

## UNIT-I

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QUANTUM MECHANICSINTRODUCTION:

classical physics is dominated by two fundamental concepts. The first is the concept of a "particle", discrete entity (consisting apart) with position and momentum which moves in accordance with "Newton's Law Of Motion". The second is the concept of an "electromagnetic wave" and "physical entity" with a presence at every point in space that is provided by electric and magnetic fields which change in accordance with "Maxwell's Law" of electromagnetism.

The classical picture began to crumble (fall apart) in 1900 when Max Plank published a theory of black body radiation; i.e. a theory of thermal radiation in equilibrium with perfectly absorbing body. Plank provided an explanation of the observed properties of black-body radiation by assuming that atoms emit and absorb discrete quanta of radiation with energy " $E=h\nu$ ", where ' $\nu$ ' is the frequency of the radiation and 'h' is a fundamental constant of nature with value, " $h=6.6 \times 10^{-34} \text{ Js}$ ".

This concept is now called "plank's constant". This constant has a strange role of linking wave-like and particle-like properties.

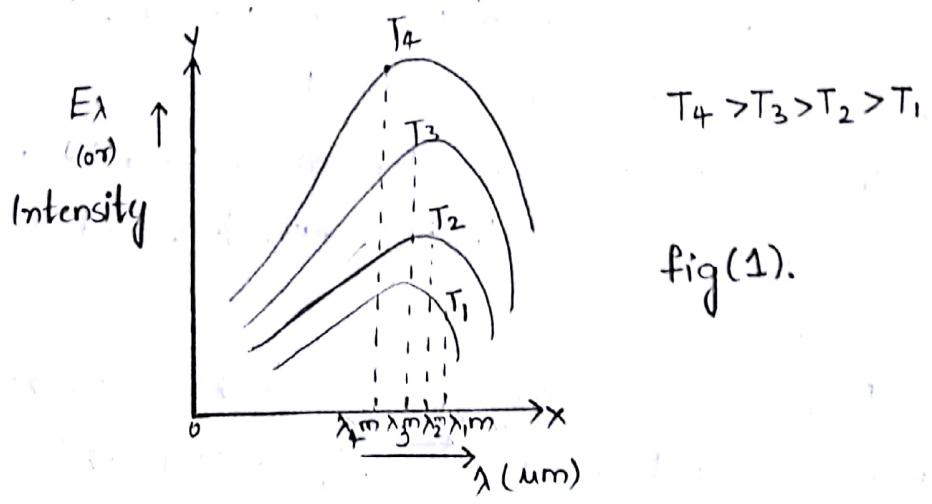
Photon's are particle-like quanta of electro magnetic radiation. They travel at the speed of light 'c' with

momentum 'p' and energy 'E' given by " $E = \frac{hc}{\lambda}$ " and " $p = \frac{h}{\lambda}$ ", where ' $\lambda$ ' is the wavelength of the electroscopic standards, the momentum and energy of a photon are tiny.

We know that an electron volt,  $1\text{eV} = 1.602 \times 10^{-19}\text{J}$  is a useful unit for the energy of a photon. Visible photons have energies of the order of an electron volt and x-ray photon have energies of the order of 10kev.

### BLACK-BODY RADIATION:

A body that completely absorbs all wavelengths of radiation incident on it at low temperatures (or) emits different wavelengths of radiation at higher temperatures known as black body. A black body may be idealized by a small hole drilled in a cavity. A graph has been plotted between intensity versus wavelength of radiation from a black body. The temperature of the body is raised to different values and distribution curves are plotted for different temperatures as shown in fig(1).



fig(1).

From the graph it has been observed that;

1. The intensity of radiation increases for each wave length as temperature of the body increases.
2. At any given temperature, the intensity of radiation from the body is maximum for a particular wave length represented as  $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \dots$ . This wave length shifts towards shorter wavelength region with increase of temperature.

### PLANK'S LAW:

A mathematical relationship formulates in 1900 by german physicist "Max Plank" to explain the spectral energy distribution of radiation emitted by a black body. Plank's law for the energy " $E_\lambda$ " radiated per unit value by a cavity of a black body in the wave length interval  $\lambda$  to  $\lambda + d\lambda$  can be written in terms of Plank's constant ( $h$ ), the speed of light ( $c$ ) the boltzmen constant ( $k$ ) and the absolute temperature ( $T$ ).

$$E_\lambda = \frac{8\pi hc}{\lambda^5} \left[ \frac{1}{e^{hc/kT\lambda}} - 1 \right] d\lambda$$

The wavelength of emitted radiation is inversely proportional to its frequency or  $\lambda = \frac{c}{\nu}$ . The value of Plank's constant is found to be  $6.626 \times 10^{-34} \text{ Js}$ .

## PHOTO ELECTRIC EFFECT:

The ejection of electrons from a surface of a metal under the action of light is called the "photo electric effect". The materials that exhibit photo electric effect are called "photo sensitive materials". The emitted electrons are called "photo electrons". This effect was observed by "Heinrich Hertz" in 1887.

To study the photo electric effect, the photo sensitive material is made as cathode of a vacuum tube and it is connected into a circuit as illustrated in fig.

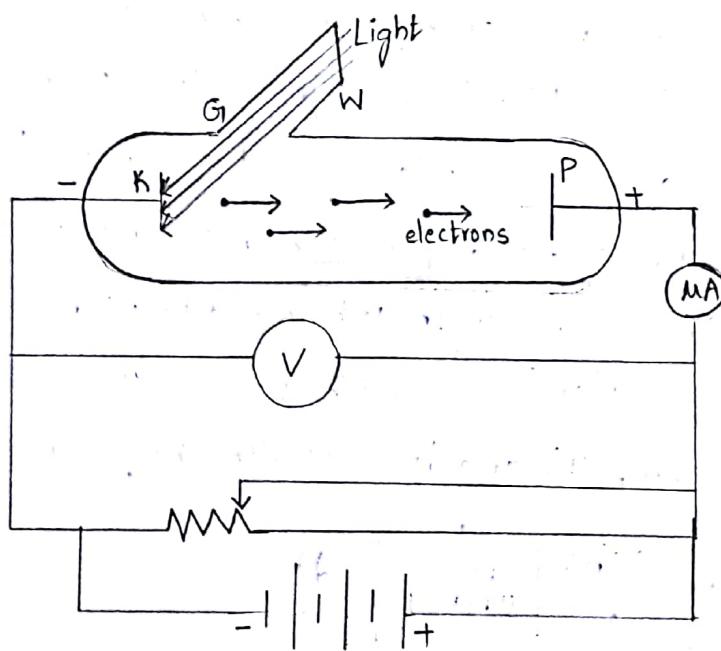


fig: Schematic of the arrangement to study photo electric effect.

The vacuum tube 'G' is made of glass or quartz. The cathode 'K' is coated with the photosensitive material and is connected to the negative terminal of a battery. When the tube is kept in the dark, there will be no current in the circuit. When it is exposed to light, photoelectrons are liberated

from the cathode 'K'. They are attracted to the anode 'P' and flow through the circuit. The strength of the current read by the meter 'A' is a measure of the number of photo electrons. The potential difference across 'K' and 'P' is varied with the help of a potentiometer. The polarity of the voltage can be varied through a suitable arrangement. The experiment showed that the photocurrent and energy of photo electrons depend on the intensity and frequency of the incident light. We shall study the results in detail:

### i) EFFECT OF INTENSITY ON PHOTO CURRENT:

The photo sensitive surface is exposed to a monochromatic light of different intensities and the variation of photocurrent with anode voltage is studied.

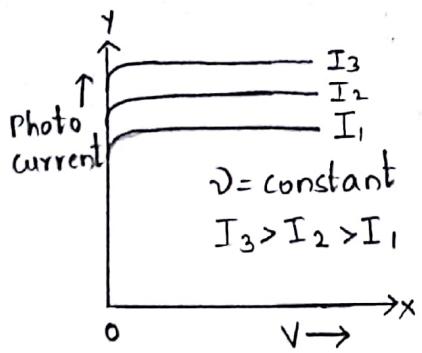


Fig 2(a)

In each case the photocurrent increases with voltage and reaches a saturation stage

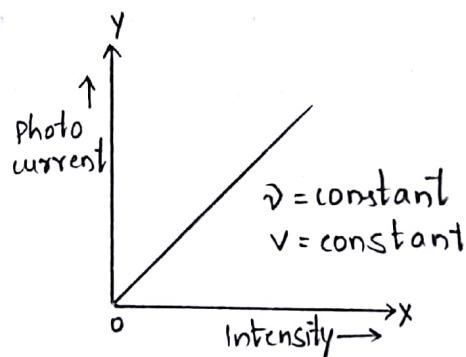


Fig 2(b)

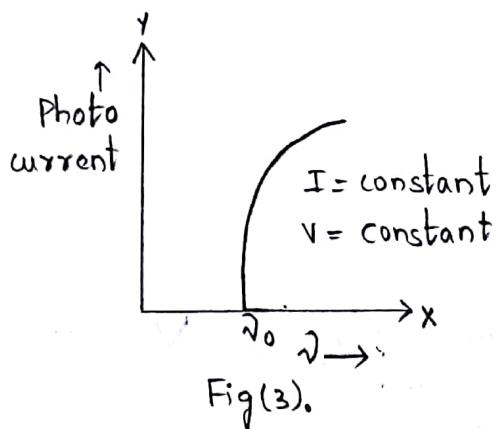
Variation of photo current with increasing intensity

Fig 2(a): Variation of photocurrent with increasing voltage

- \* A finite photo current is observed even at zero anode and it increases slightly as the voltage is increased reaching a saturation value at a higher voltage. The saturation region indicates that all the photo electrons emitted by the cathode are collected by the anode.
- \* The amount of photo current is higher, the higher is the intensity of the incident light. To bring out this feature more clearly, the variation of photo current with intensity by holding anode voltage constant is plotted in fig 2(b). It is seen that the photo current is proportional to the intensity of light.

## ii) EFFECT OF FREQUENCY ON PHOTO CURRENT:

The photo sensitive cathode is illuminated by the light of different colours but same intensity. The variation of photo current as a function of frequency is shown in fig(3).



Fig(3).

It is seen that photo emission occurs only when the light has a frequency greater than a certain value  $\nu_0$ . Photo current is zero when the frequency of light is below

the value of  $\nu_0$ . The limiting frequency is called the "threshold frequency". The corresponding wavelength  $\lambda_0 (\lambda_0 = \frac{c}{\nu_0})$  is called threshold wavelength. Different metals are found to have different threshold frequencies. Photo emission does not take place when the incident frequency is below the threshold value, however intense the light may be, on the other hand photoemission radially takes place when the frequency is above the threshold. However, feeble (weak) the light may be. Thus, it appears that photo electric effect is a frequency dependent phenomenon and it is not an intensity dependent phenomenon.

#### MEASUREMENT OF KINETIC ENERGY OF PHOTO ELECTRONS:

From fig 2(a) it is seen that a finite current flows through the circuit even when the anode potential is zero. It means that in the absence of an attracting field, the electrons are reaching the anode. It can happen only if electrons are emitted with some initial kinetic energy. The kinetic energies of the electrons can be measured when the battery in the circuit is reversed to make cathode positive and anode negative.

Now, only those electrons having a kinetic energy greater than eV will reach the anode. As the magnitude of negative anode potential is increased, the electrons are inhibited and the current in the circuit falls. Finally, when 'V' is ~~greater~~ greater than ' $v_s$ ' [ $V > v_s$ ] no electrons will reach anode.

And the current will be zero. The potential ( $-V_s$ ) which is required to halt the most energetic electrons is called the stopping potential or retarding potential. The maximum KE of photo electrons is thus,

$$KE_{\max} = eV_s \quad \text{or} \quad KE_{\max} = \frac{1}{2}mv_{\max}^2$$

### iii) EFFECT OF INTENSITY ON STOPPING POTENTIAL:

The photo sensitive cathode is illuminated with a monochromatic light whose intensity can be varied. The variation of stopping potential with a change in light intensity is studied. The result is shown in fig 4.

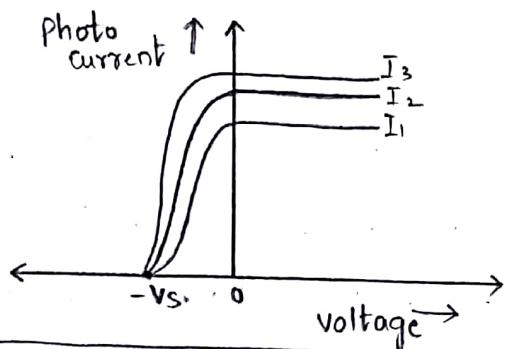
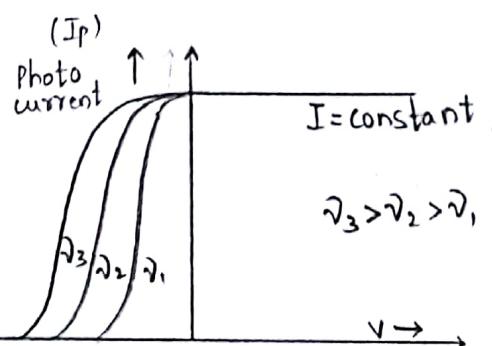


Fig 4: Photo current has a finite value at  $V=0$ , but it reaches zero at stopping Potential.

An increase in the intensity of the light is found only to increase the value of current but it does not influence the magnitude of the stopping potential  $V_s$ .

#### W EFFECT OF FREQUENCY ON STOPPING POTENTIAL:

The intensities of light of different frequencies are adjusted to give the same amount of photo current holding the anode at a suitable positive potential. Then the anode potential is reduced and made negative. The variation of stopping potential is studied varying the light frequency. The result is shown in fig(5). It is seen that the stopping potential increases with increase in frequency of the incident light.



Fig(5): Variation of stopping Potential with frequency

The stopping potentials are expressed in terms of maximum kinetic energies using the equation ① " $KE = eVs$ " and the variation of maximum kinetic energy is plotted against the frequency, the graph is a straight line as shown in fig(6) cutting the frequency axis at the threshold value ( $\nu_0$ ).

The graph shows that the maximum KE of the photo electrons increases in direct proportion to the frequency of incident light.

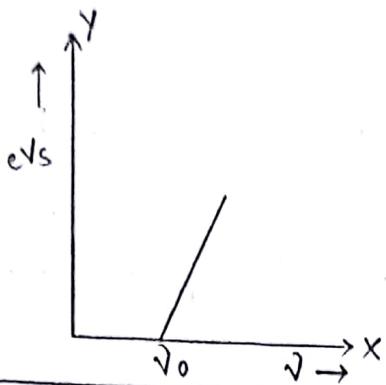


Fig (6): The  $k \cdot E$  is equal to zero at  $\nu_0$ . Above threshold, the  $kE \propto \nu$  of incident light.

#### V) NO TIME DELAY:

The photo current is observed in the circuit immediately on illumination of cathode, without any time lag. Even, though the intensity of the incident light is low.

#### CHARACTERISTICS OF THE PHOTO ELECTRIC EFFECT:

The experimental observations on photo electric effect can be summed up as follows;

1. Electrons are emitted from photo sensitive surfaces.  
Each material has a definite minimum frequency ' $\nu_0$ ' below which the photo electric emission does not occur. The minimum frequency is called the "Threshold frequency" ( $\nu_0$ ).
2. For frequencies  $\nu > \nu_0$ , the number of electrons emitted per second is proportional to the intensity of light.
3. For frequencies  $\nu > \nu_0$ , photo electrons are emitted with kinetic energies ranging upto maximum value. The maximum kinetic energy can be determined in terms of stopping potential. The stopping potential ' $V_s$ ' does not depend on intensity of light.

4. The stopping potential ' $V_s$ ' is a linear function of the incident light.
5. Emission of photo electrons is instantaneous no matter how feeble the incident light may be.

### EINSTEIN'S THEORY OF PHOTOELECTRIC EFFECT:

Einstein assumed that the light of frequency illuminating the photo sensitive material may be regarded as a stream of photons, each photon carrying an energy ' $h\nu$ '. When a photon encounters an electron in the material, it gives up all its energy to the electron and the electron acquires the energy. Each free electron in the material is prevented from leaving it, because of a potential barrier. The amount of energy required by an electron to overcome the potential barrier is equal to the work function ( $\omega_0$ ) of the material. Therefore, when an electron absorbs the photon energy ' $h\nu$ ', part of the energy is spent in overcoming the potential barrier and the remaining part of energy goes into its kinetic energy.

According to the law of conservation of energy;  
 Energy of photon = Energy needed to liberate the electron + Maximum kinetic energy of liberated electrons

$$h\nu = \omega_0 + KE_{max}$$

or

$$h\nu = \omega_0 + \frac{1}{2}mv_{max}^2 \quad \text{--- ①}$$

(1)

But,  $kE_{\text{max}} = eV_s$

$$\therefore h\nu = W_0 + eV_s \quad \text{--- (2)}$$

This equation is called photo electric equation.

Here,  $W_0$  = Work function

$$W_0 = h\nu_0.$$

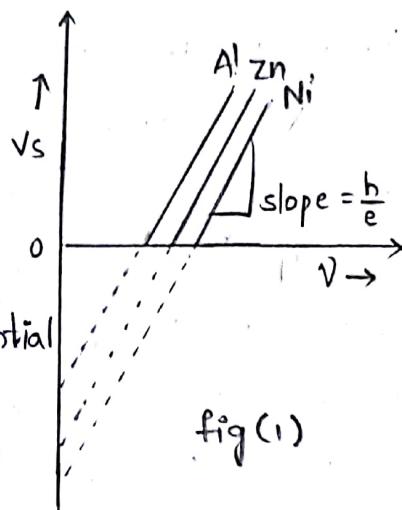
### EXPERIMENTAL VERIFICATION OF EINSTEIN'S THEORY:

Eq(2) may be rewritten as

$$eV_s = h\nu - W_0$$

$$V_s = \frac{h\nu}{e} - \frac{W_0}{e} \rightarrow (3)$$

The relation between the stopping potential ( $V_s$ ) and the incident frequency ( $\nu$ ) is linear as seen in fig(1).



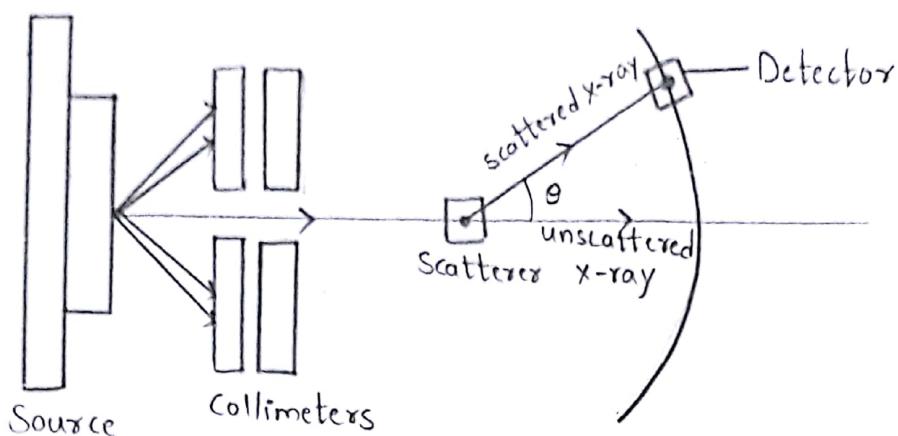
The slope of the line gives the value of  $\frac{h}{e}$  from which Plank's constant can be calculated and compared with the values obtained from other experiments and it is agreed very well. The intercept of the line on frequency axis gives threshold frequencies. For different metals, the different lines are parallel and slope is same. The lines intercept the frequency axis at different frequency values.

### THE COMPTON EFFECT:

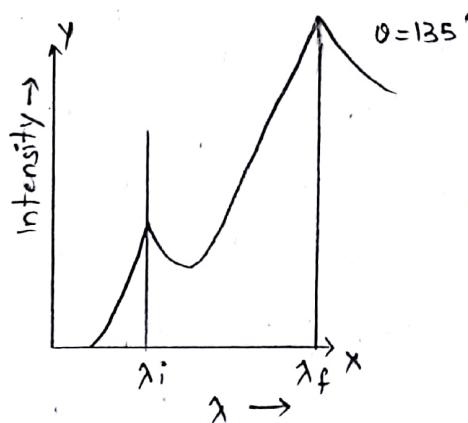
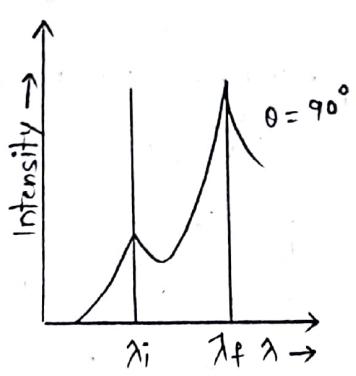
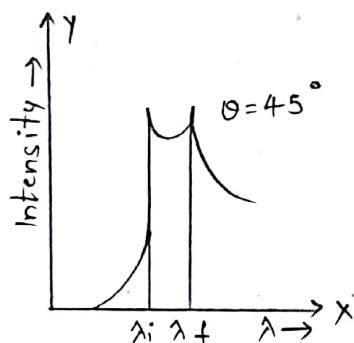
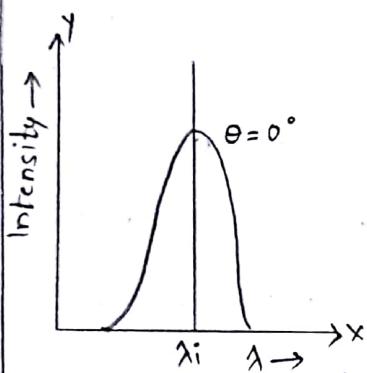
When a beam of monochromatic x-rays strike a target, the x-rays will be dispersed in all possible directions after interacting with the atoms in the target. The phenomenon is called scattering and the angle between the directions of incident and scattered rays is called scattering angle. Scattering process differs from reflection, refraction and diffraction in that there is no fixed angular relationship between the incident and scattered rays.

In an experiment conducted by the american physicist "A.H Compton" in 1923 a beam of monochromatic x-rays were allowed to fall on a block of graphite , as illustrated in fig(1). The intensity of scattered x-rays was measured as a function of wavelength of x-rays and at different scattering angles. At each angle , two peaks appear corresponding to scatter x-ray photons with two different wave lengths. The wavelength of one peak doesnot change as the angle is varied. This is called primary or unmodified component. We denote it by ' $\lambda_i$ '. The wavelength of otherpeak varies strongly with the angle and hence it is called modified component. It is denoted by ' $\lambda_f$ '. This effect is called the compton effect. The change in wavelength ( $\Delta\lambda$ ) is called the compton shift.

COMPTON'S Original results are illustrated in fig(1)



Fig(1): Schematic of the experimental arrangement for the study of compton effect.



The variation of intensity of x-rays as function of wavelength at different angles,  $\lambda_i$  corresponds to the unmodified (primary) component and  $\lambda_f$  to modified component.

NOTE:

The compton shift  $\Delta\lambda = \lambda_f - \lambda_i$  increases with increasing scattering angle ' $\theta$ '.  $\Delta\lambda = \frac{h}{m_ec} (1 - \cos\theta)$

WAVE - PARTICLE DUALITY: where  $\frac{h}{m_ec} = \lambda_c$  i.e. compton wavelength  
 $\lambda_c = 0.024,26 \text{ \AA}$

The photo electric effect and the compton effect established that light behaves as a flux of photons. On the otherhand, the phenomenon of interference, diffraction and polarization can be explained only when light treated as a continuous wave. It suggests that on the hand light resembles a collection of particles having energy 'E' and momentum 'P' and on the other hand it may be regarded as a continuous electromagnetic wave of frequency ' $\nu$ '. The "corpuscular nature" (particle) and wave nature appears to be mutually exclusive. However, it is observed that light behaves as a stream of particles in some phenomenon and as an advancing wave in some other phenomenon. It implied that light behaves both as a stream of particles and as a continuous wave. Therefore, we say that light exhibits wave-particle duality.

It is seen that according to quantum theory the energy and momentum of photon is given by " $E = h\nu$ " and " $P = \frac{h}{\lambda}$ ".

The left hand side parameters E and P in these equations are characteristics of particles. Where as, the right hand side parameters  $\nu$  and  $\lambda$  are characteristics of waves. These

equations reflect the wave-particle duality of light.

### de-BROGLIE HYPOTHESIS:

In 1924, Louis de-Broglie suggested that since, radiation has been shown to possess dual nature i.e wave and particle, matter (material particles) must also possess dual nature.

de-Broglie's suggestions were based on the following facts:

1. The entire universe consists of radiation (energy) and matter only.
2. Nature is symmetrical in so many respects. Therefore, matter and radiation must be mutually symmetrical. If radiation has dual nature, matter must be also hence dual nature i.e. both particle and wave natures. This is known as "de-Broglie hypothesis". The waves associated with the particles of matter are known as matter waves or de-Broglie waves.

Based on plank's theory of radiation, the energy of photon (quantum) is given by,

$$E = h\nu = \frac{hc}{\lambda}$$

where  $c$  = vacuum velocity of light.

$\lambda$  = wave length.

According to Einstein's Mass energy relation

$$E = mc^2$$

$$\therefore mc^2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{mc} = \frac{h}{P}$$

$$\therefore P = mc$$

Where  $P$  = momentum

de-Broglie suggested that this equation for wave length is a perfectly general one, applying to material particles as well as to photons.

In the case of particles, momentum  $(P) = mv$ , and hence the de-Broglie wave length of a particle is,

$$\lambda = \frac{h}{P} = \frac{h}{mv}$$

Where,  $m$  = mass

$v$  = velocity of particle.

If ' $E$ ' is the kinetic energy of material particle then,

$$E = \frac{P^2}{2m}$$

$$\therefore P^2 = 2mE$$

$$P = \sqrt{2mE}$$

Hence, de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

### de-BROGLIE WAVELENGTH ASSOCIATED WITH ELECTRONS:

Let us consider the case of an electron of rest mass ' $m_0$ ' and charge ' $e$ ' being accelerated by a potential ' $V$ ' volts. If ' $v$ ' is the velocity attained by the electron due to acceleration then,

$$\frac{1}{2} m_0 v^2 = ev$$

$$\therefore v^2 = \frac{2ev}{m_0}$$

$$v = \sqrt{\frac{2ev}{m_0}}$$

$\therefore$  de-Broglie wavelength  $\lambda = \frac{h}{m_0 v}$

$$\therefore \lambda = \frac{h}{m_0} \left[ \sqrt{\frac{m}{2eV}} \right]$$

$$\boxed{\lambda = \frac{h}{\sqrt{2m_0 eV}}}$$

Substituting  $h = 6.625 \times 10^{-34} \text{ Js}$ ,  
 $e = 1.602 \times 10^{-19} \text{ C}$  and  
 $m_0 = 9.1 \times 10^{-31} \text{ kg}$

We get,  $\lambda = \frac{12.26 \times 10^{-10}}{\sqrt{V}}$

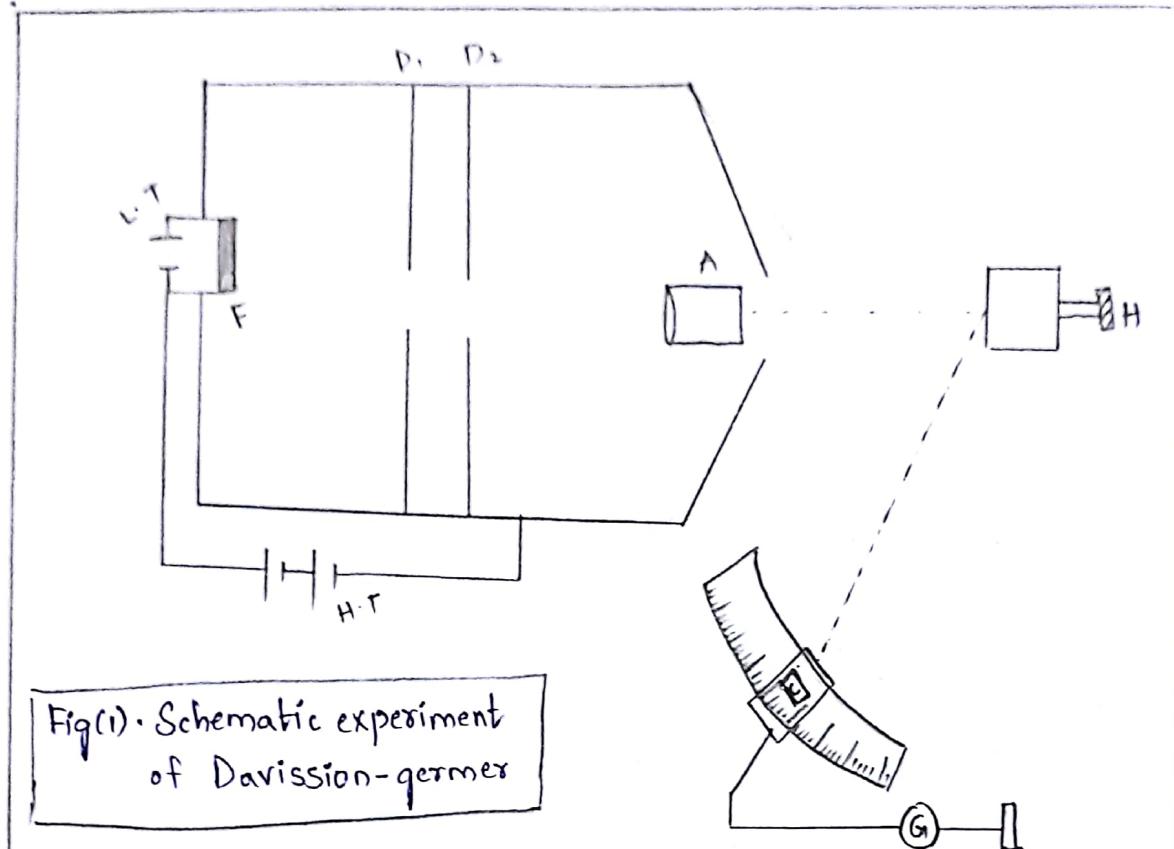
$$\therefore \boxed{\lambda = \frac{12.26}{\sqrt{V}} \text{ Å}}$$

If the voltage applied to accelerate the electron 'V' is 100 volts then,

$$\begin{aligned} \lambda &= \frac{12.26}{\sqrt{V}} \\ &= \frac{12.26}{\sqrt{100}} \\ &= \frac{12.26}{10} \\ &= 1.226 \text{ Å} \end{aligned}$$

### DAVISSON - GERMER EXPERIMENT:

In 1927, Davisson - Germer made an attempt to prove the wave nature of atomic particles. The schematic representation of the experimental setup used to demonstrate the dual nature of matter is shown in fig(1)



The electrons are produced by hot filament 'F' by applying the necessary potential through L.T. The electrons allow to pass through the two thin  $\alpha$  aluminium diaphragms  $d_1$  and  $d_2$  in order to get a fine parallel beam of electrons. By applying necessary potential 'V' to the aluminium cylinders 'A', the electrons are accelerated. The accelerated electrons are allowed to strike the single crystal of nickel 'n' which is fixed to hand 'H'. In order to get diffraction of different points on the surface of Nickel 'n', one can rotate the nickel crystal through the handle 'H' at any desired access of the incident electron beam.

The scattered electron will be collected with the help of detector known as the Faraday Cylinder 'C'.

The detector is moving on a circular scale 's'. The Faraday cylinder collects the scattered electrons in all directions by moving the same on the scale. The amount of current produced by the collected electrons by 'c' is measured through the galvanometer 'G'. The position of the cylinder on the scale measures the angle of scattering of the electrons. The experiment can be repeated for different accelerating voltages 'v'. The above observation can be plotted by taking the angles along the x-axis and the number of electrons scattered along the y-axis as shown in fig(a).

A peak in the number of scattered electrons appear at an angle of  $50^\circ$  for an applied voltage of  $54V$ . This means that the diffraction effect which is exhibited from the surface layer of crystal acting as a plane grating produces the first order spectrum at an angle of  $50^\circ$ .

The wave nature of the electron can be verified by determining the wave length of reflected waves by considering both particle and wave nature. The wave length of the electron by considering the particle nature can be determined from "Bragg's" diffraction formula. The arrangement of atoms in Nickel crystal are shown in fig 2(b). The inter atomic distance for nickel is  $2.15 \times 10^{-10} m$ . From the top surface of the crystal, the

crystal, the reflecting planes in the crystal are inclined at an angle of  $25^\circ$ . Therefore, the interplanar distance

$$d = a \sin \theta$$

$$d = 2.15 \times 10^{-10} \sin 25^\circ$$

$$d = 0.909 \times 10^{-10} \text{ m.}$$

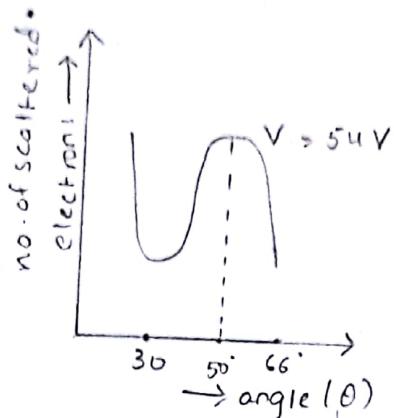


fig 2(a)

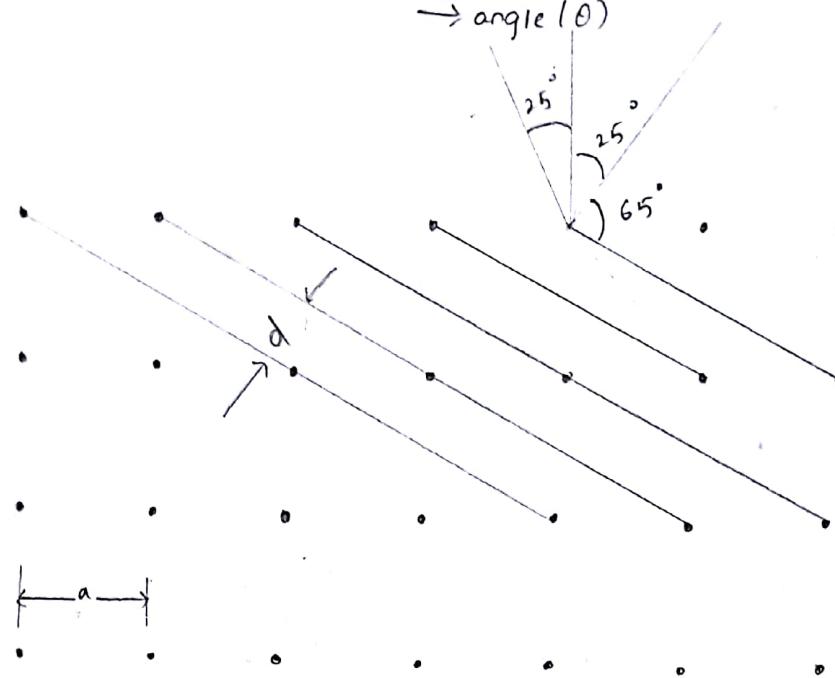


fig 2(b)

According to Bragg's Law,

$$2ds \sin \theta = n\lambda$$

$$\therefore 2 \times 0.909 \times 10^{-10} \sin(90^\circ - 25^\circ) = (1) \cdot \lambda$$

$$1.818 \times 10^{-10} \sin 65^\circ = \lambda$$

$$\therefore \lambda = 1.65 \times 10^{-10} \text{ m}$$

$$\lambda = 1.65 \text{ Å}$$

According to de-Broglie wavelength of an electron which is accelerated by a potential difference of  $54 \text{ V}$ , de-Broglie

$$\text{wavelength } \lambda = \frac{12.26}{\sqrt{v}} \text{ Å}$$

$$\lambda = \frac{12.26}{\sqrt{54}} \text{ Å}$$

$$\lambda = 1.67 \text{ Å}$$

As the two values are in good agreement this experiment confirms the de-Broglie concept of matter waves.

### HEISENBERG'S UNCERTAINTY PRINCIPLE:

The uncertainty principle was derived in 1927 by "Werner Heisenberg", the german physicist. It states that, "it is impossible to know both the exact position and exact momentum of an object at the same time".

If  $\Delta x, \Delta p$  are the uncertainties in the position and momentum measurements, then according to heisenberg's uncertainty principle the product of  $\Delta x$  and  $\Delta p$  always greater than plank's constant.

$$\Delta x \cdot \Delta p \approx h$$

$$\Delta x \cdot \Delta p \geq \frac{h}{2}$$

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi} \quad (\because h = \frac{h}{2\pi})$$

Similary, let  $\Delta E$  and  $\Delta t$  to respectively the uncertainty in the measurement of energy and time, then according to heisenberg's uncertainty principle.

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi} \text{ (or) } \frac{h}{2}$$

## BORN'S INTERPRETATION OF THE WAVE FUNCTION:

Waves represent the propagation of a disturbance in a medium. Light waves, sound waves and water waves are examples of waves. They are all characterised by some quantity that varies with position and time. Light waves are represented by electromagnetic field variations and sound waves are represented by pressure variations. In the case of de-Broglie waves associated with electrons we cannot specify in a similar manner. Electrons sometimes behave like particles, sometimes like waves depending on the experimental arrangement. Since, electrons have wave properties, it may be assumed that a quantity ' $\psi$ ' (psi read as si) represents a de-Broglie wave just as the electric vector " $E$ " represents a light wave. The quantity " $\psi$ " is called the "wave function". The wave function ' $\psi$ ' mathematically describes the motion of an electron. Further, ' $\psi$ ' is usually a complex quantity. It has no direct physical meaning (significance) as it is not an observable quantity.

"Max Born", a german physicist showed in 1926 that the square of the absolute value of the wave function  $|\psi|^2$ , is proportional to probability of a particle

being in unit volume of space centered at a point, where ' $\psi$ ' is evaluated at time 't'. Thus the probability of finding a particle between  $x$  and  $x+dx$ ,  $y$  and  $y+dy$ ,  $z$  and  $z+dz$  is  $p \propto |\psi|^2 dx dy dz$  (or)  $p \propto \psi \psi^* dx dy dz$ , where  $\psi^*$  is complex conjugate of  $\psi$ . The complex conjugate of a function is obtained by reversing the sign of the imaginary is that is in the function, the product  $\psi \psi^*$  is real and is also written as  $|\psi|^2$ , where the 'II' mean absolute value.

It follows that if  $dv = dx dy dz$  is an infinitesimally small value element so surrounding the point  $(x, y, z)$ , then the probability of finding the particle in that value element is given by  $|\Psi(x, y, z)|^2 dv$  (or)  $P \propto |\Psi|^2 dv$ .

### NORMALIZATION CONDITION:

$\int_{-\infty}^{\infty} |\Psi|^2 dx dy dz = 1$  (or)  $\int_{-\infty}^{\infty} \Psi \Psi^* dx dy dz = 1$ , this condition is known as normalization condition for 1-D;  $\int_{-\infty}^{\infty} |\Psi(x)|^2 dx = 1$

### SCHRODINGER WAVE EQUATION:

Schrodinger developed a mathematical equation in two forms to represent the dual nature of wave equations, namely time dependent wave equation and time-independent wave equation.

The Schrodinger time independent-equation is .

applicable to the wave function ' $\psi$ ' which depends only on position coordinates  $(x, y, z)$  and not with the time coordinate 't'.

### TIME - INDEPENDENT WAVE EQUATION:

Consider a system of stationary waves associated with a moving particle. Let us suppose  $\psi$  as a wave function of the particle along  $x, y, z$  coordinate axes at any time 't'.

Based on the cartesian coordinates the differential wave equation of a progressive wave with wave velocity 'u'. As

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} = \frac{1}{u^2} * \frac{d^2\psi}{dt^2} \quad \dots \dots (1)$$

where,  $u = \lambda\nu$

According to de-Broglie's second assumption, the one dimensional wave function associated with a particle is,

$$\psi = \psi_0 e^{-i\omega t} \quad \dots \dots (2)$$

where  $\psi_0$  is the amplitude of wave function at the point

$x, y, z$  and  $\omega = 2\pi\nu$

where  $\nu$  is the frequency of radiation.

The value of  $\frac{d^2\psi}{dt^2}$  can be obtained by differentiating eq(2)

w.r.t 't' twice.

$$\frac{d\psi}{dt} = (-i\omega) \psi_0 e^{-i\omega t}$$

$$\frac{d\psi}{dt} = -i\omega\psi$$

$$\frac{d^2\psi}{dt^2} = (-i\omega)(-i\omega)\psi_0 e^{-i\omega t}$$

$$\frac{d^2\psi}{dt^2} = i^2\omega^2\psi_0 e^{-i\omega t}$$

$$\frac{d^2\psi}{dt^2} = -\omega^2\psi \quad \dots \quad (3) \quad \begin{aligned} \because (i^2 = -1) \\ \text{from eq(2)} \end{aligned}$$

Substituting eq(3) in eq(1), we get

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} = \frac{1}{u^2} \times -\omega^2\psi$$

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} = -\frac{(2\pi\nu)^2\psi}{(\nu\lambda)^2}$$

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} = -\frac{4\pi^2\nu^2\psi}{\nu^2\lambda^2}$$

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} = -\frac{4\pi^2\psi}{\lambda^2} \quad \dots \quad (4)$$

According to de-Broglie wavelength,  $\lambda = \frac{h}{P}$

$$\text{total energy} = P \cdot E + K \cdot E$$

$$E = P \cdot E + K \cdot E$$

$$E + V + \frac{P^2}{2m} \quad \left[ KE = \frac{P^2}{2m} \right]$$

$$\therefore E - V = \frac{P^2}{2m}$$

$$P^2 = 2m(E - V)$$

(14)

$$P = \sqrt{2m(E-V)}$$

$$\text{Now } \lambda = \frac{h}{\sqrt{2m(E-V)}}$$

From eq(4)

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} = -\frac{4\pi^2\psi}{h^2}$$

$$\frac{2m(E-V)}{h^2}$$

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} = -\frac{8\pi^2m(E-V)\psi}{h^2} \quad \dots \quad (5)$$

We know that  $\nabla^2$  is the laplacian operator and is given by

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad \dots \quad (6)$$

the laplacian operator  $\rightarrow$ 

The Laplacian operator operates on the wave function and represents the corresponding wave function and its energy.

eq(5) can be written as,

$$\nabla^2\psi = -\frac{8\pi^2m(E-V)\psi}{h^2}$$

$$\therefore \nabla^2\psi = -\frac{2m(E-V)\psi}{h^2}$$

$$\left[ \begin{array}{l} h' = \frac{h}{2\pi} \\ h'' = \frac{h^2}{4\pi^2} \\ \frac{1}{h^2} = \frac{4\pi^2}{h^2} \end{array} \right]$$

$$\boxed{\nabla^2\psi + \frac{2m(E-V)\psi}{h^2} = 0}$$

(27)

The above equation is known as schrodinger's time independent equation.

⇒ For 1-D

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m(E-V)\psi(x)}{\hbar^2} = 0$$

- 1.Q. Calculate the de-Broglie wavelength of an electron which has been accelerated from rest on application of potential of 400v.

Sol Given:

$$V = 400 \text{ volts}$$

de-Broglie wavelength ( $\lambda$ )=?

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ A}^\circ$$

$$\lambda = \frac{12.26}{\sqrt{400}} \text{ A}^\circ$$

$$= \frac{12.26}{20} \Rightarrow 0.613 \text{ A}^\circ$$

- 2.Q. An electron is moving under a potential field of 15kV. Calculate the wavelength of the electrons.

Sol Given:

$$\text{Potential field} = 15 \text{ kV}$$

de-Broglie wavelength ( $\lambda$ )=?

$$\lambda = \frac{12.26}{\sqrt{V}} \Rightarrow \frac{12.26}{\sqrt{15000}}$$

$$\lambda = \frac{12.26}{\sqrt{150} \times \sqrt{100}} = \frac{12.26}{\sqrt{150}}$$

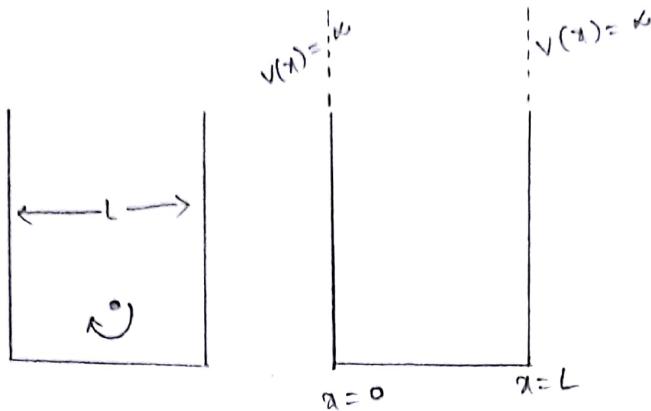
$$= 0.10 \text{ A}^\circ$$

### PARTICLE IN A POTENTIAL Box:

A free electron (particle) in a metallic crystal may move freely inside the crystal from one place to another place. but will not come out of the crystal the electron experiences very large potential called potential barrier. The potential barrier present at the surface will act as a potential barrier present at the surface will act as a potential box for free electrons. This potential box can be <sup>also</sup> called as potential well. Because the electron will remain in that region only.

