Signal Inpainting Using Graph

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***Abstract*—In this paper, we compare two graph variation control methods. One is GTVR(Graph Total Variation Regularization) and other is GTVM(Graph Total Variation Minimization). This was done using 40 blog sites data. The graph is of weighted directed type. The direction was determined by the links to other blogs that a blog had. The blogs were categorised into liberals and conservatives and gived weight -1 and 1 accordingly.**

Keywords*—Signal inpainting, graphs, graph total variation regularization, graph total variation minimization.

I. Introduction

Signal inpainting is a technique used to estimate or reconstruct the missing parts of a signal corrupted by noise from the known part of the signal. Signal processing on graphs is the newly emerged technique in which we assign signal coefficients to graphs nodes and connection are represented by a adjacency matrix. Using graphs in signal inpainting makes processing and estimating easier. In this paper, we discuss how signal inpainting works on graphs using two techniques namely, Graph total variation regularization and Graph total variation minimization and a comparison between them with different known and unknown parts of the signal.

II. Graph representation of a signal

Signals can be representation in graphs in many ways but we choose the following method to implement.

A. Signal to Graph

For this we assume that each point is connected to every other point in the signal and the relationship is done on basis of amplitude, therefore a n - point discrete signal can be represented by an nxn adjacency matrix xn is connected to xi by this relation : exp((xn - xi)2)

Each i-th row in adjacency matrix denote i-th point in the given discrete signal and each j-th column in the adjacency matrix denotes the relationship of that j-th point with other points depending on the row

B. Graph to Signal

For this the function takes three argument adjacency matrix(size nxn), one correct know value value from the signal and its position. Algorithm 1st we chose i-th row from the matrix, where i is the third argument given to the function 2nd take natural log of all the values of that row 3rd take square root of all the values after 2nd step 4th Subtract the each value (received from 3rd step) from the second argument

The result after 4th step is the original 1xn signal

III. Graph total variation regularization

In discrete signal processing, a signal or an image is assumed to be smooth. Smoothness of a graph signal is given by

$$\mathrm{TV_A}(\mathbf{x}) = \left\| \mathbf{x} - \frac{1}{|\lambda_{max}(\mathrm{A})|}\,\mathrm{A}\,\mathbf{x} \right\|_1 \text{[1]}$$

Quadratic form:

$$\mathrm{S}_2(\mathbf{x}) = \frac{1}{2}\,||\mathbf{x} - \mathrm{A}\,\mathbf{x}||_2^2\,. \text{[1]}$$

In order to apply this techique on graphs the signal must of the following form:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_{\mathcal{M}} \\ \mathbf{x}_{\mathcal{U}} \end{bmatrix}, \text{[1]}$$

where xm is the known part of the signal and xu is the unknown part of the signal. In this method we assume the smooth signal which was formulated above and with it's variation is minimum and solve for missing part of the signal.

$$\begin{aligned} \mathbf{x}^* &= \mathbf{argmin}\ \mathrm{S}_2(\widehat{\mathbf{x}}), \\ \text{subject to}\quad & ||\widehat{\mathbf{x}}_{\mathcal{M}} - \mathbf{x}_{\mathcal{M}}||_2^2 \le \epsilon^2. \end{aligned} \text{[1]}$$

This above statement make sures that the x is reconstructed such that the total variation in the reconstructed signal is minimum. In other words the value of x for which x* is minimum. And the error in the known part must also be minimum as the technique is applied to whole signal. Hence we end with the following expression:

$$\mathbf{x}^* = \mathbf{argmin}\ ||\widehat{\mathbf{x}}_{\mathcal{M}} - \mathbf{x}_{\mathcal{M}}||_2^2 + \lambda\,\mathrm{S}_2(\widehat{\mathbf{x}}), \text{[1]}$$

Where lambda is the parameter which ensures the smoothness of the signal. Larger values of lambda gives smoother solutions of x* So we differentiate the equation to minimize the total variation in the signal inorder to get the unknown part of the signal. And find the minima by setting the solved equation to zero. A closed form solution can be obtained which is as follows:

$$\mathbf{x}^* = \left(\begin{bmatrix} \mathrm{I}_M & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \lambda\widetilde{\mathrm{A}} \right)^{-1} \begin{bmatrix} \mathbf{x}_{\mathcal{M}} \\ \mathbf{0} \end{bmatrix}. \text{[1]}$$

IV. Graph Total Variation Minimization

In this method we assume that the known part of the signal kept preserved and we solve for unknown part by keeping error = 0; We divide the adjacency matrix into four parts known-known, known-unknown, unknown-known, unknown-unknown.

$$\widetilde{A} = \begin{bmatrix} \widetilde{A}_{\mathcal{MM}} & \widetilde{A}_{\mathcal{MU}} \\ \widetilde{A}_{\mathcal{UM}} & \widetilde{A}_{\mathcal{UU}} \end{bmatrix},$$

[1]

