ENGINEERING MATHEMATICS - [17MAT41]

MODULE-4 PROBABILITY DISTRIBUTIONS

SEM-4

[Part-1]

INTRODUCTION: (COURSE OUTCOME : [Develop probability distribn of discrete, continuous random variable, & joint distrbn prob in digital signal processing, inform theory & design eng])

Probability :- In real time statements, we make where we use probability unknowingly, here are some normal conversations we make; where there is existence of probability.

Ex: 1) I have a good chance of being selected to the job.
2) It might rain today.
3) I might get 100/100 in mathematics.
4) Toss of a coin (Heads or tail).

Many Examples gives an insight of probability.

Mathematicaly;

Probability :- If the outcome of a trial consist n exhaustive, mutually exclusive, equally possible cases, of which m of them are favorable cases to an event E, then probability of happening of every event E, usually denoted by: P(E) / p is defined by;

$$P(E) = p = \frac{\text{no/. of favorable cases}}{\text{no/. of possible cases}} = \frac{m}{n}.$$

*MODULE-1 (NUM. METHODS)
[CO: Solve 1st & 2nd order ODE, using Single & multistep Numerical methods]

Note :

1) If $P(E)=1$, E is called a sure event.
If $P(E)=0$, E is called an impossible event.

2) $p+q=1$ and $P(E)+P(\bar{E})=1$, where; $q=\frac{n-m}{n}$

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Examples :-

1) The probability of getting : (a) a number greater than 2.
(b) an ~~even~~ odd number
(c) an even number

when a die is thrown.

Soln : (a) Number of possible outcomes = $\boxed{6 = n}$ // die has 6 faces.
Number of favorable outcomes = $\boxed{4 = m}$

↳ Since; no's greater than 2 are : $\underbrace{3, 4, 5, 6}$ = 4 outcomes

∴ probability of getting no greater than 2 $= \frac{m}{n} = \frac{\cancel{4}2}{\cancel{6}3} = \frac{2}{3}$ //.

(b) Number of favorable outcomes (m) = 3, Since ; odd no's are $\underbrace{1, 3, 5}$ = 3 outcomes

∴ probability of getting odd number $= \frac{m}{n} = \frac{\cancel{3}1}{\cancel{6}2} = \frac{1}{2}$ //.

(c) Number of favorable outcomes (m) = 3, Since, even no's are $\underbrace{2, 4, 6}$ = 3 outcomes

∴ probability of getting even number $= \frac{m}{n} = \frac{\cancel{3}}{\cancel{6}2} = 1/2$ //.

PROBABILITY DISTRIBUTIONS & JOINT PROBABILITY

DISTRIBUTIONS.

Define Random variable :-

In a random experiment, if a real variable is associated with every outcome then, it is called a Random variable / Stochastic variable.

Ex:- Consider an experiment of tossing 2 coins :-

$S = \{HH, HT, TH, TT\}$.

Define, X = no of heads, then.

$X = \{0, 1, 2\}$ is the random variable on S.

Random Experiment :- An Experiment whose outcome is unpredictable is known as Random Experiment.

Sample space :- The set of all possible outcomes of a random experiment is known as the sample space.

Eg:- Tossing a coin : $S = \{H, T\}$

Event : An event is a subset of the sample space.

Probability :- If E is an event of sample space S, then the probability of E is defined by;

$$P(E) = \frac{\text{Favorable no. of events}}{\text{Total no of outcomes}} = \frac{O(E)}{O(S)}$$

$\rightarrow$ order of E

order of S.

Discrete random variable :-

If a random variable takes finite or countably infinite number of values, then it is called discrete random variable.

Ex:- 1) Throwing a die & observing the numbers on the face.

2) Tossing a coin & observing the outcome.

Continuous random variable :- If a random variable takes ~~non-~~ non-countable infinite numbers of values, then it is non-discrete (or) Continuous random variable.

Ex:- 1) Observing a pointer on a Voltmeter

2) Conducting a survey on the life of electric bulbs.

Probability function :-

If for each value x_i of a discrete random variable X, we assign a real number $p(x_i)$ $\ni$: i) $p(x_i) \geq 0$

ii) $\sum_i p(x_i) = 1$.

then the function $p(x)$ is probability function.

Discrete probability function :-

If the probability that X takes the values x_i is p_i, then $P(X=x_i) = p_i$ (or) $p(x_i)$, the set of values $[x_i, p(x_i)]$ is called discrete probability fn.

* The function : $P(X)$ is called probability density function. (pdf)

(or) * The function : $P(X)$ is called probability mass function (pmf)

The mean and Variance of the discrete distribution is :-

Mean : $\boxed{\mu = E[X] = \sum_i x_i P(x_i)}$

Variance : $V = E[X^2] - (E[X])^2$

$$\boxed{V = \sum_i (x_i - \mu)^2 . p(x_i)}$$

Standard deviation : $\boxed{\sigma = \sqrt{V}}$, where V is the variance.

Problems & Solutions :-

1) Show that the following distributions represent a discrete probability distribution (pdf), Find its mean & Variance.

x	10	20	30	40
$p(x)$	1/8	3/8	3/8	1/8

probability discrete / density fn :-

A function $P(x)$ is said to be a probability density/discrete function; if i) $P(x_i) \geq 0$.
ii) $\sum_i P(x_i) = 1$.

Soln :- Given

	x_1	x_2	x_3	x_4
x	10	20	30	40
$p(x)$	1/8	3/8	3/8	1/8
	$p(x_1)$	$p(x_2)$	$p(x_3)$	$p(x_4)$

Since, WKT;

$P(x)$ is probability discrete fn ; if i) $P(x_i) \geq 0$
ii) $\sum_i P(x_i) = 1$.

Consider; $i=1$, $p(x_1) = 1/8 > 0$.
$i=2$, $p(x_2) = 3/8 > 0$.
$i=3$, $p(x_3) = 3/8 > 0$
$i=4$, $p(x_4) = 1/8 > 0$.

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$\therefore$ clearly; $P(X) > 0$.

Now, to find : $\sum_i p(x_i)$

Consider; $\sum_{i=1}^{4} (p(x)) = p(x_1) + p(x_2) + p(x_3) + p(x_4)$

$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$

$\therefore \boxed{\sum (p(x)) = 1}$

$\therefore$ Since 2 Conditions are satisfied.

$\therefore$ $p(x)$ is a probability density function.

Now, we find : Mean & Variance.

$\sum_{i=1}^{4} x_i = x_1 + x_2 + x_3 + x_4$

$10 + 20 + 30 + 40$

Mean $= \mu = E[X] = \sum_i x_i P(x_i)$

$= [x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + x_3 p(x_3) + x_4 p(x_4)]$

$= \cancel{10}^{5} \cdot \frac{1}{\cancel{8}_4} + \cancel{20}^{5} \times \frac{3}{\cancel{8}_4} + \cancel{30}^{15} \times \frac{3}{\cancel{8}_4} + \cancel{40}^{5} \times \frac{1}{\cancel{8}}$

$\therefore \boxed{\mu = 25}$, Mean.

Now, Variance $V = E[X^2] - (E[X])^2$

Since; $\mu = E[X]$, therefore; $E[X^2] = \sum x^2 p(x)$

$E[X] = \mu = \sum x\, p(x)$

$\therefore E[X^2] = \sum x^2 P(x) = [x_1^2 p(x_1) + x_2^2 p(x_2) + x_3^2 p(x_3) + x_4^2 p(x_4)]$

$\therefore \boxed{E[X^2] = 700}$

$\therefore V = E[X^2] - (E[X])^2$

$= 700 - (25)^2$

$\therefore \boxed{V = 75}$

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2) The probability function of a finite random variable x, is given by the table; ⑤ 4

x	-2	-1	0	1	2	3
$p(x)$	0.1	K	0.2	$2K$	0.3	K.

Find the value of K, mean & Variance.

Soln:-

	x_1	x_2	x_3	x_4	x_5	x_6
x	-2	-1	0	1	2	3
$p(x)$	0.1	K	0.2	$2K$	0.3	K.
	$p(x_1)$	$p(x_2)$	$p(x_3)$	$p(x_4)$	$p(x_5)$	$p(x_6)$.

Since, Given that; $P(X)$ is ~~the~~ probability discrete/density function;

$\therefore$ i) $p(X) \geq 0$. ii) $\sum p(X) = 1$. } satisfies.

$\therefore$ Consider; $\sum p(X) = 1$

$\therefore p(x_1) + p(x_2) + p(x_3) + p(x_4) + p(x_5) + p(x_6) = 1.$

$\Rightarrow [0.1 + K + 0.2 + 2K + 0.3 + K] = 1.$

$4K = 0.4$

$\boxed{K = 0.1}$

$\therefore P(X) \geq 0.$

Now, we Compute, Mean: $\mu = E[X] = \sum x \cdot P(x)$.

$[x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4) + x_5 p(x_5) + x_6 p(x_6)]$

$= [x_1 p(x_1) + x_2 p(x_2) + x_3 P(x_3) + 1 \times (2K) + 2(0.3) + 3(K)]$

$= [2 \times 0.1 + (-1) \times (K) + (0)(0.2) + 1 \times (2K) + 2(0.3) + 3(K)]$, $K = 0.1$

$\therefore \boxed{\mu = 0.8}$, Mean..