### PARTICLE IN 1-D POTENTIAL Box:

consider, a one dimensional potential box with  $x=0$  to  $x=L$  and infinite height as shown in fig(i). Let  $E$  be the total energy of the particle with mass ' $m$ ' that is moving freely in the  $x$ -direction inside the potential box. The Particle is moving inside the box with limitation  $x=0$  to  $x=L$  and hence its motion is restricted by the walls. The Particle will be inside the box and hence, its potential energy is zero. Hence, one can use the schrodinger's time independent wave equation to obtain the wave function and energy of the particle inside the box.



fig(1)

Inside the potential box, the PE = 0 i.e., for  $0 < x < L$ ,  $V(x) = 0$

The PE outside the box is ' $\infty$ ', i.e. for  $L \leq x \leq 0$ ,  $V(x) = \infty$

According to schrodinger's time independent equation

$$\text{for 1-D, } \frac{d^2\psi(x)}{dx^2} + \frac{2m(E-V)\psi(x)}{\hbar^2} = 0$$

$$\therefore \frac{d^2\psi(x)}{dx^2} + \frac{2mE\psi(x)}{\hbar^2} = 0$$

Since, for a free particle inside a potential well  $V=0$ .

$$\text{Therefore, } \frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0 \quad (1)$$

$$\text{where } k^2 = \frac{2mE}{\hbar^2}$$

A possible solution for eq(1) is,

$$\psi(x) = A \sin kx + B \cos kx$$

where A and B are constants. Since the particle cannot penetrate an infinitely high potential barrier, for  $\psi(x) = 0$  at  $x=0$ , B must be zero.

for  $\psi(x) = 0$  at  $x=L$ ,  $kL$  must be an integral multiple of  $\pi$

$$\therefore kL = n\pi$$

$$\therefore k = \frac{n\pi}{L}$$

$$\therefore \psi(x) = A \sin kx \quad \text{--- (2)}$$

From  $k^2 = \frac{2mE}{\hbar^2}$

$$\left(\frac{n\pi}{L}\right)^2 = \frac{2mE}{\hbar^2}$$

$$\frac{n^2\pi^2}{L^2} = \frac{2mE}{\hbar^2}$$

$$\therefore E = \frac{n^2\pi^2}{L^2} \times \frac{\hbar^2}{2m}$$

$$E = \frac{n^2\pi^2}{L^2} \times \frac{1}{2m} \times \frac{\hbar^2}{4\pi^2}$$

$$\therefore E_n = \boxed{\frac{n^2\hbar^2}{8mL^2}} \quad \text{--- (3)}$$

The value of A in eq(2) can be obtained by applying normalization condition . i.e  $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$

$$\int_{-\infty}^{\infty} A^2 \sin^2 kx dx = 1$$

$$\int_0^L A^2 \sin^2 \left(\frac{n\pi x}{L}\right) dx = 1$$

$$A^2 \int_0^L \sin^2 \left(\frac{n\pi x}{L}\right) dx = 1$$

$$\therefore \int_0^L \sin^2 \left(\frac{n\pi x}{L}\right) dx = \frac{1}{A^2}$$

$$\int_0^L \frac{1 - \cos \left(\frac{2n\pi x}{L}\right)}{2} dx = \frac{1}{A^2}$$

$$\boxed{\sin^2 \theta = \frac{1 - \cos 2\theta}{2}}$$

Integrating and substituting the limits in the above equation , we get

$$\therefore \frac{1}{\Lambda^2} = \frac{1}{2}$$

$$\Lambda^2 = \frac{2}{L}$$

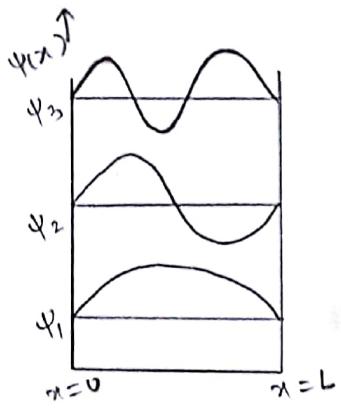
$$\Lambda = \sqrt{\frac{2}{L}}$$

Therefore the normalization wave function is

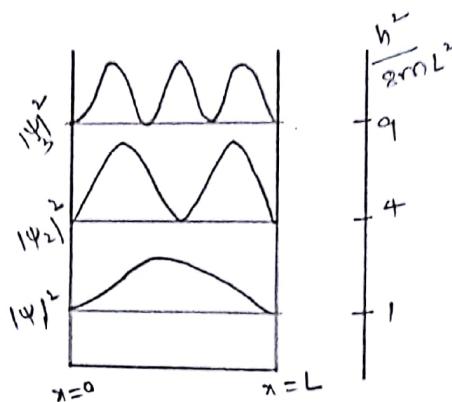
$$\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x \quad (4)$$

The wave function  $\psi_n$  and corresponding energies  $E_n$  which are often called eigen function and eigen values respectively.

The lowest eigen value is called the ground state.



Fig(2a)



Fig(2b)

Fig 2: ground state and first two excited states of an electron in a potential well.

Fig 2(a) : represents the electron wave function and

Fig 2(b) : represents the corresponding density functions.

The energies of the three states are shown on the right.

Q. Calculate the wavelength associated with an electron with energy 2000 eV.

Sol Given:

$$E = 2000 \text{ eV}$$

$$= 2000 \times 1.6 \times 10^{-19} \text{ J}$$

$$h = 6.626 \times 10^{-34} \text{ JS}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

de-Broglie wave length

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2000 \times 1.6 \times 10^{-19}}}$$

$$\lambda = 0.0275 \times 10^{-9} \text{ m}$$

$$\lambda = 0.0275 \text{ nm}$$

4. Q. An electron is bound in one dimensional potential box of size  $4 \times 10^{-10} \text{ m}$ . What will be its minimum energy.

Sol Given:

$$L = 4 \times 10^{-10} \text{ m}$$

Energy of electron in 1-D potential box,  $E_n = \frac{n^2 h^2}{8mL^2}$

for min energy  $n=1$  then,  $E_1 = \frac{h^2}{8mL^2}$

$$h = 6.626 \times 10^{-34} \text{ JS}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$E = \frac{(6.626 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (4 \times 10^{-10})^2} \Rightarrow 0.346 \times 10^{-18} \text{ J}$$

5.Q. An electron is bound in 1-D infinite well of width  $1 \times 10^{-10}$  m.  
 Find the energy values in the ground state and first two excited states.

Sol

$$\text{Given; } L = 1 \times 10^{-10} \text{ m}$$

$$\hbar = 6.626 \times 10^{-34} \text{ Js}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$E_n = \frac{n^2 h^2}{8 m L^2}$$

For ground state  $n=1$ , for 1st excited state  $n=2$   
 and for second excited state  $n=3$

∴ Energy in ground state

$$E_1 = 0.6031 \times 10^{-17} \text{ J}$$

∴ Energy in first excited state

$$E_2 = \frac{4h^2}{8 m L^2}$$

$$E_2 = 4E_1 = 2.412 \times 10^{-17} \text{ J}$$

∴ Energy in second excited state

$$E_3 = \frac{9h^2}{8 m L^2} = 9E_1$$

$$E_3 = 5.428 \times 10^{-17} \text{ J.}$$

## Laws Of Black Body Radiation.

The following are various laws of black body radiation

### i) STEFAN - BOLTZMANN LAW.

Stefan-Boltzmann law is an empirical relationship obtained by Stefan and later derived theoretically by Boltzmann. It connects the intensity of radiation to the temperature.

#### STATEMENT.

The total radiation emitted from a blackbody at temperature is proportional to the fourth power of the absolute temperature of the body ( $T^4$ ).

$$E = \sigma T^4$$

where  $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$  and it is called Stefan's constant.

### ii) WIEN'S LAW:

The peak wavelength  $\lambda_{max}$  at which the maximum emission occurs for any given temperature is inversely proportional to the absolute temperature of the body.

$$\text{Thus, } \lambda_{max} \propto \frac{1}{T}$$

$$\therefore \lambda_{max} T = \text{constant}$$

$$\lambda_{max} \cdot T = 2.8978 \times 10^{-3} \text{ mK.}$$

iii) RAYLEIGH - JEANS LAW:

According to Boltzmann's principle of equipartition of energy, each simple harmonic oscillator has an average thermal energy of  $kT$  at thermal equilibrium. Rayleigh assumed that, each of the standing waves should have energy  $kT$ . He derived an expression for the energy density of radiation distribution within the cavity.

$$E(\nu) = \frac{8\pi \nu^2 kT}{c^3}$$

Therefore, this equation is known as Rayleigh-Jeans law.

DERIVATION OF PLANCK'S LAW:

The number of oscillations in an energy state  $E_n = nh\nu$  is determined by the Maxwell-Boltzmann distribution law.

$$N_n = N_0 e^{-E_n/kT} \\ = N_0 e^{-nh\nu/kT}$$

where  $N_0$  is the no. of oscillators in the ground state ( $E_0=0$ ). Let there be 'N' oscillators in the system in equilibrium at absolute temperature  $T$ . According to Max-well-boltzmann distribution law, the no. of oscillators  $N_n$  with energy  $E_n$  is given by

$$N_n = N_0 e^{-E_n/kT} \\ = N_0 e^{-nh\nu/kT}$$

Since the energies of oscillators assume discrete values we have to sum over all possible oscillator states to determine  $E$ .

∴ The average energy 'E' of an oscillator is given by

$$E = \frac{\sum_{n=0}^{\infty} E_n N_n}{\sum_{n=0}^{\infty} N_n}$$

$$E = \frac{\sum_{n=0}^{\infty} E_n e^{-En/kT}}{\sum_{n=0}^{\infty} e^{-En/kT}}$$

$$E = \frac{\sum_{n=0}^{\infty} nh\nu e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-nh\nu/kT}}$$

$$E = \frac{h\nu e^{-h\nu/kT} + 2h\nu e^{-2h\nu/kT} + \dots}{1 + e^{-h\nu/kT} + e^{-2h\nu/kT} + \dots}$$

Substituting

$$e^{-h\nu/kT} = x, \text{ we get}$$

$$E = \frac{h\nu x + 2h\nu x^2 + 3h\nu x^3 + \dots}{1 + x + x^2 + \dots}$$

$$E = \frac{h\nu x (1 + 2x + 3x^2 + \dots)}{1 + x + x^2 + \dots}$$

$$\therefore E = \frac{h\nu x (1-x)^{-2}}{(1-x)^{-1}}$$

$$\therefore E = \frac{h\nu x}{(1-x)^1 (1-x)^2}$$

$$E = \frac{h\nu x}{(1-x)}$$

$$E = \frac{h\nu e^{-h\nu/kT}}{1 - e^{-h\nu/kT}}$$

$$E = \frac{h\nu}{\frac{1 - e^{h\nu/kT}}{e^{-h\nu/kT}}}$$

$$E = \frac{h\nu}{\frac{1}{e^{h\nu/kT} - 1}}$$

$$E = \frac{h\nu}{e^{h\nu/kT} - 1}$$

The no. of oscillator in the frequency  $\nu$  and  $\nu + d\nu$  are estimated to be  $\frac{8\pi\nu^2 d\nu}{c^3}$

Multiplying this number by the average energy of an oscillator, Planck obtained the energy density in blackbody radiation as, then he got energy density of electron.

$$E(\nu) = \frac{8\pi\nu^2 d\nu}{c^3} \left[ \frac{h\nu}{e^{h\nu/kT} - 1} \right]$$

$$\therefore E(\nu) = \frac{8\pi h\nu^3}{c^3} \left[ \frac{1}{e^{h\nu/kT}} \right] d\nu$$

(or)

$$E(\lambda) = \frac{8\pi hc}{\lambda^5} \left[ \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \right] d\lambda$$

This equation is Planck's radiation law.

## UNIT-II

Dr. MN Avadhanulu  
S. ChannabasappaSEMICONDUCTOR PHYSICSINTRODUCTION:

Semiconductors are materials having electrical conductivity considerably greater than that of an insulator but significantly lower than that of a conductor. Of all the elements in the periodic table eleven elements are semiconductors. Germanium and silicon are elemental semiconductors are widely used in semiconductor devices. Ga, As, In, P etc are compound semiconductors which are formed from the combinations of the elements of groups III and V or II and VI are widely used in fabrication of optoelectronic devices such as lasers, LED's etc.

In semiconductors they are bipolar and two charge carriers, namely electrons and holes transport current in these materials. The electrical conductivity of a pure semiconductor is significantly low and not be used in device fabrications. Through the technique of doping, the conductivity of a semiconductor can be increased in magnitude to a desired value and can be made independent of temperature in a certain temperature interval. Doped semiconductors are known as extrinsic semiconductors. The remarkable feature of extrinsic semiconductors is that current is transported in them by two different charge carriers



electrons and holes ; and through two different processes , drift and diffusion. Extrinsic semiconductors are widely used in fabrication of solid state devices.

### INTRINSIC SEMI CONDUCTORS:

chemically pure semiconductors are known as intrinsic semiconductors. A semiconductor is considered to be pure when there is less than one impurity atom in a billion host atoms. A two dimensional representation of silicon crystal is shown in fig(1)a . Each "Silicon" atom forms covalent bonds with four surrounding atoms . There is no free electron . In terms of energy band diagram, a conduction band and a valency band separated by a smaller energy gap characterised a semiconductor (Fig 1(b)).

In a real crystal,

$$\text{The concentration of atoms } N = \frac{N_A P}{M}$$

where ,  $N_A$  is the Avagadro number ,  $P$  is the density and  $M$  is the atomic weight of the material.

$$\therefore N = \frac{(6.02 \times 10^{26} \text{ atm/kmol})(2330 \text{ kg/m}^3)}{28.09 \text{ kg/kmol}}$$

$$\therefore N = 5 \times 10^{28} \text{ atoms/m}^3$$

T = 0K

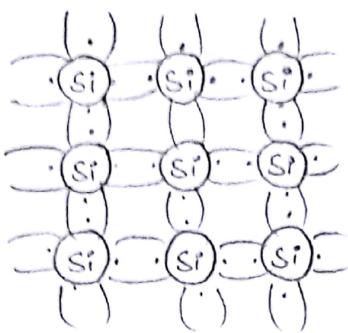
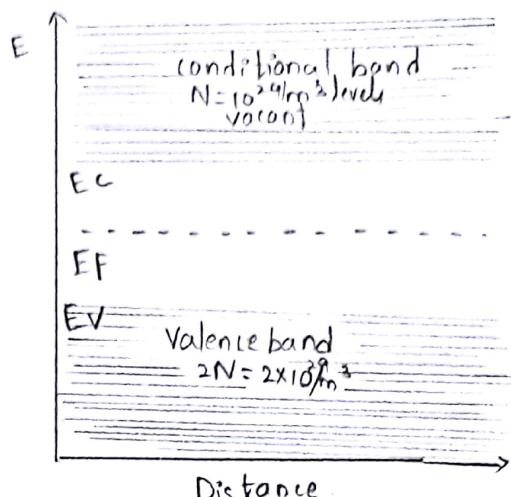


Fig (1) Silicon crystal  
at 0K.



(b)

The valency band and conduction band of silicon crystal contains  $2N$  energy levels each. The number of levels in each band is  $10^{29}$  levels  $\text{m}^{-3}$ . In other words there are  $2 \times 10^{29}$  states/ $\text{m}^{-3}$ . The number of valency electrons in silicon crystal is  $2 \times 10^{29}$  electrons/ $\text{m}^{-3}$ . These electrons completely occupy the valency band and leave the conduction band vacant.

AT 0K AN INSULATOR SEMI CONDUCTOR BEHAVES AS A  
PERFECT INSULATOR:

At 0 kelvin and temperatures closed to zero kelvin, all the valency electrons are in covalent bonds, the bonds are complete. The total no. of valency electrons completely occupies the  $10^{29}$  energy levels  $\text{m}^{-3}$  available in the valency band. At 0K electrons in the valency band does not possess enough energy to jump into the conduction band [fig 1(b)]. As free electrons donot exist in the conduction band, an extremely applied

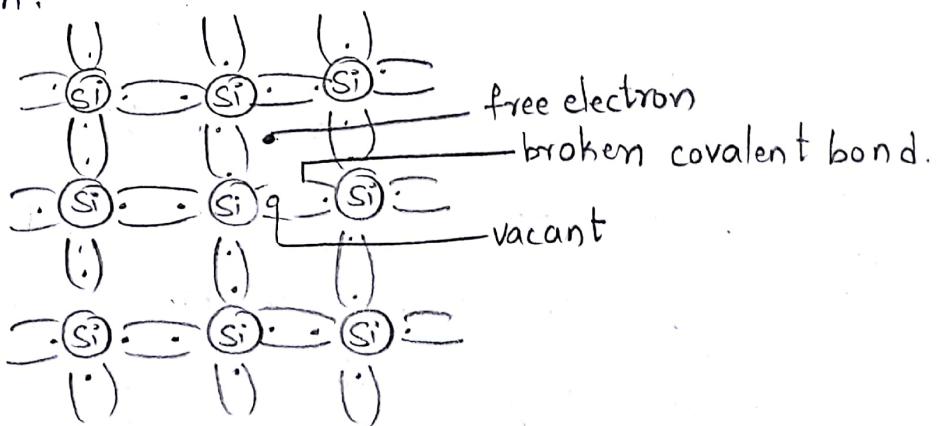
(3)

electric field cannot cause flow of current through the crystal. Hence, the intrinsic semiconductor behaves as a perfect insulator at 0K.

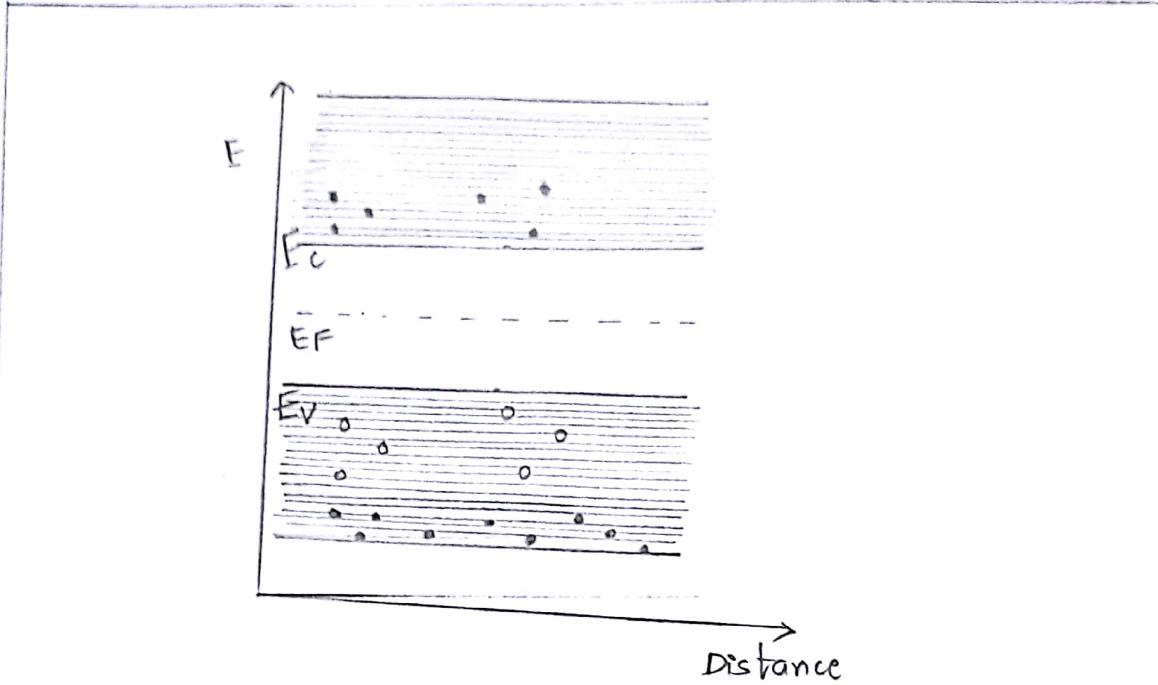
### AT TEMPERATURES ABOVE ABSOLUTE ZERO

Some of the electron acquires sufficient energy and break away from covalent bonds fig(2a) and becomes free. If the electric field is applied, these free electrons cause electrical conduction. There are  $1.5 \times 10^{16}$  electrons/m<sup>3</sup> in the conduction band at 300K and the conduction band can accommodate  $2 \times 10^{29}$  electrons/m<sup>3</sup> and hence it is partially filled. In silicon crystal at 300K,  $1.5 \times 10^6$  vacant states/m<sup>3</sup> appear in the valency band, which are very small in number compared to the number of electrons remain in the valency band. Thus, now both the bands are partially filled.

As, thermally generated electron-hole pairs causes electrical conduction.



2(a) fig.



### DEF:

An intrinsic semiconductor is a semiconductor crystal in which electrical conduction arises due to the thermally excited (above 0 K) electrons and holes.

### SIGNIFICANCE OF BAND GAP:

The band gap energy ' $E_g$ ' is the minimum amount of energy required to excite an electron from valency band to conduction band.  $E_g$  is characteristic of the material. The energy required to break a covalent bond in a germanium crystal is above 0.72 eV at 300 K and that in silicon is 1.12 eV.

### GENERATION AND RECOMBINATION:

Covalent bond breaking leads to the generation of two charge carriers, an electron in the conduction band and a hole in the valency band, electron-hole pair generation

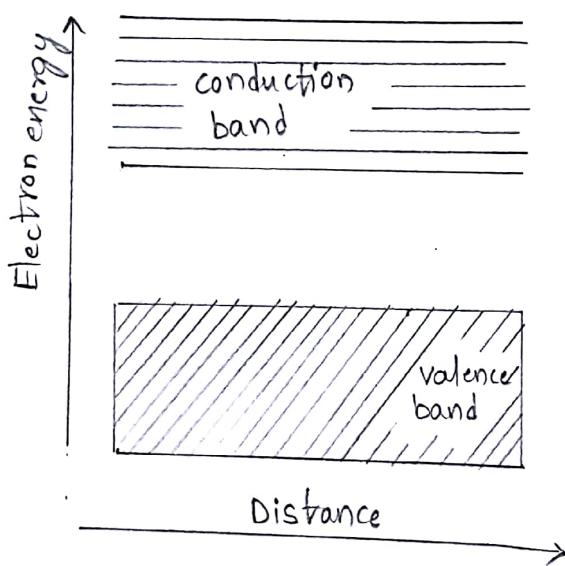
fig (1) . Electron-hole pairs generated if the frequency  $\nu'$

(S)

of the radiation is more than energy band gap ( $\hbar\nu > E_g$ )

### GENERATION:

In the process of generation a covalent bond is broken and a bound electron is transformed into a free electron. Thermal energy and optical illumination causes pair generation. At any temperature 'T' the number of electrons generated would be equal to the number of holes produced. After generation the charge carriers move independently. The electrons move in the conduction band and the holes move in the valency band. Their motion is random in the respective bands as long as an external electric field is not applied.



Fig(1)  
generation and  
recombination of  
electron-hole pair.

### RECOMBINATION:

It is likely that the electron in the conduction band may loss its energy due to collision with other particles in the lattice and fall into the valency band [fig(1)]. When a free electron falls into a valence band, it merges with a hole.

## CARRIER TRANSPORT: INTRINSIC CONDUCTIVITY ( $\sigma_i$ )

At any temperature 'T', the no. of electrons generated will be equal to the number of holes per unit volume. As the two carrier concentrations are equal, they are denoted by a common symbol 'n<sub>i</sub>', which is called intrinsic density ( $\sigma_i$ ) intrinsic concentration. Therefore,

$$n = p = n_i$$

At a temperature 'T', charge carriers possess ~~an~~ an average kinetic energy ( $\frac{1}{2}mv_{th}^2 = \frac{3}{2}kT$ ) and mean thermal velocity  $v_{th}$ .

When a potential difference is applied across the solid, net movement of electrons and holes is the drift and the corresponding mean velocity is known as drift velocity ' $v_d$ '. The drift motion is directional and causes drift current flow, which is more often called conduction current.

$$\therefore \text{Drift velocity } (v_d) = \mu E$$

where  $\mu$  = mobility of charge carriers.

As we denote the drift velocity of electron of with  $v_{de}$  and that the hole with " $v_{dh}$ " and mobility of electron and hole with  $\mu_e$  and  $\mu_h$  respectively. Therefore,  $v_{de} = \mu_e E$  and  $v_{dh} = \mu_h E$

The current density due to electrons is given by

$$J_e = n_e v_{de} = n_e \mu_e E$$

The current density due to holes,

$$J_h = PeV_{dh} = PeN_h e$$

From ohm's Law,

$$J = \sigma E$$

where,  $\sigma$  = conductivity (intrinsic)

electronic and holes conductivities are,

$$\sigma_e = n_e N_e \text{ and}$$

$$\sigma_h = P_e N_h$$

Therefore, current density

$$J = J_e + J_h$$

$$J = (n_e N_e + P_e N_h) E$$

The intrinsic conductivity,

$$\sigma = \sigma_e + \sigma_h$$

$$\sigma = n_e N_e + P_e N_h$$

As  $n = P = n_i$

In an intrinsic semiconductor the above equation may be

written as  $\sigma = n_i e N_e + n_i e N_h$

$$\therefore \sigma = e n_i (N_e + N_h)$$

### CARRIER - CONCENTRATION:

We now intend to calculate the electron concentration ' $n$ ' in the conduction band and the hole concentration ' $p$ ' in the valency band.

#### DEF:

The number of electrons in the conduction band per unit volume ( $n$ ) and the number of holes in the valency band per unit volume ( $p$ ) of the material is known as carrier concentration or density of charge carriers.

### 1. CALCULATION OF ELECTRON - DENSITY:

Let ' $d_n$ ' be the number of electrons available between energy interval  $E$  and  $E+dE$  in the conduction band.

$$\text{Therefore, } dn = z(E) f(E) dE \quad (1)$$

where,  $z(E)$  is the density of states in the energy interval ' $E$ ' and ' $E+dE$ '.

$f(E)$  is the probability that the state of energy is occupied by an electron (electron occupancy probability)

The electron density in the conduction band is given by integrating eq(1) between the limits  $E_c$  to  $\infty$  where  $E_c$  is the energy corresponding to the bottom edge of the conduction band and  $\infty$  is the energy corresponding to top edge of the conduction band.

$$\therefore n = \int_{E_c}^{\infty} dn = \int_{E_c}^{\infty} Z(E) f(E) dE \quad (2)$$

The density of states for  $E > E_c$  in the conduction band is,

$$Z(E) dE = \frac{4\pi}{h^3} (2m_e^*)^{3/2} E^{1/2} dE \quad (3)$$

where,  $m_e^*$  is the effective mass of the electron. The kinetic energy of the conduction electron.

$$E = E - E_c,$$

where,  $E_c$  corresponds to the potential energy of an electron at rest.

Hence, eq(3) becomes

$$Z(E) dE = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E - E_c)^{1/2} dE \quad (4)$$

The probability of an electron occupying an energy level is given by

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

when, the number of particles is very small compared to the available energy levels,

$$\{(E - E_F) \gg 3kT\}$$

the fermidirac function be approximated to boltzmann function.

$$f(E) = \frac{1}{e^{(E - E_F)/kT}} = e^{-(E - E_F)/kT} \quad (5)$$

Using (4) and (5) into eq(2) we obtain,

$$n = \int_{E_c}^{\infty} \frac{4\pi}{h^3} (2me^*)^{3/2} (E - E_c)^{1/2} e^{-(E-E_F)/kT} dE$$

$$\therefore n = \frac{4\pi}{h^3} (2me^*)^{3/2} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{-(E-E_F)/kT} dE$$

$$\therefore n = \frac{4\pi}{h^3} (2me^*)^{3/2} e^{(E_F - E_c)} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{-(E-E_c)/kT} dE \quad (6)$$

The integral in eq(6) is of the standard form which has a solution of the following form.

$$\int_0^{\infty} x^{1/2} e^{-ax} dx = \frac{\sqrt{\pi}}{2a\sqrt{a}}$$

$$\text{where } a = \frac{1}{kT} \text{ and } x = E - E_c$$

Therefore, from (6)

$$n = \frac{4}{h^3} (2me^*)^{3/2} e^{(E_F - E_c)/kT} \left[ \frac{\sqrt{\pi}}{2} \times (kT)^{3/2} \right]$$

rearranging the terms, we get

$$n = 2 \left[ \frac{2\pi m e^* k T}{h^2} \right]^{3/2} e^{-(E_c - E_F)/kT} \quad (7)$$

eq(7) is the expression for electron concentration in the conduction band of an intrinsic semi-conductor

$$\therefore N_c = 2 \left[ \frac{2\pi m e^* k T}{h^2} \right]^{3/2}$$

$$n = N_c e^{-(E_c - E_F)/kT} \quad \text{--- (8)}$$

$N_c$  is a temperature dependent material constant known as the effective density of states in the conduction band.

In silicon, at 300K,  $N_c = 2.8 \times 10^{25}/\text{m}^3$

Eq(8) relates the equilibrium electron concentration to the Fermilevel ' $E_F$ '.

## 2. CALCULATION OF HOLE DENSITY:

Let  $d_p$  be the number of holes (vacancies) whose energy lies in the energy interval  $E$  and  $E+dE$  in the valency band,

$$d_p = z(E) [1-f(E)] dE \quad \text{--- (1)}$$

where

$z(E)$  = density of states in the energy interval  $E$  and  $E+dE$  and  $(1-f(E))$  is the probability that a state of energy is vacant and not occupied by an electron.

If  $f(E)$  is the probability for occupancy of an energy state ' $E$ ' by an electron, then the probability that the energy state is vacant is given by  $(1-f(E))$ . Since, a hole represents a vacant in valency band, the probability for occupancy of a state at ' $E$ ' by a hole is equal to the probability of absence of electron at that state. We can write

$$[1-f(E)] = \left[ 1 - \frac{1}{1+e^{(E-E_F)/kT}} \right]$$

(12)

$$= 1 - \left[ 1 + e^{-(E_F - E)/kT} \right]^{-1}$$

Since  $E - E_F \gg kT$ ,

$$\left[ 1 + e^{-(E_F - E)/kT} \right] \approx e^{-(E_F - E)/kT} \quad (2)$$

The density of states in the valency band is given by,

$$z(E) dE = \frac{4\pi}{h^3} (2m_h^*)^{3/2} E^{1/2} dE \quad (3)$$

The top edge of the valency band ' $E_V$ ' corresponds to the potential energy of a hole at rest.

Therefore,  $(E_V - E)$  will be the kinetic energy of hole at the lower energy levels, eq(3) is to be modified as follows;

$$z(E) dE = \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_V - E)^{1/2} dE \quad (4)$$

Using (2)(4) in eq(1) we get

$$dp = \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_V - E)^{1/2} e^{-(E_F - E)/kT} dE \quad (5)$$

In order to calculate, the number of holes in the valency band eq(5) is to be integrated between the limits  $-\infty$  to  $E_V$ . The hole density in the valency band therefore is given by

$$P = \int_{-\infty}^{E_V} \left( \frac{4\pi}{h^3} \right) (2m_h^*)^{3/2} (E_V - E)^{1/2} e^{-(E_F - E)/kT} dE$$

$$P = \frac{4\pi}{h^3} (2m_h^*)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{-(E_F - E)/kT} dE$$

$$P = \frac{4\pi}{h^3} (2m_h^*)^{3/2} e^{-(E_F - E_V)/KT} \int_{-\infty}^{E_V} (E_V - E) e^{-(E_V - E)/KT} dE \quad \text{--- (6)}$$

The integral in eq(6) is of the standard form which has a solution of the following form.

$$\int_0^x x^{1/2} e^{-ax} = \frac{\sqrt{\pi}}{2a\sqrt{a}} \text{ where ,}$$

$$a = \frac{1}{KT} \quad \text{and} \quad x = E_V - E$$

from eq(6);

$$P = \frac{4\pi}{h^3} (2m_h^*)^{3/2} e^{-(E_F - E_V)/KT} \left[ \frac{\sqrt{\pi}}{2} \times (KT)^{3/2} \right]$$

rearrange , the terms we get ,

$$P = \frac{4\pi}{h^3} (2m_h^*)^{3/2} e^{-(E_F - E_V)/KT} \left( \frac{\sqrt{\pi}}{2} \times (KT)^{3/2} \right)$$

$$P = 2 \left[ \frac{2\pi m_h^* KT}{h^2} \right]^{3/2} e^{-(E_F - E_V)/KT} \quad \text{--- (7)}$$

eq(7) is the expression for the hole concentration in the valence band of an intrinsic semi-conductor denoting ,

$$N_V = 2 \left[ \frac{2\pi m_h^* KT}{h^2} \right]^{3/2}$$

$$\therefore P = N_V e^{-(E_F - E_V)/KT} \quad \text{--- (8)}$$

$N_V$  is called the effective density of states in the valency band for silicon , at 300 K  $N_V = 10^{25}/m^3$ . It is seen

that  $\frac{N_c}{N_v} = \frac{2.8 \times 10^{25} / m^3}{10^{25} / m^3}$

$$\left[ \frac{N_c}{N_v} = 2.8 \right]$$

To a first approximation  $N_c$  is taken with equal to  $N_v$ .

The symbol  $m_h^*$  represents the effective mass of hole.

### INTRINSIC CARRIER CONCENTRATION:

A single event of bound breaking in a pure semiconductor leads to generation of an electron-hole pair. At any temperature

$$'T', \quad n = p = n_i \quad (1)$$

From eq(1), we can write that

$$n_i^2 = np \quad (2)$$

$$\text{But } n = N_c e^{-(E_c - E_F)/kT}$$

$$p = N_v e^{-(E_F - E_v)/kT}$$

from eq(2)

$$n_i^2 = \left[ N_c e^{-(E_c - E_F)/kT} \right] \left[ N_v e^{-(E_F - E_v)/kT} \right]$$

$$n_i^2 = [N_c N_v] \left[ e^{-(E_c - E_v)/kT} \right] \quad (3)$$

The term  $E_c - E_v$  stands to the band gap Eg.

$$\therefore E_c - E_v = E_g,$$

$$\therefore n_i^2 = [N_c N_v] e^{-E_g/kT} \quad (4)$$

$$\text{But } N_C = 2 \left[ \frac{2\pi m_e^* kT}{h^2} \right]^{3/2}$$

$$N_V = 2 \left[ \frac{2\pi m_h^* kT}{h^2} \right]^{3/2}$$

substituting the values of  $N_C$  and  $N_V$  in eq(4) we obtain,

$$n_i^2 = 2 \left[ \frac{2\pi m_e^* kT}{h^2} \right]^{3/2} \times 2 \left[ \frac{2\pi m_h^* kT}{h^2} \right]^{3/2} e^{-E_g/kT}$$

$$n_i^2 = 4 \left[ \left( \frac{2\pi kT}{h^2} \right)^{3/2} \right]^2 (m_e^*)^{3/2} (m_h^*)^{3/2} e^{-E_g/kT}$$

$$n_i^2 = 4 \left[ \frac{2\pi kT}{h^2} \right]^3 (m_e^* m_h^*)^{3/2} e^{-E_g/kT}$$

Intrinsic carrier concentration  $n_i =$

$$n_i = \sqrt{4 \left[ \frac{2\pi kT}{h^2} \right]^3 (m_e^* m_h^*)^{3/2} e^{-E_g/kT}}$$

$$n_i = 2 \left[ \frac{2\pi kT}{h^2} \right]^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/2kT} \quad (5)$$

The experimental value of  $n_i$  in silicon at room temperature

is  $n_i = 1.5 \times 10^{16}$  carriers/m<sup>3</sup> and in germanium,  $n_i = 2.5 \times 10^{19}$  carriers/m<sup>3</sup>

## VARIATION OF INTRINSIC CARRIER CONCENTRATION WITH TEMPERATURE:

from eq(5)

$$n_i = 2 \left[ \frac{2\pi kT}{h^2} \right]^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g / (2kT)}$$

$$\Rightarrow n_i = 2 \left[ \frac{2\pi k}{h^2} \right]^{3/2} (m_e^* m_h^*)^{3/4} T^{3/2} e^{-E_g / (2kT)}$$

The above equation shows that the free charge carrier concentration varies with temperature and it may be approximated to

$$n_i = 10^{21.7} T^{3/2} e^{-2500 E_g / T}$$

The following important points may be inferred from it :

1. The intrinsic concentration is independent on fermilevel.
2. The intrinsic concentration has an exponential dependence on the band gap value 'Eg'.
3. It strongly depends on the temperature.
4. The factor '2' in the exponent indicates that two charge carriers are produced for one covalent bond broken.

## FERMI LEVEL IN INTRINSIC SEMICONDUCTOR:

In a pure semiconductor the electrons are located right at the bottom edge of the conduction band as shown in fig 1. Similarly, the holes are at the top edge of the

vacency band.

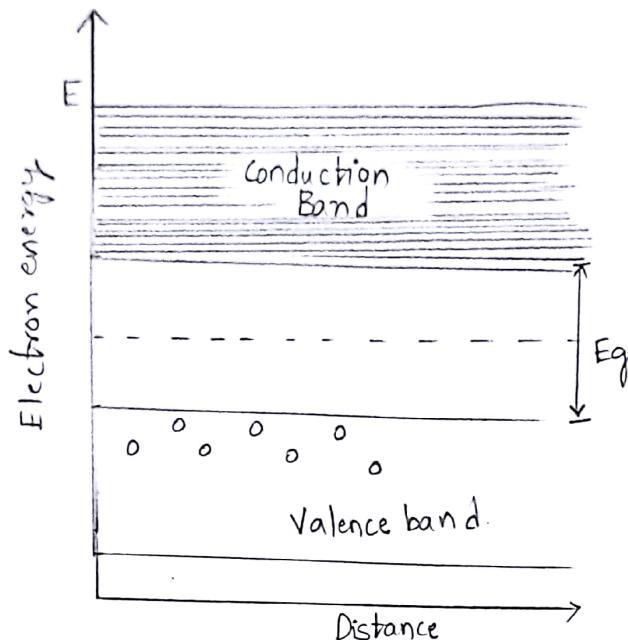


Fig (1)  
Fermilevel in  
Intrinsic semi-  
conductor.

The electron concentration in the conduction band.

$$n = N_c e^{-(E_c - E_F)/kT}$$

The holes concentration in the valency band,

$$p = N_v e^{-(E_F - E_v)/kT}$$

In an intrinsic semiconductor, the electron and hole concentrations are equal. Thus,  $n = p$

$$N_c e^{-(E_c - E_F)/kT} = N_v e^{-(E_F - E_v)/kT} \quad (1)$$

Taking logarithm on both sides, we get

$$\ln N_c - \frac{(E_c - E_F)}{kT} = \ln N_v - \frac{(E_F - E_v)}{kT}$$

$$-\frac{(E_c - E_F)}{kT} = \ln \left( \frac{N_v}{N_c} \right) - \frac{(E_F - E_v)}{kT}$$

Multiply with  $kT$

$$-(E_C - E_F) = kT \ln \left( \frac{N_V}{N_C} \right) - (E_F - E_V)$$

$$-E_C + E_F = kT \ln \left( \frac{N_V}{N_C} \right) - E_F + E_V$$

$$2E_F = (E_C + E_V) + kT \ln \left( \frac{N_V}{N_C} \right)$$

$$\therefore E_F = \frac{(E_C + E_V)}{2} + \frac{1}{2} kT \ln \left( \frac{N_V}{N_C} \right) \quad (2)$$

But  $N_C = 2 \left[ \frac{2\pi m_e^* kT}{h^2} \right]^{3/2}$  and

$$N_V = 2 \left[ \frac{2\pi m_h^* kT}{h^2} \right]^{3/2}$$

$$\frac{N_V}{N_C} = \frac{2 \left[ \frac{2\pi m_h^* kT}{h^2} \right]^{3/2}}{2 \left[ \frac{2\pi m_e^* kT}{h^2} \right]^{3/2}}$$

$$\frac{N_V}{N_C} = \frac{\left[ \frac{2\pi kT}{h^2} \right]^{3/2} (m_h^*)^{3/2}}{\left[ \frac{2\pi kT}{h^2} \right]^{3/2} (m_e^*)^{3/2}}$$

$$\frac{N_V}{N_C} = \left( \frac{m_h^*}{m_e^*} \right)^{3/2}$$

$$\ln = \left( \frac{N_V}{N_C} \right) = \frac{3}{2} \ln \left( \frac{m_h^*}{m_e^*} \right)$$

from eq(2)

$$E_F = \left( \frac{E_C + E_V}{2} \right) + \frac{1}{2} kT \left[ \frac{3}{2} \ln \left( \frac{m_h^*}{m_e^*} \right) \right]$$

$$E_F = \frac{E_C + E_V}{2} + \frac{3}{4} kT \ln \left( \frac{m_h^*}{m_e^*} \right) \quad (3)$$

We can also write,

eq(3) as,

$$E_F = \frac{E_C + E_V}{2} - \frac{3}{4} kT \ln \left( \frac{m_e^*}{m_h^*} \right) \quad (4)$$

if the effective mass of the free electron is assumed to be equal to the effective mass of a hole,

$$m_h^* = m_e^*$$

$$\ln \left( \frac{m_h^*}{m_e^*} \right) = 0$$

from eq(3)

$E_F = \frac{E_C + E_V}{2}$

— (5)

To make the meaning of eq(5) more explicit we can write

$$E_F = \frac{E_C - E_V}{2} + E_V$$

$$E_F = \frac{E_g}{2} + E_V \quad (\because E_C - E_V = E_g)$$

If we denote the top of the valency band ' $E_V$ ' as zero level,  $E_V = 0$

$\therefore E_F = \frac{E_g}{2}$

— (6)

Eq(6) shows that in an intrinsic semiconductor the fermilevel lies in the middle of the forbidden gap. The fermilevel is not an allowed energy level in semiconductors. It only serves as a reference energy with reference to which we specify the energies of electrons and holes in a semiconductor. We made the following assumptions in obtaining the eq(6);

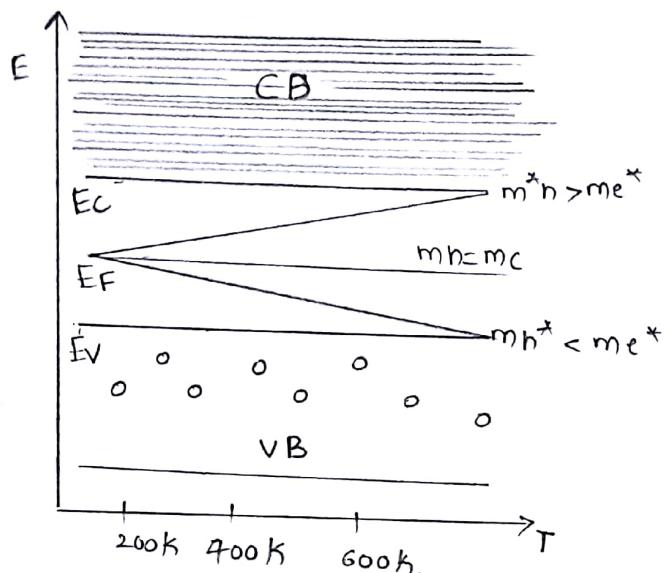
- i) It was assumed that the electrons undergo transition from the top edge level ' $E_V$ ' of the valency band to the bottom edge level ' $E_C$ ' of the conduction band. In reality, transitions are possible between the others also. However, the above result does not appreciably differ if other transitions are also taken into account.
- ii) It was assumed that the effective mass of electrons in the conduction  $m_e^*$ , is exactly equal to the effective mass of the holes in the valency band  $m_h^*$ . In practise, the effective masses differ from each other. However, the difference does not alter the above result significantly.

#### VARIATION OF FERMI LEVEL WITH TEMPERATURE:

With an increase in temperature the fermi level gets displaced upward to the bottom edge of the conduction band if  $m_h^* > m_e^*$  or downward to the top edge of the valency band

If  $m_h^* < m_e^*$  as indicated in fig(2). In most of the materials, the shift of fermilevel on account of  $m_h^* \neq m_e^*$  is insignificant.

The fermilevel in an intrinsic semiconductor may be considered as independent of temperature and as staying in the middle of the band gap.



Fig(2) Variation of fermilevel with temp in an intrinsic semiconductor.

### LIMITATIONS OF INTRINSIC SEMICONDUCTOR

We summarize the limitations as follows:

1. Conductivity is low, germanium has a conductivity of  $1.67 \text{ S/m}$ , which is nearly  $10^7$  times smaller than that of copper.
2. Conductivity is a function of temperature and increases exponentially as temperature increases.
3. Conductivity cannot be controlled from outside.

## THE FRACTION OF ELECTRONS IN CONDUCTION BAND.

The Fermi-Dirac probability function  $f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$  gives

the probability that an electron occupies the energy state  $E$ .

The probability that an electron occupies the energy state  $E_c$  may then be found from

$$f(E_c) = \frac{1}{1 + e^{(E_c - E_F)/kT}} \quad (1)$$

But according to the definition of probability,

$$f(E_c) = \frac{n}{N} \quad (2)$$

where;  $n$  is the number of electrons excited to conduction band levels.

$N$  is the total number of electrons available in the valency band initially

$$\therefore \frac{n}{N} = \frac{1}{1 + e^{(E_c - E_F)/kT}} \quad : \text{equated } (1) \& (2)$$

As  $(E_c - E_F) = \frac{E_g}{2}$ , we write

$$\frac{n}{N} = \frac{1}{1 + e^{E_g/2kT}}$$

Since  $E_g > 2kT$ , the factor unity may be neglected in comparison to the exponential term

$$\therefore \frac{n}{N} = \frac{1}{e^{E_g/2kT}}$$

$$\therefore \frac{n}{N} = e^{-Eg/2kT} \quad \text{--- (3)}$$

Eq(3) gives the fraction of electrons in the conduction band

### EXTRINSIC SEMI CONDUCTOR:

The controlled amount of impurity added into an intrinsic semi-conductor is known as "doping".

