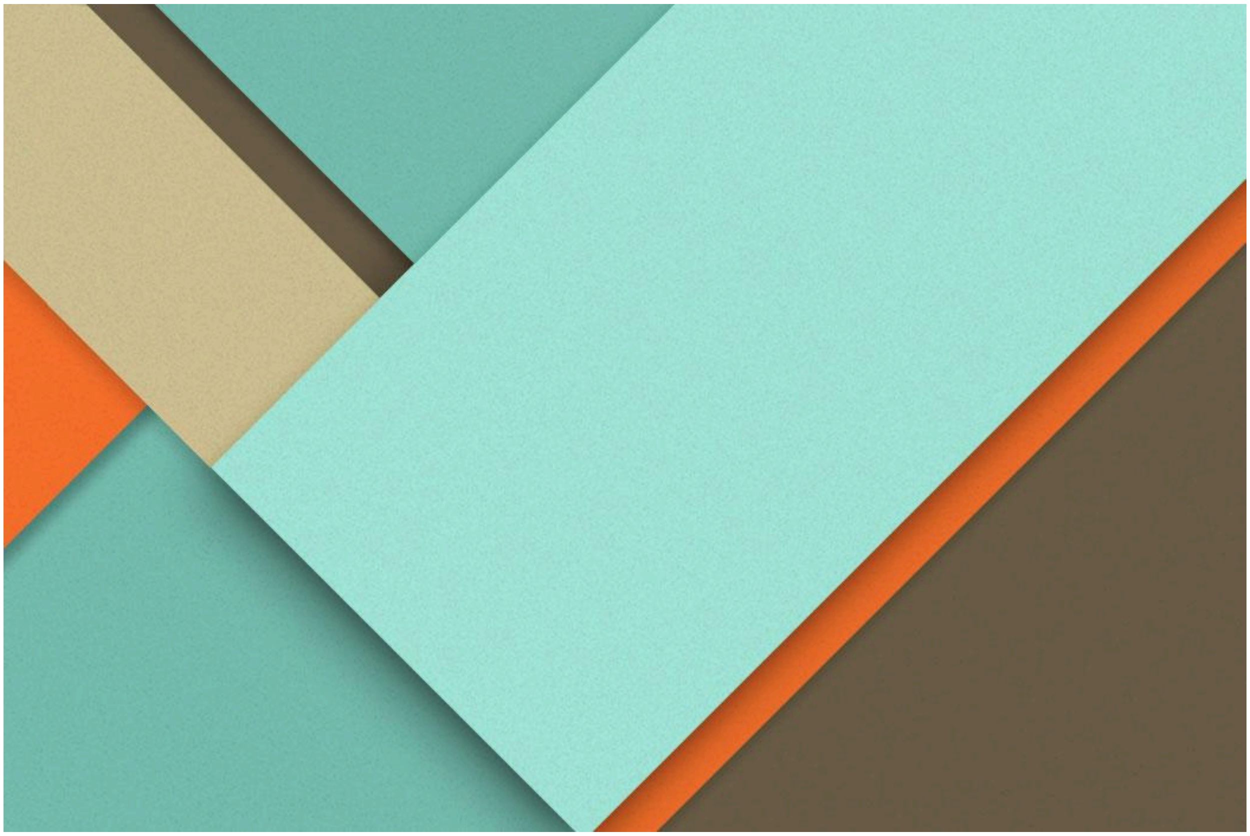

CFD LAB-3

Numerical solution of the 2D Lid driven cavity flow

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Overview

This report explores the numerical solution of the 2D Lid driven cavity flow equation using the Finite Volume method and the SIMPLE Algorithm for velocity coupling. As we get into solving the two-dimensional Navier–Stokes equations using our MATLAB code we will observe how velocity fields, pressure, vorticity change with respect to change in Reynolds number(1, 100, 1000).

Goals

1. Objective of this task is to solve the 2D Lid driven cavity flow equation.
2. Write MATLAB programs to obtain figures for velocity fields (both u and v), pressure, vorticity and streamlines of the flow for three Reynolds numbers of Re = 1, 100, and 1000.

Context:

Consider the Continuity equation and u & v momentum equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial (u^2)}{\partial x} + \frac{\partial uv}{\partial y} = -\frac{\partial \rho}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial (uv)}{\partial x} + \frac{\partial (v^2)}{\partial y} = -\frac{\partial \rho}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Task 3.0

The initial and boundary conditions are:

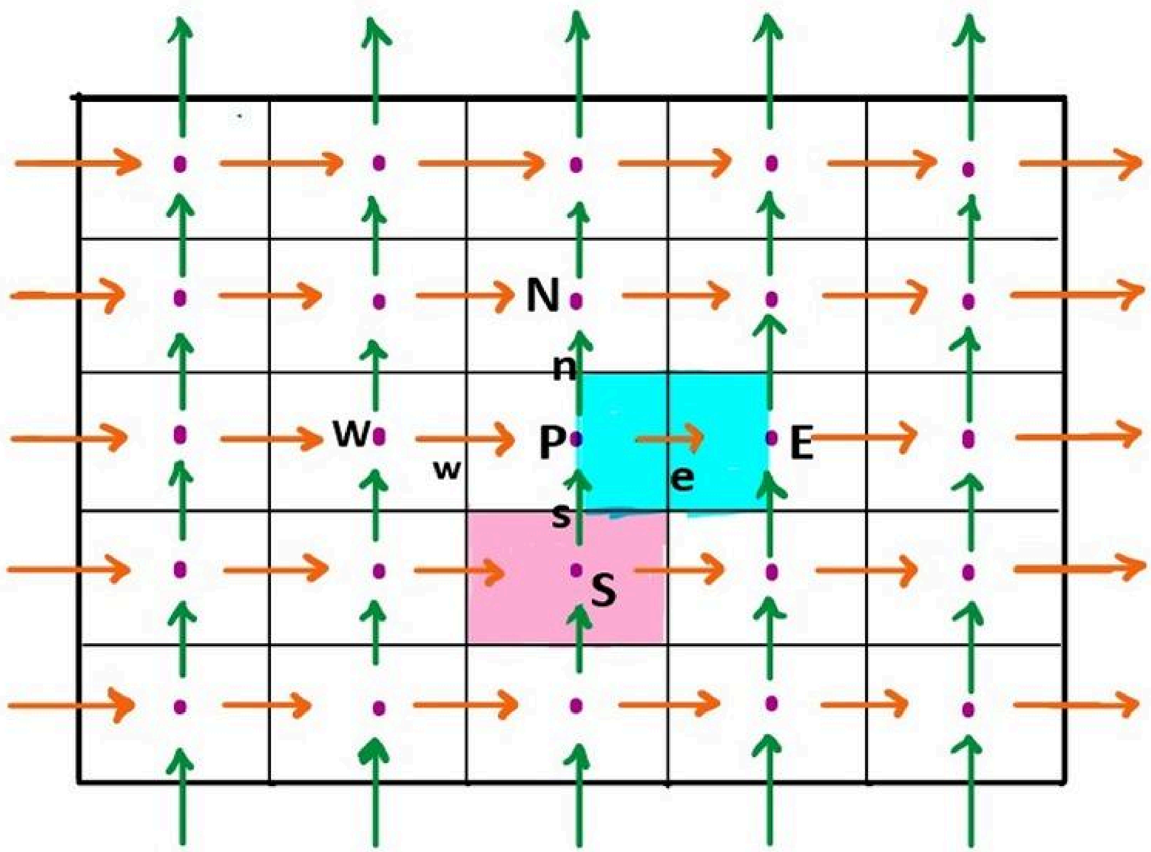
u(1, :) =1, u(i_max, :) = u(:, 1) = u(:, j_max) = 0 and,

v(1, :) = v(i_max, :) = v(:, 1) = v(:, j_max) = 0

$\frac{\partial p}{\partial n} = 0$ at all boundaries

As for an incompressible flow, the pressure field adjust to ensure div(u) = 0

so by considering Neumann boundary conditions ($\frac{\partial p}{\partial n} = 0$) ensure that the pressure boundary values are consistent with the incompressibility condition and the no slip velocity conditions.



Staggered Grid Fig(1) (Ref 1)

Task 3.1 and 3.2

Discretizing x and y momentum equations using finite volume method

$$\frac{\partial(u^2)}{\partial x}+\frac{\partial(uv)}{\partial y}=-\frac{\partial P}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad (1)$$

$$\frac{\partial(uv)}{\partial x}+\frac{\partial(v^2)}{\partial y}=-\frac{\partial P}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad (2)$$

For discretized equations (1) and (2) by taking integration of the control volume as described in Fig(1). Now we solve x and y momentum equations.

$$\int_s^n\int_w^e\left[\frac{\partial(u\cdot u)}{\partial x}+\frac{\partial(u\cdot v)}{\partial y}\right]dx\,dy=\int_s^n\int_w^e\left[-\frac{\partial p}{\partial x}\right]dx\,dy+\frac{1}{Re}\int_s^n\int_w^e\left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)\right]dx\,dy+\frac{1}{Re}\int_s^n\int_w^e\left[\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)\right]dx\,dy.$$

$$[(u\cdot u)_e\Delta y-(u\cdot u)_w\Delta y]+[(u\cdot v)_n\Delta x-(u\cdot v)_s\Delta x]=-\frac{\partial p}{\partial x}\Delta x\Delta y+\frac{1}{Re}\left[\left(\frac{\partial u}{\partial x}\right)_e\Delta y-\left(\frac{\partial u}{\partial x}\right)_w\Delta y\right]$$
$$+\frac{1}{Re}\left[\left(\frac{\partial u}{\partial y}\right)_n\Delta x-\left(\frac{\partial u}{\partial y}\right)_s\Delta x\right].$$

$$\int_s^n\int_w^e\left[\frac{\partial(uv)}{\partial x}+\frac{\partial(v\cdot v)}{\partial y}\right]dx\,dy=\int_s^n\int_w^e\left[-\frac{\partial P}{\partial y}\right]dx\,dy+\frac{1}{Re}\int_s^n\int_w^e\left[\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right)\right]dx\,dy+\frac{1}{Re}\int_s^n\int_w^e\left[\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y}\right)\right]dx\,dy.$$

$$[(uv)_e\Delta y-(uv)_w\Delta y]+[(vv)_n\Delta x-(vv)_s\Delta x]=-\frac{\partial P}{\partial y}\Delta x\Delta y+\frac{1}{Re}\left[\left(\frac{\partial v}{\partial x}\right)_e\Delta y-\left(\frac{\partial v}{\partial x}\right)_w\Delta y\right]$$
$$+\frac{1}{Re}\left[\left(\frac{\partial v}{\partial y}\right)_n\Delta x-\left(\frac{\partial v}{\partial y}\right)_s\Delta x\right].$$