$E[X^2] = \sum x^2 p(x)$, $E[X^2] = x_1^2 p(x_1) + x_2^2 p(x_2) + \cdots$, $\boxed{E[X^2] = 2.8}$

$\therefore$ Variance, $V = E[X^2] - (E[X])^2$

$= 2.80 - (0.8)^2$

$\therefore \boxed{V = 2.16}$

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3) Find the value of K such that the following distribution represents a finite probability distribution, Hence find its mean & standard deviation. Also find:

$p(x \leq 1)$, $p(x > 1)$, $P(-1 < x \leq 2)$.

x	-3	-2	-1	0	1	2	3
$p(x)$	K	$2K$	$3K$	$4K$	$3K$	$2K$	K.

Soln :- Given :

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x	-3	-2	-1	0	1	2	3
$p(x)$	K	$2K$	$3K$	$4K$	$3K$	$2K$	K.
	$p(x_1)$	$p(x_2)$	$p(x_3)$	$p(x_4)$	$p(x_5)$	$p(x_6)$	$p(x_7)$

Since, Given that, the following distribution represents a finite probability, $\Rightarrow$ 1) $p(X) \geq 0$

2) $\Sigma p(x) = 1$.

Consider $\Rightarrow$ $\sum_{i=1}^{7} p(x_i) = 1$

$\therefore [p(x_1) + p(x_2) + p(x_3) + p(x_4) + p(x_5) + p(x_6) + p(x_7)] = 1$.

$\Rightarrow [K + 2K + 4K + 3K + 3K + 2K + K] = 1$.

$16K = 1 \quad \Rightarrow \boxed{K = 0.0625}$

Now, we find mean, $\mu = E[x] = \sum_i x_i p(x_i)$.

$\mu = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4) + x_5 p(x_5) + x_6 p(x_6) + x_7 p(x_7)$

$\boxed{\mu = 0}$, Mean.

$E[x^2] = \Sigma x^2 p(x)$, $\boxed{E[x^2] = 40K = 2.5}$

$\therefore \quad V = E[x^2] - (E[x])^2$

$= 2.5 - 0$, $\boxed{V = 2.5}$, Variance.

$\therefore$ S.D, $\sigma = \sqrt{V} = \sqrt{2.5}$

$\therefore \boxed{\sigma = 1.5811}$, Standard deviation.

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Now, we find ;

i) $P(x \leq 1)$, Since from the table, the values of x, which is ≤ 1 are:-

$x_1 = -3$, $x_2 = -2$, $x_3 = -1$, $x_4 = 0$, $x_5 = 1$ $\longrightarrow$ (less than or Equal to 1)

$\therefore P(x \leq 1) = P(-3) + P(-2) + P(-1) + P(0) + P(1)$

$= K + 2K + 3K + 4K + 3K$

$= 13K$

$\therefore \boxed{P(x \leq 1) = 0.8125}$.

ii) $P(x > 1)$, Since, from table, values of $x > 1$ are:-

$x_6 = 2$, $x_7 = 3$

$\therefore P(x > 1) = P(x_6) + P(x_7)$

$= P(2) + P(3)$

$= 2K + K = 3K$

$\therefore \boxed{P(x > 1) = 0.1875}$

iii) $P(-1 < x \leq 2) = P(0) + P(1) + P(2)$

$= 4K + 3K + 2K$

$= 9K$

$\therefore \boxed{P(-1 < x \leq 2) = 0.5625}$

4) A random variable x has the following probability function for various values of x.

x	0	1	2	3	4	5	6	7
$p(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

Find: i) K

ii) $P(x < 6)$

iii) $P(x \geq 6)$

iv) $P(3 < x \leq 6)$.

Soln :- Since, following distribution represents discrete probability fn : 5)

By defn ; we have : i) $P(X) \geq 0$

ii) $\sum p(x) = 1$.

Consider : $\sum p(x) = 1$.

i) $\Rightarrow p(x_1) + p(x_2) + p(x_3) + p(x_4) + p(x_5) + p(x_6) + p(x_7) + p(x_8) = 1$.

$0 + K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$.

$9K + 10K^2 = 1$.

$10K^2 + 9K - 1 = 0$.

$10K^2 + 10K - 1K - 1 = 0$.

$10K(K+1) - 1(K+1) = 0$.

$(10K-1)(K+1) = 0$, $10K = 1$ $\boxed{K = 1/10}$ $\boxed{K = -1}$

-10
$+10 \; -1$

Now, we find ;

ii) $P(x<6)$

$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$= 8K + K^2$

$= 0.8 + 0.01$

$\boxed{P(x<6) = 0.81}$

iii) $P(x \geq 6)$

$P(x \geq 6) = P(6) + P(7)$

$= 2K^2 + 7K^2 + K$

$\boxed{P(x \geq 6) = 0.19}$

iv) $P(3 < x \leq 6) = P(4) + P(5) + P(6)$

$= 3K + K^2 + 2K^2$

$= 0.3 + 0.03$

$\therefore \boxed{P(3 < x \leq 6) = 0.33.}$

5) A coin is tossed 3 times, let X denote the number of heads, showing up, find the distribution of X. Also find the mean and variance.

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Soln:- Let $S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$.

where; X = number of heads.

no head, 1 head, 2 heads, 3 heads

$\therefore X = \{0, 1, 2, 3\}$ // (where X contains no of possibilities of heads, when 3 coins are tossed)

Now, we Compute :-

$P(X=0) = \frac{\text{Fav no of Events}}{\text{Total no of Events}} = \frac{1}{8}$

$P(X=1) = \frac{3}{8}$.

$P(X=2) = \frac{3}{8}$.

$P(X=3) = \frac{1}{8}$.

$P(X=0) \rightarrow$ probability of getting 0 heads.

$P(X=1) \rightarrow$ probability of getting 1 heads

$P(X=2) \rightarrow$ probability of getting 2 heads.

$\therefore$ The probability distribution table is given by;

X	0	1	2	3
P(x)	1/8	3/8	3/8	1/8.

$\therefore$ By defn; i) $P(X) \geq 0$

ii) $\sum P(X) = 1$.

Now, we Compute : Mean; $\mu = E[X] = \sum x \cdot P(x)$.

$= x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3) + x_4 P(x_4)$

$\boxed{\mu = 1.5}$, Mean.

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$E[X^2] = \sum x^2 p(x)$

$\boxed{E[X^2] = 3}$

Now, we Compute ; Variance; $V = E[X^2] - (E[X])^2$

$= 3 - (1.5)^2$

$\boxed{V = 0.75}$, Variance.

6) Find : $E[X]$, $E[X^2]$ and σ^2 for the probability function defined by the table.

x	1	2	3	-----	n
$p(x)$	K	2K	3K	-----	nK.

Soln :- By defn :- i) $P(X) \geq 0$

ii) $\sum p(x) = 1$.

Consider; $\sum_{i=1}^{n} p(x_i) = 1$.

$[p(x_1) + p(x_2) + p(x_3) + ----- + p(x_n)] = 1$.

$\Rightarrow K + 2K + 3K + ----- + nK = 1$.

$K \underbrace{[1+2+3+ ----- + n]} = 1$. // summation of n-series.

$K \frac{n(n+1)}{2} = 1$

$\therefore \boxed{K = \frac{2}{n(n+1)}}$

Now, Mean, $\mu = E[X] = \sum x \cdot p(x)$

$= x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + --- + x_n p(x_n)$.

$= 1(K) + 2(2K) + 3(3K) + ---- + n(nK)$.

$= K + 4K + 9K + ----- + n^2 K$.

$= K[1^2 + 2^2 + 3^2 + --- + n^2]$, Since, $K = \frac{2}{n(n+1)}$

$$= \frac{2}{n(n+1)}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{2(2n+1)}{6}$$

$$\therefore \boxed{\mu = \frac{2n+1}{3}}$$

$$E[X^2] = \sum x^2 p(x). = [k + 8k + 27k + \dots + n^3 k.]$$

$$= k[1 + 8 + 27 + \dots + n^3]$$

$$= k[\underbrace{1^3 + 2^3 + 3^3 + \dots + n^3}]$$

$$= \frac{2}{n(n+1)}\left[\left[\frac{n(n+1)}{2}\right]^2\right] = \frac{n^2(n+1)^2}{4}\cdot\frac{2}{n(n+1)}.$$

$$\boxed{E[X^2] = \frac{n(n+1)}{2}}$$

$$\therefore \sigma = \sqrt{V}, \quad \sigma^2 = V$$

$$= E[X^2] - (E[X])^2$$

$$= \frac{n(n+1)}{2} - \left(\frac{2n+1}{3}\right)^2$$

$$= \frac{n(n+1)}{2} - \frac{4n^2+1+4n}{9}.$$

$$= \frac{9n^2+9n-8n^2-2-8n}{18}$$

$$= \frac{n^2+n-2}{18}.$$

$$\therefore \boxed{\sigma^2 = \frac{(n-1)(n+2)}{18}}$$

7) A random variable x has probability f_n, $p(x) = 2^{-x}$.

$, x = 1, 2, 3, \dots,$ ST : $p(x)$ is a probability f_n,

Also find :- i) $P(x$ even$)$ iii) $P(x \geq 5)$

ii) $P(x \div 3)$

Soln: Given; $P(x) = 2^{-x} = \frac{1}{2^x}$.

clearly; $p(x) \geq 0$.

Consider; $\sum p(x) = \sum_{x=1}^{\infty} \frac{1}{2^x}$.

$\frac{1}{2} = \frac{1}{2}$

$\frac{1}{2^2} = \frac{1}{4}$ ....

$= \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\cdots$ } Geometric series.