The impurity which is introduced is known as a "dopant". A semiconductor doped with impurity atoms is called as "extrinsic semiconductor".

Typical doping levels range from  $10^{20}$  to  $10^{27}$  impurity atoms/ $m^2$ . Pentavalent elements from group V or trivalent elements from group III are used as dopants. Depending on the different types of doping, two types of extrinsic semiconductors are possible. They are 'n' type and 'p' type semiconductors.

Common dopant elements for Silicon and germanium

n-type	p type
Phosphorous	Aluminium
Arsenic	Boron
Antimony	Gallium

## ADVANTAGES OF EXTRINSIC SEMI CONDUCTOR

- Conductivity is high
- Conductivity can be tailored to the desired value through the control of doping concentration.
- Conductivity is not a function of temperature

## n-TYPE SEMICONDUCTOR

n-type semiconductor is produced when a pure semiconductor is doped with a pentavalent impurity such as phosphorous. Doped atom is called donor atom.

## ENERGY BAND DIAGRAM:

The energy band diagram of 'n'-type semiconductor is shown in fig(1). If the donor atom density is low, the donor atoms cannot interact with each other and their energy levels are discrete (separate entity) levels, ' $E_D$ '. They are called donor levels and represent the ground state of the fifth electron of impurity atom. As even small amount of thermal energy can readily liberate the fifth electron from the atom and send into the conduction band, the donor levels are expected to be located very near to the bottom edge of the conduction band.

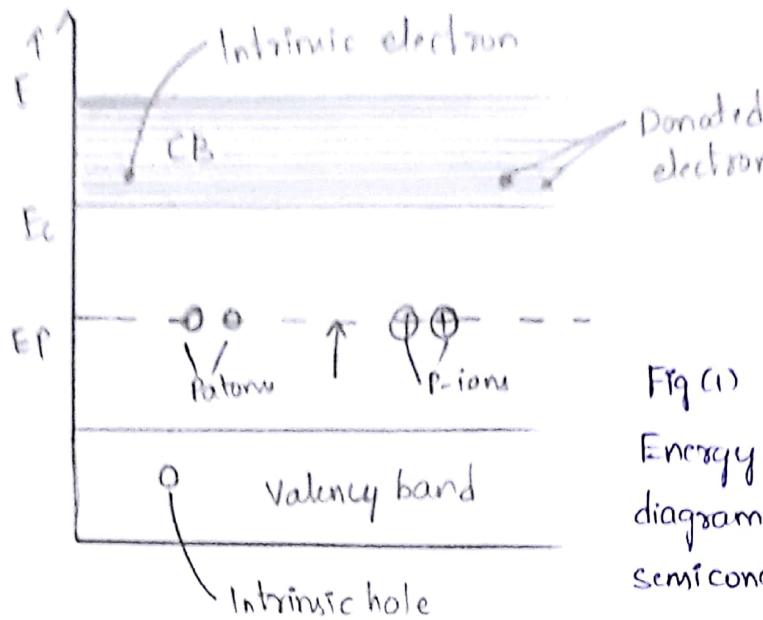


Fig (1)

Energy band diagram of n-type semiconductor

### TEMPERATURE VARIATION AND CARRIER CONCENTRATION:

At '0K' the material, behaves as an insulator means at '0K' donor electrons bound to the donor atoms. As temperature increases, the donor atoms are ionized (converted into ions) and the donor electrons go into the conduction band. (fig<sup>2</sup> ionization). By getting energy from lattice vibrations. In this process holes are not produced in the valency band. At normal temperature, the donor levels are all ionized and some electrons from the valency band are excited into the conduction band (Fig<sup>2</sup>, depletion region).

The conduction band now contains, electrons from

- donor atom ionization and
- intrinsic process.

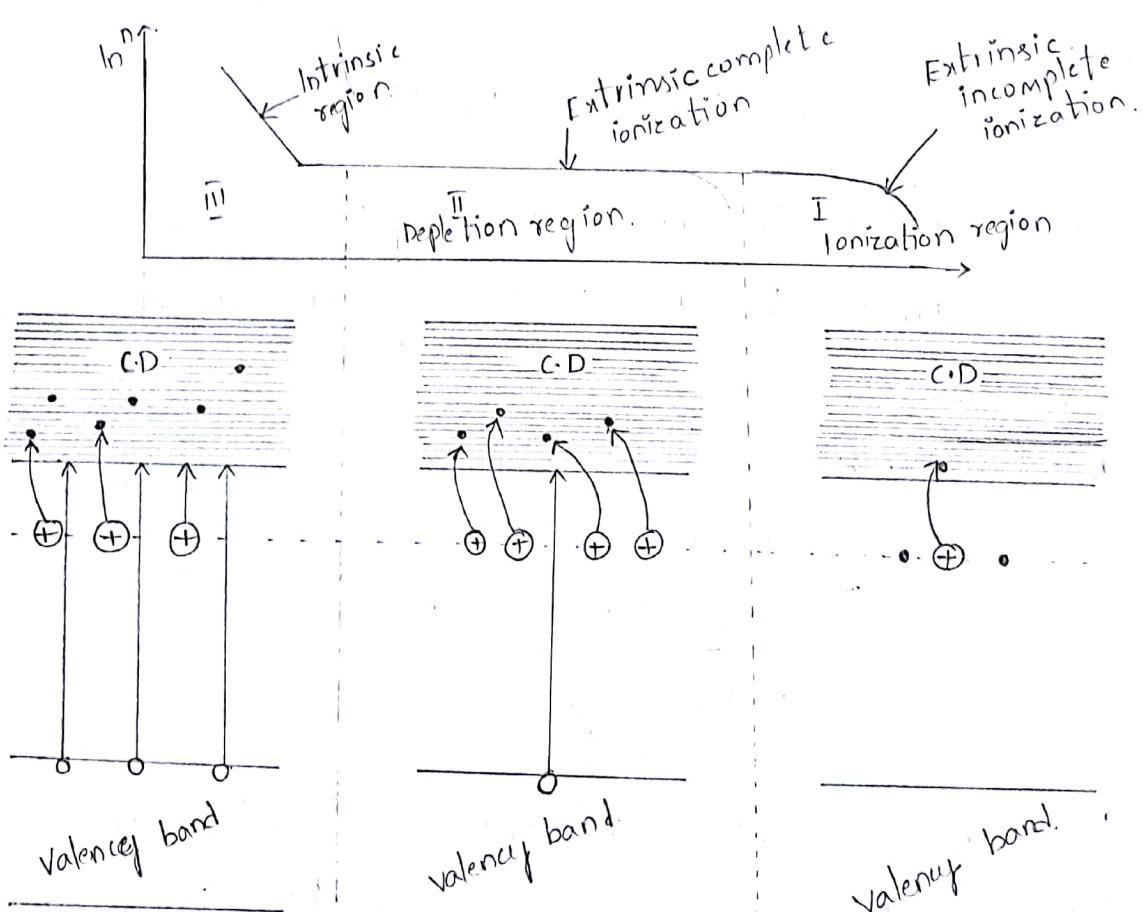
Intrinsic process generates holes also in the valency band. At moderate temperatures, there are some  $10^{10}$  electrons

free from bonds and  $10^{16}$  free electrons received from donors.

In 'n' type material the electrons out number (very large) the holes. And constitute the majority charge carriers. hole are minority carriers the current in this type of crystal is mainly due to the negatively charged electrons and hence, the material is called n-type semi-conductor. If 'N<sub>D</sub>' is the concentration of donor atoms then,  $n_n \approx N_D$ ; where  $n_n$ =electron concentration in the conduction band of 'n' type material.

As, temperature increases further some electrons from the valency band are exited into the conduction band.

In intrinsic region;  $n_n = n_i$



Fig(2): Variation of electron concentration in an 'n' type Semiconductor as a function of the inverse of temperature.

## CARRIER CONCENTRATION IN N-TYPE SEMI CONDUCTOR:

Let  $N_D$  be the concentration of donors in the material at '0K' the donor atoms are not ionized and are at the level  $E_D$  which is very near to  $E_F$ . When the temperature is raised above '0K' the donor atoms get ionized and free electrons go <sup>to</sup> the conduction band with increase in temperature the electron concentration in the conduction band increases.

For transition of electrons to the conduction band from the donor levels electrons require an energy ' $E_D$ '

We assume the electron concentration,  $n$  in the conduction band is

$$n = N_D^+ \text{ or } n = N_D - N_D^\circ$$

where,  $N_D^+$  is the no. of ionized donor atoms.

$N_D^\circ$  is the no. of unionized atoms at the energy level ' $E_D$ '

The concentration of ionized donors,  $N_D^+ = N_D - N_D^\circ$

$$\begin{aligned} N_D^+ &= N_D - N_D^\circ \\ &= N_D [1 - f(E_D)] \\ &= \frac{N_D}{1 + e^{(E_D - E_F)/kT}} \end{aligned}$$

From the operational definition of fermi level it is expected that the fermi level in n-type semiconductor lies a few  $kT$  above ' $E_D$ '. Therefore, the above equation

may be simplified as,

$$n = \frac{N_D}{e^{-(E_D - E_F)/kT}}$$

$$\therefore n = N_D e^{(E_D - E_F)/kT} \quad \text{--- (1)}$$

But the electron concentration in the conduction band is given by

$$n = N_C e^{(E_C - E_F)/kT} \quad \text{--- (2)}$$

Therefore, from eq (1) and (2)

$$N_D e^{(E_D - E_F)/kT} = N_C e^{(E_C - E_F)/kT}$$

Taking logarithm and rearranging the terms, we get

$$\ln N_D + \left( \frac{E_D - E_F}{kT} \right) = \ln N_C - \left( \frac{E_C - E_F}{kT} \right)$$

$$\left( \frac{E_D - E_F}{kT} \right) + \left( \frac{E_C - E_F}{kT} \right) = \ln \frac{N_C}{N_D}$$

Multiply with  $kT$

$$E_D - E_F + E_C - E_F = kT \ln \frac{N_C}{N_D}$$

$$-2E_F + (E_D + E_C) = kT \ln \frac{N_C}{N_D}$$

$$2E_F - (E_D + E_C) = -kT \ln \frac{N_C}{N_D}$$

$$2E_F = (E_D + E_C) - kT \ln \frac{N_C}{N_D}$$

$$\therefore E_F = \frac{E_D + E_C}{2} - \frac{kT}{2} \ln \frac{N_C}{N_D}$$

$$E_F = \frac{E_D + E_C}{2} + \frac{kT}{2} \ln \frac{N_D}{N_C}$$

(29)

Sub  $N_c$  value.

$$\therefore E_F = \frac{E_D + E_C}{2} + \frac{kT}{2} \ln \left[ \frac{N_D}{2(2\pi m_e^* kT/h^2)^{3/2}} \right] \quad (3)$$

$$\therefore N_C = 2 \left[ \frac{2\pi m_e^* kT}{h^2} \right]$$

It is seen that, from eq (3) at  $T=0K$  (it acts as insulator)

$$\therefore E_F = \frac{E_D + E_C}{2} \quad (4)$$

i.e., the equilibrium fermilevel lies between the bottom of the conduction band and donor levels.

$$\begin{aligned} \exp \left[ \frac{E_F - E_C}{kT} \right] &= \exp \left[ \frac{E_F}{kT} - \frac{E_C}{kT} \right] \\ &= \exp \left[ \frac{E_D + E_C}{2kT} + \frac{1}{2} \ln \left[ \frac{N_D}{2(2\pi m_e^* kT/h^2)^{3/2}} \right] - \frac{E_C}{kT} \right] \\ &= \exp \left[ \frac{E_D - E_C}{2kT} + \frac{1}{2} \ln \frac{N_D}{2(2\pi m_e^* kT/h^2)^{3/2}} \right] \\ &= \exp \left[ \left( \frac{E_D - E_C}{2kT} \right) + \ln \sqrt{\frac{N_D}{2(2\pi m_e^* kT/h^2)^{3/2}}} \right] \end{aligned}$$

$$\boxed{\because \frac{x}{2} \log x = \log \sqrt{x}}$$

$$= \exp \left[ \frac{E_D - E_C}{2kT} \right] \exp \left[ \ln \sqrt{\frac{N_D}{2(2\pi m_e^* kT/h^2)^{3/2}}} \right]$$

$$\boxed{\begin{aligned} &\because \exp(a+b) \\ &= \exp(a) \exp(b) \end{aligned}}$$

$$\exp \left[ \frac{E_F - E_C}{kT} \right] = \exp \left[ \frac{E_D - E_C}{2kT} \right] \left[ \sqrt{\frac{N_D}{2(2\pi m_e^* kT/h^2)^{3/2}}} \right]$$

$$\boxed{\therefore \exp(\log x) = x}$$

From ②

$$n = N_C e^{-(E_C - E_F)/kT}$$

$$n = N_C \exp\left[\frac{E_F - E_C}{kT}\right]$$

$$\therefore n = 2 \left[ \frac{2\pi m e^* k T}{h^2} \right]^{3/2} \exp\left[\frac{E_D - E_C}{2kT}\right] \times \left[ \sqrt{\frac{N_D}{2(2\pi m e^* k T/h)^{3/2}}} \right]$$

$$\therefore n = (2N_D)^{1/2} \left( \frac{2\pi m e^* k T}{h^2} \right)^{3/4} \exp\left[\frac{E_D - E_C}{2kT}\right] \quad (5)$$

eq(5) gives the majority carrier concentration. The electron concentration in the conduction of an 'n' type semiconductor is proportional to the square root of the donor concentration at fairly low temperatures

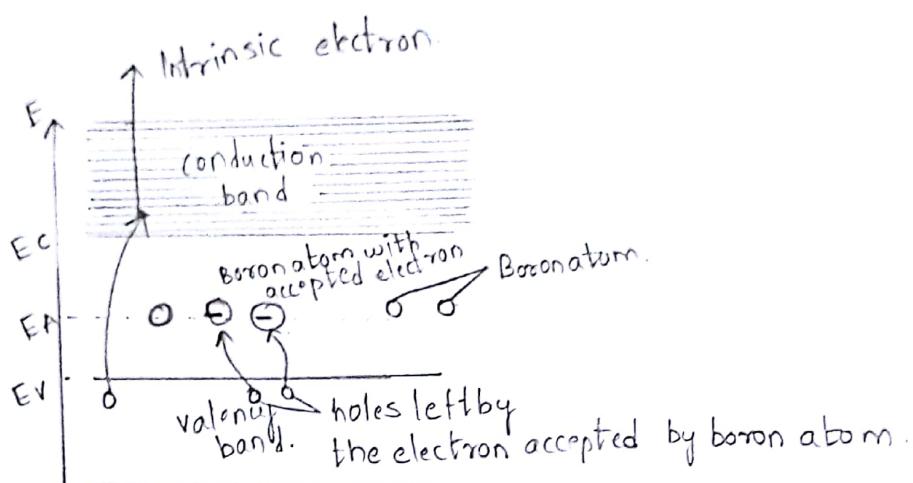
### P-TYPE SEMI CONDUCTOR:

A p-type semi conductor is produced when a pure semi-conductor is doped with a trivalent impurity such as boron. As the boron atom accepted an electron from the valency band, it is called an acceptor atom. The acceptor impurity atoms produce holes. Without the simultaneous generation of the electrons in the conduction band.

The energy band diagram of p-type semi-conductor is shown in fig(1). If the acceptor atom density is low, the acceptor atoms cannot interact with each other and their energy levels are discrete levels, 'EA'. They are called acceptor levels and represent the ground state of hole. As even small amount of thermal energy can make

③

an electron in the valency band jump into the acceptor level, acceptor levels are expected to be very near to the top edge of the valency band. They are at about 0.01 ev above the valency band. The hole may be said to have moved from the acceptor atom to the valency band.



Fig(1): Energy band diagram for p-type conductor.

#### TEMPERATURE VARIATION AND CARRIER CONCENTRATION:

At zero Kelvin, the acceptor levels are vacant and the valency band is full. The conduction band is also vacant. The material behaves essentially as an insulator at slightly elevated temperature, holes are generated without the simultaneous generation of electrons. At about 100K, once all the acceptor atoms ionized. Further, increase in temperature does not produce holes and we say, the acceptor levels are saturated. The region is called the saturation region.

In the saturation region, the hole concentration in the valency band is nearly identical to the concentration of the acceptor atoms. If 'NA' is the concentration of the

acceptor atoms, then  $P_p \approx N_A$ , where,  $P_p$  is the hole concentration in the valency band of P-type material. As temperature is increased further, the valency band contains holes that have been generated by two different processes, namely.

1. Acceptor atom ionization and

2. Intrinsic process

The intrinsic process causes electrons to appear, in the conduction band. At high enough temperatures, the material behaves as an intrinsic semiconductor. In the intrinsic region,  $P_p = n_i$ . In P-type material the holes outnumber the electrons and constitute the majority carriers, electrons are minority carriers. The number of majority carriers is independent of temperature in the depletion region. The current in this type of crystal is mainly due to the positively charged holes and hence, the material is P-type semiconductor.

#### CARRIER CONCENTRATION IN P-TYPE SEMICONDUCTOR:

Let ' $N_A$ ' be the concentration of acceptors in the material. At '0 K', the acceptor atoms are not ionized and are at the levels ' $E_A$ ' which is very near to ' $E_V$ '. When the temperature is raised above zero Kelvin, the acceptor atoms get ionized and holes appear in the valency band. With increase in temperature more and more acceptor atoms get ionized and hole concentration in the valency band increases. Since, transition of electrons to the acceptor levels

from the valency band requires energy ' $E_A$ ', they can go to acceptor levels and ionize acceptor atoms. Therefore, we may assume that the hole concentration ' $p$ ' in the valency band is given by

$p = N_A^-$  where,  $N_A$  is the no. of acceptor atoms that are ionized.

The concentration of ionized acceptors,

$$N_A^- = N_A e^{-E_A/kT}$$

$$N_A^- = \frac{N_A}{e^{(E_F - E_A)/kT}}$$

But  $-(E_F - E_A) \gg kT$

$$\therefore N_A^- = \frac{N_A}{e^{(E_F - E_A)/kT}}$$

$$\therefore p = N_A e^{(E_F - E_A)/kT} \quad \text{--- (1)}$$

But hole concentration ' $p$ ' in the valency band is given by

$$p = N_V e^{(E_F - E_V)/kT}$$

$$\therefore p = N_V e^{(E_V - E_F)/kT} \quad \text{--- (2)}$$

From eq(1) and eq(2)

$$N_A e^{(E_F - E_A)/kT} = N_V e^{(E_V - E_F)/kT}$$

taking logarithm and rearranging the terms we get,

$$\ln N_A + \left( \frac{E_F - E_A}{kT} \right) = \ln N_V + \left( \frac{E_V - E_F}{kT} \right)$$

$$\left( \frac{E_F - E_A}{kT} \right) - \left( \frac{E_V - E_F}{kT} \right) = \ln \left( \frac{N_V}{N_A} \right)$$

multiply  $kT$  on b.s

$$E_F - E_A - E_V + FF = kT \ln \left( \frac{N_V}{N_A} \right)$$

$$2E_F - (E_V - E_A) = kT \ln \frac{N_V}{N_A}$$

$$2E_F = E_V + E_A + kT \ln \frac{N_V}{N_A}$$

$$\therefore E_F = \frac{(E_V + E_A)}{2} + \frac{kT}{2} \ln \frac{N_V}{N_A}$$

$$E_F = \frac{(E_V + E_A)}{2} - \frac{kT}{2} \ln \frac{N_A}{N_V}$$

$$\therefore E_F = \frac{E_V + E_A}{2} - \frac{kT}{2} \ln \frac{N_A}{2(2\pi m_h^* kT/h^2)^{3/2}} \quad (3)$$

it is seen from eq(3) that at  $T=0^\circ K$

$$\boxed{\therefore E_F = \frac{E_V + E_A}{2}} \quad (4)$$

i.e., the equilibrium Fermi level lies midway between the top of the valency band and acceptor levels.

$$\exp \left[ \frac{E_V - E_F}{kT} \right] - \exp \left[ \frac{E_V}{kT} - \frac{E_F}{kT} \right]$$

$$= \exp \left[ \frac{E_V}{kT} - \left( \frac{E_V + E_A}{2kT} \right) + \frac{1}{2} \ln \frac{N_A}{2(2\pi m_h^* kT/h^2)^{3/2}} \right]$$

$$= \exp \left[ \frac{E_V - E_A}{2kT} \right] + \ln \sqrt{\frac{N_A}{2(2\pi m_h^* kT/h^2)^{3/2}}}$$

$$\boxed{\therefore \frac{1}{2} \log x = \log \sqrt{x}}$$

$$= \exp \left[ \frac{E_V - E_A}{2kT} \right] + \exp \left[ \ln \sqrt{\frac{N_A}{2(2\pi m_h^* kT/h^2)^{3/2}}} \right]$$

$$\boxed{\because \exp(a+b) = \exp(a)\exp(b)}$$

$$\exp\left(\frac{E_V - E_F}{kT}\right) = \exp\left(-\frac{E_V - E_A}{2kT}\right) \sqrt{\frac{N_A}{2(2\pi m_h^* kT/n^*)^{3/2}}} \cdot \frac{\exp(\log f_x)}{f_x}$$

From eq (2)

$$P = N_V \cdot e^{(E_V - E_F)/kT}$$

$$P = N_V \cdot \exp\left[\frac{E_V - E_F}{kT}\right]$$

$$P = 2 \left[ \frac{2\pi m_h^* kT}{h^2} \right]^{3/2} \exp\left[\frac{E_V - E_A}{2kT}\right] \left[ \sqrt{\frac{N_A}{2(2\pi m_h^* kT/n^*)^{3/2}}} \right]$$

$$\therefore P = (2N_A)^{1/2} \left( \frac{2\pi m_h^* kT}{h^2} \right)^{3/4} \exp\left[\frac{E_V - E_A}{2kT}\right] \quad (B)$$

eq (5) gives the majority carrier concentration of a p-type semiconductor. The hole concentration in the valency band is proportional to the square root of the acceptor concentration at fairly low temperatures.

COMPARISON OF INTRINSIC AND EXTRINSIC SEMI CONDUCTOR

### CONDUCTOR:

So No	Intrinsic Semiconductor	Extrinsic Semiconductor
1.	chemically pure semiconductors with less than one impurity atom in a billion host atoms	Large quantity of impurity atoms Present can be from group III or group V elements.

S.No	Intrinsic Semiconductor	Extrinsic Semiconductor
2.	Electrical conduction arises due to thermally excited electrons and holes .	The impurity produced carriers are not temperature dependent but are voltage dependent and amount of impurity added . In n-type semiconductors electrons are majority carriers and in p-type semiconductors holes are majority carriers.
3.	Fermilevel exists at midway of energy gap	In n-type semiconductors fermilevel is closed to conduction band and in p-type semiconduction fermilevel is closed to valency band.
4.	Electrical conductivity is low	Electrical conductivity is high.
5.	Not useful for device manufacture because of low conductivity and the strong dependence of conductivity on temperature	Useful for semiconductor devices such as diodes, transistors , LED , photodiodes etc

## CARRIER-TRANSPORT - EXTRINSIC CONDUCTIVITY:

In extrinsic region, all the impurity atoms are ionized and the carrier concentration is essentially independent of temperature. In this region any desired conductivity can be achieved by controlling the amount of impurities added from outside.

conductivity of 'n' type semiconductor

$$\sigma_n = nne N_{\text{eff}} p_n e M_n$$

as  $p_n \ll n_n$ , the second term is negligible and it is the electrons that contribute to the conductivity.

$$\therefore \sigma_n = nne M_e$$

As  $n_n = N_D$ , we can write the above relation is

$$\boxed{\sigma_n = N_D e M_e}$$

conductivity of P-type semiconductors

$$\sigma_p = p p e M_n$$

$$\boxed{\therefore \sigma_p = N_A e M_h}$$

$$\boxed{\therefore p_p = N_A}$$

A general dependence of conductivity on temperature in extrinsic semiconductors is shown in figure.

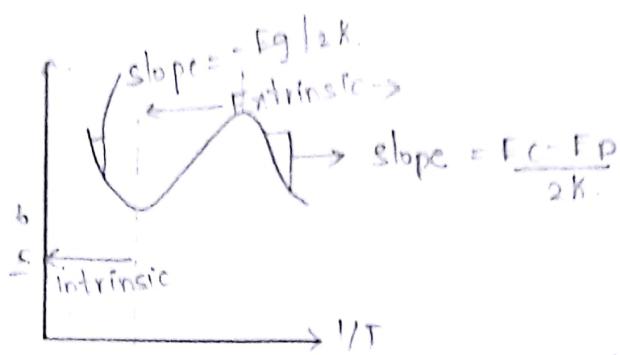


Fig: Dependence of conductivity on temperature in an extrinsic semiconductor.

### LAW OF MASS ACTION:

Electron concentration in n-type semiconductor,

$$n_n = N_c e^{-(E_c - E_F)/kT}$$

Hole concentration in n-type semiconductor,

$$P_n = N_v e^{-(E_F - E_V)/kT}$$

$$\therefore n_n \times P_n = N_c N_v e^{-(E_c - E_V)/kT}$$

$$n_n P_n = N_c N_v e^{-E_g/kT}$$

$$\because E_c - E_V = E_g$$

$$n_n P_n = n_i^2$$

$$\therefore n_i^2 = N_c N_v e^{-E_g/kT}$$

Hole concentration in p-type semiconductor,

$$P_p = N_v e^{-(E_F - E)/kT}$$

$$P_p = N_v e^{-(E_F - E_V)/kT}$$

Electron concentration in p-type semiconductor,

$$n_p = N_c e^{-(E_c - E_F)/kT}$$

$$\therefore P_p n_p = N_c N_v e^{-(E_c - E_V)/kT}$$

$$P_p n_p = N_c N_v e^{-E_g/kT}$$

$$\boxed{P_p n_p = n_i^2}$$

The product of majority and minority carrier concentrations in an extrinsic semiconductor at a particular temperature is a constant and is equal to the square of intrinsic carrier concentration at that temperature.

### CALCULATION OF MINORITY CARRIER CONCENTRATION:

In the case of an n-type semiconductor, the majority carrier concentration  $n_n$  is given by

$$n_n = N_D$$

where,  $N_D$  is the donor impurity concentration & all the donor atoms are assumed to have got ionized

Above, relation indicates that the majority carrier concentration,  $n_n$  in an n-type semiconductor is equal to the donor impurity concentration,  $N_D$ .

The minority carrier concentration ' $p_n$ ' is given

$$P_n = \frac{n_i^2}{n_n}$$

$$\therefore n_i^2 = n_n P_n$$

$$\boxed{\therefore P_n = \frac{n_i^2}{N_D}}$$

Similarly, in case of a p-type semiconductor the minority carrier concentration,  $n_p$  is given by

$$n_p = \frac{n_i^2}{P_p}$$

$$\boxed{\therefore n_p = \frac{n_i^2}{N_A}}$$

$$\boxed{\therefore P_p = N_A}$$

The charge neutrality condition, applied to the n-type to the n-type semiconductor implies that the total negative charge of mobile electrons is equal to the total positive charge created in the crystal. It means that

$$n_n = N_D + P_n$$

where  $N_D$  = donor impurity concentration and all the donor atoms are assumed to have got ionized. But

$$n_n \gg P_n$$

$$\therefore n_n = N_D$$

The above eqn indicates that, the majority carriers concentration  $n_n$  in an n-type semiconductor is equal to the donor impurity concentration  $N_D$ . The charge neutrality for a p-type semiconductor requires that  $P_p = N_A + 2P_n$ .

$$\text{As } P_p \gg N_A$$

$$\boxed{\therefore P_p = N_A}$$

### FERMI LEVEL IN EXTRINSIC SEMICONDUCTORS:

In an n-type semiconductor the fermilevel lies in the half of the gap, the majority carriers reside in the conduction band. And their average energy is more than  $E_Fn$ .

In a p-type semiconductor the fermilevel lies in the

lower half of gap, as the majority carriers reside in the valency band and their average energy is less than E<sub>FP</sub>.

### a) Variation of fermilevel with temperature in N-type:

The donor atoms in the n-type semiconductor at low temperatures are ionized and supply electrons to the conduction band at T=0K, E<sub>Fn</sub> lies midway between the donor levels and the bottom of the conduction band. Therefore,

$$E_{Fn} = \frac{E_c + E_D}{2} \quad \text{at } T=0K$$

As temperature increases the donor levels gradually get depleted and the fermilevel moves downward.

At the temperature 'T' of complete depletion of donor levels i.e T<sub>d</sub>.

$$E_{Fn} = E_D \text{ at } T=T_d$$

At high temperatures, the n-type semiconductor loses its extrinsic character and behaves as an intrinsic semiconductors.

$$\therefore E_{Fn} = E_{Fi} = \frac{E_g}{2} \text{ at } T \geq T_i$$

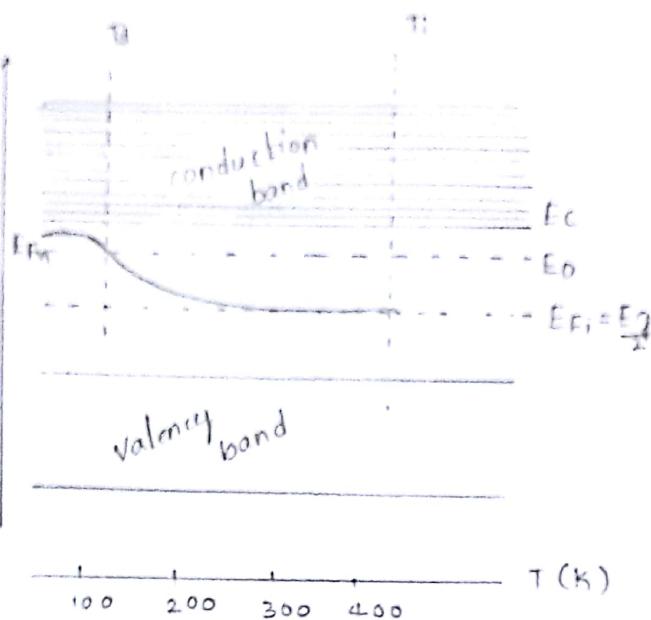


Fig 10: Qualitative dependence of fermilevel on temperature in an n-type Semiconductor.

The variation of fermilevel  $E_{Fn}$  in n-type semiconductor with temperature is illustrated in Fig 10a.

b. Variation of Fermilevel with temperature in P-type Semiconductor:

In the low temperature region holes in the valency band due to the transition of electrons from valency band to the acceptor levels.

When  $T=0K$ , the fermilevel lies midway between the acceptor levels and the top of the valency band.

$$\therefore E_{FP} = \frac{E_V + E_A}{2} \text{ at } T=0K.$$

As the temperature increases the acceptor levels gradually get filled and the fermi level moves upward. At the temperature of saturation  $T_s$ , the fermilevel

coincides with the acceptor level EA.

$$\therefore E_{FP} = \frac{E_V + EA}{2} \text{ at } T=0K$$

As the temperature increases the acceptor levels gradually get filled and the fermilevel moves upward. At the temperature of saturation  $T_s$ , the fermilevel coincides with the acceptor level EA.

$$\therefore E_{FP} = EA \text{ at } T=T_s$$

At high temperatures, the p-type semiconductor loses its extrinsic character and behaves as an intrinsic semiconductor.

$$\therefore E_{FP} = E_{FI} = \frac{E_g}{2} \text{ at } T=T_i$$

The variation of fermilevel  $E_{FP}$  in a P-type semiconductor with temperature is illustrated in fig 1(b).

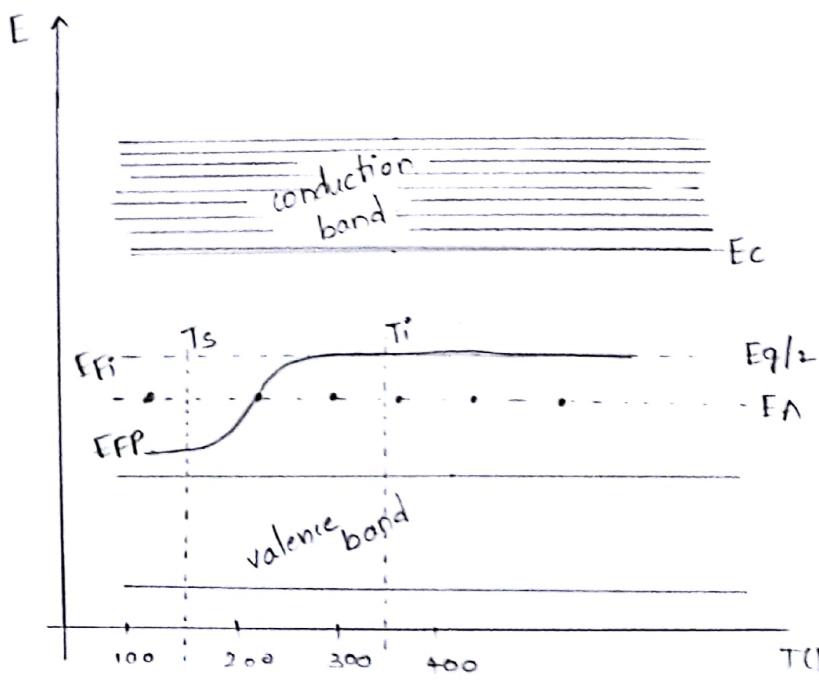


Fig 1(b): Qualitative dependence of fermilevel on temperature in a P-type semiconductor

## VARIATION OF FERMILEVEL WITH IMPURITY CONCENTRATION:

### a) N-type Semiconductor:

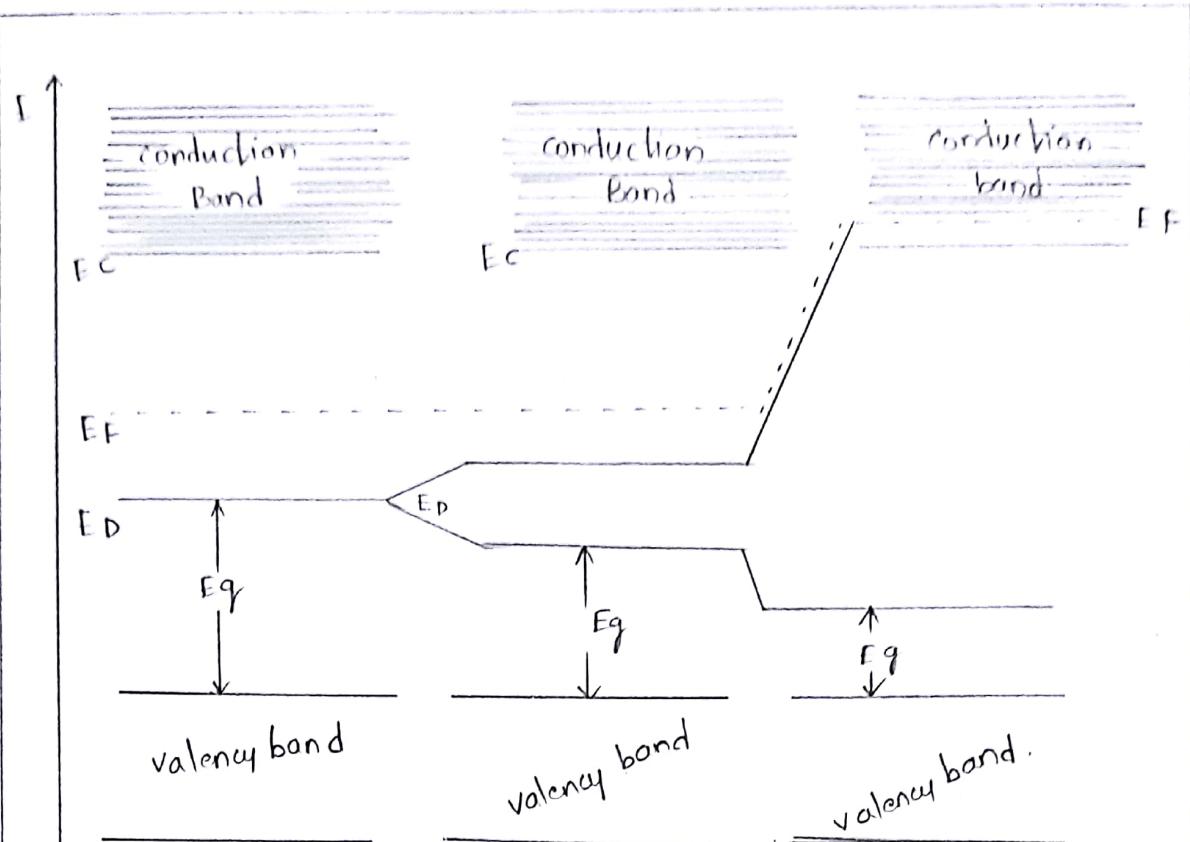
At low concentrations the impurity atoms are far apart and they do not interact with each other. On increase in the impurity concentration, the separation between impurity atoms reduces and they tend to interact. As a result, the donor levels undergo splitting and form an energy band below the conduction band as shown in figure.

The larger the doping concentration, the impurity band overlaps on the conduction band and the width of the forbidden gap decreases and fermilevel moves upwards.

The fermilevel shifts closer and closer to the conduction band with increase in impurity concentration and finally moves into the conduction band when the donor band overlaps on the conduction band.

### b) P-type Semiconductors:

In P-type semiconductor the acceptor levels broaden and form into a band with increase in impurity concentration. The acceptor band ultimately overlaps on the valency band. The fermilevel moves down closer to the valency band and finally at very high impurity concentration it will shift into the valency band.



Energy band diagram of an n-type semiconductor at three different levels of doping.

- a) Low level doping
- b) medium doping
- c) heavy doping

#### DRIFT AND DIFFUSION CURRENT:

DEF:

When an electric field applied across a semiconductor the charge carriers acquire a directional motion over and above their thermal motion and produce drift current. Drift current occurs only when external field is present across the solid electrons diffusing in the conduction band produce current density i.e  $J_{el(drift)} = ne\mu_e E$

holes drifting in the valency band cause,

$$J_h(\text{drift}) = p e N_h E$$

∴ Total drift current density,

$$J_{dr} = J_e(\text{drift}) + J_h(\text{drift})$$

$$J_{dr} = n e N_e E + p e N_h E$$

$$J_{dr} = e [n N_e + p N_h] E$$

DEF:

Any moment of charge carriers due to concentration gradient constitutes an electric current known as diffusion current.

Concentration gradient (the rate of change of carrier concentration per unit length) may be produced in an extrinsic semiconductors by applying heat or light locally at one region.

The current component due to electron diffusion,

$$J_e(\text{diff}) = e D_e \frac{dn}{dx}$$

The current component due to hole diffusion,

$$J_h(\text{diff}) = -e D_h \frac{dp}{dx}$$

∴ The total current density due to drift and diffusion of electrons

$$\therefore J_e(\text{drift}) + J_e(\text{diff}) = n e N_e E + e D_e \frac{dn}{dx}$$

$$J_e = e \left[ n N_e E + D_e \frac{dn}{dx} \right]$$

The total current density due to drift and diffusion for holes.

$$\therefore J_n(\text{drift}) + J_n(\text{diff}) = p_e N_h E - e D_n \frac{dp}{dx}$$

$$J_n = e \left[ p_e N_h E - D_n \frac{dp}{dx} \right]$$

where,  $D_e$  and  $D_n$  are diffusion coefficients for electrons and holes respectively.

E.H. Hall effect:

E.H. Hall effect

E.H. Hall discovered Hall effect in 1879. It helps to determine the

- type of semiconductor
- majority charge carrier concentration
- sign of majority charge carriers.
- drift velocity of majority charge carriers.
- mobility of charge carriers.

STATEMENT:

If a metal or a semiconductor carrying a current 'I' subjected to a transverse magnetic field 'B' a potential difference  $V_H$  is produced in a direction normal to the both the magnetic field and current directions.

Experimental arrangement:

The experimental setup for the measurement of Hall voltage and determination of Hall coefficient is shown

shown in fig (a). A thin rectangular semiconductor wafer is mounted on an insulating strip and two pairs of electrical contacts are provided on opposite sides of the wafer one pair of contacts is connected to a constant current source. And the other pair is connected to a sensitive voltmeter. This arrangement is mounted in between two pole pieces of an electromagnet such that, the magnetic field acts perpendicular to the lateral faces of the Semiconductor wafer.

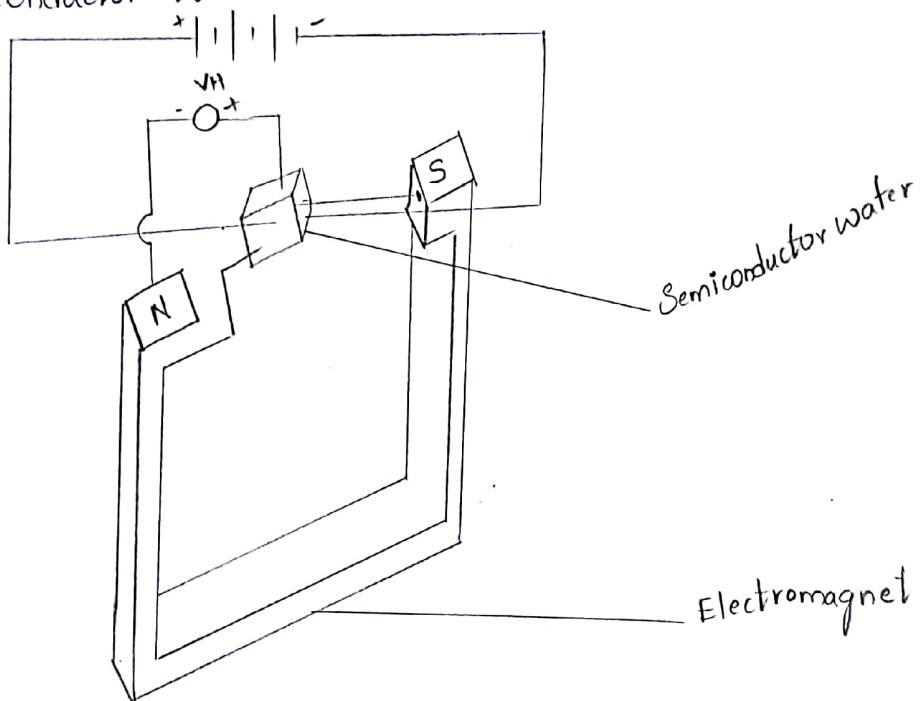


fig (a): Basic experimental arrangement to study hall effect .

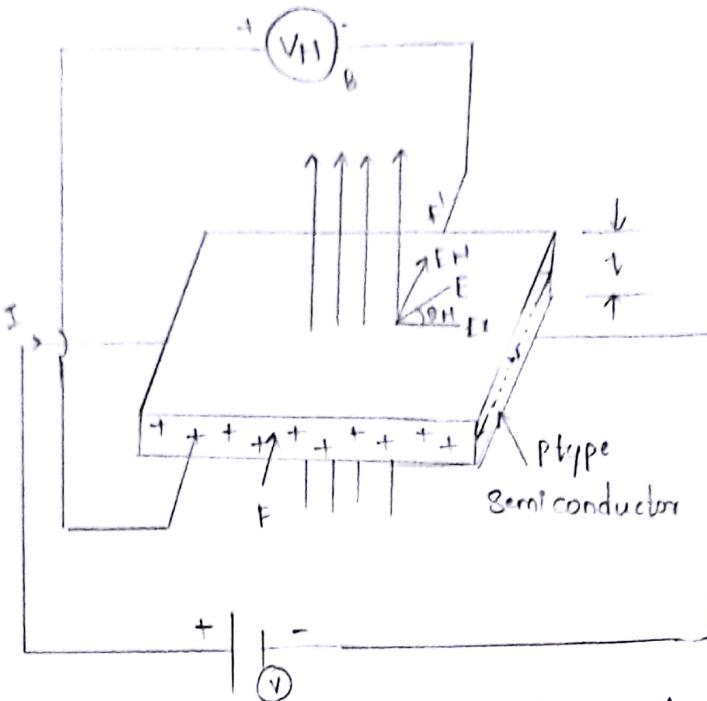


Fig ①b: Generation of hall voltage.

## EXPRESSION FOR HALL COEFFICIENT:

Let us assume that the semiconductor is a p-type semiconductor. Let a potential difference 'V' be applied across its ends. A current of strength  $I$  flows through it along the x-direction (fig ①b) holes are the majority carriers in P-type semiconductors.

$$\text{The current density } J_x = \frac{I}{A} = p e v_d \quad \text{---(1)}$$

where,  $I$  is current through the wafer

$A$  is area of cross section of end face of Semiconductor

$p$  is hole concentration

$e$  is the electrical charge associated with a hole

$v_d$  is the average drift velocity of holes.

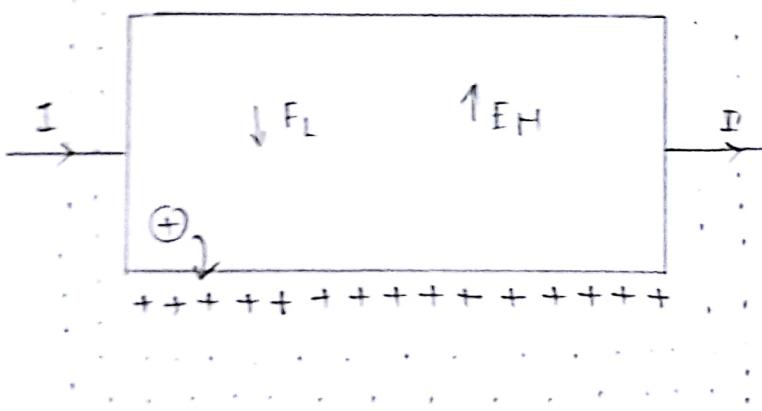
Any plane perpendicular to the current flow direction is

an equipotential surface therefore, the potential difference between the front and rear faces F and F' is zero (Fig 1b)

Now, if a magnetic field 'B' is applied normal to wafer surface and hence through the direction of current flow in it then a transverse potential is produced between faces F and F'. It is known as Hall voltage 'V<sub>H</sub>'.