Convective terms

Convective terms represent the advection of velocity u in x direction and v in y direction.

$$[((u\cdot u)_e\Delta y-(u\cdot u)_w\Delta y)+((u\cdot v)_n\Delta x-(u\cdot v)_s\Delta x)]=\frac{1}{2}u_E(u_E+u_P)\Delta y-\frac{1}{2}u_W(u_P+u_W)\Delta y$$
$$+\frac{1}{2}u_N(v_N+u_P)\Delta x-\frac{1}{2}u_S(u_P+v_S)\Delta x.$$

$$[(uv)_e\Delta y-(uv)_w\Delta y]+[(vv)_n\Delta x-(vv)_s\Delta x]=\frac{1}{2}v_E(u_E+u_P)\Delta y-\frac{1}{2}v_W(u_P+u_W)\Delta y$$
$$+\frac{1}{2}v_N(v_N+v_P)\Delta x-\frac{1}{2}v_S(v_P+v_S)\Delta x.$$

These terms are appropriate using linear interpolation and computed at the cell faces, using interpolated values from adjacent cell centers.

Diffusive terms

These terms represent the viscous diffusion in the x and y directions. The second derivative of the velocity is discretized using CDS.

$$\frac{1}{Re}\left[\left(\frac{\partial u}{\partial x}\right)_e-\left(\frac{\partial u}{\partial x}\right)_w\right]\Delta y=\frac{1}{Re}(u_E+u_W).$$
$$\frac{1}{Re}\left[\left(\frac{\partial u}{\partial y}\right)_n-\left(\frac{\partial u}{\partial y}\right)_s\right]\Delta x=\frac{1}{Re}(u_N+u_S).$$

$$\frac{1}{Re}\left[\left(\frac{\partial v}{\partial x}\right)_e-\left(\frac{\partial v}{\partial x}\right)_w\right]\Delta y=\frac{1}{Re}(u_E+u_W).$$

$$\frac{1}{Re}\left[\left(\frac{\partial v}{\partial x}\right)_n-\left(\frac{\partial v}{\partial x}\right)_s\right]\Delta x=\frac{1}{Re}(v_N+v_S).$$

Diffusive fluxes calculated at the cell faces using values from the neighbouring cells.

Pressure terms

These terms account for the pressure force in x and y direction w.r.t u and v and the pressure gradient is approximated using CDS.

$$\int_s^n\int_w^e\left(-\frac{\partial P}{\partial x}\right)dx\,dy=-(P_E-P_P)\Delta y$$

$$\int_s^n\int_w^e\left(-\frac{\partial P}{\partial y}\right)dx\,dy=-(P_N-P_P)\Delta x$$

Pressure values are stored at the cell centers and the gradients are computed at cell faces by differencing neighbouring pressures.

Discretized form of governing Equations

The coefficients aE, aW, aN, aS represent the influence of neighbouring cells on the current cell. The term de represents the contribution of the pressure gradient.

The x-momentum equation velocity discretization form

$$a_{e,u}\,u_E=a_E\,u_E+a_W\,u_W+a_N\,u_N+a_S\,u_S+(p_P-p_E)\Delta y.$$

where,

$$a_{e,u}=a_E+a_W+a_N+a_S.$$

$$a_E=-\frac{u_E\Delta y}{2}+\frac{1}{Re},\quad a_W=\frac{u_W\Delta y}{2}+\frac{1}{Re}$$

$$a_N=-\frac{v_N\Delta x}{2}+\frac{1}{Re},\quad a_S=\frac{v_S\Delta x}{2}+\frac{1}{Re}.$$

In a similar manner, The y-momentum equation velocity discretization form

$$a_{e,v}\,v_n=a_E\,v_E+a_W\,v_W+a_N\,v_N+a_S\,v_S+(p_P-p_N)\Delta x.$$

where,

$$a_{e,v} = A_E + A_W + A_N + A_S.$$

$$A_e = -\frac{u_E \Delta x}{2} + \frac{1}{Re}, \quad A_w = \frac{u_W \Delta x}{2} + \frac{1}{Re}.$$

$$A_n = -\frac{v_N \Delta y}{2} + \frac{1}{Re}, \quad A_s = \frac{v_S \Delta y}{2} + \frac{1}{Re}.$$

The x-momentumEqn in terms of the general discretized form of neighboring nodes is

$$a_{e,u}\, u_E = \sum a_{nb,u}\, u_{nb,u} + b + (p_P \Delta y - p_E \Delta y).$$

Eqn(1)

$$u_E = \frac{\sum a_{nb,u}\, u_{nb,u} + b}{a_{e,u}} + d_c(p_P - p_E).$$

where b is source term that we got from pressure gradient and $d_e = \frac{\Delta y}{a_{e,u}}$.

