MODULE-1 [NUMERICAL METHODS-1]

NUMERICAL PREDICTOR AND CORRECTOR METHODS:

In these methods the value of y at a desired value of x is estimated from a set of four values of y corresponding to 4 equally spaced values of x.

We discuss 2 predictor & Corrector methods namely;

1) Milne's method
2) Adams-bashforth method.

Consider the dE; $y' = \frac{dy}{dx} = f(x,y)$ with a set of 4 pre-determined values of y: $y(x_0) = y_0$, $y(x_1) = y_1$, $y(x_2) = y_2$ & $y(x_3) = y_3$.

Here; x_0, x_1, x_2, x_3 are equally spaced values of x with width "h".

Also: $x_4 = x_3 + h$

Therefore, the predictor & Corrector formulae to compute $y(x_4) = y_4$ are Given as follows;

Milne's predictor and Corrector formulae:-

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3') \quad // \text{Predictor formula}$$

$$y_4^{(C)} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4') \quad // \text{Corrector formula.}$$

Adam's-Bashforth predictor and Corrector formulae:-

$$y_4^{(P)} = y_3 + \frac{h}{24}(55y_3' - 59y_2' + 37y_1' - 9y_0') \quad // \text{Predictor}$$

$$y_4^{(C)} = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1') \quad // \text{Corrector.}$$

Working procedure :-

- We 1st prepare the table showing values of y corresponding to 4 equidistant values of x and the computation of $y' = f(x, y)$
- We compute y_4 from the predictor formula.
- We use this value of y_4 to compute $y_4' = f(x_4, y_4)$.
- We apply corrector formula to obtain the corrected value of y_4.
- This value is used for computing y_4' to apply corrector formula again.
- The process is continued till we get consistency in 2 consecutive values of y_4.

TUTORIAL QUESTIONS :

1) Using 4th order R-K method, Compute $y(0.2)$ for Eqⁿ:
$\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.2$ [Ans: $y(0.2) = 1.1679$]

2) Using 4th order R-k method, Compute $y(0.4)$, $y(0) = 1$,
for: $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ [II Stages : $y(x_1) = y(0.2) = 1.196$
$y(x_2), y(0.4) = 1.3753$]
$\boxed{h = 0.2}$

Problems & Solutions :-

1) Apply Milne's method & Compute y at $x = 0.8$, Given that $\frac{dy}{dx} = x - y^2$ and $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$.

Soln : Given : $\frac{dy}{dx} = x - y^2$ Also Given that ; $y(x_0) = y_0$, $y(0) = 0$, $x_0 = 0$, $y_0 = 0$

$y(0.2) = 0.02$, $x_1 = 0.2$, $y_1 = 0.02$

$y(0.4) = 0.0795$, $x_2 = 0.4$, $y_2 = 0.0795$

$y(0.6) = 0.1762$, $x_3 = 0.6$, $y_3 = 0.1762$.

$\therefore x_1 = x_0 + h$, $\boxed{x_4 = 0.8}$

$0.2 - 0 = h$

$\boxed{h = 0.2}$

We Compute the table ;

x	y	$\frac{dy}{dx} = y' = x - y^2$
$x_0 = 0$	$y_0 = 0$	$y_0' = 0 - 0^2 = 0$
$x_1 = 0.2$	$y_1 = 0.02$	$y_1' = 0.2 - (0.02)^2 = 0.1996$.
$x_2 = 0.4$	$y_2 = 0.0795$	$y_2' = 0.4 - (0.0795)^2 = 0.3937$.
$x_3 = 0.6$	$y_3 = 0.1762$	$y_3' = 0.6 - (0.1762)^2 = 0.5689$
$x_4 = ?$	$y_4 = ?$	

W.K.T ; Milne's predictor formula is Given by ;

$$y_4^{(p)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$

$$y_4^{(p)} = 0 + \frac{4(0.2)}{3}[2(0.1996) - 0.3937 + 2(0.5689)]$$

$$y_4^{(p)} = 0.3049$$

Now, we Compute : $y_4' = f(x_4, y_4)$

$\Rightarrow y_4' = x_4 - (y_4)^2 = 0.8 - (0.3049)^2$ // $y_4^{(p)} = y_4$

$\therefore \boxed{y_4' = 0.707}$

Now, we use Milne's Corrector formula;

$$y_4^{(c)} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4').$$

$$y_4^{(c)} = 0.0795 + \frac{0.2}{3}[0.3937 + 4(0.5689) + 0.707]$$

$$\boxed{y_4^{(c)} = 0.3046.}$$

Again, we find ; $y_4' = x_4 - y_4^2 = 0.8 - (0.3046)^2$

$$\boxed{y_4' = 0.7072}$$

Now; Substituting the value of y_4' again in the corrector formula;

$$y_4^{(c)} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4')$$

$$y_4^{(c)} = 0.0795 + \frac{0.2}{3}[0.3937 + 4(0.5689) + 0.7072]$$

$$\boxed{y_4^{(c)} = 0.3046}$$

// We continue to use corrector method until we get same values of y_4

Therefore; $y_4 = y(0.8) = 0.3046.$

2) Apply Milne's method, to compute $y(1.4)$ correct to 4 decimal places

Given ; $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$
$y(1.3) = 2.7514$.

Soln : Given ; $\frac{dy}{dx} = f(x,y) = y' = x^2 + \frac{y}{2}$.

$y(1) = 2$, $x_0 = 1$, $y_0 = 2$. , $y(1.1) = 2.2156$
$y(x_0) = y_0$ $\quad x_1 = 1.1$, $y_1 = 2.2156$

$y(1.2) = 2.4649$, $y(1.3) = 2.7514$
$x_2 = 1.2$, $y_2 = 2.4649$ $\quad x_3 = 1.3$, $y_3 = 2.7514$.