Without the application of magnetic field holes move parallel to faces F and F' and on the application of magnetic field 'B' the holes experiences the side way deflection due to magnetic force - F<sub>L</sub> (Lorentz force) which is given by

$$F_L = eBvd \quad \text{--- (2)}$$



Fig(2) Top view of wafer:

The directions of magnetic force (F<sub>L</sub>) and hallfield (E<sub>H</sub>) in P-type semi conductor.

Holes are deflected towards the front face 'F' and pile up there. Due to this a corresponding equivalent negative charge is left on the rarer face 'F'. These opposite charges produce a transverse electric field  $E_H$ , whose direction is from front to rarer face. Due to the action of  $E_H$ , holes experience an electric force in addition to the low range force ' $F_L$ '.

In the equilibrium condition the force ' $F_E$ ' due to this transverse electric field balances the Lorentz force  $F_L$

$$\therefore F_E = F_L$$

$$eE_H = eBvd \quad (\because F = eE)$$

$$\therefore E_H = Bvd$$

If 'w' is the width of the semiconductor wafer,

$$E_H = \frac{V_H}{w} \quad (\because E = \frac{V}{d})$$

$$\therefore \frac{V_H}{w} = Bvd \quad \text{--- (3)}$$

From eq(1);

$$v_d = \frac{J_x}{P_e}$$

We can write eq(3) as

$$\therefore \frac{V_H}{w} = B \left( \frac{J_x}{P_e} \right) \quad \text{--- (4)}$$

$$\therefore V_H = \frac{W_B J_A}{P_e}$$

$$V_H = \frac{W_B}{P_e} \left( \frac{1}{t} \right) \quad [\because \text{eqn ①}]$$

$H$  't' is the thickness of the semiconductor wafer plate,  $A = w t$

$$\therefore W = \frac{A}{t}$$

$$\therefore V_H = \frac{B I}{P_e A} \left( \frac{A}{t} \right)$$

$$\boxed{\therefore V_H = \frac{B I}{P_e t}} \quad — (5)$$

Hall coefficient ' $R_H$ ' is defined as hall field per unit current density per unit magnetic field thus  $R_H = \frac{E_H}{J \pi B}$

$$\therefore R_H = \frac{V_H / W}{J \pi B}$$

$$R_H = \frac{1}{J \pi B} \left( \frac{B J_A}{P_e} \right) \quad [\because \text{eq ④}]$$

$$\boxed{\therefore R_H = \frac{1}{P_e}} \quad — (6)$$

Hall voltage ' $V_H$ ' can now be written as,

$$V_H = \frac{B I}{P_e t} \quad [\text{eq 5}]$$

$$V_H = \frac{1}{P_e} \left( \frac{B I}{t} \right)$$

$$\boxed{V_H = R_H \frac{B I}{t}} \quad — (7)$$

$$\therefore R_H = \frac{V_H I}{B I} \quad \text{--- (8)}$$

eq (8) is derived assuming that the P-type semiconductor contains only holes. When the two types of charge carriers taken into account, it is shown that the hall-coefficient

is given by  $R_H = \frac{P nh^2 - n N^2 e}{e (P nh + n Ne)^2} \quad \text{--- (9)}$

from eq (9), we see that the hall coefficient and hall voltage are small for intrinsic materials than for extrinsic materials.

Drift Velocity:

from (5)

$$\frac{V_H}{w} = B v_d$$

$$\therefore V_d = \frac{V_H}{B w} \quad \text{--- (10)}$$

From eq (10) with experimentally measured the hall voltage, known magnetic field 'B' and width of the water, the mean drift velocity of charge carriers can be obtained.

CARRIER CONCENTRATION:

From measured of hall voltage 'V<sub>H</sub>' and current (I)

through the wafer and knowing the strength of magnetic induction and thickness of wafer ( $w$ ). Hall coefficient ( $R_H$ ) can be computed using eq (8). Once  $R_H$  is known the carrier concentration can be determined with the help of eq (6)

$$R_H = \frac{1}{Pe}$$

$$\therefore P = \frac{1}{R_H e}$$

In case of conductors and n-type semiconductors, the expression for hall-coefficient is

$$R_H = -\frac{1}{ne}$$

Therefore, carrier concentration

$$n = -\frac{1}{R_H e}$$

#### DOPING LEVEL:

It is possible to estimate the doping concentration ' $N_A$ ', from the value of ' $P$ ' using the relation  $P \propto N_A$

#### CARRIER SIGN AND TYPE OF SEMICONDUCTOR:

With the direction of magnetic field and current as depicted in fig(1)b, the sign of the hall voltage is positive for an n-type semiconductor the hall voltage will be negative. When the directions of  $B$  and  $I$  are kept same. Therefore, knowing the sign of hall voltage, the type

of the semiconductor and hence the sign of the majority carrier can be determined.

Hall mobility:

$$\text{Mo} : \mu = \frac{V_d}{E}$$

mobility is defined as the drift velocity acquired in unit electric field. We know that the current density  $J$  is given by the following expressions, along  $x$ -axis say  $b$ .

$$J = p e V_d \quad \text{and} \quad J = \sigma E$$

$$\therefore p e V_d = \sigma E$$

$$\therefore \frac{V_d}{E} = \frac{\sigma}{p_e}$$

$$\boxed{\mu_h = \sigma R_H} \quad \left( \because R_H = \frac{1}{p_e} \Rightarrow \mu_h = \frac{V_d}{E} \right)$$

Hall angle:

The net electric field 'E' in the semiconductor is a vector sum of  $E_x$  and  $E_H$ . It acts at an angle ' $\theta_H$ ' to the  $x$ -axis where,  $\theta_H$  is called the hall angle

from fig ① b we have

$$\tan \theta_H = \frac{E_H}{E_x}$$

$$\text{But } E_H = \frac{V_H}{W} = \frac{B_j x}{p_e} \quad (\because \text{eq ⑤})$$

$$\text{Also, } E_x = \frac{J_x}{\sigma}$$

$$E_x = P J_x \quad (\because P = \frac{1}{\sigma})$$

$$\text{Thus, } \tan \theta_H = \left( \frac{B J_x}{P_e} \right) \left( \frac{1}{P J_x} \right)$$

$$\therefore \tan \theta_H = \frac{B}{P_e P}$$

(or)

$$\frac{E_H}{E_x} = \frac{B}{P_e P}$$

$$\text{Using } R_H = \frac{1}{P_e} \text{ and } \frac{1}{P} = \sigma$$

$$\therefore \tan \theta_H = R_H \times B \times \sigma$$

$$\therefore \tan \theta_H = \sigma R_H B$$

or

$$\tan \theta_H = \mu_h B$$

### FACTORS EFFECTING HALL VOLTAGE;

⇒ Hall voltage is directly proportional to the magnetic field strength and current passing through wafer.

⇒ Hall voltage is larger the smaller is the carrier concentration and the thinner is the wafer.

⇒ Hall voltage is larger the smaller is the carrier concentration and thinner is the wafer.

→ The hall voltage is about  $10^5$  greater than in semiconductors (Si or Ge) than in metals.

### VARIATION OF HALL COEFFICIENT:

1. In metals it was found that the hall coefficient does not depends on temperature.
2. In semiconductors, the hall coefficient ( $R_H$ ) decreases sharply with increase in temperature  
It indicates that the carrier concentration of charge carriers in semi conductors increase with increase in temperature.

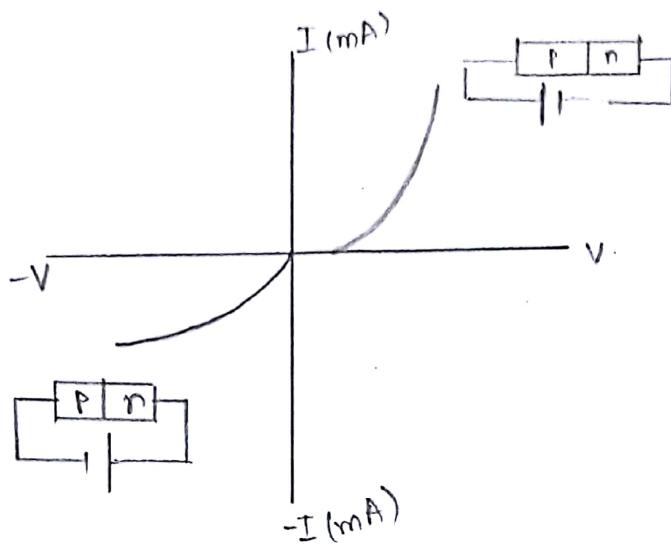
### P-N Junction diode:

A P-n junction is formed when a p-type and n-type semiconductor are joined. A p-njunction diode may be obtained in any of three methods.  
i) Grown junction type  
ii) fused junction type  
iii) diffused junction type.

The p-n junction has a non-linear (fig ①), conduction characteristic and allows current in one direction and opposes it in the opposite direction, p-n junction diodes are widely used in rectifiers which converts ac voltages to dc voltages.

A p-n junction is the basic component of solid state devices and is used as a tunnel diode, vary cap, a gun diode,

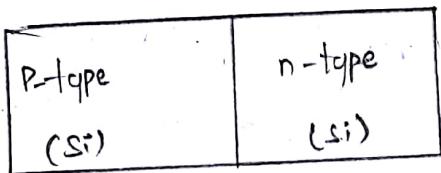
a zener diode , a photo diode, a solar cell, a LED, a laser etc



fig① Non-Linear conduction characteristic  
rectifying property

#### FORMATION OF P-N JUNCTION:

The P-region contains holes as the majority carriers which are produced by the acceptor atoms and very few (thermally generated) minority electrons in p-type. The n-side contains electrons as the majority carriers contributed mainly by the donor atoms and very few (thermally generated) minority holes. Before contact each side is electrically neutral.



(a)

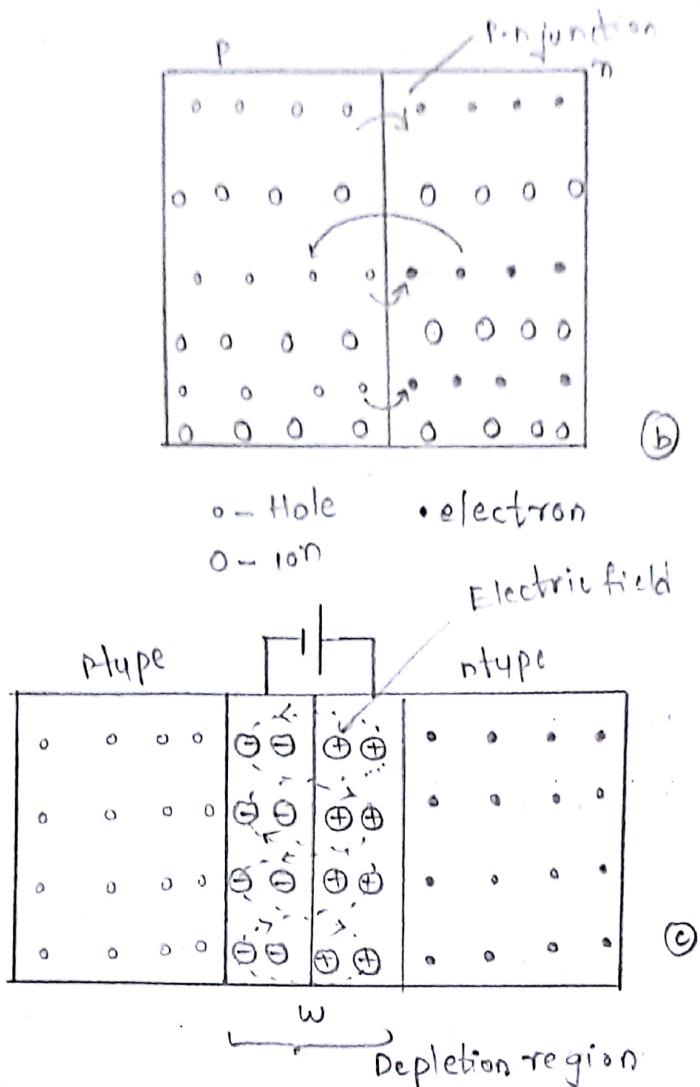


Fig 2 : @ a p-n junction

(b) majority carrier diffusion across junction

(c) formation of depletion region.

When the two sections are brought together because of the concentration difference of charge carriers, holes near to the junction begin to diffuse from p-side to n-side (fig 2b). Similarly, electrons in the n-region are more than the electrons in the p-region. Thus, majority carriers produce electron diffusion current ( $J_{en}$ ) and hole diffusion current ( $J_{hp}$ ) respectively. At the junction the holes and electrons meet each other.

and undergo recombination. The majority holes diffusing out of the P-region leave behind acceptor ions ( $N_A$ ) on P-side of the junction. Similarly, the electrons diffusing from the n-side leave behind the uncompensate positively ionized donor atoms. The double layer of ions around the junction is known as the space charge region.

#### DEPLETION REGION:

The narrow space charge region which is depleted of mobile charges and contains only the immobile uncompensated ions which is about  $1\mu m$  is called depletion region. The array of fixed ions produces electric field ' $E$ ' which is directed from donor ions on n-side towards the acceptor ions on p-side. The space charge layers reduce the diffusion of holes and electrons. The holes leaving p-region must overcome the repulsion of the positive ion layer on n-side of the junction to move further. Similarly, the electrons diffusing from the n-side must overcome the repulsion of the negative ion layer on p-side of the junction. The electric field establishes an equilibrium potential difference  $V_0$  across the depletion region and is known as the internal potential barrier. The net diffusion current density flowing across the junction is

$$J_{(diff)} = J_{hp} - J_{en}$$

## DIFFER OF MINORITY CARRIERS

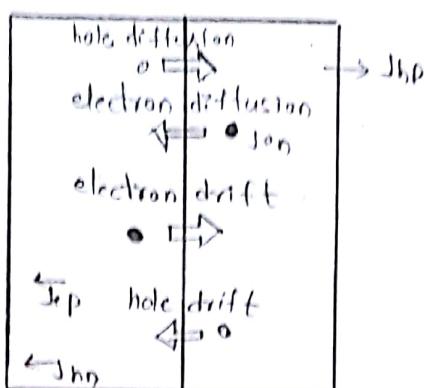
Space charge has the right direction for the flow of minority carriers across the junction and is called as drift current. The net drift current through the junction is,

$$J_{\text{drift}} = \text{hole drift current density } (J_{hp}) + \text{electron drift current density } (J_{ep}) \\ = J_{hp} + J_{ep}$$

At thermal equilibrium

$$J_{\text{diff}} = J_{\text{drift}}$$

$$J_{hp} + J_{en} = J_{hp} + J_{ep}$$



Fig② Thermal equilibrium

## THE ENERGY BAND DIAGRAM:

The energy band diagram of the individual P and n-type semiconductors are shown in (fig 4(a)). Note that the Fermilevel  $E_{FP}$  and  $E_{Fn}$  are at different levels. Fig(5) shows the energy band diagram of a P-n junction in equilibrium condition each side takes up at different electrostatic potential.

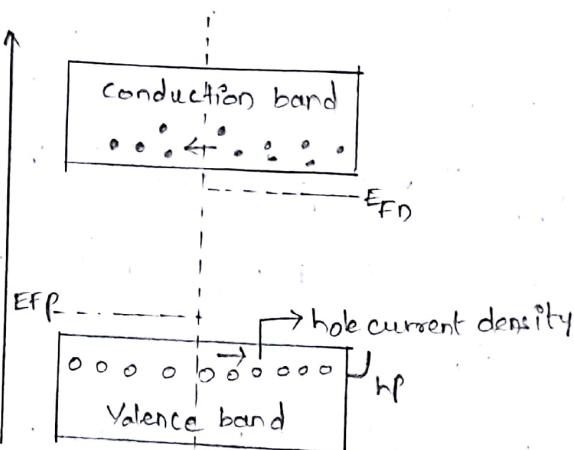
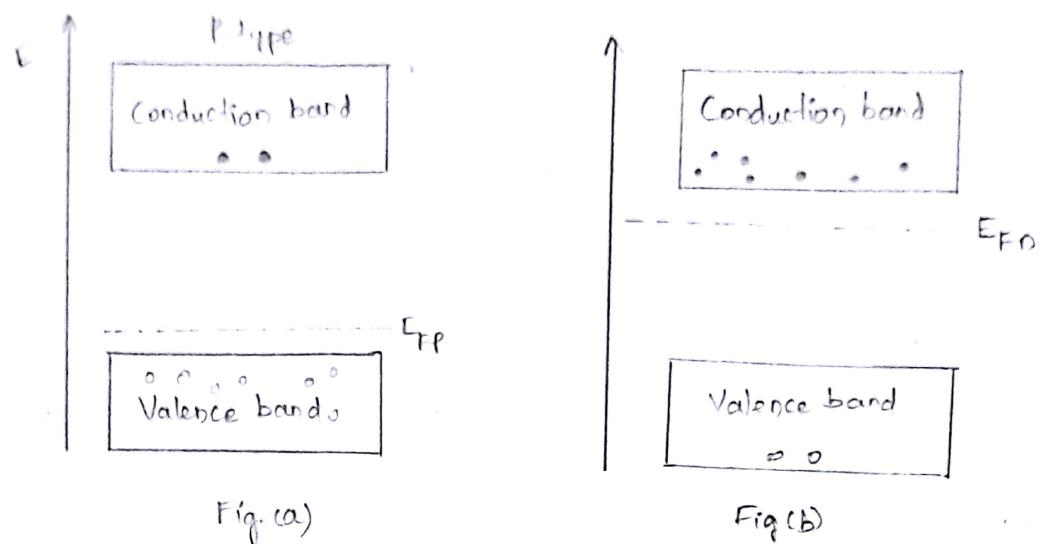
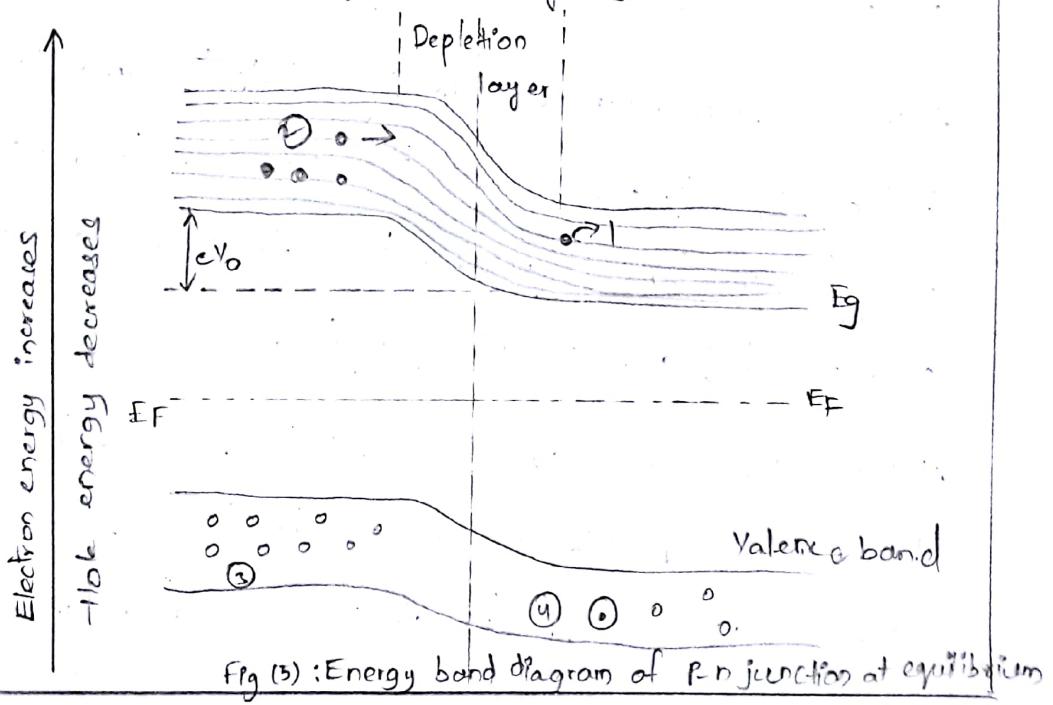


Fig.(4): The energy band diagrams



It results a potential barrier ' $V_0$ ' or an energy hill of height ' $eV_0$ '. Electrons in the conduction band of n-region face an energy hill, namely conduction hill. ~~appro~~ Electrons approaching the junction region cannot pass conduction hill unless they have a minimum energy of  $eV_0$ .

On the otherhand , the electrons near the junction in p-region can roll down. The conduction hill effortlessly and pass into n-region.

As the direction of increasing energy is downward for holes , the holes in the valency band of p-region encounter an energy hill namely valency hill. The holes on the p-side cannot go into the n-region unless they have a minimum energy of  $eV_0$ .

On the otherhand , holes near the junction on the n-side can readily float up the hill irrespective of their energy.

i) The electron marked ① in fig 5 fails to climb the conduction hill occationally , a few of the electrons that have kinetic energy equal to or greater than  $eV_0$  overcome the conduction hill and go into p-region.

ii) The electron marked ② can roll down easily the conduction hill . The components of current due to such migrations of electrons are in opposite directions and balance each other.

iii) The hole marked ⑤ in fig(5) fails to surmount (get ontop) the valency hill. Whereas, a few holes having kinetic energy equal to greater than  $eV_0$ , succeed in going into n-region.

### INTERNAL POTENTIAL BARRIER $V_0$ :

The magnitude of the potential barrier ' $V_0$ ' can be estimated from the knowledge of the electron concentration in P and n-region of the diode.

$$V_0 = \frac{V_T \ln N_p N_A}{n_i^2}$$

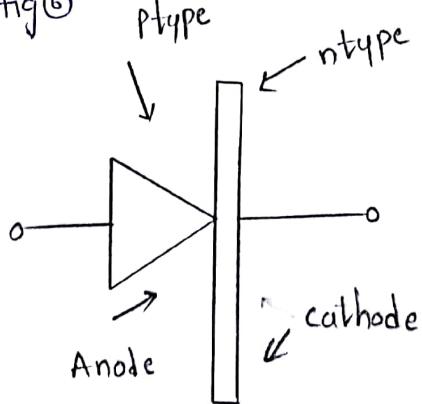
where  $V_T = \frac{kT}{e}$

$N_p$  = the electron concentration in the conduction band on the n-side.

$N_A$  = the electron concentration on p-side

### CIRCUIT - SYMBOL:

A p-n junction diode is schematically represented by the symbol shown in fig ⑥



Conventional current direction in forward bias condition

fig(6) Circuit symbol of p-n diode

**BIAS:**

When a dc voltage is applied to a device, the device is said to be biased. The p-n junction can be biased in two ways:

i) forward bias

ii) Reverse bias

**FORWARD BIAS:**

If the positive terminal of the source is connected to the p-region and negative terminal to the n-region, then the junction is said to be forward biased.

**REVERSE BIAS:**

The junction is said to be reverse biased when the positive terminal of the external source is connected to the n-region and the negative terminal to the p-region.

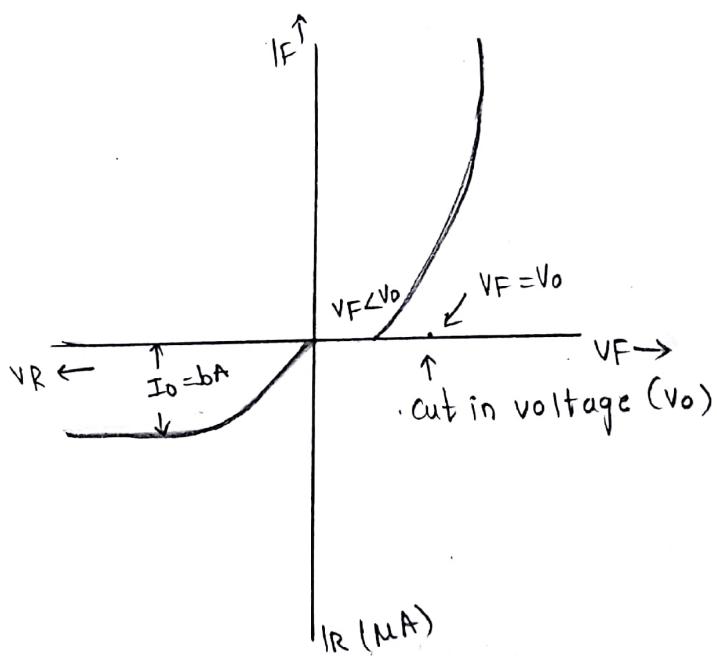
**VOLTAGE - AMPERE (V-I) CHARACTERISTICS:**

fig1 : Voltage - Ampere characteristics

A graph that shows the variation in current in a device with the variation of voltage applied across it is called voltage ampere (V-I) characteristics.

Above figure shows voltage-ampere characteristics of p-n junction diode. It is seen that the characteristics is non-linear device. In forward bias it acts as a closed switch and allows large current, in reverse bias it act as an open switch and low current passing through it.

For a forward bias voltage  $V_F < V_0$ , current through the junction is negligibly small the voltage at  $V_0$  at which the current raises sharply is called the cut-in voltage ( $V_0$ ).

As  $V_F$  increases, beyond  $V_0$  the barrier disappears and an exponential increase in forward current ' $I_F$ ' occurs. The forward current is equal to the difference between the diffusion current and drift current.

$$\therefore I_F = I_{\text{diff}} - I_{\text{drift}}$$

When  $V_F$  exceeds  $V_0$ , the barrier collapsed and the drift current becomes nearly zero. However, the barrier does not totally vanished. The reverse current through the junction is very much smaller than the forward current. The reverse current ' $I_R$ ' is given by

$$I_R = I_{\text{drift}} - I_{\text{diff}}$$

In the initial stage the reverse current increases rapidly

due to an exponential decrease of diffusion current with increase in voltage. At higher 'V<sub>R</sub>' values, the diffusion current drops to zero and the reverse current becomes equal to drift current due to the minority carriers. The drift current due to minority carriers is known as reverse saturation current. It is denoted by I<sub>0</sub>. It is very small as the number of minority carriers very small.

#### REVERSE BREAK DOWN:

When the reverse bias voltage is increased steadily the junction breaks down at a particular stage and starts conducting heavily. The critical value of voltage is called the breakdown voltage.

Once the breakdown occurs, even a small increase in voltage causes large change in the reverse current. The breakdown occurs in reverse bias diode only mainly due to two mechanisms. They are

- i) Avalanche breakdown
- ii) Zener breakdown.

#### AVALANCHE BREAKDOWN:

In lightly doped junctions on large reverse voltage across the junction avalanche breakdown occurs. When large reverse bias is applied to the diode, minority electrons acquire high kinetic energy and knocks off (pull out) valency electrons from

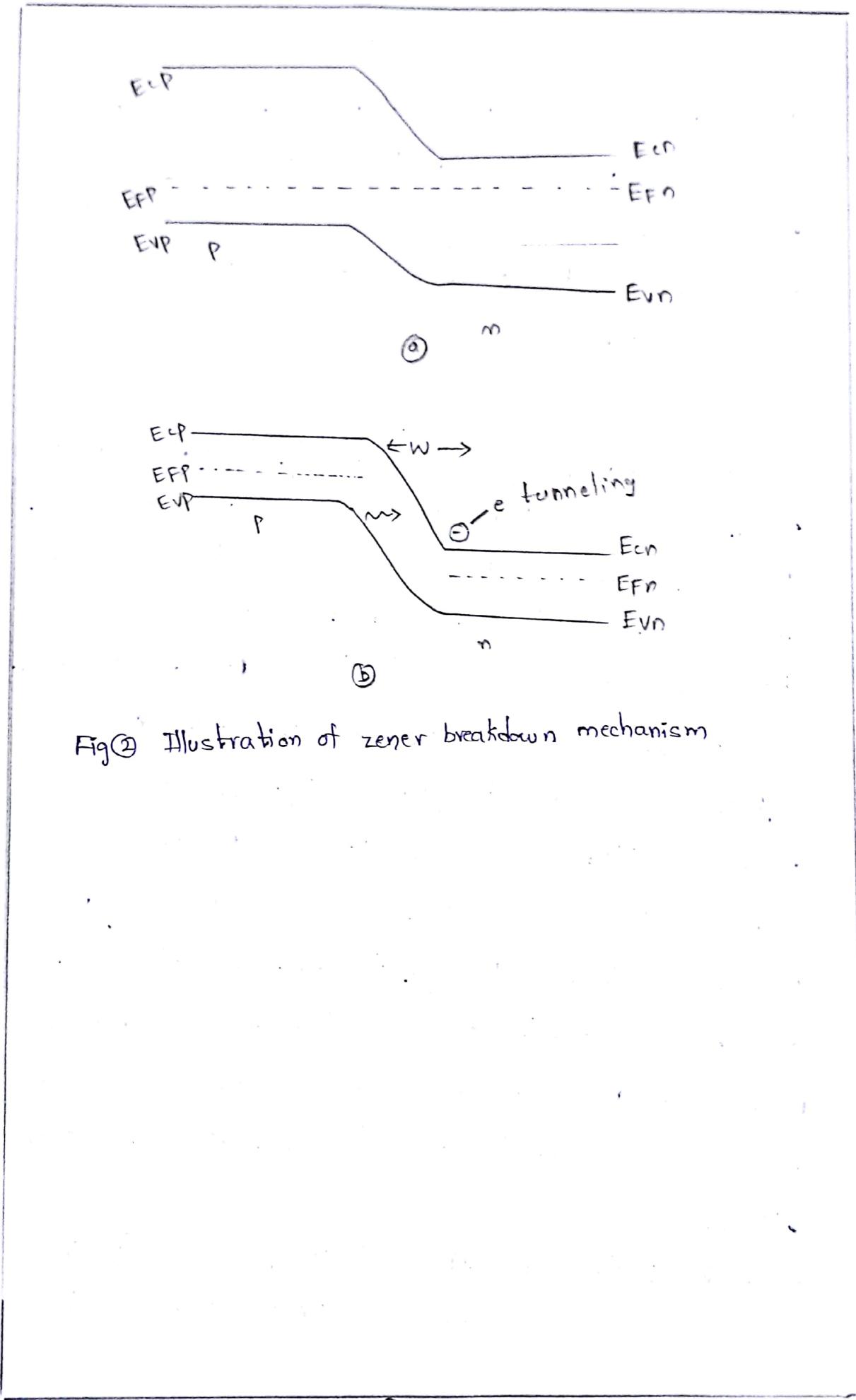
some of the covalent bonds. These new carriers produce additional charge carriers and the process continues. A flood of carriers is produced in a very short time and causes large reverse current. This is known as Avalanche breakdown. The Breakdown voltage is found to increase with temperature.

#### ZENER TEMPERATURE:

When p and n regions are heavily doped, the depletion region becomes very thin, of the order of 10 nanometers. Application of reverse bias voltage causes the conduction and valency band to bend to extend that the n-side conduction band appears opposite to the p-side valency band. As a result, a number of vacant states in the n-side conduction band are brought directly opposite to large number of filled states in p-side valency band.

As the barrier separating the two bands is very narrow, the electrons tunnel through it under the effect of a small reverse bias. The electrons from the valency band on p-side reach the conduction band on n-side, thus a sudden large reverse current from n-top side. This is the zener effect.

~~zener effect~~ Zener breakdown takes place usually at low reverse voltages of order of 4V or less in silicon devices



Fig(2) Illustration of zener breakdown mechanism

## ZENER DIODE:

Zener diode is a semiconductor specially designed to operate in the breakdown region of the reverse bias. Zener diodes are always operated in reverse bias condition. By varying the impurity concentration and other parameters, it is possible to design the breakdown voltage to suit specific applications. In zener diodes, the breakdown phenomenon is reversible and harmless.

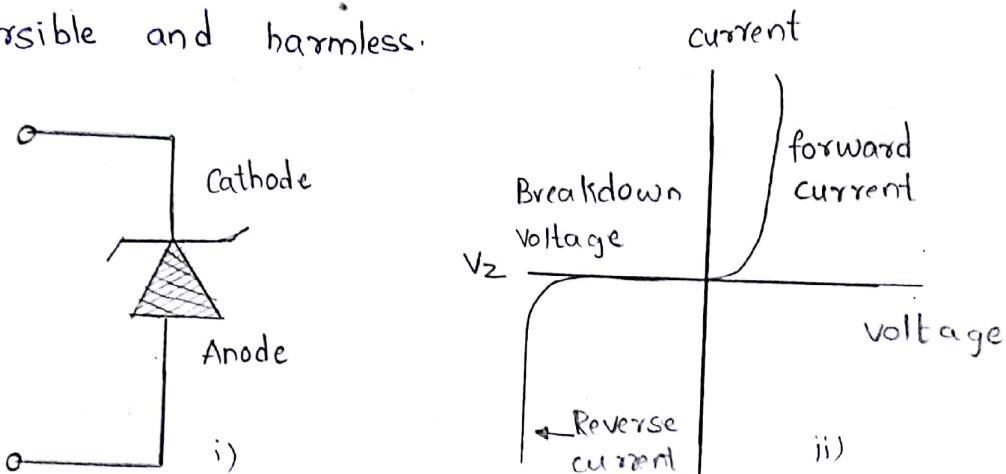


Fig 1(a) Zener diode  
 i) Symbol  
 ii) V-I characteristics

Fig 1(a)i) Shows the symbol of a zener diode. It may be seen that it is like that of an ordinary diode except the bar is turned into z-shape. The volt-ampere characteristic of the zener diode is shown in fig 1(a)

ii) Zener diode acts very much similar to ordinary diode under forward bias condition. However, the zener diode is generally not used in the forward bias condition. The

The reverse bias characteristic is very much different from that of an ordinary diode. As the reverse voltage is increased, the reverse current remains constant till a certain value is reached. At that value, the reverse current increases abruptly. The voltage at which such sudden increase in reverse current occurs is called zenerbreakdown voltage or zener voltage,  $V_z$ . If a reverse current above breakdown value is allowed the diode is permanently damaged. Zener diodes are designed to lower zener voltage such as 2.4V. In the zener region the voltage across zener diode remains constant but the current changes depending on the supply voltage. The voltage drop across the zener diode is equal to the zener voltage of that diode. Doping levels decides the position of zener region, an increase in doping will decrease the zener potential. Zener diodes are available in the range from 2V to 200V.

#### APPLICATIONS:

Zener diodes are widely used in electronic circuits. Some of the important uses are in

- i) Voltage regulators
- ii) reference element
- iii) meter protection.

## BIPOLAR JUNCTION TRANSISTOR.

The transistor was invented in 1947 by the american Physicists John Bardeen, Walter Brattain, and William Shockley at Bell telephone laboratories.

### Construction:

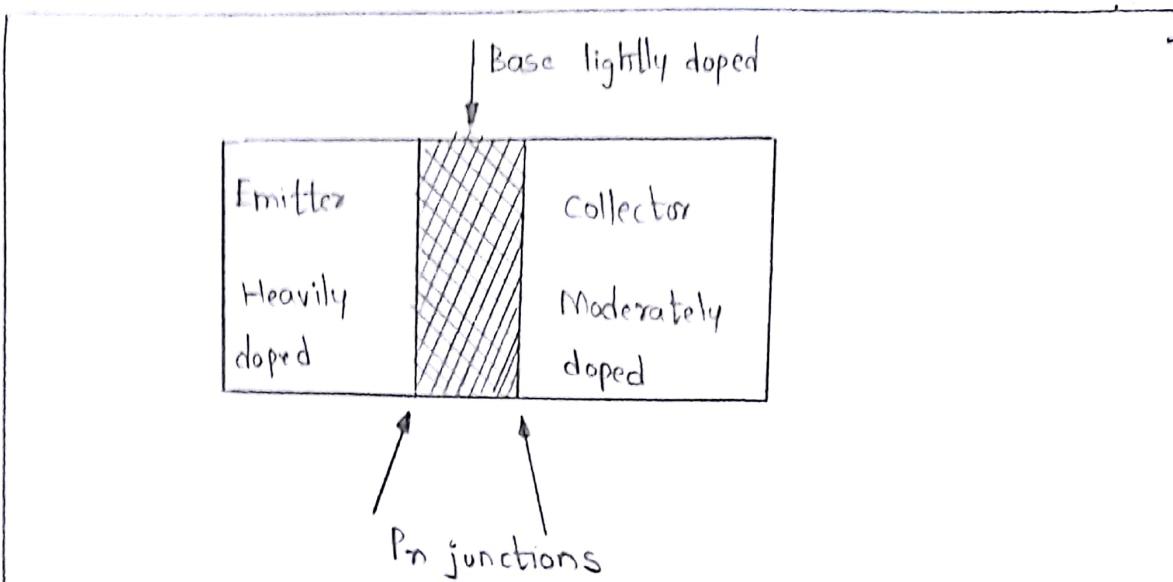
A transistor is a semiconductor device consisting of three regions separated by two distinct p-n junctions. The central region is called the base. It may be a p-type or n-type semiconductor. The two outer regions are called emitter and collector. They are of the same type extrinsic Semiconductor but different from that of base. Thus, if the base is p-type the emitter and collector are n-type and if the base is n-type the emitter and collector are p-type. Two types of transistors are available, as shown in fig 1(a). They are called npn and pnp transistors.

### npn transistor:

It is constructed using n-type material as the emitter and collector while the base is made of p-type material.

### PnP transistor:

It is constructed using p-type material as the emitter and collector while the base is made of n-type.



Fig(2(a)) Transistor junctions

The n-region contains free electrons which are negative charge carriers and p-region contains mobile holes which are positive carriers. Thus, two types of charge carriers, namely electrons and holes, are involved in current flow through an npn or pnp transistor. Therefore, these transistors are known as bipolar junction transistors. The function of each element is as follows.

- i) Emitter: It provides the majority carriers necessary to support current flow.
- ii) Base: It controls the flow of the majority carriers within all elements of the transistor.
- iii) Collector: It supports the majority of the current flow in the transistor.

Schematic Representation: The schematic symbols of npn and pnp transistors are shown in fig. 2(b)

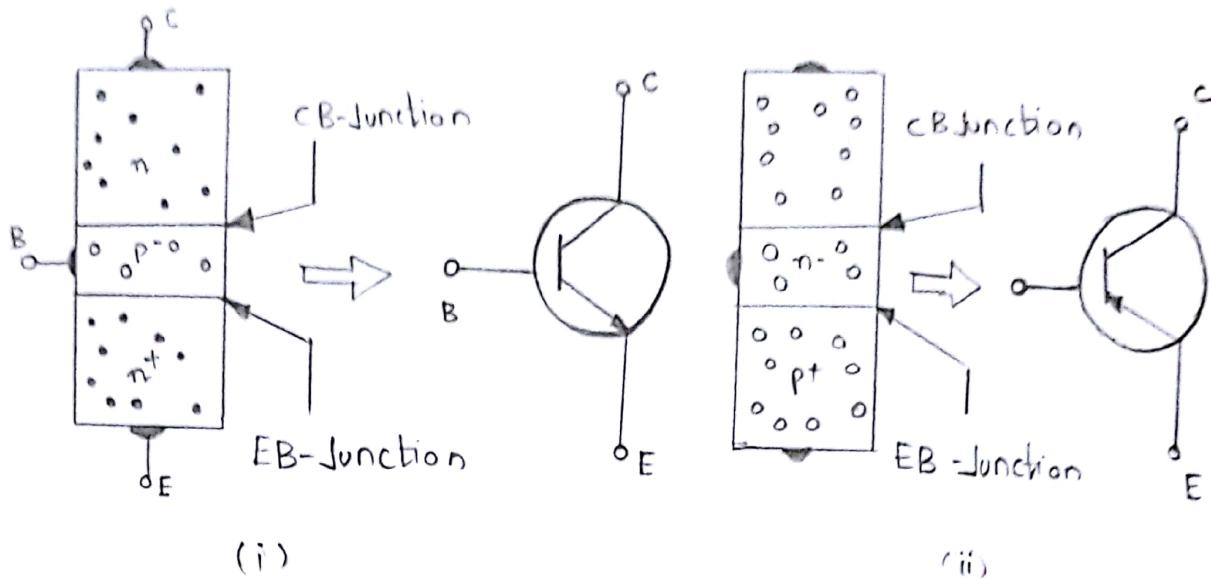


Fig 2(b) Circuit symbol of transistor i) n-p-n transistor

ii) p-n-p transistor . In the symbols , the emitter is always indicated by an arrow head

npn.... n outside , arrow outward

pnp... n inside , arrow inward

This arrowhead serves to tell us three things:

- i) location of the emitter
- ii) type of the transistor that is being represented
- iii) direction in which the conventional current flows.

#### FORMATION OF DEPLETION REGIONS:

Each transistor has two p-n junctions. The junction that separates the base and the emitter is called the emitter-base (EB) junction and the one separating the base

and the collector-base ((B) junction. Each transistor is actually one piece of crystalline material that has been doped to create the three elements.

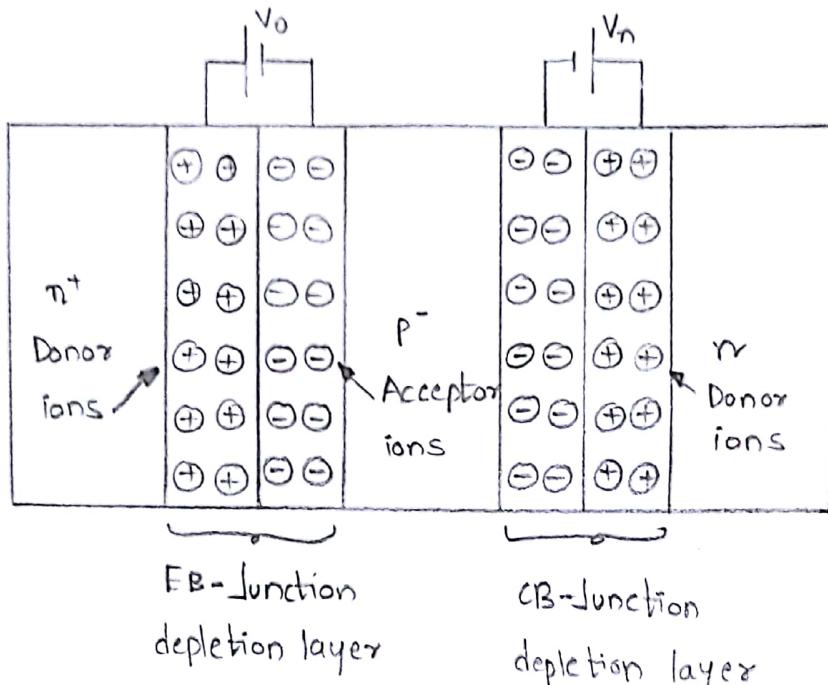


Fig: ① Formation of Depletion Regions

During the process of formation of junctions, diffusion of majority carriers takes place and depletion layers from fig ①. As the doping levels in the three regions are different, the two depletion layers form with different widths. Because the emitter is heavily doped and the base is lightly doped, results in a narrow depletion layer at EB-junction and a wide depletion layer at CB-junction. The base region becomes thinner compared to its actual physical dimension, as the two depletion layers encroach on it. The built-in barrier voltages across the two depletion layers are the same and will be of the order of

0.7V in case of silicon transistor.

The two p-n junctions can be viewed as two diodes. Therefore, a transistor may be regarded as two p-n junction diodes arranged back-to-back with the base being common to both the diodes. As both the diodes have the base is common, they influence each other strongly.

### ROLES OF EMITTER

Roles of emitter, Base and collector: i) Base: The base current is minimized through the following steps in an npn transistor.

a) Base region is lightly doped:

If base region is heavily doped, more holes would be present in the base and the incoming electrons would have more chance of undergoing recombination. The number of electrons flowing into collector would have thus decreased. It leads to lesser collector current and more base current. To reduce this possibility, base region is lightly doped.

b) Base region is made narrow:

It enables the electrons injected into base to quickly diffuse and come under the action of electric field due to reverse bias across junction, which sweeps them into the collector. Thus, the chance of electrons recombining with holes and causing a base current is precluded.

ii) Emitter is heavily doped.

In a transistor, the emitter is the source of current. It is required that a maximum of the majority carriers is injected into the base so that the emitter current ie will be large. The function of the emitter is to provide charge carriers in large number. Hence emitter is heavily doped compared to base and collector.

iii) Collector is wider.

Collector current is produced by minority carriers. Current by minority carriers is a drift current and requires only the presence of electric field acting in a favourable direction. Whatever may be the strength of the electric field, minority carriers are accelerated into the collector region. The minority carriers are in fact rolling down the barrier. Whether the barrier is high or low it does not matter for rolling down it. Therefore whether the reverse bias (collector voltage) is large or small it does not influence the strength of the collector current.

The minority carriers rolling down the high potential barrier acquire large kinetic energy. They produce large amount of heat while transferring part of their energy to the lattice through collisions. In order to

dissipate away the heat, the collector region is made larger.