In a similar manner, The y momentum equation

$$a_{n,v}\, v_n = \sum a_{nb,v}\, v_{nb,v} + b + (p_P \Delta x - p_E \Delta x).$$

Eqn(2)

$$v_n = \frac{\sum a_{nb,v}\, v_{nb,v} + b}{a_{n,v}} + d_n(p_P - p_N).$$

where b is source term that we got from pressure gradient and $d_n = \frac{\Delta x}{a_{n,v}}$.

Let us guess pressure p as p* , the approximate velocity u* and v *

$$a_{e,u} u_e^* = \sum a_{nb,u} u_{nb,u}^* + b + (p_P^* \Delta y - p_E^* \Delta y)\ .$$

Eqn(3)

$$a_{n,v}\, v_n^* = \sum a_{nb,v}\, v_{nb,v}^* + b + (p_P^* \Delta x - p_N^* \Delta x).$$

Eqn(4)

Subtract Equations (1) and (3) , Equations (2) and (4) we get

$$a_{e,u} u_e' = \sum a_{nb,u} u_{nb,u}' + (p_P' \Delta y - p_E' \Delta y)\ .$$

Eqn(5)

$$a_{n,v}\, v_n' = \sum a_{nb,v}\, v_{nb,v}' + (p_P' \Delta x - p_N' \Delta x).$$

Eqn(6)

Where $u_e = (u_e^* + u_e')$, $v_n = (v_n^* + v_n')$, $p = (p^* + p')$. u' and v' are correction velocity.

For the SIMPLE algorithm the summation terms in Equations (5) and (6) are removed.

Consequently, the velocity correction can be written as,

$$u_e' = (d_e p_P' - d_e p_E')\ . \qquad v_n' = (d_n p_P' - d_n p_N')\ .$$

Now, the Continuity Equation is discretized over the main control volume and then evaluated pressure correction p'.

$$\int_{s}^{n}\int_{w}^{e}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dx\,dy=(u_e\Delta y-u_w\Delta y)+(v_n\Delta x-v_s\Delta x)$$

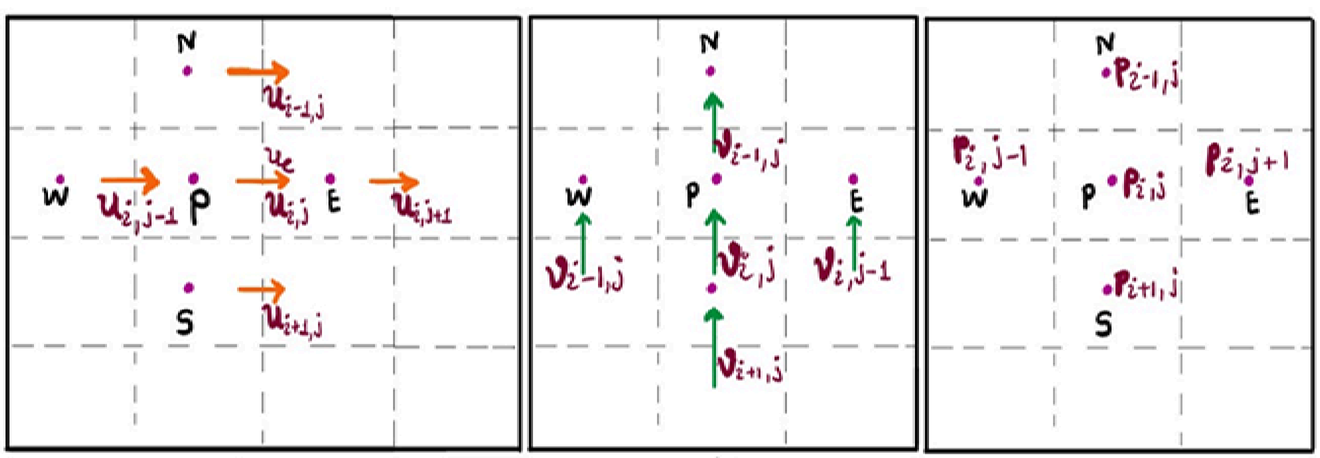
Inserting the values given below in the above continuity equation results in equation (7)

$$u_e = u_e^* + d_e\left(P_P' - P_E'\right)$$
$$u_w = u_w^* + d_w\left(P_w' - P_P'\right)$$
$$v_n = v_n^* + d_n\left(vP_P' - P_N'\right)$$
$$v_s = v_s^* + d_s\left(P_S' - P_P'\right)$$

$$a_p P_p' = a_E P_E' + a_W P_W' + a_N P_N' + a_S P_S' + q \qquad \text{Eqn(7)}$$

where,

$$a_E = d_e \Delta y \quad a_W = d_w \Delta y$$
$$a_N = d_n \Delta x \quad a_S = d_s \Delta x$$
$$q = (u_w^* - u_e^*)\Delta y + (v_s^* - v_n^*)\Delta x$$
$$a_p = a_E + a_W + a_N + a_S$$



A B C

We took Grid discretization for (A) u-velocity (B) v- velocity and (C) p-pressure as shown in above image [Ref - Manoj R. Patel, Jigisha U. Pandya,and Vijay K. Patel, Numerical Analysis of Fluid Flow Behaviour in Four-Sided Square Lid-Driven Cavity Using the Finite Volume Technique]

In Eqs. (5) and (6) removing summation terms. So, for simplification, we required some relaxation operator α_p such that pressure become as $p = p^* + \alpha_p p'$ for the optimal solution relaxation factor α_p is taken 0.8.