$= \frac{a}{1-r}$, $\underline{a = 1/2}$, $\underline{r = 1/2}$

$= \frac{1/2}{1-1/2}$

$\therefore \boxed{\sum p(x) = 1}$ $\therefore$ p(x) is a probability function.

i) $P(x \text{ even}) = p(2) + p(4) + p(6) + \cdots$

$= 1/2^2 + 1/2^4 + 1/2^6 + \cdots$ [Infinite G P].

$= \frac{a}{1-r}$, $\underline{a = 1/4}$, $\underline{r = 1/4}$

$= \frac{1/4}{1-1/4} = \frac{1}{4} \times 4/3$ $\therefore \boxed{P(x \text{ even}) = 1/3}$.

ii) $P(x \div \text{by } 3) = p(3) + p(6) + p(9) + \cdots$

$= 1/2^3 + 1/2^6 + 1/2^9 + \cdots$

$= \frac{a}{1-r} = \frac{1/8}{1-1/8} \Rightarrow \therefore \boxed{P(x \div 3) = 1/7}$.

iii) $P(x \geq 5) = P(5) + P(6) + P(7) + P(8) + \cdots$

$= 1/2^5 + 1/2^6 + 1/2^7 + 1/2^8 + \cdots$

$= \frac{1/32}{1-1/2} \Rightarrow \therefore \boxed{P(x \geq 5) = 1/16}$.

——— * ———

BINOMIAL DISTRIBUTION :-

If p is the probability of success and q is the probability of failure, the probability of x successes out of n trials is given by; $\boxed{P(x) = {}^nC_x\, p^x . q^{n-x}}$

where, $q = 1-p$ is called Binomial distribution / Bernoulli's distribution.

(Binomial distribution): formula.

$$\sum p(x) = q^n + {}^nC_1 q^{n-1} p + {}^nC_2 q^{n-2} p^2 + \cdots\cdots + p^n = (q+p)^n = 1^n = 1$$

That is : $\sum P(x) = (q+p)^n = 1^n = 1$

Mean of Binomial distribution :-

$$\text{Mean}, (\mu) = \sum_{x=0}^{n} x . p(x) \quad , \text{ WKT ; } p(x) = {}^nC_x\, p^x . q^{n-x}.$$

$$= \sum_{x=0}^{n} x . {}^nC_x\, p^x . q^{n-x} \qquad // \; {}^nC_r = \frac{n!}{r!(n-r)!} \; , \; {}^nC_x = \frac{n!}{x!(n-x)!}$$

$$\mu = \sum_{x=0}^{n} x \left[\frac{n!}{x!(n-x)!}\right] p^x . q^{n-x}. \qquad // \; n! = n(n-1)! \quad x! \;\; x(x-1)!$$

$$= \sum_{x=0}^{n} \cancel{x} \left[\frac{n(n-1)!}{(n-x)!\,\cancel{x}(x-1)!}\right] p^x . q^{n-x}.$$

// if $\sum_{x=0}^{n} \frac{n(n-1)!}{(n-0)!\,(0-1)!} p^0 q^{n-0} = \frac{n(n-1)!\, q^n}{n(n-1)!\,(-1)}$

if $x=0$, $\sum_{x=0}^{n} = (-1)$ is not possible.

if $x=1$ $\sum_{x=1}^{n} \frac{n(n-1)!}{(n-1)!\,(1-1)!} p^1 q^{n-1} = \frac{npq^{n-1}}{0!} = \frac{npq^{n-1}}{1}$

$\therefore$ if $x=1$, $\sum_{x=1}^{n} =$ +ve value

$$\mu = \sum_{x=1}^{n} \frac{n(n-1)!}{(x-1)!\,[(n-1)-(x-1)]!} p^{x-1} . q^{(n-1)-(x-1)}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,[(n-1)-(x-1)]!} . p^{x-1} . q^{(n-1)-(x-1)}.$$

// $p^x = p^{x-1} . p$

$q^{n-x} = q^{(n-1)-(x-1)}$

$(n-x)! = [(n-1)-(x-1)]!$

$$= np \sum_{x=1}^{n} {}^{(n-1)}C_{x-1}\, p^{x-1} . q^{(n-1)-(x-1)}$$

// ${}^nC_r = \frac{n!}{r!(n-r)!}$

$$\mu = np \underbrace{\sum_{x=1}^{n} (n-1)C_{x-1}\, p^{x-1} \cdot q^{(n-1)-(x-1)}}$$ — ①

WKT; By Binomial distribution formula; $\sum p(x) = (q+p)^n = 1^n = 1$. — ($*$)$_1$

Also; $p(x) = nC_x\, p^n . q^{n-x}$ // $\sum_{x=1}^{n} n-1C_{x-1} . p^{x-1} . q^{(n-1)-(x-1)} = \underline{\underline{\sum p(x-1)}}$

$\sum p(x) = \sum nC_x p^n q^{n-x}$

Eq^n ($*$)$_1$ $\Rightarrow$ $\sum (p(x-1)) = (q+p)^{n-1}$ — ($*$)$_2$

$\Rightarrow$ By using ($*$)$_2$ in Eq^n ①; we get;

$\mu = np(q+p)^{n-1} = np\underline{\underline{(1)}}^{n-1}$

$\therefore \boxed{\mu = np.}$, Mean.

<u>Variance</u>:- $V = \sum_{x=0}^{n} x^2 p(x) - \mu^2$ // $E[X^2] = \sum_{x=0}^{n} x^2 p(x)$.

Now; $\sum_{x=0}^{n} x^2 p(x) = \sum_{x=0}^{n} [x(x-1)+x]\, p(x)$.

$= \sum_{x=0}^{n} x(x-1).\underline{\underline{p(x)}} + \underbrace{\sum_{x=0}^{n} x.p(x)}$ // $p(x)$

$= \sum_{x=0}^{n} x(x-1) [\underline{\underline{nC_x}} . p^x . q^{n-x}] + \mu$ // $\sum_{x=0}^{n} x p(x) = \underline{\underline{\mu = np}}$

$= \sum_{x=0}^{n} x(x-1) \left[\frac{n!}{x!(n-x)!}\right] p^x q^{n-x} + np.$

$= \sum_{x=0}^{n} \cancel{x(x-1)} \left[\frac{n(n-1)(n-2)!}{\cancel{x(x-1)}(x-2)!(n-x)!}\right] p^{x-2} . p^2 . q^{n-2-x-2} + np.$ // $p^x = p^{x-2} . p^2$; $q^{n-x} = q^{(n-2)-(x-2)}$; $n! = n(n-1)(n-2)!$; $x! = x(x-1)(x-2)!$; $(n-x)! = (n-2-x-2)!$

$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!\; p^{x-2} q^{n-2-x-2}}{(x-2)!\,[(n-2)-(x-2)]!} + np$

$$= n(n-1)p^2 \sum_{x=2}^{n} \underbrace{\frac{(n-2)!}{(x-2)!\,[(n-2)-(x-2)]!}} p^{x-2} \cdot q^{(n-2)-(x-2)} + np$$

// if $\underline{x=0}$, $\sum_{x=0}^{n} \frac{(n-2)!}{(-2)!\,[n]!} p^{-2} q^{n-2} = \underline{(-2)}$

(-ve value).

$$= n(n-1)p^2 \sum_{x=2}^{n} \underbrace{{}^{n-2}C_{x-2}\, p^{x-2} \cdot q^{n-2-x-2}} + np$$

// if $\underline{x=2}$, $\sum_{x=2}^{n} \frac{(n-2)!}{(2-2)!\,(n-2)!} p^0 q^{n-2}$

$= \underline{q^{n-2}}$ +ve

(+ve value)

$$= n(n-1)p^2 \cdot [(q+p)^{n-2}] + np.$$

// $(q+p)^n = 1^n$

$$= n(n-1)p^2 [(1)^{n-2}] + np.$$

$$\boxed{\sum x^2 P(x) = n(n-1)p^2 + np.}$$

$\therefore$ Variance, $V = \sum x^2 P(x) - \mu^2$ // $\underline{\mu = np}$

$$= n(n-1)p^2 + np - (np)^2$$

$$= \cancel{n^2p^2} \underbrace{-np^2 + np}_{\text{Common}} - \cancel{n^2p^2}$$

$$= np - np^2$$

$$= np(1-p) \quad // \ \underline{q = 1-p}$$

$$\therefore \boxed{V = npq.} \text{, Variance.}$$

Standard deviation :- (σ)

$$\boxed{\sigma = \sqrt{V} = \sqrt{npq} = S.D.}$$

Thus, the proof of μ, V and σ for Binomial distribution.

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Problems & Solutions :-

1) when a Coin is tossed 4 times, find the probability of getting

i) Exactly one head.

ii) Almost 3 heads.

iii) Atleast 2 heads.

Soln:— Since, a coin is tossed 4 times, $\underline{n=4}$ (Total 4 outcomes)

W.K.T; The probability of getting head is = $\boxed{p=\frac{1}{2}}$ // In one trial, probability or chance of getting head is 1 time out of 2 possibilities

Since, $q = 1-p$

$q = 1-\frac{1}{2}$, $\boxed{q=\frac{1}{2}}$

∴ By Binomial distribution, we have;

$p(x) = nC_x . p^x q^{n-x}$ —① , $\underline{n=4}$, $\underline{p=\frac{1}{2}}$, $\underline{q=\frac{1}{2}}$

$= 4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$

$= 4C_x \cancel{\left(\frac{1}{2}\right)^x} \left(\frac{1}{2}\right)^4 . \cancel{\left(\frac{1}{2}\right)^{-x}}$

∴ $\boxed{p(x) = \frac{4C_x}{16}}$

i) The probability of getting exactly one head is ;

[That is : Out of 4 possibilities, all 3 outcomes should be tails and only one (1) outcome, should be head]

// $nC_r = \frac{n!}{(n-r)!\,r!}$

∴ $\underline{x=1}$ (1 head)

// $\underline{nC_1 = n}$

∴ $p(x) = \frac{4C_1}{16} = \frac{\cancel{4}^{\,1}}{\cancel{16}_{\,4}} = 0.25$

∴ $\boxed{p(x) = \frac{1}{4} = 0.25}$

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ii) probability of getting atmost 3 heads.
[That is : out of 4 outcomes ; 3 possible outcomes should be heads, remaining one(1) possibility should be tail.]

