Heat Transfer
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Lecture - 17
Equations of Change for Non-isothermal Systems

I would like to introduce a concept which we started discussing in the last class, which is called the shell balance. Now, this shell balance can be of momentum, it can be of energy it can also be of a species. When we do a shell momentum balance we get a governing equation that describes the change in velocity as a function of x, y, z and time. When we write the shell heat balance we should get what is known as the equation of energy. Similarly, if I write it for a species, which is let us say reacting with another species in a flowing fluid field, then we would get the species conservation equation which is going to be very important in mass transfer. Let us try to concentrate on how we can write a shell heat balance, the trick is to define a shell of let us say some size del x, del y, del z which is fixed in space.

So, a cuboid shape of size del x, del y, del z we have 6 faces: one is the x face which is perpendicular to the x direction, one is the y face which is perpendicular to the y direction and the other is going to be that z face which is perpendicular to the z direction. Each of these faces will have areas associated with them and through these areas heat energy can come into the control volume. Because of which the internal energy of the box would change. When we talk about energy will we are not going to talk only about the internal energy, we also must consider the kinetic energy. So, a fluid may come through the x face with certain velocity and therefore, certain kinetic energy, at a temperature which is different form the temperature of the fluid contained in the box. Therefore, the entering fluid will have some internal energy and some kinetic energy associated with it. So, it will enter the x face and will leave the x plus delta x face, similarly it would come through y, leave at y plus delta y, come at z and leave at z plus delta z.

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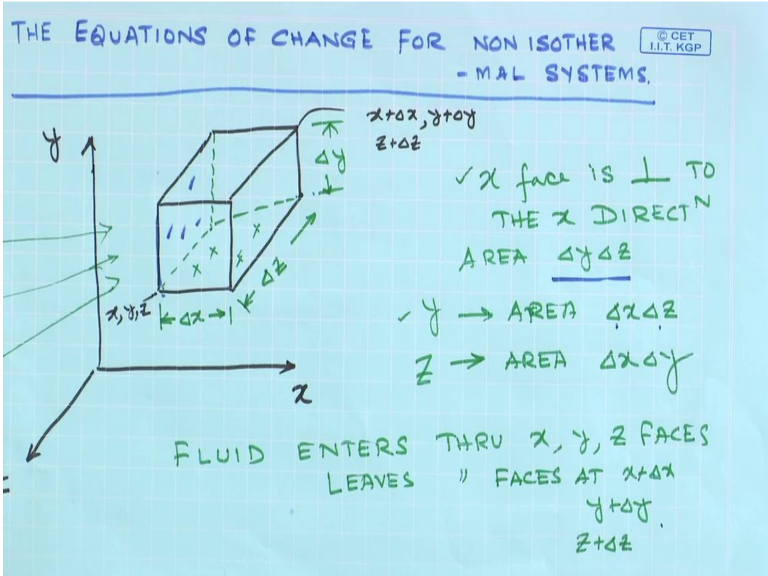

Let us once again go through the equation of change for a non-isothermal system, here I have drawn this box which is del x, del z and del y. These are the x, y and z directions and the x face which is this one is perpendicular to the x direction and therefore, its area is $\Delta y \cdot \Delta z$. Similarly, the y face which is perpendicular to the y direction would should have area of $\Delta x \cdot \Delta z$; and the z face would have an area of $\Delta x \cdot \Delta y$. We are going to now write what would be the form of the energy equation for a system of size del x, del y, del z which is which is fixed in space and this is our coordinate system.

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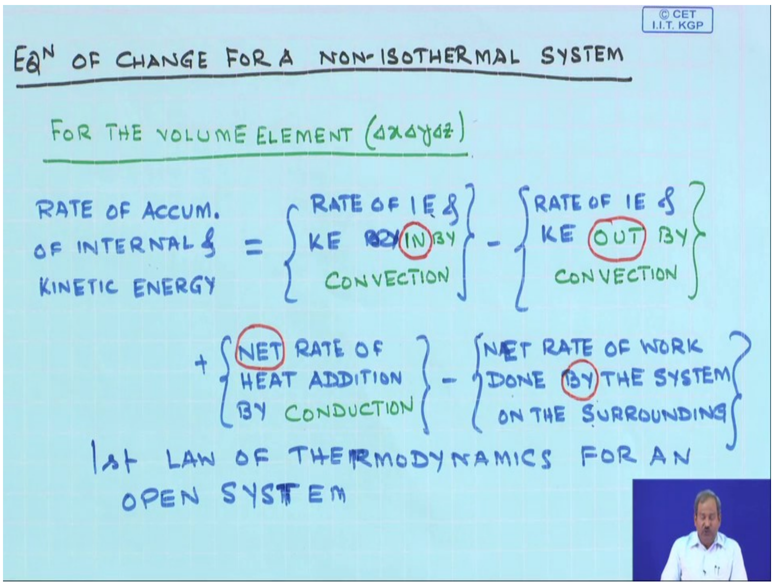

