. Calculate the smallest thickness of plate which would appear dark by reflection.

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Sol:- $\lambda = 5890 \times 10^{-10} m$; $\mu = 1.5$; $r = 60^\circ$

thickness $(t) = \dfrac{n\lambda}{2\mu \cos r}$

$n = 1$ (Minimum thickness)

$t = \dfrac{1 \times 5890 \times 10^{-8} cm}{2 \times 1.5 \times 0.5}$

$\boxed{t = 3.926 \times 10^{-5} cm}$

=(Ans)

③. Two narrow & parallel slits 0.08cm apart are illuminated by light of frequency 8×10^{11} KHz. It is desired to have a fringe width of 6×10^{-4}m. where the screen should be placed from the slits?

Sol:- $d = 0.08cm$; $\beta = 6 \times 10^{-4} m$; $V = 8 \times 10^{11}$ KHz

Distance $(D) = \dfrac{\beta d}{\lambda} \rightarrow (1)$ $\left[\because \beta = \dfrac{\lambda D}{d}\right]$

where, $\lambda = \dfrac{c}{V}$

$= \dfrac{3 \times 10^{8}}{8 \times 10^{11} \times 10^{3}} m$

Sub the corresponding values in eq (1)

$D = \dfrac{6 \times 10^{-4} \times 0.08 \times 10^{-2} \times 8 \times 10^{14}}{3 \times 10^{8}}$

Scanned with CamScanner

$$\boxed{D = 1.25m}$$

=(Ans)

4. In Newton's rings experiment, the dia of the 4th & 12th dark rings are 0.40cm & 0.70 cm respectively. Find the diameter of the 20th dark ring.

Sol:-

Dia of 4th ring, $D_4 = 0.4cm = 4.0\times10^{-3}m$

" " 12th ", $D_{12} = 0.7cm = 7.0\times10^{-3}m$

The relation b/w the dia & nth ring in Newton's ring experiment is,

$$D \propto \sqrt{n}$$

$$\frac{D_1}{D_2} = \sqrt{\frac{n_1}{n_2}}$$

$$D_2 = D_1\sqrt{\frac{n_1}{n_2}}$$

Let $D_1 = D_4$ for $n_1 = 4$

$D_2 = D_{20}$ for $n_2 = 20$

$$D_2 = 0.40 \times \sqrt{\frac{20}{4}}$$

$$D_2 = 0.40 \times 2.236$$

$$\therefore \boxed{D_2 = 0.894cm}$$

=(Ans)

5. In a Newton's rings experiment the dia of 15^{th} ring was found to be 0.59cm & that of 5^{th} ring was 0.336cm. If the radius of curvature of lens is 100cm find the wavelength of the light.

Sol:- $D_{15} = 0.59cm$; $D_5 = 0.336cm$; $R = 100cm$

$$\text{wavelength } (\lambda) = \frac{D^2_{n+p} - D^2_n}{4PR} \longrightarrow (1)$$

Here, $n+p = 15$, $n = 5$

$p = 15 - 5 = 10$

Sub, the corresponding values in eq(1)

$$(1) \Rightarrow \lambda = \frac{(0.59 \times 10^{-2})^2 - (0.336 \times 10^{-2})^2}{4 \times 10 \times 100 \times 10^{-2}}$$

$= 5880 \times 10^{-10} m$

$$\boxed{\lambda = 5880 A^o}$$

=(Ans)

6. Distance b/w the slits is 0.1mm & the width of fringes formed on the screen is 5mm. What would be the wavelength of light used, if the distance b/w the screen & the slit is one meter?

Sol:-

Distance b/w the slits, $2d = 0.1mm$

width $(\beta) = 5mm = 5\times10^{-3}m$

$D = 1m$

Wavelength of the light $(\lambda) = \frac{\beta \times 2d}{D} \rightarrow (1)$

$$\lambda = \frac{5\times10^{-3}\times0.1\times10^{-3}}{1}$$

$$= 0.5\times10^{-6}m$$

$$= 5000\times10^{-10}m$$

$$\boxed{\lambda = 5000A^{o}}$$

(Ans)

⑦. How many orders will be visible, if the wavelength of light is 5000 A°? Given that the no. of lines per cm of the grating is 6655.

Sol:-

Wavelength of light $(\lambda) = 5000A^{o}$

No. of lines of grating, $\frac{1}{e+d} = 6655$ per cm

The condition for max. no. of orders possible for a plane diffraction grating is

$$(e+d)\sin\theta = n\lambda$$

For max. diffraction, $\theta = 90^\circ$

$$(e+d) = n\lambda \quad [\because \sin 90^\circ = 1]$$

$$\Rightarrow \frac{1}{6655} = n \times 5000 \times 10^{-8}$$

$$\boxed{n = 3}$$

=(Ans)

⑧. In a Michelson's interferometer, a mica sheet of thickness 0.005cm is placed infront of fixed mirror & then in order to bring back the fringes to their original position, the movable is moved by a distance of 0.0025cm. Calculate the refractive index of material.

Sol:- Let x_1, x_2 are initial & final positions of movable mirror, distance moved by movable mirror,

$$(x_2 - x_1) = 0.0025\text{cm}$$

$$= 0.0025 \times 10^{-2}\text{m}$$

Thickness of mica sheet, $t = 0.005\text{cm}$

$$= 0.005 \times 10^{-2}\text{m}$$

Refractive index $(\mu) = 1 + \frac{(x_2 - x_1)}{t}$

$$= 1 + \frac{0.0025 \times 10^{-2}}{0.005 \times 10^{-2}}$$

$$\therefore \boxed{\mu = 1.5}$$

=(Ans)

(9). In a Michelson's Interferometer, the movable mirror is moved through a distance of 0.06cm & 200 fringes crossed the field of view. Find the wavelength of light used.

Sol:- Let x_1, x_2 are initial & final positions of mirror

Distance moved by movable mirror,

$$(x_2 - x_1) = 0.06mm = 0.00006m$$

No. of fringes crossed field of view, $N = 200$

From Michelson's experiment,

$\therefore$ wavelength of light is $\frac{\lambda N}{2} = (x_2 - x_1)$

$$\lambda = \frac{2(x_2 - x_1)}{N}$$

$$\lambda = \frac{2(0.00006)}{200}$$

$$= 6 \times 10^{-10} m$$

$$\boxed{\lambda = 6000 A^0}$$

=(Ans)

(10). A source of light with wavelength 6000 Å is incident on a slit of width 1 μm. Find the angular separation b/w the first order minima & the central maxima.

Sol:- For a Fraunhofer single slit diffraction,

$$\lambda = 6000\,Å = 6000 \times 10^{-10}\,m$$

$$\text{width of slit } (a) = 1\,\mu m = 1 \times 10^{-6}\,m$$

The expression for angular width of minima in Fraunhofer single slit diffraction is :-

$$(e+d)\sin\theta = n\lambda, \quad n = \pm 1, \pm 2 \ldots$$

But,

$$e+d \cong a$$

$$a\sin\theta = n\lambda \longrightarrow (1)$$

For first order minima ($n=1$), eq (1) becomes,

$$a\sin\theta = \lambda \qquad [\because n=1]$$

$$\Rightarrow 1\times 10^{-6}(\sin\theta) = 6000 \times 10^{-10}\,m$$

$$\sin\theta = \frac{6000\times 10^{-10}}{1\times 10^{-6}}$$

$$= 6\times 10^{-1}$$

$$\sin\theta = 0.6$$

$$\theta = \sin^{-1}(0.6)$$

$$\boxed{\theta = 36.87°}$$

(Ans)

(11). Determine the radius of the central airy disk of a circular aperture, if a wavelength of light 6000A° is incident & the focal length of the lens is 100 cm. The diameter of circular aperture is 0.01cm.

Sol:- $\lambda = 6000A^{\circ} = 6000 \times 10^{-10} m$

$f = 100cm = 1m$

$d = 0.01cm = 0.01 \times 10^{-2} m$

According to Fraunhofer diffraction due to circular aperture method,

Radius of airy disc, $x = \frac{1.22 \lambda f}{d}$

$= \frac{1.22 \times 6000 \times 10^{-10} \times 1}{0.01 \times 10^{-2}}$

$\boxed{x = 7.32 \times 10^{-3} m}$

(Ans)

(12). A diffraction grating used at normal incidence gives a line 5400A° in a certain order superimposed on the violet line (4050 A°) of the next higher order. If the angle of diffraction is 30°, how many lines/cm are there in the grating?

Sol:— Wave length of line, $\lambda_1 = 5400 A^\circ$

" " violet line, $\lambda_2 = 4050 A^\circ$

$\theta = 30^\circ$

For a diffraction grating, $(e+d)\sin\theta = n\lambda$,
Consider the nth order maxima of λ_1 coincides with $(n+1)^{th}$ order maxima of λ_2.

$$(e+d)\sin\theta = n\lambda \rightarrow (1)$$

$$(e+d)\sin\theta = (n+1)\lambda_2 \rightarrow (2)$$

From eq (1) & (2)

$$n\lambda_1 = (n+1)\lambda_2$$

$$n\lambda_1 = n\lambda_2 + \lambda_2$$

$$n(\lambda_1 - \lambda_2) = \lambda_2$$

$$\therefore n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

Sub, the value of 'n' eq (1)

$$(e+d)\sin\theta = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

The no. of lines/cm $= \frac{1}{e+d} \Rightarrow \frac{\sin\theta(\lambda_1 - \lambda_2)}{\lambda_1\lambda_2}$

$$= \frac{\sin 30^\circ (5400\times10^{-8} - 4050\times10^{-8})}{5400\times10^{-8}\times4050\times10^{-8}}$$

$$= \frac{0.5(5400-4050)\times10^{-8}}{2187\times10^4\times10^{-16}} = \frac{0.5\times1350\times10^4}{2187}$$

$$= 3086.41$$

$$\simeq 3086 \text{ (Ans)}$$

———— o ————

THE END

UNIT – V

CHAPTER – I

LASERS

SYLLABUS

- Introduction to interaction of radiation with matter.
- Coherence.
- Principle and working of Laser.
- Population inversion.
- Pumping.
- Types of Lasers: Ruby laser, Carbon dioxide (CO_2) laser, He-Ne laser.
- Applications of laser.

INTRODUCTION TO LASERS

- Laser is an acronym for Light Amplification by Stimulated Emission of Radiation.
- In 1917, based on thermodynamic equilibrium between atoms and radiation, Albert Einstein predicted that there are two kinds of light emission from matter, namely spontaneous and stimulated emissions.
- He further proved that both spontaneous emission and stimulated emission are necessary to derive Planck's Quantum theory of radiation, which is the basis for theoretical prediction of Laser.
- In 1960, Charles Towner demonstrated experimentally stimulated emission for first time at Microwave frequencies as MASER and received Noble prize in 1964.
- In the same year, Theodore Maiman demonstrated stimulated emission based LASER in optical frequencies using Ruby rod as lasing medium, and Ali Javan and his co-workers constructed laser device using He-Ne gas as lasing medium.
- In 1962, lasing action using semiconductor medium was invented. Since then a variety of materials were used to demonstrate lasing action using liquids, ionized gases, dyes etc.
- **Characteristics of Laser:** Some of the unique characteristics of lasers which are different from ordinary incoherent light are:
    1. Directionality.
    2. High intensity.
    3. Monochromacity.