Atmost 3 heads (max) 3 heads $\Rightarrow x \leq 3$, where , $x=0$, $x=1$, $x=2$, $x=3$. $x=4$
$\downarrow$ tail. $\downarrow$ Tail, Head Head Head ; $\underbrace{\qquad\qquad}_{4 \text{ outcomes}}$.

$\therefore p(x \leq 3) = p(0) + p(1) + p(2) + p(3)$

$= \frac{4C_0}{16} + \frac{4C_1}{16} + \frac{4C_2}{16} + \frac{4C_3}{16}$ // $nC_0 = 1$, $nC_r = \frac{n!}{(n-r)!\,r!}$

$= \frac{1}{16} + \frac{1}{4} + \frac{6}{16} + \frac{4}{16}$

$\boxed{p(x \leq 3) = 0.9375}$

iii) Probability of Getting atleast 2 heads ;
[That is : out of outcomes, 2 outcomes should be head.

$\therefore p(x \leq 2) = p(0) + p(1) + p(2)$

$= \frac{4C_0}{16} + \frac{4C_1}{16} + \frac{4C_2}{16}$

$= \frac{1}{16} + \frac{1}{4} + \frac{6}{16}$.

$\therefore \boxed{p(x \leq 2) = \frac{11}{16} = 0.6875}$

$p(x \geq 2) = p(2) + p(3) + p(4)$

$= \frac{4C_2}{16} + \frac{4C_3}{16} + \frac{4C_4}{16}$

$= \frac{6}{16} + \frac{4}{16} + \frac{1}{16}$

$\boxed{p(x \geq 2) = 11/16 = 0.6875}$

2) A pair of dice is thrown twice, find the probability of scoring 7 points,

1) once
2) Twice
3) ~~Thrice~~ Atleast once.

Soln :- Given ; $n = 2$ [A dice is thrown twice].

∴ The probability of scoring 7 points is : Since, the dice is thrown twice.

* 1st time : 3 outcomes
* 2nd time : 3 outcomes

$\Big\}$ $3+3 = \underline{6 \text{ outcomes}}$

∴ The prob of scoring 7 pts is $= \frac{\cancel{6}\,1}{\cancel{36}\,6}$

$\boxed{p = 1/6}$

WKT; $q = 1 - p$.

$q = 1 - 1/6$, $\boxed{q = 5/6}$

By Binomial distribution ; $p(x) = nC_x p^x q^{n-x}$

$p(x) = 2C_x (1/6)^x (5/6)^{2-x}$ — (1).

i) The prob of scoring 7 points once is ; $x = 1$

$P(1) = 2C_1 (1/6)^1 (5/6)^{2-1} = \cancel{2}(1/\cancel{6}_3)(5/6)$.

∴ $\boxed{P(1) = \frac{5}{18} = 0.2778.}$

ii) prob. of scoring 7 points twice is ; $x = 2$

$P(2) = 2C_2 (1/6)^2 (5/6)^{2-2} = 1.(1/36)(1)$

$\boxed{P(2) = 1/36 = 0.0278.}$

iii) Prob. of scoring 7 points atleast once;

$P(x \geq 1) = P(1) + P(2)$

$= 0.2778 + 0.0278$

$\boxed{P(x \geq 1) = 0.3056.}$

That is : when pair of dice (3)

|| If thrown once (1st time) : prob of scoring 7 points is : $\{\underbrace{\boxed{6}\ \boxed{1}}_{\text{pair}}, \underbrace{\boxed{5}\ \boxed{2}}_{\text{pair}}, \underbrace{\boxed{4}\ \boxed{3}}_{\text{pair}}\} = 3$ outcomes

pair of dice ....

|| when pair of dice is thrown 2nd time : prob of scoring 7 points is : $\{\boxed{6}\boxed{1}, \boxed{5}\boxed{2}, \boxed{4}\boxed{3}\}$ = 3 outcomes

|| Total no of outcomes : 1st time : $\{\underbrace{6,1 / 5,2 / 4,3 / 3,4 / 1,6 / 5,2}_{\text{6 outcomes}}\}$

|| 2nd time : 6 outcomes.

$\Rightarrow 6 \times 6 = 36$ total outcomes.

dice

point

12

3) The probability that a person aged 60 years will live upto 70 is 0.65 is : 0.65, what is the probability that out of 10 persons aged 60 atleast 7 of them will live up to 70.

Soln :- Let Given ; $\boxed{n=10}$ 10 persons are aged 60.

Given that ; The probability that a person aged 60 will live up to 70 is : 0.65 $\Rightarrow \boxed{p=0.65}$

$\therefore q = 1-p$

$q = 1-0.65$, $\boxed{q=0.35}$

Now, By binomial distribution :- $p(x) = nC_x p^x q^{n-x}$.

$p(x) = 10C_x (0.65)^x (0.35)^{10-x}$ — ①.

Now, to find : $(p(x \geq 7))$, ie : The probability, that out of 10 persons aged 60 atleast 7 of them will live up to 70

is : $p(x \geq 7) = p(7)+p(8)+p(9)+p(10)$ // Since ; only 10 persons are there.

$= 10C_7 (0.65)^7 (0.35)^{10-7} + 10C_8 (0.65)^8 (0.35)^{10-8} + 10C_9 (0.65)^9 (0.35)^{10-9}$

$+ 10C_{10} (0.65)^{10} (0.35)^{10-10}$.

But $\Rightarrow$ $10C_7 = \frac{10(10-1)(10-2)}{(10-7)!}$ // $nC_r = \frac{n!}{(n-r)!r!} = 10C_7 = \frac{10!}{(10-7)!7!}$

$\Rightarrow 10C_7 = \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} = 120$. // $= \frac{10(10-1)(10-2)(10-3)\ldots}{7!\ 3!} \ldots = \frac{10(10-1)(10-2)\cancel{7!}}{\cancel{7!}\ 3!}$

$\Rightarrow 10C_8 = \frac{10 \cdot 9}{1 \cdot 2} = 45$ (or) $10C_2$; $10C_9 = 10$ (or) $10C_1$, $10C_{10} = 1$

$\therefore p(x \geq 7) = 120(0.65)^7 (0.35)^3 + 45(0.65)^8 (0.35)^2 + 10(0.65)^9 (0.35)^1 + 1(0.65)^{10} (1)$

$\therefore \boxed{p(x \geq 7) = 0.5138}$

4) The number of telephone lines busy at an instant of time is a binomial variable, with probability 0.2, If at a instant, 10 lines are chosen at random, what is the probability that ; i) 5 lines are busy ii) 2 lines are busy. iii) All lines are busy.

Soln:- $n=10$ (No of telephone lines choosed).

Given ; $\boxed{p=0.2}$ $q=1-p$, $q=1-0.2$, $\boxed{q=0.8}$

By binomial distribution ; $p(x) = nC_x\, p^x . q^{n-x}$.

$\therefore p(x) = 10C_x . (0.2)^x (0.8)^{10-x}$ — 1.

(i) Prob. of that, 5 lines are busy ; $x=5$

$p(5) = 10C_5\,(0.2)^5(0.8)^{10-5}$ = $\dfrac{10(10-1)(10-2)(10-3)(10-4)\cancel{(10-5)!}}{5!\,(10-5)!} = 10C_5$

$10C_5 = \dfrac{10(9)(8)(7)(6)}{5\times4\times3\times2\times1} = 36\times7 = 252$

$\therefore p(5) = 252(0.2)^5(0.8)^5$

$\boxed{p(5) = 0.0264}$

(ii) Prob that 2 lines are busy; $x=2$; $p(2) = 10C_2\,(0.2)^2(0.8)^{10-2}$.

$p(2) = 45(0.2)^2(0.8)^8$

$\boxed{p(2) = 0.3020}$

// $10C_2 = \dfrac{10(10-1)(10-2)\cancel{8}\cdots\cancel{(10-8)!}}{2!\,(10-2)!}$

// $10C_2 = \dfrac{10(9)\cancel{(8)(7)(6)(5)(4)(3)}}{8!\ 2}$

$10C_2 = 45$

(iii) prob that All lines are busy ;

$p(10) = 10C_{10}\,(0.2)^{10}(0.8)^0$

$p(10) = 1.024\times10^{-7}$

5) 2 persons A & B play a game in which their chances of winning are in the ratio 3:2, Find A's chance of winning atleast 3 games out of 6 games played.

$3:2 = 3+2 = 5$

Soln :- $n = 6$.

The probability that A wins the game :- $\boxed{p = \frac{3}{5}}$ → A wins game 3 times, A and B wins game

$q = 1 - p = 1 - 3/5$

$\boxed{q = 2/5}$

By Binomial distribution ;

$p(x) = nC_x \cdot p^x \cdot q^{n-x}$.