So, let us see how the equation of change would look like for a non-isothermal system. When there is a flow, some amount of energy comes with the flow and when I talk about energy I speak about both the internal and the kinetic energy. So, some internal and kinetic energy can come into this volume element by convection and it is going to go out again by convection, from the x plus del x, y plus del y and z plus delta z faces. Let us say I have a temperature difference, which exists in the x direction; then obviously, I am going to have some flow of heat through conduction through this x face. So, a difference in temperature either in x or in y or in z even if the fluid is still, would give rise to conductive heat transfer. So, if there is a temperature difference, there is going to be a conductive heat transfer. I have combined the terms together to write it in the form of net rate of heat addition to the volume element by conduction. So, this takes care of all the heat that comes to the system by conduction or by convection. However, there is one missing term that I should consider at this point, that is the rate of work done by the system on the surrounding; and hence, the total energy of the system should reduce, and that is why we have a minus sign. Had this been a case of work being done on the system, then this sign should be positive. So, what I have written over here is nothing, but the first law of thermodynamics, for an x and since I am allowing fluid to enter and leave, this must be for an open system.

Now, from this generalized energy equation one should be able to deduct the commonly available equation for kinetic energy of a system. And therefore, what you would left out with is the energy equation where we are only considering internal energy which is manifested by a change in temperature.

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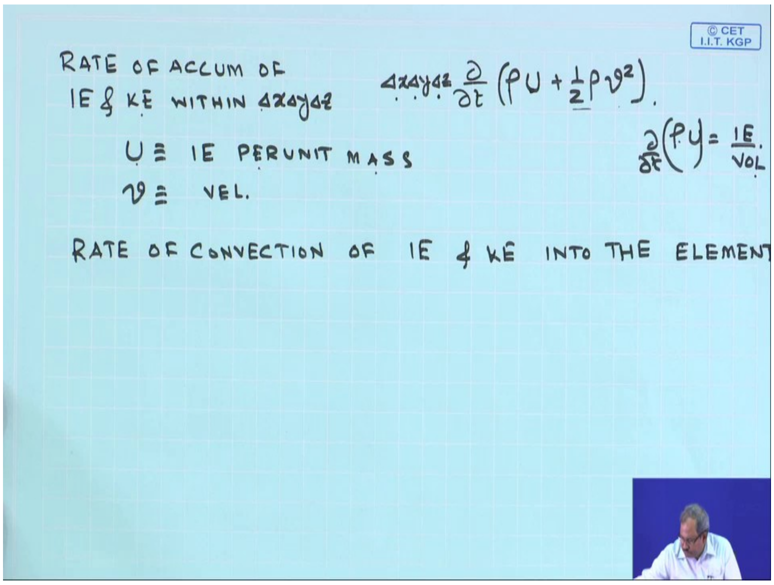

So, first of all let us see what is going to be the rate of accumulation of internal energy and kinetic energy within the system, which is defined as del x del y del z. So, this must be $\frac{\partial}{\partial t}\left(\rho U + \frac{1}{2}\rho v^2\right)$ where, U is the internal energy per unit mass and v is the velocity. This internal energy per unit mass, here I have multiplied it with rho. So, this becomes internal energy per unit volume. So, if I would like to find out what is the total rate of accumulation of internal and kinetic energy within x, y, z this must be multiplied by del x del y del z, which makes it the rate of change of internal energy for a system, whose dimensions are del x del y and del z. As long as my U is defined as internal energy per unit mass, rho is the density.