4. High degree of coherence.

1. Directionality:

- Any conventional light source like incandescent light emits radiations in all direction whereas a laser source emits radiation only in one direction.
- The directionality of the laser beam is generally expressed in terms of full angle beam divergence which is twice the angle that the outer edge of the beam makes with the axis of the beam.
- The outer edge is defined as a point at which the intensity (I) of the beam drops to 1/e times its value at the centre.

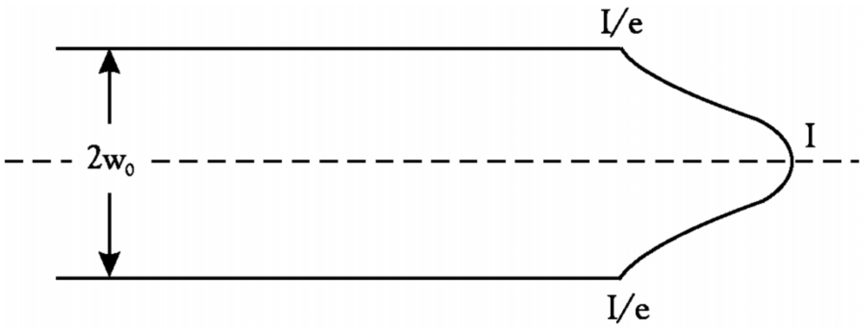


Fig: Gaussian beam

A Gaussian shape of laser beam is shown above and the full angle divergence in terms of minimum spot size of radius w_0 is given by

$$\phi = 1.27\ \lambda / 2w_0$$

where λ is the wavelength of the beam. For a typical planar wavefront emerging from an aperture of diameter d, it propagates as a parallel beam for a distance of d_2/λ called the Rayleigh's range, beyond which the beam due to diffraction diverges with an angular spread of $\Delta\Phi = \lambda/d$. For a typical laser the beam divergence is less than 0.01 milliradian, i.e. a laser beam spreads less than 0.01 millimeter for every metre. However, on the other hand, for ordinary light the spread is 1m for every 1m of travel. If a_1 & a_2 are the diameters of laser radiation at distances d_1 and d_2 from a laser source respectively, then the angle of beam divergence in degrees is given by

$$\phi = (a_2 - a_1) / 2(d_2 - d_1)$$

2. Intensity:

- A laser emits light radiation into a narrow beam, and its energy is concentrated in a small region.
- This concentration of energy both spatially and spectrally accounts for the great intensity of lasers.
- It can be shown that even a one-watt laser would appear many thousand times more intense than a 100 watt ordinary lamp.
- If we compare the number of photons emitted in one second from a square centimetre of a surface of a laser source with those from an ordinary source, the ratio is of the order of l028 to 1012.

3. Monochromacity:

- The light from a laser source is highly monochromatic compared to light from a conventional incoherent monochromatic source.
- The monochromacity is related to the wavelength spread of radiation given by

$$\Delta\lambda = (-c/f^2)\ \Delta f$$

The value of $\Delta\lambda$ is in the order of 300 nm for white light, 0.01 nm for gas discharge lamp, while it is 0.0001 nm for laser.

4. Coherence:

- Laser radiation is characterized by a high degree of ordering of the light field compared to radiation from other sources.
- In other words, laser light has a high degree of coherence, both spatial and temporal.
- Spatial coherence, also called transverse coherence, describes how far apart two sources or two portions of the same source can be located in a direction transverse to the direction of observation and still exhibit coherent properties over a range of observation points.
- The high degree of coherence of laser radiation makes it possible to realise a tremendous spatial concentration of light power such as 1013 watt in a space with linear dimensions of only 1μm.
- The temporal coherence on the other hand, normally refers to the relative phase or the coherence of two waves at two separate locations along the propagation direction of the two beams.

- The relationship between coherence length, wavelength and wavelength spread is given by:

$$l_{coh} = \frac{\lambda^2}{\Delta\lambda}$$

The above characteristics of lasers supported their unprecedented scientific and technological application. Thus lasers have been used in telecommunications, meteorology, metrology, biology, cybernetics, optical computations etc.

INTRODUCTION TO INTERACTION OF RADIATION WITH MATTER [OR] INTERACTION BETWEEN MATTER AND LIGHT [OR] ABSORPTION, SPONTANEOUS & STIMULATED EMISSION

- In lasers, the interaction between matter and light is of three different types.
- They are:
    1. Absorption.
    2. Spontaneous emission.
    3. Stimulates emission.
- Let E_1 and E_2 be ground and excited states of an atom. The dot represents an atom.
- Transition between these states involves absorption and emission of a photon of energy $E_2 - E_1 = h\nu_{12}$. Where ‘h’ is Planck’s constant.

1. Absorption: As shown in fig, if a photon of energy $h\nu_{12}(E_2 - E1)$ collides with an atom present in the ground state of energy E_1 then the atom completely absorbs the incident photon and makes transition to excited state E_2. It is represented as follows,

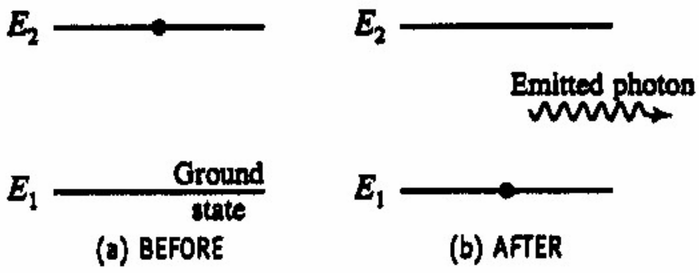
Photon + Atom = Atom*.

Fig. 1(a): Absorption

2. Spontaneous emission: As shown in fig, an atom initially present in the excited state makes transition voluntarily on its own. Without any aid of external stimulus or an agency to the ground. State and emits a photon of energy h = ν

$_{12}(E_2 - E_1)$. This is called spontaneous emission. These are incoherent. Spontaneous emission is represented as follows,

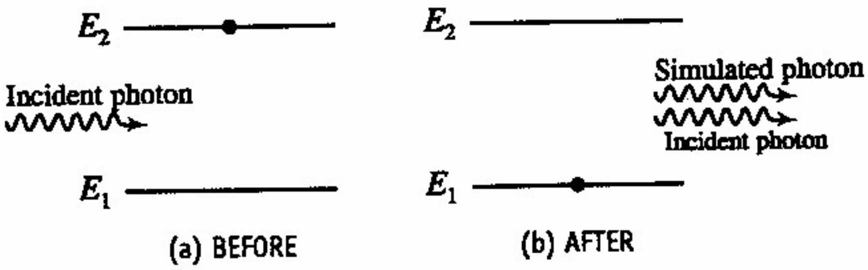
Atom* = Atom + Photon.



SPONTATNEOUS EMISSION

3. Stimulated emission: As shown in fig, a photon having energy $h\nu_{12}(E_2 - E_1)$ impinges on an atom present in the excited state and the atom is stimulated to make transition to the ground state and gives off a photon of energy $h\nu_{12}$. The emitted photon is in phase with the incident photon. These are coherent. This type of emission is known as stimulated emission. Stimulated emission can be represented as follows.

Photon + atom* = Atom + (photon + photon).



STIMULATED EMISSION

Differences Between Spontaneous Emission & Stimulated Emission of Radiation:

SNO.	Spontaneous emission	Stimulated emission
1.	Polychromatic radiation.	Monochromatic radiation.
2.	Less intensity.	High intensity.
3.	Less directionality, more angular spread during propagation.	High directionality, so less angular spread during propagation.

4.	Spatially and temporally in coherent radiation.	Specially and temporally coherent radiation.
5.	Spontaneous emission takes place when excited atoms make a transition to lower energy level voluntarily without any external stimulation.	Stimulated emission takes place when a photon of energy equal to $h\nu_{12} = E_2 - E_1$ stimulates an excited atom to make transition to lower energy level.
6.	One photon released.	Two photons released.
7.	It is independent of incident radiation.	It is dependent on incident radiation.
8.	It was postulated by Bohr.	It was postulated by Einstein.
9.	**Ex:** Light from sodium.	**Ex:** Light from a laser source.

PRINCIPLE AND WORKING OF LASER [OR] PRINCIPLE AND PRODUCTION OF LASER

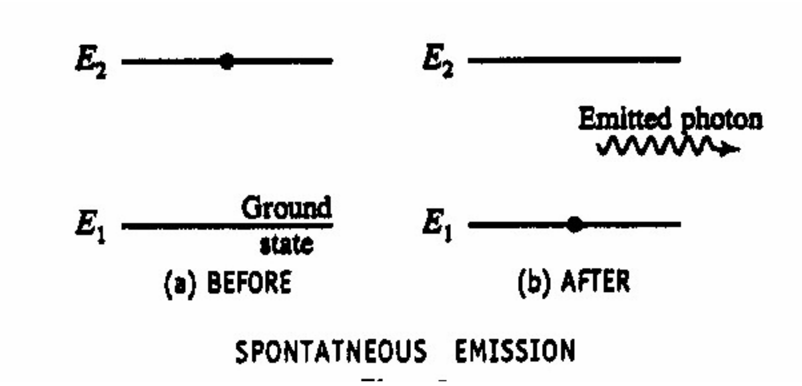
1. Absorption: As shown in fig, if a photon of energy $h\nu_{12}(E_2 - E1)$ collides with an atom present in the ground state of energy E_1 then the atom completely absorbs the incident photon and makes transition to excited state E_2. It is represented as follows,

Photon + Atom = Atom*.

Fig. 1(a): Absorption

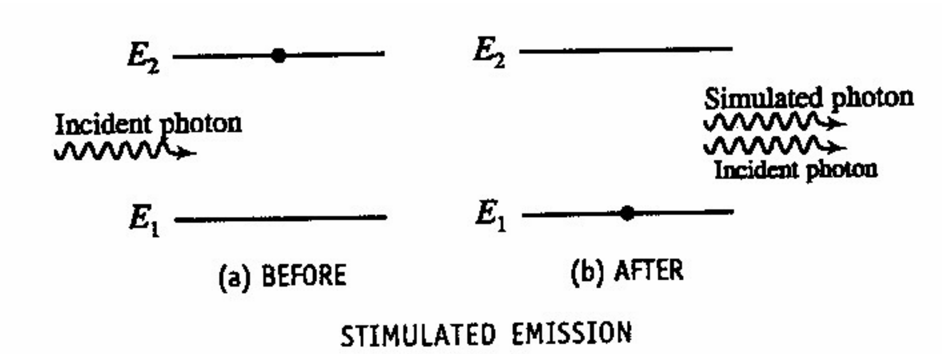
2. Spontaneous emission: As shown in fig, an atom initially present in the excited state makes transition voluntarily on its own. Without any aid of external stimulus or an agency to the ground. State and emits a photon of energy $h = \nu_{12}(E_2 - E_1)$. This is called spontaneous emission. These are incoherent. Spontaneous emission is represented as follows,

Atom* = Atom + Photon.



3. Stimulated emission: As shown in fig, a photon having energy $h\nu_{12}(E_2 – E_1)$ impinges on an atom present in the excited state and the atom is stimulated to make transition to the ground state and gives off a photon of energy $h\nu_{12}$. The emitted photon is in phase with the incident photon. These are coherent. This type of emission is known as stimulated emission. Stimulated emission can be represented as follows.