$p(x) = 6C_x (3/5)^x (2/5)^{6-x}$ — ①.

Now, To find prob of A's chance of winning atleast 3 games out of 6 games played is ; $p(x \geq 3)$

$p(x \geq 3) = p(3) + p(4) + p(5) + p(6)$

$$\therefore p(x \geq 3) = 6C_3 (3/5)^3 (2/5)^3 + 6C_4 (3/5)^4 (2/5)^2 + 6C_5 (3/5)^5 (2/5) + 6C_6 (3/5)^6 (1)$$

$$6C_3 = \frac{6(6-1)(6-2)\cancel{(6-3)!}}{3!\cancel{(6-3)!}} = \frac{6(5)(4)}{3 \times 2 \times 1} = 20.$$

$\boxed{p(x \geq 3) = 0.8208}$

6) In a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2, Out of 1000 such samples, how many should be expected to contain atleast 3 defective parts.

Soln :- $n = 20$ (Mean no of defective samples).

Since, Mean no of samples defectives = 2

$\boxed{\text{Mean} = \mu = 2}$, $n = 20$

WKT $\Rightarrow \mu = np$

Also; $p = \frac{\mu}{n} = \frac{2}{20} = \boxed{0.1 = p}$

Also; $q = 1 - p$

$q = 1 - 0.1 \Rightarrow \boxed{q = 0.9}$

By Binomial distribution; $p(x) = nC_x\, p^x . q^{n-x}$

$p(x) = 20C_x (0.1)^x (0.9)^{20-x}$ — ①

Now, Probability of atleast 3 defective parts is ; // (3 or more defects) atleast

$P(x \geq 3) = 1 - (P(x < 3))$

$= 1 - \{P(0) + P(1) + P(2)\}$

$= 1 - \{20C_0 (0.1)^0 (0.9)^{20} + 20C_1 (0.1)^1 (0.9)^{19} + 20C_2 (0.1)^2 (0.9)^{18}\}$

$\boxed{P(x \geq 3) = 0.3231}$

$nC_0 = 1$
$nC_n = 1$
$nC_1 = n$

∴ Out of 1000 samples, the expected no of samples that contain atleast 3 defective parts $= 0.3231 \times 1000$

$= 323.1$

$\simeq 323$

7) If the mean & SD of no of correctly answered questions given to 4096 students are 2.5 & $\sqrt{1.875}$, Find an estimate of no of student answering correctly, (a) 8 or more questions (atleast)
(b) 2 or less question
(c) 5 questions.

Soln :- Given ; $\mu = 2.5$, Mean.

$S.D = \sigma = \sqrt{1.875}$

WKT; $\mu = np$, $\sigma = \sqrt{npq}$.

$\sigma = \sqrt{\mu q} \Rightarrow q = \frac{\sigma^2}{\mu} = \frac{1.875}{2.5}$, $\boxed{q = 0.75}$

WKT; $p = 1-q$.

$\boxed{p = 0.25}$

By Binomial distribution; $P(x) = nC_x\, p^x q^{n-x}$

$P(x) = 10C_x\, (0.25)^x (0.75)^{10-x}$ — (1)

(i) Prob. of no of stdnt answering Correctly 8 or more questions ;

$P(x \geq 8) = P(8) + P(9) + P(10) = 10C_8 (0.25)^8 (0.75)^2 + 10C_9 (0.25)^9 (0.75)^1 + 10C_{10} (0.25)^{10} (0.75)^0$.

$\therefore \boxed{P(x \geq 8) = 0.0004}$ $\therefore$ Out of 4096 stds, expected value no of stds who answered 8 or more questions correctly $= 0.0004 \times 4096$
$= 1.6384 \simeq \boxed{2 = P(x \geq 8)}$

ii) $P(x \leq 2) = P(0) + P(1) + P(2)$.

$= 10C_0 (0.25)^0 (0.75)^{10} + 10C_1 (0.25)^1 (0.75)^9 + 10C_2 (0.25)^2 (0.75)^8$

$\therefore P(x \leq 2) = 0.5256$. $\therefore$ Out of 4096 stds, expected no of stds who answer 2 or less questions correctly $= 0.5256 \times 4096$

$\boxed{P(x \leq 2) = 2153}$

iii) $P(5)$

$= 10C_5 (0.25)^5 (0.75)^5 = \boxed{0.0584 = P(5)}$

$\therefore$ out of 4096 stds, the expected no of stds who answer 5 questions correctly $= 0.0584 \times 4096 \simeq \boxed{239 = P(5)}$

— * —

POISSON' DISTRIBUTION :-

The probability function defined by; $p(x) = \dfrac{e^{-\mu}\mu^{x}}{x!}$

where; $\mu = np$ is called poisson's Distribution.

Poisson's Distribution :-

Poisson's distribution is regarded as the limiting form of the binomial distribution, when n is very large $(n \to \infty)$ and probability of success p is very small $(p \to 0)$, so that np tends to a fixed finite constant μ.

Consider, the Binomial distribution; $p(x) = {}^{n}C_{x}\, p^{x}.q^{n-x}$ — (1)

$$p(x) = \frac{n!}{(n-x)!\,x!}\, p^{x}.q^{n-x} \qquad // \; {}^{n}C_{x} = \frac{n!}{(n-r)!\,r!}$$

(n common)

$$p(x) = \frac{n(n-1)(n-2)\ldots\ldots[n-(x-1)]\cancel{(n-x)!}\ldots(1)}{\cancel{(n-x)!}\; x!\;.q^{x}}.p^{x}.q^{n}$$

$$p(x) = \frac{n.n[1-1/n][1-2/n]n\ldots\ldots n[1-(\frac{x-1}{n})]}{x!\,q^{x}}\, p^{x}.q^{n}$$

$$// \; n.n^{2}\ldots.n^{x} = n^{x}$$

$$p(x) = n^{x}\frac{[1-1/n][1-2/n]\ldots[1-(\frac{x-1}{n})]}{x!\,q^{x}}.p^{x}.q^{n}$$

$$= \frac{(np)^{x}[1-1/n][1-2/n]\ldots\ldots[1-(\frac{x-1}{n})]}{x!\,q^{x}}.q^{n} \qquad // \; \mu = np$$

$$p(x) = \frac{(\mu)^{x}(1-1/n)(1-2/n)\ldots\ldots(1-(\frac{x-1}{n}))}{x!\,q^{x}}.q^{n} \sim (2)$$

Consider; $\lim_{n\to\infty} q^n = \lim_{n\to\infty} (1-p)^n$ // $\mu = np$

// $p = \frac{\mu}{n}$.

$\lim_{n\to\infty} q^n = \lim_{n\to\infty} \left(1 - \frac{\mu}{n}\right)^n$

$\boxed{\lim_{n\to\infty} q^n = e^{-\mu}}$

// $\lim_{n\to\infty} (1+1/n)^n = e$

$\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x$

$\lim_{n\to\infty} \left(1+\left(-\frac{\mu}{n}\right)\right)^n = e^{-\mu}$

$\lim_{p\to 0} q^x = \lim_{p\to 0} (1-p)^x$.

$\boxed{\lim_{p\to 0} q^x = 1}$

Also, the factors; $(1-1/n)(1-2/n)\dots\left(1-\left(\frac{x-1}{n}\right)\right)$ tends to $\to 1$

as $n \to \infty$.

$\therefore$ The Eqn (2) reduces to :- $\boxed{P(x) = \frac{e^{-\mu}\mu^x}{x!}}$

MEAN :- $\mu = \sum_{x=0}^{\infty} x \cdot p(x) = \sum_{x=0}^{\infty} \frac{e^{-\mu}\mu^x}{x!} x$.

$= \sum_{x=1}^{\infty} \frac{\cancel{x} \cdot e^{-\mu}\mu^x}{\cancel{x}(x-1)!} = \sum_{x=1}^{\infty} \frac{e^{-\mu}\mu^{x-1}\mu^1}{(x-1)!} = e^{-\mu} \cdot \mu \sum_{x=1}^{\infty} \frac{\mu^{x-1}}{(x-1)!}$

$= e^{-\mu} \cdot \mu \left\{ 1 + \frac{\mu}{1!} + \frac{\mu^2}{2!} + \dots \right\}$ // $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$

$= e^{-\mu} \cdot \mu \left[e^{\mu} \right]$

$= e^{-\mu+\mu} \mu = e^0 \mu$

$\boxed{\text{Mean} = \mu}$

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Variance:- $V = E[X^2] - (E[X])^2$

$V = E[X^2] - \mu^2 \quad -(3)$

WKT;

$$E[X^2] = \sum_{x=0}^{\infty} x^2 . p(x)$$

$$= \sum_{x=0}^{\infty} [x(x-1)+x]\, p(x)$$

$$= \sum_{x=0}^{\infty} x(x-1)\, p(x) + \underbrace{\sum_{x=0}^{\infty} x . p(x)} \quad // \ \mu = \sum x . p(x)$$

$$= \sum_{x=0}^{\infty} x(x-1)\, p(x) + \mu$$

$$= \sum_{x=0}^{\infty} x(x-1) . \frac{e^{-\mu} \mu^{x}}{x!} + \mu .$$

$$= \sum_{x=2}^{\infty} \cancel{x(x-1)} \frac{e^{-\mu} \mu^{x}}{\cancel{x(x-1)}(x-2)!} + \mu$$

$$= \sum_{x=2}^{\infty} \frac{e^{-\mu} . \mu^{(x-2)+2}}{(x-2)!} + \mu .$$

$$= e^{-\mu} . \mu^2 \left[\sum_{x=2}^{\infty} \frac{\mu^{x-2}}{(x-2)!} \right] + \mu .$$

$$= e^{-\mu} \mu^2 \left\{ 1 + \frac{\mu}{1!} + \frac{\mu^2}{2!} + \frac{\mu^3}{3!} + \cdots \right\} + \mu .$$

$E[X^2] = e^{-\mu} \mu^2 e^{\mu} + \mu$ — use in (3), we get.

$$\therefore V = e^{-\mu} . \mu^2 e^{\mu} + \mu - \mu^2$$

$$\therefore \boxed{V = \mu}$$

Hence, the proof of Mean & Variance of Poisson's Distribution.