WKT; $x_1 = x_0 + h$.
$h = x_1 - x_0$, $\boxed{h = 0.1}$

Now,

3) The following table gives the solution of : $5xy' + y^2 - 2 = 0$, Find the value of y at $x = 4.5$ using Milne's predictor & Corr formulae, use Corrector formula twice.

x	4	4.1	4.2	4.3	4.4
y	1	1.0049	1.0097	1.0143	1.0187.

$5x\frac{dy}{dx} = 2 - y^2$

$\therefore \frac{dy}{dx} = \frac{2-y^2}{5x}$.

Soln : Given, $x_0 = 4$, $x_1 = 4.1$, $x_2 = 4.2$, $x_3 = 4.3$, $x_4 = 4.4$

$y_0 = 1$, $y_1 = 1.0049$, $y_2 = 1.0097$, $y_3 = 1.0143$, $y_4 = 1.0187$

$x_5 = ?, y_5 = ?$

Now we Compute table;

x	y	$y' = \frac{2-y^2}{5x}$.
$x_0 = 4$	$y_0 = 1$	$y_0' = 0.05$
$x_1 = 4.1$	$y_1 = 1.0049$	$y_1' = 0.0483$
$x_2 = 4.2$	$y_2 = 1.0097$	$y_2' = 0.0467$
$x_3 = 4.3$	$y_3 = 1.0143$	$y_3' = 0.0452$
$x_4 = 4.4$	$y_4 = 1.0187$	$y_4' = 0.0437$.
$x_5 = 4.5$?	$y_5 = ?$	$y_5' = ?$

WKT; $x_1 = x_0 + h$.

$x_1 - x_0 = h$, $\boxed{h = 0.1}$

Now; WKT Milne's predictor formula; $y_4^{(p)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$

But; we need : $y_5^{(p)} \Rightarrow y_5^{(p)} = y_1 + \frac{4h}{3}(2y_2' - y_3' + 2y_4')$

$\Rightarrow y_5^{(p)} = 1.0049 + \frac{4(0.1)}{3}[2(0.0467) - (0.0452) + 2(0.0437)]$

$\boxed{y_5^{(p)} = 1.023.} = y_5$

Now, $y_5' = \frac{2 - y_5^2}{5x_5} = 0.0424$

Now, By using Milne's Corrector formula : $y_5^{(c)} = y_3 + \frac{h}{3}(y_3' + 4y_4' + y_5')$

$\boxed{y_5^{(c)} = 1.023}$

$\therefore y_4^{(p)} = y_4^{(c)}$

$\therefore \boxed{y(4.5) = 1.023.}$

Q. If $y' = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$, $y(0.3) = 2.090$, find $y(0.4)$, correct to 4 decimal places by using milne's predictor-Corrector method.

Soln: Given; $y' = 2e^x - y$. $y(0) = 2$ $x_0 = 0$, $y_0 = 2$.

$y(0.1) = 2.010$, $x_1 = 0.1$, $y_1 = 2.010$.

$y(0.2) = 2.040$

$x_2 = 0.2$, $y_2 = 2.040$, $y(0.3) = 2.090$.

$x_3 = 0.3$, $y_3 = 2.090$. $\boxed{h = 0.1}$

Now, We Compute table;

x	y	$y' = 2e^x - y$
$x_0 = 0$	$y_0 = 2$	$y'_0 = 0$
$x_1 = 0.1$	$y_1 = 2.010$	$y'_1 = 0.2003$
$x_2 = 0.2$	$y_2 = 2.040$	$y'_2 = 0.4028$
$x_3 = 0.3$	$y_3 = 2.090$	$y'_3 = 0.6097$
$x_4 = ?$	$y_4 = ?$	

WKT; $x_1 = x_0 + h$.

$x_1 - x_0 = h$, $h = 0.1$

WKT; By Milne's predictor's Method; $y_4^{(p)} = y_0 + \frac{4h}{3}(2y'_1 - y'_2 + 2y'_3)$

$\boxed{y_4^{(p)} = 2.1623}$

Now; $y' = 2e^x - y \Rightarrow y'_4 = 2e^{x_4} - y_4^{(p)}$

$y'_4 = 2.e^{0.4} - 2.1623$.

$\boxed{y'_4 = 0.8213}$

Now, by Milne's Corrector method; $y_4^{(c)} = y_2 + \frac{h}{3}(y'_2 + 4y'_3 + y'_4)$.

$\boxed{y_4^{(c)} = 2.1621}$, Now; $y' = 2e^x - y$. $\Rightarrow y'_4 = 2e^{x_4} - y_4^{(c)}$

$\boxed{y'_4 = 0.8215}$

Applying Corrector method again; $\boxed{y_4^{(c)} = 2.1621}$ // $y_4^{(c)} = y_4^{(p)}$

$\therefore \boxed{y(0.4) = 2.162}$

Adam's - Bashforth Predictor and Corrector Formula

Apply A and

$$\boxed{y_4^{(p)} = y_3 + \frac{h}{24}\left(55y_3' - 59y_2' + 37y_1' - 9y_0'\right)}$$ // predictor formula

$$\boxed{y_4^{(c)} = y_3 + \frac{h}{24}\left(9y_4' + 19y_3' - 5y_2' + y_1'\right)}$$ // Corrector formula.

Working

- We 1st prepare the table showing values of y corresponding to 4 equidistant values of x & $y' = f(x, y)$.
- We compute y_4 from predictor formula.
- we use this value of y_4 to compute $y_4' = f(x, y)$.
- we apply Corrector formula to obtain corrected value of y_4.
- This value is used for Computing y_4' to apply corrector formula again.
- This process is continued until we get consistency in 2 Consecutive values of y_4.