Photon + atom* = Atom + (photon + photon).



4. Einsteins Equations [Or] Einstains Co – Efficients

Let N_1 be the number of atoms per unit volume with energy E_1 and N_2 the number of atoms per unit volume with energy E_2. Let ‘n’ be the number of photons per unit volume at frequency ν such that $h\nu = E_2 – E_1$. Then the energy density of interacting photons ρ (v) is given by

$$\rho(\nu) = nh\nu \qquad (1)$$

When these photons interact with atoms, both upward (absorption) and downward (emission) transitions occur. At equilibrium these transition rates must be equal.

Upward Transition: Stimulated absorption rate depends on the number of atoms available in the lower energy state for absorption of these photons as well as the energy density of interacting radiation. i.e. stimulated absorption rate α N_1

$$\alpha\ \rho\ (\nu)$$

$$= B_{12} N_1\ \rho\ (\nu) \qquad (2)$$

Where the constant of proportionality B_{12} is the Einstein coefficient of stimulated absorption

Downward transition: Once the atoms are excited by stimulated absorption, they stay in the excited state for a short duration of time called the lifetime of the excited state. After their life time they move to their lower energy level spontaneous by emitting photons. This spontaneous emission rate depends on the number of atoms in the excited energy state. i.e., spontaneous emission rate $\alpha\ N_2$

$$= N_2\ A_{21} \qquad (3)$$

Where the constant of proportionality A_{21} is the Einstein coefficient of spontaneous emission.

Stimulated emission: Before excited atoms de excites to their lower energy states by spontaneous emission they may interact with photons resulting in stimulated emission of photons. Therefore stimulated emission rate depends on the number of atoms available in the excited state as well as energy density of interacting photons**.** i.e., stimulated emission rate $\alpha\ N_2$

$$\alpha\ \rho(\nu)$$

$$= N_2 \rho\ (\nu)\ B_{21} \qquad (4)$$

Where the constant of proportionality B_{21} is the Einstein coefficient of stimulated emission.

During stimulated emission, the interacting photon called the stimulating photon and the photon due to stimulated emission are in phase with each other. Please note that during stimulated absorption, the photon density decreases where as during stimulated emission it increases. For a system in equilibrium, the upward and down word transition rates must be equal and hence we have

$$N_1\ \rho\ (\nu)\ B_{12} = N_2 \rho(\nu) B_{21} + N_2 A_{21} \qquad \rightarrow (5)$$

Hence $$\rho(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_1 B_{21}} \rightarrow (6)$$

$$\rho\ (\nu) = \frac{A_{21}/B_{21}}{\left(\frac{B_{12}}{B_{21}}\right)\left(\frac{N_1}{N_2}\right) - 1}$$

The population of various energy levels in thermal equilibrium is given by Boltzmann distribution law.

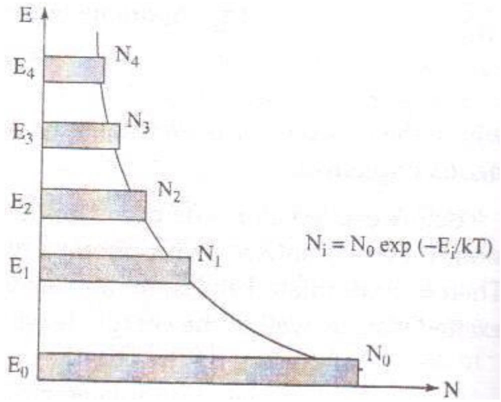


Fig: Boltzmann distribution for several energy levels

$$N_i = g_i N_o \exp(-E_i/KT)$$

Where N_i is the population density of the energy level E_i, N_0 is the population density of the ground state at temperature T_1, g_1 is the degeneracy of the i^th level and k is the Boltzmann constant ($=1.38\times10^{-23}$ joule/k). The concept of degeneracy occurs since more than one level has the same energy.

Hence $N_1 = g_1 N_o \exp(-E_1/kT)$

$$\frac{N_1}{N_2} = \frac{g_1}{g_2} \exp\left[\frac{(E_2 - E_1)}{kT}\right]$$

$$= \frac{g_1}{g_2} \exp\left[\frac{h\nu}{kT}\right] \qquad \rightarrow (7)$$

Substituting e q (6) in e q (4) $\rho(\nu) = \dfrac{[A_{21}/B_{21}]}{\left[\frac{B_{12}}{B_{21}}\frac{g_1}{g_2}\exp\left[\frac{h\nu}{kT}\right]\right]}$ $\rightarrow (8)$

From Planck's law of blackbody radiation, the radiation density is given by

$$\rho(\nu) = \frac{8\pi h\nu^3}{C^3} \frac{1}{\exp\left[\frac{h\nu}{kT}\right] - 1} \qquad \rightarrow (9)$$

Comparing equations (8) and (9), we get

$$\frac{B_{12}}{B_{21}} \frac{g_1}{g_2} = 1$$

$$g_1 B_{12} = g_2 B_{21} \qquad \rightarrow (10)$$

And $\dfrac{A_{21}}{B_{21}} = \dfrac{8\pi h\nu^3}{C^3}$ $\rightarrow (11)$

Equation's (10) and (11) are referred to as the Einstein relations.

The ratio of spontaneous emission rate to the stimulated emission rate is given by

$$R = \frac{N_2 A_{21}}{N_2 \rho(\nu) B_{21}} = \frac{A_{21}}{\rho(\nu) B_{21}}$$

From equation (9)

$$R = \frac{A_{21}}{\rho(\nu) B_{21}} = [exp^{[\frac{h\nu}{kT}]} -1] \qquad \rightarrow (12)$$

In practice, the absorption and emission phase occur simultaneously. Let us calculate the rates of spontaneous emission and stimulating emission for a tungsten filament lamp operating at a temperature of 2000k. Taking the average frequency to be $5x10^{14}$ Hz, this ratio is

$$R = e^{\frac{(6.6\times10^{-34})\times(5\times10^{14})}{(1.38\times10^{-23}\times2000)}} - 1 \qquad \rightarrow (13)$$

This confirms that under conditions of thermal equilibrium, even for sources operating at higher temperatures and lower frequencies, spontaneous emission predominates.

From equation (12), we understand that to make R smaller ρ(ν) the energy density of interacting radiation has to be made larger. Let us consider the relation of stimulated emission rate to stimulated absorption rate.

Thus at thermal equilibrium stimulated absorption predominates over stimulated emission. Instead if we create a situation that N2 > N1. Stimulated emission will predominate over stimulated absorption. If stimulated emission predominates the photon density increases and light amplifies the photon density increases and light amplification by stimulated emission of radiation (LASER) occurs. Therefore, in order to achieve more stimulated emission, population of the excited state (N_2) should be made larger than the population of the lower state (N_1) and this condition is called population inversion. Hence if we wish to amplify a beam of light by stimulated emission, then we must create population inversion and increase the energy density of interacting radiation.

POPULATION INVERSION & PUMPING

- The no of atoms in higher energy level is less than the no of atoms in lowest energy level.
- The process of making of higher population in higher energy level than the population in lower energy level is known as population inversion.
- Population inversion is achieved by pumping the atoms from the ground level to the higher energy level through optical (or) electrical pumping.

- It is easily achieved at the matastable state, where the life time of the atoms is higher than that in other higher energy levels.
- The states of system, in which the population of higher energy state is more in comparision with the population of lower energy state, are called "Negative temperature state".
- A system in which population inversion is achieved is called as an active system.
- The method of raising the particles from lower energy state to higher energy state is called "Pumping".
- Population inversion is associated with three Phenomenon:
    1. Stimulated emission.
    2. Amplification.
    3. Pumping Process.

1. Stimulated Emission: If majority of atoms are present in higher energy state than the process becomes very easy.

2. Amplification: If 'N', represents number of atoms in the ground state and 'N_2' represents number of atoms in the excited state than the amplification of light takes place only when $N_2 > N_1$.

$$\frac{N1}{N2} = \exp(E_2\text{-}E_1/KT) = \exp(\Delta E/KT)$$

$$\frac{N1}{N2} = \exp(hv/KT)$$

3. Pumping Process:

- This process is required to achieve population inversion.
- Pumping process is defined as: "The process which excites the atoms from ground state to excited state to achieve population inversion".
- Pumping can be done by number of ways:
    i. **Optical Pumping:** Excitation by strong source of light (flashing of a Camera).
    ii. **Electrical Pumping:** Excitation by electron impact.
    iii. **Chemical Pumping:** Excitation by chemical reactions.
    iv. **Direct Conversion:** Electrical energy is directly converted into radiant Energy in devices like LED's, population Inversion is achieved in forward bias.

v. **Inelastic atom-atom collision:** In electric discharge one type of atoms are raised to their excited state. These atom collide inelastically with another type of atoms. The latter atom provide the population inversion needed for laser emission. The example is He–Ne laser.

Explanation:

- Consider two energy levels E_1 and E_2 in a system.
- Let N_1, N_2 be the populations (number) of atoms per unit volume in the energy levels E_1, E_2.
- According to the Boltzmann's distribution law, the population of atoms in E_1, E_2 levels written as,

$$N_1 = N_0 exp^{\frac{-E_1}{K_B T}} \qquad (1)$$

$$N_2 = N_0 exp^{\frac{-E_2}{K_B T}} \qquad (2)$$

(2)/(1),

$$\frac{N_2}{N_1} = \frac{N_0 exp^{\frac{-E_1}{K_B T}}}{N_0 exp^{\frac{-E_2}{K_B T}}}$$

$$N_2 = N_1 \frac{exp^{\frac{-E_1}{K_B T}}}{exp^{\frac{-E_2}{K_B T}}}$$

$$N_2 = N_1 exp^{-(E_2 - E_1)/K_B T}$$

Since $E_2 > E_1$ so $N_2 < N_1$

- But to get stimulated emission continuously, $N_2 > N_1$ i.e the population of higher energy level should be more than in the population in the lower energy level.
- The process of sending more atoms in the higher energy level than in the population of atoms in the lower energy level is known as population inversion.
- In a three level system, $E_1 < E_2 < E_3$ and $N_1 > N_2 > N_3$

Where,

E_1 is the lower energy state with more lifetime of atoms.

E_3 is the highest energy state with less lifetime of atoms.

E_2 is the intermediate energy state with more lifetimes an atom (10^{-3} sec) and is known as metastable state.

- This state provides necessary population inversion for the emission of laser radiation.
- When a sufficient energy is supplied to the system then, the ground atoms excited to the E_3 level and then transit to the E_2 level which has more lifetime of atoms.
- Due to the continuous pumping, at one stage the E_2 level becomes more populated than lower energy level.
- It is the desired condition to get laser radiation.