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* **NOTE:-**

(1) $P(x \geq n) = 1 - P(x < n)$

(2). Avg. rate = Mean = μ

(3). $P(x > n) = 1 - P(x \leq n)$

(4).

POISSON'S DISTRIBUTION :-

1) Alpha particles are emitted by a radioactive source at an average rate of 5 in 20 minutes interval, using Poisson's distribution, find probability that there will be:

i) 2 Emissions ii) Atleast 2 emissions in 20 minutes interval.

Soln :- Given ; Avg. rate $= \mu = 5$, Mean.

WKT ; By Poisson's distribution ; $p(x) = \frac{e^{-\mu} \cdot \mu^{x}}{x!}$

$$p(x) = \frac{e^{-5} \cdot 5^{x}}{x!} \quad — ①.$$

i) Probability that, there will be 2 emissions is ;

$$x = 2, \quad p(2) = \frac{e^{-5}\, 5^{2}}{2!} = \frac{e^{-5}(25)}{2}$$

$$\therefore \boxed{p(2) = 0.0842}$$

ii) probability that there will be atleast 2 emissions ;

$$p(x \geq 2) = 1 - p(x<2)$$
$$= 1 - \{p(0) + p(1)\}$$
$$= 1 - \{e^{-5} + e^{-5} \cdot 5\}$$
$$\boxed{p(x \geq 2) = 0.9596.}$$

2) A car hire firm has 2 cars, which it hires out day by day, The demand of a car on each day is distributed as a poisson distribution with mean 1.5. Calculate the probability that on a certain day

i) Neither car is used.

ii) Some demands are refused.

*) Probability : WKT; Given; $\mu = 1.5$

WKT; Poison's distribution is : $p(x) = \frac{e^{-\mu} \mu^{x}}{x!}$

i) Probability that neither car is used. (no car is used).

$\therefore x = 0, \ p(0) = \frac{e^{-1.5} \cdot (-1.5)^{0}}{0!}$ // $0! = 1$

$\boxed{p(0) = 0.2231.}$

ii) Probability that some demands are refused.

= prob that there will be more than 2 demands.

$p(x > 2) = 1 - (p(x \leq 2))$

$= 1 - \{ p(0) + p(1) + p(2) \}$

$= 1 - \{ 0.223 + 0.3347 + 0.2510 \}$

$\boxed{p(x > 2) = 0.1912.}$

18) 4

3) The probability that an individual suffers a bad reaction from certain injection is : 0.002, Using Poisson's distribution, determine probability that out of 1000 individuals : (i) Exactly 2.
(ii) More than 2. , will suffer from bad reaction.

Soln :- Given; $p = 0.002$
$n = 1000$.

$\therefore \mu = np$.
$\mu = 1000 \times 0.002$. , $\boxed{\mu = 2}$

WKT; By poisson's distribution; $p(x) = \frac{e^{-\mu} \mu^{x}}{x!}$

$$p(x) = \frac{e^{-2} (2)^{x}}{x!} \quad \text{--- (1)}.$$

(i) Prob. that out of 1000 individuals, Exactly 2 suffer from bad reaction is; $x=2$

$$P(2) = \frac{e^{-2} 2^{2}}{2!}$$

$$\boxed{P(2) = 0.2707}$$

(ii) Prob that out of 1000 individuals, more than 2 suffer from bad reaction is; $(x>2)$

$$\Rightarrow p(x>2) = 1 - P(x \leq 2)$$
$$= 1 - \{p(0) + p(1) + p(2)\}$$
$$= 1 - \left\{ \frac{e^{-2} 2^{0}}{0!} + \frac{e^{-2} 2^{1}}{1!} + \frac{e^{-2} 2^{2}}{2!} \right\}$$

$$\therefore \boxed{p(x>2) = 0.3233.}$$

4) Given that, 2% of the fuses manufactured by a firm are defective, Find by poisson's distribution, the probability that a box containing 200 fuses has:

i) Atleast 1 defective fuse.

ii) 3 or more defective fuses.

Soln:- Given; $p = 2\% = \frac{2}{100} = \boxed{0.02 = p}$

$\boxed{n = 200}$

WKT; $\mu = np$

$\mu = 200 \times 0.02$

$\boxed{\mu = 4}$

WKT; By Poisson's distribution;

$\Rightarrow P(x) = \frac{e^{-4} 4^{x}}{x!} \sim (1)$.

(i) The prob, that box containing 200 fuses, having atleast 1 defective fuse is; $p(x \geq 1) = 1 - P(x < 1)$

$1 - p(0)$

$\Rightarrow 1 = \frac{e^{-4} 4^{0}}{0!}$

$\therefore \boxed{p(x \geq 1) = 0.9817}$

(ii) The prob., that box containing 200 fuses, have 3 or more defective fuses is; $p(x \geq 3) = 1 - P(x < 3)$.

$= 1 - \{p(0) + p(1) + p(2)\} \cancel{+ p(}$

$= 1 - \{ e^{-4} + \frac{e^{-4} 4}{1} + \frac{e^{-4} 4^{2}}{2} \}$

$\therefore \boxed{p(x \geq 3) = 0.7619}$

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5) The probability that a news reader commits no mistakes, is : $\frac{1}{e^3}$, Find the probability that; on a particular news broadcast, he commits , i) Only 2 mistakes.
ii) More than 3 mistakes.
iii) Atmost 3 mistakes.

Soln:- Given ; $p(0) = \frac{1}{e^3}$ // probability that, newsreader commits no mistakes.

WKT; By Poison's distribution ; $p(x) = \frac{e^{-\mu}\mu^x}{x!}$ ~ ①

put, $x=0$; $p(0) = \frac{e^{-\mu}\mu^0}{0!} = e^{-\mu}$ // $p(0) = \frac{1}{e^3}$.

$p(0) = e^{-\mu}$.

$\Rightarrow \frac{1}{e^3} = e^{-\mu} \Rightarrow \frac{1}{e^3} = \frac{1}{e^{\mu}}$ (Bases are same ; powers are equal).

$\Rightarrow \boxed{\mu = 3.}$, Mean.

$\therefore ① \Rightarrow p(x) = \frac{e^{-3}(3)^x}{x!}$ -②.

(i) prob. that ; on particular news, he commits only 2 mistakes
is ; $p(2) = \frac{e^{-3}3^2}{2!} \Rightarrow \boxed{p(x=2) = 0.2240.}$

(ii) prob, that on particular news, he comits more than 3 mistakes
is ; $p(x>3) = 1 - p(x \leq 3)$
$= 1 - \{p(0)+p(1)+p(2)+p(3)\}$
$\therefore \boxed{p(x>3) = 0.3528.}$

(iii) prob, that on particular news, he comits Atmost 3 mistake
is ; $p(x \leq 3) = p(0)+p(1)+p(2)+p(3)$
$\therefore \boxed{p(x \leq 3) = 0.6472}$.

6) The no of accident in a year by taxi drivers in a city follows a poisson distribution, with mean 3, Out of 1000 taxi drivers, find approximately the no of drivers with; i) No accident
ii) More than 3 accidents in a year.

Soln :- Given; $\mu = 3$
$n = 1000$

WKT; By poisson's distribution; $p(x) = \frac{e^{-\mu}\mu^x}{x!}$

$p(x) = \frac{e^{-3}3^x}{x!}$ ~ (1).

(i) Prob that; no of drivers; with no accident in a year,

$\Rightarrow p(0) = \frac{e^{-3}3^0}{0!}$, $\boxed{p(0) = 0.0498}$ (for 1 taxi driver).

$\therefore$ The no of drivers with no accident in year $= 0.0498 \times 1000$
$= 49.8 \simeq 50$

$\therefore \boxed{p(0) = 50}$

(ii) $p(x>3) = 1 - P(x \leq 3)$
$= 1 - \{p(0) + p(1) + p(2) + p(3)\}$
$= 1 - \{0.0498 + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2} + \frac{e^{-3}3^3}{6}\}$.
$= 0.3528$

$\therefore$ The no of taxi drivers with 3 accidents in a year

is ; 0.3528×1000

$\boxed{p(x>3) = 352.}$

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4) In a certain factory, turning out razor blades, there is a chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, using Poisson distribution, find the approximate no of packets containing : i) No defective blade ii) 1 def. blade iii) 2 def. blade in a consignment of 10,000 packets.

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Soln : Given : $p = 0.002$.

$n = 10$.

WKT; $\mu = np$

$= 10 \times 0.002$, $\boxed{\mu = 0.02}$, Mean.