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Apply Adams - Bashforth method to compute : $\frac{dy}{dx} = x - y^2$

and Given: $y(0)=0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$

$y(0.6) = 0.1762$, ~~y~~ Compute y at $x = 0.8$.

Soln : Given ; $y' = x - y^2$ $\quad y(0) = 0$, $x_0 = 0$, $y_0 = 0$

$y(0.2) = 0.02$, $x_1 = 0.2$, $y_1 = 0.02$, $\boxed{h = 0.2}$

$y(0.4) = 0.0795$; $x_2 = 0.4$, $y_2 = 0.0795$, $y(0.6) = 0.1762$

$x_3 = 0.6$, $y_3 = 0.1762$

Now, We Compute table;

x	y	$y' = x - y^2$
$x_0 = 0$	$y_0 = 0$	$y_0' = 0$
$x_1 = 0.2$	$y_1 = 0.02$	$y_1' = 0.1996$
$x_2 = 0.4$	$y_2 = 0.0795$	$y_2' = 0.3937$
$x_3 = 0.6$	$y_3 = 0.1762$	$y_3' = 0.5689$
$x_4 = 0.8$	$y_4 = ?$	

By Applying Adam's - Bashforth predictor formula;

$$y_4^{(p)} = y_3 + \frac{h}{24}(55y_3' - 59y_2' + 37y_1' - 9y_0')$$

$$\boxed{y_4^{(p)} = 0.3049.}$$

Now, we Compute ; $y_4' = x_4 - (y_4^{p})^2$

$$\boxed{y_4' = 0.7072}$$

Now, Applying A-B Corrector formula ; $y_4^{(c)} = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1')$

$$\boxed{y_4^{(c)} = 0.3046.} \quad // \; y_4^{(c)} = y_4^{(p)}$$

$$y_4 = y(0.8) = 0.3046$$

2) Employing Adams-Bashforth method, find approximate solution DE : $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ at point $x = 1.4$. Given that

again,

$y(1) = 1$, $y(1.1) = 0.996$, $y(1.2) = 0.986$, $y(1.3) = 0.972$.

Soln :- Given; $\frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x}$. $\boxed{h = 0.1}$

$x_0 = 1$, $x_1 = 1.1$ $x_2 = 1.2$ $x_3 = 1.3$

$y_0 = 1$, $y_1 = 0.996$ $y_2 = 0.986$ $y_3 = 0.972$.

Now, we Compute table,

x.	y.	$y' = \frac{1}{x^2} - \frac{y}{x}$
$x_0 = 1$	$y_0 = 1$	$y_0' = 0$
$x_1 = 1.1$	$y_1 = 0.996$	$y_1' = -0.079$
$x_2 = 1.2$	$y_2 = 0.986$	$y_2' = -0.12722$.
$x_3 = 1.3$	$y_3 = 0.972$	$y_3' = -0.15598$.
$x_4 = ?$	$y_4 = ?$	$y_4' = ?$

Now, we apply Adams-Bashforth predictor formula;

$$y_4^{(p)} = y_3 + \frac{h}{24}(55y_3' - 59y_2' + 37y_1' - 9y_0')$$

$$\boxed{y_4^{(p)} = 0.95535.}$$

We Compute; $y_4' = \frac{1}{x_4^2} - \frac{(y_4)^p}{x_4}$, $y_4' = -0.172189$.

Now, we apply Adam's-bashforth Corrector formulae;

$$y_4^{(c)} = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1')$$

$$\boxed{y_4^{(c)} = 0.95552}$$

Now, we use, $y_4^{(c)}$ in ; $y_4' = \frac{1}{x_4^2} - \frac{(y_4)^c}{x_4}$, $\boxed{y_4' = -0.17231}$

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again, we use y_4' in A-B. Corrector method;

$$y_4^{(c)} = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1')$$

$\boxed{y_4^{(c)} = 0.95551}$ // we take this twice corrected value, y^c as soln

$y_4 = 0.95551$

3) If $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$, $y(0.3) = 2.090$, find $y(0.4)$, correct to 4 decimal places by A-B. method.

Soln: Given; $\frac{dy}{dx} = 2e^x - y$,

we compute table;

x	y	$y' = 2e^x - y$
$x_0 = 0$	$y_0 = 2$	$y_0' = 0$
$x_1 = 0.1$	$y_1 = 2.010$	$y_1' = 0.2003$
$x_2 = 0.2$	$y_2 = 2.040$	$y_2' = 0.4028$
$x_3 = 0.3$	$y_3 = 2.090$	$y_3' = 0.6097$
$x_4 = 0.4$	$y_4 = ?$	$y_4' = ?$

Now, by A-B's predictor formula;

$$y_4^{(p)} = y_3 + \frac{h}{24}\cancel{(9y_4' + 19y_3' - 5y_2' + y_1')}.(55y_3' - 59y_2' + 37y_1' - 9y_0')$$

$$y_4^{(p)} = 2.09 + \frac{0.1}{24}\left[\cancel{9(0.822) + 19(0.6097) - 5(0.4028) + 0.2003}\right]$$

$\boxed{y_4^{(p)} = 2.1615}$

$\therefore\ y_4' = f(x_4, y_4) = 2.e^{x_4} - y_4^{(p)}$

$\boxed{y_4' = 0.822}$

Substituting in AB's Corrector formula.;

$$y_4^{(c)} = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1')$$

$$\boxed{y_4^{(c)} = 2.1615}$$

$\therefore y_4' = f(x_4, y_4) = \boxed{y_4' = 0.82215}$

Substituting again in Corrector formula;