Principle of Operation - Transistor Action:

Let the emitter-base junction is forward biased as shown in fig 2(a) Forward bias at the E-B junction lowers the potential barrier. Majority carriers, electrons in the n-type emitter, are injected in large number across the junction. As p-type base region is doped lightly, electron current becomes much larger than the hole current in the base region. Therefore, the emitter current is due to electrons flowing from emitter to base. The emitter current  $I_E$  is the sum of electron and hole currents. The ratio of the electron current to the total emitter current  $I_e/I_E$  is known as emitter injection ratio,  $\gamma$ ,  $\gamma$  is typically of the order of 0.995. It means that only 0.5% of  $I_E$  consists of hole current.

Under forward bias, an intense injection of electrons into base region takes place and as a result the electron concentration in the base region nearer to EB-junction steeply rises to a value many times higher than the equilibrium value. Because of reverse bias at the CB-junction, the electron concentration in the base region nearer to

CB-junction is practically zero. Therefore a large concentration gradient is established for electrons in the base region.

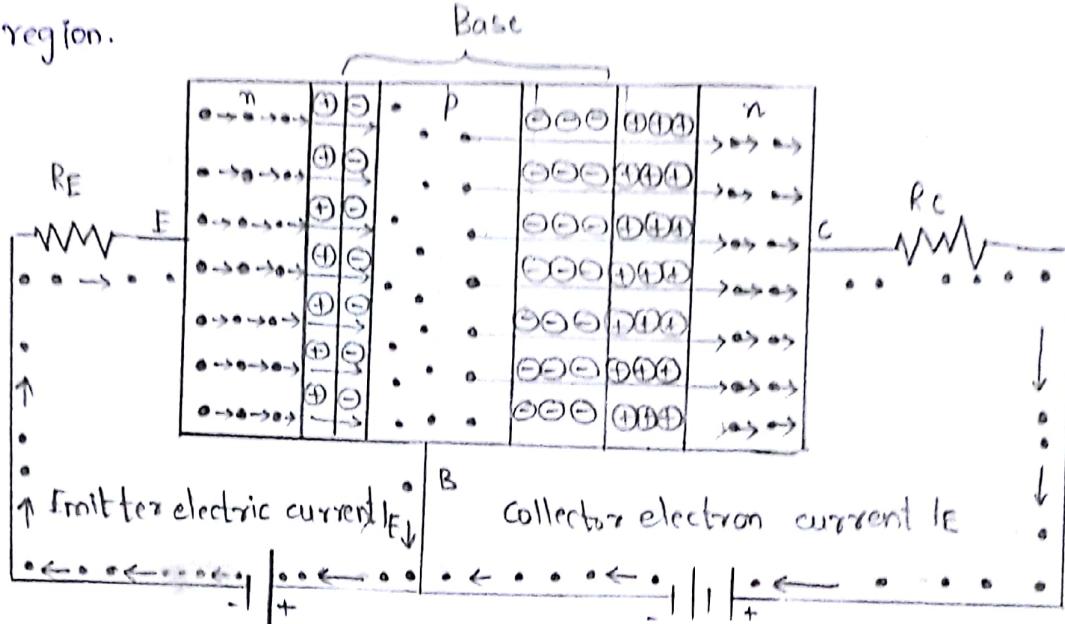


Fig 2(a) Action of the Bias on NPN transistor

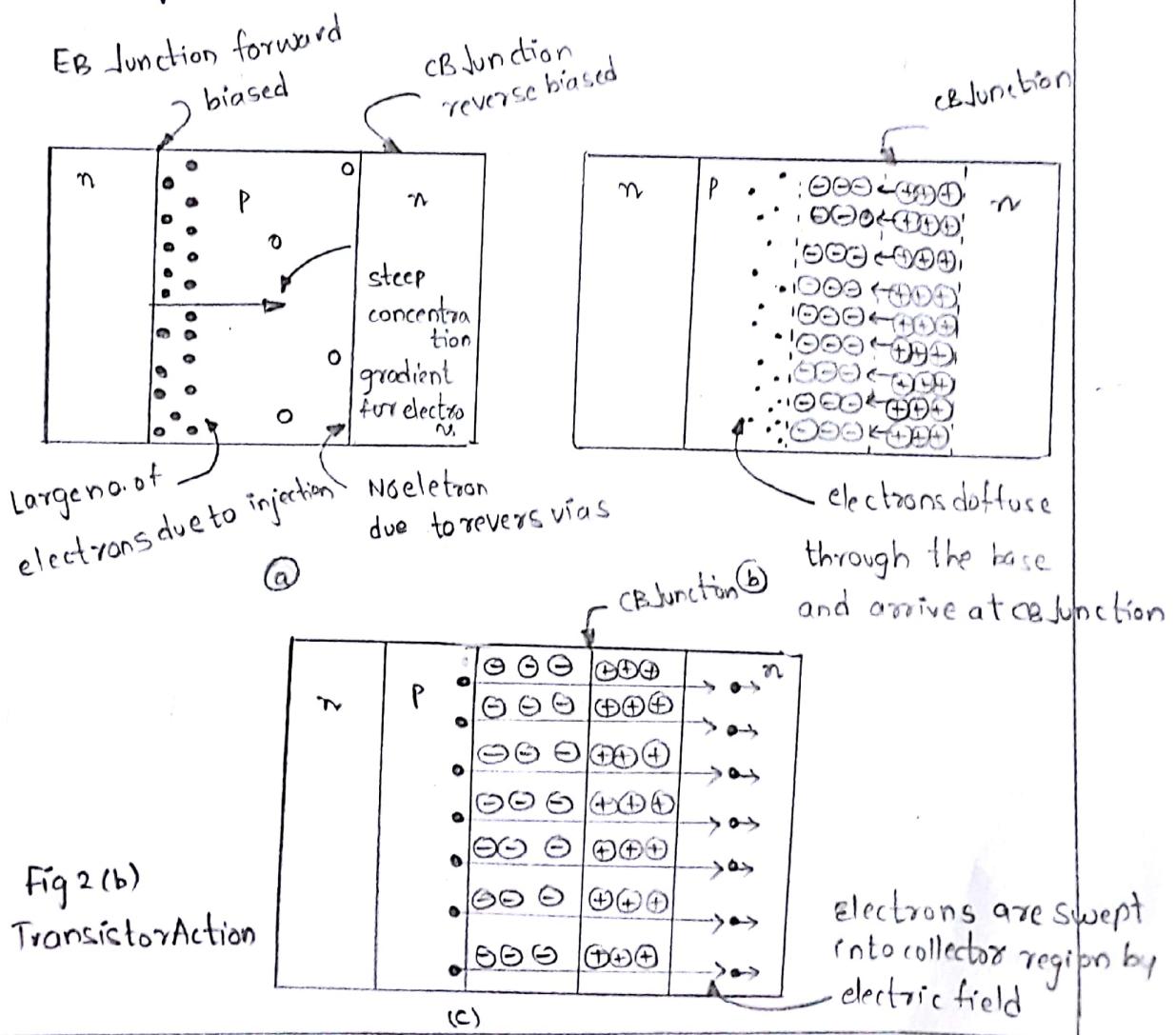


Fig 2(b)  
Transistor Action

Now the electrons in the base region have two options.

i) One is that they may recombine with the holes in the base causing large base current. But due to light doping of the base region, sufficient number of holes is not available; recombination cannot take place in large way and the base current  $I_B$  is very small.

ii) As electron concentration is very high on the emitter side and zero on the collector side of the base region, the possibility is that electrons swiftly diffuse towards the collector-base junction under the influence of the concentration gradient across the base. The base region is narrow originally and is made further narrower due to the encroachment of depletion layers into the base and due to the action of the biases applied. Owing to this electrons quickly reach the CB-Junction.

Once they arrive in the vicinity of the junction they will be acted upon by the strong electric field due to reverse bias and get swept electrons emitted by the emitter flow into the collector. It causes a large reverse current  $I_C$  which is nearly equal to  $I_E$  to flow across CB-Junction.

A small base current  $I_B$  is caused by the few electrons that undergo recombination in the base. The emitter-base junction is forward biased and therefore, it has a low resistance. The collector base junction is reverse

biased and has a high resistance. Almost the same current flows through the two junctions. Thus, the current is transferred from a lower resistance to high resistance level. Hence, the device is called trans-resistor, which is shortened to transistor.

The principle particle flows in an npn transistor biased in active mode are shown in fig 2(c)

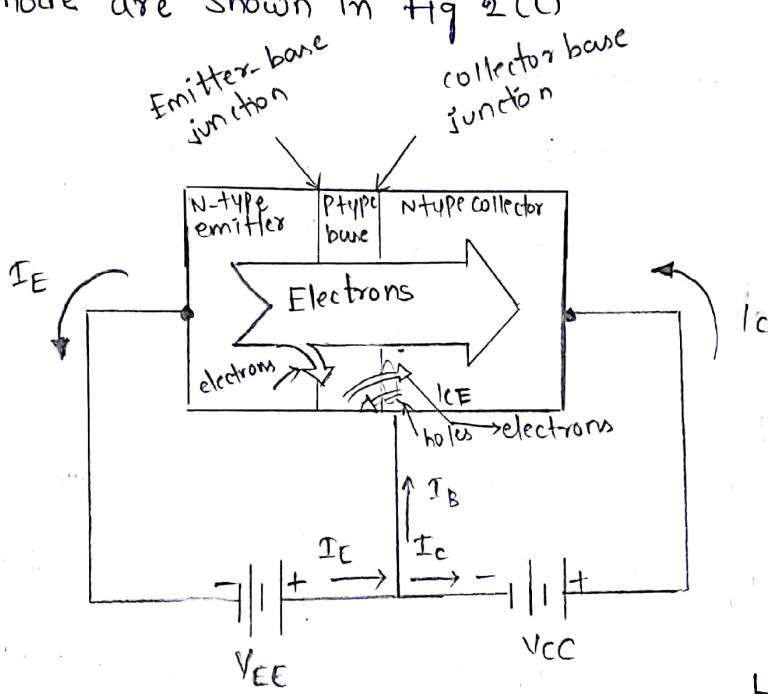
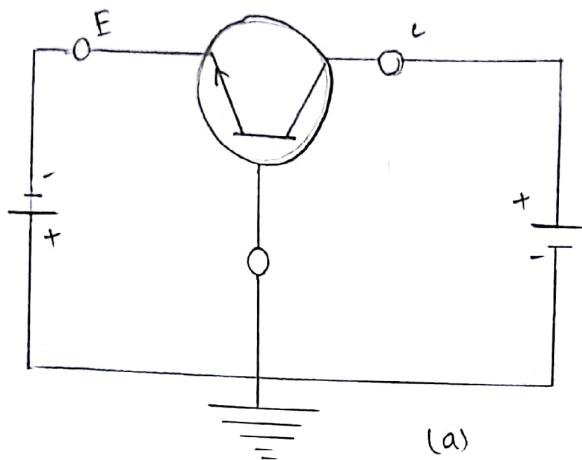


fig 2(c) Principal particle flows when an n-p-n transistor is biased in the normal mode

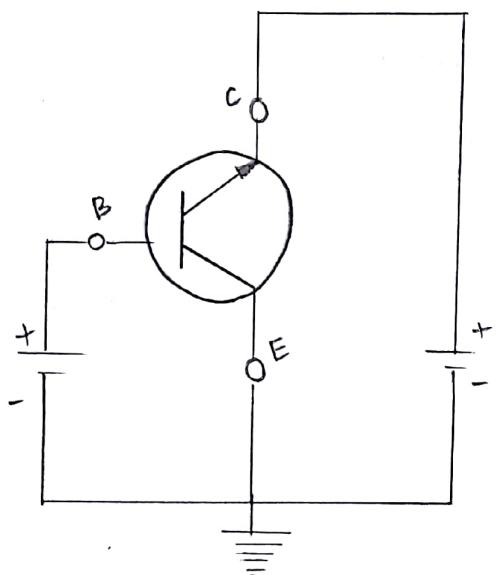
#### CIRCUIT CONFIGURATIONS:

A transistor is a three-terminal device. There are three possible ways in which it may be connected into a circuit. They are known as circuit configurations. When the transistor is connected with its base terminal common to both the EB-Junction and CB Junction, the configuration is known as

common-base (cB) configuration. The other configurations are known as common emitter and common collector configurations. The circuit configurations for an npn transistor are shown in fig 3.



(a)



(b)

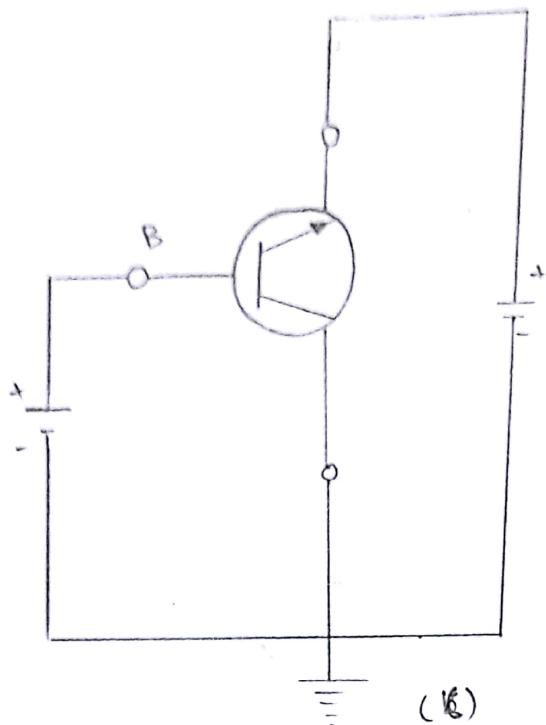


Fig (3)

The three different ways of connecting a transistor

- a) Common - base configuration
- b) Common - emitter configuration
- c) Common - collector configuration.

## UNIT - III

### Optoelectronics

#### Radiative and Non-radiative recombination Mechanisms in Semiconductors:

In generation process, a covalent bond is broken and an electron-hole pair produced. In Recombination process a free electron transforms into a Valence electron and that a captured covalent bond is re-bridged.

Recombination reversed process of generation whereby charge carriers, electrons and holes are destroyed. There are two types of recombination, they are,

- (1) Radiative recombination associated with photons
- (2) Non-radiative recombination associated with lattice vibration (or) phonon,

Radiative recombination occurs when an electron in the conduction band recombines with a hole in the valence band and the excess energy is emitted in the form of photon. Optical processes associated with radiative transitions such as absorption, spontaneous emission and stimulated emission,

Non radiative recombination occurs when an electron in the conduction band recombines with holes in the valence band and the excess energy is emitted in the form of heat in the semiconductor crystal lattice. Non-radiative recombination affects the performance by increasing threshold current and heterojunction carrier leakage.

### Light Emitting Diode (LED):

The light emitting diode (LED) is a solid-state light source. It is an optoelectric device in which a forward biased P-n junction emits light i.e. it converts electrical energy into light energy. The amount of light output is directly proportional to the forward current. A LED is fabricated using III-V compound semiconductors, such as GaAs, which have a direct band gap.

Injection electroluminescence is the principle of LED, when a P-n junction is forward biased, minority carriers flow in large numbers into regions where they can recombine with majority carriers producing light in the visible or infrared region. The wavelength of light is given by,

$$\lambda = \frac{hc}{E_g} = \frac{1.24}{E_g(\text{eV})} \mu\text{m.} \quad (2)$$

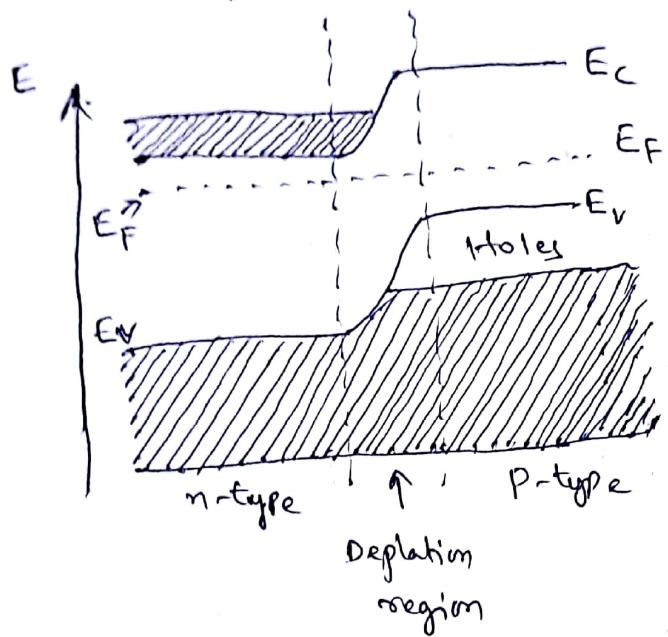
This effect is known as injection electroluminescence. As the p and n regions are heavily doped, a significant output is obtained due to a large number of electron-hole recombinations occurring per second. To get the visible light of wavelength lying in the region  $0.38$  to  $0.78\mu m$ , a semi-conductors should have  $E_g$  between  $1.7\text{eV}$  to  $3\text{eV}$ .

Def:- The light emitting diode (LED) emits photons as visible light. Its purpose is for indication and other intelligible displays. Various impurities are added during the doping process to vary the color output.

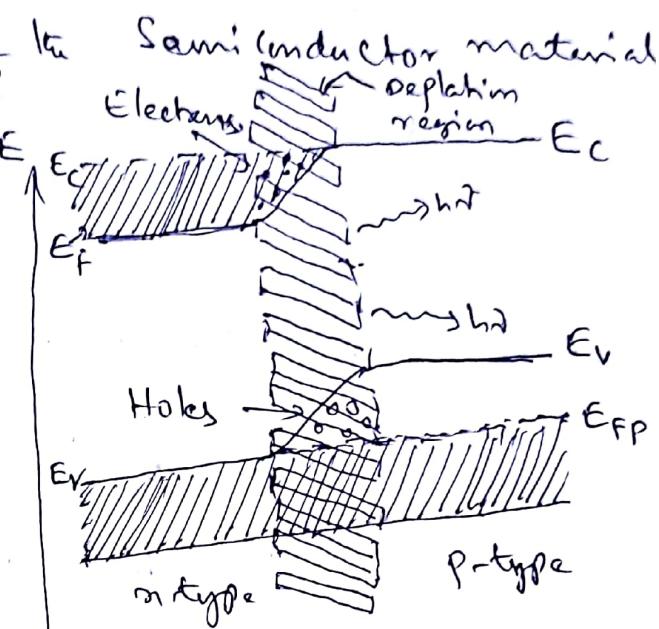
Theory:- Fig 1(a) Shows the energy band diagram of a heavily doped p-n junction. The p-n junction has a large concentration of electrons in the conduction band of n-region and a large concentration of holes in the valence band of p-region. Under forward bias conditions, the electrons move forward into the depletion region and occupy energy levels in the conduction band.

Similarly, holes push forward into the depletion region and occupy energy levels in the valence band. The electrons in the conduction band are directly above the holes at the

edge of the Valence band (Fig 1 (b)). When an electron from the conduction band jumps into the hole in the Valence band, recombination occurs and the excess energy is emitted in the form of a light photon. The colour of the emitted light depends on the type of the Semiconductor material.



1 @



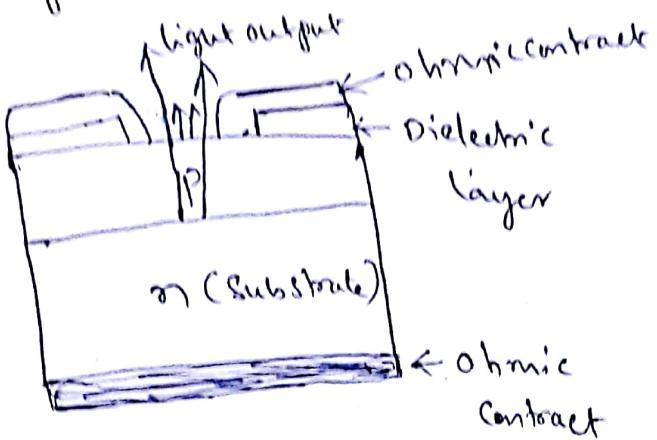
1 (b)

Fig: Energy band diagram of an LED

(a) Without bias (b) Under forward bias.

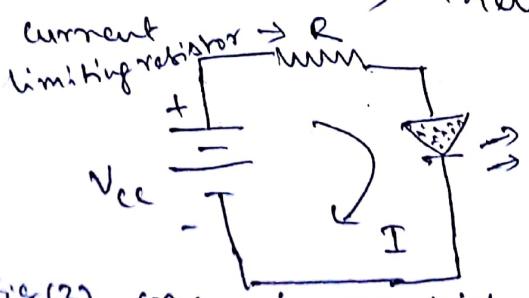
Construction :- The construction of a surface emitting LED is shown in fig ②. These LEDs emit light in a direction perpendicular to the p-n junction plane. An n-type layer is grown on a substrate and a p-type layer is grown on it by the process of diffusion. The p-layer is made very thin to prevent loss of photons due to absorption in the layer. Metal connections are made at the edges of the p-layer in

Order to allow more central surface for the light to escape. A metal film is deposited at the bottom of the substrate for reflecting as much light as possible towards the surface of the device and also provide electrode connection.

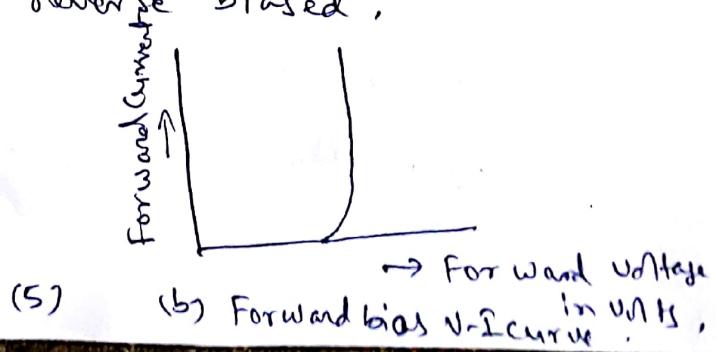


Fig(2): LED Construction.

Working:- The circuit symbol of LED and a simple circuit to illustrate the Working of an LED are shown in fig(2). LED is always forward biased, The forward Voltage across an LED is significantly greater than an Ordinary diode, it is in between 1.2V and 3.2V. The amount of light emitted is directly proportional to the forward current (Fig(3b)) The reverse breakdown voltage of LED is of the order of 3V and an LED is never reverse biased.



Fig(3) (a) LED in FB bias



(b) Forward bias V-I curve

## Applications

- i, The LEDs operate at low voltage i.e from 1.5V to 2.5V
- ii, They have a long life of about 10,000 hours and can be switched ON and OFF at a very fast speed (1ns) (max)
- iii, LEDs are used in 7 segment, 16 segment and dot matrix displays which are used to indicate alphanumeric characters and symbols in digital clock, metric characters and symbols in digital clock, microwave ovens, stereo turners, calculators etc.
- iv, They are used for indicating ON/OFF conditions for solid state video displays, which are rapidly replacing CRTs. IR LEDs are used in burglar alarm systems

## Semiconductor Laser:-

A Semiconductor diode laser is a specifically made P-N junction diode that emits coherent light under forward bias. R.N. Hall and his co-workers made the first Semiconductor laser in 1962. P-N junction lasers are emits light almost anywhere in the spectrum from UV to IR

## ② Semiconductor Materials :-

- i, Semiconductors are two types, different groups, direct band gap Semiconductors and indirect band gap Semiconductors,
- ii, Direct band gap Semiconductors are formed by group III-V elements and group IV-VI elements. Most of the compound Semiconductors belong to this group.
- iii, Lasers are made using direct band gap Semiconductors. Gallium Arsenide (GaAs) diode is an example of semiconductor diode laser.
- iv, Direct band gap Semiconductor is the one in which a conduction band electron can recombine directly with a hole in the valence band. The recombination process leads to emission of light.

③ Principle :- i, The energy band structure of a Semiconductor consists of a Valence band and a Conduction band Separated by an energy gap Eg. The Conduction band Contains and the Valence band Contains holes and electrons,

ii, When an electron from the Conduction band jumps into a hole in the Valence band, the excess energy  $E_g$  is given out in the form of a photon,

iii, Electron - hole recombination is the basic mechanism responsible for emission of light.

iv, The wavelength of the light is given by the relation  $\lambda = hc/E_g$ .

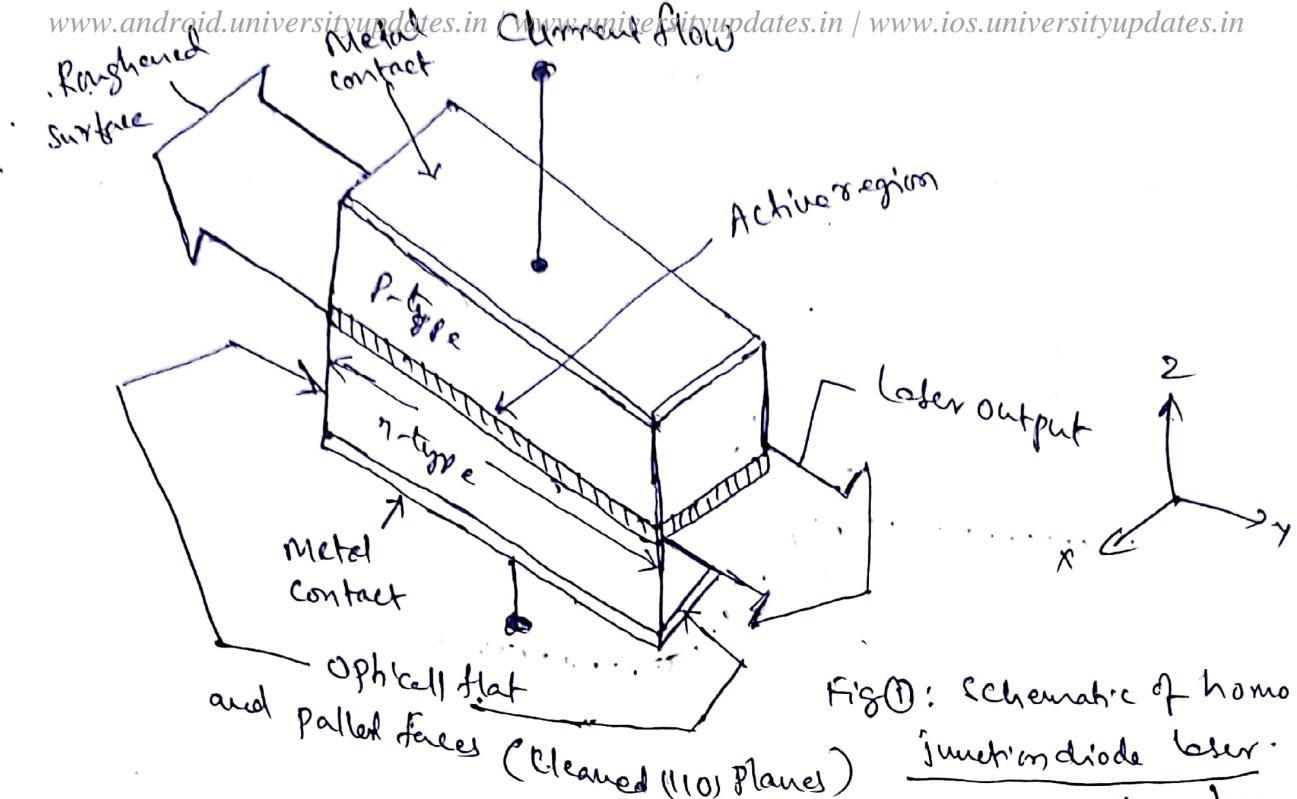
v, Semiconductors having a suitable value of  $E_g$  emit light in the optical region,

### (c) Types of Semiconductor Diode Lasers:-

Broadly there are two types of Semiconductor diode lasers. They are known as homojunction semiconductor lasers and heterojunction semiconductor lasers.

#### Homojunction Semiconductor laser:-

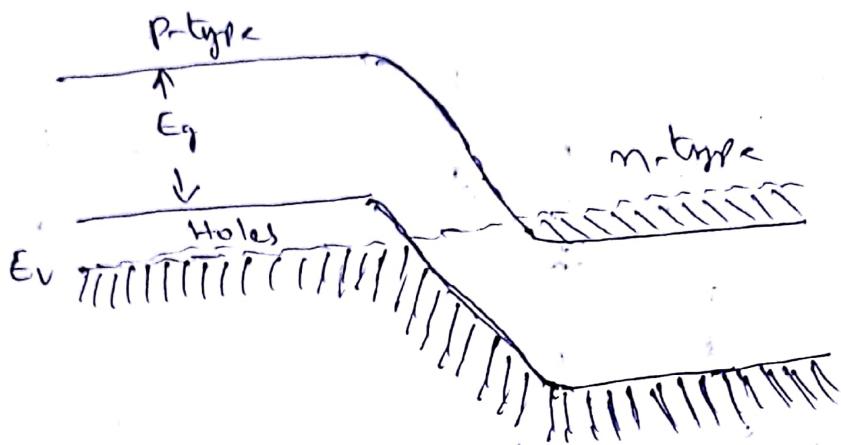
A simple diode laser which makes use of the same Semiconductor material on both sides of the junction is known as a homojunction diode laser. Example is Gallium Arsenide (GaAs) laser.



Fig(1): Schematic of homo-junction diode laser.

(a) Construction: - Fig(1) shows the schematic of a homo-junction diode laser. Starting with a heavily doped n-type GaAs material, a p-region is formed on its top by diffusing zinc atoms into it. A heavily zinc doped layer constitutes the heavily doped P-region. The diode is extremely small in size. Typical diode chips are 500 μm long and about 100 μm wide and thick. The top and bottom faces are metallized and metal contacts are provided to pass current through the diode. A pair of parallel planes cleaved at the two ends of the PN junction provides the required reflection to form the cavity. The two remaining sides of the diode are roughened to remove lasing action in that direction. The

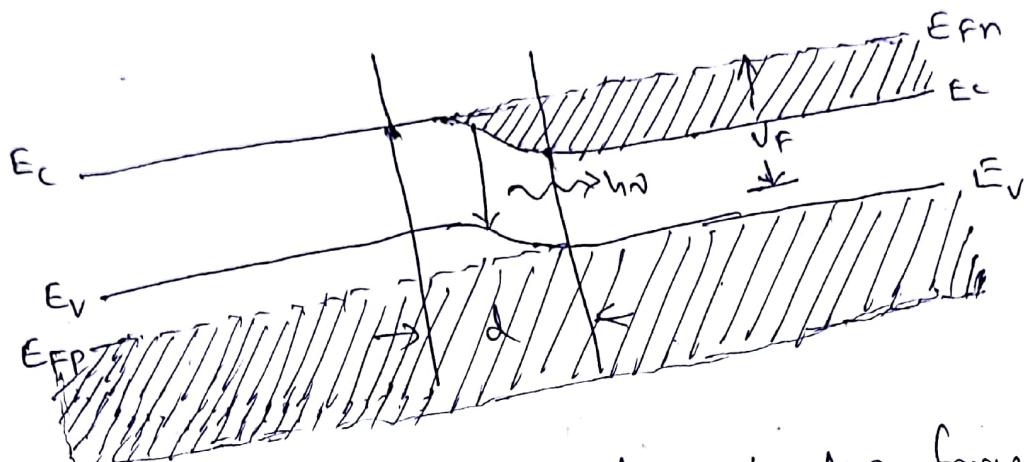
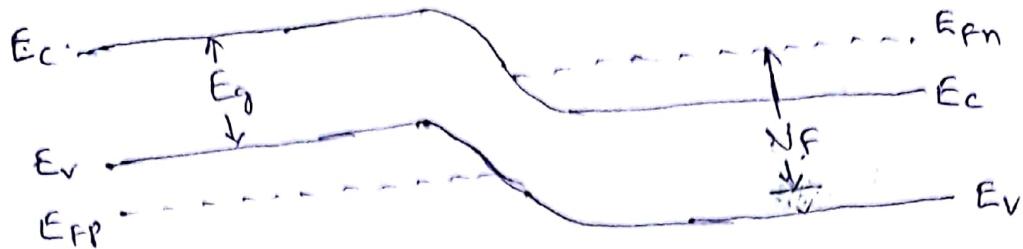
entire structure is packaged in small case which looks like the metal case used for discrete transistors.



Fig(2): Energy band diagram of a heavily doped P-n junction without bias.

(b) Working:- The energy band diagram of a heavily doped p-n junction is shown in fig(2).

- i, Heavily doped p- and n-regions are used. Because of very high doping on n-side, the donor levels are broadened and extend into the conduction band.
- ii, The Fermi level is pushed into the conduction band and electrons occupy the levels lying below the Fermi level.
- iii, Similarly, on the heavily doped p-side the Fermi level lies within the Valence band and holes occupy the portion of the Valence band that lies above the Fermi level.
- iv, At thermal equilibrium, the Fermi level is uniform across the junction,



fig(3) : Laser diode under forward bias .

(c) Pumping Mechanism:-

- i, When the junction is forward-biased, electrons and holes are injected into the junction region in high concentrations ,
- ii, When the diode current reaches a threshold value (Fig 3), the carrier concentrations in the junction region will rise to a very high value.

(d) Population inversion:-

- i, The region d' shown in fig 3 contains a large concentration of electrons within the conduction band and

(ii)

ii, The upper energy levels in the narrow region are having a high electron population while the lower energy levels in the same region are vacant. It creates the condition of population inversion and is called an inversion region or active region.

(P) Lasing action :-

- i, Change recombination leads to emission of spontaneous photons which stimulate the conduction electrons to jump into the Vacant <sup>states</sup> of Valence band.
- ii, This stimulated electron-hole recombination produces coherent radiation,
- iii, GaAs laser emits light at a wavelength of  $9000\text{Å}$  in IR region,

Applications :-

- i, Semiconductor lasers are the cheapest and smallest lasers available.
- ii, The laser output can be easily modulated by modulating the current through the laser diode.
- iii, They are small in size and highly efficient.
- iv, They are mass produced and easily fabricated into arrays using the same techniques developed for transistors.

## Figure of merit:-

A figure of merit is a quantity used to characterize the performance of a device, system or method, relative to its alternatives. In engineering, figures of merit are often defined for particular materials or devices in order to determine their relative utility for an application. Different approaches are available for computing figures of merit. The application requirement on lasers can be expressed by a figure of merit.

Def:- Figure of merit is a measure of the target performance; it is inversely proportional to the running time of an experiment.

For example, high-power, high-energy laser systems includes an optical train (consisting of mirrors and windows) to transport and direct the beam without reducing the performance. figures of merit (FOM) are useful for rating the thermal lensing performance of mirror-substrate materials as well as window-material candidates.

## Semiconductor photodetectors:-

Defn.: photo detectors are devices that convert optical energy to electrical energy and use the principle of internal photoelectric effect.

When light is incident on an intrinsic semiconductor, electrons are excited from the Valence band to the Conduction band leaving behind holes in the Valence band. The free electrons and holes generated in material do not leave the material and so free charge carrier concentration increases. This is known as internal photo electric effect.

The frequency of the photon satisfies the condition  $\nu \geq \frac{E_g}{h}$  and terms of wavelength as  $\lambda \leq \frac{hc}{E_g}$ .

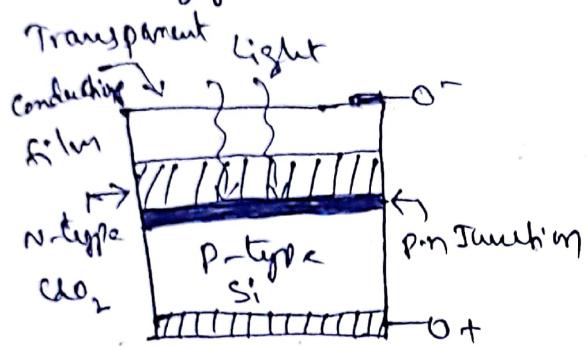
The largest wavelength that can cause the electron transition is,  $\lambda_g = \frac{1.24}{E_g(\text{eV})} \mu\text{m}$

The application of an electric field to the photo detector causes current in the circuit, photo conductivity is arises in the availability of free charge carriers and the conductivity of the semiconductors.

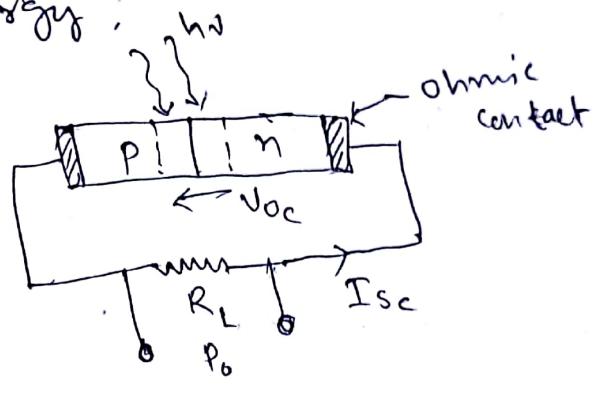
There are three main types of photo detectors, namely, photo diodes, p-i-n diodes and avalanche photodiodes, which are widely used in optical communication systems.

### SOLAR CELL :

A Solar cell is basically a p-n junction that can generate electrical power, when illuminated. Solar cells are usually large area devices typically illuminated with sunlight and are intended to convert the solar energy into electrical energy.



1 (a)



(b)

Fig: Schematic of a Solar cell

The schematic of a solar cell is shown in fig 1@. It consists of a p-type chip on which a thin layer of n-type material is grown. When the solar radiation is incident on the cell, electron-hole pairs are generated in the 'n' and 'p' regions. The majority of them cannot recombine in the

regions. They reach the depletion region at the junction where an electric field due to the space charge separates them. Electrons in the p-region are drawn into the n-region and holes in the n-region are drawn into the p-region. This gives charge accumulation on the two sides of the junction and produce a potential difference called Photo emf of magnitude of order of 0.5V.

Generally Power-conversion efficiency of single-crystalline Solar cell is in range of 10 to 30%. Fig 1(6) shows the sign convention of the current and voltage. Current coming out of the cell is taken as positive as it leads to electrical power generation. The power generated depends on the Solar cell itself and the load connected to it.

I-V characteristic of a Solar cell is shown in fig 2). The open-circuit voltage  $V_{oc}$  is the voltage across the illuminated cell at zero current and the short-circuit current  $I_{sc}$  is the current

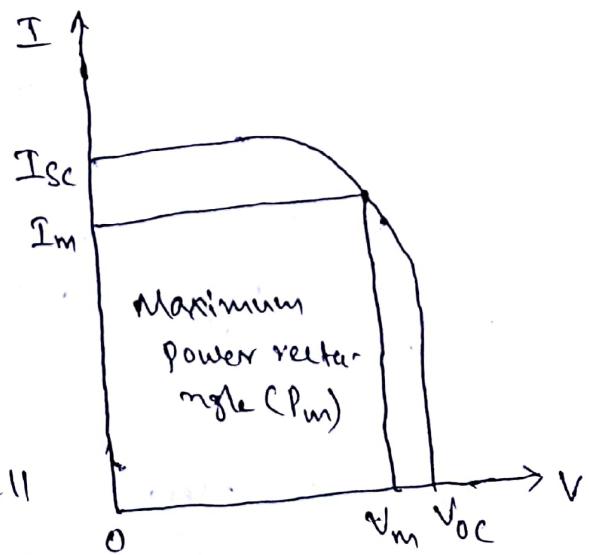


Fig 2) I-V characteristic of Solar cell

through the illuminated cell <sup>if the</sup> at zero voltage across the cell is zero. The short-circuit current is close to the photo-current while the open-circuit voltage is close to the turn-on voltage of the diode. The power equal the product of the diode voltage and current and at first increases linearly with diode voltage but then rapidly goes to zero around the turn-on voltage of the diode. The maximum power is attained at  $V_m$  with  $I_m$ .

Solar cell can be connected in parallel or series into Solar Panels, which can deliver power output of several kilowatts. Solar panels are used in numerous applications in remote locations and in space.

- Applications:
- ① Solar systems are used to power railway signal, alarm systems and etc.
  - ② Solar cells are used in satellites and space vehicles to supply power to electronic and charge storage batteries.
  - ③ Solar cells are used to provide commercial electricity.
  - ④ Solar cells are used to power calculators and watches.

- ⑤ Solar cells are used in mobile telephone, remote instruments etc.
- ⑥ In telecommunication systems radio transceivers on mountain tops or telephone boxes in the country can often be solar powered.

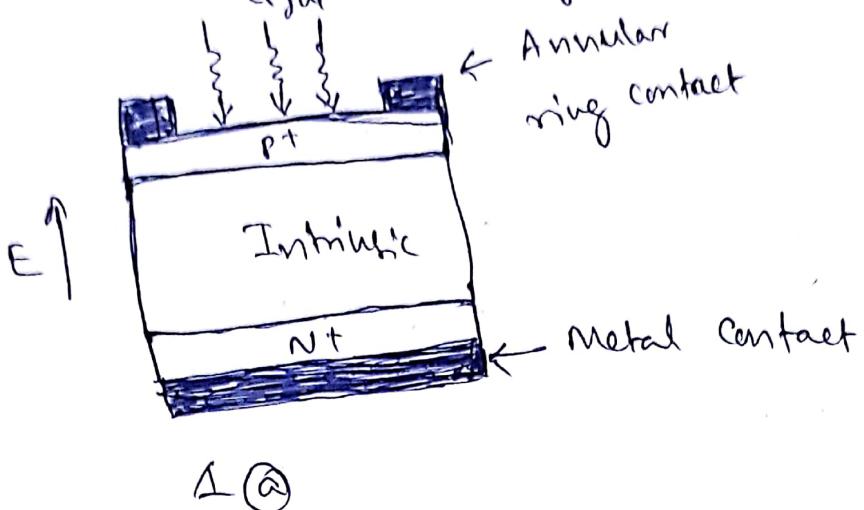
### P-I-N Photodiode :

Positive Intrinsic Negative diode is a device that consists of 'p' and 'n' regions separated by a very lightly doped intrinsic region ('i'). In the P-i-n photodiode the depletion region extends well in the intrinsic region, as it is lightly doped. Under sufficient large reverse bias, the depletion region could extend through the intrinsic region, whereby the entire intrinsic region could be made free of charge carriers, so, the intrinsic layer widens the depletion region and therefore increases area available for capturing light.

Structure:- The P-i-n Structure of the Photodiode enables us to increase the width of the depletion region to a value far greater than what it could be in a

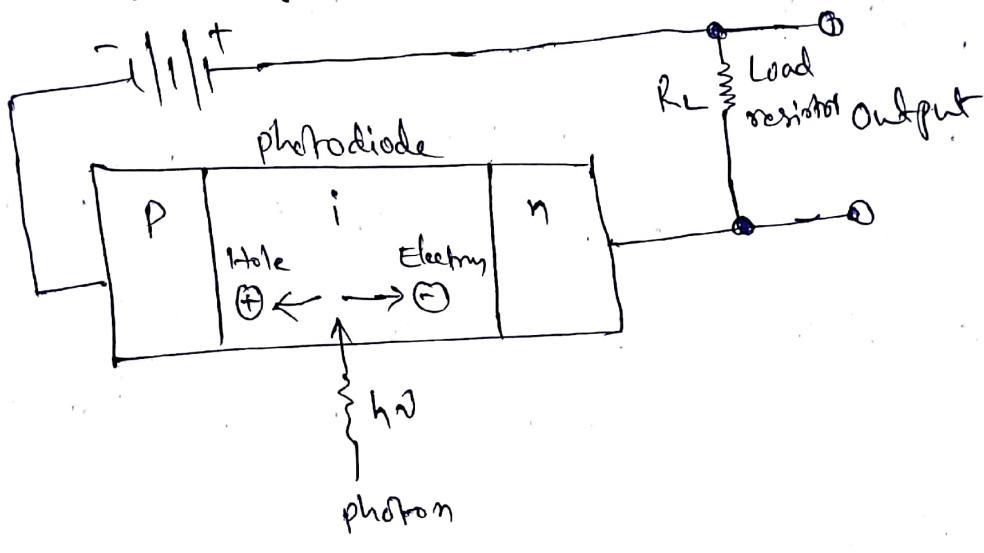
Simple p-n junction, The structure of a p-i-n

photo diode is shown in fig ①.



②

Bias Voltage



④

Fig ①: p-i-n photodiode ② Structure ③ Circuit diagram

The pin diode has a broad intrinsic

Semiconductor layer between p-and n-regions. As the intrinsic layer has

no free charges its resistance is high

and the largest part of the bias voltage drops across it. Incident photon produces free electron-hole pairs in the



Fig ④: Symbol of PIN diode.

depleted intrinsic region where most of the incident light is absorbed. The high electric field in the depletion region causes the free carriers to separate and move across the reverse biased junction. This gives rise to a current flow in the external circuit. As the intrinsic layer is wide enough, most of the photons are absorbed and larger photocurrent is produced. Hence,  $p-n-p$  photo diode is more sensitive than  $p-n$  photo diode. Schematic of PIN diode is shown in Fig ②.

PIN diodes are prepared two ways, a planar structure and mesa structure. In the planar structure an epitaxial film is grown onto the substrate material and the  $p+$  region is introduced either by diffusion or ion implantation.

The mesa structure has layers grown onto the substrate. These layers have the dopants to control the thickness of the layers more accurately.

### PIN diode Characteristics:

- (i) The intrinsic layer gives properties such as a high reverse breakdown voltage, a low level of capacitance and carrier storage when it is forward biased.

(2.) When the PIN diode is forward biased both types of current carrier are injected into the intrinsic layer where they combine. It is this process that enables the current to flow across the layers.

Advantages :- ① The depletion region is very wide and extends throughout the intrinsic region and hence the reverse bias need not be varied to widen the depletion region.