SIMPLE Algorithm

1. Initialize flow field u,v,p u*=u, v*=v and p*=p

2. Solve momentum equations (updated values of u* and v*)

3.Solve pressure correction equation(3) which is obtained from continuity equation to get P'

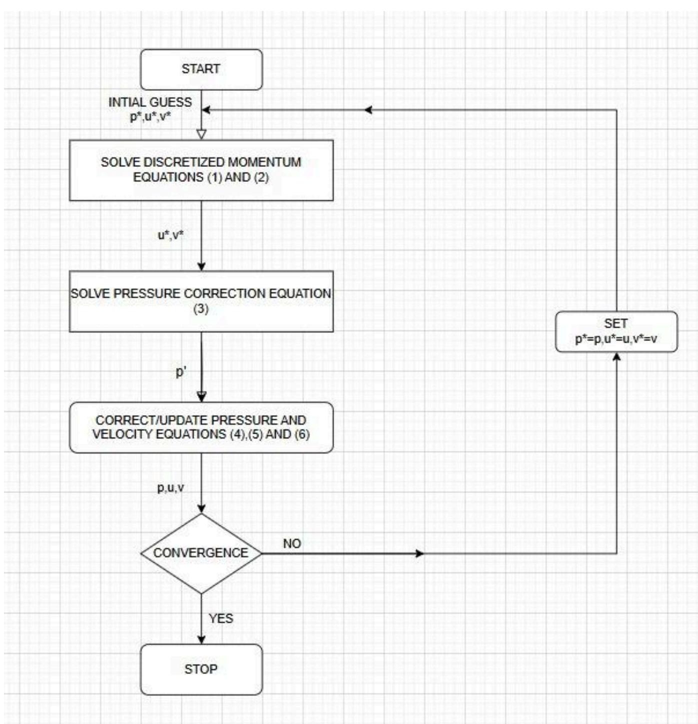
4. Update/Correct the flow field $u' = de\left(P'_{i+1,j} - P'_{i,j}\right)$ $u = u^\star + u'\alpha_u$

$v' = dn\left(P'_{i,j-1} - P'_{i,j}\right)$ $v = v^\star + v'\alpha_v$ $\alpha_u, \alpha_v \in [0,1]$

5.Pressure updation / correction $P_{i,j} = P^\star_{i,j} + P'_{i,j}\alpha_p$ $\alpha_p \in [0,1]$

The pressure correction equation is subjected to divergence unless some under relaxation is used during the iterative process. The velocities are also under relaxed to ensure stable computations.

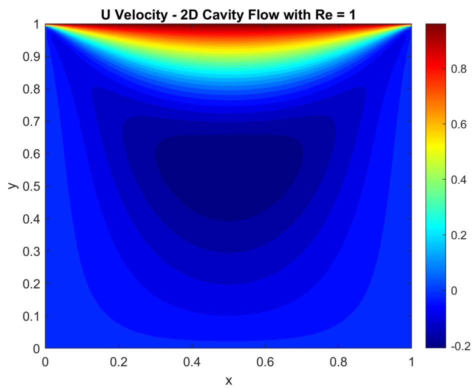
Ref: Lec-12 pg 17-41, Manoj R. Patel, Jigisha U. Pandya,and Vijay K. Patel, Numerical Analysis of Fluid Flow Behaviour in Four-Sided Square Lid-Driven Cavity Using the Finite Volume Technique, Versteeg Chap-6 pg(180-190) and flow chart constructed using draw.io

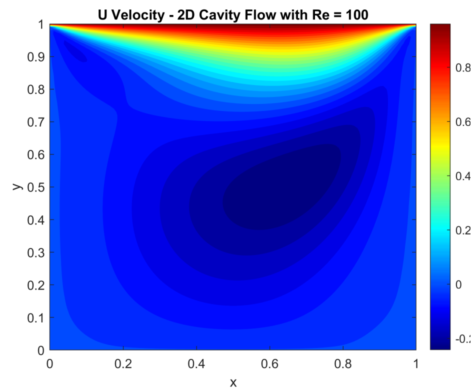


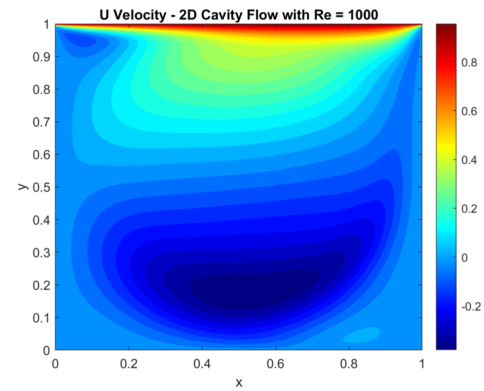
Task 3.3 and 3.4

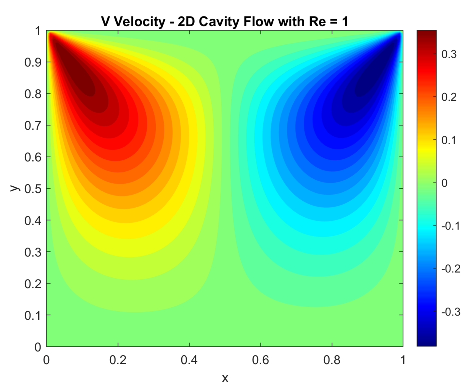
The code that needs to be run is Two_D_LID_Cavity.m (task 3.3)