$y_4^{(c)} = 2.1615$, therefore; $\boxed{y(0.4) = 2.1615}$

Asmt

4) Given; $\frac{dy}{dx} = x^2(1+y)$, $y(1.1) = 1.233$, $y(1.2) = 1.548$
$y(1.3) = 1.979$, determine $y(1.4)$ by AB method, Carryout 2 Correctors for soln

Soln :- $\frac{dy}{dx} = x^2(1+y)$. $h = 0.1$

$y_0' = 2$, $y_1' = 2.702$, $y_2' = 3.669$, $y_3' = 5.035$

Pred $\rightarrow$ $y_4^{(p)} = y_3 + \frac{h}{24}(55y_3' - 59y_2' + 37y_1' - 9y_0')$

$$\boxed{y_4^{(p)} = 2.572}$$

$y_4' = f(x_4, y_4^{(p)}) = \boxed{y_4' = 7.001}$

Corre $\rightarrow$ $y_4^{(c)} = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1')$

$$\boxed{y_4^{(c)} = 2.575}$$

$$\boxed{y_4' = 7.007}$$

Again, Corre $\rightarrow$ $\boxed{y_4^{(c)} = 2.5752}$

$\therefore \boxed{y(1.4) = 2.575}$

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MODULE-2 NUMERICAL METHODS-II

Numerical Solution of second order ordinary differential equations :-

The Given differential equation will be second order Ordinary differential Equation with 2 initial Conditions, which will be reduced to 2 first order simultaneous ODE's, further the obtained dE is solved by : IInd order R-K method (or) Milne's Method.

Let $y''=g(x,y,y')$ with initial conditions, $y(x_0)=y_0$ and $y'(x_0)=y_0'$ be the Given 2nd order ODE.

Now, let $y'=\frac{dy}{dx}=z$

Therefore; $y''=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}(z)$

$$\therefore y''=\frac{dz}{dx}.$$

*(The given 2nd order D.E assumes the form : $\frac{dz}{dx}=g(x,y,z)$ with conditions : $y(x_0)=y_0$ & $z(x_0)=z_0$) (where; y_0' is denoted by z_0.)

Hence, we now have 1st order ODE's;

ie; $\frac{dy}{dx}=z$ & $\frac{dz}{dx}=g(x,y,z)$ with $y(x_0)=y_0$ & $z(x_0)=z_0$.

Taking $f(x,y,z)=z$, we have follo system of Equations for solving;

ie; $\frac{dy}{dx}=f(x,y,z)$, $\frac{dz}{dx}=g(x,y,z)$; $y(x_0)=y_0$ and $z(x_0)=z_0$.

I Runge-Kutta method :-

We have to Compute $y(x_0+h)$ and if required

$y'(x_0+h) = z(x_0+h)$

We need to 1st Compute the following :-

$K_1 = h.f(x_0, y_0, z_0)$

$K_2 = h.f\left(x_0+\frac{h}{2}, y_0+\frac{K_1}{2}, z_0+\frac{l_1}{2}\right)$

$K_3 = h.f\left(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}, z_0+\frac{l_2}{2}\right)$

$K_4 = h f\left(x_0+h, y_0+k_3, z_0+l_3\right)$

$l_1 = h.g(x_0, y_0, z_0)$

$l_2 = h.g\left(x_0+\frac{h}{2}, y_0+\frac{K_1}{2}, z_0+\frac{l_1}{2}\right)$

$l_3 = h.g\left(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}, z_0+\frac{l_2}{2}\right)$

$l_4 = h.g\left(x_0+h, y_0+k_3, z_0+l_3\right).$

The required ; $y(x_0+h) = y_0 + \frac{1}{6}(K_1+2K_2+2K_3+K_4)$

$y'(x_0+h) = z(x_0+h) = \frac{1}{6}(l_1+2l_2+2l_3+l_4).$

Problems & Solutions :-

1) Given : $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$, $y(0)=1$, $y'(0)=0$, Evaluate $y(0.1)$ using Runge-kutta method of order 4.

Soln :- Given , $\frac{d^2y}{dx^2} - x^2 \cdot \frac{dy}{dx} - 2xy = 1$ ~ ①, $y(0)=1$, $y(x_0)=y_0$

$x_0=0$, $y_0=1$.

$y'(x_0)=y_0'$, $y'(0)=0$

$x_0=0$, $y_0'=0$

$\Rightarrow$ putting : $\frac{dy}{dx}=z$ and diff wrt x,

we obtain ; $\frac{d^2y}{dx^2}=\frac{dz}{dx}$ so that the Given Eqn assumes the form ;

By using in Eqn ①; we get : $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$

$\frac{dz}{dx} - x^2z - 2xy = 1$, $\frac{dz}{dx} = x^2z + 2xy + 1$

Hence, we have a system of Equations ; (of 1st order) :-

$\frac{dy}{dx} = z$, $\frac{dz}{dx} = 1+2xy+x^2z$, where : $x_0=0$, $y_0=1$, $y_0'=z_0=0$

Let $f(x,y,z) = z$, $g(x,y,z) = 1+2xy+x^2z$

WKT; $x_0=0$, $y_0=1$, $z_0=0$, $h=0.1$

$x_1 = x_0+h$.

$h = 0.1-0$

$\boxed{h=0.1}$

We shall 1st Compute ;

K_1, K_2, K_3, K_4.