TYPES OF LASERS: RUBY LASER, CARBON DIOXIDE (CO_2) LASER, HE – NE LASER

- Lasers are divided into different types based on the lasing materials used.
- Accordingly the important types of lasers are Solid state lasers, gas lasers, and semiconductor lasers.
- Most of the lasers emit light in IR or visible region, work in Continuous wave (CW) mode or in pulsed mode.
- Table gives some important types of Lasers with examples:

S. No.	Type of lasers	Examples with Wavelength of emission
1	Solid state lasers	Ruby laser (λ = 0.6928 μm) CaF_2 laser (λ = 2.49 μm) Nd:YAG laser (λ = 1.064 μm) Nd:Glass laser (λ = 1.6928 μm)
2	Gas lasers	He-Ne laser ($\lambda \approx$ 0.633 μm) Cu–vapour laser (λ = 0.5106 μm)
3	Ion lasers	Argon ion laser (λ = 0.4881 μm and λ = 0.5145 μm. Power up to 100 W)
4	Metal vapour laser	He–Cd laser (λ = 0.4416 μm. Power up to 300 mW)
5	Molecular gas laser	CO_2 laser–invented by C.K.N. Patel in 1963. (λ = 10.6 μm, Power = 1 kW) Nitrogen laser (λ = 0.3371 μm) Hydrogen laser (λ = 0.116 μm)
6	Excimer laser	Xenon excimer laser (λ = 0.172 μm), power = 15 MW Argon excimer laser (λ = 0.126 μm) power = several MW
7	Semiconductor lasers	GaAs laser (λ = visible and IR region)

Ruby Laser

Introduction:

- Ruby Laser is a solid state pulsed, three level lasers.
- It consists of a cylindrical shaped ruby crystal rod of length varying from 2 to 20 cms and diameter varying 0.1 to 2 cms.
- This end faces of the rod are highly flat and parallel.
- One of the faces is highly silvered and the other face is partially silvered so that it transmits 10 to 25% of incident light and reflects the rest so as to make the rod-resonant cavity.
- Basically, ruby crystal is aluminum oxide [Al_2O_3] doped with 0.05 to 0.5% of chromium atom.
- These chromium atoms serve as activators.
- Due to presence of 0.05% of chromium, the ruby crystal appears in pink color.
- The ruby crystal is placed along the axis of a helical xenon or krypton flash lamp of high intensity.

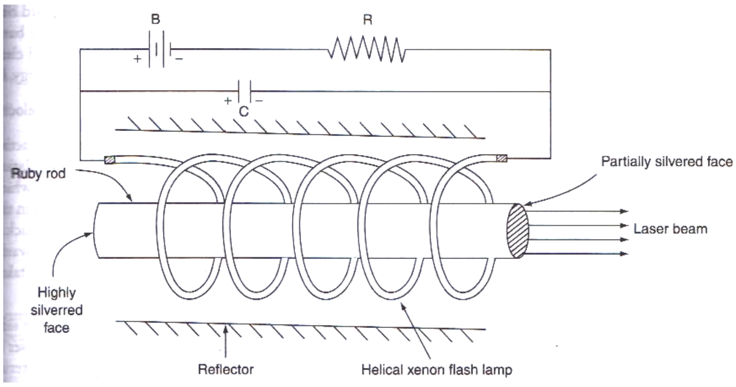


Fig: Ruby laser

Construction:

- Ruby ($Al_2O_3 + Cr_2O_3$) is a crystal of Aluminum oxide in which some of Al^{+3} ions are replaced by Cr^{+3} ions.
- When the doping concentration of Cr^{+3} is about 0.05%, the color of the rod becomes pink.
- The active medium in ruby rod is Cr^{+3} ions.
- In ruby laser a rod of 4cm long and 5mm diameter is used and the ends of the rod are highly polished.
- Both ends are silvered such that one end is fully reflecting and the other end is partially reflecting.

- The ruby rod is surrounded by helical xenon flash lamp tube which provides the optical pumping to raise the Chromium ions to upper energy level (rather energy band).
- The xenon flash lamp tube which emits intense pulses lasts only few milliseconds and the tube consumes several thousands of joules of energy.
- Only a part of this energy is used in pumping Chromium ions while the rest goes as heat to the apparatus which should be cooled with cooling arrangements as shown in fig.
- The energy level diagram of ruby laser is shown in fig:

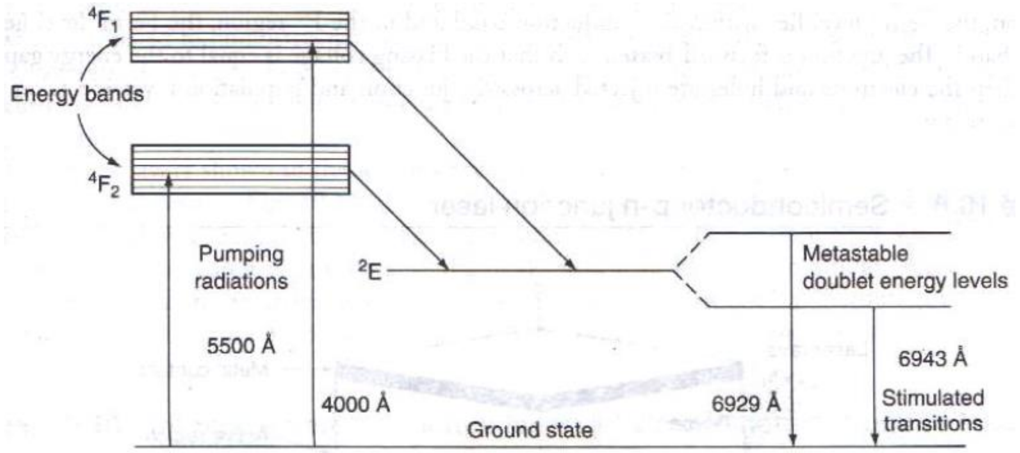


Fig: Energy level diagram of chromium ions in a ruby crystal

Working:

- Ruby crystal is made up of aluminum oxide as host lattice with small percentage of Chromium ions replacing aluminum ions in the crystal chromium acts as do pant.
- A do pant actually produces lasing action while the host material sustains this action.
- The pumping source for ruby material is xenon flash lamp which will be operated by some external power supply.
- Chromium ions will respond to this flash light having wavelength of $5600A^0$.
- When the Cr^{+3} ions are excited to energy level E_3 from E_1 the population in E_3 increases.
- Chromium ions stay here for a very short time of the order of 10^{-8} seconds then they drop to the level E_2 which is mat stable state of life time 10^{-3}s.
- Here the level E_3 is rather a band, which helps the pumping to be more effective.
- The transitions from E_3 to E_2 are non-radioactive in nature.

- During this process heat is given to crystal lattice.
- Hence cooling the rod is an essential feature in this method.
- The life time in mete stable state is 10^5 times greater than the lifetime in E_3.
- As the life of the state E_2 is much longer, the number of ions in this state goes on increasing while ions.
- In this state goes on increasing while in the ground state (E_1) goes on decreasing.
- By this process population inversion is achieved between the exited Meta stable state E_2 and the ground state E_1.
- When an excited ion passes spontaneously from the metastable state E_2 to the ground state E_1, it emits a photon of wave length $6943A^0$.
- This photon travels through the rod and if it is moving parallel to the axis of the crystal, is reflected back and forth by the silvered ends until it stimulates an excited ion in E_2 and causes it to emit fresh photon in phase with the earlier photon.
- This stimulated transition triggers the laser transition.
- This process is repeated again and again because the photons repeatedly move along the crystal being reflected from its ends.
- The photons thus get multiplied.
- When the photon beam becomes sufficiently intense, such that part of it emerges through the partially silvered end of the crystal.

Drawbacks of ruby laser:

1. The laser requires high pumping power to achieve population inversion.
2. It is a pulsed laser.

Carbon dioxide (CO_2) Laser

Construction and Working:

- We know that a molecule is made up of two or more atoms bound together.
- In molecule in addition to electronic motion, the constituent atoms in a molecule can vibrate in relation to each other and the molecule as a whole can rotate.
- Thus the molecule is not only characterized by electronic levels but also by vibration and rotational levels.
- The fundamental modes of vibrations of CO_2 molecule shown fig.

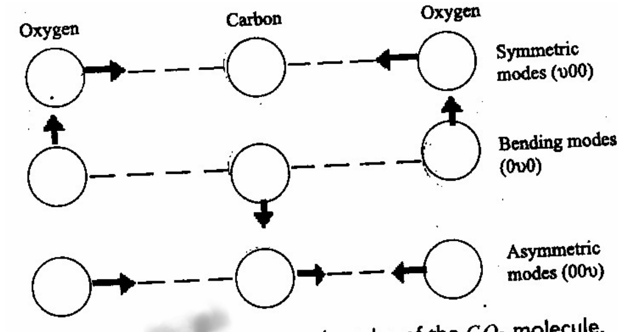
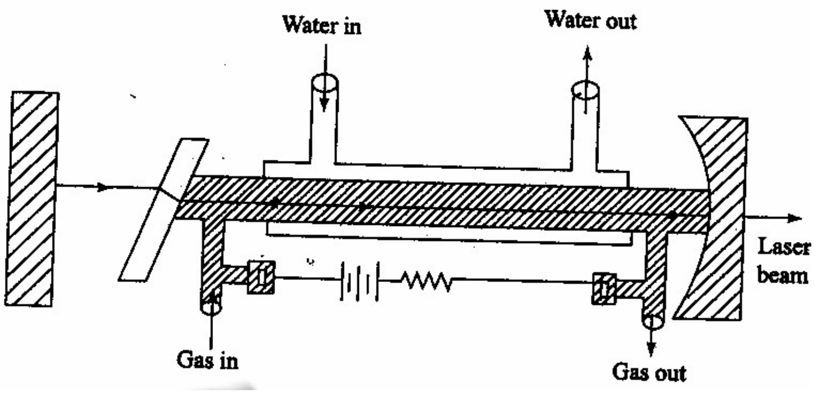
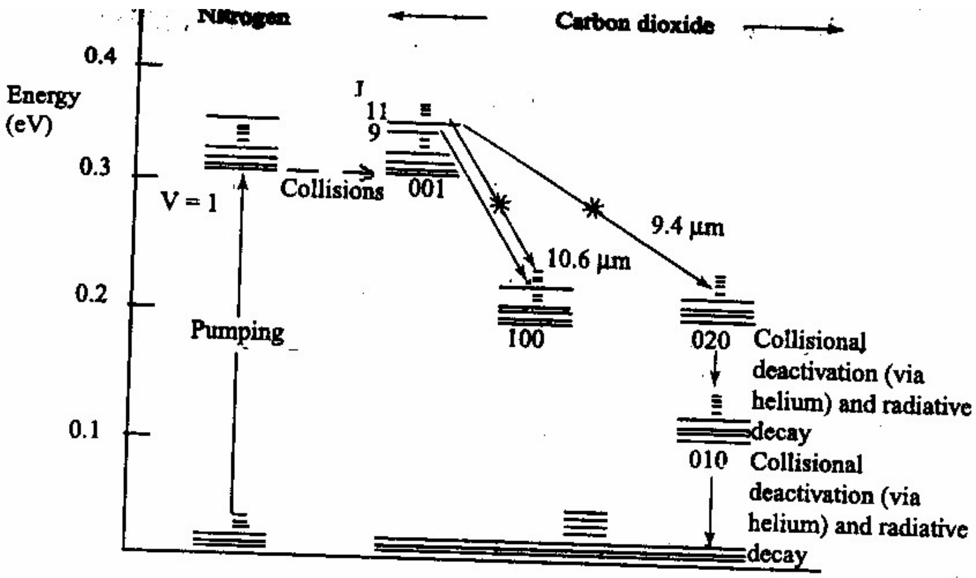


Fig. Vibrational modes of the CO_2 molecule.