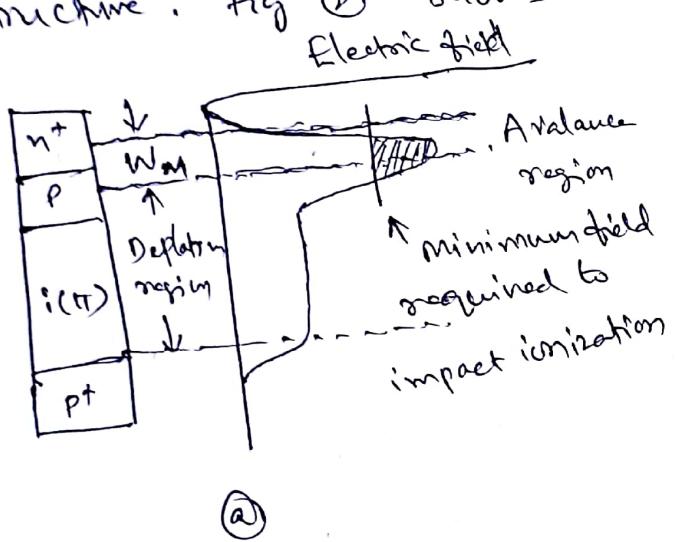
- ② The reverse bias applied is small, of the order of 5V.
- ③ The reverse bias is low.
- ④ As the depletion area is wider, most of the incident photons are absorbed in this region and hence the efficiency of this device is high.
- ⑤ The dark current in this device is smaller.

Major Application :- ① The PIN diode can be used as a high voltage rectifier.

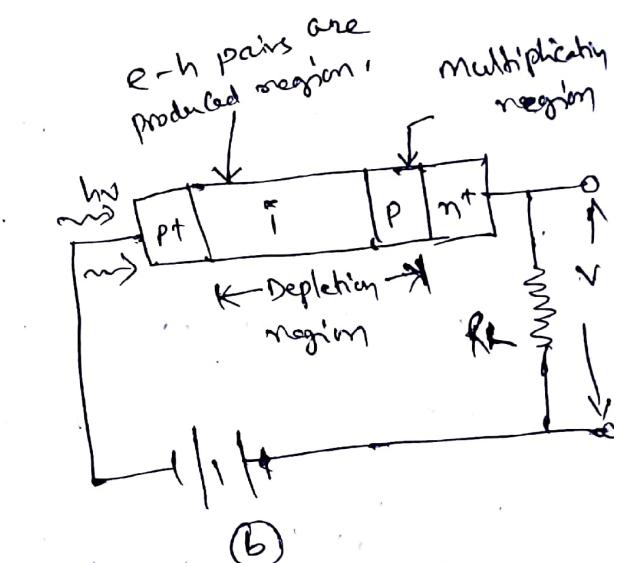
- ② The PIN diode makes an ideal RF switch.
- ③

## AVALANCHE PHOTODIODE

An Avalanche photodiode (APD) is more sophisticated than a p-i-n diode and incorporates internal gain mechanism so that the photoelectric current is amplified within the detector. It will be very much useful when very low levels of light are to be detected. The structure of a typical APD is shown in fig (a). This configuration is known as  $p^+ \text{--} i \text{--} p^-$  reach-through structure. fig (b) shows the energy band diagram of PIN



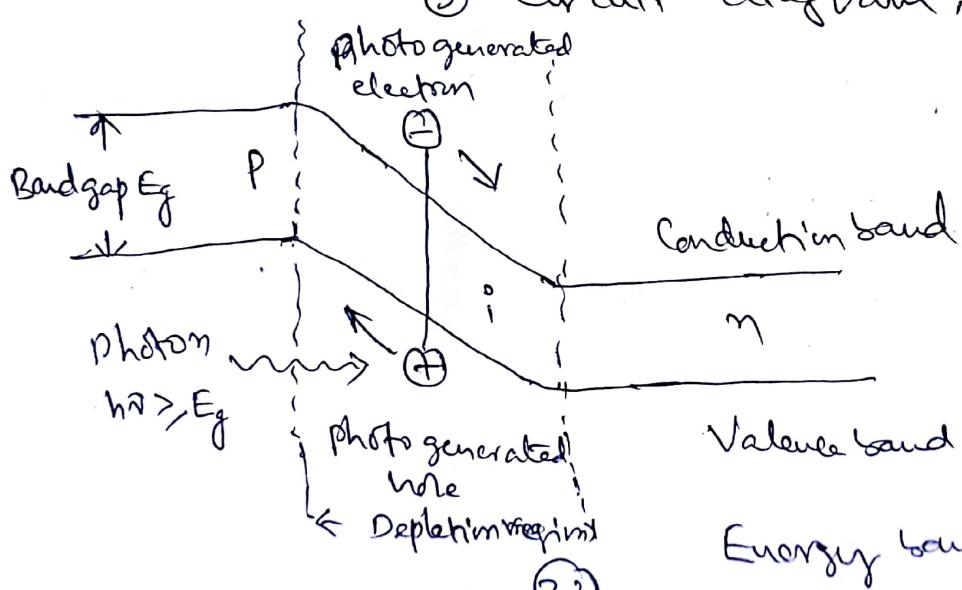
(a)



(b)

Fig (a): APD Photodiode (b) Side view of the structure.

(c) Circuit diagram,



Energy band diagram of PIN

The device is essentially a reverse-biased P-n junction. The n<sup>+</sup> and p<sup>+</sup> are heavily doped semiconductors and very low resistance. The i<sup>-</sup> region is very lightly doped and hence is nearly intrinsic. Most of the incident light passes into the intrinsic region through the thin p<sup>+</sup> region and electron-hole pairs are generated in the intrinsic region. Under reverse bias most of the applied voltage drops across the P-n junction. With increase in reverse bias voltage, the depletion region across this junction widens. Under sufficient reverse bias, the depletion region widens enough to reach through to the intrinsic layer.

In this condition, the internal field intensity near the junction becomes very high and the junction approaches the breakdown condition. Therefore, the electrons and holes photo-generated in the depletion layer acquire sufficient energy from the field to liberate secondary electrons and holes within the layer by a process of impact ionization. The newly generated carriers are also accelerated by the high electric field,

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thus gaining enough energy to cause further impact ionization. In this device multiplication is initiated by electrons. Holes generated in the 'P' region drift to Pt electrode and hence do not take part in the multiplication process.

Def:- The number of carriers multiplies in geometrical progression and this phenomenon is called Avalanche effect.

Def:- The Avalanche photo diode possesses a similar structure to that of the PN (or) PIN photodiode. It operates under a high reverse bias condition. As a photon enters the depletion region and creates a hole-electron pair, these charge carriers will be pulled by the very high electric field away from one another. With increased velocities they will create further hole-electron pairs and the process will repeat. The avalanche action enables the gain of the diode to be increased many times, providing a very much greater levels of sensitivity.

• A photon that enters through the p+ region is absorbed in the intrinsic region and the resulting electron - hole pair is separated by the electric field in the ii region. The hole drift towards the p+ and donor take part in the multiplication process. The electron drifts through the ii region to the p+n junction. There, the electric field due to high reverse bias accelerates the electron. The electron acquires enough kinetic energy to ionize neutral atoms in its path. The electrons thus produced get in turn accelerated and ionize atoms lying in their paths. The effect is cumulative and builds up into an avalanche. As a result, one electron-hole pair will on an average produce  $m$  electron-hole pairs in the process, where  $m$  is the multiplication factor. Thus there occur a carrier multiplication and internal amplification. This internal amplification process enhances the responsibility of the detector -

## UNIT-IV

### Lasers and Fiber Optics

#### Introduction:

LASER is an outstanding achievement of science and Technology in the twentieth century. The theoretical prediction for laser oscillations was given by A. Shim low and C.H. Townes in

The year 1958 using the process of stimulated emission.

→ The first laser was demonstrated by T.H.M. maiman in the year 1960 using a Ruby crystal as the active material. lasting has been extended upto 8-days. P-ray laser are called GRASERS.

#### \* Characteristics of Laser:

Laser when compared with any conventional light (sunlight or tube light) laser possesses few outstanding characteristics they are

- ① Directionality
- ② Monochromaticity
- ③ Intensity
- ④ Coherence

①

① Directionality := laser beam travels as a parallel beam over very long distances but conventional light source emits in all directions. the directionality of laser beam is usually expressed in terms of Divergence.

$$\text{Divergence } (\Delta\theta) = \frac{\pi_2 - \pi_1}{D_2 - D_1} \text{ (or)} \frac{\lambda}{d}$$

Where  $\pi_1, \pi_2$  are the ratio of laser beams pots of distance of  $D_2$  and  $D_1$  respectively.

- for laser beam  $\Delta\theta = 0.01$  milli radian and for light  $\Delta\theta = 0.5$  meter
- ⇒ Because of low spread, lasers are used in the satellite communication.

2) Monochromaticity :=

The Property of exhibiting a single wavelength by a light is called monochromaticity.

the Degree of monochromaticity is defined as

$$\frac{d_f}{f} = \frac{\Delta V}{V_0}$$

where  $\Delta V$  is the bandwidth,  $V_0$  is avg. frequency.

- for laser beam  $\Delta V = 500 \text{ Hz}$  and  $V_0 = 5 \times 10^4 \text{ Hz}$ .

$$\frac{d_f}{f} = \frac{\Delta V}{V_0} = \frac{500}{5 \times 10^4} = 10^{-4}$$

- ⇒ for conventional light source  $\frac{d_f}{f} = \frac{\Delta V}{V_0} = \frac{10^9}{10^4} = 10^5$

Hence, the monochromaticity of the conventional light source is poorer than the laser source.

(2)

(3) Intensity:

Laser beam gives out light in to a narrow beam of light and its energy is concentrated in a small region even a 1 watt laser light is more intense than 100 watt ordinary lamp.

→ The number of photons coming out from a laser per second per unit area is given by.

$$N_m = \frac{P}{h\nu\pi r^2} \approx 10^{22} \text{ to } 10^{34} \text{ photons/m}^2\text{-sec.}$$

→ The number of photons coming out from a black body per second per unit area with temp., T.

$$N_m = \frac{2\pi c}{\lambda^4} \left[ \frac{1}{e^{h\nu/k_B T} - 1} \right] d\lambda \approx 10^{16} \text{ photons/m}^2\text{-sec.}$$

This comparison shows that laser light is highly intensive beam.

(4) Coherence:

The property of existing either zero (or) constant phase angle difference b/w two (or) more waves is known as coherence.

In case of conventional light, the property of coherence exists b/w a source and its virtual source. whereas

In case of laser the property of coherence exists b/w any two (or) more light waves of same type.

Coherence is of two types.

(2)

Ordinary light

- ① It is not coherent
- ② It is not directional
- ③ It is less intense
- ④ The angles spread is more
- ⑤ Examples: = Sun light  
mercury vapour lamp

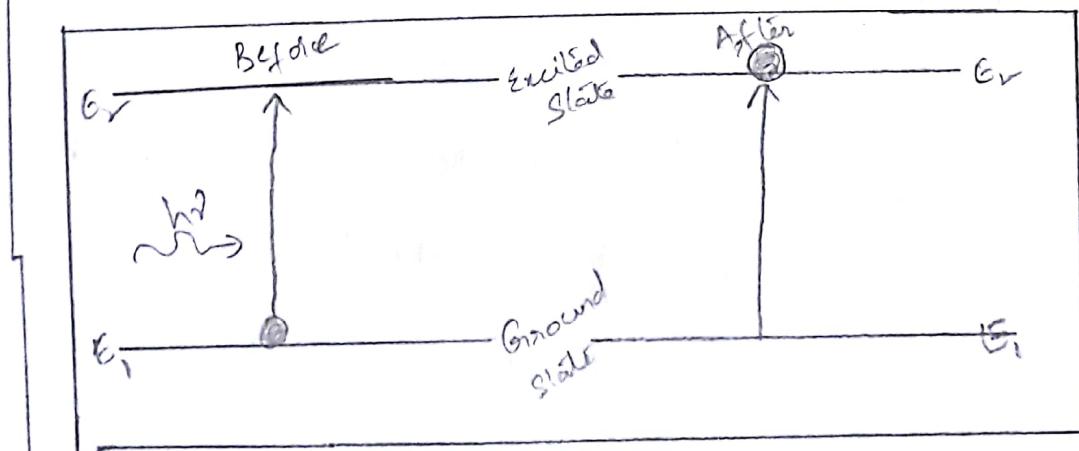
Laser light

- ① It is highly coherent
- ② It is more directional
- ③ It is highly intense
- ④ The angular spread is less
- ⑤ Examples: = Helium, Neon laser  
CO<sub>2</sub> laser.

⇒ To understand the working of laser system, the following concepts have to be explained.

- ① Absorption
- ② Spontaneous emission
- ③ Stimulated emission
- ④ Lifetime
- ⑤ Metastable state
- ⑥ Population
- ⑦ Population inversion
- ⑧ Pumping.

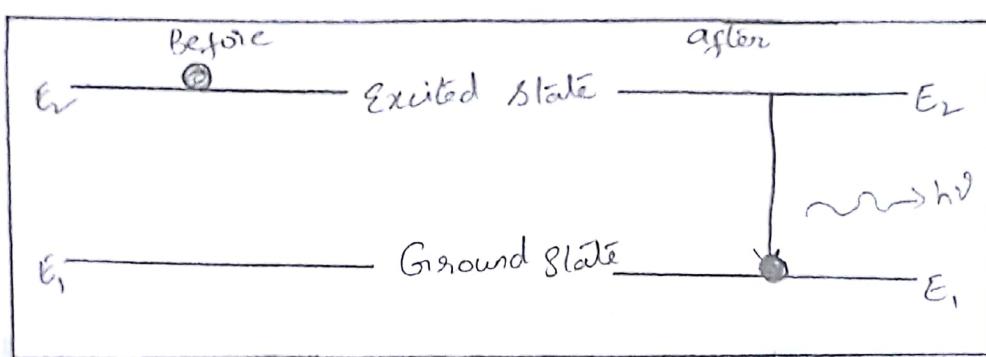
① Stimulated absorption    ② absorption



(4)

Let us consider a system in which two active energy levels are present whose energies are  $E_1$ ,  $E_2$ . where  $E_1$  is ground state and  $E_2$  is excited state as shown in fig usually atoms are in the ground state, then if excited to higher state  $E_2$ , this phenomenon is known as absorption.

## ② Spontaneous emission:

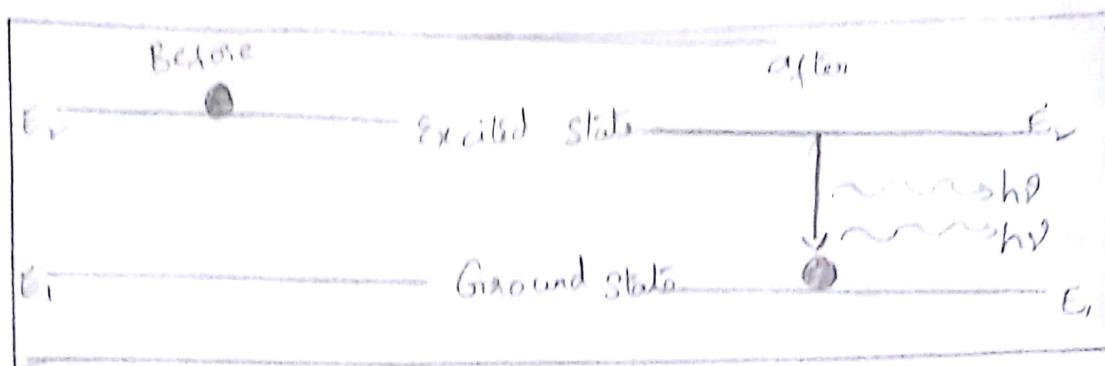


Let us assume that the atom is in the excited state  $E_2$ . After the lifetime, the atom deexcites to its ground state spontaneously emitting a photon of Energy  $h\nu_{E_2-E_1}$ , this phenomenon is known as spontaneous emission.

$$\text{the frequency of emitted photon } \nu = \frac{E_2 - E_1}{h}$$

→ The photons in this case have various wavelengths and they are out of phase. Thus the photons are in coherent.

⇒ The radiation emitted spontaneously by each atom has a random direction and random phase.



→ Let us assume that the atom is in the excited state  $E_2$ , if a photon of energy  $h\nu = E_2 - E_1$  is incident on it before the life-times it stimulates the atom from  $E_2$  to  $E_1$  then a photon of energy  $h\nu$  release along with incident photon as shown in fig. These two photons will have same energy and phase. This phenomenon is known as stimulated emission, the emitted photons in this case have the single wavelength and they are in phase. These photons are coherent.

\* Distinguish b/w spontaneous emission and stimulated emission.

#### Spontaneous emission

- ① Emission takes place without any stimulus energy.
- ② Incoherent radiation.
- ③ Low intense and less directional.

#### Stimulated emission

- ① Emission takes place with the help of stimulus energy.
- ② Coherent radiation.
- ③ High intense and more directional.
- ④

⑤ This emission postulated by Bohr

⑥ This emission postulated by Einstein.

⑦ Examples:- light from Sodium (or) mercury lamp

⑧ Examples:- light from Ruby (or) He-Ne laser.

⑨ Life-time :- The duration of time spent by an atom in the excited state is known as life-time of that energy state for example the life time for hydrogen atom is  $10^{-8}$  sec.

⑩ Metastable state :-

The excited state, which has long life time is known as metastable state.

The life time of the electrons in the metastable state will be very large due to its narrow size and hence it is easy to achieve population inversion at this level to start lasing action.

⑪ Population :-

The number of atoms per unit volume in an energy level is known as population of that energy level.  
→ If  $N$  is number of atoms per unit volume in an

energy state ' $E$ ' then expression for population can be

(7)

Written as

$$N = N_0 \exp\left(\frac{E}{k_B T}\right)$$

where  $N_0$  is the population in the ground state and  $k_B$  is the Boltzmann's constant,  $T$  is temp.  
 $\Rightarrow$  population is maximum in the ground state and decreases exponentially as energy level increases.

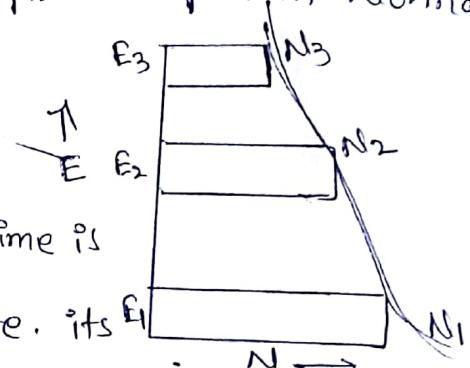
### ⑦ population inversion:

consider three level system in which three active energy levels  $E_1, E_2$  and  $E_3$  are present and population in those energy level are  $N_1, N_2$  and  $N_3$  respectively. in Normal condition.

$$E_1 < E_2 < E_3 \text{ and } N_1 > N_2 > N_3$$

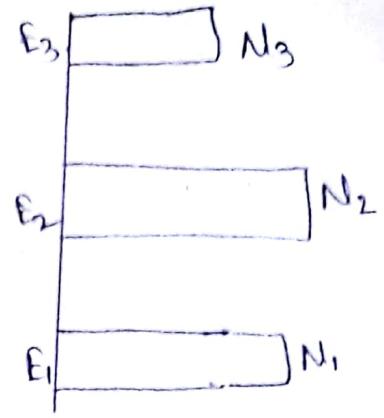
$\Rightarrow E_1$  is the ground state its life time is unlimited,  $E_3$  is highest energy state. its life time is very less. and  $E_2$  is an metastable state. it has more life time.

$\Rightarrow$  when a suitable form of energy is supplied to the system in a suitable way, then the atom excite from ground state  $E_1$  to excited states  $E_2 \& E_3$ . Due to instability, excited atoms will come back to ground state with respective energy states  $E_2$  and  $E_3$ .



atoms will fall into  $E_2$  immediately

At a stage the population in  $E_2$  will become more than the population in ground state. This situation is called population inversion which is shown in fig.



### ② pumping :-

To achieve population inversion suitable form of energy must be supplied. The process of supplying suitable form of energy to a system to achieve population inversion is called pumping.

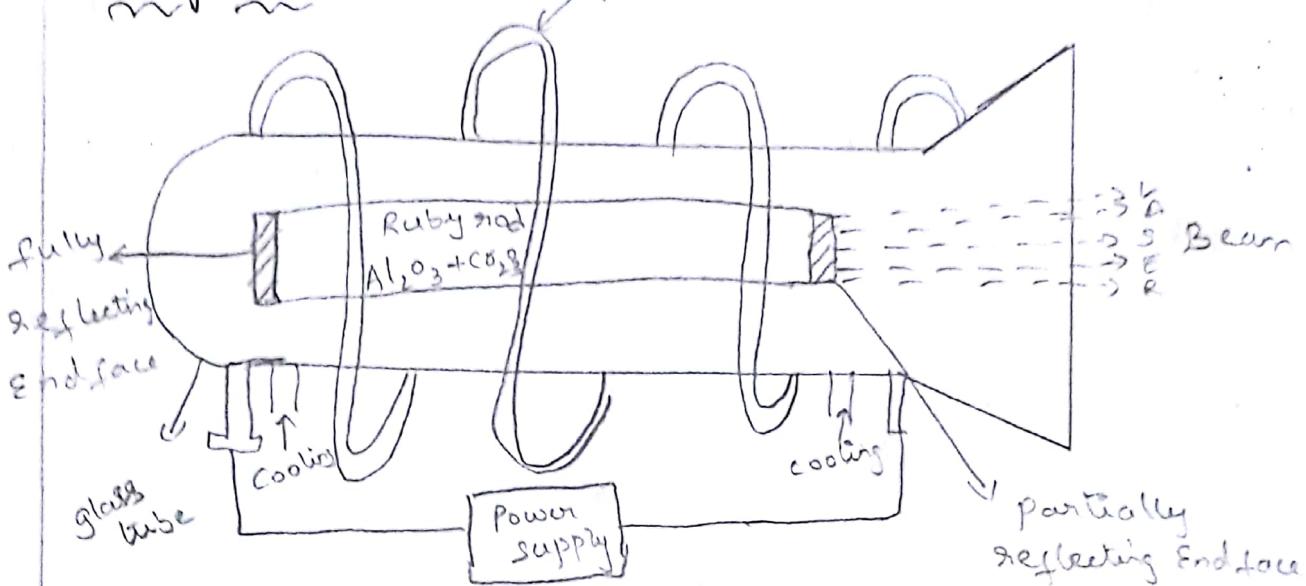
- Several pumping methods are ① optical pumping
- ② electric discharge method ③ inelastic atom-atom collision
- ④ Direct conversion ⑤ chemical reaction.

### \* Different types of laser systems :-

On the basis of active medium used in the lasers system, lasers are classified into several types. and most popular kinds of lasers are.

- ① Solid state lasers: Ruby laser, Nd:YAG lasers
- ② Liquid lasers: Seoch<sub>2</sub> laser, Eupropium chelate lasers.
- ③ Gaseous lasers: He-Ne laser, CO<sub>2</sub> lasers, Argon-ion lasers
- ④ Dye lasers: Coumarin dye lasers, Rhodamine 6G lasers
- ⑤ Semiconductor lasers: Ga-As laser, In-P laser.

# \* Ruby Laser



→ Ruby laser is a solid state three level laser system demonstrated first by manmohan in 1960. It produces pulsed laser each pulse will come out in duration of 10 nanoseconds

Source of energy: Xenon flash light

Active medium: Ruby crystal rod ( $\text{Al}_2\text{O}_3 + \text{Cr}_2\text{O}_3$ )

Optical cavity: arrangement of silver polished surfaces on either sides of the ruby rod.

Construction:

Ruby laser is made up of a cylindrical ruby crystal rod of composition  $\text{Al}_2\text{O}_3$  and 0.05%  $\text{Cr}_2\text{O}_3$ . Then the colour of the rod becomes pink.

⇒ Ruby crystal rod length is 4cm and diameter is 0.5 cm and both the ends of the rod is highly polished with a silver film one end becomes fully reflecting and another end becomes partially reflecting.

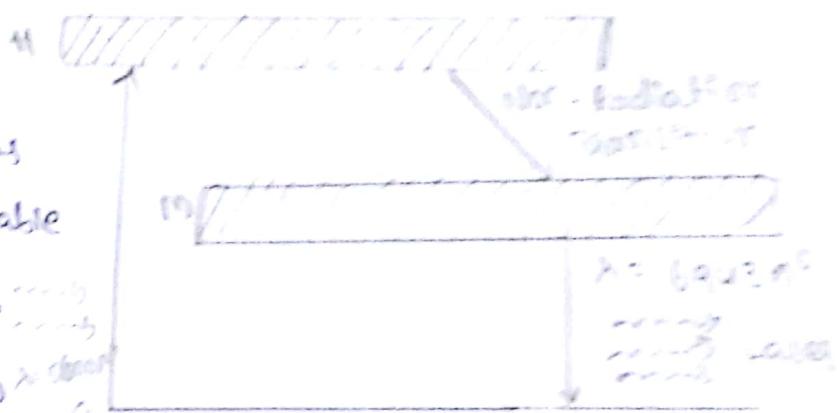
$\text{Cr}^{+3}$  ions can be active medium in Ruby rod or  $\text{Cr}^{+3}$  laser.

→ The Ruby rod is surrounded by a Helical Xenon flash lamp which supplies green colour flash light of wave length  $5600 \text{ Å}$  to the active medium to achieve population inversion.

→ only one part of the energy is used in pumping  $\text{Cr}^{+3}$  ion while the rest goes as heat which should be cooled with cooling arrangement as shown in fig.

### Working principle:

→ The Chromium atom have three active energy levels they are named as Ground state (g), metastable state (m) and higher state (h). The energy diagram is shown in fig.



⇒ Due to supply of xenon flash light to the Ruby rod, the Chromium atoms begin to excite from g state to excited states m and h.

⇒ once chromium atoms are excited to an upper energy level 'h' They require two steps to return back to their ground state

⇒ first step is from higher state h to metastable state m.

To Metastable state M. which is a longer pump and the energy is not radiated in the form of photons

This transition is called "Radiation less Transition (0<sub>2</sub>)

Non-Radiative Transition".

→ Second step is the chromium atoms returned to M level can remain in this state for several milliseconds. At a particular stage population of excited chromium atoms are more at M than at 'G' hence there is population inversion.

→ After getting population inversion, the chromium atoms are returned to ground state by emitting a photons of wavelength  $6943\text{ A}^{\circ}$  This results in stimulated emission.

→ The emitted photons are in phase and have same frequency in the range of  $4.32 \times 10^{14}\text{ Hz}$ . Its power is more than few hundred mw

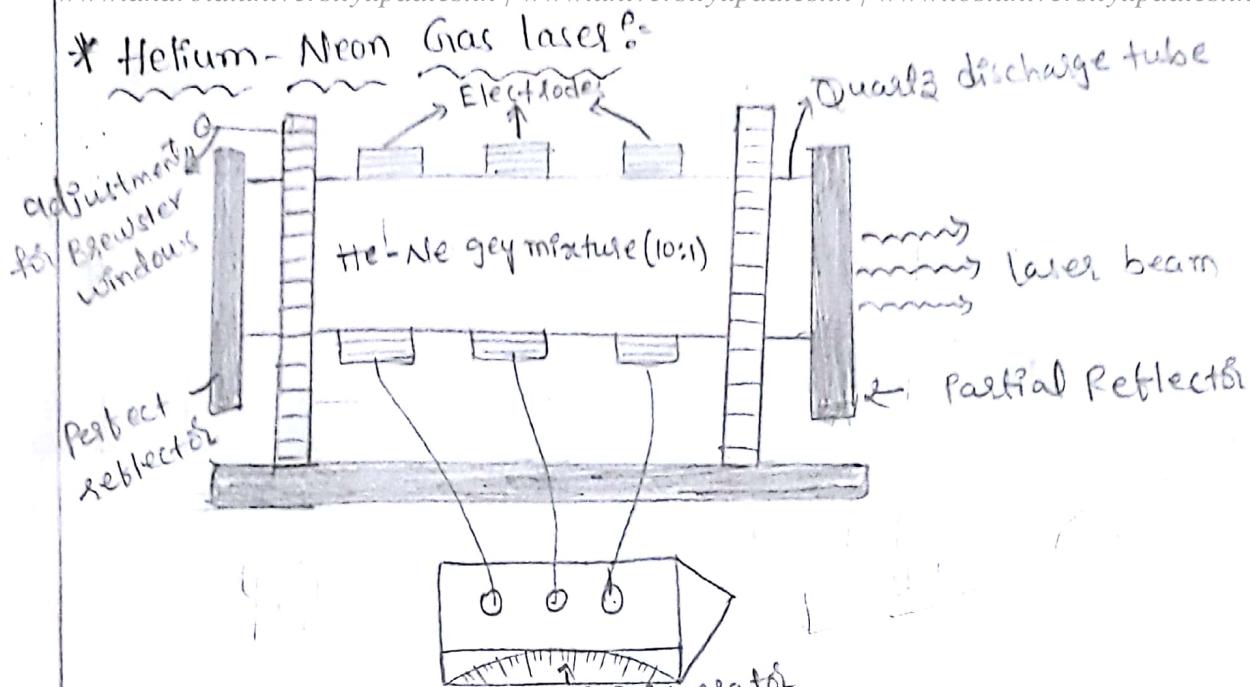
Applications:-

① Ruby lasers are used in optical holography

② Ruby lasers can be used for measurement of plasma properties. such as electron density and temperature.

③ Ruby lasers are used to remove melanin of the skin and tattoos

④ Ruby lasers can be used for recording holograms of large volume.



→ He-Ne laser is a <sup>R.F. generator</sup> gaseous laser, and it was used to produce a continuous laser constructed by Javan in 1961. Source of Energy: R.F. oscillator.

Active medium : Helium and Neon gas mix -ture

Optical cavity: arrangement of silver polished mirrors

Construction:= the experimental arrangement is shown in

fig.

⇒ The He-Ne laser consists of quartz-tube with a diameter of about 15 cm and 80 cm length. which has three electrode to connect R.F. oscillator.

⇒ Inside the tube there is a mixtube of Neon under a pressure of 0.1 mm of Hg and Helium under a pressure of 1mm of Hg. the ratio of He-Ne mixtube is 10:1

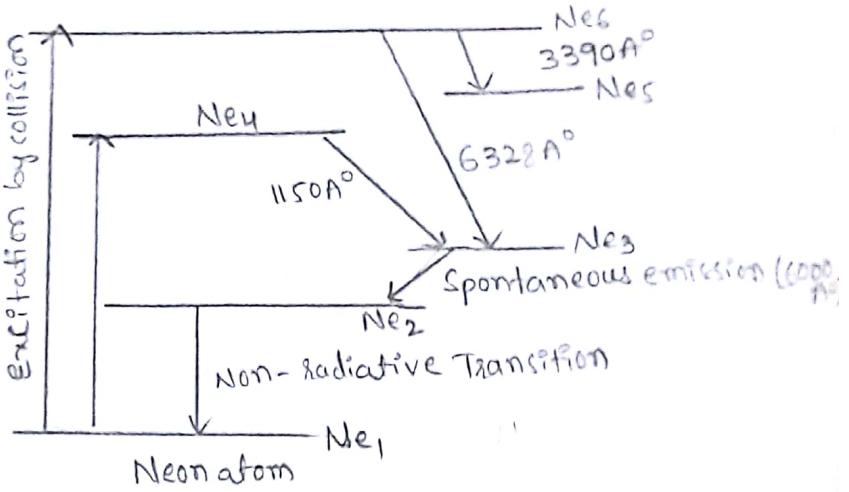
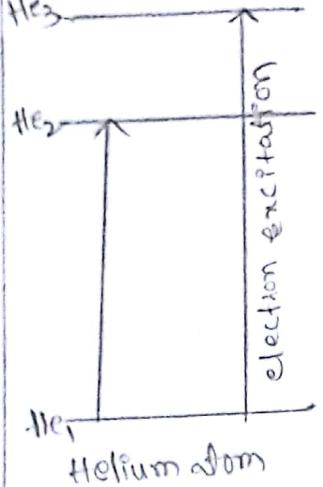
⇒ At both ends of the tube is polished with Silver material such that one end of the tube is a perfect

→ the active material is excited due to energy discharge by means of Radio frequency generator.

Working: - the schematic energy diagram of He-Ne gas mixture is shown in fig.

→ for Helium atom three active energy levels are present they are named as  $\text{He}_1$ ,  $\text{He}_2$ ,  $\text{He}_3$ . whereas for Neon atom six active energy levels are present named as  $\text{Ne}_1$ ,  $\text{Ne}_2$ ,  $\text{Ne}_3$ ,  $\text{Ne}_4$ ,  $\text{Ne}_5$  and  $\text{Ne}_6$ .

He<sub>3</sub>



⇒ By the electrical discharge in the gas tube, the ground level helium atoms are excited to higher levels  $\text{He}_2$  and  $\text{He}_3$ . This process is called electron excitation. These are the metastable states. for helium atoms

⇒ By Resonance collisional transfer method, the helium atoms at  $\text{He}_3$  give up their excitation energy to the ground state neon atoms and the neon atoms are excited to another higher energy level  $\text{Ne}_4$ .

⇒ As the energy exchange continues b/w He and Ne atoms, the population <sub>(14)</sub> of neobatoms in the

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Excited states  $N_{e_1}$  and  $N_{e_2}$  indicates more and  
more and at a stage population inversion will be achieved  
after achieving population inversion:-

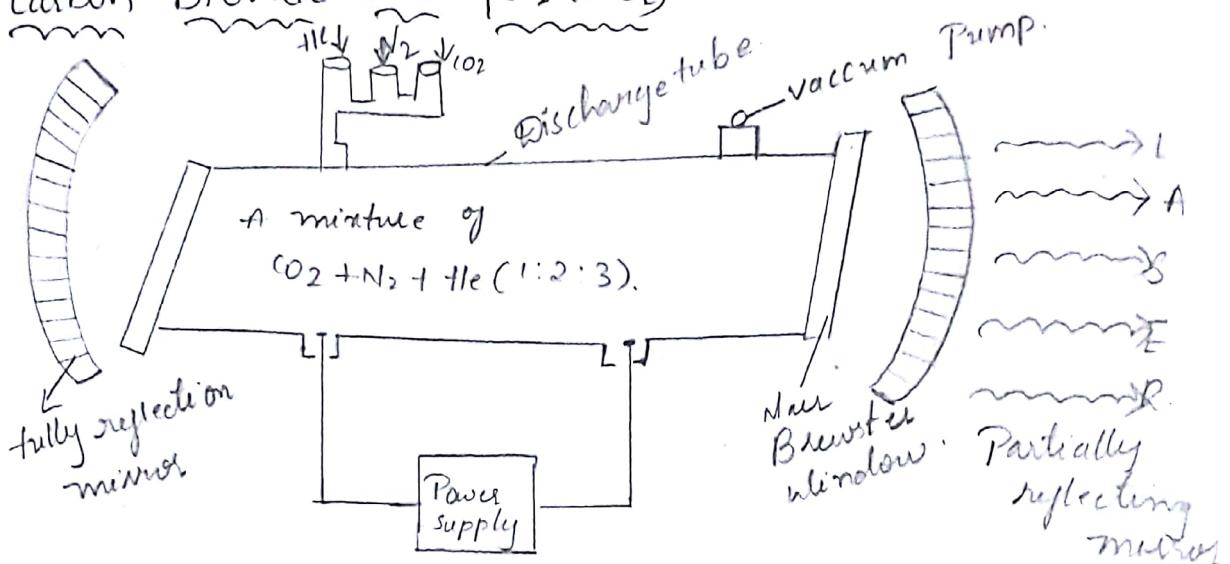
- ① few neon atoms de-excites from  $N_{e_2}$  to  $N_{e_1}$ . During this electromagnetic radiation of wavelength of  $3390\text{ \AA}^\circ$  will be emitted.
- ② Many other neon atoms deexcites from  $N_{e_2}$  and  $N_{e_1}$ . During this transition, electromagnetic radiation of wavelength  $6328\text{ \AA}^\circ$  will be emitted.
- ③ In continuation, atoms in the excited state  $N_{e_1}$  will be de-excite  $N_{e_2}$ , where an electromagnetic radiation of wavelength  $1150\text{ \AA}^\circ$  is emitted
- ④ After reaching all the neon atom to  $N_{e_2}$  spontaneously those will be de-excited to  $N_{e_1}$ . During this transition electromagnetic radiation of wavelength  $6000\text{ \AA}^\circ$  is emitted.
- ⑤ Finally, neon atoms take non-radiative transition by making collision with wall of the tube from  $N_{e_2}$  to  $N_{e_1}$ .

Applications of He-Ne laser:-

- ① In the field of Interferometry, the He-Ne laser are used to identify optically characteristics of material like smoothness and the fig. of the surface.
- ② In the field of Laser printing, the He-Ne lasers beam is used for writing on the photo sensitive material.

(3) In the field of Bar-code reading, the He-Ne laser is used to check out (or) for scanning the digital codes, which are imprinted on the product.

⇒ \* Carbon Dioxide laser ( $\text{CO}_2$  laser)



⇒  $\text{CO}_2$  laser is a four level molecular laser system. It was proposed by C. Kumar. N. Patel in the Year of 1963  
source of Energy: D.C excitation (Discharge method)  
active medium: mixture of  $\text{CO}_2$ ,  $\text{N}_2$ , He, gases (1:2:3)  
optical cavity: arrangement of silicon mirrors  
waffled with aluminum.

Construction :-

⇒  $\text{CO}_2$  laser consist of a discharge tube of 2.5 cm in diameter and 5 cm long in length and discharge is produced by D-C

excitation

⇒ the tube is filled with a mixture of  $\text{CO}_2$ ,  $\text{N}_2$ , and He gases in the ratio of 1:2:3 and partial pressures of  $\text{CO}_2$ ,  $\text{N}_2$  and He are around  $0.33 \text{ torr}$ ,  $1.2 \text{ torr}$  and  $\approx 10.88$  respectively,

⇒ the ends of the tube is closed with alkali halide,  $\text{NaCl}$

wlth aluminium to make silicon mirrors.

⇒ In CO<sub>2</sub> laser along with CO<sub>2</sub> there are also N<sub>2</sub> and the gasses N<sub>2</sub>. helps to increase the population of the upper level of CO<sub>2</sub>. while He helps to depopulate the lower level.

Working Principle :-

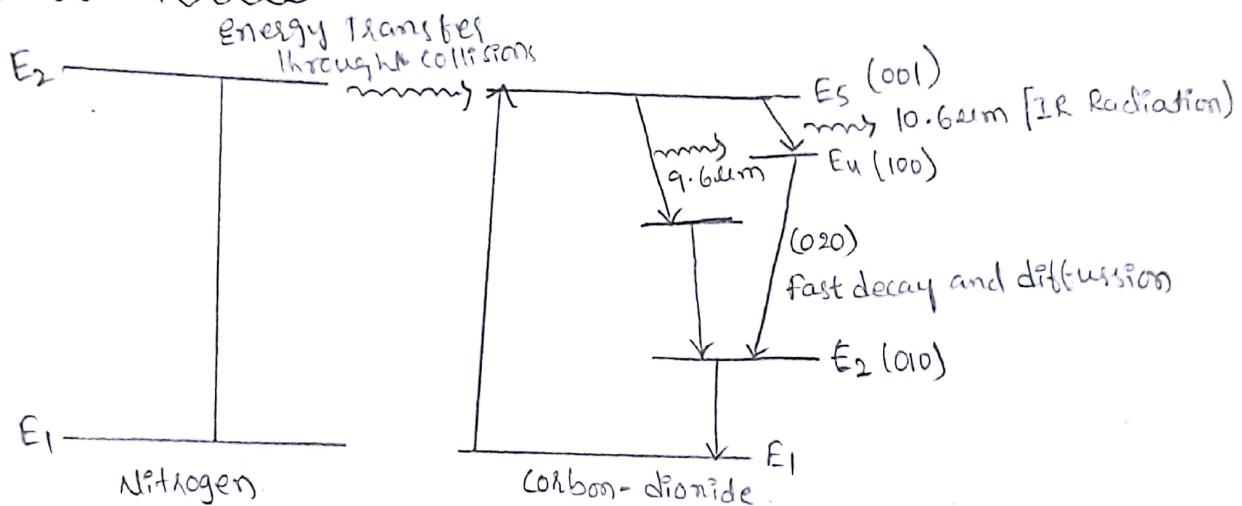


Fig shows the various vibrational energy levels taking in the laser transition. In the fig E<sub>5</sub> refers to (001) level, Eu refers to (100) level and E<sub>2</sub> refers to (012,0) level, E<sub>2</sub>, refers to (0,1,0) level and E<sub>1</sub> refers to ground state energy level.

⇒ When a discharge is passed through a tube, the N<sub>2</sub> molecules are excited and are raised to higher excited states.

⇒ The excited energy of N<sub>2</sub> molecule is transferred to ground state CO<sub>2</sub> molecular by inelastic collision and CO<sub>2</sub> molecules are raised to their excited energy levels

E<sub>5</sub>

→ The energy level E<sub>5</sub> is a metastable energy level

Hence there is population inversion.

## FIBER OPTICS

Introduction : Fiber optics deals with light propagation through thin glass fibres. The transmission of light along the thin cylindrical glass fibre by total internal reflection was first demonstrated by John Tyndall, a British physicist in 1870. Fibrosopes made of optical fibres are widely used in a variety of forms in medical diagnostics. Sensors for detecting electrical, mechanical, thermal energies are made using optical fibres.

### Optical fibre as a dielectric wave guide

An optical fibre is a cylindrical wave guide made of transparent, dielectric, (glass or clear plastic), which guides light waves along its length by total internal reflection. It is approximately 70 $\mu$ m (or) 0.003 inch diameter.

Def:- Fibre optics is a technology in which signals are converted from electrical into optical

signals, transmitted through a thin glass fibre and reconverted into electrical signals.

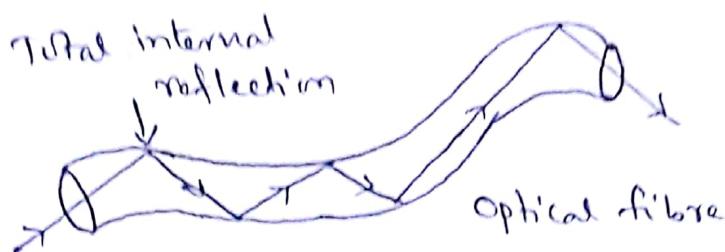


Fig ①. Illustration of optical fibre as a dielectric waveguide.

### Principle of optical fibre as a dielectric waveguide:

Principle: The propagation of light in an Optical fibre from one of its ends to the other end is based on the principle of total internal reflection. When light enters one end of the fibre, it undergoes successive total internal reflections from sidewall and travels down the length of the fibre along a zigzag path, as shown in Fig ①.

Structure: A practical optical fibre is cylindrical in shape and has in general three coaxial regions [Fig 2 (a) and (b)].

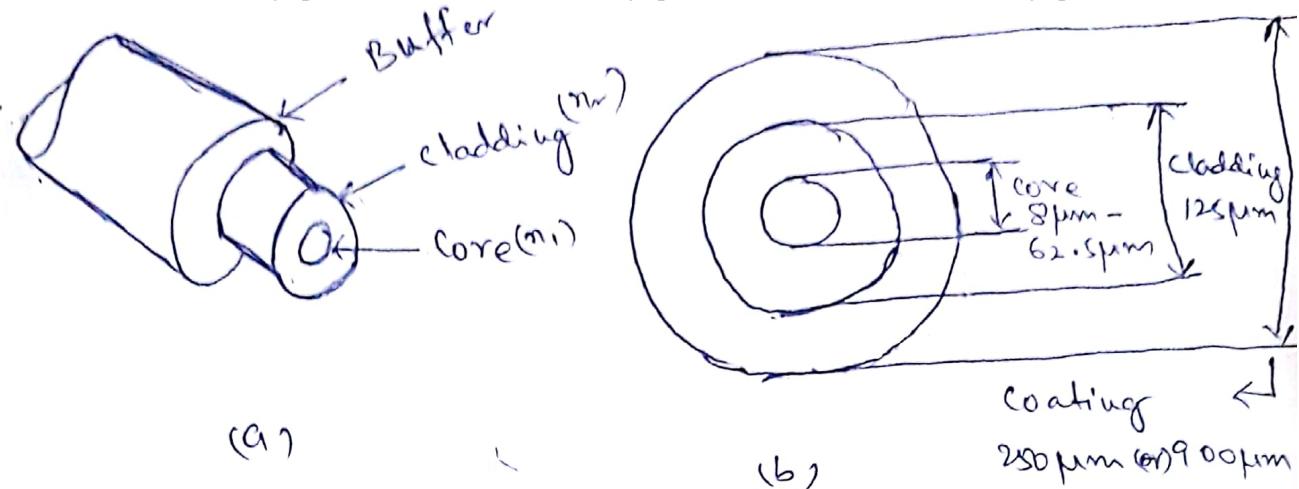


Fig (2): Illustration of Side view and Cross sectional view of an optical fibre.