$K_1 = h.f(x_0,y_0,z_0)$

$K_1 = 0.1.f(0,1,0)$ // $z_0=0$

$K_1 = 0.1[z_0] = 0.1\times 0$, $\boxed{K_1=0}$

$l_1 = h.g(x_0, y_0, z_0)$

$l_1 = 0.1(1 + 2(0)(1) + (0)^2(0))$

$\boxed{l_1 = 0.1}$

By

x

$K_2 = h.f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}, z_0 + \frac{l_1}{2}\right)$

$K_2 = 0.1.f(0.05, 1, 0.05)$

$\boxed{K_2 = 0.005}$

$l_2 = h.g\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}, z_0 + \frac{l_1}{2}\right)$

$l_2 = 0.1(1 + 2(0.05)(1) + (0.05)^2(0.05)) = \boxed{0.11 = l_2}$

$K_3 = h.f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}, z_0 + \frac{l_2}{2}\right).$

$K_3 = 0.1.f(0.05, 1.0025, 0.055)$

$\boxed{K_3 = 0.0055}$

$l_3 = h.g\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}, z_0 + \frac{l_2}{2}\right).$

$l_3 = 0.1(1 + 2(0.05)(1.0025) + (0.05)^2(0.055)).$

$\boxed{l_3 = 0.11004}$

$K_4 = h.f(x_0 + h, y_0 + K_3, z_0 + l_3).$

$\boxed{K_4 = 0.011}$

$l_4 = h.g(x_0 + h, y_0 + K_3, z_0 + K_3)$

$\boxed{l_4 = 0.12022.}$

$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$K = \frac{1}{6}(0 + 2(0.005) + 2(0.0055) + 0.011)$

$\boxed{K = \quad}$

We have; $y(x_0 + h) = y(x_1) = y_0 + K.$

$y(x_0 + h) = y(x_1) = 1 + K$

$\therefore \boxed{y(x_1) = y(0.1) = 1.0053.}$

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By RK method, solve : $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for $x = 0.2$, correct to 4 decimal places, using initial conditions ; $y = 1$ & $y' = 0$ when $x = 0$.

Soln : By data... $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ — ①, $y_0 = 1$, $y_0' = 0$, $x_0 = 0$.

$x_1 = x_0 + h$.

$h = 0.2$.

putting ; $\frac{dy}{dx} = z$ and diff wrt x

we obtain; $\frac{d^2y}{dx^2} = \frac{dz}{dx}$.

$\therefore$ The Given Eqn becomes ① $\Rightarrow \frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$

$\frac{dz}{dx} = xz^2 - y^2$, where: $\boxed{y_0 = 1}$, $\boxed{y_0' = z = 0}$ $\boxed{x_0 = 0}$.

Hence, we have system of equations ; (of 1st order)

$\frac{dy}{dx} = z$, $\frac{dz}{dx} = xz^2 - y^2$

Now, Let ; $f(x, y, z) = z$, $g(x, y, z) = xz^2 - y^2$

$x_0 = 0$, $y_0 = 1$, $z_0 = 0$.

We shall compute the following ;

$k_1 = h.f(x_0, y_0, z_0)$

$k_1 = 0.2, f(0, 1, 0)$

$\boxed{k_1 = 0}$

$l_1 = h.g(x_0, y_0, z_0)$

$l_1 = 0.2\left[0(0) - 1\right]$

$\boxed{l_1 = -0.2}$

$k_2 = h.f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$

$\boxed{k_2 = -0.02.}$

$l_2 = h.g\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$

$\boxed{l_2 = -0.1998.}$

$K_3 = h.f\left(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}, z_0+\frac{l_2}{2}\right)$

$\boxed{K_3 = -0.01998}$

$l_3 = h\,g\left(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}, z_0+\frac{l_2}{2}\right)$

$\boxed{l_3 = -0.1958}$

$K_4 = h.f(x_0+h, y_0+K_3, z_0+l_3)$

$\boxed{K_4 = -0.03916}$

$l_4 = h.\,g(x_0+h, y_0+K_3, z_0+l_3)$

$\boxed{l_4 = -0.19055}$

$K = \frac{1}{6}(K_1+2K_2+2K_3+K_4)$

$K =$

$\therefore\ y(x_0+h) = y(x_1) = y_0 + K$

$\boxed{y(x_1) = y(0.2) = 0.9801}$

3) Compute $y(0.1)$ Given; $\frac{d^2y}{dx^2} = y^3$ & $y=10$, $y'=5$ at $x=0$ — (1)
by R.K method of order 4.

Soln:- put; $\frac{dy}{dx} = z$ dwt ... $\frac{d^2y}{dx^2} = \frac{dz}{dx}$ use in (1)

$\frac{dz}{dx} = y^3$, $y_0 = 10$, $y_0' = z = 5$, ~~z_0~~ $x_0 = 0$

$\therefore \frac{dy}{dx} = z$ & $\frac{dz}{dx} = y^3$

$\Rightarrow f(x,y,z) = z$ $\quad g(x,y,z) = y^3$

$K_1 = 0.5 \quad l_1 = 100$

$K_2 = 5.5 \quad l_2 = 107.7$

$K_3 = 5.885 \quad l_3 = 207.27$

$K_4 = 21.227 \quad l_4 = 400.83$

$\therefore\ y(x_0+h) = y(x_1) = y_0 + k.$

$\boxed{y(x_1) = y(0.1) = 17.4162}$

Given

$y'(0)$

4) Given that : $y'' - xy' - y = 0$ with the initial conditions : $y(0) = 1$, $y'(0) = 0$, Compute : $y(0.2)$ & $y'(0.2)$ using R-K method of order 4.

Soln :- $y'' - xy' - y = 0$. —①.

Putting ; $y' = z$ and $y'' = \frac{dz}{dx}$.