Therefore, $\left(\frac{\partial}{\partial t}(\rho U)\right)\cdot(\Delta x \cdot \Delta y \cdot \Delta z)$ would simply give us the rate of accumulation of internal energy within del x del y del z and similarly the same logic will also be applied to $\frac{1}{2}\rho v^2$. The next one is, I am going to write would be the convection of internal energy and kinetic energy into the element.

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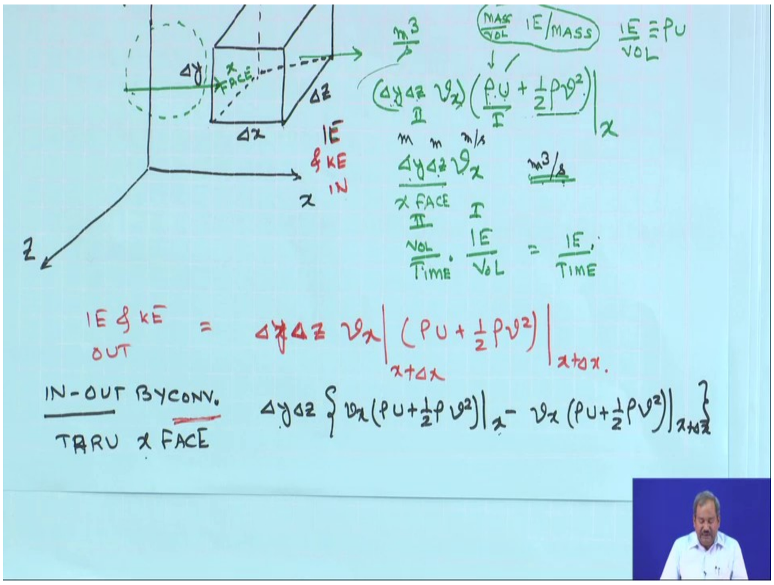

In this figure, some amount of let us, this is my x face, which has an area of $\Delta y \cdot \Delta z$, and some amount of fluid is going to enter through this x face. Let us say, this velocity with which it comes into the control volume is v_x. So, when you multiply $\frac{1}{2}\rho v^2$, with $(\Delta y \cdot \Delta z) \cdot v_x$ it is going to give you the kinetic energy per unit time. So, together this whole term gives is evaluated at x on this face. So, it comes in through the x face and goes out through the face at x plus del x.

So, the entire thing is going to be at $(\Delta y \cdot \Delta z) \cdot \left(\rho U + \frac{1}{2}\rho v^2\right) \cdot v_x$, all evaluated at x plus delta x. So, the amount of energy, internal and kinetic energy in due to convection. So, when I consider the net energy balance due to convection, the equation would be:

$$\Delta y \cdot \Delta z \left\{ \underbrace{\left(\rho U + \frac{1}{2}\rho v^2\right) \cdot v_x}_{at\ x} - \underbrace{\left(\rho U + \frac{1}{2}\rho v^2\right) \cdot v_x}_{at\ x+\Delta x} \right\}$$

So, this is what the expression for in minus out by convection through the x face would look like. Now, I can write the same thing for the y face, the only thing which will be different here is the y face has an area equal to $\Delta x \cdot \Delta z$ and since we are talking about y then the v_x must be replaced with v_y and instead of evaluating it at x, it is going to be evaluated at y and y plus delta y, similarly for z face this area is going to be $\Delta x \Delta y$, v_x is to be replaced by v_z and everything else will remain same.

So, in total to consider the convective flow of internal and kinetic energy into the volume element, del x del y del z, I will have 6 terms. Two terms each for x, y, and z face would give me total of 6 terms that would signify what is the total amount of heat which comes into the system by convection.

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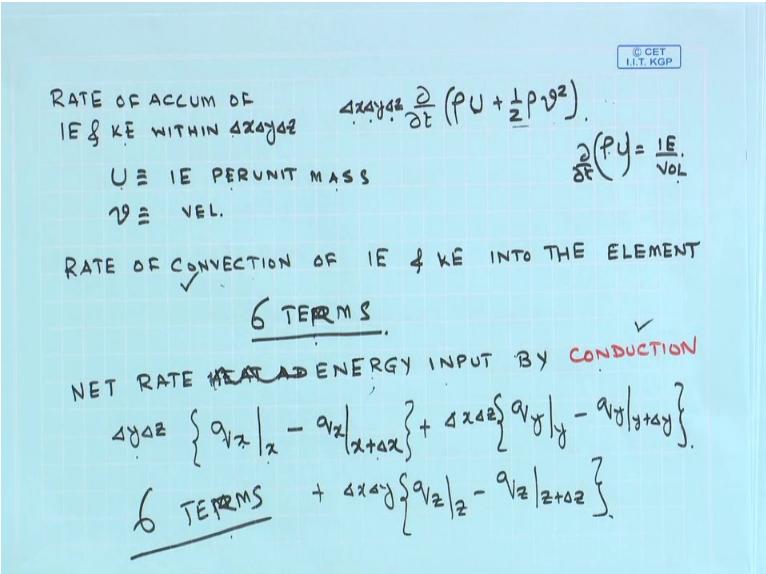