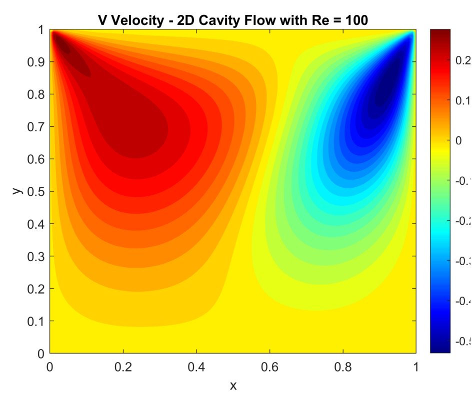
The below attached plots are regarding (task 3.4) velocity fields u and v , pressure,stream lines and vorticity of the flow for three Reynolds number Re =1 , 100, 1000

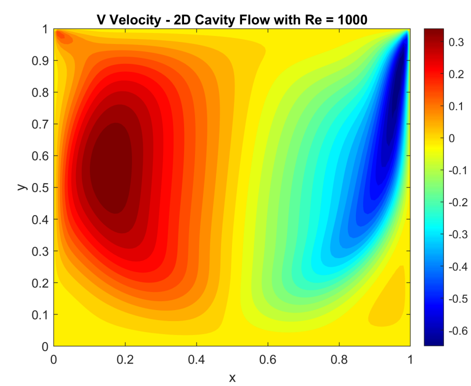


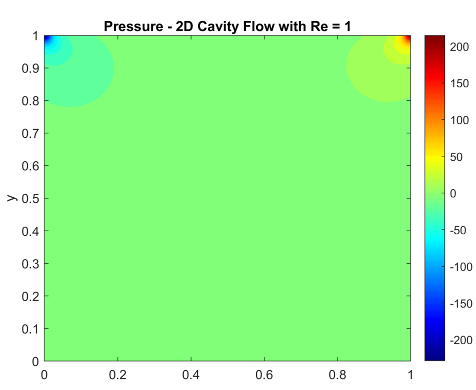


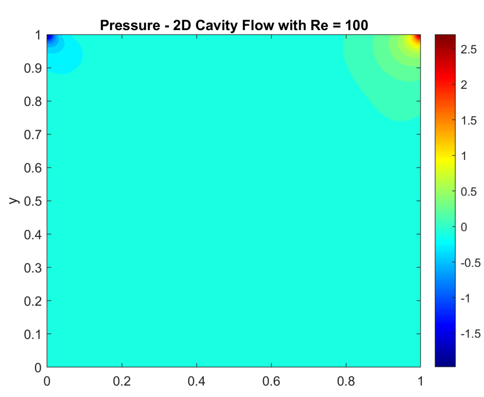


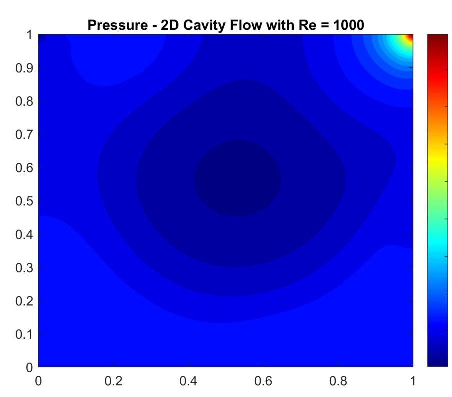


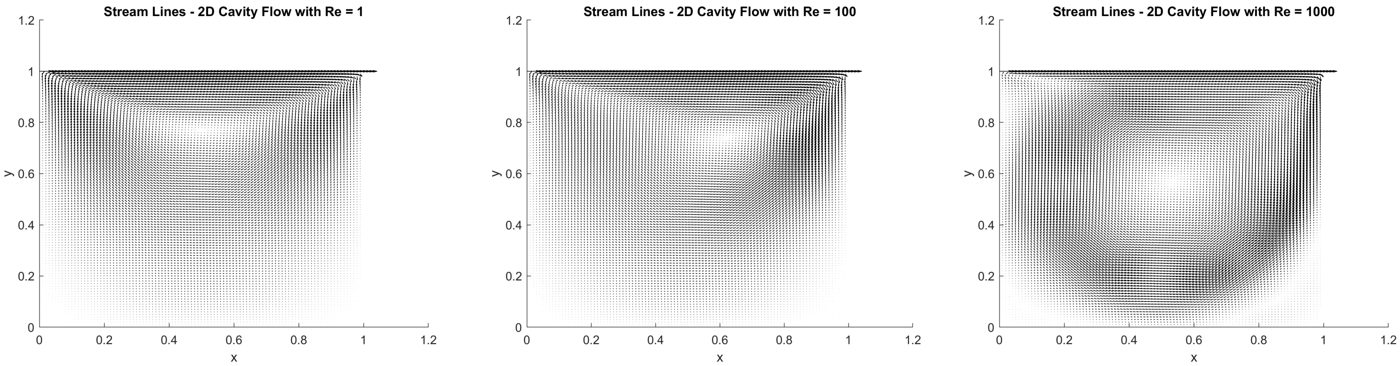


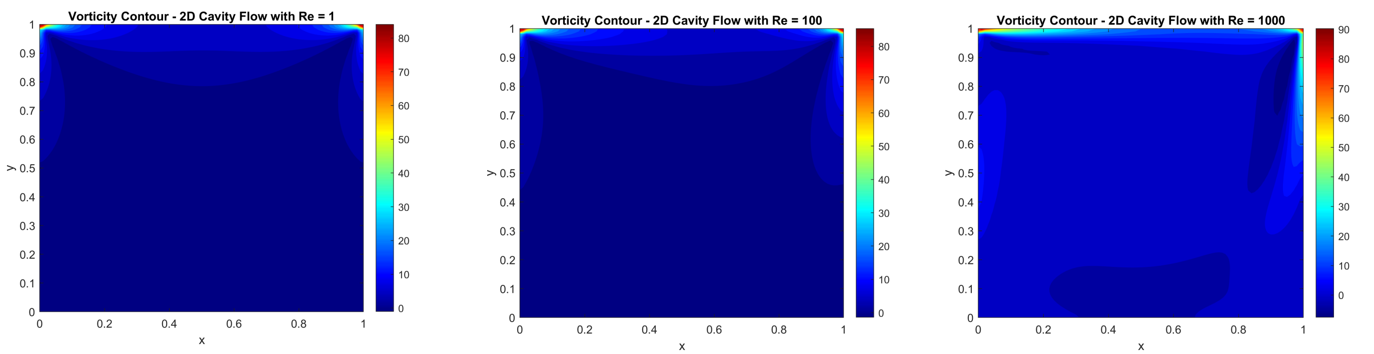










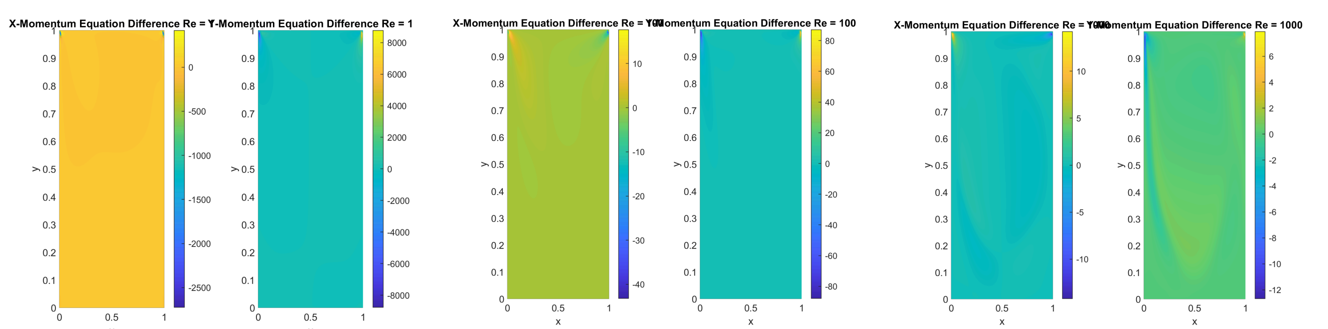


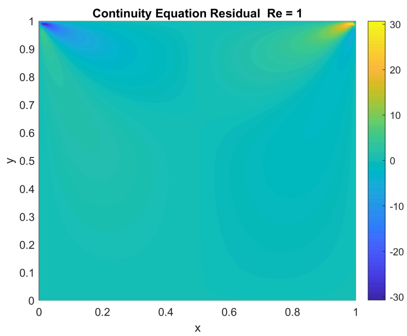
The velocity gradient is evident as we move from top towards the bottom of the cavity. When the lid is moved with a unit velocity of 1, the velocity of the fluid near the lid is maximum and it keeps reducing