$y(0) = 1$, $x_0 = 0$, $y_0 = 1$

$y_0' = z_0 = 0$

Sub in Eqn ①, we get ; $\frac{dz}{dx} - xz - y = 0$.

$\therefore \frac{dy}{dx} = z$, $\frac{dz}{dx} = xz + y$ are obtained a system of Eqns

Let : $f(x,y,z) = z$, $g(x,y,z) = xz + y$.

where ; $x_0 = 0$, $y_0 = 1$, $z_0 = 0$, $h = 0.2$

Now, we shall Compute ; $K_1 = h.f(x_0, y_0, z_0)$

$K_1 = 0.2.f(0,1,0)$, $\boxed{K_1 = 0}$

$l_1 = h.g(x_0 y_0, z_0)$

$\boxed{l_1 = 0.2}$

$K_2 = h.f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}, z_0 + \frac{l_1}{2}\right)$

$\boxed{K_2 = 0.02}$

$l_2 = h.g\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}, z_0 + \frac{l_1}{2}\right)$.

$\boxed{l_2 = 0.202}$

$K_3 = h.f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}, z_0 + \frac{l_2}{2}\right)$

$\boxed{K_3 = 0.0202}$

$l_3 = h.f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}, z_0 + \frac{l_2}{2}\right)$

$\boxed{l_3 = 0.204}$

$\boxed{K_4 = 0.0408}$

$\boxed{l_4 = 0.2122}$

We have ; $y(x_0 + h) = y(x_1) = y(0.2) = y_0 + K = 1 +$

$\boxed{y(0.2) = 1.0202}$

$y'(x_0 + h) = z(x_0 + h) = y'(0.2) = y_0 + l$.

$\boxed{y'(0.2) = 0.204}$

5) Obtain the value of x and $\frac{dx}{dt}$ when $t=0.1$, Given that x satisfies the equation : $\frac{d^2x}{dt^2} = t\frac{dx}{dt} - 4x$ and $x=3$, $\frac{dx}{dt}=0$ when $t=0$ initially. Use 4th order R-K method.

Soln : Given ; $\frac{d^2x}{dt^2} = t.\frac{dx}{dt} - 4x$. —(1)

$\underline{x_0 = 3}$, $\underline{x_0' = y_0 = 0}$

putting ; $\frac{dx}{dt} = y$ dwt wrt t, we obtain ; $\underline{t_0 = 0}$

$\frac{d^2x}{dt^2} = \frac{dy}{dt}$ ⇒ substituting in Eqn (1) ; we get ;

$$\Rightarrow \frac{dy}{dt} = ty - 4x.$$

Hence, we have obtained a system of Equations ;

$$\frac{dx}{dt} = y. \qquad \frac{dy}{dt} = ty - 4x.$$

where $t_0=0$, $\underline{x_0 = 3}$, $\underline{y_0 = 0}$, ~~$t_0=0$~~

Let : $f(x,y,z) = y$. $g(x,y,z) = ty - 4x$.

where $\underline{t_0=0}$, $\underline{x_0 = 3}$, $\underline{y_0 = 0}$, ~~$t_0=0$~~ and $\underline{h = 0.1}$

Now, we will Compute ;

$K_1 = h.f(t_0, x_0, y_0)$ $l_1 = h.g(t_0, x_0, y_0)$

$K_1 = 0.1 f(0,3,0)$ $l_1 = 0.1\, g(0,3,0)$

$\boxed{K_1 = 0}$ $\boxed{l_1 = -1.2.}$

$K_2 = h.f\left(t_0 + \frac{h}{2}, x_0 + \frac{K_1}{2}, y_0 + \frac{l_1}{2}\right)$ $l_2 = h.g\left(t_0 + \frac{h}{2}, x_0 + \frac{K_1}{2}, y_0 + \frac{l_1}{2}\right)$

$\boxed{K_2 = -0.06}$ $\boxed{l_2 = -1.203}$

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that

$\therefore K_3 = h.f\left(t_0+\frac{h}{2},\ x_0+\frac{K_2}{2},\ y_0+\frac{l_2}{2}\right)$

$\boxed{K_3 = -0.06015}$

$l_3 = h.g\left(t_0+\frac{h}{2},\ x_0+\frac{K_2}{2},\ y_0+\frac{l_2}{2}\right)$

$\boxed{l_3 = -1.191}$

$K_4 = h.f\left(t_0+h,\ x_0+K_3,\ y_0+l_3\right)$

$\boxed{K_4 = -0.1191}$

$l_4 = h.g\left(t_0+h,\ x_0+K_3,\ y_0+l_3\right)$

$\boxed{l_4 = -1.18785}$

$\therefore K = \frac{1}{6}\left(K_1+2K_2+2K_3+K_4\right)$

$K =$

$\therefore l = \frac{1}{6}\left(l_1+2l_2+2l_3+l_4\right)$

$l =$

$\therefore x(t_0+h) = x(t_1) = x(0.1) = x_0 + K.$

$x(0.1) = 3 + -$

$\boxed{x(0.1) = 2.9401}$

$\therefore y(t_0+h) = y(t_1) = y(0.1) = y_0 + l.$

$\underline{x_0'(0.1)} = \boxed{y(0.1) = -1.196}$

Asgnt 5) Solve : $y''+4y=xy$, $y(0)=3$, $y'(0)=0$, Compute ; $y(0.1)$ using 4th order R-K method. [$y(0.1)=2.94$ Ans]

2) MILNE'S METHOD :-

We have milne's predictor & Corrector formula's; as follows :-

$$y_4^{(P)} = y_0 + \frac{4h}{3}\left(2y_1' - y_2' + 2y_3'\right) \quad // \text{Predictor}$$

$$y_4^{(C)} = y_2 + \frac{h}{3}\left(y_2' + 4y_3' + y_4'\right) \quad // \text{Corrector}$$