As I have explained before, we must think about net rate of heat addition by conduction, I would not say heat here, energy because I also have the kinetic energy to take care of, net rate of energy input by conduction. And here I am going to express it in terms of the component of heat flux in the x direction.

So, q_x is the heat in per unit area per unit time. So, I must multiply it with the appropriate area since it is x face this must be equal to $\Delta y \Delta z$. So, this denotes the heat that comes to the control volume through the x face and the one that goes out through the x plus delta x face. So, these two terms together one at x and one at x plus del x multiplied by $\Delta y \Delta z$, together they tell us about the net rate of heat addition by conduction through the x face.

Similarly, I am going to have the y face which will have $\Delta x \Delta z$, the area and the heat flux is are going to be q_y, at y minus q_y at y plus delta y and for the z face it is going to be $\Delta x \Delta y$ times q_z, evaluated at z minus q_z at z plus delta z. So, these 6 terms again, 6 terms for each of the faces would tell me about the net energy input by conduction and so I have taken care of the convection and I have taken care of conduction. So, what is left is work done by the system on the surrounding.

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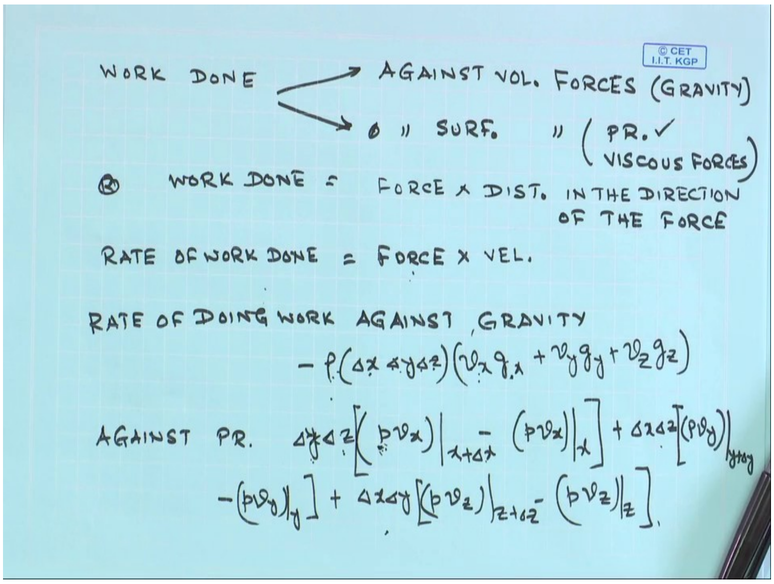

Now, work done as we all know it simply, it can be against volumetric forces, volumetric forces that which are acting on the entire volume of the volume element. The common example would be gravity and the second one is against surface forces, surface forces which could be against pressure, which could be against viscous forces. So, these two are again the common examples of surface forces.

Now, I am not interested in work done, because work done is, force times distance in the direction of the force. So, what would be rate of work done that is the time rate of work done, it would be force times distance by time. So, the work done is given by:

$\rho\left(\Delta x \Delta y \Delta z\right)\left(v_x g_x + v_y g_y + v_z g_z\right)$. We know that, distance by time; obviously, would give you the velocity. So, this is what you are going to get for the rate of work done. So, rate of work done would simply be expressed as force times velocity. So, let us quickly write the expression for the force against gravity forces, so rate of doing work against gravity would simply be equal to minus since, it is against work is done against gravity.