as we move away from the lid surface. This is due to the clock-wise re-circulations of the fluid formed due to the movement of the lid in positive x direction.

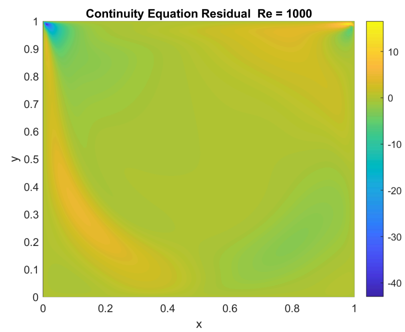
It is also noted that the magnitude of average dynamic pressure has reduced significantly with increase in Reynolds number. From the very definition of Reynolds number, the density is directly proportional to the Reynolds number itself. So as the Reynolds number increases the density increases.

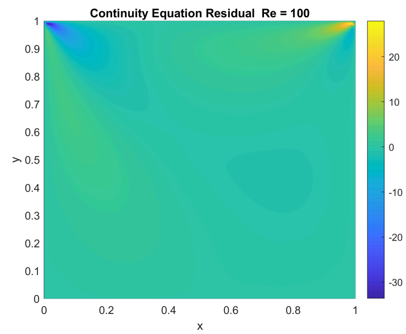
As the density increases the circulations inside the cavity reduces as there is not much movement as before. This implies that the static pressure is increased and to account for that the dynamic pressure automatically drops.