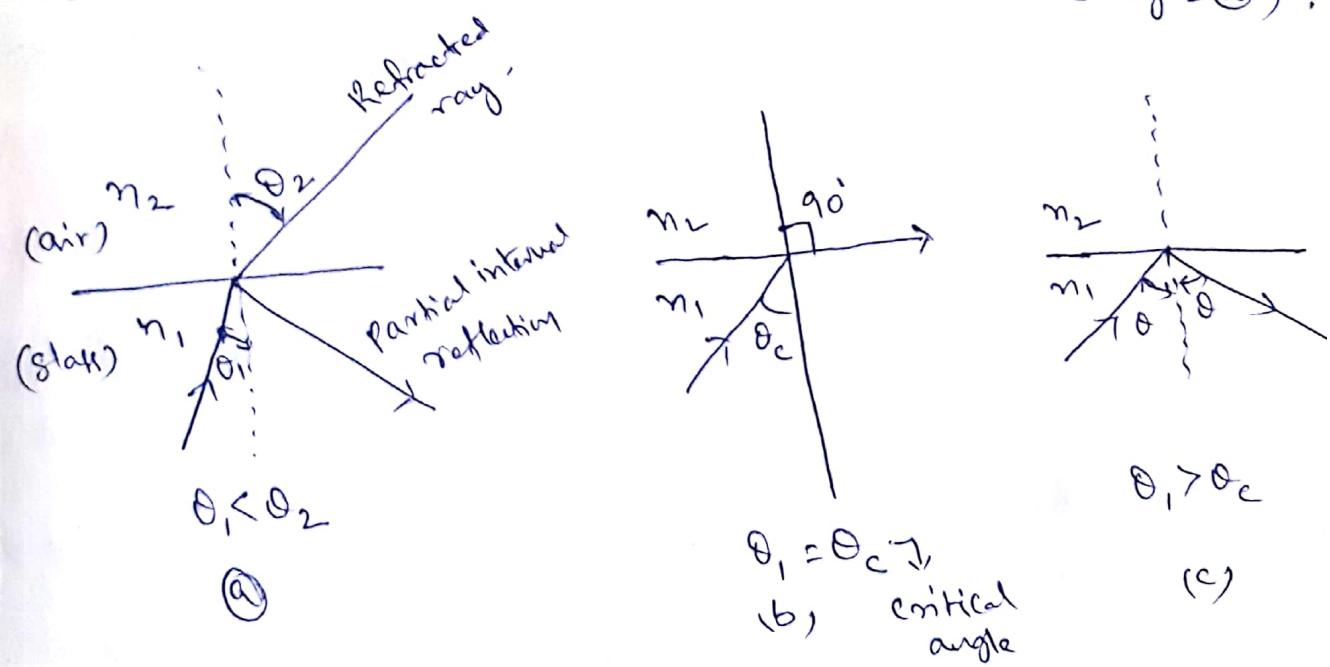
i, Core:- It is the inner light - carrying layer. In general, the diameter of the core is of the order of  $8.5\mu m$  to  $62.5\mu m$ .

ii, Cladding:- It is the middle layer, which serves to confine the light to the core. The diameter of the cladding is of the order of 125 mm, the refractive index of cladding ( $m_2$ ) is always lower than that of the core ( $m_1$ ). Light launched into the core and striking the core-cladding interface at an angle greater than critical angle will be reflected back into the core. Since the angles of incidence and reflection are equal, the light will continue to rebound and propagate through the fibre.

iii, Sheath:- Sheath is the outermost region and a protective buffer coating given to the cladding for extra environmental protection. The coating can vary in size from 250µm (0.7900ftm).

### Total Internal Reflection

A medium having a lower refractive index is said to be an optically rarer medium while a medium having a higher refractive index is known as an optically denser medium. When a ray of light passes from denser medium to a rarer medium, it is bent away from the normal in the rarer medium. (Fig 3@).



Fig(3) : Illustration of total internal reflection,

④

Snell's law for this case may be written as,

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{n_1}{n_2}$$

$$\therefore \sin \theta_2 = \left( \frac{n_1}{n_2} \right) \sin \theta_1 \quad \text{--- (1)}$$

Where  $\theta_1$  is the angle of incidence of light ray in the denser medium and  $\theta_2$  is the angle of refraction in the rarer medium and  $n_1 > n_2$ .

When  $\theta_1$  is increased,  $\theta_2$  increases and the refracted rays bend more and more away from the normal. At some particular angle  $\theta_c$  the refracted ray glides along the boundary surface so that  $\theta_2 = 90^\circ$ , as seen in fig 3(b). At angles greater than  $\theta_c$  the rays are reflected back into the denser medium (Fig 3(c)). Thus,

- If  $\theta_1 < \theta_c$ , the ray refracts into the rarer medium,
- If  $\theta_1 = \theta_c$ , the ray just grazes the interface of rarer-to-denser media. [ $\theta_c$  = critical angle]
- If  $\theta_1 > \theta_c$ , the ray is reflected back into the denser medium.

(5)

Def: The phenomenon in which light is totally reflected from a denser-to-rarer medium boundary is known as total internal reflection.

The rays that experience total internal reflection obey the laws of reflection. Therefore, the critical angle can be determined from Snell's law,

$$\text{When } \theta_1 = \theta_c, \theta_2 = 90^\circ$$

∴ From Eqn ①, we get,

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$$

$$\therefore n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

$$\therefore \sin \theta_c = \frac{n_2}{n_1} \quad \text{--- ②}$$

When the rarer medium is air,  $n_2 = 1$  and writing  $n_1 = n$ , we obtain

$$\sin \theta_c = \frac{1}{n} \quad \text{--- ③}$$

If critical angle  $\phi_c$ , then,

$$\sin \phi_c = \frac{n_2}{n_1}$$

(6)

## Acceptance Angle

Let us consider a step index optical fibre into which is launched at one end, as shown in fig (4). Let the refractive index of the core be  $n_1$ , and the refractive index of the cladding be  $n_2$  ( $n_1 > n_2$ ). Let  $n_0$  be the refractive index of the medium from which light is launched into the fibre.

Assume that a light ray enters the fibre at an angle ' $\theta_i$ ' to the axis of the fibre. The ray refracts at an angle ' $\theta_r$ ' and strikes the core-cladding interface at angle ' $\phi$ '. If ' $\phi$ ' is greater than critical angle ' $\phi_c$ ', the ray undergoes total internal reflection at the interface. Since  $n_1 > n_2$ .

As long as, the angle ' $\phi$ ' is greater than ' $\phi_c$ ', the light will stay within the fibre.

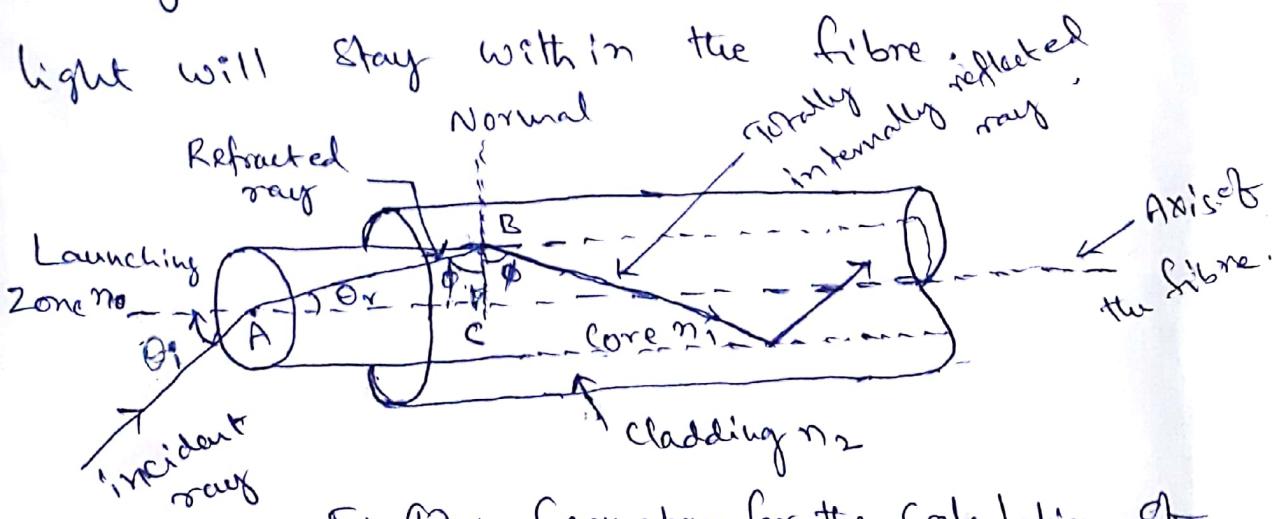


Fig (4) : Geometry for the calculation of acceptance angle of the fibre. (7)

Applying Snell's law to the launching face of the fibre, we get,

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{n_1}{n_0} \quad \text{--- (1)}$$

If  $\theta_i$  is increased beyond a limit,  $\phi$  will drop below the critical value  $\phi_c$  and the ray escapes from the sidewalls of the fibre. The largest value of  $\theta_i$  occurs when  $\phi = \phi_c$ . [  $\theta_i = \theta_{i\max}$  ]

From the  $\Delta ABC$ , it is seen that

$$\theta_r = 90^\circ - \phi$$

$$\sin \theta_r = \sin(90^\circ - \phi) = \cos \phi \quad \text{--- (2)}$$

Using Eqn (2) into Eqn (1), we obtain,

$$\sin \theta_i = \frac{n_1}{n_0} \cos \phi$$

When  $\phi = \phi_c$ ,

$$\sin(\theta_{i\max}) = \frac{n_1}{n_0} \cos \phi_c \quad \text{--- (3)}$$

$$\text{But } \sin \phi_c = \frac{n_2}{n_1}$$

$$\therefore \cos \phi_c = \sqrt{1 - \sin^2 \phi_c}$$

$$\cos \phi_c = \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\cos \phi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \quad \text{--- (4)}$$

(8)

Substituting Eqn (4) into Eqn (3), we get,

$$\sin[\theta_{i(\max)}] = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \times \frac{n_1}{n_0}$$

$$\therefore \sin[\theta_{i(\max)}] = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \quad \text{--- (5)}$$

Quite often the incident ray is launched from air medium, for which  $n_0 = 1$

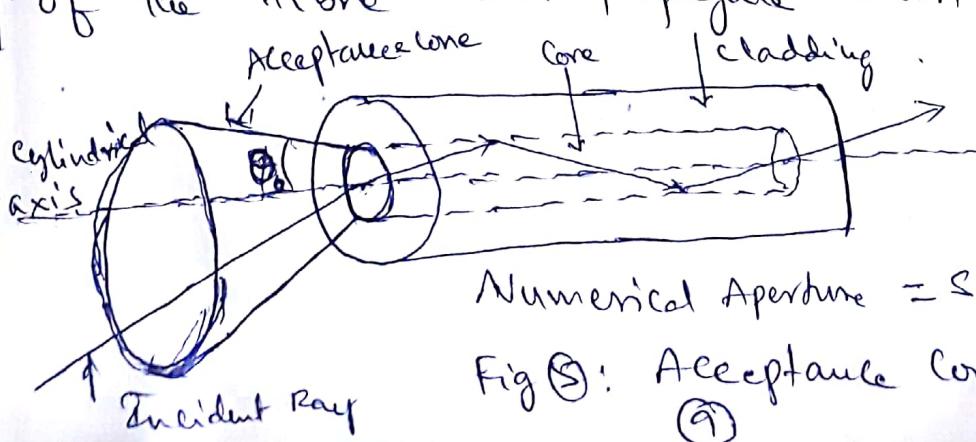
Designating  $\theta_{i(\max)} = \theta_o$ , Eqn (5) may be simplified to,

$$\sin \theta_o = \sqrt{n_1^2 - n_2^2} \quad \text{--- (6)}$$

$$\theta_o = \sin^{-1} (\sqrt{n_1^2 - n_2^2}) \quad \text{--- (7)}$$

The angle  $\theta_o$  is called the acceptance angle of the fibre.

Def:- Acceptance angle is the maximum angle that a light ray can have relative to the axis of the fibre and propagate down the fibre.



$$\text{Numerical Aperture} = \sin \theta_o$$

Fig (5): Acceptance cone  
⑨

Def:- In three dimensions, the light rays contained within the cone having a full angle  $2\theta_0$  are accepted and transmitted along the fibre (Fig 5). The cone is called the acceptance cone.

### Fractional Refractive Index Change

Def:- The fractional difference  $\Delta$  between the refractive indices of the core and the cladding is known as fractional refractive index change ( $\Delta$ ),

$$\Delta = \frac{n_1 - n_2}{n_1}$$

### Numerical Aperture

The light gathering ability of a fiber depends on two factors, namely, core size and the numerical aperture. The acceptance angle and the fractional refractive index change determine the numerical aperture of fibre.

Def:- The numerical aperture (NA) is defined as the sine of the acceptance angle. If  $\theta_0$  is the acceptance angle,

$$NA = \sin \theta_0 = \sqrt{n_1^2 - n_2^2}$$

(10)

## Relation between numerical aperture and fractional refractive index change :-

From Eqn (6),

$$\sin \theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$\therefore NA = \sqrt{n_1^2 - n_2^2}$$

$$NA = \sqrt{(n_1 + n_2)(n_1 - n_2)}$$

$$NA = \sqrt{(n_1 + n_2) \Delta n_1} \quad [ \because \Delta = \frac{n_1 - n_2}{n_1}$$

$$\therefore n_1 - n_2 = \Delta n_1 ]$$

Since  $n_1 \approx n_2$ ,

$$\text{So } n_1 + n_2 \approx 2n_1$$

$$\therefore NA = \sqrt{(2n_1)(\Delta n_1)}$$

$$NA = \sqrt{2n_1^2 \Delta}$$

$$\boxed{\therefore NA = n_1 \sqrt{2 \Delta}}$$

Numerical aperture determines the light gathering ability of the fibre. It is a measure of the amount of light that can be accepted by a fibre. It is seen from above equation that 'NA' is dependent only on the refractive indices of the core and cladding materials and does not depend on the physical dimensions of the

(11)

fibre. The value of 'NA' ranges from 0.13 to 0.50. A large 'NA' implies that a fibre will accept large amount of light from the source.

### Propagation of Light Through An Optical Fibre:-

Light propagates as an EM wave through an optical fibre. Light rays entering the fibre strike the core-cladding interface at different angles. As the refractive index of the cladding is less than that of the core, majority of the rays undergo total internal reflection at the interface and the angle of reflection equal to the angle of incidence in each case.

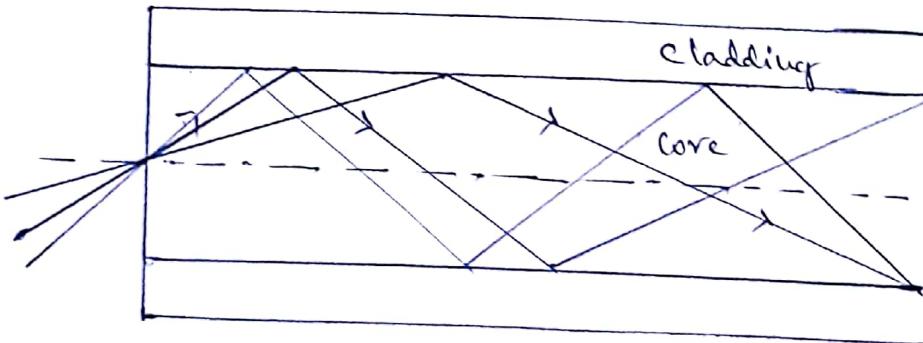


Fig: Propagation of light rays through optical fibre due to total internal reflection,

The rays travel through travel the fibre via a series of total internal reflections and emerge out from

the exit end of the fibre (fig). Because of the negligible loss during the total internal reflections, optical fibre can carry the light waves over very long distances. Thus, the optical fibre acts essentially as a wave-guide and is often called a light guide. At the exit end of the fibre, the light is received by a photo-detector.

### Types of Optical Fibers:-

Optical fibers are classified as follows into various types based on different parameters.

Fig ① shows the classification of optical fibers based on different parameters.

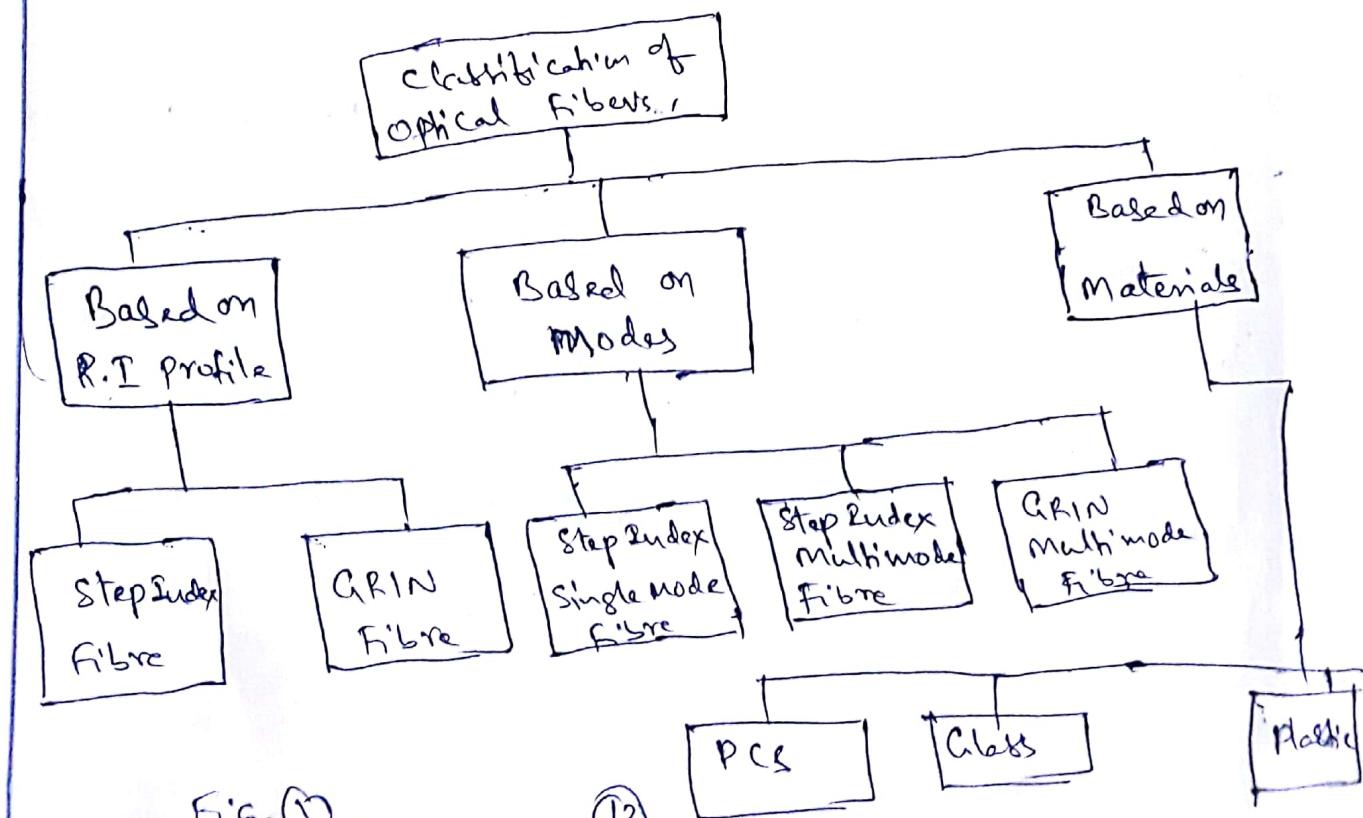


Fig ①,

① 13.

PCF

Cladding

Plastic

### i) Classification based on refractive index profile -

Refractive index profile of an optical fibre is a plot of refractive index drawn on one of the axes and the distance from the core axis drawn on the other axis. Optical fibres are classified into the following two categories on the basis of refractive index profile :-

- (a) Step index fibers
- (b) Graded index (GRIN) fibers.

(a) Step index means the refractive index of the core is constant along the radial direction and abruptly falls to a lower value at the cladding and core boundary (Fig 2(a)).

(b) Graded index means the refractive index of the core is not constant but varies smoothly over the diameter of the core (Fig 2(b)). It has a maximum value at the center and decreases gradually towards the outer edge of the core; At the core-cladding interface the refractive index of the core matches with the refractive index of the cladding. The refractive index of the cladding is constant.

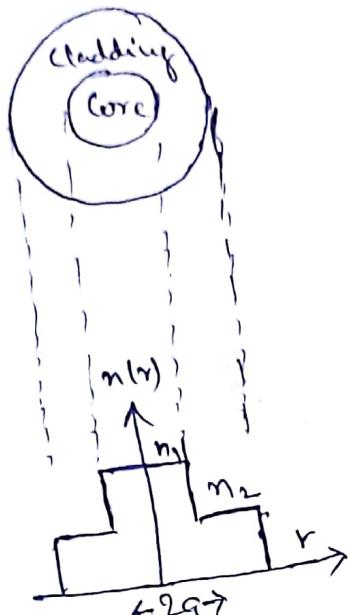
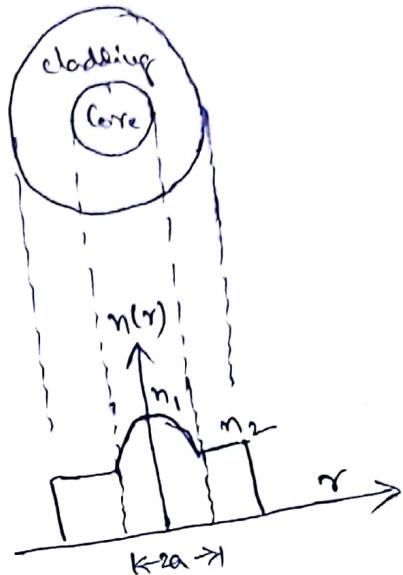


Fig ② @



2 ⑤

(a) Step index fibre

(b) GRIN fibre

iii Classification based on the modes of light propagation! —

On the basis of the modes of light propagation, optical fibers are classified into two categories as (a) Single mode fibres (SMF) and (b) multimode fibres (MMF)

- (a) A single mode fibre (SMF) has a smaller core diameter and can support only one mode of propagation.
- (b) A multimode fibre (MMF) has a larger core diameter and supports a number of modes.

15

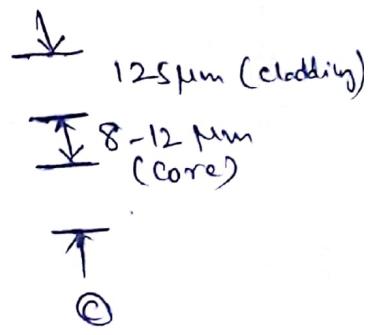
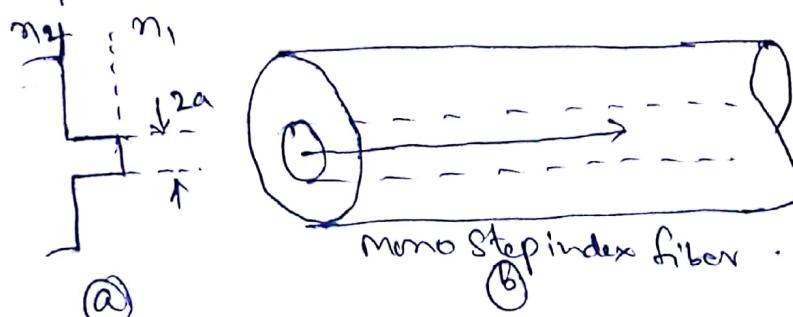
Thus, on the whole, the optical fibers are classified into three types:

- i. Single mode - Step index fibre (SMF)
- ii) Multimode Step index fibre (MMF)
- (iii) Graded index (multimode) fibre (GRIN),

### Single mode Step index fibre :

Structure: A single mode step index fibre has a very fine thin core of diameter of 8  $\mu\text{m}$  to 12  $\mu\text{m}$  (Fig③). Core is generally made of germanium doped silicon and is enclosed by a thick cladding of lower refractive index. The cladding is composed of silica lightly doped with phosphorous oxide. The external diameter of the cladding is of the order of 125  $\mu\text{m}$ . The fibre is surrounded by a non-transparent shielding sheath. The refractive index of the fibre changes abruptly at the core-cladding boundary as shown in fig ③@.

Typical dimensions,



Fig③: Single mode Step index fibre ① RI profile  
② ray paths ③ typical dimensions

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The variation of the refractive index of a step index fibre as a function of radial distance can be mathematically represented as,

$$n(r) = \begin{cases} n_1 & r \leq a \text{ [inside core]} \\ n_2 & r > a \text{ [cladding]} \end{cases}$$

Propagation of light in SMF: Light travels in SMF along a single path that is along the axis. Both A and NA are very small for single mode fibres. It can be achieved by reducing the fibre radius and the relative refractive index change ( $\Delta$ ). The low NA indicates a low acceptance angle and the difficulty in light coupling into the fibre.

### Multimode Step Index fibre:

Structure:- A multimode step index fibre is very much similar to the single mode step index fibre except that its core is of larger diameter. The core diameter is of the order of 50 to 100  $\mu\text{m}$ , which is very larger compared to the wavelength of light. The external diameter of cladding about 150 to 200  $\mu\text{m}$ . (Fig(4)(c))

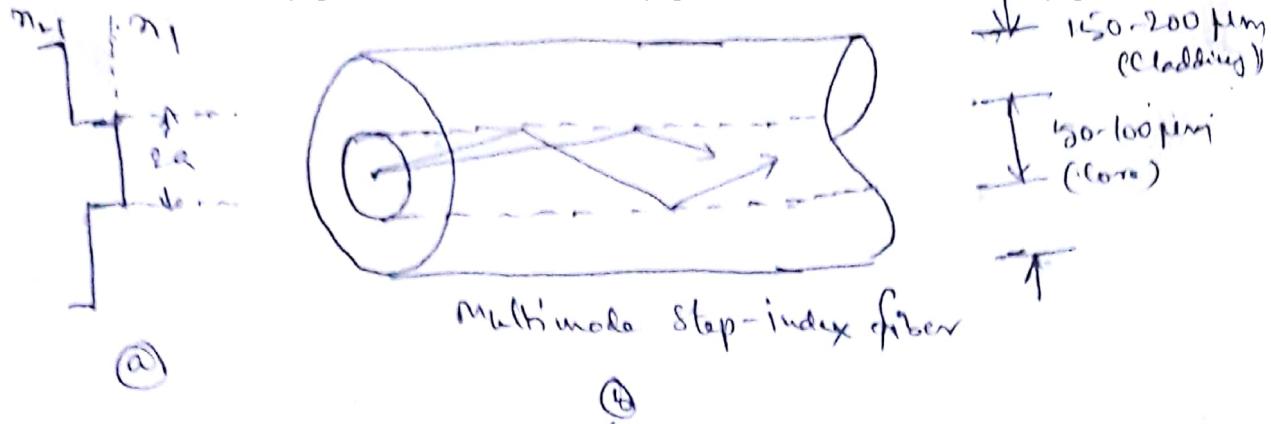


Fig (4) Multimode Step index fiber @ R-L profile

(b) Ray paths (c) typical dimensions,

### Propagation of light in MMF :- multimode Step

index fibres allow finite number of guided modes. The direction of polarization, alignment of electric and magnetic fields will be different in rays of different modes. In other words, many zigzag paths of propagation are permitted in a MMF. The path length along the axis of the fibre

is shorter while the other zigzag paths are longer.

Because of this difference, the lower order modes reach the end of the fiber earlier while the high order modes reach after some time delay (Fig (b)).

### Graded Index (GRIN) Fibre:-

Structure:- A graded index fibre is a multimode fibre with a core consisting of concentric layers of different refractive indices. Therefore, the refractive

index of the core varies with distance from the axis. It has a high value at the centre and falls off with increasing radial distance from the axis. A typical structure and its index profile are shown in fig(s) (a), such a profile causes a periodic focusing of light propagating through the fibre. The size of the graded index fibre is about the same as the step index fibre.

The variation of the refractive index of the core with radius measured from the centre is given by,

$$n(r) = \begin{cases} n_1 \sqrt{1 - \left[ 2 A \left( \frac{r}{a} \right)^{\alpha} \right]}, & r \leq a \text{ inside core} \\ n_2 & r > a \text{ in cladding} \end{cases}$$

where  $n_1$  is maximum refractive index at the core axis, 'a' is the core radius and  $\alpha$  the grading profile index number which varies from 1 to  $\infty$ .

When  $\alpha = 2$ , the index profile is parabolic and is preferred for different applications.

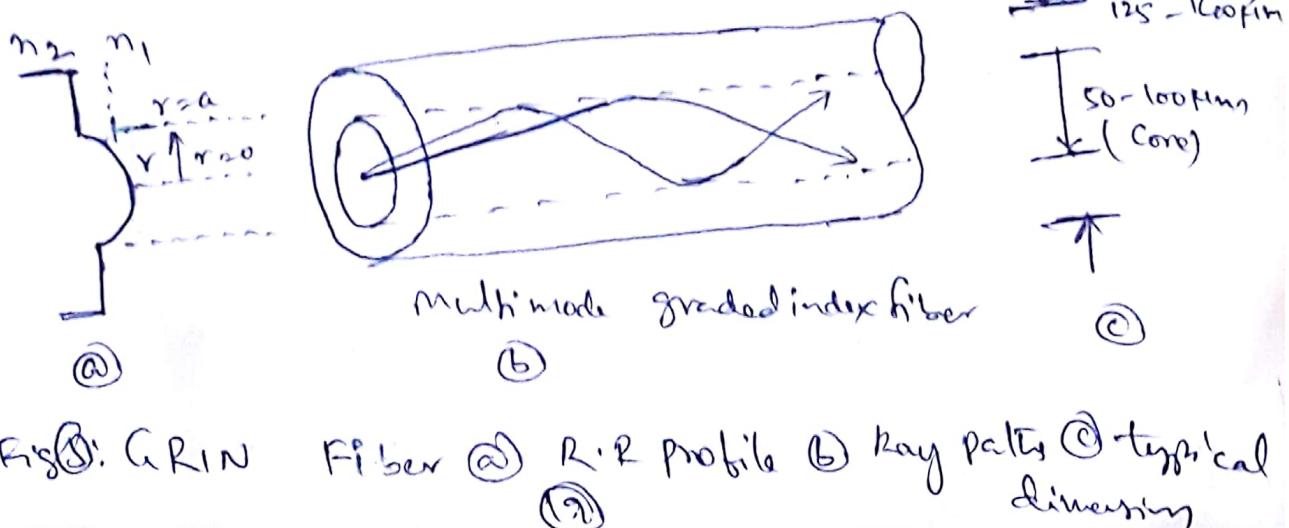


Fig (a): GRIN Fiber @ R.I.R profile (b) Ray Paths (c) typical dimensions

Propagation of light in GRIN fibre: When light ray travels from a region of higher refractive index to a region of lower refractive index, it is bent away from the normal till the condition of total internal reflection. Then they travel back towards the core axis, again being continuously refracted [Fig 5(6)]. The turning around of the ray may happen even before reaching the core-cladding boundary. Accordingly, continuous refraction is followed by total internal reflection and again continuous refraction towards the axis. In GRIN fibre, rays making larger angles with the axis travel longer paths. However, as they travel in a region of lower refractive index and hence at a higher speed of propagation, consequently, all rays travelling through the fibre, irrespective of their mode of travel, will have almost the same optical path length and reach the output end of the fibre at the same time.

In case of GRIN fibers, the acceptance angle and NA decreases with radial distance from the axis, the NA of a graded index fibre is given by,

$$\text{NA} = \sqrt{n^2(r) - n_2^2} = n_1 (2A)^{\frac{1}{2}} \sqrt{1 - \left(\frac{r}{a}\right)^2}$$

$$(20) \quad = n_1 \sqrt{2A \left[1 - \left(\frac{r}{a}\right)^2\right]}$$

## Losses in optical fibers:

### Attenuation in optical fibers:

The power of the light at output end is found to be always less than the power launched at the input end. This is called attenuation in fibers. The attenuation is found to be a function of fiber material, wavelength of light and length of the fiber.

Attenuation in an optical fiber is caused by absorption, scattering and bending losses. Signal attenuation is defined as the ratio of optical output power to the optical input power.

Loss in optical fiber.

$$= -10 \log \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right) \text{ dB.}$$

Most fiber manufacturers characterize attenuation loss by the no of dBs. Loss per kms of the fibers. This value can be calculated by

$$\text{loss/km} = \frac{-10}{L} \log \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right) \text{ dB/km}$$

where  $P_{\text{out}}$  = power coming out of the fibers

$P_{\text{in}}$  = the power launched into the fibers

$L$  = the length of the fibers.

Absorption is major cause of signal loss in an optical fiber. Absorption is defined as the portion of a attenuation resulting from the conversion of optical power into another energy form such as heat. Absorption in OFS is cause by three factors. they are

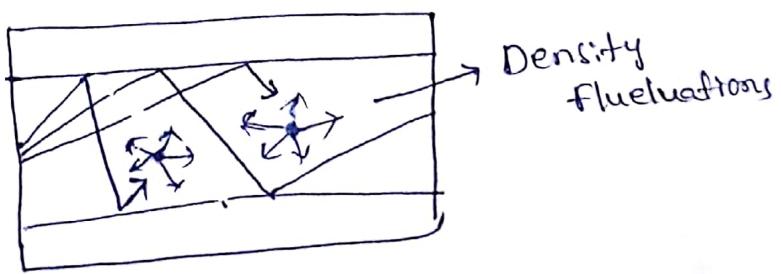
- a) Imperfection in the atomic structure of the material
- b) the intrinsic (or) basic fiber properties
- c) the extrinsic (presence of impurities) fiber material properties.

ii) Scattering:= Basically scattering losses are caused by

interaction of light with density fluctuation with in a fiber. Density changes are produced when optical fibers are manufactured

During manufacturing, regions of higher and lower molecular density area, relative to the average density of the fiber are created

Light travelling through the fiber interacts with the density areas as shown in fig light is scattered in all directions. then partly



III) Bending loss: Bending the fibers also cause attenuation. From, bending loss is classified according the bend radius of curvature, microbend loss and macro bend loss.

micro bends are small microscopic bends of the fiber axis that occur mainly when the fiber axis bend occurs in cabled.

Macrobends are bends having large radius of curvature relative to the fiber diameter.

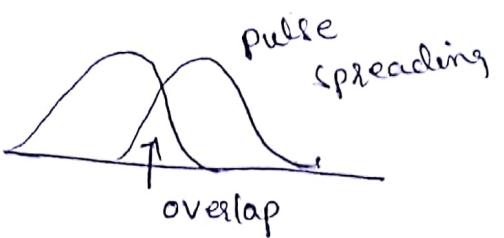
Fiber loss caused by micro bending can still occur even if the fiber is cabled for correctly during installation. If fibers are bent too sharply macro bend losses will occur.

Dispersion:= spreading of pulse width during transmission through the fiber is called dispersion. The dispersion occurs in all types of fibers. There are two different types of dispersion in OFS. They are

i) intra modal and ii) inter modal dispersion.

Intra modal dispersion occurs

because different colours of light travel through different materials and different wave guide structure at different splice



Inter modal dispersion causes the input light pulse to spread. The input signal pulse is made up of a group of modes. This dispersion occurs because each mode travels at different distance over the same time span as shown in fig.

The mode of the light pulse that enter the fiber at one time exit the fiber at different times. This condition causes the light pulse to spread. As the length of fiber increases modal dispersion increases.

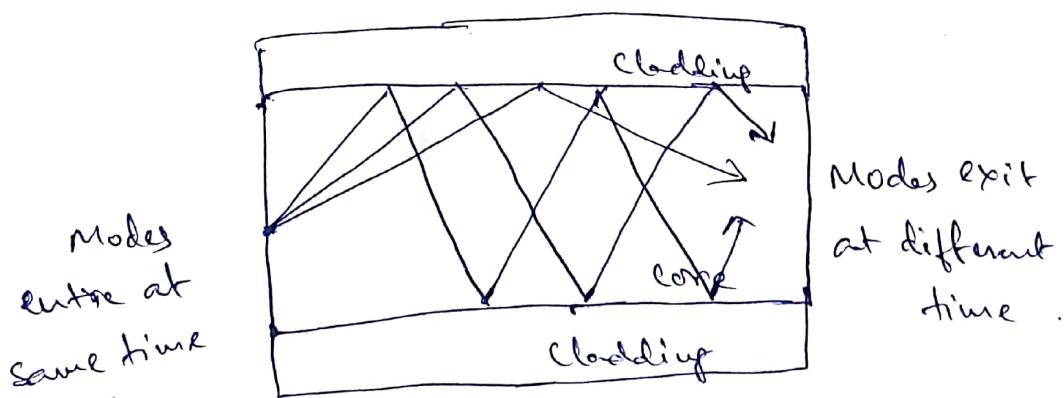


Fig: Distance travelled by each mode over the same time span.

## Applications of Optical fibers.

Transmission of light via an optical fibre has a wide variety of applications.

### i, Fibre optic communication system:

A fibre optic communication system is very much similar to a traditional communications system and has three major components. A transmitter converts electrical signal to light signals, an optical fibre transmits the signals and a receiver captures the signals at the other end of the fibre and converts them to electrical signals.

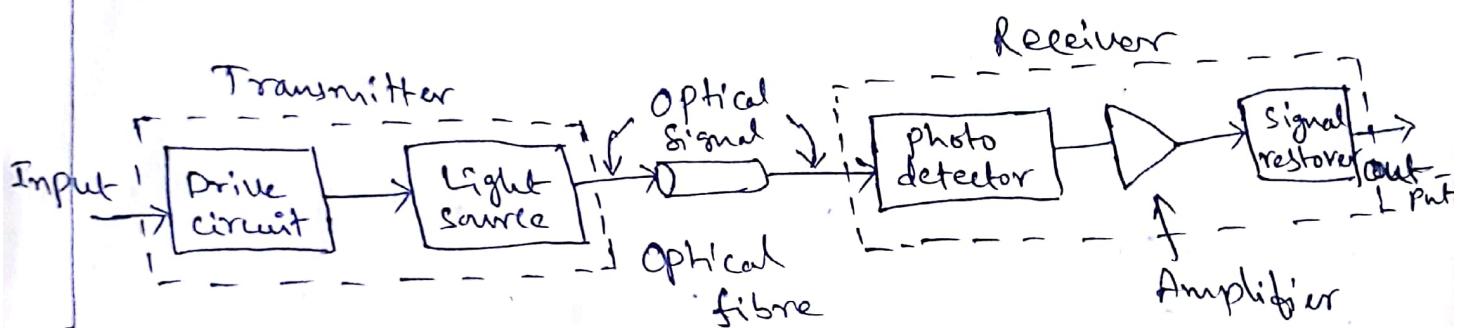


Fig: Illustration of a typical fibre optic communication link.

A basic communication system consists of a transmitter, a receiver and an information pathway. Normally, the information to be communicated is to be converted first into an electrical form. The

Conversion is done by a transducer. These electrical messages are superposed on a carrier wave of very high frequency. The process of imposing a message signal on a carrier wave is called modulation. The message travels along the transmission channel and is received at the receiver. The receiver demodulates the modulated wave and separates out the message and feeds to a transducer such as a loud speaker. The bandwidth requirement of the message and the bandwidth of the carrier determine the number of messages that can be simultaneously transmitted on an information channel. Thus, the use of light waves expands our communication capabilities immensely.

ii) Fibre optic Sensors: Fibre Optic Sensors are transducers, which generally consist of end. The fibres used could be either multimode (Or) single mode type. The sensors can be used to measure pressure, temperature, strain, the acoustic field, magnetic field, etc physical parameters. The advantages of these sensors are that they are lighter, occupy lesser volume and are cheaper. The optical fiber merely carries the

light beam in some of the lenses and in others  
of the fibre it self acts as the lens.

### iii, Medical applications:

(a) Fibre optic technology is used in medical diagnostics as well as in medical procedures. The fibre optic endoscope is used to inspect internal organs for diagnostic purposes.

(b) In ophthalmology, a laser beam guided by optical fibers is used to reattach detached retina and to correct defective vision.

(c) In cardiology, optical energy transmitted through a optical fibre is used to evaporate built-up plaque that is blocking an artery.

(d) In the treatment of cancer also the optical fibre technology is used.

### iv, Military applications:

② An aircraft, a ship or a tank needs tons of copper wire for wiring of the communication equipment, control mechanisms, instrument panel illumination etc. Use of Optical fibre in place of copper

Reduces weight and further maintains true communication silence to the enemy.

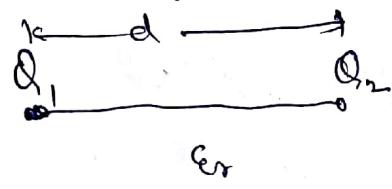
- ⑥ Fibre guided missiles are used in recent wars. Sensors mounted on the missile transmit video information through the optical fibre to a ground control van and receive commands from the van again. The control van continuously monitors the course of the missile and if necessary corrects its course to ensure that the missile precisely hits the target.

Electromagnetismand Magnetic Properties of Materials:-Laws of Electrostatics:-

1<sup>st</sup> law:- like charges of electricity repel each other, whereas unlike charges attract each other.

2<sup>nd</sup> law:- This is known as coulomb's law.

- i. is directly proportional to the product of their strengths,
- ii. is inversely proportional to the square of the distance between them and
- iii. is inversely proportional to the absolute permittivity of the surrounding medium.



$$\therefore F = \frac{Q_1 Q_2}{4\pi\epsilon_0 \epsilon_r d^2}$$

Where  $\epsilon_0$  = permittivity of free space.

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$\text{Therefore } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$$\epsilon_r = \text{Relative permittivity of medium}, \quad \textcircled{1}$$

## Electric current and continuity equation :-

Electric charges in motion constitute a current.

Current is symbolized by  $I$ .

$$I = \frac{dq}{dt}$$

Current is defined as the motion of positive charges, although conduction in metals takes place through the motion of electrons.

In the field theory, we are interested in current density  $\mathbf{J}$  at a point rather than in current within some large region.

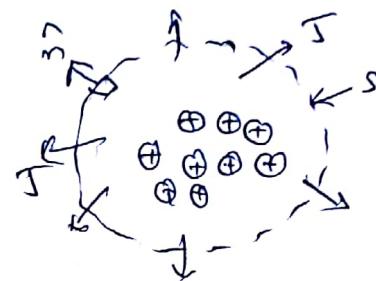
The incremental current  $\Delta I$  crossing an incremental surface  $\Delta S$  is,

$$\Delta I = \mathbf{J} \cdot \Delta \mathbf{S}$$

Total Current is given by,

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

Let us imagine a closed surface 'S' in a conducting medium through which a current is passing. Fig shows current flowing out of the region 'S'



∴ This current must be equal to the rate of decrease of positive charge within the volume.

$$\therefore \int_S J \cdot dS = - \frac{\partial Q}{\partial t}$$

The total charge inside a volume can be expressed in terms of charge density  $\rho$  as,

$$Q = \int_V \rho dV$$

$$\therefore \int_S J \cdot dS = - \frac{\partial}{\partial t} \int_V \rho dV$$

Applying divergence theorem to the left side of above equation, we get,

$$\therefore (\nabla \cdot J) dV = - \frac{\partial}{\partial t} \int_V \rho dV$$

This equation must be true for any shape of volume. Hence, the integrands can be equated.

They, we get,

$$\nabla \cdot J = - \frac{\partial \rho}{\partial t}$$

The above equation is the well known Continuity equation. It states that the amount of current diverging from an infinitesimal volume element is equal to the time rate of decrease of charge contained within it.

(3)

## Ampere's law and Faraday's law:-

Ampere's law:- In 1820 Ampere established that the line integral of magnetic field taken about any given closed path must be equal the current enclosed by the path.

$$\oint \mathbf{H} \cdot d\mathbf{l} = I$$

Where  $H$  is the magnetic field strength.

Suppose the current is distributed through space with a current density  $J$ , then the above equation may be written as,

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int J \cdot d\mathbf{s}$$

Applying Stokes' theorem, we convert the equation to the differential form,

$$\nabla \times \mathbf{H} = J$$

The Ampere's law implies that a magnetic field is produced only by a motion of charges.

Faraday's law:- In 1831 Michael Faraday made one of the most fundamental discoveries. He established that a

time-varying magnetic field induces an emf in a closed conducting loop. The induced emf does not depend on how the change in magnetic flux was caused but depends only on the rate of its variation.

$$\text{emf} = -\frac{d\phi}{dt}$$

Where ' $\phi$ ' is the magnetic flux linked with the loop and  $d\phi/dt$  is the rate of change of magnetic flux. The minus sign is an indication that the emf is in such directions, as to produce a current whose flux, if added to the original flux, would reduce the magnitude of the emf.

If we consider a loop of wire, the magnetic flux through the loop is given by integrating the normal components of the magnetic flux density ' $B$ ' over any surface ' $S$ ', which has the circuit as a boundary, as shown in fig.

$$\phi = \int_S B \cdot dS$$

The emf voltage is related to the work done in moving a charge around a closed path. If an emf voltage is produced in the circuit, it implies that a force must exist on the electric charges, in the conductor. The force

could arise due to an electric field  $\vec{E}$  which acts tangential to the loop. The work per unit charge due to  $\vec{E}$  when added around the loop, must be equal to the induced emf in it. Thus,

$$\text{emf} = \oint_{\text{loop}} \vec{E} \cdot d\vec{l} \quad \dots \textcircled{1}$$

$$\oint_{\text{loop}} \vec{E} \cdot d\vec{l} = - \frac{d\Phi}{dt}$$

$$\therefore \oint_{\text{loop}} \vec{E} \cdot d\vec{l} = - \frac{\partial}{\partial t} \oint_{\text{surface}} \vec{B} \cdot d\vec{s}, \dots \textcircled{2}$$

Eqn  $\textcircled{1}$  is known as the Faraday's law. This law states that the line integral of the electric field around any closed path equals the rate of change of magnetic flux through any surface area bounded by that path.