* Method to Solve : By Milne's predictor, Corrector formulas

Step 1: Let $y'' = f(x, y, y')$

Given; $y(x_0) = y_0$ & $y'(x_0) = y_0'$ be diff Equation of 2nd order given

Step 2: We put ; $y' = \frac{dy}{dx} = z$ and $y'' = \frac{dz}{dx}$.

the given DE becomes : $z' = f(x, y, z)$

Step 3: We compute table for Computing values ;

x	x_0	x_1	x_2	x_3
y	y_0	y_1	y_2	y_3
$y' = z$	$y_0' = z$	$y_1' = z_1$	$y_2' = z_2$	$y_3' = z_3$
$y'' = z'$	$y_0'' = z_0'$	$y_1'' = z_2'$	$y_2'' = z_2'$	$y_3'' = z_3'$

Step 4 : We first apply predictor formula to Compute : $y_4^{(P)}$ & $z_4^{(P)}$

where;
$$y_4^{(p)} = y_0 + \frac{4h}{3}(2z_1 - z_2 + 2z_2), \quad \text{Since : } \underline{y' = z}$$
$$y_4'^{(p)} = z_4^{(p)} = z_0 + \frac{4h}{3}(2z_1' - z_2' + 2z_3').$$

Step 5: We compute : $z_4' = f(x_4, y_4, z_4)$ and then apply corrector formula where ;
$$y_4^{(c)} = y_2 + \frac{h}{3}(z_2 + 4z_3 + z_4)$$
$$y_4'^{(c)} = z_4^{(c)} = z_2 + \frac{h}{3}(z_2' + 4z_3' + z_4').$$

Step 6: Corrector formula can be applied repeatedly for better accuracy.

Obtain the solution of the Equation, $\frac{d^2y}{dx^2} = 1-2y\frac{dy}{dx}$ by computing the value of dependent variable corresponding to the value of 0.8 of independent variable by applying Milne's method using follo data., (Apply corrector formula twice)

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689.

Soln :- Given ; $\frac{d^2y}{dx^2} = 1-2y.\frac{dy}{dx}$ — (1)

putting : $\frac{dy}{dx} = z$, diwt wrt x , we obtain.

$(y''=z')$, $\frac{d^2y}{dx^2} = \frac{dz}{dx}$, Eqn (1) becomes... $\left[\begin{array}{l} y'=z \\ y''=z' \end{array}\right]$

$$\frac{dz}{dx} = z' = 1-2yz.$$

Now, we Compute ; $z_0' = z(0) = 1-2(0)(0) = 1$

$z_1' = z(0.2) = 1-2(0.02)(0.1996) = 0.992$

$z_2' = z(0.4) = 1-2(0.0795)(0.3937) = 0.9374$

$z_3' = z(0.6) = 1-2(0.1762)(0.5689) = 0.7995$

Now, we Compute table;

x	$x_0=0$	$x_1=0.2$	$x_2=0.4$	$x_3=0.6$
y	$y_0=0$	$y_1=0.02$	$y_2=0.0795$	$y_3=0.1762$
$y'=z$	$z_0=0$	$z_1=0.1996$	$z_2=0.3937$	$z_3=0.5689.$
$y''=z'$	$z_0'=1$	$z_1'=0.992$	$z_2'=0.9374$	$z_3'=0.7995.$

Now, by Milne's predictor formula;

$$y_4^{(p)} = y_0 + \frac{4h}{3}(2z_1 - z_2 + 2z_3).$$

$$y_4^{(p)} = 0 + \frac{4(0.2)}{3}\left[2(0.1996) - (0.3937) + 2(0.5689)\right]$$

$$\boxed{y_4^{(p)} = 0.3049}$$

also; $z_4^{(p)} = \cancel{z}_0 + \frac{4h}{3}\left[2z_1' - z_2' + 2z_3'\right]$

$$z_4^{(p)} = 0 + \frac{4(0.2)}{3}\left[2(0.992) - (0.9374) + 2(0.7995)\right]$$

$$\boxed{z_4^{(p)} = 0.7055} \Rightarrow z_4' = 1 - 2y_4^{(p)} z_4^{(p)}$$

$$\boxed{z_4' = 0.5698}$$

Now, we compute Milne's Corrector formula;

$$y_4^{(c)} = y_2 + \frac{h}{3}(z_2 + 4z_3 + z_4^{(p)})$$

$$\boxed{y_4^{(c)} = 0.3045}$$

$$z_4^{(c)} = z_2 + \frac{h}{3}(z_2' + 4z_3' + z_4')$$

$$\boxed{z_4^{(c)} = 0.7074}$$

Now, By Applying corrector formula again; we have;

$$y_4^{(c)} = y_2 + \frac{h}{3}(z_2 + 4z_3 + z_4^{(c)})$$

$$y_4^{(c)} = 0.0795 + \frac{0.2}{3}(0.3937 + 4(0.5689) + 0.7074)$$

$$\boxed{y_4^{(c)} = 0.3046.}$$

$\therefore$ Thus, the required solution is : $\boxed{y(0.8) = y_4^{(c)} = 0.3046.}$

—·—

Problems & Solns :-

Q) Apply Milne's method to solve : $\frac{d^2y}{dx^2} = 1+\frac{dy}{dx}$ given the following table of initial values.

x	$x_0=0$	$x_1=0.1$	$x_2=0.2$	$x_3=0.3$
y	$y_0=1$	$y_1=1.1103$	$y_2=1.2427$	$y_3=1.399$
$z=y'$	$z_0=1$	$z_1=1.2103$	$z_2=1.4427$	$z_3=1.699$.