- CO_2 Laser is a gas discharge, which is air cooled.
- The filling gas within the discharge tube consists primarily of, CO_2 gas with 10 – 20%, Nitrogen around 10 – 20 % H_2 or Xenon (Xe) a few percent usually only in a sealed tube.
- The specific proportions may vary according to the particular application. The population inversion in the laser is achieved by following sequence:
  1. Electron impact excites vibration motion of the N_2. Because N_2 is a homo nuclear molecule, it cannot lose this energy by photon emission and it is excited vibration levels are therefore metestable and live for long time.
  2. Collision energy transfer between the N_2 and the CO_2 molecule causes vibration excitation of the CO_2, with sufficient efficiency to lead to the desired population inversion necessary for laser operation.
- Because CO_2 lasers operate in the infrared, special materials are necessary for their construction.
- Typically the mirrors are made of coated silicon, molybdenum or gold, while windows and lenses are made of either germanium or zinc sulenide.
- For high power application gold mirrors and zinc selenide windows and lenses are preferred.
- Usually lenses and windows are made out of salt NaCl or KCl.
- While the material is inexpensive, the lenses windows degraded slowly with exposure to atmosphere moisture.
- The most basic form of a CO_2 laser consist of a gas discharge (with a mix close to that specified above) with a total reflector at one end and an output coupler (usually a semi reflective coated zinc selendine mirror) at the output end.
- The reflectivity of the output coupler is typically around 5 – 15 %.
- The laser output may be edge coupled in higher power systems to reduce optical heating problems.



- CO_2 lasers output power is very high compared to Ruby laser or He – Ne lasers.
- All CO_2 lasers are rated in Watts or kilowatts where the output power of Ruby laser or He – Ne laser is rated in mill watts.
- The CO_2 laser can be constructed to have CW powers between mill watts and hundreds of kilowatts.



Helium – Neon Laser [Or] He–Ne Laser

Introduction:

- The Helium-Neon laser was the first continuous wave (CW) laser.
- It was invented by Ali Javan and his co-workers in 1961 at the Bell Telephone Laboratory, New Jersey.
- The most common and inexpensive gas laser, the helium-neon laser is a four level laser and usually constructed to operate in the red at 632.8 nm.
- It can also be constructed to produce laser action in the green at 543.5 nm and in the infrared at 1523 nm.

- The collimation of the beam is accomplished by mirrors on each end of the evacuated glass tube which contains about 85% helium and 15% neon gas at 1/300 atmospheres pressure.

Principle:

- The Helium-Neon laser is a four level laser.
- The energy level diagram is shown in below figure.
- The left side of the representation shows the lower levels of the helium atoms.
- A characteristic of helium is that its first states to be excited, 2^1S_1 and 2^1S_0 are metastable, i.e. optical transitions to the ground state 1^1S_0 are not allowed.
- The atoms can be excited to metastable state by means of electron collision provided by electric discharge.
- Apart from the electron collision, the electric discharge pumping also supports the atomic collision in which, an excited helium atom reaches back to the ground state by transferring its energy to excite Ne atom and creates population inversion in the Ne system.
- The population inversion in Ne leads to laser transition.

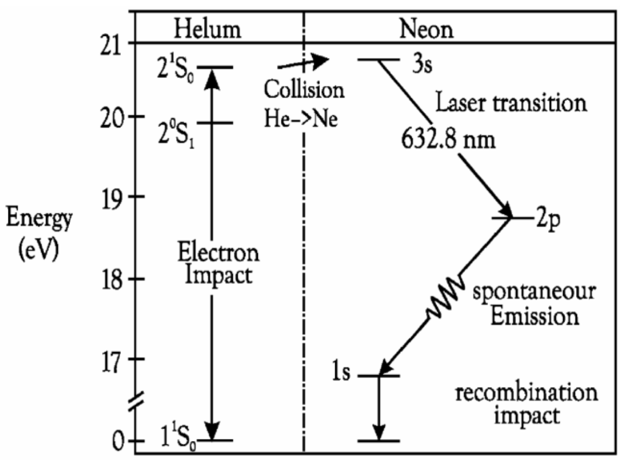


Fig: Excitation and laser process for the visible laser emission

Construction:

- The setup consists of a discharge tube of length 50 cm and bore diameter of 0.5 cm.
- The gain medium of the laser, as suggested by its name, is a mixture of helium and neon gases, in a ratio 5:1, contained at low pressure (an average of 1 torr) in a glass envelope.

- The energy or pump source of the laser is provided by an electrical discharge of around 1000 volts through an anode and cathode at each end of the glass tube.
- A current of 5 to 100 mA is typical for CW operation.
- The optical cavity of the laser typically consists of a plane, high-reflecting mirror at one end of the laser tube, and a concave output coupler mirror of approximately 1% transmission at the other end.
- He-Ne lasers are normally small, with cavity lengths of around 15 cm up to 0.5 m, and optical output powers ranging from 1 mW to 100 mW.

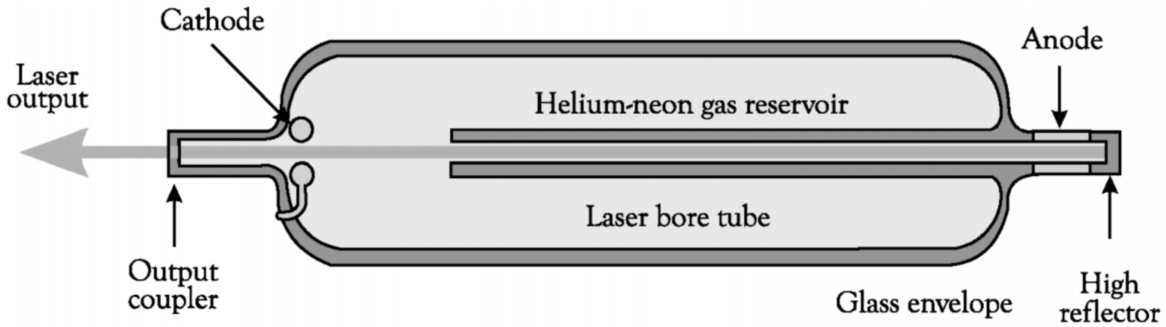


Fig: Diagrammatic representation of He-Ne laser

Working: He-Ne excitation process can be explained in terms of the following four steps:

1. When the power is switched on, an energetic electron of electric discharge collisionally excites a He atom to the state labeled 2^1S_0. This excited state is often written as He*(2^1S_0), where the asterisk means that the He atom is in an excited state.
2. The excited He*(2^1S_0) atom collides with an unexcited Ne atom and transfers internal energy to it, resulting an excited Ne atom written as Ne*($3S_2$). This energy exchange process occurs with high probability only because of the accidental near equality of the two excitation energy levels of these two atoms. The excited Ne atom Ne*($3S_2$) is metastable and no spontaneous transition directly to the ground state is allowed. Thus, population inversion is created.
3. When the excited Ne atom passes from metastable state (3s) to lower level (2p), it emits photon of wavelength 632.8nm. This photon travels through the gas mixture parallel to the axis of the tube, reflected back and forth by the mirror ends until it stimulates an excited Ne atom and causes it to emit a photon of 632.8 nm with the stimulating photon.
4. The stimulated transition from (3s) level to (2p) level is laser transition.

This process is continued and when a beam of coherent radiation becomes sufficiently strong, a portion of it escape through partially silvered end. The Ne atom passes to lower level (1s) by spontaneous emission and finally the Ne atom comes to ground state through collision with tube wall and undergoes radiation less transition.

Applications:

1. The Narrow red beam of He-Ne laser is used in supermarkets to read bar codes.
2. The He-Ne Laser is used in Holography in producing the 3D images of objects.
3. He-Ne lasers have many industrial and scientific uses, and are often used in laboratory demonstrations of optics.

APPLICATIONS OF LASER

1. Laser is used in optical fiber communication to transmit more information in less time without any losses.
2. LIDAR (Light Detection and Ranging) is used in calculation of distance between two long range objects.
3. Due to high intensity of laser beam, it can be used in laser guns, laser weapons, etc in army, military troops.
4. Laser is used in the cutting technology in large industries to cut thick/hard materials due to high intensity.
5. The laser technique is used for detection and analysis of finger prints in documentation.
6. Laser is used for measurement of water vapor concentration, temperature, humidity, pressure in the atmosphere.
7. Laser is used to detect internal defects in medical field.
8. Laser is used in CD-ROM etc. to record large information.
9. Laser is used in computer printers to get high quality prints.
10. Laser is widely using in the medical field like eye operations, cancer treatment etc.
11. Laser is used in the holography technique to record 3D pictures.

APPLICATIONS OF LASERS IN VARIOUS FIELDS

Depending on the special characteristics of laser light, it is having tremendous applications in various filed. The details are as follows:

1. In Communications:

i. Lasers are used in optical fiber communications. In optical fiber communications, lasers are used as light source to transmit audio, video signals and data to long distances without attention and distortion.
ii. The narrow angular spread of laser beam can be used for communication between earth and moon or to satellites.
iii. As laser radiation is not absorbed by water, so laser beam can be used in under water (inside sea) communication networks.

2. Industrial Applications:

i. Lasers are used in metal cutting, welding, surface treatment and hole drilling. Using lasers cutting can be obtained to any desired shape and the curved surface is very smooth.
ii. Welding has been carried by using laser beam.
iii. Dissimilar metals can be welded and micro welding is done with great case.
iv. Lasers beam is used in selective heat treatment for tempering the desired parts in automobile industry.
v. Lasers are widely used in electronic industry in trimming the components of ICs

3. Medical Applications:

i. Lasers are used in medicine to improve precision work like surgery. Brain surgery is an example of precision surgery Birthmarks, warts and discoloring of the skin can easily be removed with an unfocussed laser. The operations are quick and heal quickly and, best of all, they are less painful than ordinary surgery performed with a scalpel.
ii. Cosmetic surgery (removing tattoos, scars, stretch marks, sun spots, wrinkles, birthmarks and hairs) see lasers hair removal.
iii. Eye surgery and refracting surgery.
iv. Soft tissue surgery: Co_2 Er: YAG laser.
v. Laser scalpel (general surgery, gynecological, urology, laparoscopic).
vi. Dental procedures.

vii. Photo bio modulation (i.e. laser therapy).
viii. "No-touch" removal of tumors, especially of the brain and spinal cord.
ix. In dentistry for caries removal, endodontic/periodontic, procedures, tooth whitening, and oral surgery.

4. Military Applications: The various military applications are:

i. **Death rays:** By focusing high energetic laser beam for few seconds to aircraft, missile, etc can be destroyed. So, these rays are called death rays or war weapons.
ii. **Laser gun:** The vital part of energy body can be evaporated at short range by focusing highly convergent beam from a laser gun.
iii. **LIDAR (Light detecting and ranging):** In place of RADAR, we can use LIDAR to estimate the size and shape of distant objects or war weapons.

5. In Computers: By using lasers a large amount of information or data can be stored in CD-ROM or their storage capacity can be increased. Lasers are also used in computer printers.

6. In Thermonuclear fusion: To initiate nuclear fusion reaction, very high temperature and pressure is required. This can be created by concentrating large amount of laser energy in a small volume. In the fusion of deuterium and tritium, irradiation with a high energy laser beam pulse of 1 nano second duration develops a temperature of 1017^0c, this temperature is sufficient to initiate nuclear fusion reaction.