WKT; Poisson's distribution ; $p(x) = \dfrac{e^{-\mu}\mu^{x}}{x!}$

$$p(x) = \frac{e^{-0.02}(0.02)^{x}}{x!} \quad \text{--- (1)}.$$

(i) Probability that there is no defective blade;

$\Rightarrow p(0) = e^{-0.02} = \boxed{0.9802 = p(0)}$

$\therefore$ No of packets containing no defective blades $= 10{,}000 \times 0.9802$

$\boxed{p(0) = 9802}$

(ii) $p(2) = \dfrac{e^{-0.02}(0.02)^{2}}{2!} = 0.0002$

$\therefore$ No of packets containing 2 defective blades $= 10{,}000 \times 0.0002$

$\therefore \boxed{p(2) = 2}$

— * —

CONTINUOUS PROBABILITY DISTRIBUTION :-

The probability distributions, where the random variable varies Continuously over an interval, is called Continuous-probability distribution.

A function $P(x)$ is said to be a probability density function [Probability mass function (PMF)], if :

i) $P(x) \geq 0$

ii) $\int_{-\infty}^{\infty} P(x).dx = 1.$

For any specified variable t, the function $F(t)$ is defined by ; $F(t) = P(x \leq t) = P(x < t)$ is called Cumulative distribution function (CDF).

$\therefore$ Mean, $\boxed{\mu = E[X] = \int_{-\infty}^{\infty} x.p(x).dx.}$

$\therefore$ Variance, $V = E[X^2] - (E[X])^2$ (or)

$\therefore \boxed{V = \int_{-\infty}^{\infty} (x-\mu)^2 p(x).dx}$

$\therefore$ Standard deviation; $\boxed{\sigma = \sqrt{V}.}$

— * —

$$= \int_0^{2.5} p(x).dx$$

$$= \int_0^{2.5} e^{-x}.dx = -e^{-x}\Big|_0^{2.5}$$

$$= -\left[e^{-0.25} - e^{0}\right]$$

$$\therefore \boxed{F(2.5) = 0.9179}$$

2) A Continuous random variable x has the probability density

$$p(x) = \begin{cases} 2e^{-2x} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0. \end{cases}$$

Evaluate ; i) $E[X]$ ii) $E[X^2]$ iii) Variance iv) S.D.

Soln :-

WKT; Mean, $\mu = E[X] = \int_{-\infty}^{\infty} x.\,p(x).dx$

$$= \int_0^{\infty} x(2e^{-2x}).dx$$

$$= 2\int_0^{\infty} \underset{\text{I}}{x}.\underset{\text{II}}{e^{-2x}}.dx. = 2\left[x.\frac{e^{-2x}}{-2} - \left(-\frac{e^{-2x}}{4}\right)(1)\right]_0^{\infty}.$$

(prod. rule)

$$= \cancel{2}\left[\frac{e^{0}}{\cancel{4}_{2}}\right]$$

$$\therefore \boxed{\mu, E[X] = \frac{1}{2}}$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 p(x).dx$$

$$= 2\int_{+0}^{\infty} x^2 e^{-2x}.dx.$$

$$= 2\left[x^2\left(\frac{e^{-2x}}{-2}\right) - \left(\frac{e^{-2x}}{4}\right)2x + \frac{e^{-2x}}{8}2\right]_0^{\infty}$$

$$= 2.\frac{1}{4}$$

$$\boxed{E[X^2] = 1/2}$$

$$\therefore V = E[X^2] - (E[X])^2 \Rightarrow \boxed{V = 1/4}$$

$$= \frac{1}{2} - \frac{1}{4}$$

$$S.D = \sigma = \sqrt{V}, \quad \boxed{\sigma = 0.5}$$

$$\sigma = \sqrt{1/4}$$

3) A random variable x, has the density function;

$$P(x) = \begin{cases} Kx^2 & -3 \leq x \leq 3. \\ 0 & \text{elsewhere.} \end{cases}$$

Evaluate :- i) $P(1 \leq x \leq 2)$

ii) $P(x \leq 2)$

iii) $P(x > 1)$

Soln:- By defn :- $\int_{-\infty}^{\infty} p(x).dx = 1.$

$$= \int_{-\infty}^{\infty} Kx^2.dx = 1$$

$$= K\int_{-\infty}^{\infty} x^2.dx = 1.$$

$$K\left[\frac{x^3}{3}\right]_{-\infty}^{\infty} = K\left[\frac{3^3}{3} - \frac{(-3)^3}{-3}\right] = 1.$$

$$= K\left[\frac{x^3}{3}\right]_{-3}^{+3} = 1$$

$$= K\left[\frac{3^3}{3} - \frac{(-3)^3}{3}\right] = 1 \quad , \quad K[9+9] = 1$$

$$\therefore \boxed{K = 1/18 = 0.0556.}$$

(i) $p(1 \leq x \leq 2) = \int_1^2 p(x).dx$

$$= \int_1^2 Kx^2.dx = \frac{1}{18}\left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{(8-1)}{54} = \frac{7}{54} = 0.1296$$

(ii) $p(x \leq 2) = \int_{-3}^{2} p(x).dx$

$$= \int_{-3}^{2} Kx^2.dx = \frac{1}{18\times3} x^3 = \frac{1}{54}(8+27)$$

$$= \frac{35}{34} = 0.6481$$

(iii) $p(x>1) = \int_1^3 p(x).dx = \int_1^3 Kx^2 dx = \frac{1}{54} x^3\Big|_1^3$

$$= \frac{1}{54}[27-1] = \frac{13}{27} = 0.4815$$

3) The probability density of continuous random variable, 23 4

is : $p(x) = \begin{cases} Kx(1-x)e^{x} & 0 \leq x \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$

find K & Evaluate mean & S.D of distribution.

Soln :- By def^n : $\int_{-\infty}^{+\infty} p(x)\,dx = 1$

$$\Rightarrow \int_0^1 K(x)(1-x)e^{x}\,dx = 1$$

$$= K\int_0^1 \underset{I}{(x-x^2)}\,\underset{II}{e^{x}}.\,dx = 1.$$

$$= K\left[(x-x^2)e^{x} - (1-2x)e^{x} + (-2)e^{x}\right]_0^1 = 1$$

$$= K\left[+e - 2e - (1-2)\right] = 1$$

$$= K[-e+3] = 1$$

$$\boxed{K = \frac{1}{3-e}}$$

WKT;

$$\therefore \mu = \int_{-\infty}^{\infty} x.\,p(x)\,dx$$

$$= \int_0^1 x\cdot Kx(1-x)e^{x}.\,dx = K\int_0^1 (x^2-x^3)e^{x}.\,dx.$$

$$\left\{(x^2-x^3)e^{x} - (2x-3x^2)e^{x} + (2-6x)e^{x} - (-6)e^{x}\right\}_0^1$$

$$\mu = K\left\{(x^2-x^3)e^{x} - (2x-3x^2)e^{x} + (2-6x)e^{x} - (-6)e^{x}\right\}_0^1$$

$$= K\{e - 4e + 6e - (2+6)\}$$

$$= K\{3e-8\} \Rightarrow \mu = \frac{3e-8}{3-e}, \quad \boxed{\mu = 0.5496.}$$

$$\therefore S.D = \sigma = \sqrt{V}$$

Now, we find ; $E[x^2] = \int_{-\infty}^{+\infty} x^2 p(x)\,dx$

$$= \int_0^1 (x^2\,Kx(1-x)e^{x})\,dx$$

$$= K\int_0^1 (x^3-x^4).\,e^{x}.\,dx.$$

$$= K\left\{(x^3-x^4)e^{x} - (3x^2-4x^3)e^{x} + (6x-12x^2)e^{x} - (6-24x)e^{x} + (-24)e^{x}\right\}_0^1$$

$= K\{-11e+30\}$, $E[X^2] = \frac{30-11e}{3-e}$

$$\boxed{E[X^2] = 0.3511}$$

Variance, $V = E[X^2] - (E[X])^2$

$= 0.3511 - (0.5496)^2$

$$\boxed{V = 0.0490}$$

$\therefore$ SD, $\sigma = \sqrt{V}$

$$\boxed{\sigma = 0.2214.}$$

4) The probability density fn of a continuous random variable x is given by ; $p(x) = Ke^{-|x|}$, $-\infty < x < \infty$.
Show that ; $K = \frac{1}{2}$, find mean, variance & SD of distribution