Task 3.5



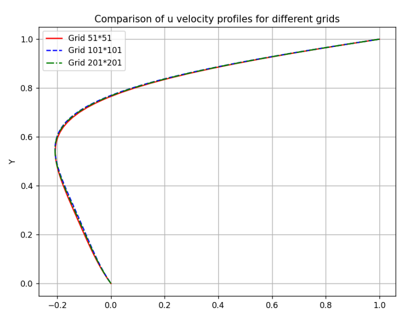


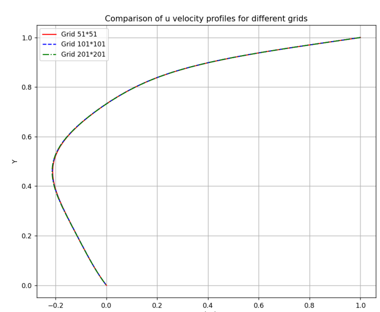


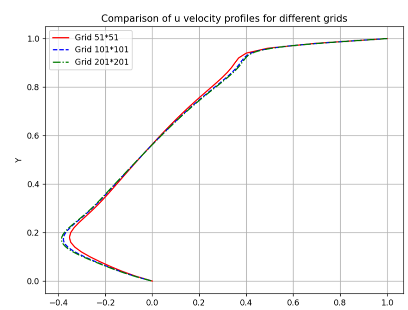


The difference between RHS and LHS of the x and y momentum equations will lead to a theoretical solution is zero but as we are using finite volume method, the expected values of the residual of momentum equations should be very close to zero but not exactly zero . For low Reynolds numbers (laminar), the flow is diffusion dominant. As we can observe, the x and y momentum differences are very close to zero that can be shown in the images and also the continuity equation residual is also close to zero with some numerical inconsistency at the edges of the moving lid due to sharp gradients indicating strong advection effects. For the continuity equation, the residual is close to zero, so we can say that the velocity field is divergence-free.

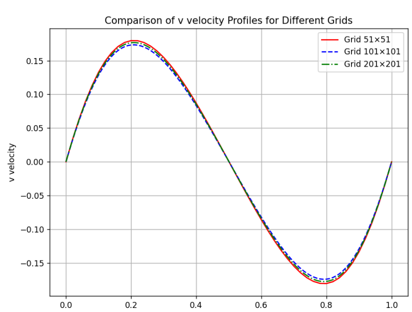
Task 3.6

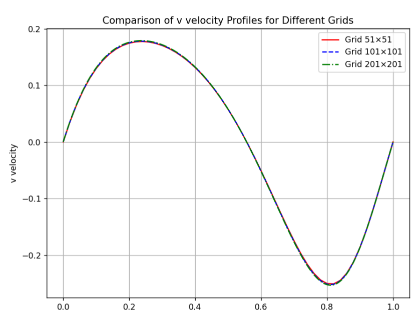


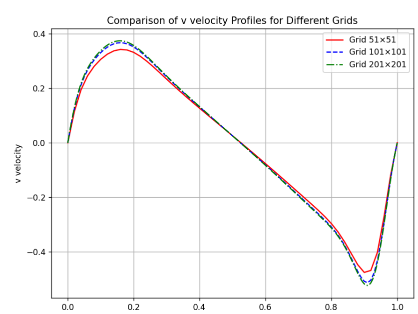




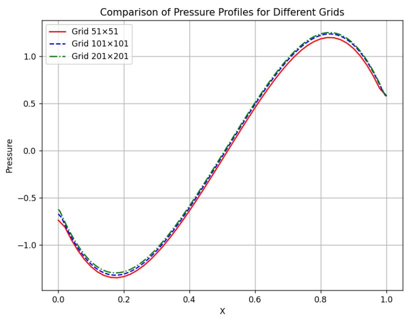
U velocities Re 1 Re 100 Re 1000



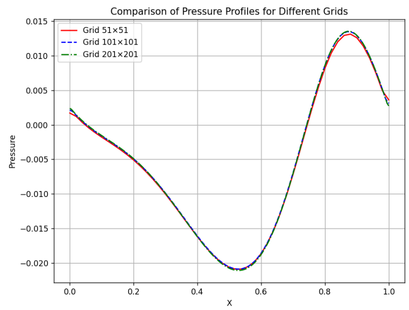




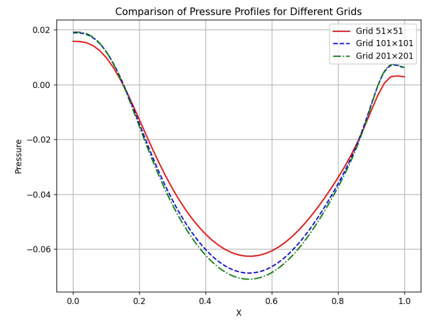
V Velocities Re 1 Re 100 Re 1000



Pressure Profile Re 1



Re 100



Re 1000

From the journal https://jmsg.springeropen.com/articles/10.1186/s40712-019-0104-7#Sec8

We have taken the location at x = 0.5 m because velocity distributions along the centre line help in observing the flow development and to check grid independency. The value at x = 0.5 is obtained by extracting the values from the matrix formed by grid points and the formulae (n_points+ 1 / (2)). n_points are max grid points i.e., i_max and j_max.