7. In Scientific Research: In scientific, lasers are used in many ways including:

i. A wide variety of interferometrie techniques.
ii. Raman spectroscopy.
iii. Laser induced breakdown spectroscopy.
iv. Atmospheric remote sensing.
v. Investigating non linear optics phenomena.
vi. Holographic techniques employing lasers also contribute to a number of measurement techniques.
vii. Laser (LADAR) technology has application in geology, seismology, remote sensing and atmospheric physics.
viii. Lasers have been used abroad spacecraft such as in the cassini-huygens mission.

Engineering physics / Engineering physics – I

Lasers

Numerical problems :-

①. A 10 mW He-Ne laser has efficiency of 1%. Assume that all input energy is utilized in pumping the atoms from the ground state to the excited state, which is 20eV above the ground state. Find how many atoms are promoted to the excited state in one second.

sol:- Efficiency of laser = 1% = $\frac{1}{100}$ = 0.01

$$\text{Power input} = \frac{\text{Power output}}{\text{Efficiency}} = \frac{10\,mW}{0.01} = 1\,W.$$

∴ Energy i/p in one second = 1 J.

$$\text{No. of atoms excited in one second} = \frac{1\,J}{20\,eV}$$

$$= \frac{1\,J}{20 \times 1.602 \times 10^{-19}\,J}$$

$$= 3.12 \times 10^{17}.$$

= Ans

②. Find the ratio of populations of the two states in a He-Ne laser that produces light of wavelength 6328 A° at 27°C.

Sol:- The ratio of population is

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/KT}$$

$$E_2 - E_1 = \frac{12400}{6328} \text{ eV} = 1.96 \text{ eV}$$

$$\therefore \frac{N_2}{N_1} = \exp\left[\frac{-1.96 \text{ eV}}{(8.61\times10^{-5} \text{ eV})(300K)}\right]$$

$$= e^{-75.88}$$

$$\boxed{\frac{N_2}{N_1} = 1.1\times10^{-33}}$$

(Ans)

③. A semi conductor laser diode has a peak emission wavelength of 1.55 um. Find its energy gap in eV.

Sol:- Energy gap $(E_g) = \frac{hc}{\lambda}$

$$= \frac{(6.63\times10^{-34}\times3\times10^{8})}{1.55\times10^{-6}} \text{ J}$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{1.55\times10^{-6}\times1.6\times10^{-19}} \text{ eV}$$

$$\boxed{E_g = 0.8\,eV}$$

=(Ans)

④. Calculate the wavelength of emitted radiation from GaAs which has a band gap of 1.44eV!

Sol:- Energy gap of a semiconductor, $E_g = h\nu$

$$E_g = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{E_g}$$

$$E_g = 1.44\,eV = 1.44 \times 1.6 \times 10^{-19}\,J$$

$$E_g = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.44 \times 1.6 \times 10^{-19}} = 8633 \times 10^{-10}\,m$$

$$= 8633\,A^\circ$$

=(Ans).

⑧.

THE END

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UNIT – V

CHAPTER – II

FIBRE OPTICS

SYLLABUS

- Introduction.
- Optical fibre as a dielectric wave guide.
- Total internal reflection.
- Acceptance angle.
- Acceptance cone and Numerical aperture.
- Step and Graded index fibres.
- Losses associated with optical fibres.
- Applications of optical fibres.

INTRODUCTION TO OPTICAL FIBER

- Optical fiber is a long cylindrical hair thin structure, which guides the information carrying light waves.
- It consists of three parts: Core, Cladding & Polyurethane protective jacket.
- The innermost part is called as core (denser) the next part is called as a cladding (rarer) and the outer part is called as shield.
- Here, the selection of core and cladding depends on their refractive indices.
- The refractive index of the core is greater than the refractive index of the cladding.
- The transmission of signal through optical fiber is depends upon the principle called total internal reflection.

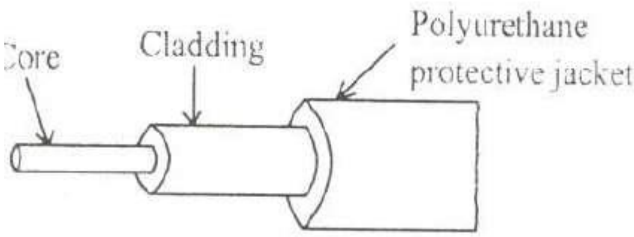


Fig. 1: Optical fibre

Advantages:

1. **Higher band width:** Optical fibers can support higher bandwidth and hence can transfer data at a higher rate.
2. **Less signal attenuation:** In optical fibers, the signal transmission distance is greater than the other transmission mediums and signal can travel 60 km without regeneration.
3. Optical fibers are immune to electromagnetic interference.
4. Optical fiber cables are much lighter than the copper cables.
5. Optical fiber cables are more immune to tapping than the copper cables.

Disadvantages:

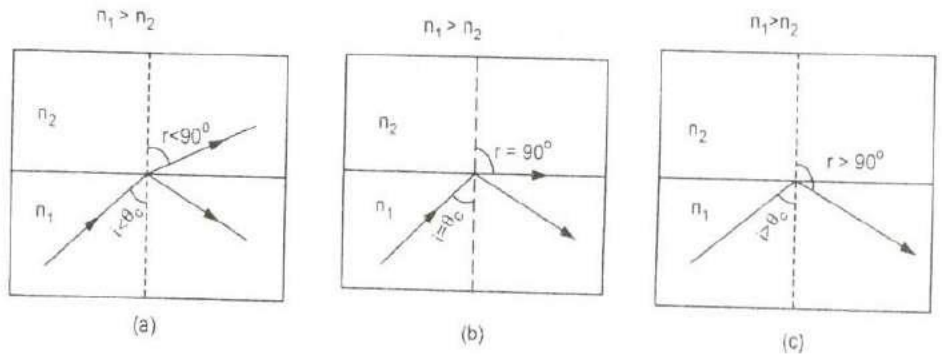
1. **Installation or maintenance:** Since the technology is new and hence needs expertise to installation and maintenance.
2. **Unidirectional:** Propagation of light in optical fiber is unidirectional and hence need two fibers for bidirectional communication.
3. **Costly:** The optical fiber cables and interconnectors used are relatively expensive.

PROPAGATION OF LIGHT THROUGH FIBER (OPTICAL FIBER AS A LIGHT GUIDE [OR] OPTICAL FIBRE AS A DIELECTRIC WAVE GUIDE)

- The main function of the Optical fiber is to accept maximum light and transmit the same with minimum attenuation.
- The incident light enters the core and strikes the interface of the Core and Cladding at large angles as shown in fig.
- Since the Cladding has lower RI than Core the light suffers multiple Total Internal Reflections.
- This is possible since by geometry the angle of incidence at the interface is greater than the Critical angle.
- Since the Total internal reflection is the reflection at the rarer medium there is no energy loss.
- Entire energy is transmitted through the fiber.
- The propagation continues even if the fiber is bent but not too sharply.
- Since the fiber guides light it is called as fiber light guide or fiber waveguide.

PRINCIPLE OF OPTICAL FIBER [OR] TOTAL INTERNAL REFLECTION

- The operation of an optical fiber is based on the principle of total internal reflection.



- We know that, when a ray of light passes from one medium to another medium then at the separation between two mediums the incident ray bends and travels into another medium. It is called refraction.
- Here, the refracted bends towards or away from the normal is depends on the denser and rarer medium.
- In this case, if the incidence angle is increases then refracted angle also increases; at a particular angle of incidence the angle of refraction becomes normal or 90^0. The respective angle of incidence is called critical angle.
- If the angle of incidence is increased above the critical angle then there is no refraction into another medium but there is reflection into same medium. This phenomenon is called as total internal reflection.
- Consider an optical fiber consisting of a core and cladding of refractive indices n_1 and n_2 (here $n_1 > n_2$).
- Let a light ray passes from core to cladding with an angle of incidence 'i' and the get refracted with angle of refraction 'r'.
- The refracted ray bends away from the normal as it travels from core to cladding.
- If the angle of incidence is increases then angle of refraction also increases.
- **Cases:**
  1. When $i < \theta c$, then the incident ray refracts into the core.
  2. When $i = \theta c$, then the incident ray passes along the interface of core and cladding.
  3. When $i > \theta c$, then the light ray will be reflected back into the core i.e undergoes total internal reflection.

Conditions for total internal reflection:

1. The ray of light must travel from denser medium towards rarer medium.
2. The angle of incidence in the denser medium must be greater than the critical angle for the pair of the media in contact.

Relation between refractive index and critical angle: Let a ray of light travels from denser medium of refractive index n_1 towards rarer medium of refractive index n_2 with an angle of incidence i_c and angle of refraction 90° as shown in Fig below. Applying Snell's

law to the interface, $n_1 \sin i_c = n_2 \sin 90$ or $\frac{n_2}{n_1} = \frac{\sin i_c}{\sin 90} = \frac{\sin i_c}{1}$

or $\sin i_c = n_2 / n_1$

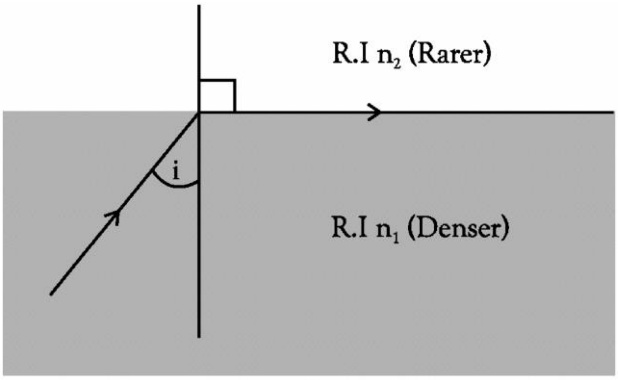


Fig: Representation of critical angle

Thus Critical angle, $i_c = \sin^{-1}(n_2/n_1)$

To confine the optical signal in the core, the refractive index of the core must be greater than that of the cladding to support total internal reflection.