So write the quadratic form with known part after GTVM is same as before GTVM and solve the expression by the same procedure followed by GTVR and a closed form is obtained of the unknown part of the signal:

$$\widehat{x}_{\mathcal{U}} = -\widetilde{A}_{\mathcal{UU}}^{-1}\widetilde{A}_{\mathcal{UM}}x_{\mathcal{M}}.$$

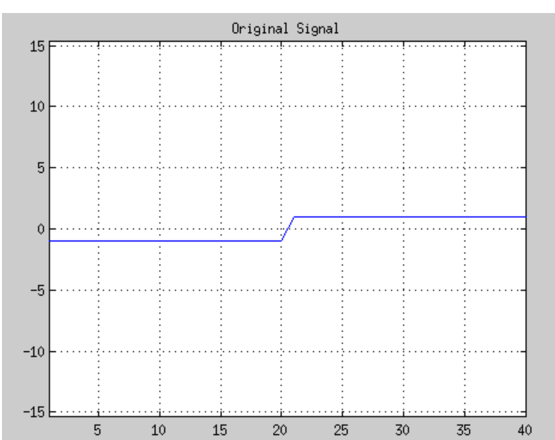
[1]

V. Application of GTVR and GTVM

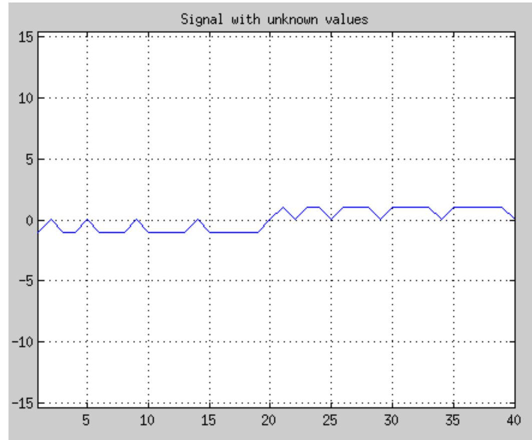
We applied these techniques on online blog data which is taken from [2]. Blogs are divided into two different categories-Conservative and liberal. Conservative blogs are represented as +1 and liberal blogs are represented as -1. Two blogs are connected if and only if one blog has the hyperlink of other. Blogs are represented in graphs by nodes as either +1 or -1 and a adjacency matrix which represents the connections between them. Since for this application we consider a directed weighted graph, the weights of the connection are given by 1/outdegree of the node. After constructing the graph we added zero into it in order to add missing parts in the signal. Hence we applied GTVR and GTVM and observed the results with different values of knowns and unknowns and lambda.

VI. Results

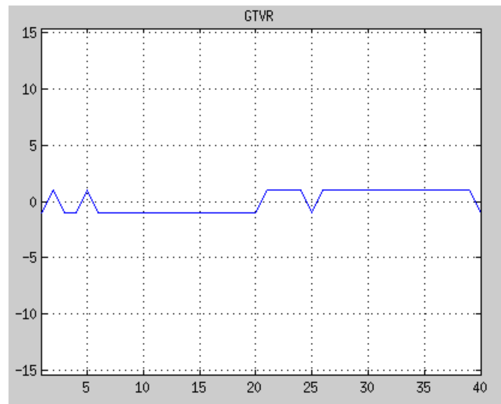
A. Original Signal



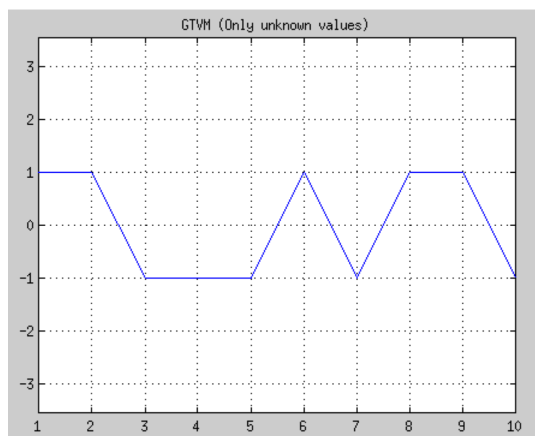
B. Singal with 10 unknown parts



C. GTVR with 30 known and 10 unknown with $\lambda = 1$



D. GTVM with 30 known and 10 unknown with $\lambda = 1$



We can observe that 90 percent of the signal is reconstructed using the these techniques(Out of 10 unknown samples 4 samples are not recovered while keeping the known part perserved in the case of GTVR). As we vary the knowns and unknowns

VII. Conclusion

We applied the closed form solutions for the problem of signal recovery using graphs on a data-set(blog) and observed the accuracy and efficiency of these methods by varying the known and unknown parts of the signal. We concluded that the accuracy of GTVR is more than GTVM and in every case, 80 percent of the signal is recovered.

References

[1] S. Chen, A. Sandryhaila, George Lederman, Zihao Wang, Jose M. F. Moura, P. Rizzo, Jacobo Beilak, James H. Garrett and Jelena Kovacevic, *Signal Inpainting on Gaphs via Total Variation Minimization.*

[2] L. A. Adamic and N. Glance, *The political blogsphere and the 2004 U.S. election: Divided they blog.*in Proc. LinkKDD,2005, pp. 36-43