Soln :- By defn :- $\int_{-\infty}^{+\infty} p(x)\,dx = 1$

$\int_{-\infty}^{\infty} K e^{-|x|}.dx = 1.$ // $e^{-|x|} = e^{-x}$

$= K.2\int_0^{\infty} e^{-x}.dx = 1.$

$= 2K\left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1.$

$= -2k(0-1) = 1$

$$\boxed{K = 1/2.}$$

$\therefore$ Mean, $\mu = E[X] = \int_{-\infty}^{\infty} x.p(x).dx$

$= \int_{-\infty}^{\infty} x.K.e^{-|x|}.dx$

$= K\int_{-\infty}^{\infty} x.e^{-|x|}.dx$ // [Since, $e^{-|x|}$ is odd function
Integration of odd fn is zero]

$$\boxed{\mu = 0}$$

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Now; $E[X^2] = \int_{-\infty}^{\infty} x^2 p(x).dx$

$$E[X^2] = \int_{-\infty}^{+\infty} x^2 K.e^{-|x|}.dx$$

$$= K.2\int_0^{\infty} \underset{I}{x^2}.\underset{II}{e^{-x}}.dx.$$

$$= 2K\left\{x^2\left(\frac{e^{-x}}{-1}\right) - 2x\left(e^{-x}\right) + 2\left(\frac{+e^{-x}}{-1}\right)\right\}_0^{\infty}$$

$$= 2K(-(-2))$$

$$= \boxed{E[X^2] = 2}$$

$$\therefore V = E[X^2] - (E[X])^2$$

$$= 2 - 0.$$

$$\boxed{V = 2}$$

$$\therefore SD, \sigma = \sqrt{V}$$

$$\boxed{\sigma = 1.4142}$$

Assgnt: Find the constant K so that; $p(x) = \begin{cases} Kxe^{-x} & 0<x<1 \\ 0 & , \text{otherwise.} \end{cases}$

is probability density fn, find, μ, V,

$$E[X^2] = \frac{6e-16}{e-2}$$

$$\mu = \frac{2e-5}{e-2}$$

$$K = \frac{e}{e-2}$$

$$V = \frac{2e^2-8e+7}{e^2-4e+4}$$

The random variable x has the density fn; $p(x) = \frac{K}{1+x^2}$

$-\infty < x < \infty$, Determine K & evaluate;

i) $P(x \geq 0)$

ii) $P(0 < x < 1)$

Soln :- By defn : $\int_{-\infty}^{+\infty} \phi(x).dx = 1$

$$= \int_{-\infty}^{+\infty} \frac{K}{1+x^2}.dx = 1$$

$$= K \int_{-\infty}^{+\infty} \underbrace{\frac{1}{1+x^2}}.dx = 1.$$

$$= K \left(\tan^{-1} x \right)_{-\infty}^{\infty} = 1$$

$$K \left[\pi/2 + \pi/2 \right) = 1$$

$$\therefore \boxed{K = 0.3183}$$

(i) $P(x \geq 0)$

$$\therefore = \int_0^{\infty} p(x).dx = K \int_0^{\infty} \frac{1}{1+x^2}.dx.$$

$$= K \tan^{-1} x \Big|_0^{\infty} = K.\pi/2 = 1/2$$

$$\therefore \boxed{P(x \geq 0) = 1/2}$$

(ii) $P[0 < x < 1)$

$$= \int_0^1 p(x).dx$$

$$= K \int_0^1 \frac{1}{1+x^2}.dx$$

$$= K \tan^{-1}(x) \Big|_0^1 = K.\pi/4$$

$$\therefore \boxed{P[0 < x < 1) = 1/4}$$

NORMAL DISTRIBUTION :-

The continuous probability distribution having the probability density function (pdf), $f(x)$ is given by;

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(x-\mu)^2/2\sigma^2}$$

where; $-\infty < x < \infty$, $-\infty < \mu < \infty$ and $\sigma > 0$ is known as the normal distribution.

Evidently; the following 2 conditions are satisfied;

1) $f(x) \geq 0$.

2) $\int_{-\infty}^{+\infty} f(x)dx = 1$.

MEAN & STANDARD DEVIATION OF NORMAL DISTRIBUTION :-

1) Mean, $\boxed{\mu = \text{Mean.}}$, The mean of normal distribution is equal to the mean of the given distribution.

2) Variance ; $\boxed{V = \sigma^2}$

3) Standard deviation ; $\boxed{\sigma = SD.}$

} Hence, Variance & S.D. of normal distribution is equal to V & SD of given distribution.

* EXPONENTIAL DISTRIBUTION :-

The continuous probability distribution having the probability density function f(x) given by;

$$f(x) = \begin{cases} \alpha \cdot e^{-\alpha x} & \text{for } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

is known as Exponential distribution.

* The 2 necessary conditions to be satisfied are :-

1) $f(x) > 0$.

2) $\int_{-\infty}^{+\infty} f(x)\,dx = 1$.

* Mean & Standard deviation of Exponential distribution :-

Mean, $\boxed{\mu = \frac{1}{\alpha}}$

Variance, $\boxed{V = \frac{1}{\alpha^2}}$

Standard deviation ; $\boxed{\sigma = \frac{1}{\alpha}}$

Problems & Solutions :-

1) Find which of the following functions is a probability density function.

Soln i) $f_1(x) = \begin{cases} 2x, & 0<x<1 \\ 0 & \text{otherwise.} \end{cases}$

iv) $f_4(x) = \begin{cases} 2x, & 0<x\leq 1. \\ 4-4x, & 1<x<2 \\ 0 & \text{otherwise.} \end{cases}$

ii) $f_2(x) = \begin{cases} 2x, & -1<x<1 \\ 0 & \text{otherwise.} \end{cases}$

iii) $f_3(x) = \begin{cases} |x|, & |x|\leq 1 \\ 0 & \text{otherwise.} \end{cases}$

i) $f_1(x) = \begin{cases} 2x, & 0<x<1 \\ 0, & \text{otherwise.} \end{cases}$

Soln:- WKT: Conditions satisfied for a "probability density function"

are :- 1) $f(x) \geq 0$.

2) $\int_{-\infty}^{+\infty} f(x)\,dx = 1$.

Clearly; $2x \geq 0$. $\because x \geq 0$. 1st condition is satisfied.

$(2) \Rightarrow \int_{-\infty}^{\infty} f_1(x).dx = \int_0^1 2x.dx. = \left[2.\frac{x^2}{2}\right]_0^1$

$= [1-0]$

$\therefore \boxed{\int_{-\infty}^{+\infty} f_1(x).dx = 1.}$

Hence, 2 Conditions are satisfied.

$\Rightarrow f_1(x)$ is a "probability density function". (pdf)

ii) $f_2(x) = \begin{cases} 2x & , -1 < x < 1 \\ 0 & , \text{otherwise.} \end{cases}$

Soln:-

$f(x) = 2x \quad , -1 < x < 1$

$f(x) = -ve < 0$

$\therefore$ $f(x) \geq 0$ Condition is not satisfied.

Consider (2) $\Rightarrow \int_{-\infty}^{+\infty} f(x)\,dx$

$= \int_{-1}^{+1} 2x \cdot dx = \left[\cancel{2} \cdot \frac{x^2}{\cancel{2}}\right]_{-1}^{+1}$

$= [\cancel{1 - 1}]^{0} = 0 \cdot \neq 1.$

$\therefore \int_{-1}^{+1} f(x)\,dx \neq 1$

$\therefore$ 2 Conditions are not satisfied.

$\therefore$ It is not pdf

iii) $f_3(x) = \begin{cases} |x| & , |x| \leq 1 \\ 0 & \text{otherwise.} \end{cases}$

Soln:- Since; $f_3(x) = |x| \geq 0$ (1) Condn. is satisfied

(2) $\Rightarrow \int_{-\infty}^{+\infty} f_3(x)\,dx = \int_{-1}^{+1} |x|\,dx$

Here; $|x| = \begin{cases} -x & , \text{if } -1 < x < 0 \\ +x & , \text{if } 0 < x < 1. \end{cases}$

$\therefore \int_{-1}^{+1} |x|\,dx = \int_{-1}^{0} -x\,dx + \int_{0}^{1} x\,dx = -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1} = \frac{1}{2} + \frac{1}{2} = 1$

$\therefore$ It is a pdf

iv) $f_4(x) = \begin{cases} 2x & , 0 < x \leq 1 \\ 4-4x & , 1 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$

Soln:- Clearly; $f_4(x) = 2x > 0$, $f_4(x) \geq 0$, ① Condn is satisfied in $0 < x \leq 1$.

Clearly; $f_4(x) = 4-4x$ is negative in : $1 < x < 2$.

$\therefore$ The 1st condition is not satisfied.

$\therefore$ $f_4(x)$ is not a pdf

2) Find the value of c such that;

$f(x) = \begin{cases} x/6 + c, & 0 \leq x \leq 3. \\ 0 & \text{elsewhere} \end{cases}$ is a probability density function.

Also find : $p(1 \leq x \leq 2)$.

Soln :- given that; $f(x)$ is a pdf

then, it satisfies :- 1) $f(x) \geq 0$.

2) $\int_{-\infty}^{+\infty} f(x).dx = 1$.

Consider; ② $\Rightarrow \int_{-\infty}^{+\infty} f(x)\,dx = 1$.

$$\int_0^3 \left(x/6 + c\right) dx = 1.$$

$$= \left[\frac{x^2}{12} + cx\right]_0^3 = 1.$$

$$= \frac{3}{4} + 3c = 1, \quad \boxed{c = 1/12}$$

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Now, to find: $P(1 \leq x \leq 2) = \int_1^2 f(x).dx.$

$$= \int_1^2 \left(\frac{x}{6} + \frac{1}{12}\right) dx$$

$$= \left[\frac{x^2}{12} + \frac{x}{12}\right]_1^2$$

$$= \frac{1}{12}\left[(4+2) - (1+1)\right]$$

$$\therefore \boxed{p(1 \leq x \leq 12) = 1/3}$$

3) Find the constant K such that;

$f(x) = \begin{cases} Kx^2 & 0<x<3 \\ 0 & \text{otherwise.} \end{cases}$ is a p.df, Also Compute:

i) $P(1<x<2)$ ii) $P(x \leq 1)$ iii) $P(x>1)$

iv) Mean v) Variance.

Soln:- Since, Given that; $f(x)$ is pdf

$\Rightarrow$ 1) $f(x) \geq 0$

2) $\int_{-\infty}^{+\infty} f(x) dx = 1.$

Consider; ② $\Rightarrow \int_0^3 Kx^2.dx = 1.$

$$\Rightarrow \left[\frac{Kx^3}{3}\right]_0^3 = \left[\frac{K}{3}(3^3-0^3)\right] = \frac{K}{\cancel{3}_1}\cancel{27}^9 = 1$$

$$\therefore \boxed{K = 1/9.}$$