Compute $y(0.4)$ numerically ~~and also theoretically~~.

Soln :- Given; $\frac{d^2y}{dx^2} = 1+\frac{dy}{dx} \sim (1)$ $\qquad x_1 = x_0+h.$ $\boxed{h=0.1}$

put ; $\frac{dy}{dx} = z$, diff wrt x..

$\frac{d^2y}{dx^2} = \frac{dz}{dx} = z'$: using in Eqn (1)...

$$\boxed{z' = 1+z}$$

Now, we compute ;
$$z_1' = 1+z_1 = 1+1.2103 = 2.2103 = z_1'$$
$$z_2' = 1+z_2 = 1+1.4427 = 2.4427 = z_2'$$
$$z_3' = 1+z_3 = 1+1.699 = 2.699. = z_3'$$

Now, we use Milne's predictor formula;

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2z_1 - z_2 + 2z_3)$$
$$= 1+ \frac{4}{3}\times 0.1 \left[2(1.2103) - 1.4427 + 2(1.699)\right]$$
$$\boxed{y_4^{(P)} = 1.5835}$$

$$z_4^{(P)} = z_0 + \frac{4h}{3}(2z_1' - z_2' + 2z_3')$$
$$z_4^{(P)} = 1+\frac{4\times 0.1}{3}\left(2\times(2.2103) - 2.4427 + (2\times 2.699)\right)$$
$$\boxed{z_4^{(P)} = 1.9835}$$

therefore; $z_4' = 1 + z_4^{(p)} = \cancel{\#}$

$\boxed{z_4' = 2.9835}$

Now, by applying Milne's Corrector formula;

$$y_4^{(c)} = y_2 + \frac{h}{3}\left(z_2 + 4z_3 + z_4^{(p)}\right)$$

$$\boxed{y_4^{(c)} = 1.58344}$$

$$z_4^{(c)} = z_2 + \frac{h}{3}\left(z_2' + 4z_3' + z_4'\right)$$

$$\boxed{z_4^{(c)} = 1.98344}$$

Thus, the approximate values of $\underline{y}$ & $\underline{y'}$ at $\underline{\underline{x = 0.4}}$ are :-

$\boxed{y(0.4) = y_4^{(c)} = 1.58344}$ $\boxed{y'(0.4) = z_4^{(c)} = 1.98344}$

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Apply milne's method to compute $y(0.4)$, for the given

dE : $y'' + xy' + y = 0$ using following table of initial values

x	$x_0 = 0$	0.1	0.2	0.3
y	$y_0 = 1$	0.995	0.9801	0.956
y'	$y_0' = 0$	-0.0995	-0.196	-0.2867

Soln:- Given; $y'' + xy' + y = 0$ ~①.

putting; $\frac{dy}{dx} = y' = z$. diff wrt x..

$$\frac{d^2y}{dx^2} = y'' = \frac{dz}{dx}.$$ using in Eqn ①...

$$\Rightarrow \frac{dz}{dx} + xz + y = 0.$$

ie; $z' = -(xz+y)$

further; we find; $z'(0) = z_0 = -[0+1] = -1$

$z'(0.1) = z_1 = -[(0.1)(-0.0995)+0.995] = -0.985.$

$z'(0.2) = z_2 = -0.941.$

$z'(0.3) = z_3 = -0.87.$

Now, we compute the table;

x	$x_0 = 0$	$x_1 = 0.1$	$x_2 = 0.2$	$x_3 = 0.3$
y	$y_0 = 0$	$y_1 = 0.995$	$y_2 = 0.9801$	$y_3 = 0.956$
$y' = z$	$z_0 = 0$	$z_1 = -0.0995$	$z_2 = -0.196$	$z_3 = -0.2867$
$y'' = z'$	$z_0' = -1$	$z_1' = -0.985$	$z_2' = -0.941$	$z_3' = -0.87.$

By Milne's predictor formula;

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2z_1 - z_2 + 2z_3)$$

$$\boxed{y_4^{(P)} = 0.9231}$$

$$z_4^{(P)} = z_0 + \frac{4h}{3}(2z_1' - z_2' + 2z_3')$$

$$\boxed{z_4^{(P)} = -0.3692}$$

WKT; $z_4' = -(x_4 z_4 + y_4^{(P)})$

$$\boxed{z_4' = -0.7754}$$

Now, By Milne's Corrector formula;

$$y_4^{(c)} = y_2 + \frac{h}{3}(z_2 + 4z_3 + z_4)$$

$$\boxed{y_4^{(c)} = 0.9230}$$

$$z_4^{(c)} = z_2 + \frac{h}{3}(z_2' + 4z_3' + z_4')$$

$$\boxed{z_4^{(c)} = -0.3692}$$

Thus, the required soln is ; $\boxed{y_4 = y(0.4) = 0.923}$ //..

4) Apply Milne's method to compute : $y(1.4)$ for : $2\frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$ using following initial values from table ,

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	3.0657

Soln:- $y'' = 2x + \frac{y'}{2}$

putting; $y' = z$, $y'' = z'$

$z_0' = 3$, $z_1' = 3.3589$, $z_2' = 3.73625$, $z_3' = 4.13285$

MPF ; $\boxed{y_4^{(P)} = 3.0793}$ $\boxed{z_4^{(P)} = 3.4996}$

MCF ; $\boxed{z_4' = 4.5498}$

$\boxed{y_4^{(c)} = 3.0794}$ $\boxed{z_4^{(c)} = 3.4997}$

$\therefore$ $\boxed{y(1.4) = 3.0794}$ //.. —x—