So, v x times g x velocity and this is acceleration due to gravity plus v y g y plus v z g z. So, when you, when you see this equation you would be able to see that it is the force in the x direction which is v x, del x del y del z times rho and this is this is multiplied by g x. So, this totally gives you the rho del x del y del z times g x is the force, because this is mass, this is mass per unit volume this is volume, and this is the acceleration. So, this gives you the force and force multiplied by the velocity in the appropriate direction. So, v x, v y, and v z would give you the rate of work, rate of doing work against gravity.

So, again what is going to be the form for against pressure, it should be the area on which let u say the x face del z times P v x, evaluated at x plus delta x minus P v x at x. So, this is one term, plus del x del z, p v y at y plus del y minus p v y at y, this is going to be the second term plus del x del y, p v z times z plus del z minus p v z. Look at these terms one more time and see what the mean, pressure is force per unit area. Whatever be the pressure at x plus del x is multiplied by the appropriate area which is del y del z, to give to give us the force in the x direction acting on the control volume, acting on the volume element at x plus del x and we understand that the rate of work done is force times velocity. So, the pressure-work is given by:

$$\left(\Delta y\Delta z\right)\left(Pv_x\Big|_{x} - Pv_x\Big|_{x+\Delta x}\right) + \left(\Delta x\Delta z\right)\left(Pv_y\Big|_{y} - Pv_y\Big|_{y+\Delta y}\right) + \left(\Delta x\Delta y\right)\left(Pv_z\Big|_{z} - Pv_z\Big|_{z+\Delta z}\right)$$

So, these 6 terms together would give us the work done against the pressure forces by the volume element del x del y del z. What is remaining here is the work done against viscous forces, now work done against viscous forces this I am going to neglect for the time-being because work done against viscous forces is something like solid friction. So, what happens when you work against friction forces, you are pulling an object over a rough surface. So, you have to overcome the frictional forces exerted by the rough surface, as a result of which there is going to be heat generation in it and any work that you do that in order to make that block move over a rough surface, is going to be converted into heat and it will change the energy of the of the system. Similarly, when fluid flows specially at high speed through a small duct, there is going to be tremendous velocity gradient which is present.

So, let us say I have a jet which very thin, and the fluid is coming at a very high velocity. So, the velocity is large and if the velocity is large and the gap is small, then the velocity gradient would be very large, and we understand that the viscous force is related to velocity gradient. The shear stress is mu times velocity gradient. So, if the velocity gradient is large or if the viscosity is large, in that case you will have a strong force that you need to overcome in order to make the fluid flow through that thin conduit at a very high velocity. If that happens then you do substantial work against the viscous forces and whenever you do that kind of work against viscous forces the temperature will increase. And that increase in temperature which is obtained at the expense of work done by the system must be considered for any form of energy equation.

However, this is only relevant in some special situations, we do not get the heat generation due to viscosity in many of the practical problems. As you would see it requires high velocity gradient and very high viscosity. So, what are the places in which they become relevant, when a rocket reentered earth's atmosphere its velocity is very large, the atmosphere is still, but the rocket is coming down with a very high velocity. So, near the boundary layer, formed close to the rocket, the velocity changes from that of the rocket which is very large, to velocity equal to 0 which is the velocity of the atmosphere. So, this thinness of the boundary layer and the very high speed of the rocket at reentry would ensure that the frictional heat generated is tremendous. And that is why you would see that the rocket comes almost like a red-hot object and there has to be special protective arrangements to ensure the safety of the astronauts inside the rocket.

So, that is an extreme example, in some cases viscous polymer is extruded by making it flow through a very thin gap, if that is the case then the viscosity is high, the velocity is large as you would like to have higher throughput of the polymer when you are making a sheet out of it. So, the velocity combined with the high viscosity of the polymer ensures that you cannot neglect viscous dissipation. However, we will neglect viscous heat dissipation, wherever the viscous heat dissipation is irrelevant; but I will tell you that how to incorporate additional terms into the energy equation which would take into account the viscous heat generation.

So, if you look at your textbook and look at the full form of the energy equation, you would see there are a bunch of terms which have which are multiplied with mu. So, the easiest way to identify which term of your energy equation in your text relates to be, relate to viscous heat generation, is to look for terms containing mu. If in your problem the viscous heat generation is negligible drop the entire set of terms containing mu and what you would have is the energy equation that we are going to use for most of the realistic applications. So, since those terms are complicated, I am dropping them for the time being, but making you aware that in some special situations you need to add them to ensure that your energy equation is complete.