ANGLE OF ACCEPTANCE [OR] ACCEPTANCE ANGLE & ACCEPTANCE CONE

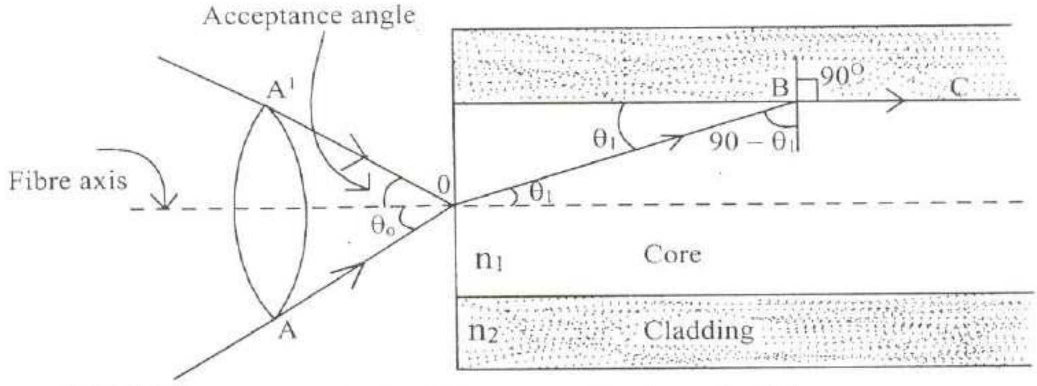


Fig. 2: Light propagation in optical fibre

- Consider an optical fiber consisting core of refractive index n_1 and cladding of refractive index n_2.
- Let a ray of light AO coming from air and is incident at 'O' and makes an angle of incidence θo.
- This ray is refracted into the core and passes along the path OB; here the angle of refraction is θ_1.
- The ray OB is incident on core-cladding interface with an angle of incident $90 - \theta_1$.
- If this angle of incidence is equal to the critical angle in core – cladding then the angle of refraction is 90^0.
- If the angle of incidence is less than θ_o then angle of refraction is less than θ_1 and angle of incidence at the core-cladding interface is larger than critical angle then that ray suffers total internal reflection at the core-cladding interface.
- Hence, all the rays which the core at an angle of incidence less than θo will have refracting angles less than θ_1.
- As a result their angles of incidence at the interface between core and cladding will be more than critical angle.
- Then total internal reflection is occurs and the ray propagates through the fiber.
- Equation for acceptance angle can be obtained by applying snell's law at the point 'O' then,

Applying Snell's law at A (core-air interface)

$$\frac{\sin\theta_i}{\sin\theta_r}=\frac{n_1}{n_0}$$

$$n_0\sin\theta_i=n_1\sin\theta_r \qquad (1)$$

Let a normal BC be drawn from the point B to fibre axis. Then from ΔABC, we get

$$\theta_r+\theta=90°$$

$$\theta_r=90°-\theta \qquad (2)$$

Substituting the above value in Eq. (1),

$$n_0\sin\theta_i=n_1\sin(90°-\theta)$$

$$n_0\sin\theta_i=n_1\cos\theta \qquad (3)$$

To get total internal reflection at point B (core, cladding interface) the incident angle θ should be greater than or equal to θ_c (critical angle).

Let the maximum angle of incidence at point A be θ_a for which $\theta\geq\theta_c$

From Eq. (3), we get

$$n_0\sin\theta_a=n_1\cos\theta_c$$

$$\sin\theta_a=\frac{n_1}{n_0}\cos\theta_c \qquad (4)$$

But $\qquad \sin\theta_c=\dfrac{n_2}{n_1}$

$$\cos\theta_c=\sqrt{1-\sin^2\theta_c}=\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$=\sqrt{\frac{n_1^2-n_2^2}{n_1^2}}=\frac{\sqrt{n_1^2-n_2^2}}{n_1} \qquad (5)$$

Substituting the value in Eq. (4),

$$\sin\theta_a=\frac{n_1}{n_0}\times\frac{\sqrt{n_1^2-n_2^2}}{n_1}$$

$$\sin\theta_a=\frac{\sqrt{n_1^2-n_2^2}}{n_0} \qquad (6)$$

For air medium, $n_0 = 1$

$$\sin \theta_a = \sqrt{n_1^2 - n_2^2} \tag{7}$$

$$\theta_a = \sin^{-1} \sqrt{n_1^2 - n_2^2} \tag{8}$$

- In order to maintain θ to be greater than Critical angle, the angle of incidence relative to Axis of the fiber (At the launching face) should not be greater than a value θ_0.
- The ray corresponding to θ_0 can be used to describe a Conical surface. The cone formed is called Acceptance cone

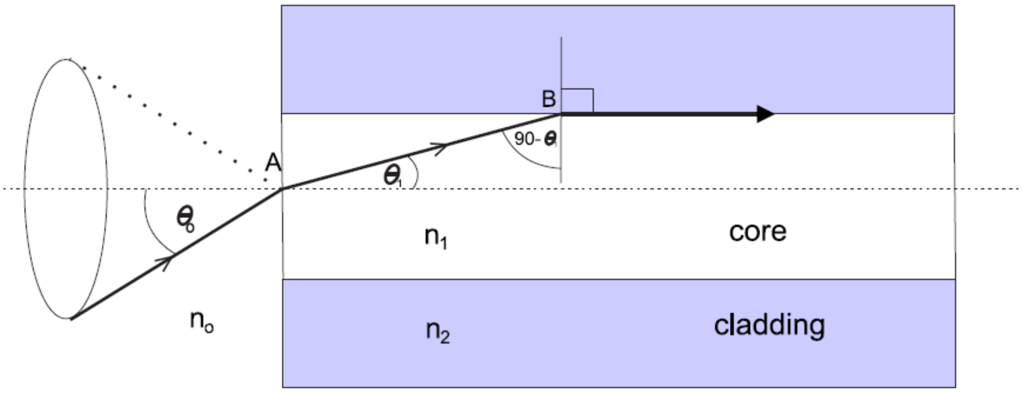


Figure 6.4: Acceptance cone

NUMERICAL APERTURE

The light gathering capacity of optical fiber is called numerical aperture. It is also defined as the sine of acceptance angle is called as numerical aperture.

$$NA = \sqrt{(n_1 + n_2)(n_1 - n_2)}$$

$$NA = \sqrt{(n_1 + n_2)n_1\Delta}$$

where $\Delta = \frac{n_1 - n_2}{n_1} =$ Fractional change in refractive indices of core and cladding.

For all optical fibres, $n_1 \approx n_2$ so $n_1 + n_2 = 2n_1$

$$NA = \sqrt{2n_1^2\Delta} = n_1\sqrt{2\Delta} \tag{10}$$

'*NA*' can be increased by increasing Δ and thus enhancing the light gathering capacity of the fibre.

STEP AND GRADED INDEX FIBRES

Depending on the refractive indices of the core and cladding again they are classified in to two types:

1. Step-index optical fibers.
2. Graded index optical fibers.

1. Step-index optical fiber:

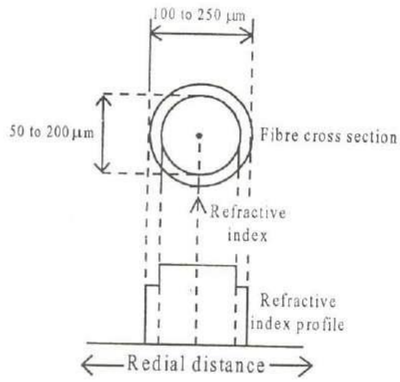
- The refractive index is uniform throughout the core of the fiber.
- As we go radially in this fiber, the refractive index under goes a step-changes at the core-cladding interface.
- Again, depending on the propagation of light through the fiber it is further classified into two types:
  - i. Single mode step-index optical fiber.
  - ii. Multimode step-index optical fiber.

i. Single mode step index optical fiber:

- The core diameter of this fiber is about 8 to 10 μm and outer diameter of cladding is 60 to 70 μm.
- There is only one path for ray propagation. So, it is called single mode step-index optical fiber.
- In this fiber the transmission of light is by successive total internal reflection.

ii. Multi mode step-index optical fiber

- The construction of multimode step index optical fiber is same as that of single mode optical fiber except that its core and cladding refractive indices are much larger than single mode.
- The core diameter changes from 50 to 200 nm and outer diameter of cladding is changes from 100 to 250 nm.
- The construction of multimode step index optical fiber is shown below:



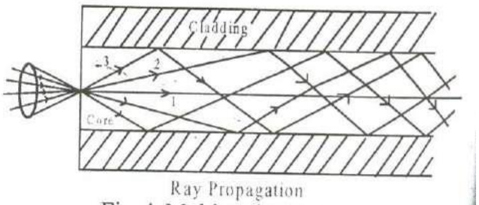


Fig. 4: Multimode step index fibre

The number of possible propagation modes in the core is given by the V- number as,

$$V = (2\pi/\lambda) * a * (N.A)$$

Where,

λ = wavelength of the light

a = radius of the core

N.A = numerical aperture

2. Graded index fiber:

- In this type optical fiber the refractive index core decreases continuously from the fiber axis to the cladding interface in a parabolic manner.
- The refractive index is maximum at the centre and minimum at the surface of the core.
- When light ray enters into the core and moves towards the cladding interface, then it encounters a more and more rarer medium due to decrease of refractive index.
- As a result, the light ray bends more away from the normal and finally bends towards the axis and moves the core-cladding interface at the bottom.
- Again, it bends in the upward direction.
- Thus, the light due to refraction takes sinusoidal paths.
- This fibre is of refractive type.

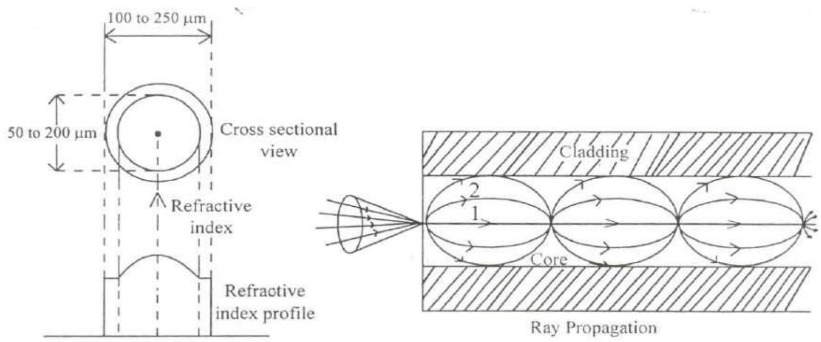


Fig.5 : Multimode graded index fibre

Differences between Step Index & Graded Index Optical Fibre:

SNO.	Step Index Fibre	Graded Index Fibre
1.	It is of reflective type.	It is of refractive type.
2.	In this fibre, signal distortion is high.	In this type fibre, signal distortion is very low.
3.	Refractive of the core is constant.	Refractive index of the core decreases parabolically.
4.	Numerical aperture is more for multimode step index fiber.	Numerical aperture is less.

LOSSES ASSOCIATED WITH OPTICAL FIBRES [OR] ATTENUATION IN FIBERS

Usually, the power of light at the output end of optical fiber is less than the power launched at the input end, then the signal is said to be attenuated.

Attenuation: It is the ratio of input optical power (P_i) in to the fiber to the power of light coming out at the output end (P_o).

Attenuation coefficient is given as, $\alpha = 10/L \log_{10} P_i/P_o$ db/km.

Attenuation is mainly due to:

1. Absorption.
2. Scattering.
3. Bending.

1. Absorption Losses: In glass fibers, three different absorptions take place.

i. **Ultra violet absorption**: Absorption of UV radiation around 0.14μm results in the ionization of valence electrons.

ii. **Infrared absorption:** Absorption of IR photons by atoms within the glass molecules causes heating. This produces absorption peak at 8μm, also minor peaks at 3.2, 3.8 and 4.4μm.

iii. **Ion resonance/OH^- absorption:** The OH^- ions of water, trapped during manufacturing causes absorption at 0.95, 1.25 and 1.39μm.

2. Scattering Losses: The molten glass, when it is converted in to thin fiber under proper tension creates sub microscopic variations in the density of glass leads to losses. The dopents added to the glass to vary the refractive index also leads to the inhomogenities in the fiber. As a result losses occur. Scattering losses are inversely proportional to λ^4

3. Bending Losses: In a bent fiber, there is a loss in power of the transmitted signal called as Bending Loss. According to the theory of light, the part of the wave front travelling in cladding (rarer medium) should travel with more velocity than the wave front in the core (denser medium).