### Maxwell's equations -

Maxwell found that all the basic principles of electromagnetism can be formulated in terms of four fundamental equations, now called Maxwell equations. The equations existed as experimental laws in the form of Gauss law, Faraday's law and Ampere's law.

$$\text{i), } \nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad \text{Faraday's law}$$

$$\text{ii), } \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{Ampere's law}$$

$$\text{iii), } \nabla \cdot \vec{D} = \rho \quad \text{Gauss's law}$$

$$\text{iv), } \nabla \cdot \vec{B} = 0$$

⑥

This integral form of the above equation are,

$$\text{i}, \quad \oint \vec{E} \cdot d\vec{l} = - \left\{ \frac{\partial B}{\partial t} \right. \cdot ds \quad \left. \right\}$$

$$\text{ii}, \quad \oint \vec{H} \cdot d\vec{l} = \emptyset + \left\{ \frac{\partial D}{\partial t} \right. \cdot ds \quad \left. \right\}$$

$$\text{iii}, \quad \oint \vec{D} \cdot d\vec{l} = \int_{VA} P dV \quad \left. \right\}$$

$$\text{iv}, \quad \oint \vec{B} \cdot d\vec{s} = 0.$$

Maxwell's equations which are in the vector form can be expressed in scalar form as follows,

$$\text{i}, \quad \frac{\partial E_2}{\partial y} - \frac{\partial E_1}{\partial z} = - \frac{\partial B_3}{\partial t}$$

$$\text{iii}, \quad \frac{\partial D_m}{\partial n} + \frac{\partial A_1}{\partial y} + \frac{\partial A_2}{\partial z} = 0$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_2}{\partial n} = - \frac{\partial B_y}{\partial t} \quad \text{(iv)} \quad \frac{\partial B_3}{\partial n} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

$$\frac{\partial E_y}{\partial n} - \frac{\partial E_1}{\partial x} = - \frac{\partial B_2}{\partial t}$$

$$\text{ii}, \quad \frac{\partial H_2}{\partial y} - \frac{\partial H_1}{\partial z} = \frac{\partial D_x}{\partial t} + j_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_2}{\partial n} = - \frac{\partial D_y}{\partial t} + j_y$$

$$\frac{\partial H_y}{\partial n} - \frac{\partial H_1}{\partial y} = \frac{\partial D_z}{\partial t} + j_z \quad \text{(v)}$$

Electric dipole :-

A system of separation of equal positive and negative charges.

Dipole moment ( $\mu$ ):- The product of magnitude of the charge and the distance of separation is known as electric dipole moment.

$$\therefore \mu = qd, \text{ SI unit: cm.}$$

Polarizability ( $\alpha$ ):- Induced dipole moment per unit electric field,

$$\alpha = \frac{\mu}{E}, \text{ SI } \rightarrow \text{cm}^2 \text{V}^{-1} (\because E^2 \frac{\mu}{\alpha})$$

Electric displacement @) Electric flux density (D)

$$D = \epsilon E \geq \epsilon_0 \epsilon_r E \quad (\because \epsilon_r = \frac{\epsilon}{\epsilon_0})$$

$$\text{When } \epsilon_r = 1, D \geq \epsilon_0 E$$

Relation between 'D' and 'P' is,

$$D = \epsilon_0 E + P$$

$$\therefore P = D - \epsilon_0 E = \epsilon_0 \epsilon_r E \quad (\because D \geq \epsilon_0 \epsilon_r E)$$

$$P = \epsilon_0 (\epsilon_r - 1) E \quad \boxed{P = \epsilon_0 (\epsilon_r - 1) E}$$

Polarizations are three types,

① Electronic Polarization ( $\alpha_e$ ), ② Ionic polarization ( $\alpha_i$ )  
and Orientation polarization ( $\alpha_o$ )

$$\text{Electric susceptibility } (\chi_e) = \epsilon_r - 1$$

(C.I.W.)

Polarization: When an electric field is applied to a dielectric (or) a glass containing positive and negative charges, the positive charges are displaced in the direction of the field while negative charges are displaced in the opposite direction. The displacement of charges produce local dipoles throughout the solid. This process of producing dipoles by the influence of an electric field is called electric-polarization. It is defined as the induced dipole moment per unit volume.

$$\therefore P = \frac{\mu}{\text{Volume}} \quad \begin{matrix} \mu = \text{dipole moment} \\ \text{SI Unit} \rightarrow \text{C/m}^2 \end{matrix}$$

$$\therefore P = N \mu e, \text{ where } N = \frac{\text{no. of molecules}}{\text{per unit volume.}}$$

Permittivity! - Permittivity is the ability of medium material. It describes how the electric field is affected by a dielectric medium.

Dielectric Constant! - The dielectric constant of the material is defined as the ratio of the permittivity of the medium ( $\epsilon$ ) to the permittivity of the free space ( $\epsilon_0$ ).

$$\therefore \epsilon_r = \frac{\epsilon}{\epsilon_0} \quad (9)$$

Where  $\epsilon_r$  = dielectric constant @ Relative permittivity.

It has no units. It is measure of polarization in the dielectric material.

The permittivity of free space ( $\epsilon_0$ ) =  $8.85 \times 10^{-12} \text{ F/m}$

The dielectric constant of air = 1

### Internal fields in a solid [Lorentz Method] [ $E_i$ ]

When a dielectric material is inside an electric field, due to polarization, the electric field inside the material increases. Total electric field present at a point (or) an atom is known as Internal field,

$\therefore E_i = E +$  The field due to all other dipoles

### Calculation of Internal field:-

Let a dielectric be placed between the plates of a parallel plate capacitor and let there be an imaginary spherical cavity around the atom 'A' inside the dielectric as shown in fig ①. It is also assumed that the radius of the cavity is large compared to the radius of an atom.

①

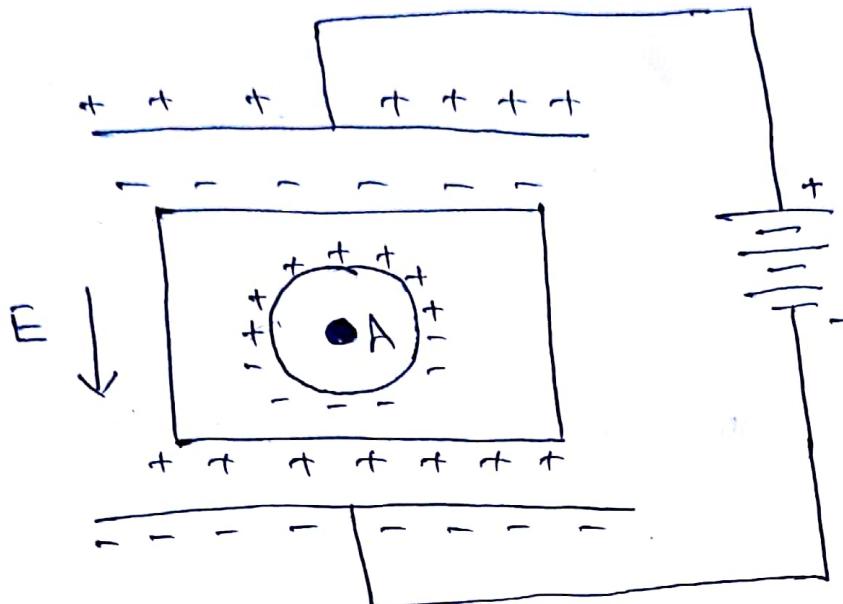


Fig 1: Calculation of internal field for a cubic structure.

The internal field at the atom site 'A' can be considered to be made up of the following four components namely,  $E_1, E_2, E_3$ , and  $E_4$ .

Field  $E_1$  :-  $E_1$  is the field intensity at 'A' due to the charge density on plates.

From field theory,  $E_1 = \frac{D}{\epsilon_0}$  [  $\because D$  = displacement vector

$$\therefore D = P + \epsilon_0 E$$

$$D = \epsilon_0 E \text{ and } D = P + \epsilon_0 E$$

$$\therefore E_1 = \frac{P + \epsilon_0 E}{\epsilon_0} = \frac{P}{\epsilon_0} + E$$

$$\therefore E_1 = E + \frac{P}{\epsilon_0} \quad \textcircled{1}$$

(11)

Field  $E_2$ : -  $E_2$  is the field intensity at 'A' due to.

The charge density induced on the two sides of the dielectric (field induced in opposite direction)

$$\therefore E_2 = -\frac{P}{\epsilon_0} \quad \left[ \because E = \frac{D}{\epsilon_0} = \frac{P}{\epsilon_0} \right]$$

Field  $E_3$ : -  $E_3$  is the field intensity at 'A' due to other atoms contained in the cavity. We are assuming a cubic structure. So  $E_3 = 0$  because of symmetry.

Field  $E_4$ : -  $E_4$  is the field intensity due to polarization charges on the surface of the cavity and was calculated by Lorentz as given below.

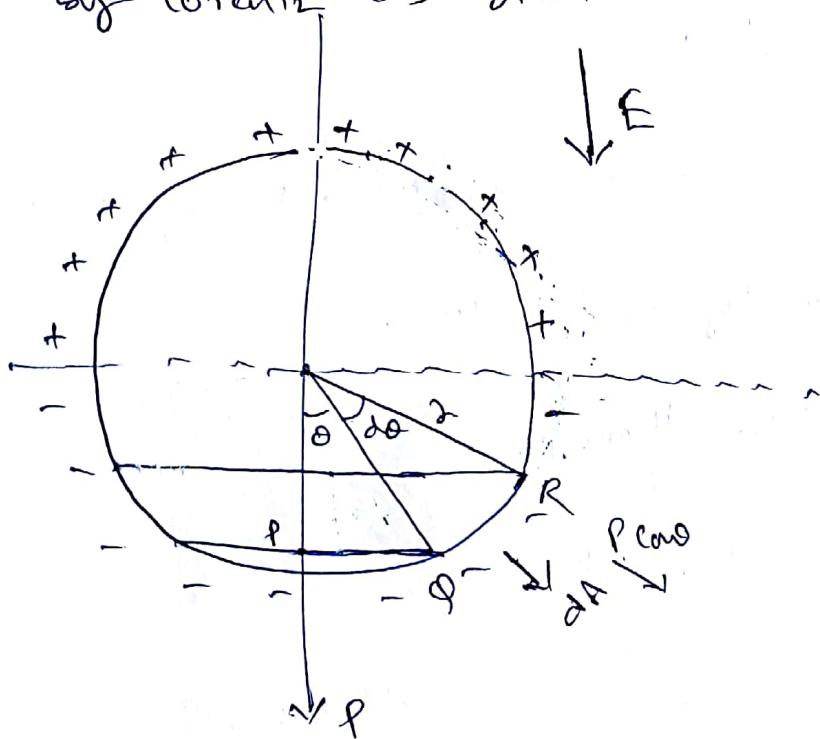


Fig ②: Cavity enlarged,

The enlarged view of the cavity is shown in fig ②.

If  $dA$  is the surface area of the sphere of radius  $r$  lying between  $\theta$  and  $\theta + d\theta$ , where  $\theta$  is the direction with reference to the direction of the applied force. Then,

$$\therefore dA = 2\pi (PQ)(QR)$$

$$\text{But } \sin\theta = \frac{PQ}{r} \Rightarrow PQ = r \sin\theta \text{ and}$$

$$d\theta = \frac{QR}{r} \Rightarrow QR = r d\theta.$$

$$\text{Hence } dA = 2\pi (r \sin\theta) (r d\theta)$$

$$\therefore dA = 2\pi r^2 \sin\theta d\theta$$

The charge  $dq$  on the surface  $dA$  is equal to the normal component of the polarization multiplied by the surface area.

$$\therefore dq = P \cos\theta dA = P (2\pi r^2 \sin\theta \cos\theta d\theta)$$

The field due to this charge at 'A', denoted by  $dE_4$ ,

$$dE_4 = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq \cos\theta}{r^2} \quad \left[ \because E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \right]$$

$$dE_4 = P \underbrace{\left( 2\pi r^2 \sin\theta \cos\theta d\theta \right) (\cos\theta)}_{4\pi\epsilon_0 r^2}$$

$$dE_4 = \frac{P \cos^2\theta \sin\theta d\theta}{2\epsilon_0}$$

(13)

$$\therefore dE_4 = \frac{P}{2\epsilon_0} \cos^2\theta \sin\theta d\Omega$$

Thus the total field  $E_4$  due to the charges on the surface of the entire cavity is obtained by integrating,

$$\int dE_4 = \frac{P}{2\epsilon_0} \int_0^{\pi} \cos^2\theta \sin\theta d\Omega$$

$$(\because \sin\theta d\theta \approx d(r\cos\theta))$$

$$E_4 = \frac{P}{2\epsilon_0} \int_0^{\pi} \cos^2\theta d(r\cos\theta)$$

$$2r\cos\theta d\theta \approx x, \\ \int r^2 dr = \frac{x^3}{3}$$

$$= -\frac{P}{2\epsilon_0} \left[ \frac{\cos^3\theta}{3} \right]_0^{\pi}$$

$$= -\frac{P}{6\epsilon_0} [-1 - 1]$$

$$= +\frac{P}{6\epsilon_0} (+2)$$

$$\therefore E_4 = \frac{P}{3\epsilon_0}$$

$$\therefore E_i = E_1 + E_2 + E_3 + E_4$$

$$E_i = E + \frac{P}{3\epsilon_0}$$

Where  $E_i$  is the total internal field or Lorentz field and it should observe that the Lorentz field is greater than the applied field.

(14)

## Clausius - Mossotti Equation:-

It is the relation between the dielectric constant of a dielectric and a polarizability of atoms.

Let us consider the elemental dielectric having cubic structure. Since there are no ions and permanent dipoles in these materials, the ionic polarizability ( $\alpha_i$ ) and Orientation polarizability ( $\alpha_o$ ) are zero.

$$\text{i.e } \alpha_i = \alpha_o = 0$$

Hence polarization  $P = N\mu$

$$P = N\alpha_e E \quad (\because \mu = \alpha_e E)$$

$$\therefore P = N\alpha_e E_i \quad \text{---(1)}$$

$$\text{But } E_i = E + \frac{P}{3\epsilon_0}$$

Substitute  $E_i$  value in Eqn(1), we get,

$$\therefore P = N\alpha_e \left[ E + \frac{P}{3\epsilon_0} \right]$$

$$P = N\alpha_e E + \frac{N\alpha_e P}{3\epsilon_0}$$

$$\therefore N\alpha_e E = \left[ P - \frac{N\alpha_e P}{3\epsilon_0} \right]$$

$$\therefore N\alpha_e E = P \left[ 1 - \frac{N\alpha_e}{3\epsilon_0} \right] \quad (15)$$

$$\therefore P = \frac{N_{de} E}{\left(1 - \frac{N_{de}}{3\epsilon_0}\right)} \quad \text{--- (2)}$$

We know that,  $P = \epsilon_0 (\epsilon_r - 1) E \quad \text{--- (3)}$

From Eqn (2) and Eqn (3), we get,

$$\epsilon_0 (\epsilon_r - 1) E = \left( \frac{N_{de} E}{\left(1 - \frac{N_{de}}{3\epsilon_0}\right)} \right)$$

$$\therefore \epsilon_0 (\epsilon_r - 1) = \frac{N_{de}}{\left(1 - \frac{N_{de}}{3\epsilon_0}\right)}$$

$$\therefore 1 - \frac{N_{de}}{3\epsilon_0} = \frac{N_{de}}{\epsilon_0 (\epsilon_r - 1)}$$

$$\therefore 1 = \frac{N_{de}}{3\epsilon_0} + \frac{N_{de}}{\epsilon_0 (\epsilon_r - 1)} \times \frac{3}{3}$$

$$\therefore 1 = \frac{N_{de}}{3\epsilon_0} \left[ 1 + \frac{3}{(\epsilon_r - 1)} \right]$$

$$\therefore \frac{N_{de}}{3\epsilon_0} = \frac{1}{\left[ 1 + \frac{3}{(\epsilon_r - 1)} \right]}$$

$$\frac{N_{de}}{3\epsilon_0} = \frac{1}{\left( \frac{\epsilon_r + 1 + 3}{\epsilon_r - 1} \right)}$$

$$\frac{N_{de}}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \quad \text{--- (16)}$$

Thus

$$\frac{\epsilon_r - 1}{\epsilon_r + 1} = \frac{Nde}{3\epsilon_0}$$

where 'N' is the no. of

molecules per unit volume. The above relation is known as, Clausius - Moatti equation. Using this relation one can determine the value of  $de$  knowing the value of  $\epsilon_r$ .

### Ferro Electric Materials :-

Ferroelectric materials constitute a very important group of dielectrics. These materials exhibit spontaneous polarization. In the absence of an electric field, if the centres of gravity of the positive and negative charges do not coincide, it results in a resultant dipole moment, which is the cause of Spontaneous Polarization. Ferro electricity was first discovered in Rochelle salt ( $\text{NaK}\text{C}_4\text{H}_4\text{O}_6 \cdot 4\text{H}_2\text{O}$ ). It exhibits spontaneous polarization over a range of temperature  $-18^\circ\text{C}$  to  $22^\circ\text{C}$ . Barium titanate, Potassium  $\text{BaTiO}_3$  phosphate and potassium niobate are other examples of ferro electrics.

Def:- The materials, which posses special structure that permits spontaneous polarization, are called ferroelectrics and the phenomenon of spontaneous polarization is called ferro electricity.

(1.7)

Spontaneous polarization is the dielectric polarization which occurs under the action of the internal processes and without the application of an electric field.

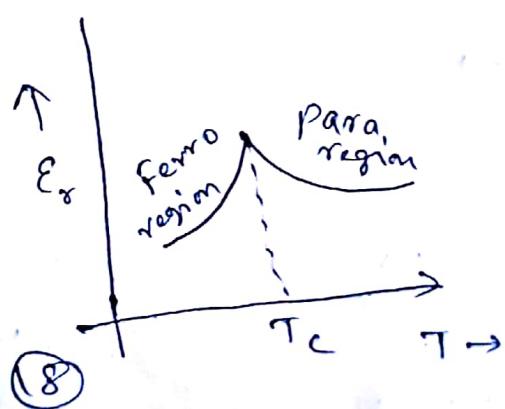
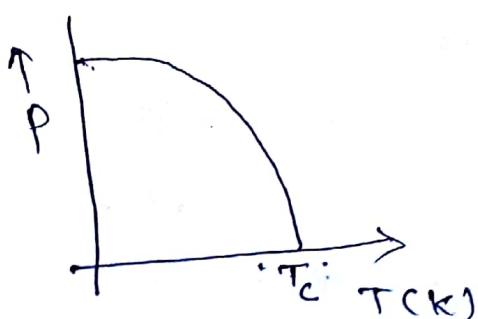
Ferroelectric materials exhibit pyroelectric and piezoelectric properties. The main characteristics of ferroelectric substances are as follows:

- i, They possess very high values of permittivity ' $\epsilon_r$ ' of the order of 1000 to 10,000.
- ii, The static dielectric constant changes with temperature according Curie-Weiss law.

$$\therefore \epsilon_r = \frac{C}{T - T_c} (T \neq T_c)$$

Where 'c' is called Curie constant and  $T_c$  is the Curie temperature.

- iii, The spontaneous polarization versus temperature is shown in fig(1). At Curie temperature, spontaneous polarization becomes zero and ferroelectric material converts into paraelectric material (fig 2).



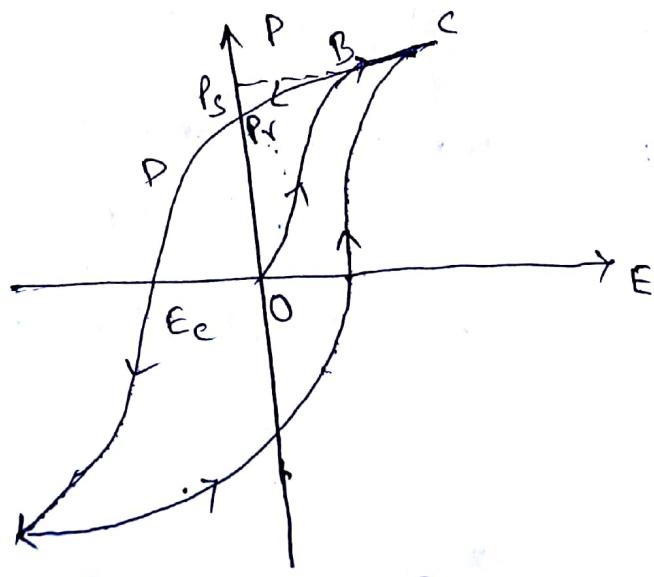
(8)

(iv) Ferro electric Hysteresis:- In a ferroelectric, the dielectric polarization depends nonlinearly on the applied electric field. They exhibit hysteresis under the action of an alternating voltage. The polarization versus electric field curve is known as a ferroelectric hysteresis loop (fig)

The polarization increases nonlinearly and reaches saturation at a certain value  $P_s$ . The polarization will not increase further even if the electric field increased.

When the electric field is switched off, the value of the polarization does not return to zero and the crystal retains a residual polarization  $P_r$ .

In order to bring back the polarization to zero, an electric field  $E_c$  may be applied in the opposite direction,  $E_c$  is known as the coercive field.



(19)

Fig: Ferroelectric Hysteresis loop.

## Applications of Ferroelectric Materials : Application of the

ferroelectric materials Utilizes the pyroelectric, piezoelectric (or) ferroelectric properties of the materials.

i, Capacitors :-  $\text{BaTiO}_3$  (Barium titanate) based ceramics Show dielectric constant values as high as 15,000 as compared to 5 (or) 10 for common Ceramic and polymer materials. The use of high dielectric constant ceramic like  $\text{BaTiO}_3$ , allows large capacitance values to be achieved in relatively small volume capacitor devices.

ii, Generation of Ultrasonic Waves :- piezoelectric crystal provide a coupling between electrical and mechanical forces and hence serve as transducers which produce (or) detect electrical (or) mechanical signals. Hence they are used to detect very small mechanical displacements and small amounts of electric charge. All commercial piezoelectric materials used today are ferroelectrics.

iii, Vibrators :- Piezo electrics are used as vibrators. When an ac voltage is applied across piezoelectric element, it vibrates and at a particular frequency of the field, it resonates.

(iv) Detectors: Piezoelectric Ceramics are used in the generation and reception of Sound Waves in water. They are used in ultrasonic cleaners and under water detectors of sounds. The ultrasound is used to find the internal cracks and other hidden defects in solid bodies like bars, rods, plates, etc.

(v) Piezoelectric Transformers: Low Voltage to High Voltage transformation can be done by using a piezo electric plate.

(vi) Ferroelectric Memories:- ferroelectric materials suddenly polarize on cooling below the  $T_c$ . The magnitude and direction of polarization can be inverted by the help of an external electric field. The ferroelectric RAMs made from ferroelectric thin films use of this phenomenon to store data. Data is stored by localized polarization switching in the microscopic regions of ferroelectric thin films.

Piezoelectric materials: -

Def:- Piezoelectricity is the ability of certain materials to generate AC voltage when subjected to mechanical stress (or) vibration, (or) to vibrate when an AC voltage is applied.

Piezoelectric effect is the accumulation of electric charge in certain solid materials in response to applied

Mechanical stress. Crystals that exhibit piezoelectric effect are called piezoelectric crystals. The most common piezoelectric material is quartz.

The French physicists Pierre Curie and Paul-Jean Curie discovered the piezoelectric effect in 1880.

The development of charges as a result of the material mechanical deformation is known as the direct piezoelectric effect.

The mechanical deformation of piezoelectric materials caused by an external electric field is known as the inverse piezoelectric effect.

Piezoelectric effect is exhibited by a crystal only if the crystalline symmetry is non-centrosymmetric. Ammonium phosphate, quartz and PZT (lead zirconate titanate) are examples of piezoelectric materials.

Magnetization :- Magnetization refers the process of converting a non-magnetic sample into a magnetic sample.

The Magnetization ( $M$ ) of the solid is defined as the induced magnetic moment per unit volume.

$$\therefore M = \frac{\sum m_i}{V} \quad \text{SI unit is A m}$$

Permeability :-

The magnetic induction (or) magnetic flux density ( $B$ ) in any material is the number of lines of magnetic force passing through unit area perpendicularly. Its unit is  $\text{Wb/m}^2$  (or) Tesla.  $B = \phi/A$

The magnetic field intensity ( $H$ ) at any point in the magnetic field is the force experienced on unit north pole placed at that point. Its unit is A/m.

The relation between  $B$  and  $H$  is given by,

$$B = \mu_0 H$$

Where  $\mu_0$  = Permeability of free space (Vacuum)

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m} \quad (2.2)$$

Instead in vacuum if the field is applied in a medium, the magnetic induction in the solid is given by,

$$B = \mu H$$

where ' $\mu$ ' is the permeability of the solid material through which the magnetic lines of the force pass.

$$\therefore \mu = \frac{B}{H}$$

Hence the magnetic permeability ( $\mu$ ) of any material is the ratio of the magnetic induction in the sample to the applied magnetic field intensity. The ratio of  $\mu/\mu_0$  is called the relative permeability.

$$\therefore \mu_r = \frac{\mu}{\mu_0}$$

Susceptibility:- The magnetization of a magnetic material is proportional to the applied field ' $H$ '. Thus,

$$M \propto H \Rightarrow M = \chi_m H$$

Where  $\chi_m$  is the proportionality constant. It is a dimensionless quantity and is called the magnetic susceptibility.

$$\therefore \chi_m = \frac{M}{H}$$

## Relation between $\mu_r$ and $\chi_m$ :-

We know,  $B = \mu_0 H$

Rearrange the above equation,

$$B = \mu_0 (H + M)$$

$$\therefore \mu_0 = \frac{B}{H+M}$$

$$\text{Relative permeability } (\mu_r) = \frac{\mu}{\mu_0}$$

$$\therefore \mu_r = \frac{B/H}{B/H+M}$$

$$\therefore \mu_r = \frac{H+M}{H}$$

$$\mu_r = 1 + \frac{M}{H} \quad \left( \because \chi_m = \frac{M}{H} \right)$$

$$\therefore \mu_r = 1 + \chi_m$$

## Classification of Magnetic Materials :

Magnetic materials are classified into two categories based on magnetic moment and response of the magnetic materials to external magnetic fields.

① Dia magnetic materials — no permanent magnetic moment

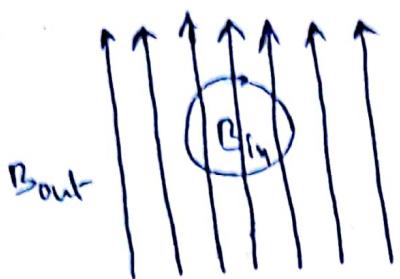
② Paramagnetic, ferromagnetic, anti-ferromagnetic

and ferrimagnetic materials — possess permanent magnetic moment.

## Dia magnetic materials:

In these materials an electron moving around the nucleus, results in magnetic moment. Due to different orientations of various orbits of an atom, the net magnetic moment is zero in dia magnetic materials. When an external magnetic field is applied the motion of electrons in their orbits changes resulting in induced magnetic moment in a direction opposite to the direction of applied field. As result, the magnetic fields are repelled from the materials. This effect is known as dia magnetism. It is a weak effect.

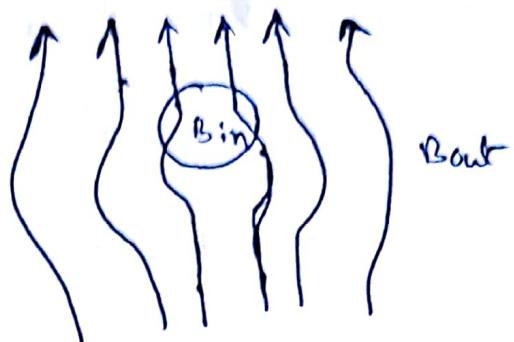
- Properties:-
- ① Permanent dipoles are absent.
  - ② Magnetic effect is very weak and often masked by other kinds of magnetism.
  - ③ When placed inside a magnetic field, magnetic lines of forces are repelled as shown in fig ①.
  - ④ The susceptibility of a diamagnetic materials is negative
  - ⑤ The susceptibility is independent of temperature and external field.
  - ⑥ The Relative permeability is slightly less than one.



$B_{in} = B_{out}$  and  
 $B_{in} \neq 0$

Fig 1@ Normal material

Eg: Copper, gold, mercury, silver and zinc.



$$B_{in} < B_{out} \text{ and } B_{in} = 0$$

(b) Dia magnetic material

E.g.: Copper, gold, mercury, silver and zinc.

Paramagnetic materials: In these materials, each electron in an orbit has orbital magnetic moment and a spin magnetic moment. When shells are unfilled there is net magnetic moment. In the absence of external field, the net moments of the atoms are arranged in random directions because of thermal fluctuations. Hence there is no magnetization.

When external field is applied, there is a tendency of dipoles to align with the field giving rise to an induced positive dipole moment. This induced dipole moment is proportional to the field. The induced magnetism is the source of paramagnetism.

Properties: ① Para magnetic materials possess permanent magnetic dipoles.

② They attracts the magnetic lines of force as shown in fig ②.

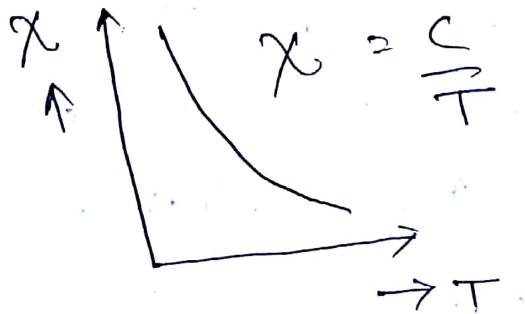
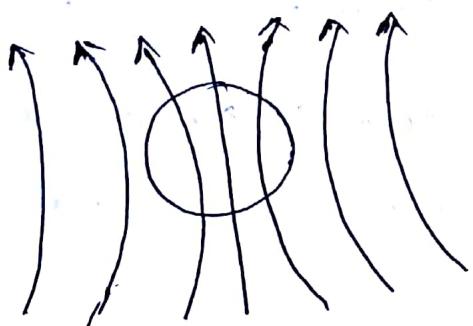
③ Susceptibility is positive, small and depends on temperature.

$$\therefore \chi = \frac{C}{T}, \text{ where 'C' is Curie}$$

constant and  $T$  is the temperature and the equation is known as the Curie law of paramagnetism.

④ Spin alignment is random. 

⑤ Relative permeability is slightly greater than one.



$B_{in} > B_{out}$

Paramagnetic material.

Eg:- Aluminium, Chromium, Sodium, titanium, Zirconium, Oxygen, etc.

Ferro magnetic materials: Ferro magnetic materials possess permanent magnetic moment which is mainly due to the spin magnetic moment. The magnetic dipoles are aligned parallel to each other due to

interaction between any two dipoles. This interaction known as exchange interaction. These materials exhibit spontaneous magnetization, even if the absence of an external field. Due to this these material has a large magnetization. When a small magnetic field is applied, it produces a large value of magnetization due to the parallel alignment of dipoles.

Properties:- ① They possess permanent dipole moment.

- ② They exhibits strong magnetization, even if the absence of magnetic field due to exchange interaction.
- ③ They attracts the magnetic lines of force very strongly when they placed inside a magnetic field.
- ④ Each ferro-magnetic material has a characteristic temperature called the ferro magnetic curie temperature ( $\theta_f$ ), Materials below  $\theta_f$  behave as a ferromagnetic materials and obey hysteresis curve. above  $\theta_f$  they acts as a paramagnetic materials.
- ⑤ Susceptibility is positive and very large.
- ⑥ Susceptibility of a ferro magnetic material is given by,

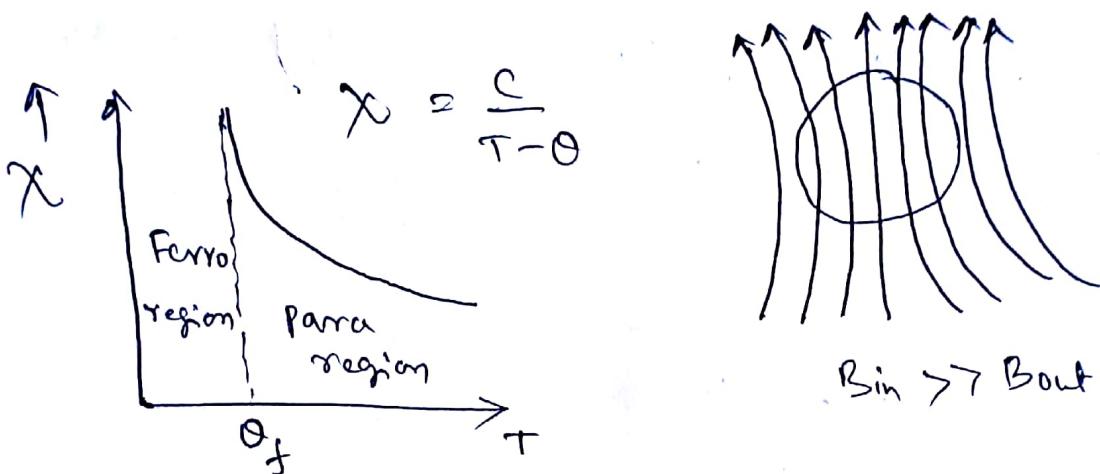
$$\chi = \frac{C}{T - \Theta}, \text{ where } 'C' \text{ is Curie Constant and}$$

' $\Theta$ ' the paramagnetic Curie temperature.

If  $T > \Theta_f$ , Paramagnetic behaviour

If  $T < \Theta_f$ , Ferromagnetic behaviour  
exhibits hysteresis

(7) Spin alignment is parallel in the same direction,



e.g.- Iron, Cobalt, nickel, Steel etc.

### Anti ferro magnetic materials:-

In Ferro magnetic materials, the magnetic dipoles are parallel to each other. This is due to an exchange interaction between the spin magnetic moments. Similarly, in few materials, the exchange interaction leads to anti parallel alignment of dipoles. The magnitude of all dipoles are equal and

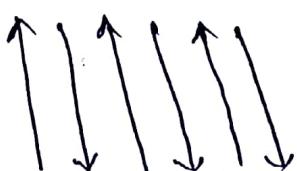
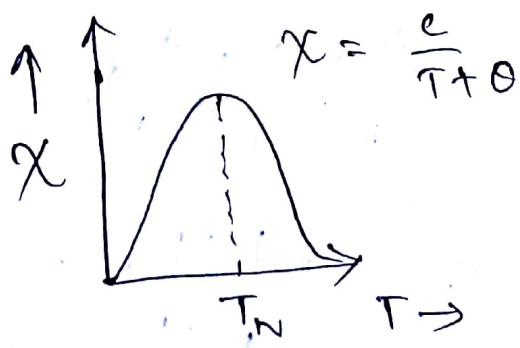
hence, resultant magnetic moment and magnetization is zero. The anti-parallel alignment exists in material below a critical temperature known as Néel temperature.

- Properties:-
- ① The dipoles are aligned anti-parallel.
  - ② Anti-parallel alignment of dipole is due to exchange interaction.
  - ③ When temperature increases, susceptibility increases and reaches a maximum at a temperature known as Néel temperature and beyond Néel temperature the susceptibility decreases with temperature.
  - ④ The value of Susceptibility is positive and very small when  $T$  is greater than the Néel temperature  $T_N$ .

$$\therefore \chi = \frac{C}{T+0} \quad \text{when } T > T_N$$

$$\chi \propto T \quad \text{when } T < T_N$$

- ⑤ Spin alignment is anti parallel.



(31)

Ferri magnetic materials: This is a special case of anti-ferro magnetic. The net magnetization of magnetic sub lattices is not zero since anti-parallel moments are of different magnitudes.

Hence ferri magnetic material possesses a net magnetic moment.

Properties: - ① The dipoles are anti parallel and they are not equal in magnitude.

② Above Curie temperature becomes paramagnetic while below it behaves as ferri magnetic material.

③ The Susceptibility is positive and very large when the temperature is higher than  $T_N$ ,

$$\chi = \frac{C}{T + \theta}, T > T_N,$$

For paramagnetic,  $T < T_N$ .

④ Spin alignment is anti-parallel of different magnitudes.



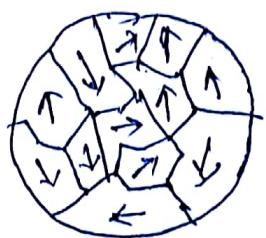
⑤ Ferri magnetic domains are used as magnetic bubbles in memory elements.



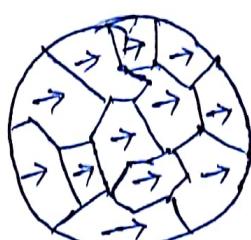
## Ferromagnetic domains:

In 1907 P. Weiss proposed the molecular field theory to explain qualitatively the salient features of ferromagnetism.

Weiss postulated that an internal molecular field causes a parallel alignment of magnetic dipoles (Fig 1⑥) and setup spontaneous magnetization in a ferromagnetic material. The magnetization in the material is at maximum temperature  $0\text{ K}$  and decreases with increasing temperature. It falls off rapidly to zero value at  $T_c$ . If the material is cooled from above the Curie temperature, the spontaneous magnetization reappears at  $T_c$ . At temperature above Curie point, a ferromagnetic material goes into paramagnetic state (Fig 1⑦),



⑥



⑦

Fig 1: ⑥ Schematic illustration of magnetic domains in a demagnetized material. ⑦ Domain Configuration in magnetized body.

According to this theory, the entire ferromagnetic volume splits into a large number of small regions of spontaneous magnetization (Fig 1@). These regions are called 'domains'. Each domain has a finite magnetic moment. In the absence of external magnetic field, the magnetic vectors of the separate volume domains are oriented in all probable directions so that the net magnetic moment of the entire body equal to zero.

When magnetic field is applied, the magnetisation of the material takes place through two processes. In weak field, domains, which are favourably oriented with respect to the applied field, increase their volume, (grow in size) at the expense of unfavourably oriented domains. In strong fields, domains rotate in an attempt to align their magnetic moments with field directions, (Fig 1@).

Hysteresis: A typical property of ferromagnetic materials is hysteresis.

Def:- Hysteresis is the lag in the changes of magnetization behind variations of the magnetic field.

Hysteresis is a loop pattern of the magnetic field induction ' $B$ ' and increasing (or) decreasing magnetic field ' $H$ '

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applied to an initially unmagnetized ferromagnetic specimen (Fig).

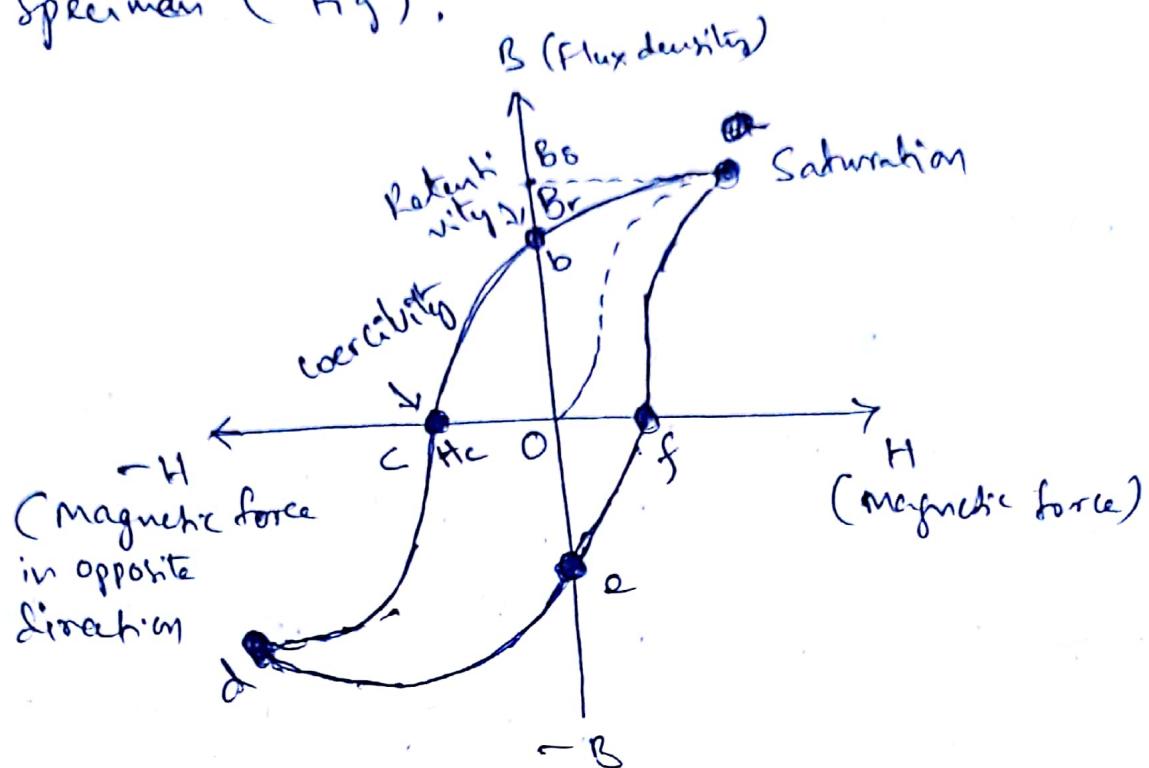


Fig: Hysteresis Loop.

The curve starts at the Origin 'O'. As ' $H$ ' is increased, the field ' $B$ ' increases slowly, then more rapidly. It finally reaches a saturation value and becomes independent of ' $H$ '. The maximum value of ' $B$ ' is known as the saturation flux density ' $B_s$ ' and the corresponding magnetization is the saturation magnetization ' $M_s$ '.

If ' $H$ ' is decreased, ' $B$ ' also decreases but following a path 'AC' instead of the original path 'AO'. Thus ' $B$ ' lags behind ' $H$ '. When ' $H$ ' becomes zero, ' $B$ ' does not become zero but has a value equal to  $O\text{B}(B_r)$ .

It indicates that the material remains magnetized even in the absence of an external applied field 'H'. The power of retaining the magnetism is called the remanability of the material.

Def:- The remanability of a material is a measure of the magnetic flux density remaining in the material when the magnetizing field is removed.

When the magnetic field 'H' is applied in the reverse direction, a field of magnitude  $-H_c$  must be applied to reduce the flux density to zero. It is called the coercivity or the coercive force.

Def:- Coercivity is a measure of the magnetic field strength required to destroy the residual magnetism in the material.

As the applied field 'H' is increased further in the negative direction, saturation is ultimately reached in the reverse direction point i.e.

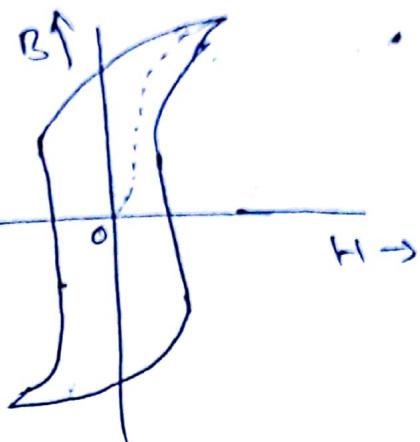
The closed curve abcdefa represents a cycle of magnetization of the specimen and is known as the hysteresis loop of the specimen.

## Magnetic materials and their applications

Based on the area of the hysteresis loop, magnetic materials are two types. They are soft magnetic materials and hard magnetic materials.

### Soft magnetic materials:

The magnetic materials which can be easily magnetized and demagnetized are known as soft magnetic materials.

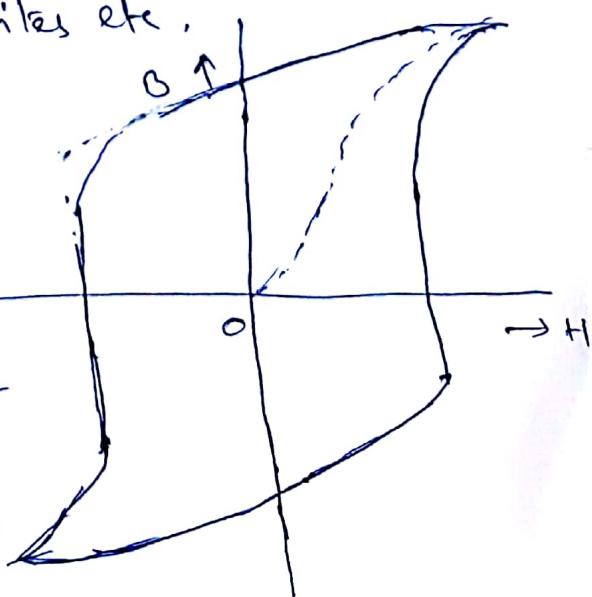


- i, Hysteresis loop area is less as shown in fig.
- ii, Permeability and Susceptibility are large.
- iii, Eddy current loss is more due to higher <sup>resistivity</sup>.
- iv, Coercivity and remanence are small.
- v, Magnetization energy is very small.
- vi, It is widely used in transformers, motors, generators, electromagnets etc.
- vii, Examples: ferrites, garnites etc.

### Hard magnetic materials:

Hard materials which cannot easily magnetized and demagnetized are known as Hard magnetic materials.

(37)



- i, Hysteresis loop area is more as shown in fig.
- ii, Permeability and Susceptibility are small.
- iii, Eddy current loss is less due to smaller resistivity.
- (iv) Coercivity and resistivity are large.
- v) Magnetization energy is large.
- vi, It is used in making permanent magnets, magnetic detectors, microphones, magnetocroptometers etc.
- vii, Examples:- plain carbon steel, cobalt steel, Alnico alloy etc.