APPLICATIONS OF OPTICAL FIBERS

1. Fiber Optic Communications: Optical fiber is the basic building block for a fiber optic based network. A model of this simple link is shown in Fig:

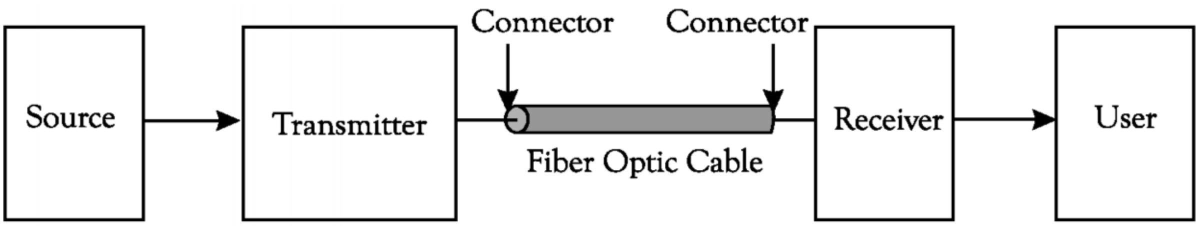


Fig: Block diagram of point to point fiber optic communication

The above illustration indicates the Source-User pair, Transmitter and Receiver. It also clearly shows the fiber optic cable constituting the Transmission Medium as well as the connectors that provide the interface of the Transmitter to the Transmission Medium and the Transmission Medium to the Receiver. All of these are components of the simple fiber optic data link.

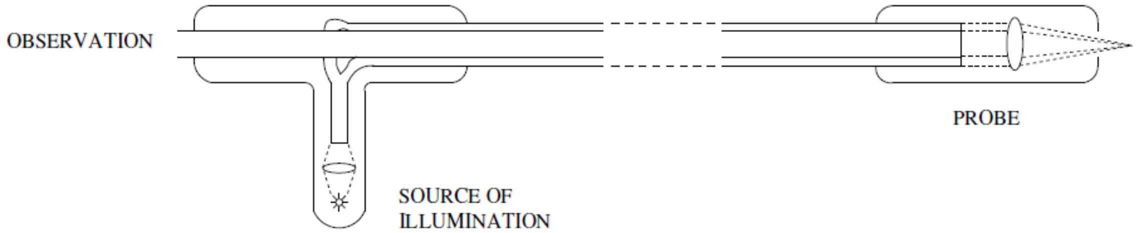
Optical fiber can be used as a medium for telecommunication and networking because it is flexible and can be bundled as cables. It is especially advantageous for long-distance communications, because light propagates through the fiber with little attenuation compared to electrical cables. This allows long distances to be spanned with few repeaters. Additionally, the per-channel light signals propagating in the fiber have been modulated at rates as high as 111 gigabits per second. Each fiber can carry many independent channels, each using a different wavelength of light using a technique called wave length division multiplexing (WDM).

2. Fiber Optic Sensors: Fibers have many uses in remote sensing. In some applications, the sensor is itself an optical fiber. In other cases, fiber is used to connect a non-fiberoptic sensor to a measurement system. Depending on the application, fiber may be used because of its small size, or the fact that no electrical power is needed at the remote location, or because many sensors can be multiplexed along the length of a fiber by using different wavelengths of light for each sensor, or by sensing the time delay as light passes along the fiber through each sensor. Time delay can be determined using a device such as an optical time-domain reflectometer.

Optical fibers can be used as sensors to measure strain, temperature, pressure and other quantities by modifying a fiber so that the quantity to be measured modulates the intensity, phase, polarization, wavelength or transit time of light in the fiber. Sensors that vary the intensity of light are the simplest, since only a simple source and detector are required. A particularly useful feature of such fiber optic sensors is that they can, if required, provide distributed sensing over distances of up to one meter.

Extrinsic fiber optic sensors use an optical fiber cable, normally a multi-mode one, to transmit modulated light from either a non-fiber optical sensor, or an electronic sensor connected to an optical transmitter. A major benefit of extrinsic sensors is their ability to reach places which are otherwise inaccessible. An example is the measurement of temperature inside aircraft jet engines by using a fiber to transmit radiation into a radiation pyrometer located outside the engine. Extrinsic sensors can also be used in the same way to measure the internal temperature of electrical transformers, where the extreme electromagnetic fields present make other measurement techniques impossible. Extrinsic sensors are used to measure vibration, rotation, displacement, velocity, acceleration, torque and twisting.

3. Applications in medicine and industry: Optical fibers are also useful for medical applications for visualization of internal portions of the human body. They can also be used for the examination of visually inaccessible regions for engineering applications. A typical example of a flexible fiberscope (endoscope) is shown in Fig.



Use of laser in combination with optical fibers is being exploited not only for the observation of internal portions but also in the treatment of malignant tissues. A similar equipment will also be useful to examine parts of machinery which are otherwise inaccessible to observation.

Optical fibers also find application in the fabrication of sensors which are devices used to measure and monitor physical quantities such as displacement, pressure, temperature, flow rate etc.

4. Other Applications of Optical Fibers:

i. Illumination: Fibers are widely used in illumination applications. They are used as light guides in medical and other applications where bright light needs to be shone on a target without a clear line-of-sight path. In some buildings, optical fibers are used to route sunlight from the roof to other parts of the building. Optical fiber illumination is also used for decorative applications, including signs, art, and artificial Christmas trees.

ii. Imaging Optics: Optical fiber is also used in imaging optics. A coherent bundle of fibers is used, sometimes along with lenses, for a long, thin imaging device called an endoscope, which is used to view objects through a small hole. Medical endoscopes are used for minimally invasive exploratory or surgical procedures (endoscopy). Industrial endoscopes (like fiberscope or borescope) are used for inspecting anything hard to reach, such as jet engine interiors.

iii. Spectroscopy: In spectroscopy, optical fiber bundles are used to transmit light from a spectrometer to a substance which cannot be placed inside the spectrometer itself, in order to analyze its composition.

iv. Laser Gain Medium: An optical fiber doped with certain rare earth elements such as erbium can be used as the gain medium of a laser or optical amplifier.

Engineering physics / Engineering physics – I

Fiber optics / optical fibers

Numerical problems:–

① In an optical fibre, the core material has refractive index 1.43 & refractive index of clad material is 1.4. Find the propagation angle.

Sol:–

$$\cos\theta_c = \frac{n_2}{n_1} = \frac{1.40}{1.43} = 0.979$$

$\therefore$ Propagation angle $(\theta_c) = \cos^{-1}(0.979) = 11.8^\circ$.

(Ans)

② In an optical fibre, the core material has refractive index 1.6 & refractive index of clad material is 1.3. What is the value of critical angle? Also calculate the value of angle of acceptance cone.

Sol:–

Critical angle $(\sin\phi_c) = \frac{n_2}{n_1} = \frac{1.3}{1.6} = 0.8125$

$$\phi_c = \sin^{-1}(0.8125) = 54.3^\circ$$

Acceptance angle, $\theta_0 = \sin^{-1}\left[\sqrt{n_1^2 - n_2^2}\right]$

$$= \sin^{-1}\left[\sqrt{1.6^2 - 1.3^2}\right]$$

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$= \sin^{-1}(0.87)$

$\theta_0 = 60.5^\circ$

Angle of acceptance cone $= 2\theta_c = 121^\circ$

= (Ans)

③ Calculate the numerical aperture & acceptance angle of an optical fibre from the following data:-

μ_1 (Core) $= 1.55$

μ_2 (cladding) $= 1.50$

Sol:-

(i). Numerical Aperture (NA) $= \sqrt{n_1^2 - n_2^2} = \sqrt{1.55^2 - 1.50^2}$

$= \sqrt{0.153} = 0.471$

(ii). Acceptance angle $(\theta_0) = \sin^{-1}\left[\sqrt{n_1^2 - n_2^2}\right]$

$= \sin^{-1}\left[\sqrt{1.55^2 - 1.50^2}\right]$

$\theta_0 = 23.02^\circ$

= (Ans)

④ What is the numerical aperture of an optical fibre cable with a clad index of 1.378 & a core index of 1.546?

Sol:- $N.A = \sqrt{n_1^2 - n_2^2}$

$= \sqrt{1.546^2 - 1.378^2}$

$= \sqrt{0.491} = 0.70$

= (Ans)

⑤. A fibre cable has an acceptance angle of 30° & a core index of refraction of 1.4. Calculate the refractive index of the cladding.

Sol:-

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$\therefore \sin^2\theta_0 = n_1^2 - n_2^2$$

$$n_2^2 = n_1^2 - \sin^2\theta_c$$

$$= (1.4)^2 - \sin^2(30^\circ)$$

$$= 1.96 - 0.25$$

$$n_2^2 = 1.71$$

$$n_2 = \sqrt{1.71} = 1.308$$

(Ans)

⑥. Calculate the angle of acceptance of a given optical fibre, if the refractive indices of the core & the cladding are 1.563 & 1.498 respectively.

Sol:-

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$= \sqrt{(1.563)^2 - (1.498)^2}$$

$$\sin\theta_c = 0.4461$$

$$\theta = \sin(0.4461) = 26.49^\circ$$

(Ans)

7. Calculate the fractional index change for a given optical fibre if the refractive indices of the core & the claddings are 1.563 & 1.498 respectively.

Sol:-

Fractional index change $(\Delta) = \frac{n_1 - n_2}{n_1}$

$= \frac{1.563 - 1.498}{1.563}$

$= \frac{0.065}{1.563}$

$\boxed{\Delta = 0.0415}$ ←(Ans)

8. Calculate the refractive ~~indices~~ indices of the core & the cladding material of a fiber from the following data:-

Numerical aperture (NA) = 0.22

$\Delta = 0.012$

where, Δ is the fractional refractive index change

Sol:- $NA = n_1\sqrt{2\Delta}$

$$0.22 = n_1\sqrt{2\times0.012} = 0.155\,n_1$$

$$n_1 = \frac{0.22}{0.155} = 1.42$$

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$\frac{1.42 - n_2}{1.42} = 0.012$$

$$n_2 = 1.42 - 1.42\times0.012$$

$$n_2 = 1.403$$

=(Ans)

Q. Find the fractional refractive index & numerical aperture for an optical fibre with refractive indices of core & cladding as 1.5 & 1.49 respectively.

Sol:-

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$= \frac{1.5 - 1.49}{1.5}$$

$$= 0.0067$$

$$NA = n_1\sqrt{2\Delta} = 1.5\sqrt{2\times0.0067} = 0.174$$

=(Ans)

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(10). Optical power of 1mW is launched into an optical fibre of length 100m. If the power emerging from the other end is 0.3mW, calculate the fibre attenuation.

Sol:- Attenuation (α) $= \frac{10}{L} \log \frac{P_i}{P_o}$

$= \frac{10}{0.1km} \log \frac{1mW}{0.3 mW}$

$\boxed{\alpha = 52.3 \, dB/km}$ (Ans)

(11). What is the attenuation in dB/km, if 15% of the power fed at the launching end of a ½ km fibre is lost during propagation?

Sol:- Attenuation (α) $= \frac{10}{L} \log \frac{P_i}{P_o}$

$= \frac{10}{0.5km} \log \frac{1}{0.15}$

$\boxed{\alpha = 16.48 \, dB/km}$ (Ans).

THE END

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